Introduction to hadronic transport

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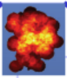
Dynamical models of HIC (from Prof. Bratkovskaya)

Macroscopic

hydro-models:
- description of QGP and hadronic phase by hydrodanamical equations for fluid
- assumption of local equilibrium
- EoS with phase transition from QGP to HG
- initial conditions (e-b-e, fluctuating)



ideal
(Jyväskylä,SHASTA, TAMU, ...)

viscous
(Romachkke,(2+1)D VISH2+1, (3+1)D MUSIC,...)

fireball models:
- no explicit dynamics: parametrized time evolution (TAMU)

Microscopic

Non-equilibrium microscopic transport models – based on many-body theory

Hadron-string models
(UrQMD, IQMD, HSD, QGSM ...)

Partonic cascades pQCD based
(Duke, BAMPS, ...)

Parton-hadron models:
- QGP: pQCD based cascade
- massless q, g
- hadronization: coalescence

(AMPT, HIJING)

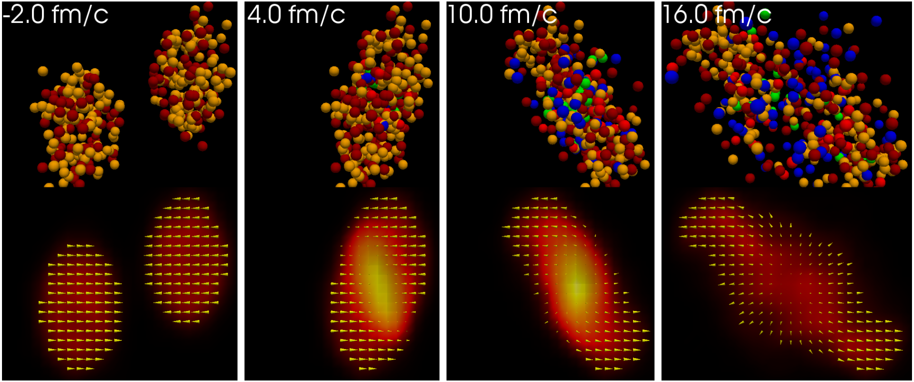
PHSD
- QGP: lQCD EoS
- massive quasi-particles (q and g with spectral functions) in self-generated mean-field
- dynamical hadronization
- HG: off-shell dynamics (applicable for strongly interacting systems)

‚Hybrid'
QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

(‚hybrid'-UrQMD, EPOS, ...)

Transport is simple: particles propagate, collide, decay

Au+Au, $\sqrt{s_{NN}} = 3$ GeV, $b = 5$ fm



...but devil is in details

Applications of hadronic transport: everywhere, where more than one scattering occurs

- Full simulations of ion collisions at lower and intermediate energies ($\sqrt{s} \lesssim 20$ GeV, e.g. SMASH, UrQMD, HSD, IQMD), at higher energy they tend to undershoot v_2
- With some partonic part: full simulations at any energies (e.g. PHSD, AMPT)
- Higher energies ion collisions ($\sqrt{s} \gtrsim 20$ GeV) as a hadronic afterburner after hydrodynamics
- $e + A$, $\nu + A$, e.g. GiBUU
- Participate in simulations of air-shower from cosmic rays, e.g. UrQMD

Outline

- Overview of some theoretical foundations

  thanks to Elena Bratkovskaya for her lectures in transport
  - Relations between equations
    - Bogolyubov chain (BBKYG)
    - Vlasov
    - Boltzmann
  - Properties of Boltzmann equation: H-theorem and detailed balance
- Application to ion collisions
  - Properties of hadrons
  - Reactions and cross-sections
  - Potentials
  - Pauli-blocking
  - Initialization of nuclei
  - Overview of existing models

Many-body density matrix

- Many-body density operator $\hat{\rho} = \sum_k P_k |\Psi_k\rangle\langle\Psi_k|$
  $k \in$ all possible pure states, P_k — probability of state k,
  Ψ_k — wavefunction
- Choosing orthogonal basis $\langle\varphi_m|\varphi_n\rangle = \delta_{mn}$.
  $\Psi_k = \sum_n a_{kn}\varphi_n$
  $a_{kn} = \langle\Psi_k|\varphi_n\rangle$
- Density matrix $\rho_{mn} = \sum_k P_k \langle\varphi_m|\Psi_k\rangle\langle\Psi_k|\varphi_n\rangle$
- Expectation value of operator $\hat{O}$:
  $\langle\hat{O}\rangle = \sum_k P_k \langle\Psi_k|\hat{O}|\Psi_k\rangle = \sum_{mn}\sum_k P_k a_{kn} a_{km} \langle\varphi_m|\hat{O}|\varphi_n\rangle =$
  $= \sum_n \langle\varphi_n|\hat{\rho}\hat{O}|\varphi_n\rangle \equiv Tr(\hat{\rho}\hat{O})$

Reduced density matrix

- Scrödinger equation: $i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$
- Liouville equation: $i\hbar\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]$
- N-body wave-function $|\Psi\rangle \equiv |\xi_1, \xi_2, \dots, \xi_N\rangle$
- Trace over particles $n+1$ to N: take equal ξ_{n+1} up to ξ_N on both sides and sum over them
- Reduced density matrix $\rho_n = \frac{1}{(N-n)!} Tr_{n+1 \dots N}\{\rho_N\}$

  Integrate out correlations of $n+1$ and more particles.

Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy aka BBGKY aka Bogolyubov chain

following S. J. Wang and W. Casing, Annals Phys. **159**, 328 (1985).

$$\hat{H} = \sum_i h^{(0)}(x_i) + \sum_{i<j} v_{ij}$$

$$i\hbar\dot{\rho}_1 = [h_0, \rho_1] + Tr_{(2)}\{[v, \rho_2]\}$$

$$i\hbar\dot{\rho}_2 = [\sum_{i=1}^{2} h_0(i), \rho_2] + [v, \rho_2] + Tr_{(3)}\{[\sum_{i=1}^{2} v(i,3), \rho_3]\}$$

$$\ldots$$

$$i\hbar\dot{\rho}_n = [\sum_{i=1}^{n} h_0(i), \rho_n] + [v, \rho_n] + Tr_{(n+1)}\{[\sum_{i=1}^{n} v(i,n+1), \rho_{n+1}]\}$$

In classical version: n-body distribution function depends on n+1-body distribution function. Needs truncation scheme (e.g. neglecting 3-body correlations). Note: truncation can break time-reversal invariance.

From BBKGY to Vlasov

Neglect 2-particle correlations: $i\hbar \frac{\partial \rho(11';t)}{\partial t} = (h_0(1) - h_0(1'))\rho(11';t)$
This is Time-Dependant Hartree-Fock equation, still quantum.
In the non-relativistic case:

$$\frac{\partial \rho(x,x',t)}{\partial t} + \frac{i}{\hbar}\left(\frac{\hbar^2}{2m}\nabla_x^2 + U(x,t) - \frac{\hbar^2}{2m}\nabla_{x'}^2 + U(x',t)\right)\rho(x,x',t) = 0$$

$\rho(x,x',t) = \langle x|\rho(t)|x'\rangle$

Applying Wigner transformation:

$$f(\vec{r},\vec{p},t) = \int d^3s \, \exp\left(\frac{i}{\hbar}\vec{p}\vec{s}\right)\rho\left(\vec{r}+\frac{\vec{s}}{2}, \vec{r}-\frac{\vec{s}}{2}, t\right)$$

Neglecting microscopic scale ($\ll$ scale of mean field potential change, further also $\ll$ mean free path):

$$U\left(\vec{r}+\frac{\vec{s}}{2}\right) - U\left(\vec{r}-\frac{\vec{s}}{2}\right) \approx \vec{s}\nabla_{\vec{r}}U(\vec{r})$$

“Quantumness” is abandoned and Vlasov equation emerges.

Vlasov Equation (non-relativistic version)

Motion of particles in self-generated mean field

$$\frac{\partial}{\partial t} f(t,\vec{r},\vec{p}) + \frac{\vec{p}}{m}\nabla_{\vec{r}} f(t,\vec{r},\vec{p}) - \nabla_{\vec{r}} U(\vec{r}) \nabla_{\vec{p}} f(t,\vec{r},\vec{p}) = 0$$
$$U(\vec{r}) = \int d^3r'\, d^3p\, V(\vec{r}-\vec{r'}) f(t,\vec{r'},\vec{p})$$

Classical single-particle equations of motion:

$$\frac{d\vec{r}}{dt} = \frac{\vec{p}}{m}$$
$$\frac{d\vec{p}}{dt} = -\nabla_{\vec{r}} U(\vec{r})$$
$$\frac{df(t,\vec{r},\vec{p})}{dt} = \left(\frac{\partial}{\partial t} + \dot{\vec{r}}\nabla_{\vec{r}} + \dot{\vec{p}}\nabla_{\vec{p}}\right) f = 0$$

Easy way to think: f is number of particles per $d^3x\, d^3p$. Conserving number of particles and phase space volume (Liouville theorem).

