

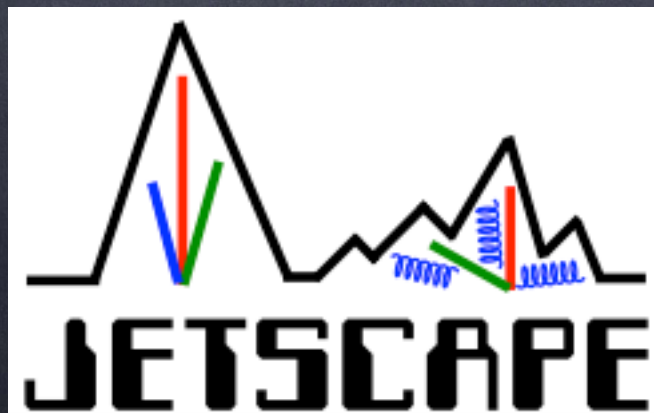


# Parton Energy Loss in Generalized High-twist Approach

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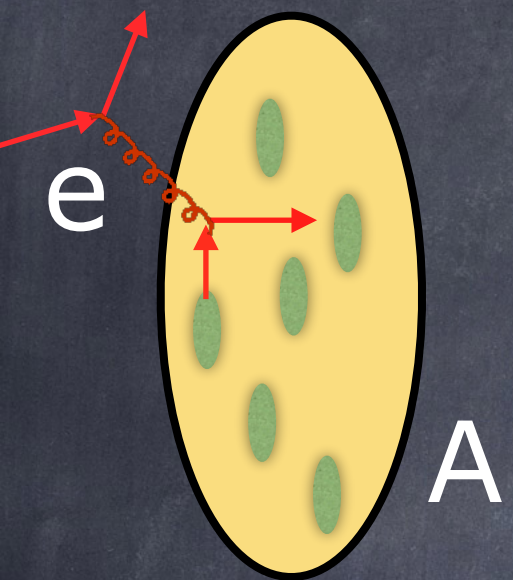
*JETSCAPE Workshop 2019*  
*Texas A&M University*



- Introduction
- Generalized High Twist approach
- Results and approximations
- Summary

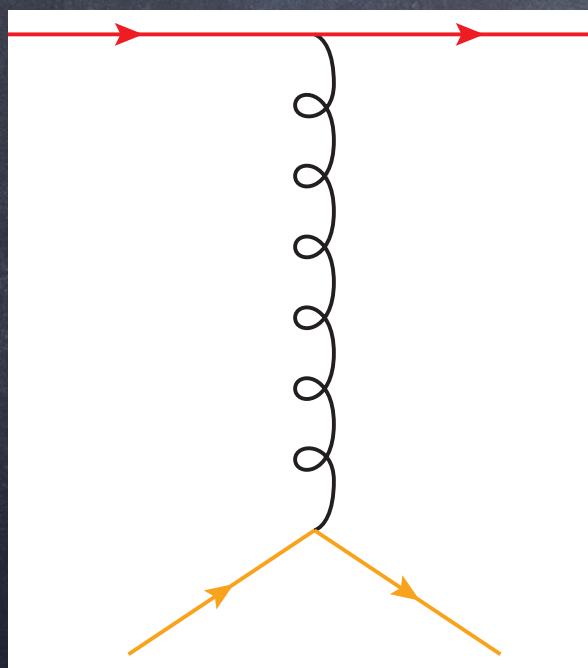


# Parton energy loss in medium

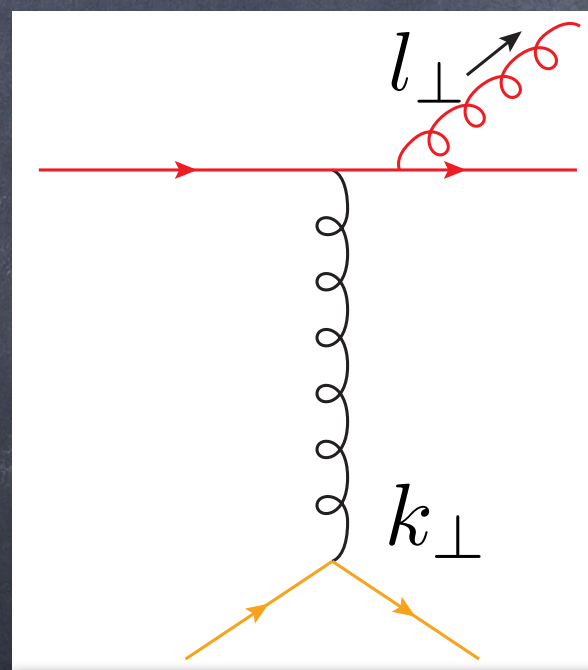


- High energy partons travel through hot QGP or cold nuclei (eA DIS process)
- Energy loss mechanisms reveal medium properties

