# Generating functional for jet observables in HIC

in preparation

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# Jets(cape) in heavy ion collisions

*A twofold approach:*

- More Carlo Implementation needed for quantitative studies due to the complexity of heavy ion environment and multi scale nature of the problem. Rely on a good understanding of the underlying physics and higher order effects (JEWEL, MARTINI, LBT, HYBRID, etc)

- Guidance from theory: (Tractable) analytic calculations permits to understand the qualitative features of jet observables and constrain modeling for MC's $\longrightarrow$ this talk

# Jet evolution (general picture)

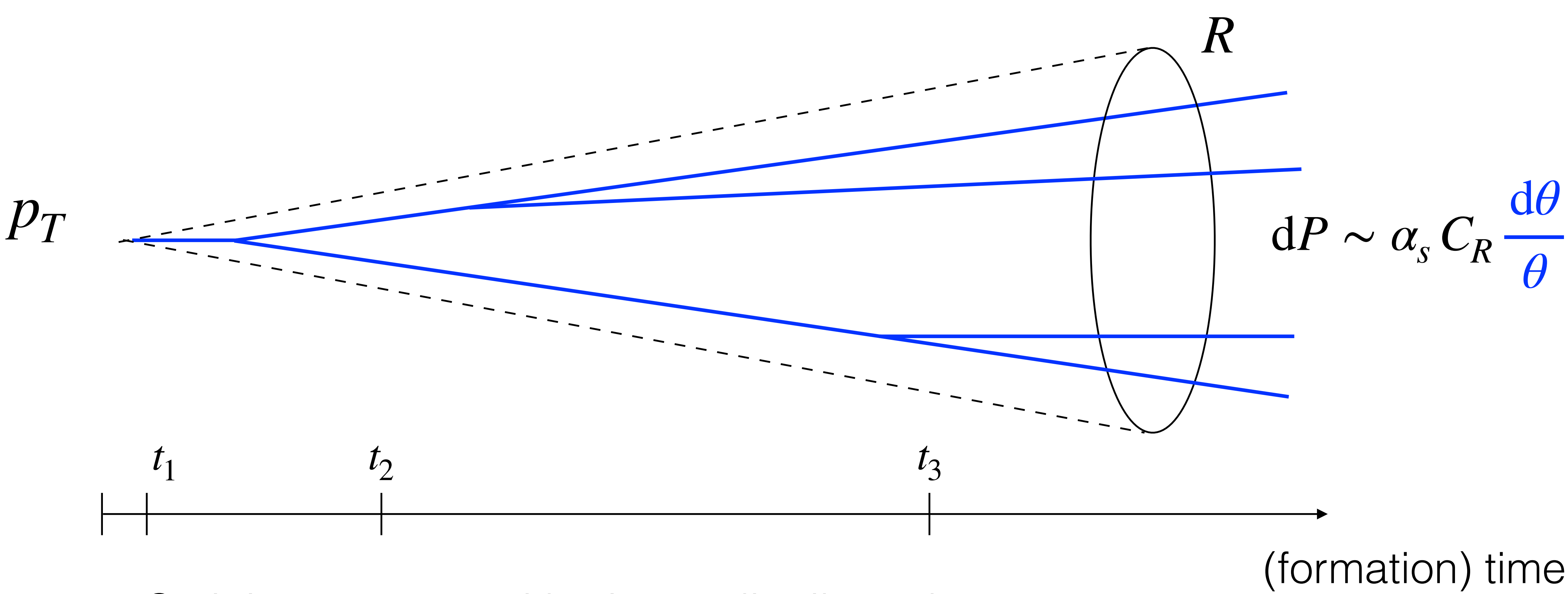
Parton showers of two kinds

- Vacuum shower. Ordering variables: angle, kt, virtuality.
  *Properties:* collimated (mass singularity), color coherence,…

- Medium-induced cascade: ordered in real time.
  *Properties:* large angle democratic branchings, color randomization

# Vacuum shower

- Soft & Collinear divergences (resummation)
- Color coherence: angular ordered shower, interjet activity
- Not uniquely defined: cone size *R*, reconst. algo, …




$$\mathrm{d}P \sim \alpha_s\, C_R \,\frac{\mathrm{d}\theta}{\theta}\,\frac{\mathrm{d}\omega}{\omega}$$

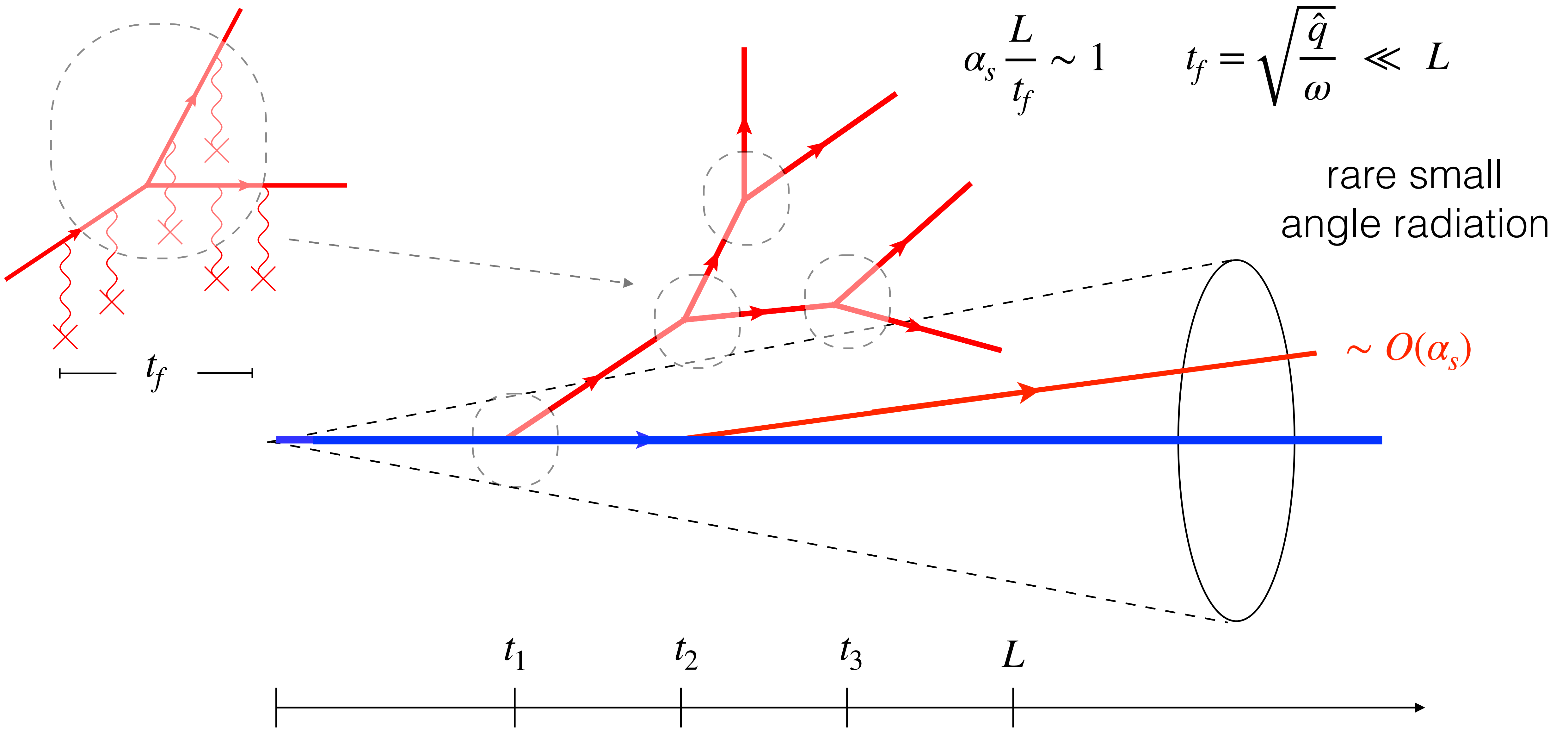
Splittings are logarithmically distributed:

$$\frac{1}{p_T R^2} \ll t \ll \frac{p_T}{\Lambda^2_{QCD}}$$

# Medium-induced shower

large angle incoherent cascade

$$\alpha_s \frac{L}{t_f} \sim 1 \qquad t_f = \sqrt{\frac{\hat{q}}{\omega}} \ll L$$