Boltzmann-Ueling-Uhlenbeck (BUU) equation (non-relativistic)

Take two first equations from BBGKY hierarchy, but assume 2-body correlations are local in space-time (space and time span of collisions $\ll$ mean free path).

$$\frac{df(t,\vec{r},\vec{p})}{dt} = \frac{\partial}{\partial t} f(t,\vec{r},\vec{p}) + \frac{\vec{p}}{m} \nabla_{\vec{r}} f(t,\vec{r},\vec{p}) - \nabla_{\vec{r}} U(\vec{r}) \nabla_{\vec{p}} f(t,\vec{r},\vec{p}) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

$$I_{coll} \equiv \left(\frac{\partial f}{\partial t}\right)_{coll} = \left(\frac{\partial f}{\partial t}\right)_{gain} - \left(\frac{\partial f}{\partial t}\right)_{loss}$$

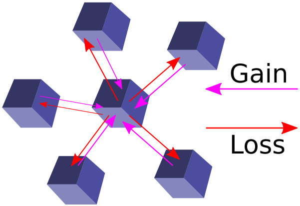
Boltzmann equation: gain and loss terms

Number of particles $dN(t,r,p)$ in the phase-space cell $d^3\vec{r}\,d^3\vec{p}$:

$$\frac{d}{dt}N(t,r,p) = dN_{coll}(p', \dots \to p, \dots) - dN_{coll}(p, \dots \to p', \dots)$$

$$dN(t,r,p) = f(t,r,p)d^3\vec{r}\,d^3\vec{p}$$

- Assumptions to calculate dN_{coll}:
  - only $2 \to 2$ scattering, neglect many-particle scatterings as rare
  - incoming particles uncorrelated "Stoßzahlansatz"
  - separation between long- and short-range interactions



$$I_{coll} = \int \frac{d^3p_2}{E_2}\frac{d^3p_1'}{E_1}\frac{d^3p_2'}{E_2'} \times W(p, p_2 \to p_1', p_2') \times (f(p_1')f(p_2') - f(p)f(p_2))$$

Detailed balance

Number of forward and reverse reactions (with Bose/Fermi factors):

$$d^4\Omega_p \equiv d^4x \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_1'}{2E_1'}\frac{d^3p_2'}{2E_2'}(2\pi)^{-2}\delta^{(4)}(P_i - P_f)$$

$$dN_{\rightarrow} = |M_{if}|^2 d^4\Omega_p\, f(p_1)f(p_2)(1 \pm f(p_1'))(1 \pm f(p_2'))$$

$$dN_{\leftarrow} = |M_{fi}|^2 d^4\Omega_p\, f(p_1')f(p_2')(1 \pm f(p_1))(1 \pm f(p_2))$$

Detailed balance:

- $|M_{fi}|^2 = |M_{if}|^2$ follows from T-invariance of S-matrix
- In equilibrium, when $\frac{df}{dt} = 0$, $dN_{\rightarrow} = dN_{\leftarrow}$ for all momenta (and all reaction channels in case of multicomponent Boltzmann)
- From $|M_{fi}|^2 = |M_{if}|^2$ follow certain relations between forward and reverse cross-sections and decay widths

H-theorem

using “Introduction to relativistic transport theory” by Hendrik van Hees

Entropy (non-relativistic):

$$S = -\int d^3p f(p)(\log[(2\pi\hbar)^3 f(p)] - 1)$$

Entropy current (relativistic):

$$S^\mu = -\int \frac{d^3p}{E} p^\mu f(p)(\log[(2\pi\hbar)^3 f(p)] - 1)$$

$$\frac{\partial S^\mu}{\partial x^\mu} = \frac{1}{4}\int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_1'}{2E_1'}\frac{d^3p_2'}{2E_2'} f f_2 \,(z - \log z - 1)\, W(p, p_2 \to p_1', p_2')$$

$$z \equiv \frac{f_1' f_2'}{f f_2}$$

H-theorem: as $z - \log z - 1$ grows for $z > 0$, entropy in Boltzmann equation is non-decreasing with time.

H-theorem (continue)

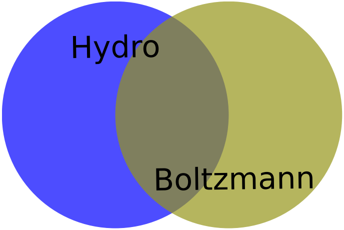
With Fermi/Bose corrections

$$z \equiv \frac{f_1' f_2' (1 \pm f)(1 \pm f_2)}{f f_2 (1 \pm f_1')(1 \pm f_2')}$$

Entropy is maximal:

- $z - \log z - 1$ is maximum: at $z = 1$
- Collision term vanishes: equilibration
- In equilibrium $\log \dfrac{f}{1 \pm f} = A_\mu p^\mu + const \rightarrow$ Fermi/Bose distribution

Ideal hydro follows from equilibrated Boltzmann



Maximal entropy, equilibrium distribution, right hand side moments of Boltzmann equation vanish:

$$f_0(r,p) = exp((-p^\mu u_\mu + \mu(r))/T(r))$$

From $p^\mu \frac{\partial f_0}{\partial x^\mu} = 0$ follows $\partial_\mu T^{\mu\nu} = 0$ and $\partial_\mu j^\mu = 0$, where $T^{\mu\nu} = \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^\mu p^\nu}{p^0} f(p)$ and $j^\mu = \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^\mu}{p^0} f(p)$.

