JETSCAPE Winter School and Workshop 2019

4D lattice calculation of $\hat{q}$ for a pure gluon plasma

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Texas A&M University, USA, January 11th, 2019

U.S. DEPARTMENT OF ENERGY

Office of Science

Outline

- Phenomenology based study of transport coefficient $\hat{q}$ in heavy-ion collisions

- Formulating $\hat{q}$ for hot QGP using Lattice gauge theory

1) Express $\hat{q}$ as a series of local operators using dispersion relation

2) Computing operators on a quenched SU(3) lattice

- Estimates of $\hat{q}$ on a quenched QGP plasma

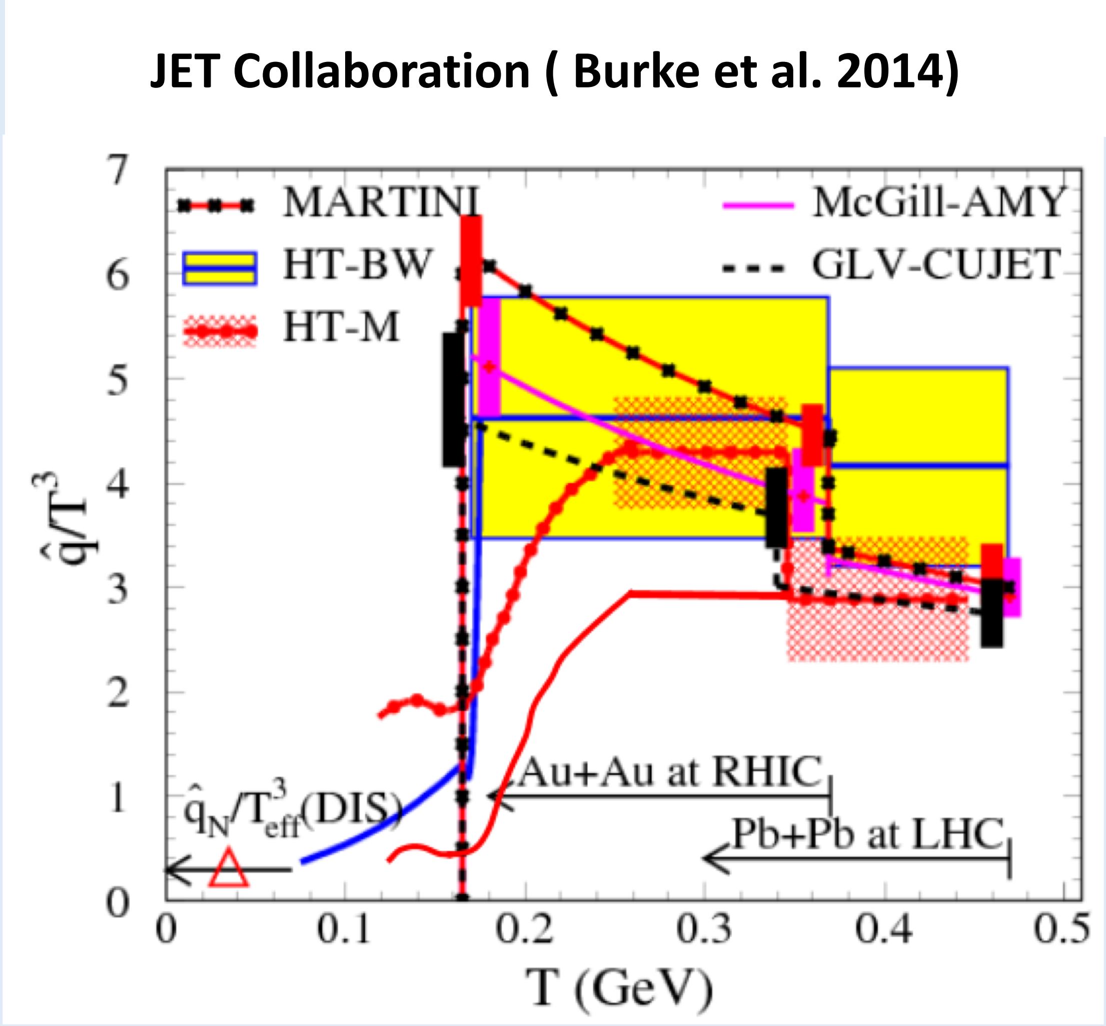
First systematic extraction of $\hat{q}$ based on phenomenology

$$\hat{q}(\vec{r},t) = \frac{< k_{\perp}^{2} >}{L}$$

(1) $\frac{\hat{q}}{T^3} \sim 3 - 5$

(2) Based on fit to experimental data

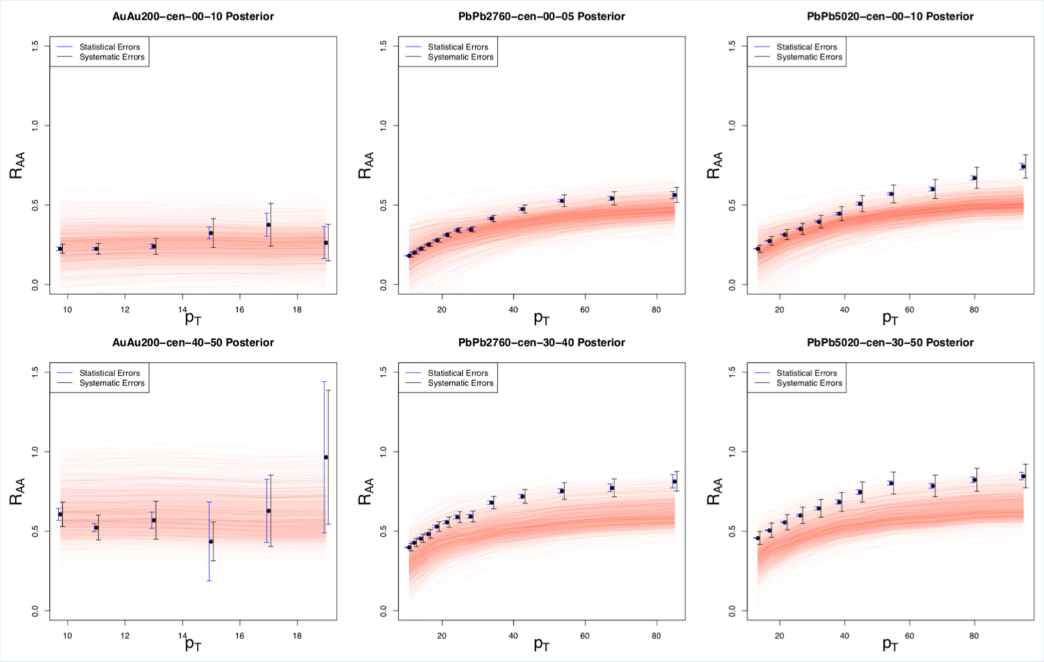
(3) Based on single energy-loss scheme

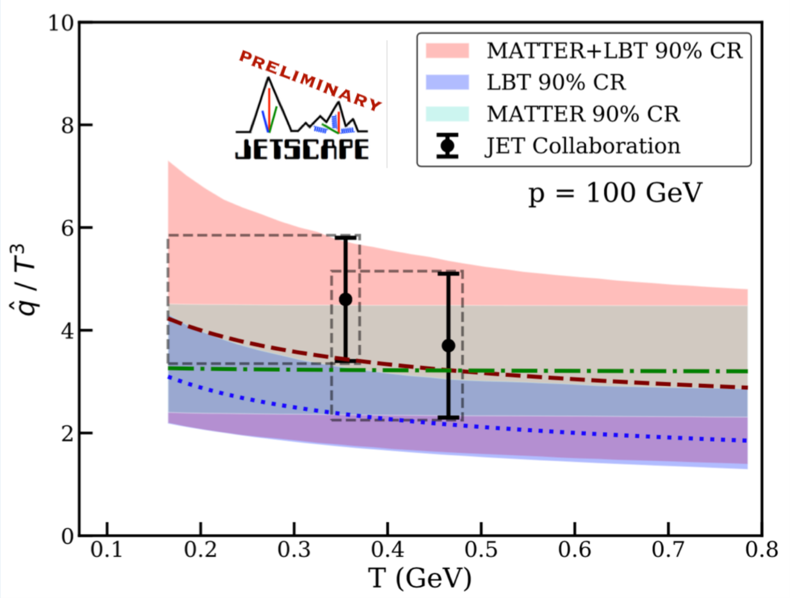


First Bayesian and multi-stage extraction of $\hat{q}$

Using JETSCAPE Instrument

(Taken from Ron Soltz's slides (HP 2018))





Multi-stage approach within JETSCAPE
Jet energy loss model: MATTER+LBT with switching virtuality Q0 ~ 1-2 GeV