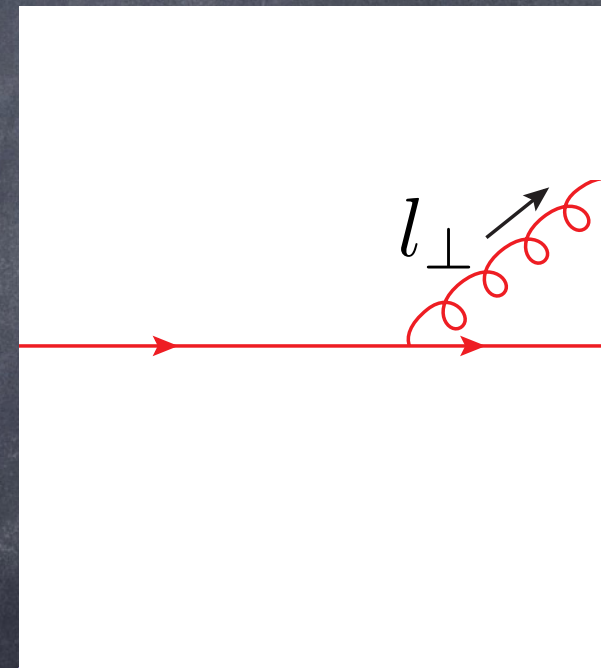
## Parton energy loss mechanisms



collisional energy loss



medium induced



vacuum

radiative energy loss



# Radiative energy loss: assumptions

Approaches to radiative energy loss : BDMPS-Z, GLV, AMY, SCET, High Twist

## Scattering Center : Static or Dynamic?

Static: no energy transfer  
(BDMPS-Z, GLV)

Extension of GLV to dynamic S.C.  
Djordjevic, Heinz PRL 101,022302

Dynamic: both momentum and energy transfer



## Radiated Gluon : Soft or hard ?

$z \rightarrow 0$  (BDMPS-Z, GLV)