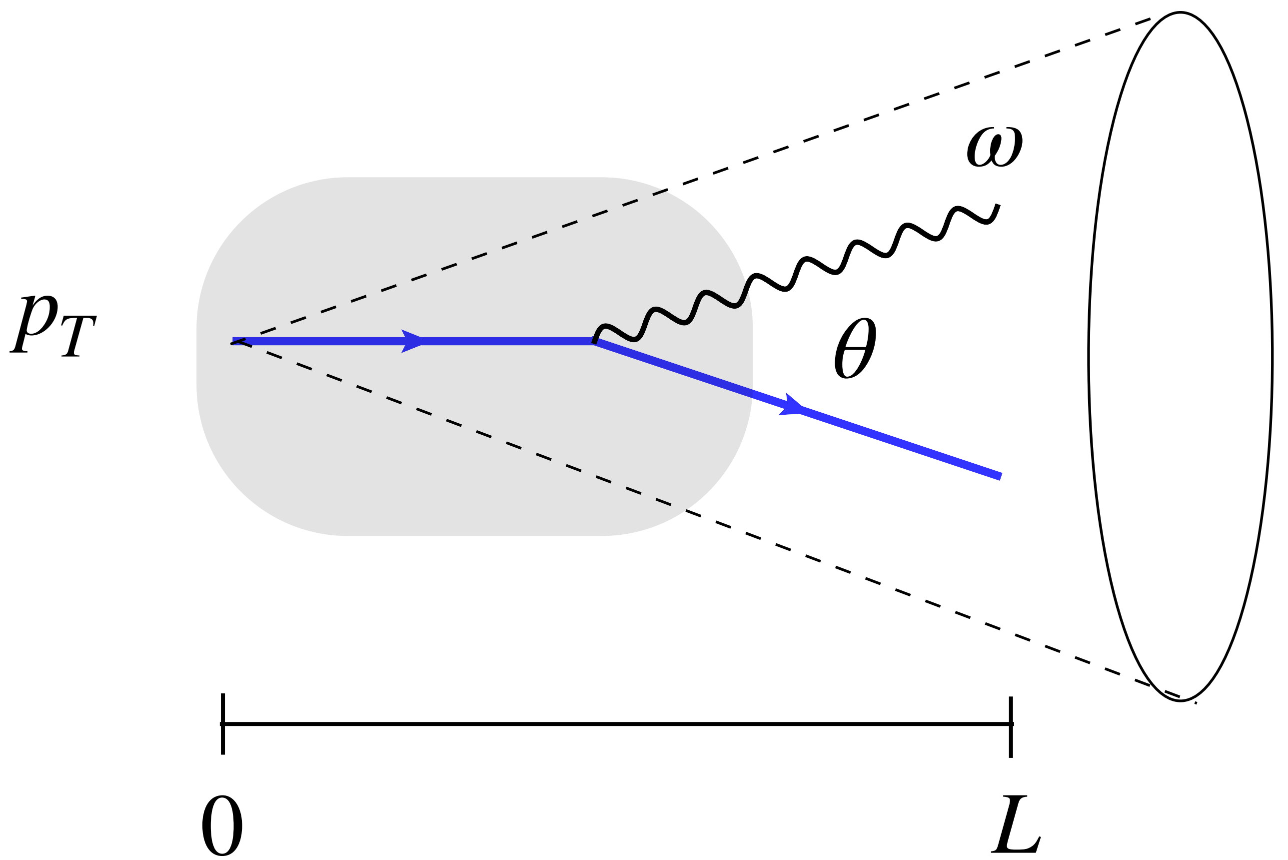

Splittings are linearly distributed

BDMPS-Z-W (1995-2000) GLV (2000) AMY (2002) GW (2002)

Jeon, Moore (2003) , Schenke et al (2009) Blaizot, Dominguez, Iancu, MT (2013)

# Phase-space analysis

- How important are jet substructure fluctuations in the medium?




formation time

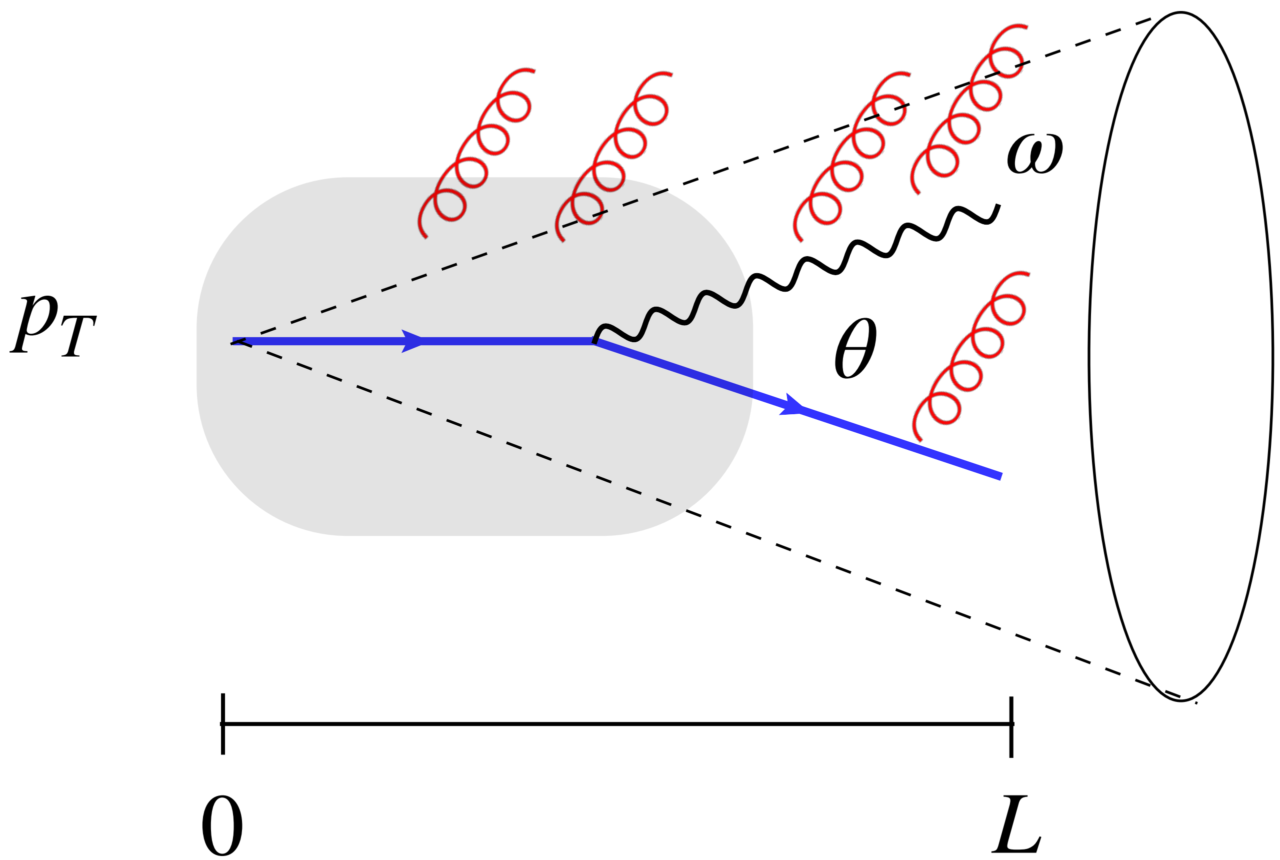
$$t_f \sim \frac{1}{\omega\theta^2}$$

$$\mathrm{PS} = \bar{\alpha} \int_0^{p_T} \frac{\mathrm{d}\omega}{\omega} \int_0^{R} \frac{\mathrm{d}\theta}{\theta}\, \Theta(t_\mathrm{f} > L) = \frac{\bar{\alpha}}{4} \ln^2\left(p_T R^2 L\right) \gtrsim 1$$