Based on fit to experimental data

First principles calculation of $\hat{q}$

QGP is locally thermalized and non-perturbative

↓

Thermal Lattice QCD to compute $\hat{\boldsymbol{q}}$

Lattice formulation of $\hat{q}$

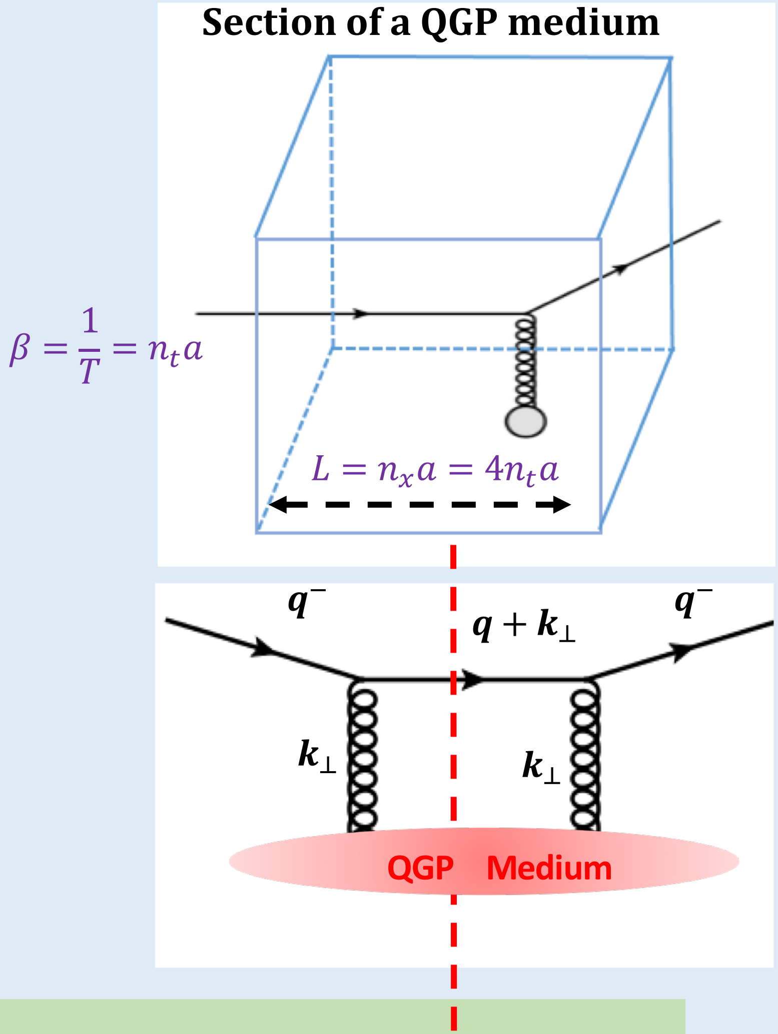
A. Majumder, PRC 87, 034905 (2013)

- Simplest process: A leading quark propagating through **hot plasma**

$$\boldsymbol{q} = \left({}^{\boldsymbol{\mu}^2}\!/_{2\boldsymbol{q}^-}, \boldsymbol{q}^-, \boldsymbol{0}\right) = \left(\boldsymbol{\lambda}^2, \boldsymbol{1}, \boldsymbol{0}\right)\boldsymbol{Q};\quad \textbf{Hard scale} = \boldsymbol{Q}; \boldsymbol{\lambda} \ll \boldsymbol{1}$$

$$\boldsymbol{k} = (\boldsymbol{k}^+, \boldsymbol{k}^-, \boldsymbol{k}_\perp \boldsymbol{0}) = \left(\boldsymbol{\lambda}^2, \boldsymbol{\lambda}^2, \boldsymbol{\lambda}\right)\boldsymbol{Q};\quad \textbf{Transverse gluon}$$

$\mathcal{M}(\boldsymbol{k})$: **Transition probability**



Lattice formulation of $\hat{q}$

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- Simplest process: A leading quark propagating through **hot plasma**

$$q = \left({}^{\mu^2}\!/_{2q^-}, q^-, 0\right) = (\lambda^2, 1, 0)Q; \quad \textbf{Hard scale} = Q; \lambda \ll 1$$

$$k = (k^+, k^-, k_\perp 0) = (\lambda^2, \lambda^2, \lambda)Q; \qquad \textbf{Transverse gluon}$$

$\mathcal{M}(k)$: **Transition probability**

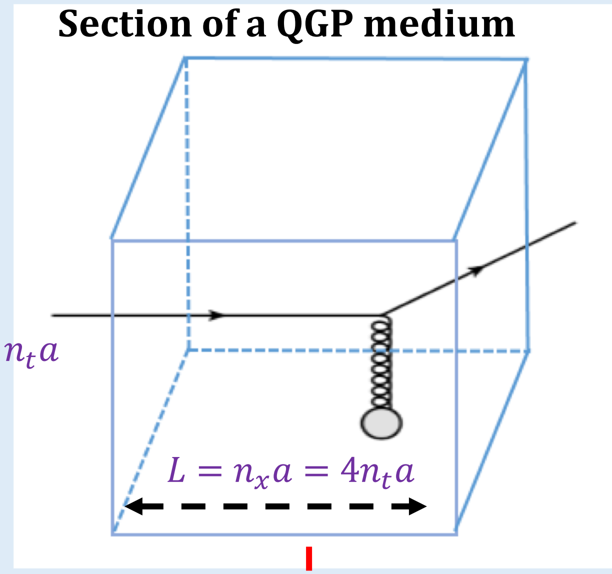
$$\boxed{\hat{q}(\vec{r}, t) = \sum_k k_\perp^2 \frac{Disc[\mathcal{M}(k)]}{L}}$$

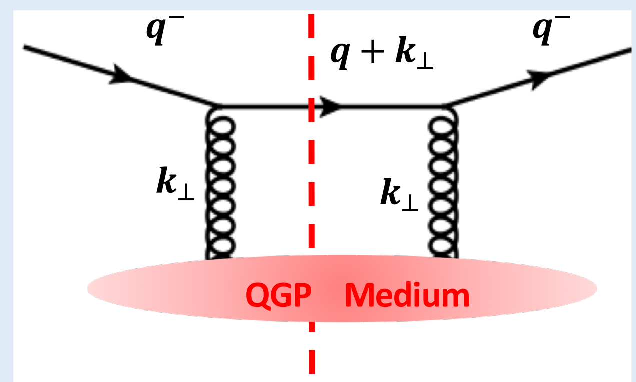
$$\hat{q} = \frac{16\sqrt{2}\pi^2\alpha_s}{N_c} \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp . \vec{y}_\perp}$$

$$\text{x} \left\langle M \middle| F_\perp^{+\mu}(y^-, y_\perp) F_{\perp\mu}^{+}(0) \middle| M \right\rangle$$

Non-perturbative part **(Lattice QCD)**

$\beta = \frac{1}{T} = n_t a$



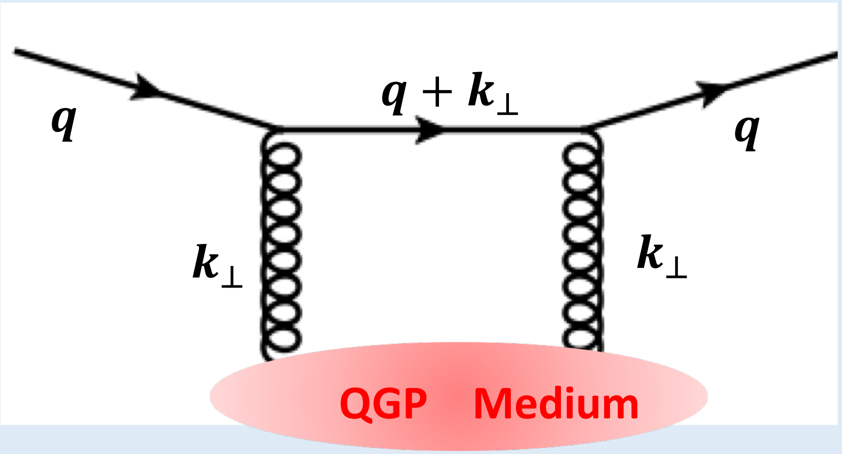


Constructing a more general expression as $\hat{Q}$

❖ **General form of $\hat{q}$: with q^- $fixed$; q^+ $is\ variable$**

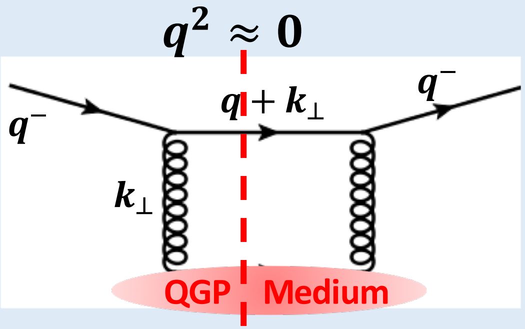
$$\hat{Q}(q^+) = \frac{16\sqrt{2}\pi\alpha_s}{N_c} \int \frac{d^4y d^4k}{(2\pi)^4} e^{iky}\, q^- \frac{\langle M | F_\perp^{+\mu}(0)\, F_{\perp\mu}^{+}(y) | M \rangle}{(q+k)^2 + i\epsilon}$$

$\boxed{\hat{Q}(q^+)}$



When $q^+ \sim T$ $\quad$ $\mathrm{Disc}\left[\hat{Q}(q^+)\right]_{at\ q^+\sim T} = \hat{q}$