$z$  finite



Beyond soft appr. of GLV

Blagojevic *et al.* arXiv:1804.07593

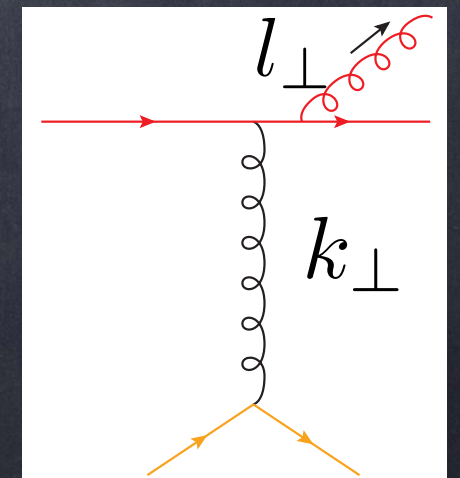
Beyond soft appr. of SCET

Ovanesyan, Vitev JHEP 2011(6), 80

## Transverse momentum transfer $k_{\perp}$ : smaller or same order $l_{\perp}$ ?

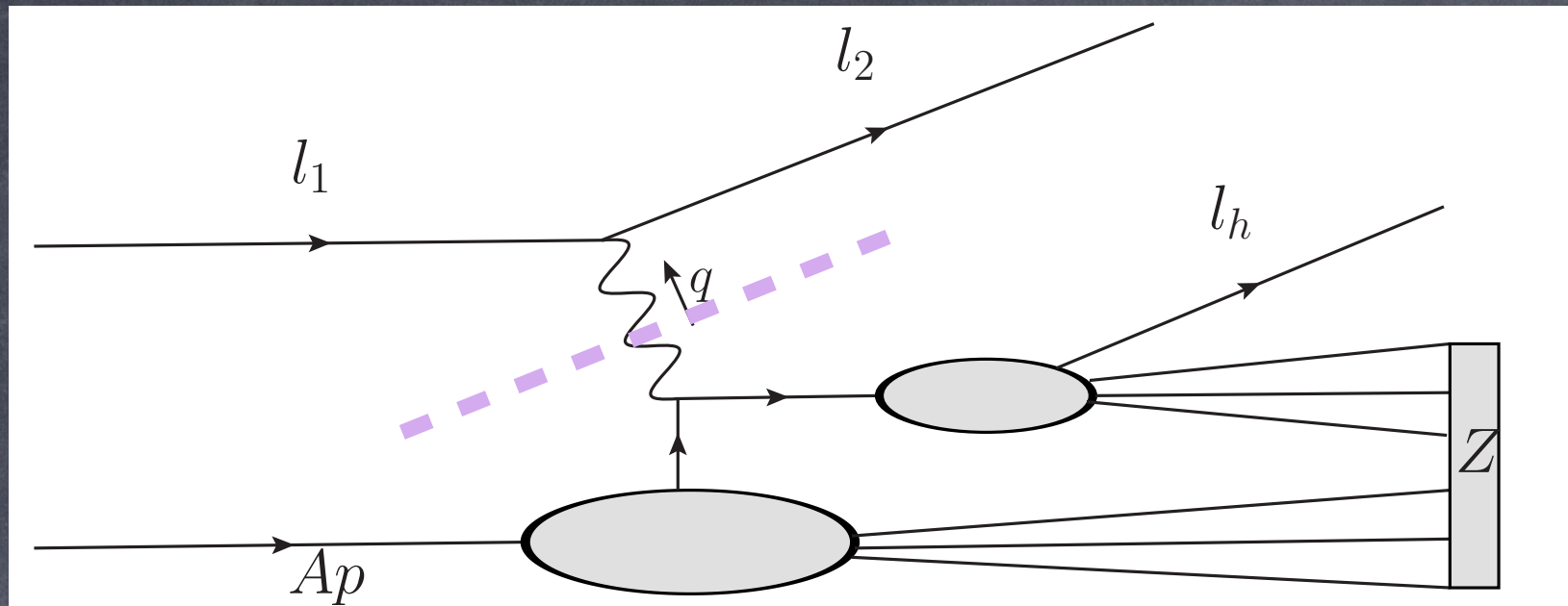
$k_{\perp} \ll l_{\perp}$  (High Twist)

$k_{\perp} \sim l_{\perp}$





# Semi-inclusive Deeply Inelastic Scattering



- lepton-nucleus scattering

$$e(l_1) + A(Ap) \rightarrow e(l_2) + h(l_h) + Z$$

identify one hadron in final state

- cross section  $d\sigma$  and hadronic tensor  $W^{\mu\nu}$

$$d\sigma = \frac{e^4}{2s} \frac{\sum_q e_q^2}{q^4} \int \frac{d^4 l_2}{(2\pi)^4} 2\pi \delta(l_2^2) \frac{1}{2} L_{\mu\nu} W^{\mu\nu}$$