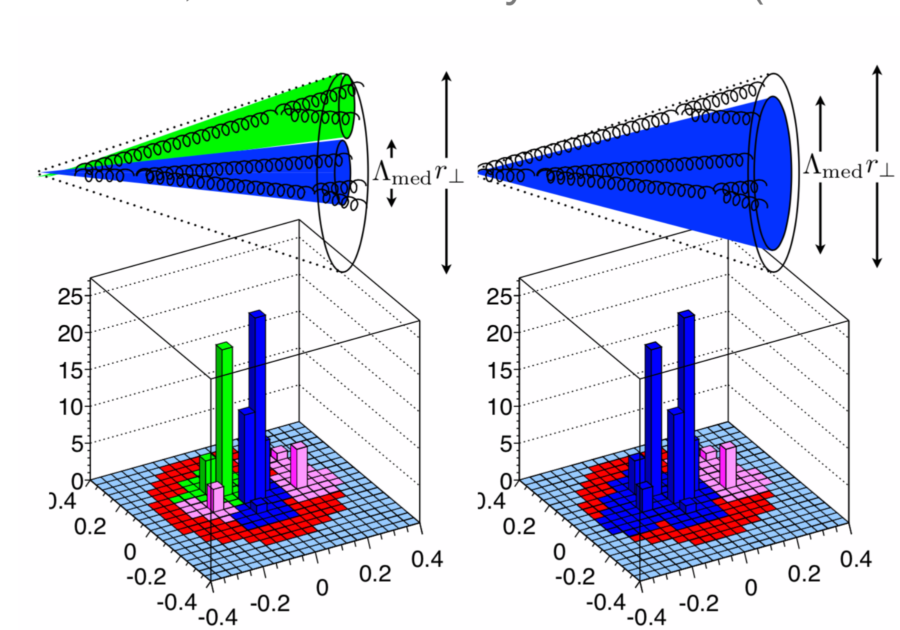
→ Large phase-space for in-medium vacuum splittings

# Color decoherence

- Double logarithmic phase-space:




MT, Salgado, Tywoniuk (2010-11)
Iancu, Casalderrey-Solana (2011)



Casalderrey-Solana, MT, Salgado, Tywoniuk (2012)

Apolinario et al (2014) MT, Tywoniuk (2017)

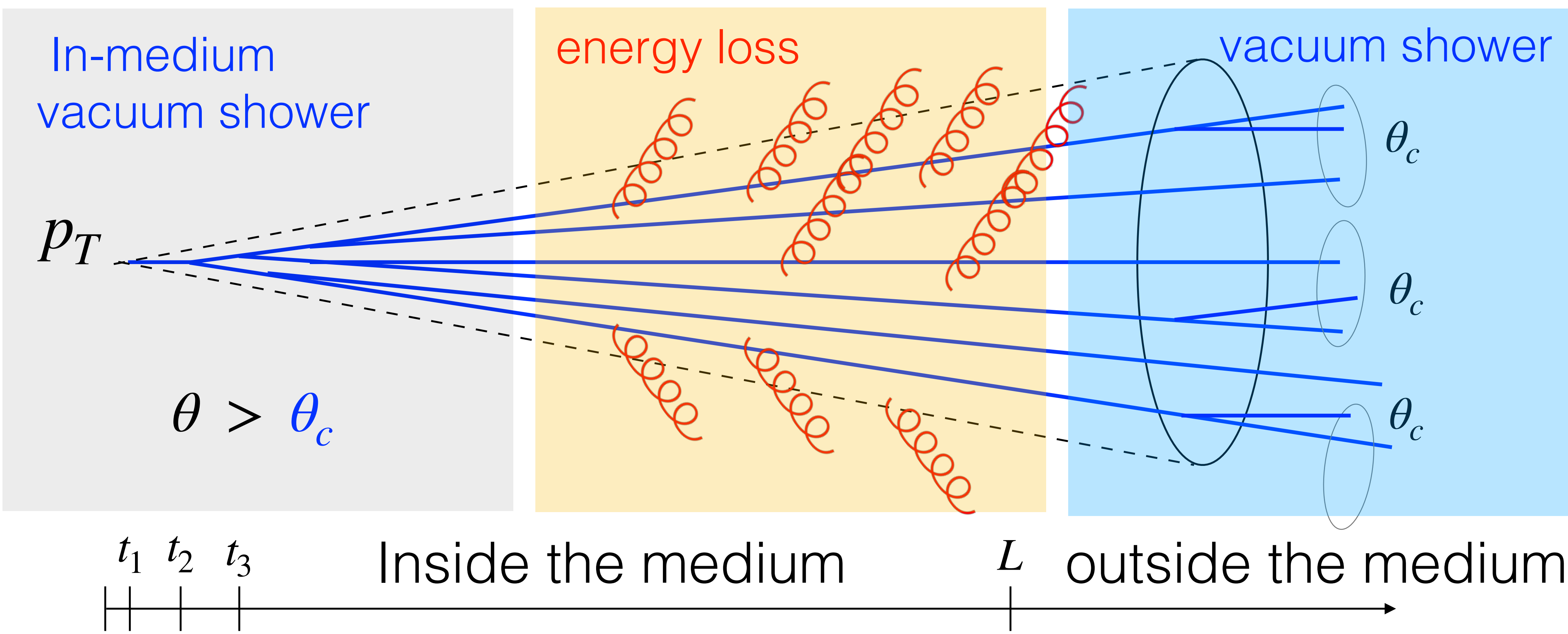
- Additional time scale: the color decoherence time associated with the resolution of subjects by the medium

$$t_{\rm d} \sim (\hat{q}\,\theta^2)^{-1/3} < L \qquad \text{or} \qquad \theta > \theta_c \sim (\hat{q}L^3)^{-1/3}$$

# General picture (3-stage evolution)

early time (angular ordered) vacuum shower with:

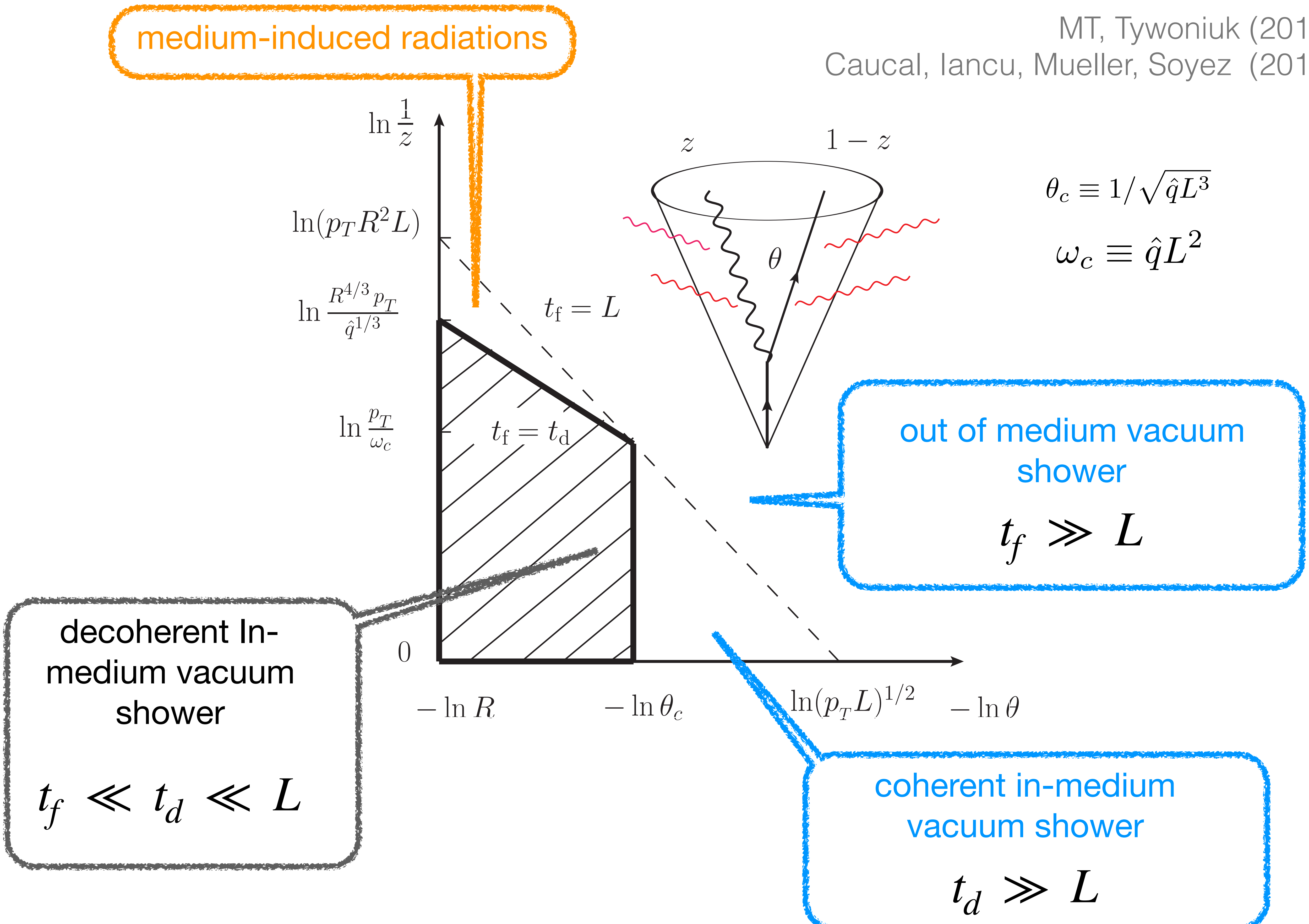
$$\frac{1}{p_T R^2} \ll t_f \ll t_\mathrm{d} \ll L$$




To leading logarithmic accuracy the first stage occurs long before energy loss takes place

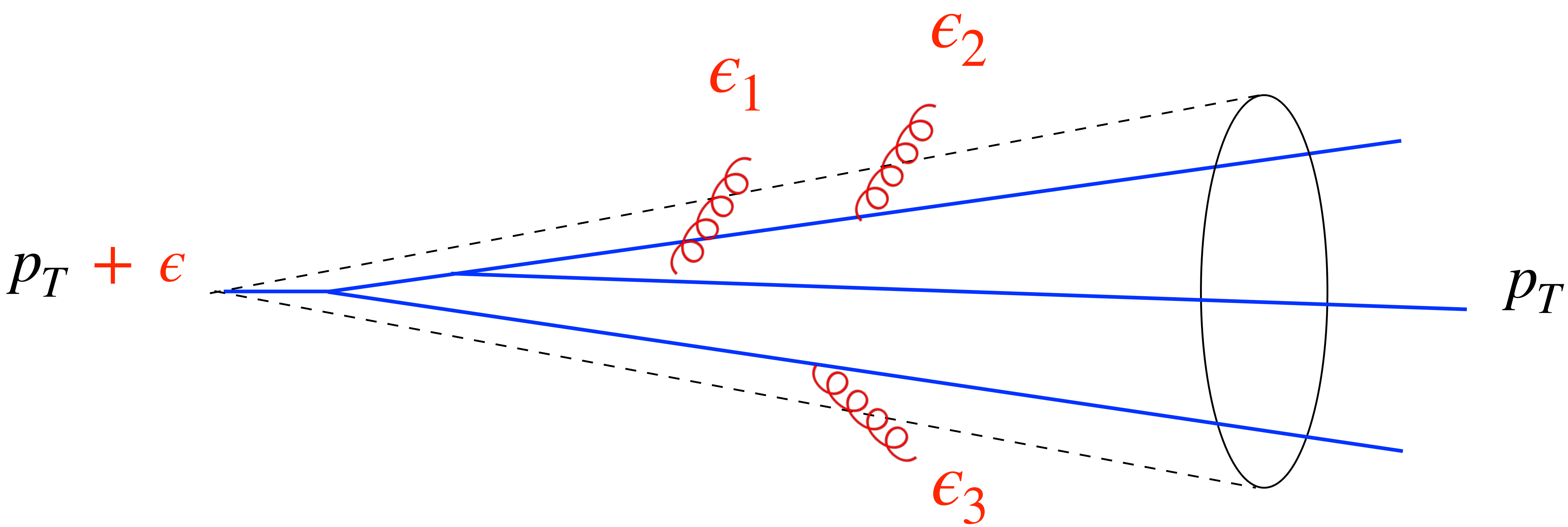
# Lund diagram

MT, Tywoniuk (2017)
Caucal, Iancu, Mueller, Soyez (2018)



# Factorization of quenching factors

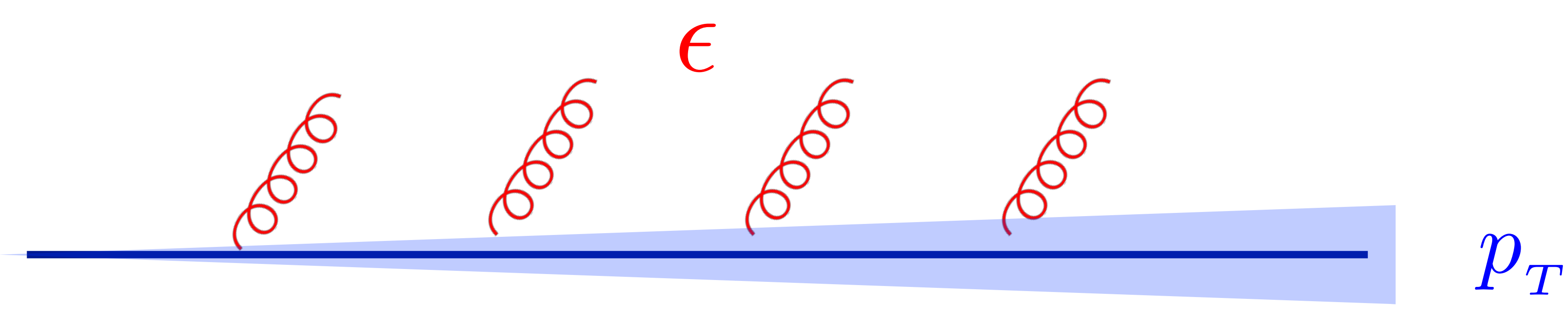
- Suitable for Monte Carlo implementation (see E. Iancu's talk)
- Analytic method? Need to deal with involved convolutions of energy loss distribution for multiple parton configurations




Total energy fluctuates with jet substructure $\epsilon = \epsilon_1 + \epsilon_2 + \epsilon_3$

# Factorization of quenching factors

- Standard analytic approach to jet quenching: single parton energy loss




- Jet spectrum: convolution of pp jet spectrum with energy loss probability distribution

$$\frac{\mathrm{d}\sigma(p_T)}{\mathrm{d}^2 p_T \mathrm{d}y} = \int_0^\infty \mathrm{d}\epsilon \, \mathcal{P}(\epsilon) \, \frac{\mathrm{d}\sigma^{\mathrm{vac}}(p_T + \epsilon)}{\mathrm{d}^2 p_T \mathrm{d}y}$$