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



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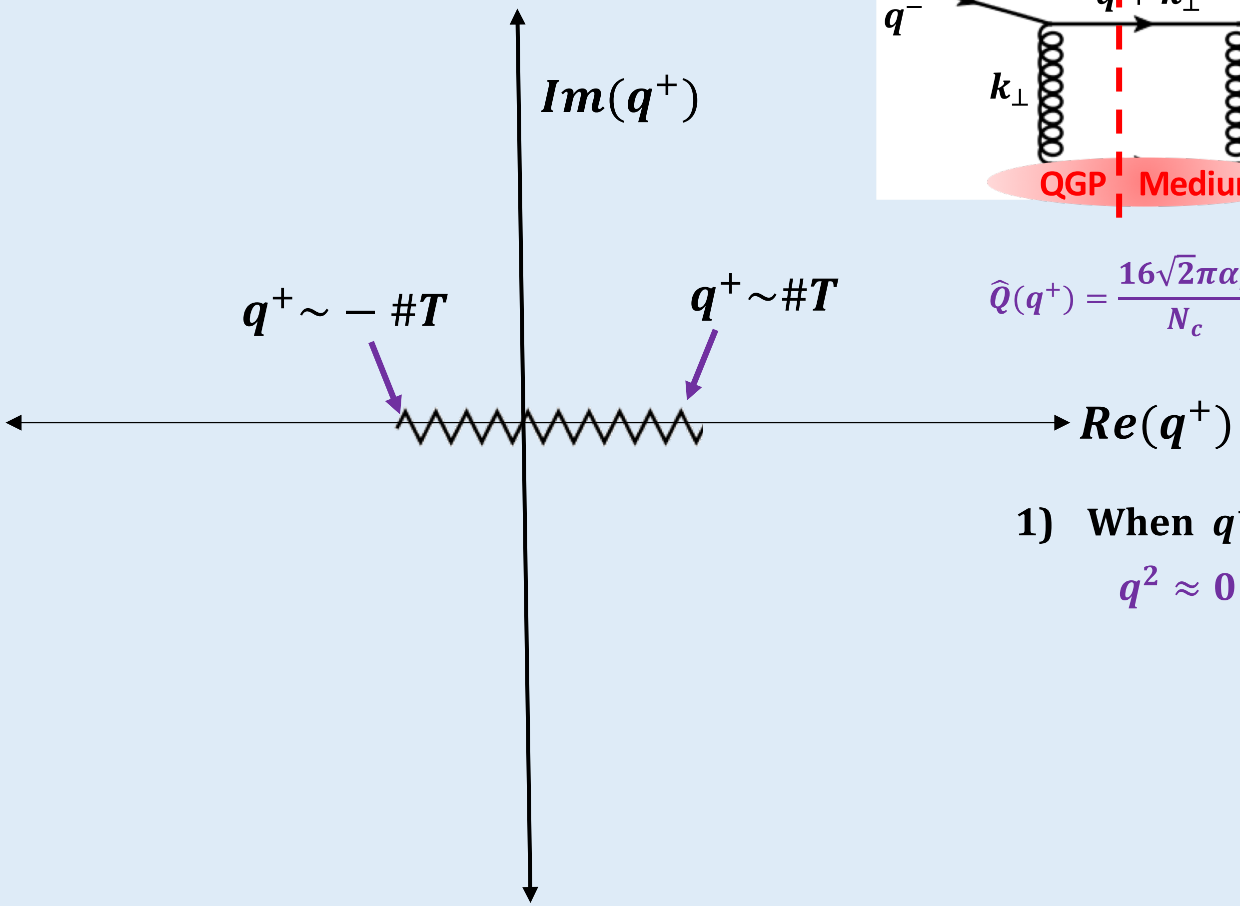
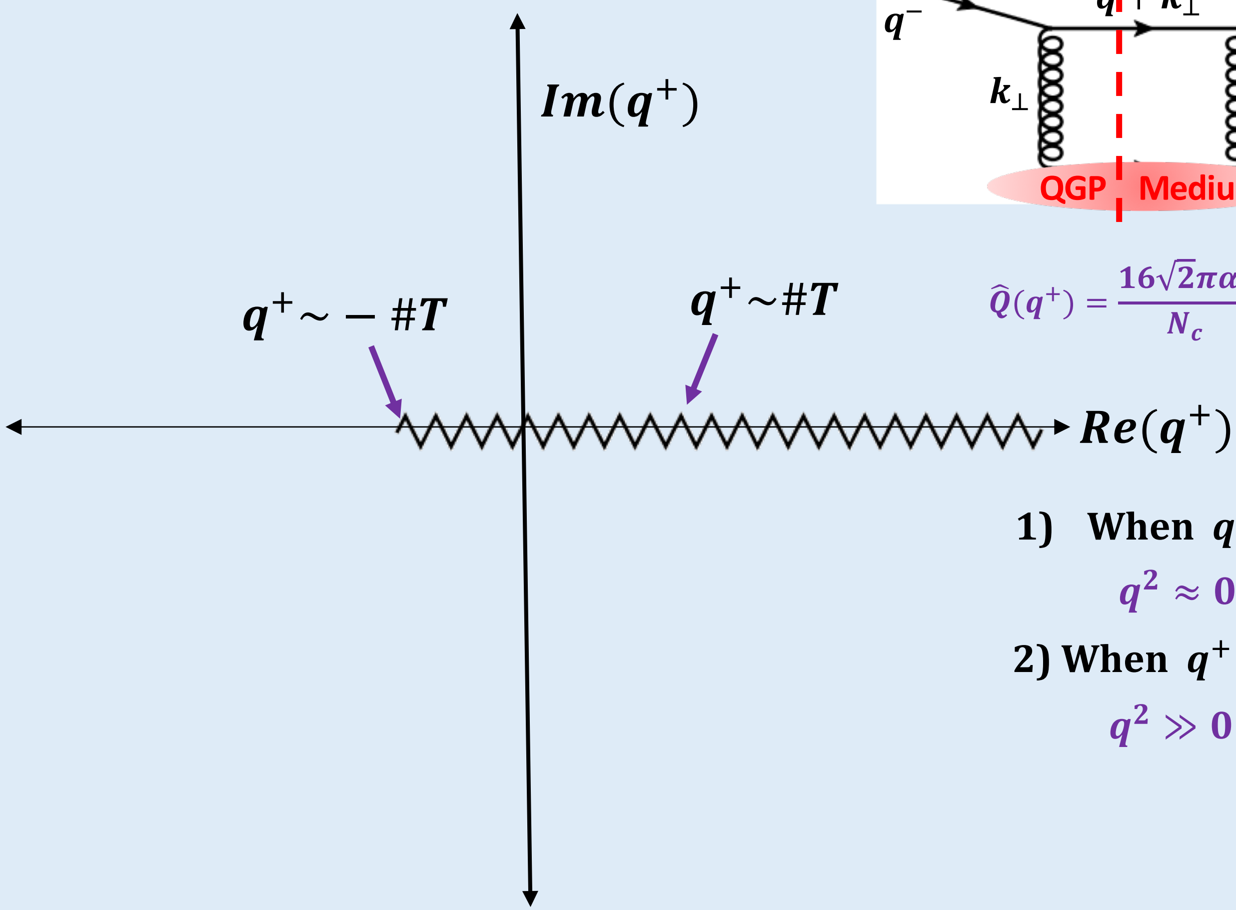
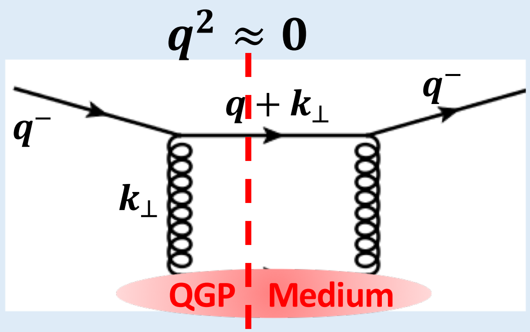


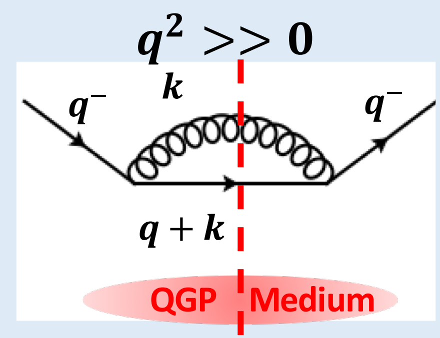
1) **When $q^+ \in [-\#T, \#T]$**

   $q^2 \approx 0$ (in-medium scattering)

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$







$$\hat{Q}(q^+) = \frac{16\sqrt{2}\pi\alpha_s}{N_c} \int \frac{d^4y\, d^4k}{(2\pi)^4} e^{iky}\, q^- \frac{\langle M | F_\perp^{+\mu}(0)\, F^+_{\perp\mu}(y) | M \rangle}{(q+k)^2 + i\epsilon}$$

1) When $q^+ \in [-\#T, \#T]$

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2) When $q^+ \in [\#T, +\infty)$

   $q^2 \gg 0$ (Bremsstrahlung radiation)