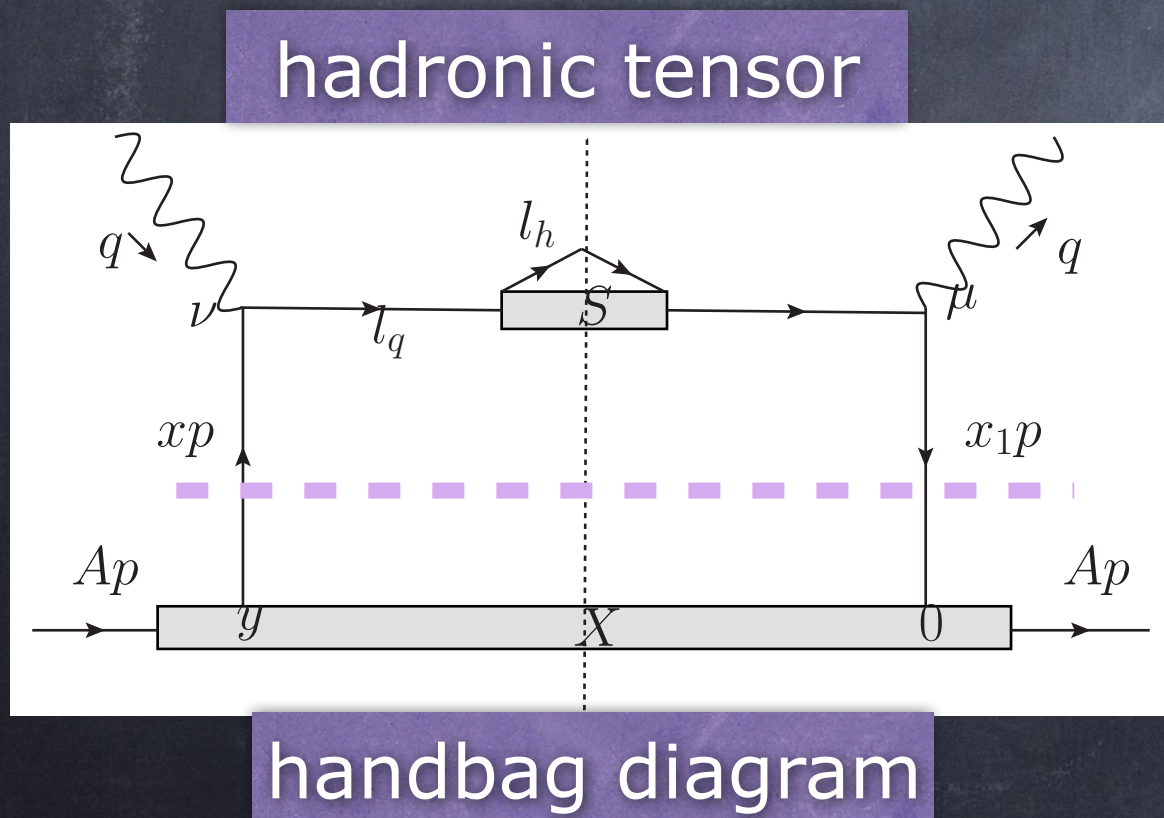
# Collinear Factorization for SIDIS

# Factorization

Separate non-perturbative part (pdf, fragmentation function)  
from perturbative part (hard scattering)

# SIDIS process

# Collinear factorization when final hadron $l_{h\perp}$ integrated



$$\frac{dW_{S(0)}^{\mu\nu}}{dz_h} = \int dx f_q^A(x) H_{(0)}^{\mu\nu} D_{q \rightarrow h}(z_h)$$

$f_q^A(x)$  nuclear quark distribution function

$D_{q \rightarrow h}(z_h)$  quark fragmentation function

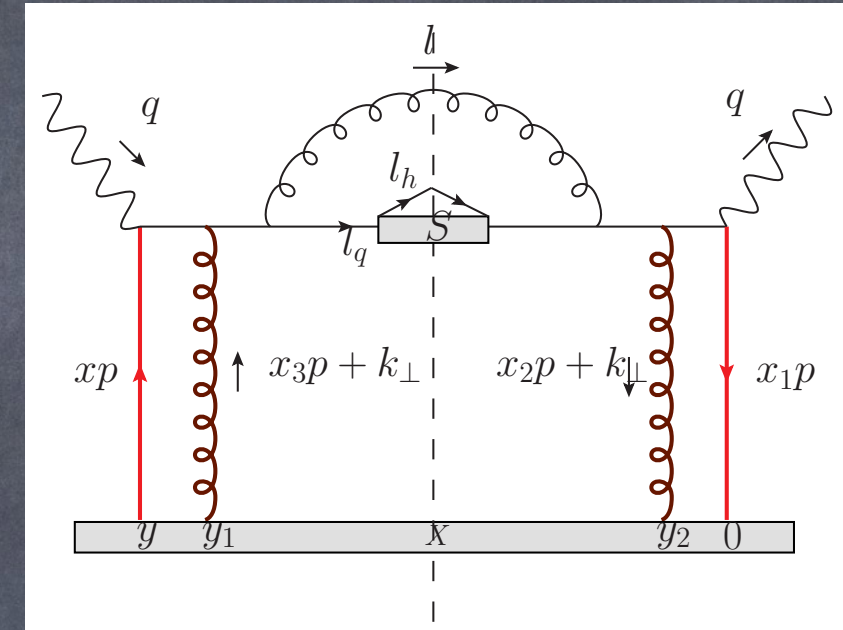


# Factorization of Medium Induced Radiation

## High Twist approach

### Collinear factorization

*XF Guo, XN Wang (2000) PRL 85(17), 3591*  
*XN Wang and XF Guo (2001) Nucl. Phys. A 696, 788*



$$\frac{dW_{D(1)q}^{\mu\nu}}{dz_h} = \int_{z_h}^1 \frac{dz}{z} D_{q \rightarrow h}(z_h/z) \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{d^2 y_{1\perp}}{(2\pi)^2} d^2 y_{2\perp} d^2 k_{\perp} e^{-i\vec{k}_{\perp} \cdot (\vec{y}_{1\perp} - \vec{y}_{2\perp})}$$

$$\frac{1}{2} < A | \bar{\psi}_q(0) \gamma^+ A^+(y_2^-, \vec{y}_{2\perp}) A^+(y_1^-, \vec{y}_{1\perp}) \psi_q(y^-) | A > H_D^{\mu\nu}(k_{\perp}, y^-, y_1^-, y_2^-, p, q, z)$$