Solving BUU equations in practice

- All (or almost all) hadron species known: π, ρ, K, a_2, f_1, ϕ, $\dots$, N, $\Delta(1232)$, $N(1440)$, $\dots$: more than 100 species without accounting for charges
- Solve coupled equations, $D \equiv \dfrac{d}{dt}$:

$$
\begin{aligned}
Df_\pi &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\
Df_N &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\
Df_\Delta &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\
\dots
\end{aligned}
$$

- Left hand side: testparticle ansatz

$$f \sim \frac{1}{N_{test}} \sum_i^{N_{test}} \delta^4(x - x_i)\delta^4(p - p_i)\delta(p^\mu p_\mu = m^2)$$

- Collision integrals: Monte-Carlo approach

Treatment of potentials in transport codes

- Boltzmann - Ühling - Uhlenbeck (BUU) approach
  - Mean-field potentials depend on densities: $U = U(\rho(\{\vec{r}_1, \vec{r}_2, \dots\}))$
  - Energy and momentum conservation on event average
  - Utilize testparticle ansatz: $N \to N \cdot N_{test}$, $\sigma \to \sigma / N_{test}$
  - Solve Boltzmann equations in the limit of $N_{test} \to \infty$
  - No fluctuations in the limit $N_{test} \to \infty$
- Quantum Molecular Dynamics (QMD) approach
  - Pairwise potentials depend on coordinates $U(r_{12})$
  - Energy and momentum conserved exactly event-by-event
  - Does not solve any particular equation for distribution function

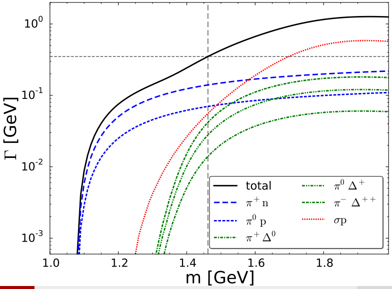
Treatment of collisions in transport codes

- A) Geometrical criterion: $d_{ij} \leq \sqrt{\sigma/\pi}$ (d_{ij} in the CM frame of colliding paricles)
  - Only allows $2 \to 2$, not $3 \to 2$ or $3 \to 1$: detailed balance violation
  - Collision time: time of closest approach
  - Sort by collision time, perform the earliest
  - Time sorting depends on frame, problems with Lorentz-invariance
  - Kodama criterion: smaller Lorentz-invariance troubles
- B) Stochastic rates: choose two random particles in cell and collide with some probability

  Cassing, NPA 700, 618 (2002); Xu and Greiner, PRC 71, 064901 (2005)
  - No problems with Lorentz-invariance
  - Allows $3 \to 2$ or $3 \to 1$ collisions
  - Not applicable with QMD
  - Needs care concerning cell size and timestep
  - So far only in BAMPS and (P)HSD, planned in SMASH

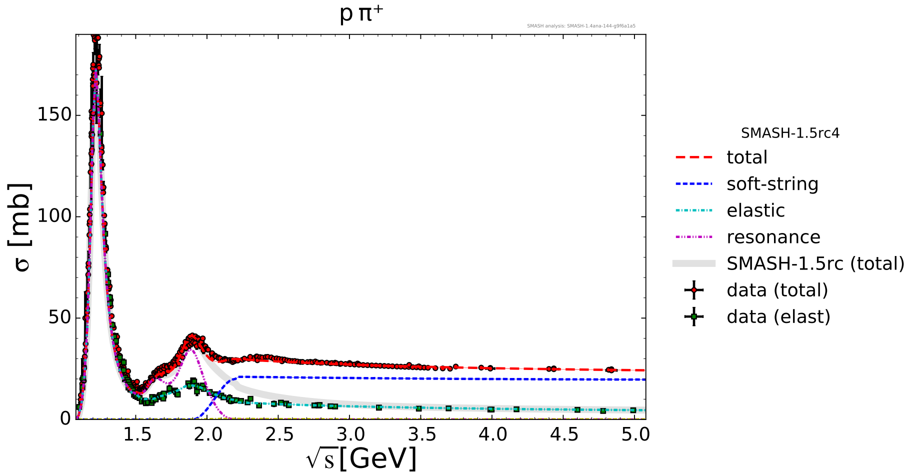
Hadronic spectral functions

- *Pole* mass m_0 and *pole* width $\Gamma(m_0)$ from experiment
- Branching ratios (and partial widths) are often unknown
- Spectral functions: typically Breit-Wigner
  - Non-relativistic: $\mathcal{A}(m) \sim \frac{\Gamma}{(m-m_0)^2 + \Gamma^2/4}$, e.g. UrQMD
  - Relativistic: $\mathcal{A}(m) \sim \frac{m^2\Gamma}{(m^2-m_0^2) + m^2\Gamma^2}$, e.g. SMASH, HSD
  - Mass dependent width: $\Gamma(m)$