# Factorization of quenching factors

- Due to the steep initial spectrum, energy loss biased towards small values (< mean energy loss)

$$\frac{d\sigma_{vac}}{d^2p_T} \sim \frac{1}{p_T^n} \qquad \epsilon \sim \frac{p_T}{n} \ll \alpha_s^2 \hat{q} L^2$$

Baier, Dokshitzer, Mueller, Schiff, JHEP (2001)

- large n approximation:

$$\frac{d\sigma(p_T+\epsilon)}{d^2p_T} \sim \frac{1}{(p_T+\epsilon)^n} = \frac{e^{-\frac{n\epsilon}{p_T}}}{p_T^n}(1+\mathcal{O}(n\epsilon^2/p_T^2))$$

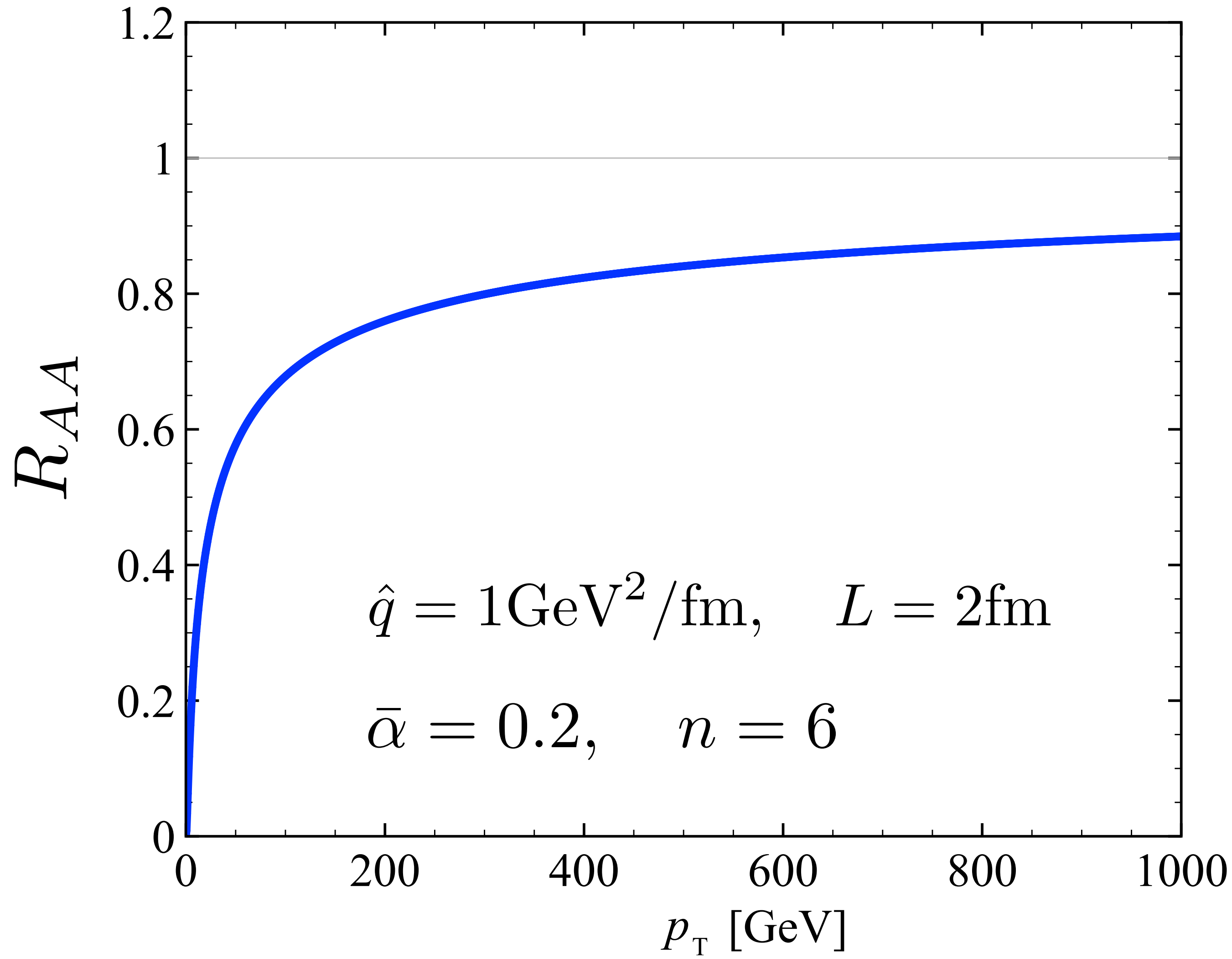
- Quenching factor as Laplace transform of loss probability

$$R_{AA} \simeq Q(p_T) \equiv \int d\epsilon\, \mathcal{P}(\epsilon)\, e^{-\frac{n\epsilon}{p_T}}$$

# Factorization of quenching factors

- Illustration: neglecting finite size effects and medium geometry

$$R_{AA} \simeq \exp\left(-\bar{\alpha}\, L \sqrt{\frac{\pi \hat{q}\, n}{p_T}}\right)$$



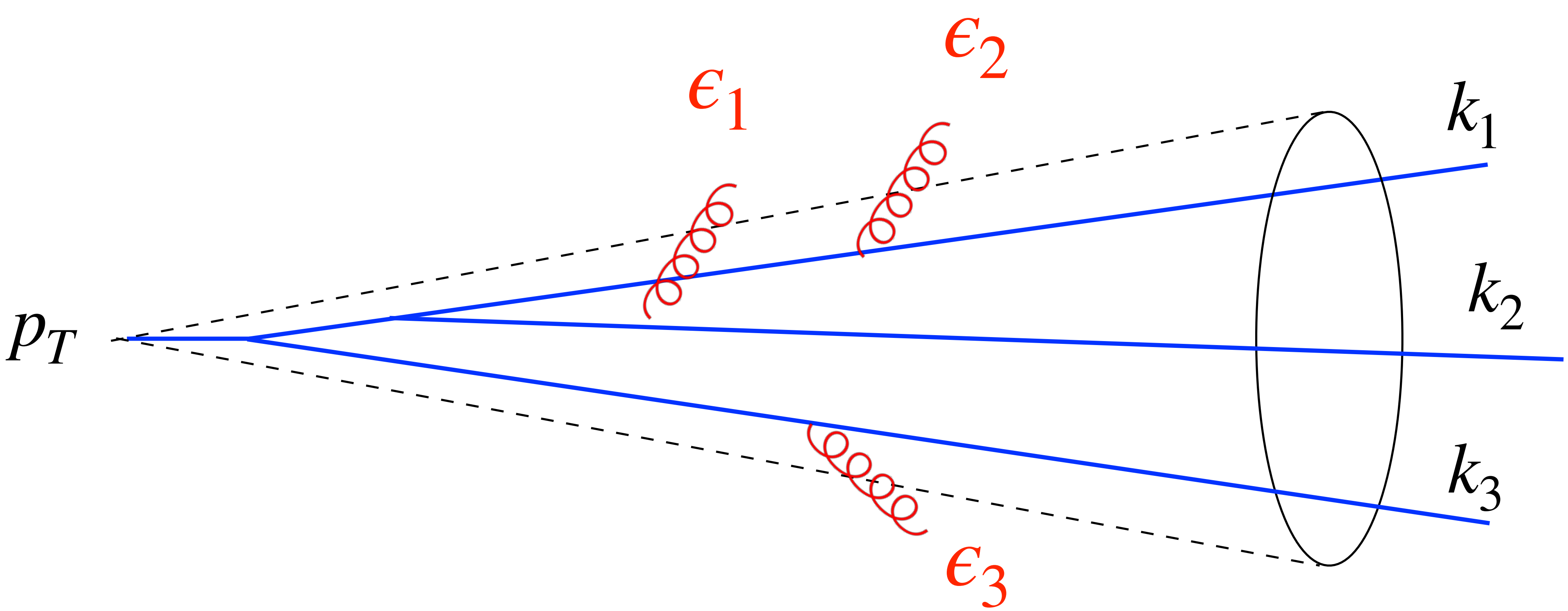

strong quenching

$$p_T \ll \pi\, n\, \bar{\alpha}^2 \hat{q} L^2$$

$$Q(p_T) \ll 1$$

# Factorization of quenching factors

Multi-parton energy loss: convolution of single parton energy loss probability distributions




$$\frac{\mathrm{d}\sigma_{\mathrm{excl}}}{\mathrm{d}k_1\mathrm{d}k_2...\mathrm{d}k_N} = \int \mathrm{d}p \left\{ \int \prod_{i=1}^{N} \mathrm{d}\epsilon_i P(\epsilon_i) \right\} f(k_1+\epsilon_1, k_2+\epsilon_2, ..., k_N+\epsilon_N|p)$$

$$\times\delta(p-\epsilon-k_1-k_2-...-k_N)\,\frac{\mathrm{d}\sigma}{\mathrm{d}p}$$