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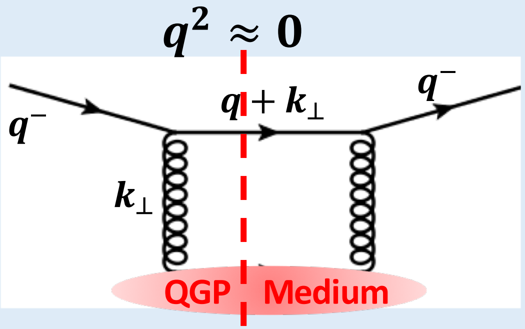

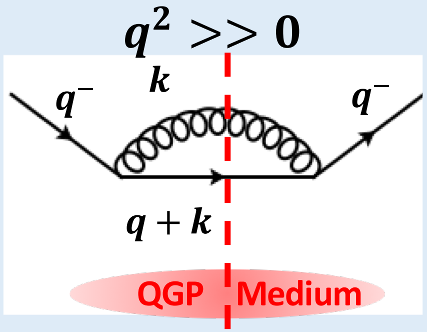

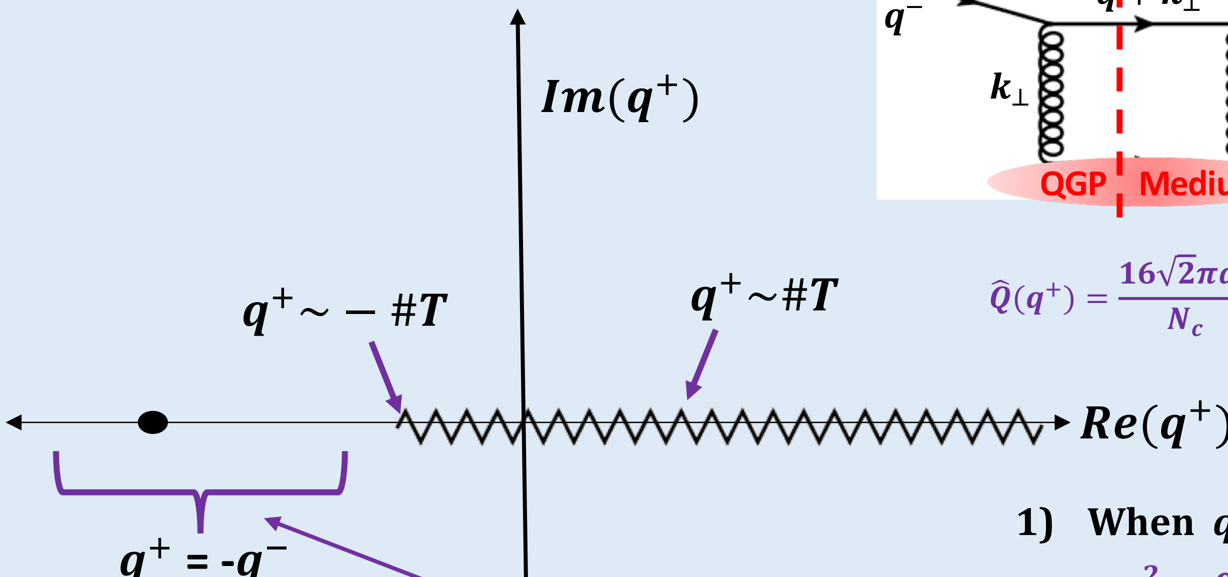

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$q^2 \approx 0$ (in-medium scattering)

2) When $q^+ \in [\#T, +\infty)$

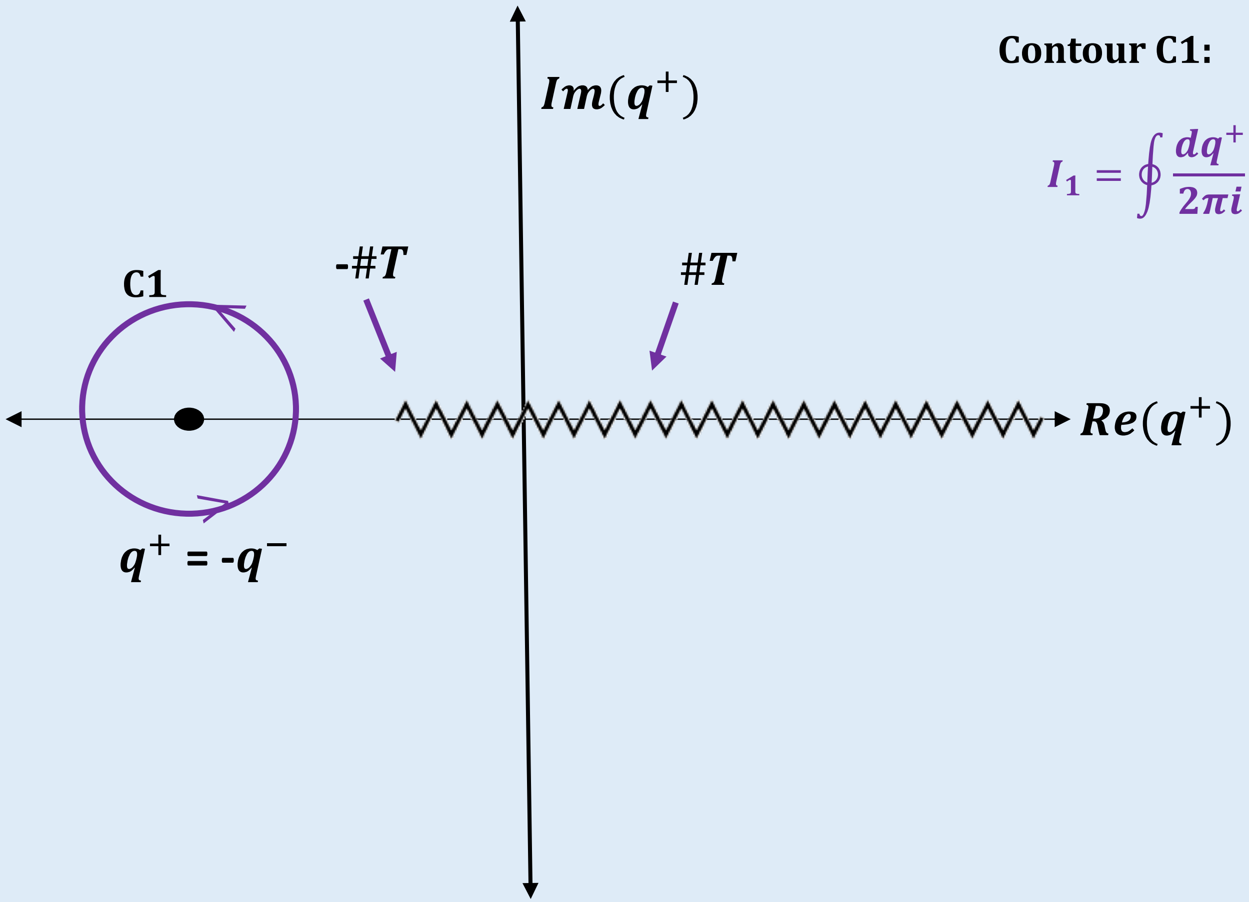
$q^2 \gg 0$ (Bremsstrahlung radiation)

3) When $q^+ \in (-\infty, -\#T]$

$q^2 \ll 0$ (Space-like) ; $\lim_{q^+ \to -q^-} \mathrm{Disc}[\hat{Q}(q^+)] = 0$

$$\hat{Q}(q^+ = -q^-) = \frac{8\sqrt{2}\pi\alpha_s}{N_c\, q^-}\langle M|F_\perp^{+\mu}(0)\sum_{n=0}^{\infty}\left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F_{\perp\mu}^{+}(0)\,|M\rangle$$