### Collinear expansion of hard part

$$H_D^{\mu\nu}(k_{\perp}, y^-, y_1^-, y_2^-, p, q, z) = H_D^{\mu\nu}(k_{\perp} = 0) + \left. \frac{\partial H_D^{\mu\nu}}{\partial k_{\perp}^{\alpha}} \right|_{k_{\perp}=0} k_{\perp}^{\alpha}$$

$k_{\perp} \ll l_{\perp}$  approximation

$$+ \frac{1}{2} \left. \frac{\partial^2 H_D^{\mu\nu}}{\partial k_{\perp}^{\alpha} \partial k_{\perp}^{\beta}} \right|_{k_{\perp}=0} k_{\perp}^{\alpha} k_{\perp}^{\beta} + \dots$$

$k_{\perp} = 0$  contribute to gauge link of initial nuclear quark PDF  
 $k_{\perp}^{\alpha}$  contribute zero for unpolarized beam



# Factorization of Medium Induced Radiation

## Generalized High Twist approach

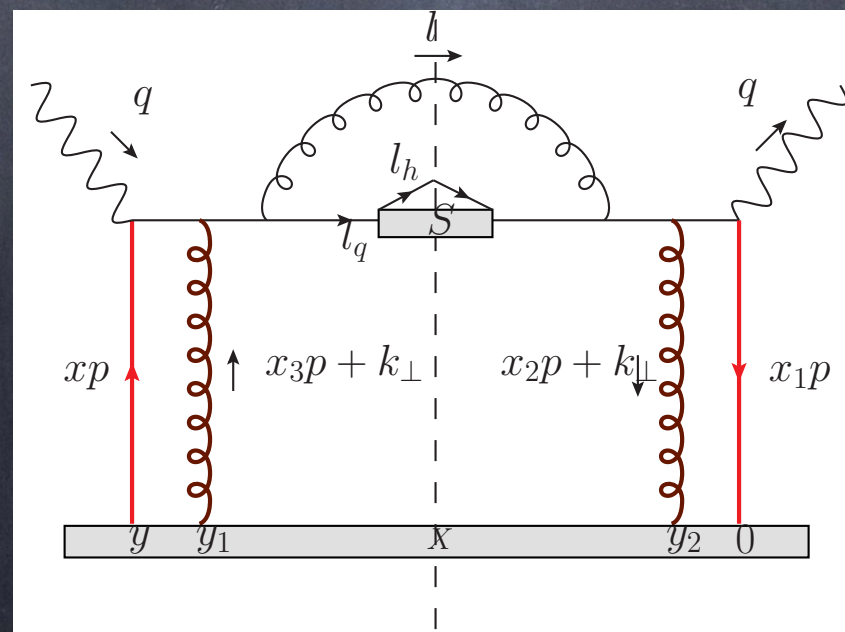
relax  $k_{\perp} \ll l_{\perp}$  without collinear expansion

factorize quark PDF and gluon PDF directly

YY Z, GY Qin, XN Wang  
(paper in preparation)

$$\frac{dW_{D(1)q}^{\mu\nu}}{dz_h} \sim \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \frac{1+z^2}{1-z} \overbrace{H_{(0)}^{\mu\nu}(x)}^{\text{splitting function}} \otimes \overbrace{\rho(y_1^-, \vec{y}_{1\perp})}^{\text{nucleon density}} \otimes \frac{D_{q \rightarrow h}(z_h/z)}{z}$$

$$\otimes \overbrace{f_q^A(x)}^{\text{quark pdf}} \frac{\pi}{[\vec{l}_{\perp} - (1-z)\vec{k}_{\perp}]^2} \frac{\phi(x_L + x_D, \vec{k}_{\perp})}{k_{\perp}^2} \overbrace{\text{TMD gluon pdf}}$$