# Factorization of quenching factors

- In the large n approximation one can neglect energy shifts except in the steep spectrum

$$\epsilon_i \sim p_T/n \ll k_i$$

- This yields the factorization of single parton quenching factors:

$$\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{excl}}}{\mathrm{d}k_1 \mathrm{d}k_2 ... \mathrm{d}k_N} &\simeq \int \mathrm{d}p \left\{ \int \prod_{i=1}^{N} \mathrm{d}\epsilon_i P(\epsilon_i) \right\} f(k_1, k_2, ..., k_N | p) \\
&\times \delta(p - k_1 - k_2 - ... - k_N) \frac{\mathrm{d}\sigma(p+\epsilon)}{\mathrm{d}p} \\
&\simeq \left( \int_0^\infty \mathrm{d}\epsilon P(\epsilon) \exp\left( -\frac{n\epsilon}{p_T} \right) \right)^N \frac{\mathrm{d}\sigma_{0,\mathrm{excl}}}{\mathrm{d}k_1 \mathrm{d}k_2 ... \mathrm{d}k_N} \\
&= \mathcal{Q}(p)^N \frac{\mathrm{d}\sigma_{0,\mathrm{excl}}}{\mathrm{d}k_1 \mathrm{d}k_2 ... \mathrm{d}k_N} .
\end{aligned}$$

# Generating functional (vacuum)

- Generating function for $P_n$:  $$Z(u) = \sum_{n=0}^{\infty} P_n \, u^n$$

normalization

$$Z(u=1) = 1$$

expectation value

$$\langle n \rangle = \frac{\mathrm{d}}{\mathrm{d}u} Z(u) \Big|_{u=1}$$

- Application to parton cascades:

$$P_n \to P(k_1, \ldots, k_n) \qquad u^n \to u(k_1) \ldots u(k_n)$$

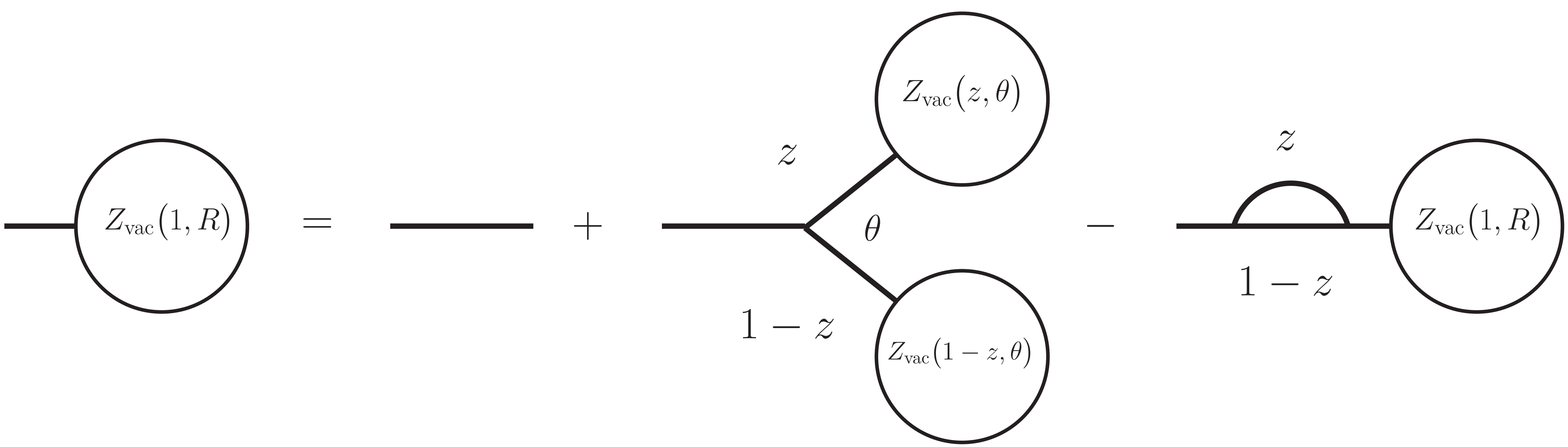
# Generating functional (vacuum)

- Coherent branching algorithm: strict angular ordering

$$R > \theta_1 > ... > \theta_n$$

- The GF of multi Parton probability distributions obeys the evolution equation

$$Z_{\rm vac}(p,R;u) = u(p) + \frac{\alpha_s}{\pi}\int_0^R \frac{{\rm d}\theta}{\theta}\int_0^1 {\rm d}z\, P(z) \times \left[Z_{\rm vac}(zp,\theta)\, Z_{\rm vac}((1-z)p,\theta) - Z_{\rm vac}(p,\theta)\right]$$




Bassetto, Ciafaloni, Marchesini, Mueller, Dokshitzer, Khoze, Toyan,… 1980's

# Generating functional (vacuum)

- Example: Fragmentation Function

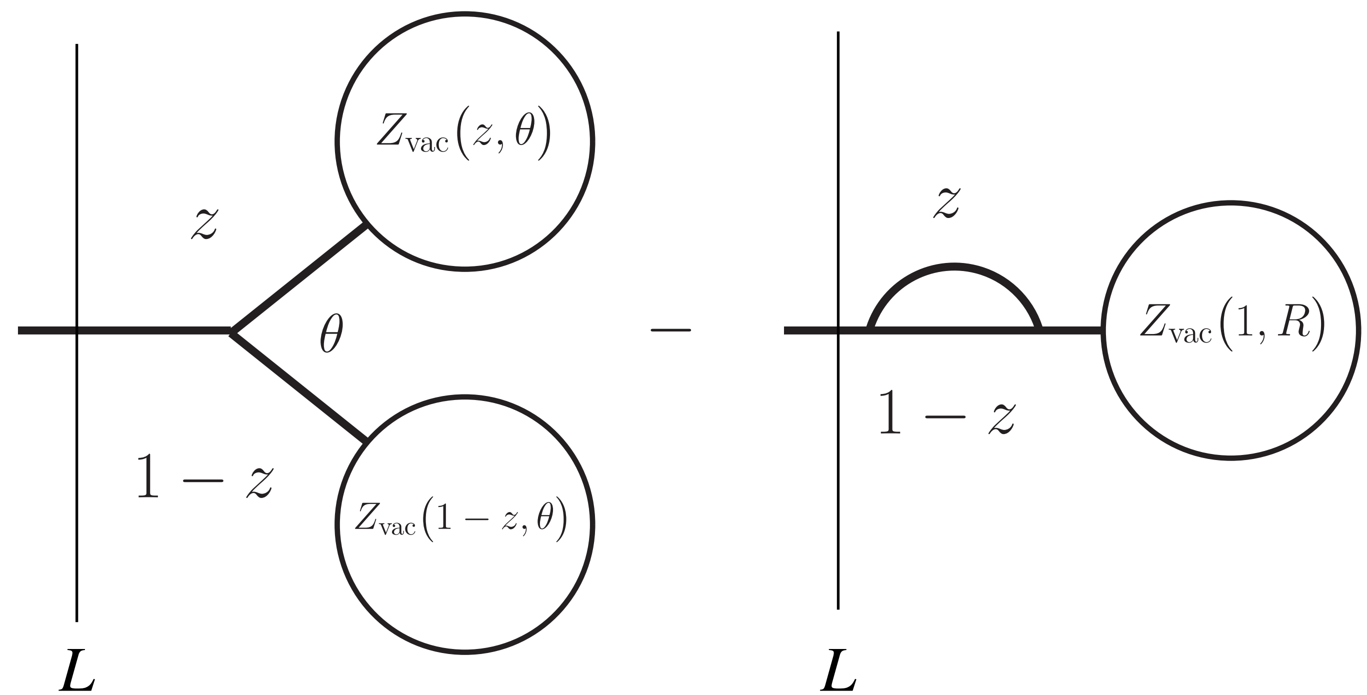
$$D(x,Q) = x\frac{\delta}{\delta u(x)}Z(Q,u)\Big|_{u=1}$$