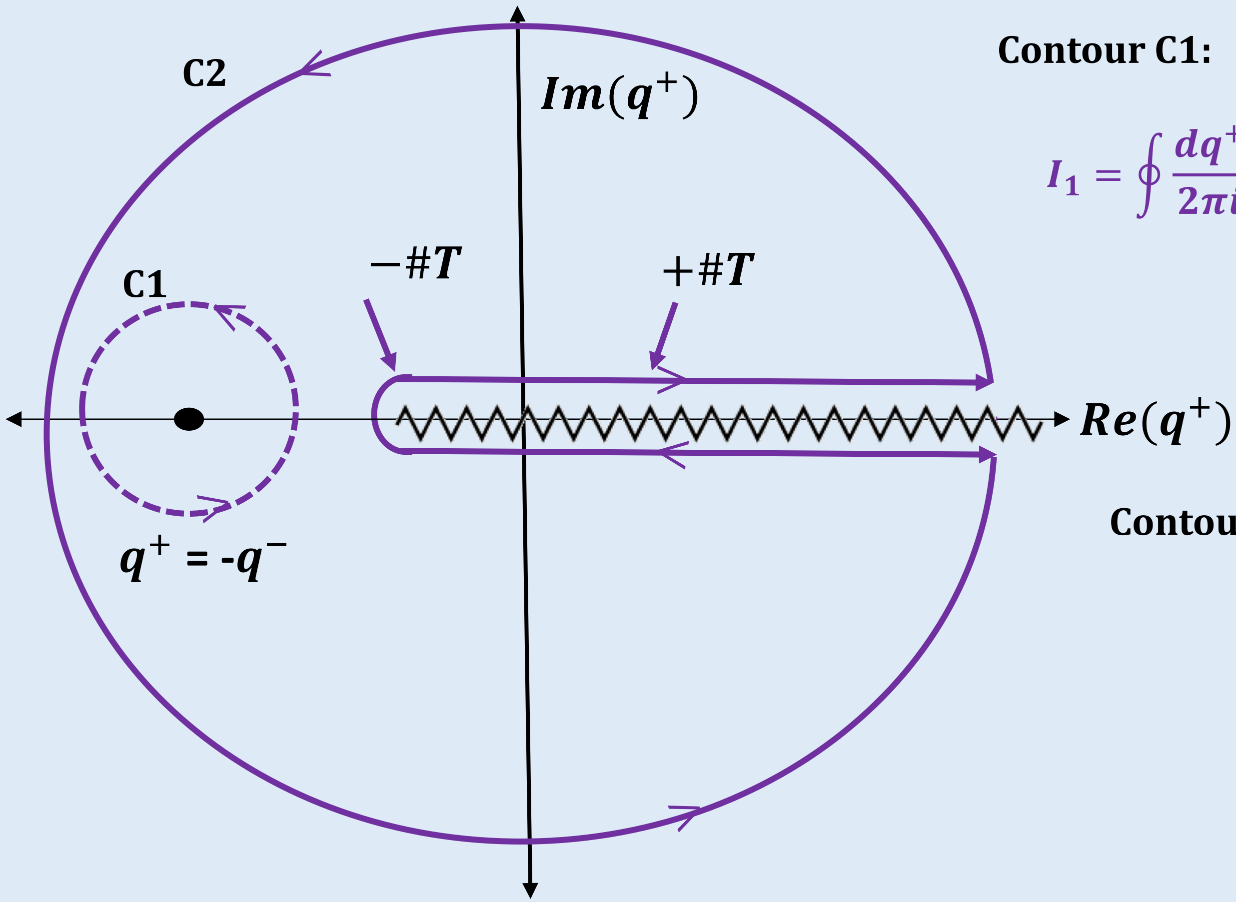
Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



Contour C1:

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



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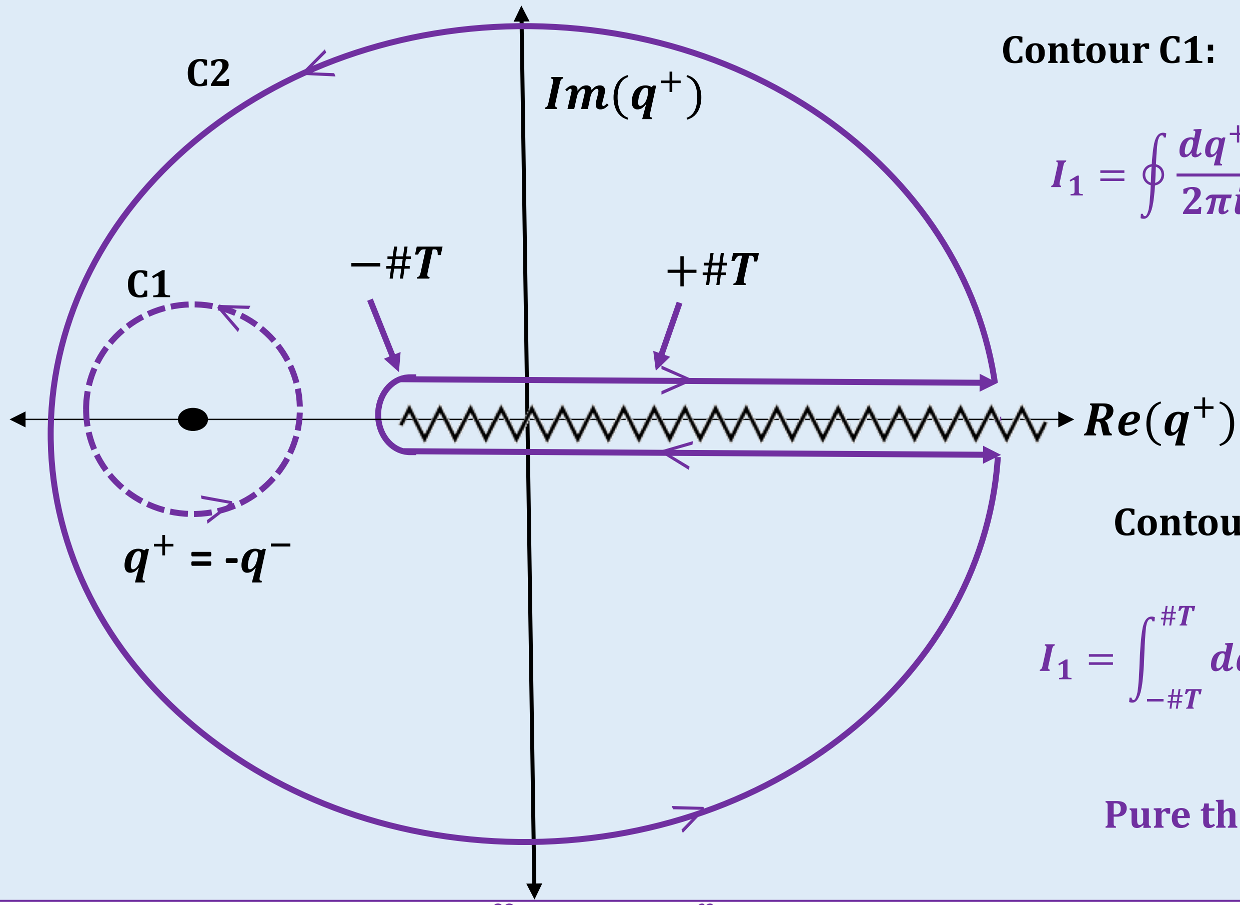
Contour C2: On extending it to infinity

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



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Contour C2: On extending it to infinity

$$I_1 = \int_{-\#T}^{\#T} dq^+ \frac{\hat{q}(q^+)}{q^+ + q^-} + \int_0^\infty dq^+ \frac{Disc[\hat{Q}(q^+)]}{q^+ + q^-}$$

Pure thermal part

Pure Vacuum part

$$\hat{q} = \frac{8\sqrt{2}\pi\alpha_s}{N_c(T_1 + T_2)} \langle M| F_\perp^{+\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-} \right)^n F_{\perp\mu}^{+}(0) \,|M\rangle_{(Thermal-Vacuum)};$$

Width of thermal discontinuity $2T$ or $4T$ (HTL analysis)
$\Longrightarrow$ Source of systematic error

$\hat{q}$ as a series of local operators

❖ **Physical form of $\hat{q}$ at LO:**

$$\hat{q} = \frac{8\sqrt{2}\pi\alpha_s}{N_c(T_1 + T_2)} \langle M | F_{\perp}^{+\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-} \right)^n F_{\perp\mu}^{+}(0) \, | M \rangle_{(Thermal-Vacuum)}$$

$\hat{q}$ as a series of local operators

Xiangdong Ji, PRL 110, 262002 (2013)
Parton PDF operator product expansion with D_z derivatives

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Rotating to Euclidean space:

$$x^0 \to -ix^4; \quad A^0 \to iA^4$$

$$\Longrightarrow \quad F^{0i} \to iF^{4i}$$

LO operators:

$$\sum_{i=1}^{2} \underbrace{Trace\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right]}_{E_i^2 + B_i^2} + 2i \sum_{i=1}^{2} \underbrace{Trace\left[F^{3i}F^{4i}\right]}_{(\vec{E}\times\vec{B})_z}$$

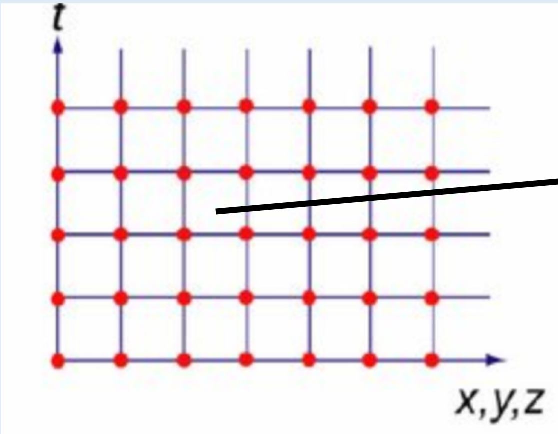
Uncrossed operator **Crossed operator**

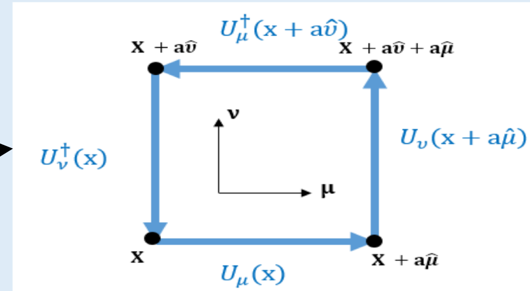
LO operators with $D_z$derivative:

$$\sum_{i=1}^{2} Trace\left[F^{3i}D_zF^{3i} - F^{4i}D_zF^{4i}\right] + i\sum_{i=1}^{2} Trace\left[F^{3i}D_zF^{4i} + F^{4i}D_zF^{3i}\right]$$

where, *Dz* is covariant derivative along leading parton direction (*z*-dir)