Cross-sections

Resonances at lower energies, string models at $\sqrt{s} \gtrsim 3 - 4$ GeV



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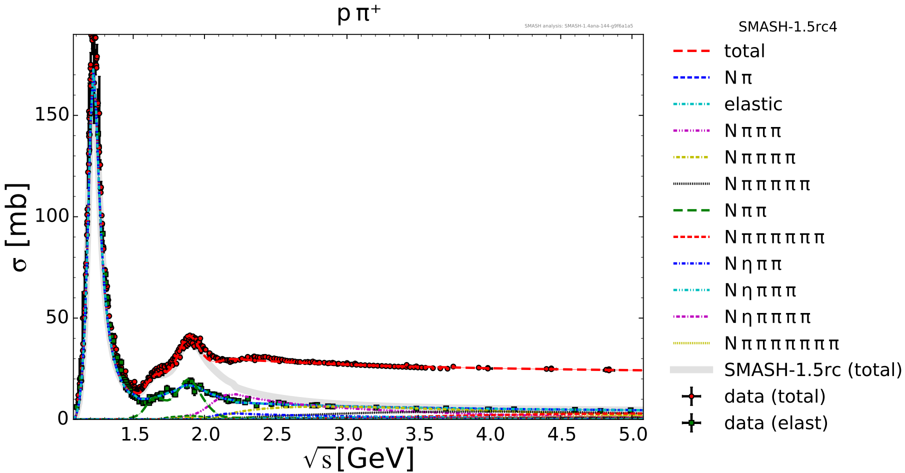

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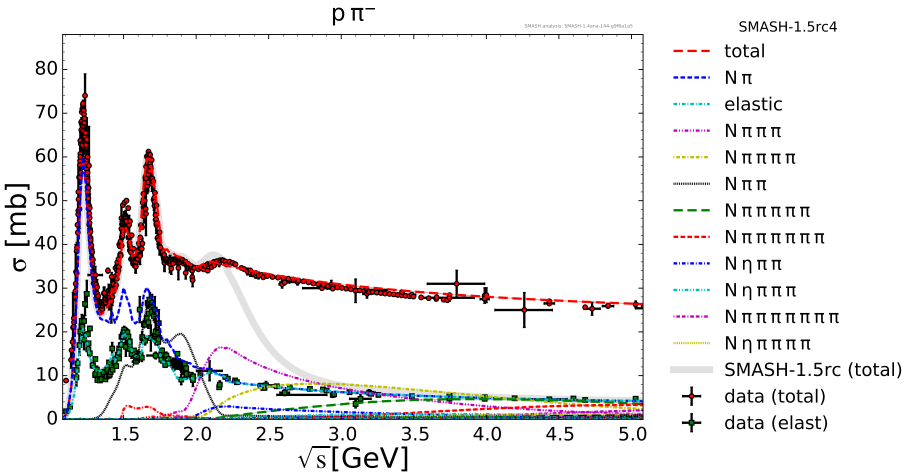

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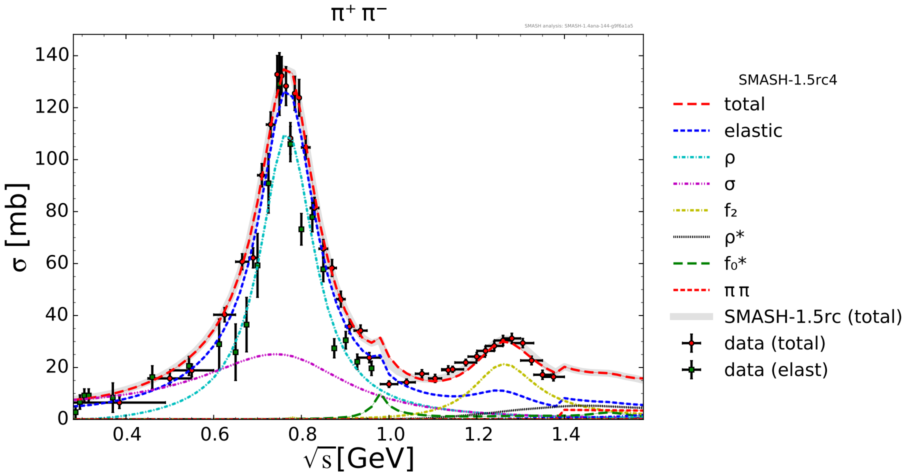