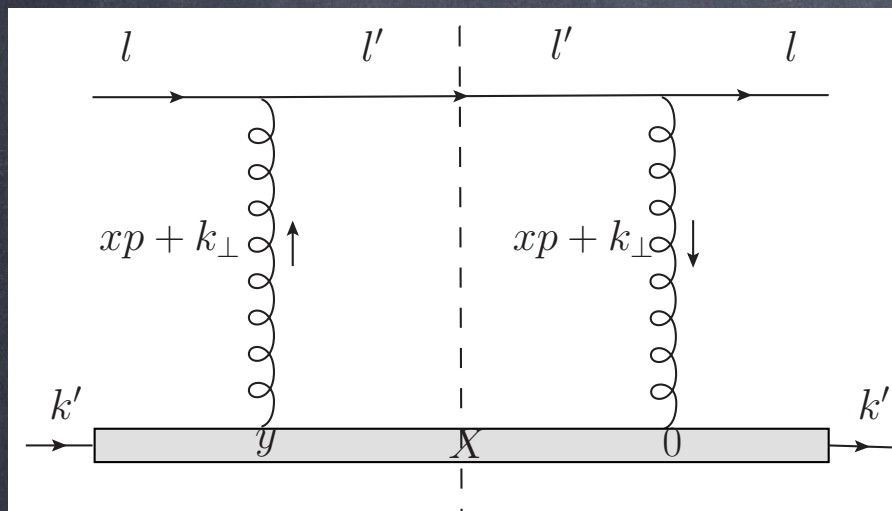


# TMD gluon distribution function

- TMD gluon distribution function emerge from TMD transport parameter  $\hat{q}(\vec{k}_\perp)$

$$\hat{q} = \rho \int dk_\perp^2 \frac{\langle d\sigma \rangle}{dk_\perp^2} k_\perp^2$$

color source density \* average kT broadening per scattering



$$\begin{aligned} \hat{q} &\equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \int dx \delta\left(x - \frac{k_\perp^2}{2p^+ l^-}\right) \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1} \rho(y) \phi(x, \vec{k}_\perp) \\ &\equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \hat{q}(\vec{k}_\perp) \end{aligned}$$