- We obtain the evolution equation in the MLL Approximation

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q}D(x,Q) = \frac{\alpha_s}{\pi}\int_x^1 \mathrm{d}z\,P(z)\,\left[D(x/z,zQ) - \frac{1}{2}\,D(x,Q)\right]$$

# Generating functional (in-medium)

- Vacuum shower outside the medium and coherent shower inside

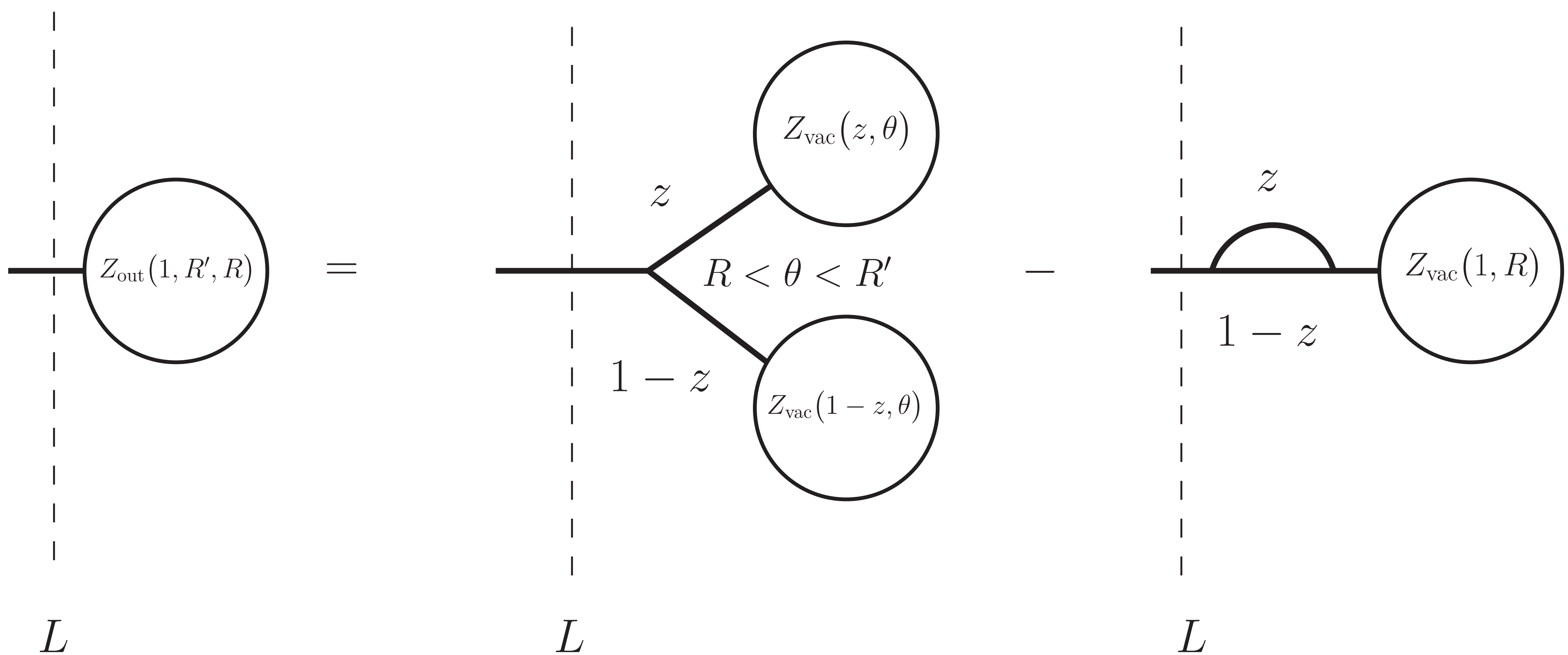



$$\Theta(t_d > L) + \Theta(t_d < L < t_f)$$

$\theta < \theta_c = (\hat{q}L^3)^{-1/3}$ $\qquad$ $\theta > \theta_c$ $\qquad$ $t_f \sim \dfrac{1}{z(1-z)p\theta^2} > L$

# Generating functional (in-medium)

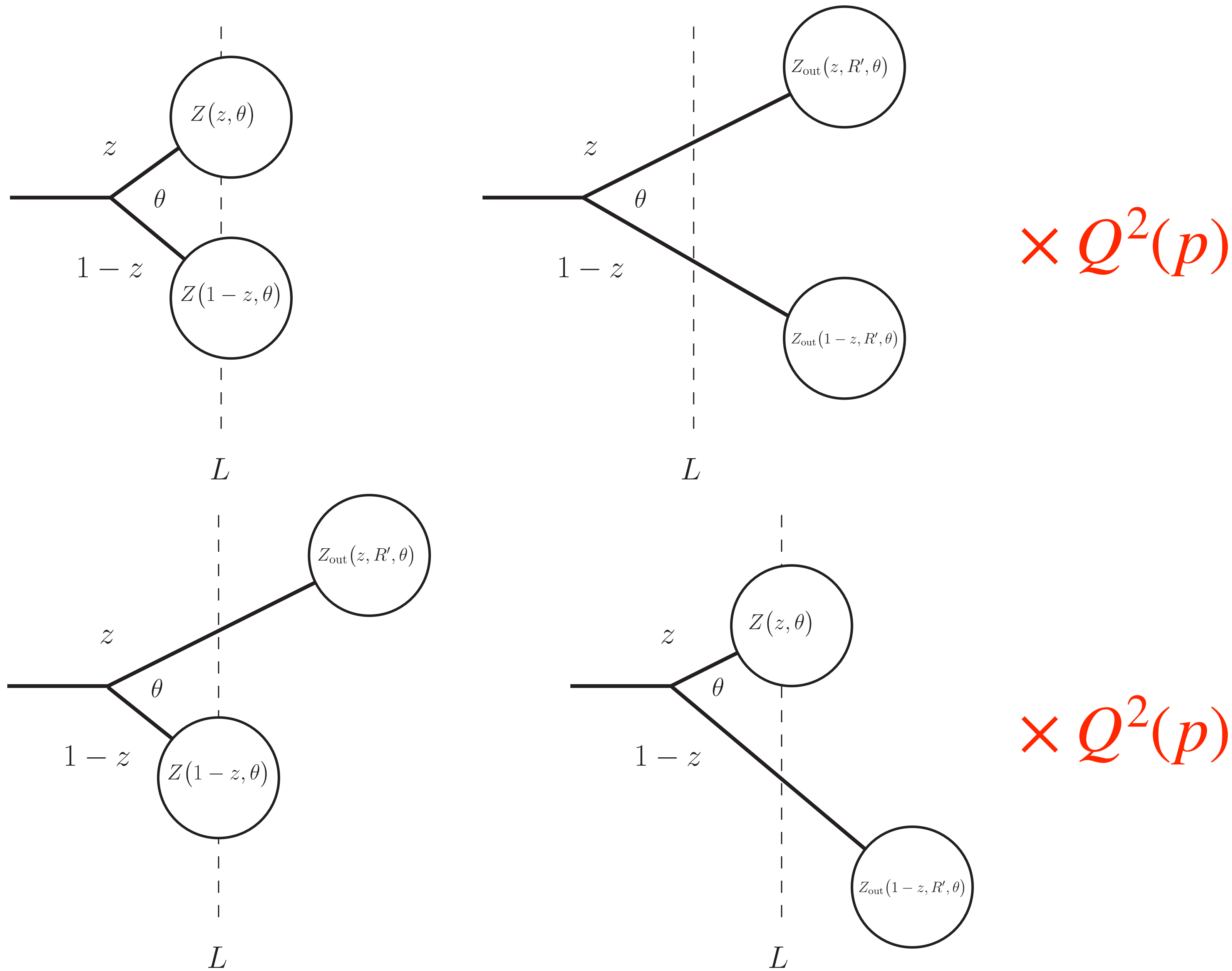
- Decoherent vacuum shower $Z_{decoh}(p, R', R)$