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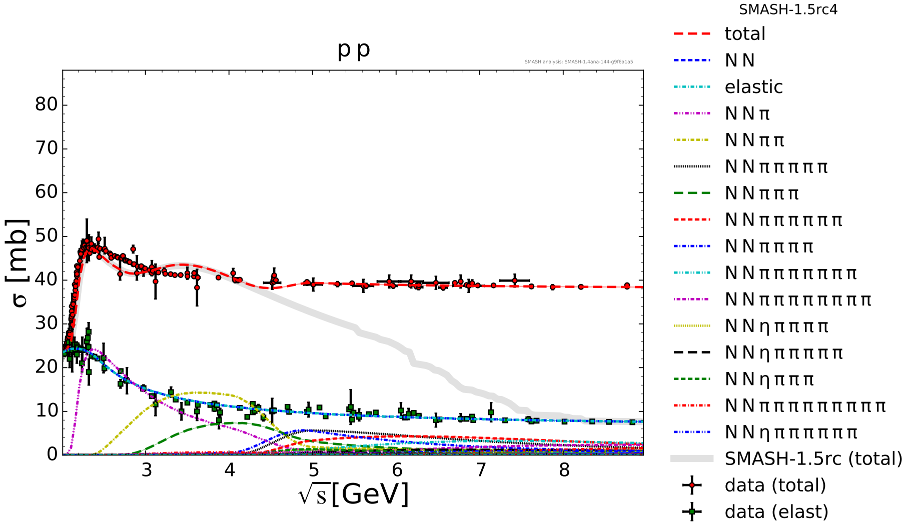
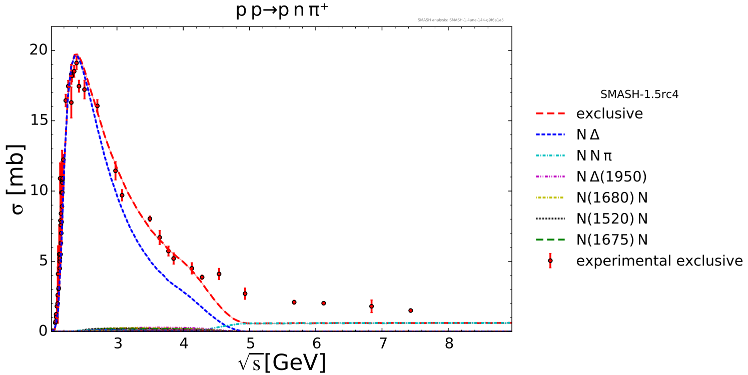


Exclusive cross-sections

Transport approaches can't account for channel interference

"Perfect fit" – some manipulations with cross-sections:

- A) Exclusive via resonances or parametrized $\rightarrow$ total is off
- B) Parametrize total, rescale exclusive OR
- C) Fill in remaining total



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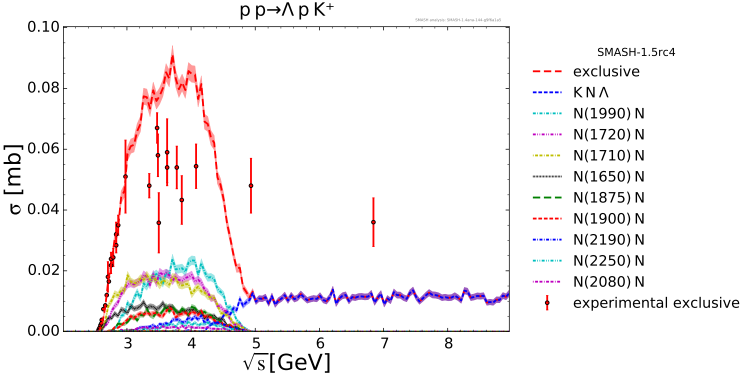

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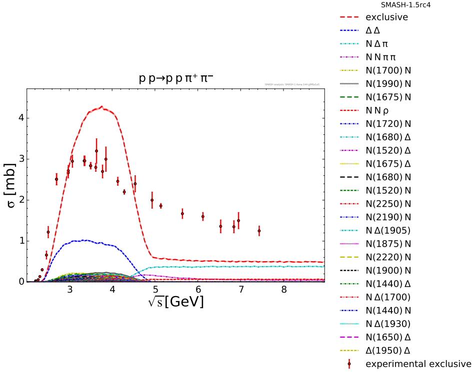