$\phi(x, \vec{k}_\perp)$  emerge naturally

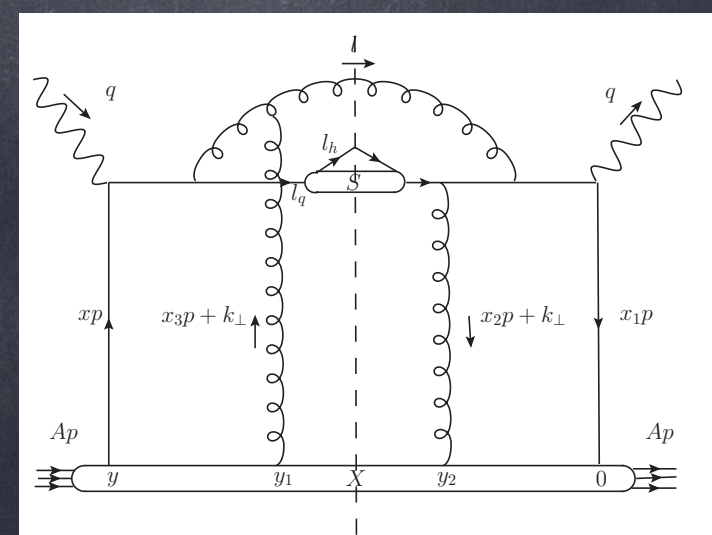
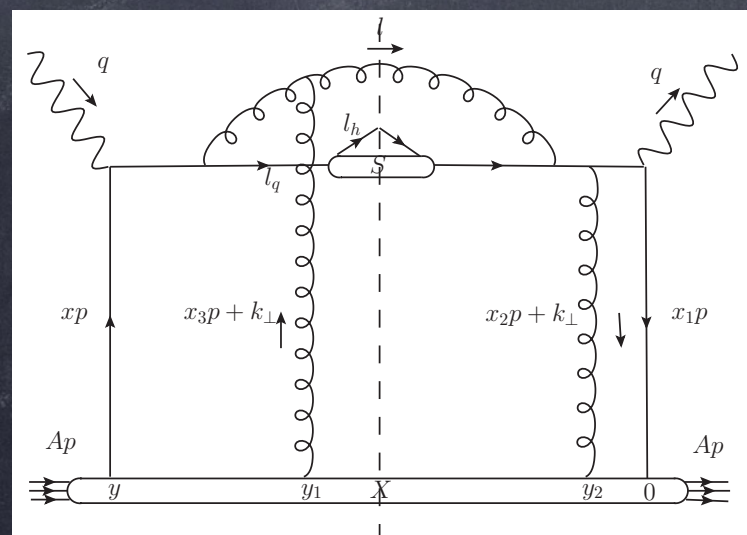
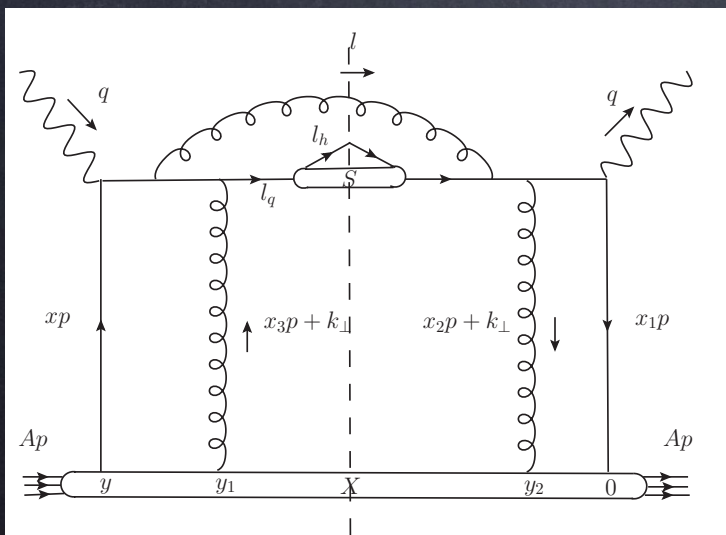
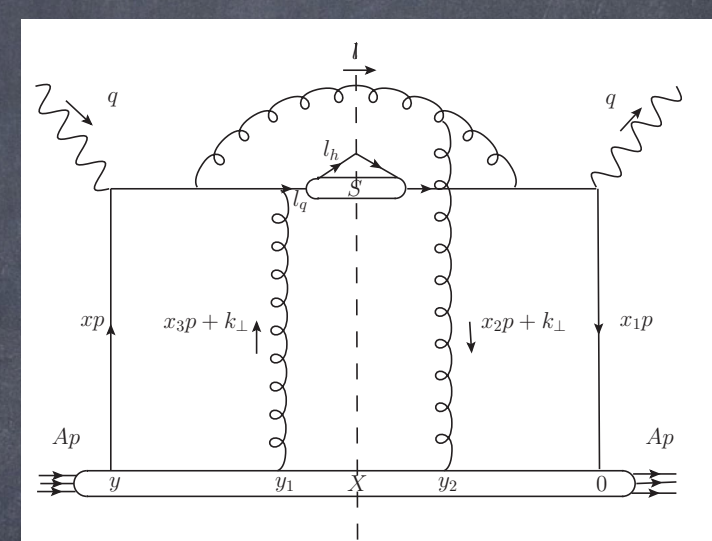
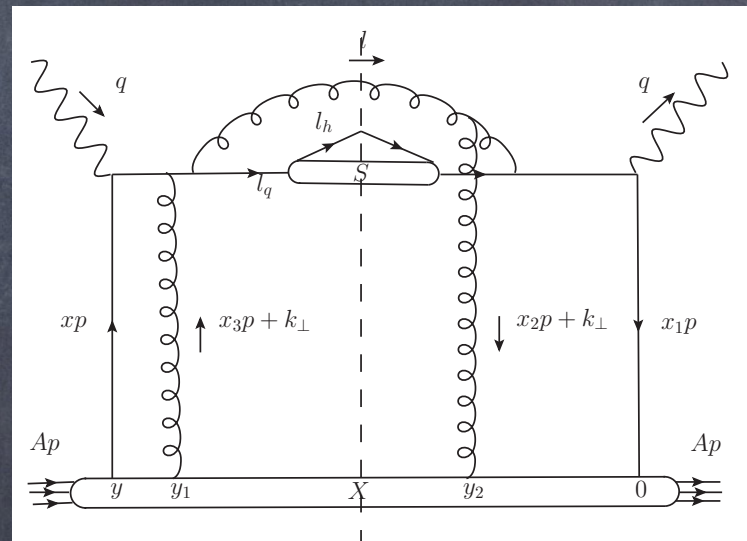
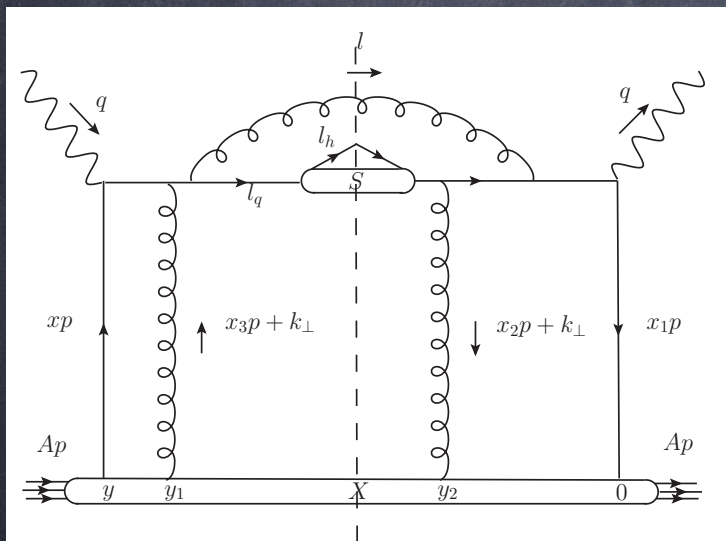