- First splitting outside the medium is decoherent initiating a subsequent coherent cascade

# Generating functional (in-medium)

- In-medium vacuum shower + energy loss




In the large-Nc limit: each in-medium splitting is reweighed by a quenching factor (in the fundamental representation) squared

# Generating functional (in-medium)

- Normalization of the GF:

$$Z(p, R; u = 1) \;\; = C(p, R) \;\; < 1$$

where the collimator function obeys the non-linear evolution equation:

$$C(p, R) = 1 + \bar{\alpha} \int_0^R \frac{\mathrm{d}\theta}{\theta} \int_0^1 \mathrm{d}z \, P(z) \; \Theta(t_f \; < \; t_d < \; L \,)$$

$$\times \left[ C(zp, \theta) \, C((1 - z)p, \theta) \, Q^2(p) - C(p, \theta) \right]$$

- It is related the the nuclear modification factor:

$$C(p, R) \equiv \frac{Q_{jet}(p)}{Q_{parton}(p)} \qquad \text{where} \qquad Q_{jet}(p) \equiv R_{AA}$$

# Sudakov suppression of inclusive jets

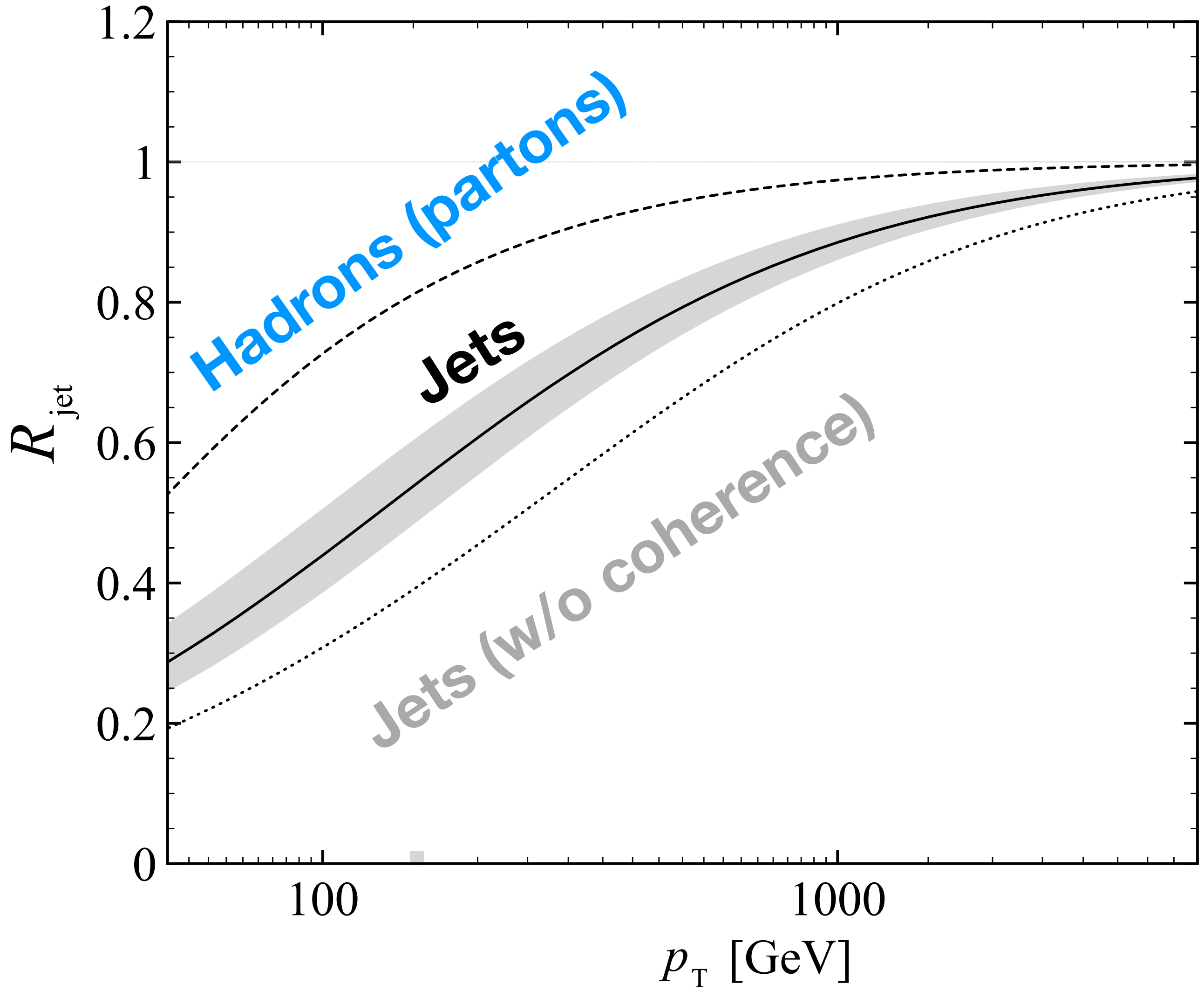
- In the strong quenching limit: only the leading particle survives albeit suppressed by a Sudakov form factor in addition to its total charge energy loss

$$C(p_T) = \exp\left[-2\bar{\alpha}\ln\frac{R}{\theta_c}\left(\ln\frac{p_T}{\omega_c}+\frac{2}{3}\ln\frac{R}{\theta_c}\right)\right]$$

- Increasing suppression with R (at large R energy must be recovered, not included here: $O(\alpha_s)$). Effect observed on groomed jets with JEWEL [Andrews et. al (2018)]

# Sudakov suppression of inclusive jets

MT, Tywoniuk PRD (2018)




$$R_{\text{jet}} = Q_{\text{tot}}(p_T) \times C(p_T)$$

$$R = 0.4$$

$$\hat{q} = 1\text{GeV}^2/\text{fm}$$

$$L = 3\text{fm}$$

Amplification of jet quenching due to increasing multiplicity at high pT qualitatively accounted for in Monte Carlo event generators: JEWEL, MARTINI, Hybrid Model, etc

Milhano, Zapp (2016) Casalderrey et al.(2017)

# Summary

- To leading logarithmic accuracy and large medium limit a complete leading order picture for jet evolution in QCD media is constructed: including color (de)coherence and multiple gluon radiation
- Analytic calculations can be performed in the limit of large power index energy loss of multiple cartons factorizes into the product of quenching weights
- Rare in cone medium induced radiation can be computed order by order in the coupling constant
- Applications: jet substructure observables mass, broadening, N-subjetiness, etc, and compare to Monte Carlo event generators