Computing operators on lattice and Scale setting





- **Fundamental variable: Link**

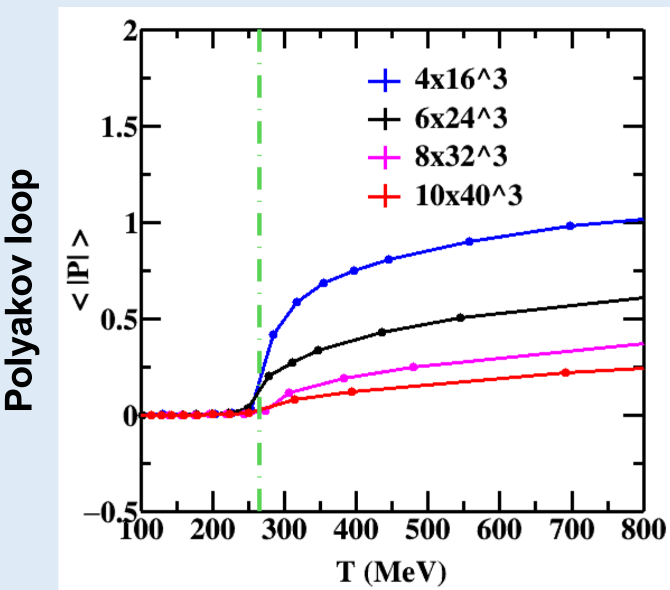
$$U_\mu(x) = e^{igaA_\mu(x)} \in SU(3)$$

- **Full QCD calculation**

$$\langle O\rangle = \frac{\int DUD\bar{\psi}D\psi\; O\; e^{-S_F-S_g}}{\int DUD\bar{\psi}D\psi\; e^{-S_F-S_g}}$$

- **Quenched calculation**

$$\langle O\rangle = \frac{\int DU\; O\; e^{-S_g}}{\int DU\; e^{-S_g}}$$



- **Nonperturbative correction to two loop beta function**

$$a_L = \frac{f}{\Lambda_L}\left(\frac{11}{16\pi^2 g^2}\right)^{-\frac{51}{121}} exp\left(-\frac{8\pi^2}{11g^2}\right); \qquad Temperature, T = \frac{1}{n_t a_L}$$

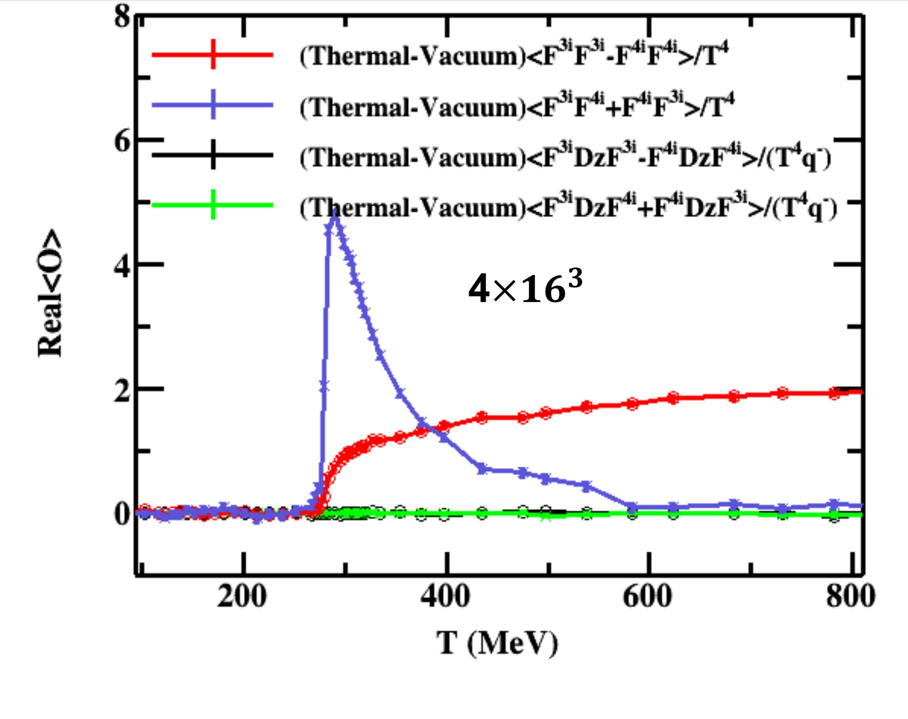
$$Critical\ Temperature, T_c = 265\ MeV$$ (Pure SU(3))

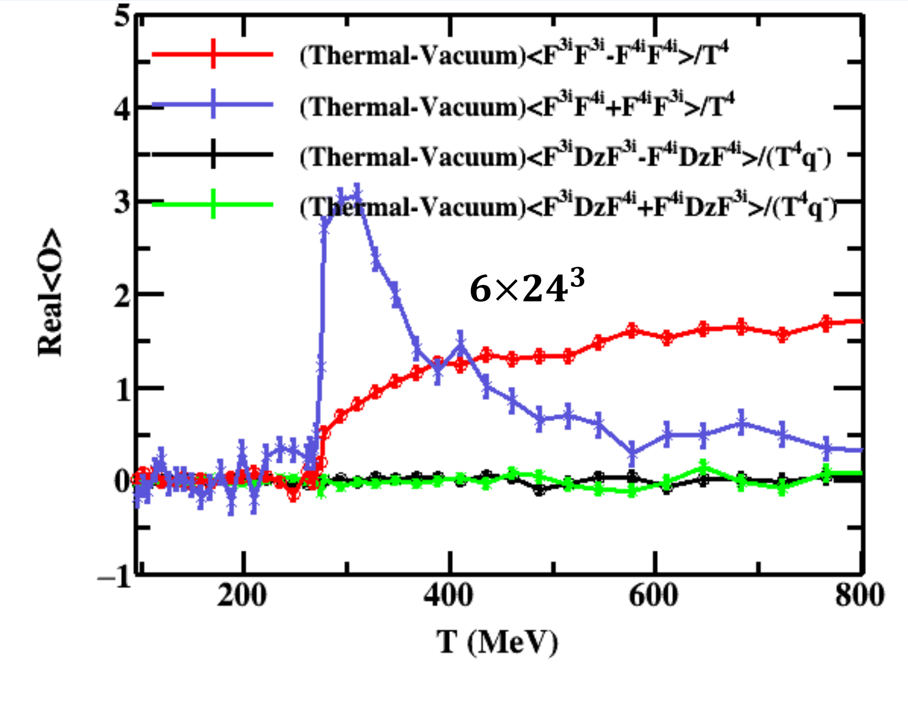
$$Tune\ \ f\ such\ that\ \frac{T_c}{\Lambda_L}\ is\ independent\ of\ g$$

- **Expectation value of Polyakov loop:**

$$P = \frac{1}{n_x n_y n_z} tr\left[\sum_{\vec{r}} \prod_{n=0}^{n_t-1} U_4(na, \vec{r})\right]$$

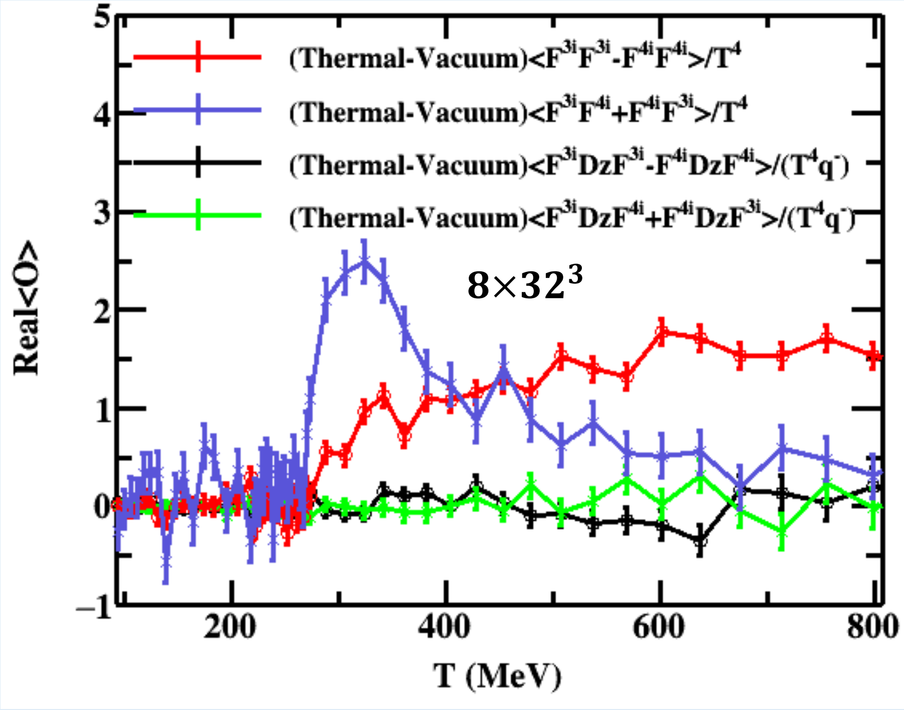
Real part of FF correlator in quenched SU(3)