String models: $\sqrt{s} > 3 - 4$ GeV

- Pick up $q\bar{q}$ or quark-diquark from colliding hadrons
- They form a string, which undergoes a sequence of decays: "string fragmentation"
- Different models of string fragmentation: FRITIOF (Lund), PYTHIA (Lund), HERWIG
- In Lund string: tunnelling through QCD potential

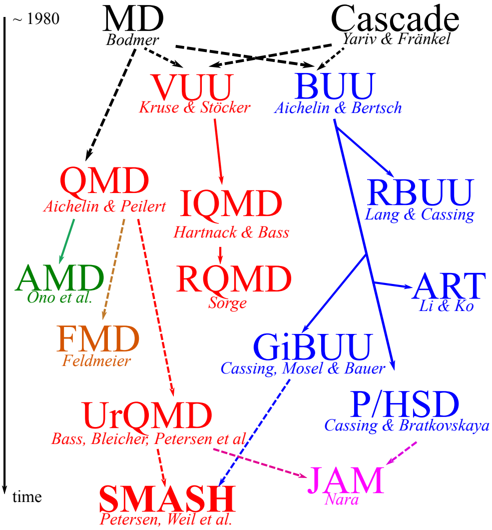
$$\mathcal{P} \sim exp\left(\frac{-\pi p_\perp^2}{\kappa}\right) exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

- Flavor composition, longitudinal momenta $\rightarrow \simeq 10$ parameters

Summary

- Rigorous transport theory
  - BBGKY $\rightarrow$ up to second order correlations $\rightarrow$ Kadanoff-Baym
  - Kadanoff-Baym $\rightarrow$ Wigner transform, first order gradient expansion $\rightarrow$ Boltzmann
  - Boltzmann or BUU $\rightarrow$ no collisions $\rightarrow$ Vlasov
  - Kadanoff-Baym $\rightarrow$ no collisions $\rightarrow$ TDHF
- Properties of Boltzmann equations: H-theorem, detailed balance
- Real-world transport has many model-dependent choices
  - Potential treatment: QMD/BUU
  - Collisions: geometric (couple of ways), stochastic
  - Cross-sections (resonances vs. strings, treating exclusive)
  - String model choices

SMASH and its ancestors



thanks to Steffen Bass

- SMASH :
  Simulating
  Many
  Accelerated
  Strongly-interacting
  Hadrons
- first C++ code in this historical chain
- written from scratch
- started coding in 2013
- under *git* version control from the very beginning

SMASH: general properties

J. Weil *et al.*, Phys.Rev. C94 (2016) no.5, 054905

- Monte-Carlo solver of relativistic Boltzmann equations
  BUU type approach, testparticles ansatz: $N \to N \cdot N_{test}, \sigma \to \sigma/N_{test}$
- Degrees of freedom
  - most of established hadrons from PDG up to mass 2.5 GeV
  - strings: do not propagate, only form and decay to hadrons
- Propagate from action to action (timesteps only for potentials)
  action $\equiv$ collision, decay, wall crossing
- Geometrical collision criterion: $d_{ij} \leq \sqrt{\sigma/\pi}$
- Interactions: $2 \leftrightarrow 2$ and $2 \to 1$ collisions, decays, potentials, string formation (soft - SMASH, hard - Pythia 8) and fragmentation via Pythia 8
- C++ code, git version control, public on github

SMASH: initialization

- "collider" - elementary or AA reactions, $E_{beam} \gtrsim 0.5$ *A* GeV
- "box" - infinite matter simulations

  detailed balance tests, computing transport coefficients, thermodynamics of hadron gas
  Rose et al., PRC 97 (2018) no.5, 055204
- "sphere" - expanding system

  testing collision term via comparison to analytical solution of Boltzmann equation,
  Tindall et al., Phys.Lett. B770 (2017) 532-538
- "list" - hadronic afterburner after hydrodynamics

SMASH: degrees of freedom

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^{-}_{1672}	π_{138}	$f_{0\,980}$	$f_{2\,1275}$	$\pi_{2\,1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^{-}_{2250}	π_{1300}	$f_{0\,1370}$	$f'_{2\,1525}$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\,1500}$	$f_{2\,1950}$	$\rho_{3\,1690}$	$K_{1\,1270}$
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\,1710}$	$f_{2\,2010}$		$K_{1\,1400}$
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\,2300}$	$\phi_{3\,1850}$	K^*_{1410}
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\,980}$	$f_{2\,2340}$		$K_0{}^*_{1430}$
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\,1450}$		$a_{4\,2040}$	$K_2{}^*_{1430}$
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\,1285}$		K^*_{1680}
N_{1710}		Λ_{1820}	Σ_{2030}			η_{1475}	ϕ_{1019}	$f_{1\,1420}$	$f_{4\,2050}$	$K_{2\,1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				ϕ_{1680}			$K_3{}^*_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\,1320}$		$K_{2\,1820}$
N_{1900}		Λ_{2100}					$h_{1\,1170}$			$K_4{}^*_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\,1400}$		
N_{2080}		Λ_{2350}				ρ_{1450}	$b_{1\,1235}$	$\pi_{1\,1600}$		
N_{2190}						ρ_{1700}				
N_{2220}							$a_{1\,1260}$	$\eta_{2\,1645}$		
N_{2250}						ω_{783}				
						ω_{1420}		$\omega_{3\,1670}$		
						ω_{1650}				