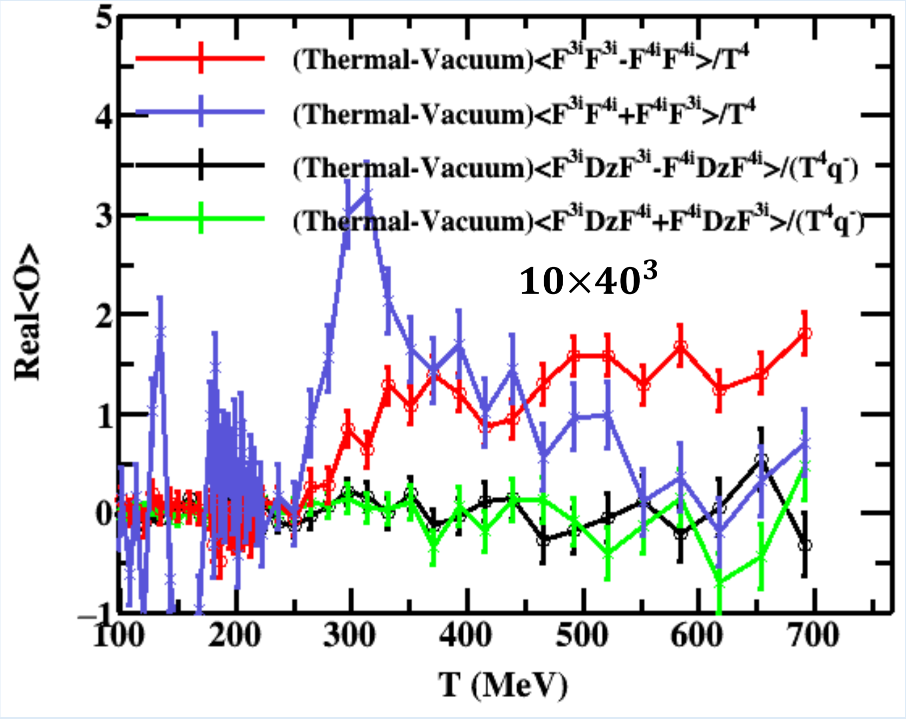




- **Uncrossed correlator is dominant at high temperature**
- **Crossed correlator goes to zero at high temperature**
- **Correlator with Dz derivative are suppressed**

Real part of FF correlator in quenched SU(3)





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$\hat{q}$ in quenched SU(3) plasma

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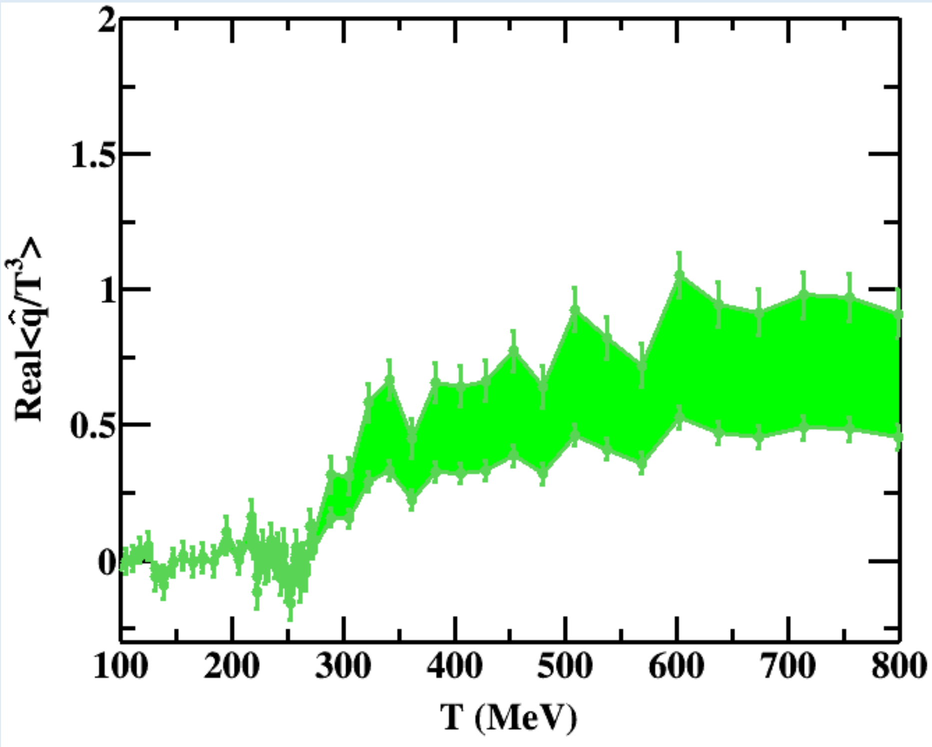
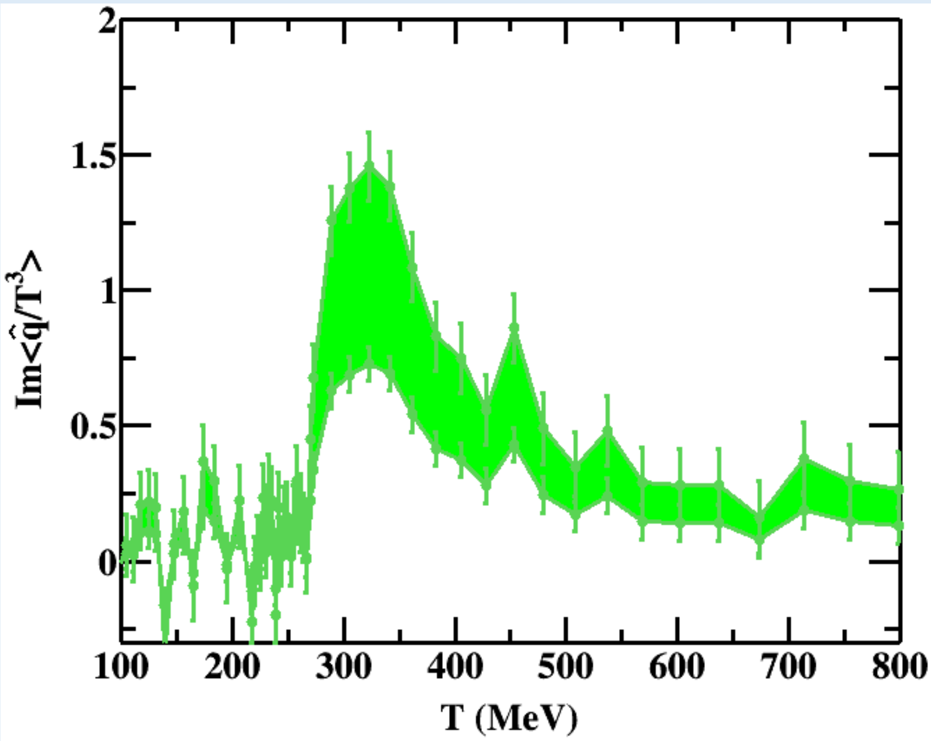
Width of thermal discontinuity $2T$ or $4T$ (HTL analysis)





At high temperature:

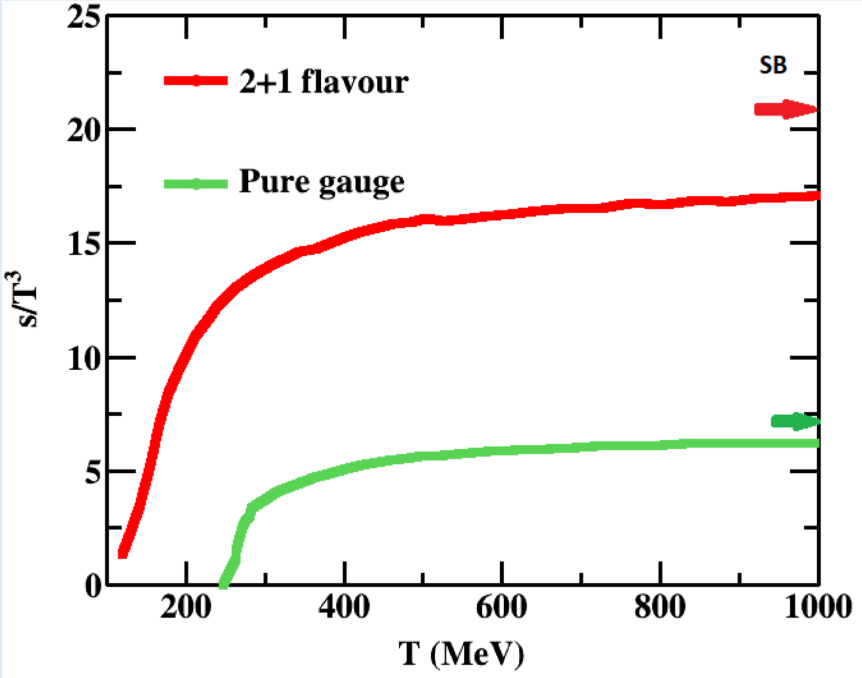
$$\frac{\hat{q}}{T^3} \sim 0.5 - 1$$

At high temperature:

$\mathrm{Im}[\frac{\hat{q}}{T^3}]$ **goes to zero**

Pure gluon plasma vs QGP plasma

- Lect.Notes Phys. 583 (2002) 209-249;
- JHEP 1011 (2010) 07



Going from pure gluon to quark-gluon system

1) Critical temperature shifts to lower temperature
2) Magnitude amplifies by factor of 3-4

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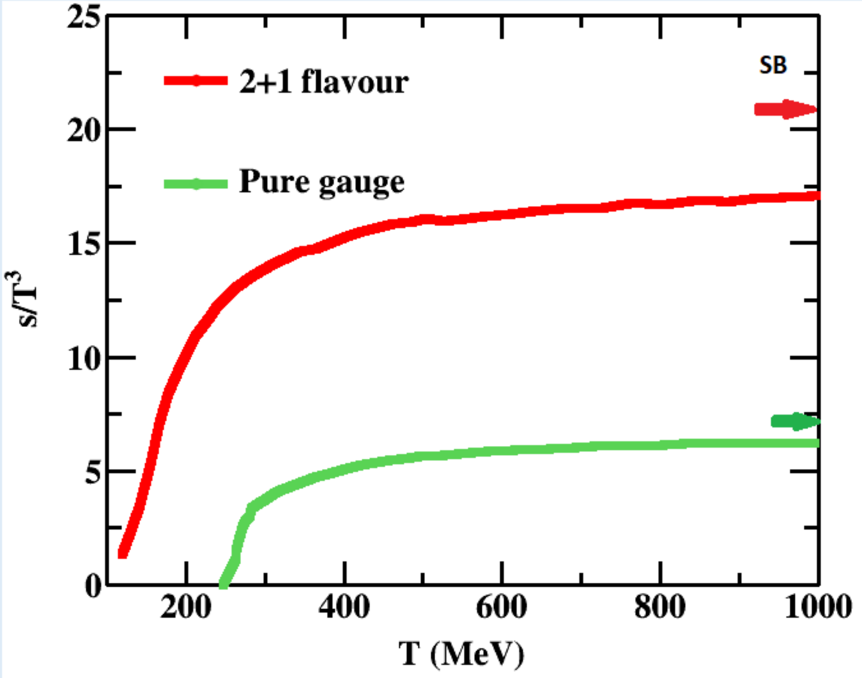
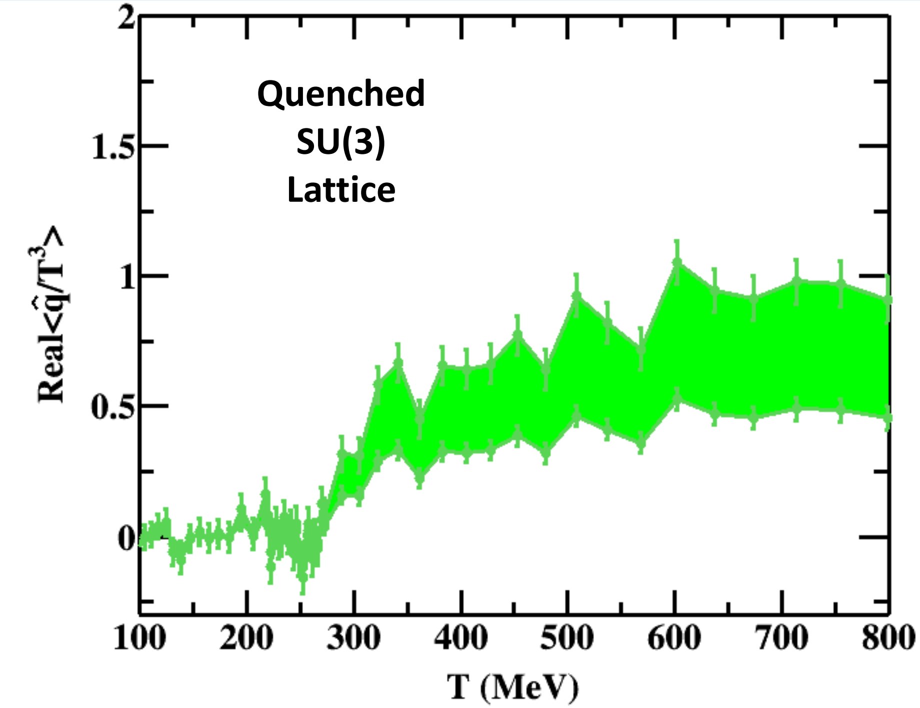


$\hat{q} \propto S$ **(Entropy density of the medium)**



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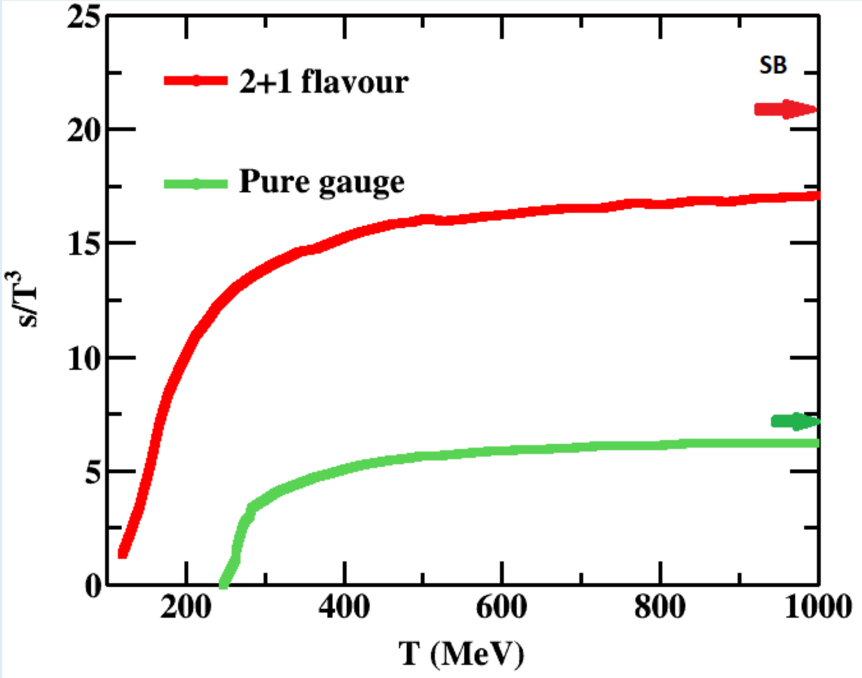

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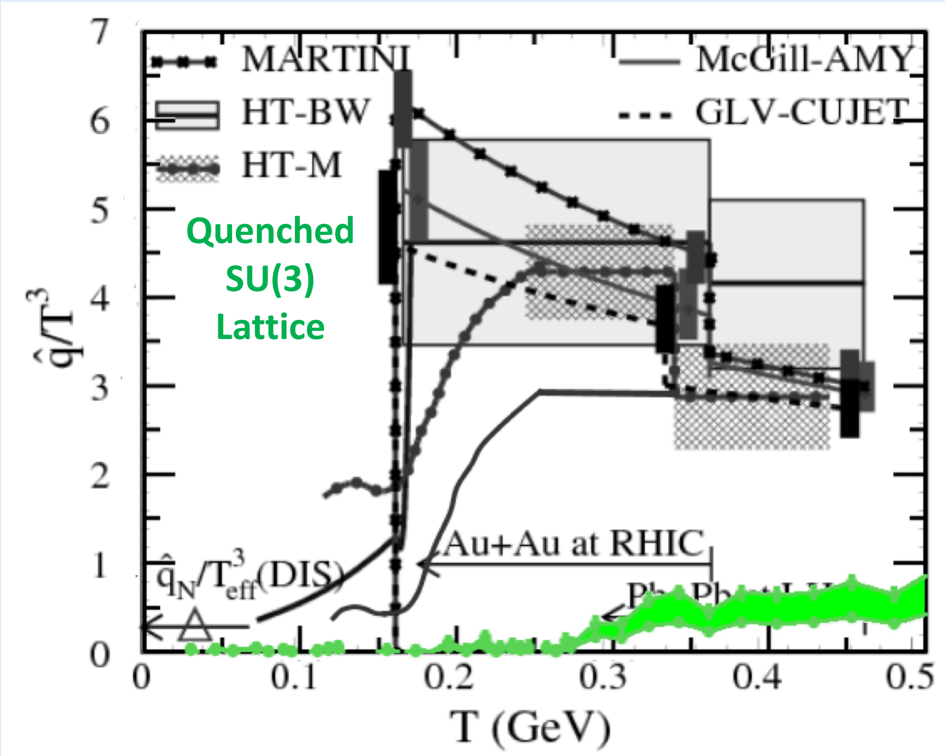

Summary and Future work

- First calculation of $\hat{q}$ on SU(3) quenched plasma
  - Lattice extraction of $\hat{q}$ is consistent with JET collaboration plot
  - Analytic continuation to deep Euclidean space and expressed as local operators
  - $\hat{q}$ shows scaling behavior (Nt=4, 6, 8 and 10)
  - Scale setting using perturbative loop beta function with non-perturbative correction using Polyakov loop.

- Future work
  - Extend calculation to unquenched plasma (QGP)
  - Extend calculation using Improved Action and bigger lattice sizes
  - Include medium induced radiation diagram contributions

Operators in quenched SU(2) plasma

- **Average over 5000 configuration**
- **Transition temperature $T_c \in [170, 350]$ MeV**
- **Crossed correlator is small for T ~ 400 MeV**

A. Majumder, PRC 87, 034905 (2013)