- Isospin symmetry
- Perturbative treatment of non-hadronic particles (photons, dileptons)

Hadrons and decay modes configurable via human-readable files

Interactions in SMASH

- Resonance formation and decay
  Ex. $\pi\pi \to \rho \to \pi\pi$, quasi-inlastic scattering
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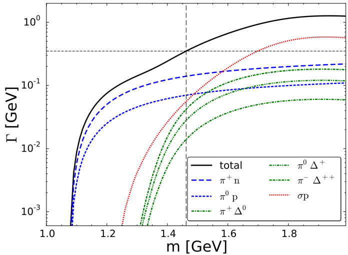
$N^*(1440)^+$



For every resonance:

- Breit-Wigner spectral function

$$\mathcal{A}(m) = \frac{2\mathcal{N}}{\pi} \frac{m^2\Gamma(m)}{(m^2 - M_0^2)^2 + m^2\Gamma(m)^2}$$

- Mass dependent partial widths $\Gamma_i(m)$
  Manley formalism for off-shell width Manley and Saleski, Phys. Rev. D 45, 4002 (1992)
  Total width $\Gamma(m) = \sum_i \Gamma_i(m)$

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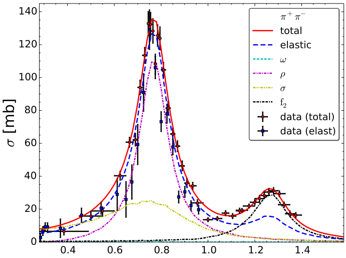

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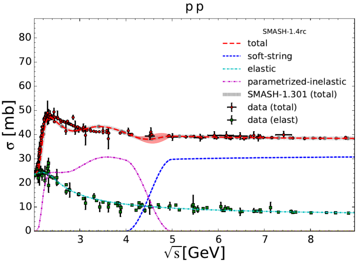

- $NN \to NN^*$, $NN \to N\Delta^*$, $NN \to \Delta\Delta$, $NN \to \Delta N^*$, $NN \to \Delta\Delta^*$

  angular dependencies of $NN \to XX$ cross-sections implemented
- Strangeness exchange $KN \to K\Delta$, $KN \to \Lambda\pi$, $KN \to \Sigma\pi$

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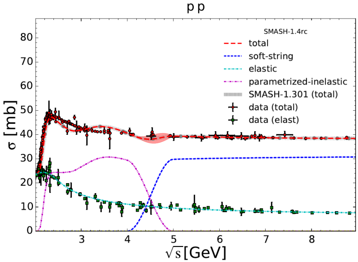

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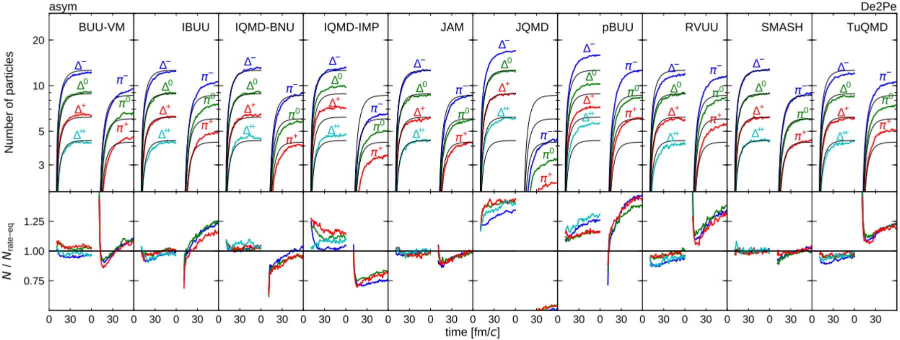
string model parameters
tuned to NA61 pp
Justin Mohs master thesis

- String (soft or hard) fragmentation: always via Pythia 8
- Hard scattering and string formation: Pythia
- Soft string formation: SMASH
  - single/double diffractive
  - $B\bar{B}$ annihilation
  - non-diffractive

Transport code comparison project

Zhang et al., PRC 97, 034625 (2018); Ono et al., in preparation

- Same “homework” for many codes, all physics identical
- Quantifying transport code uncertainty
- Compare to analytical solutions of Boltzmann equation



SMASH compares well to analytic solutions