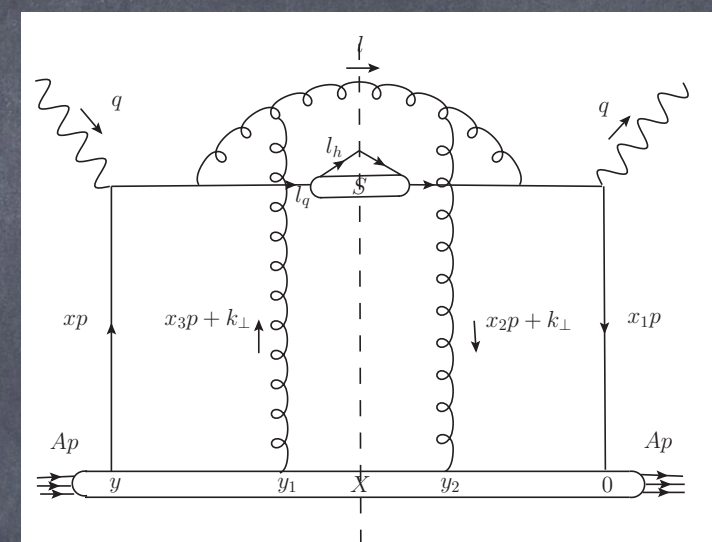
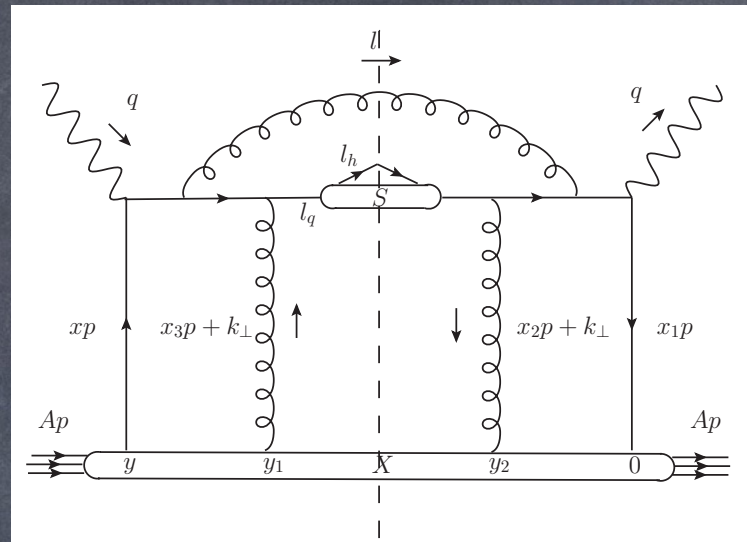
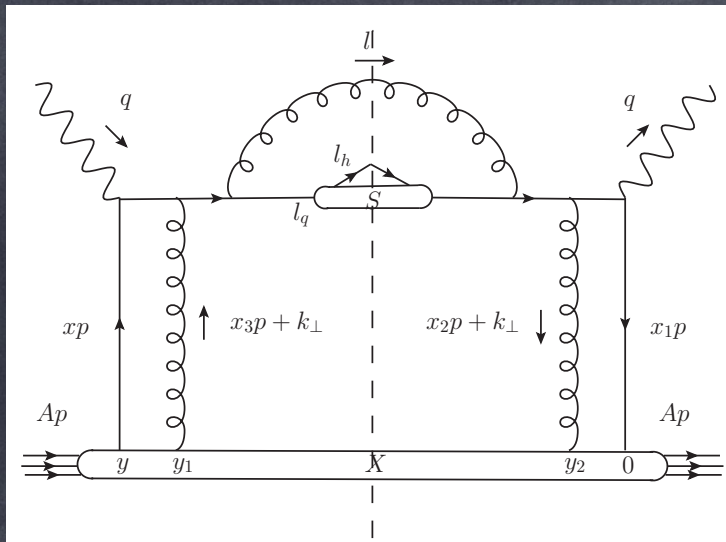
$$\phi(x, \vec{k}_\perp) \equiv \int \frac{dy^-}{2\pi p^+} \int d^2 y_\perp e^{ixp^+ y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle p | F_\alpha^{+}(0) F^{+\alpha}(y^-, y_\perp) | p \rangle$$

$\hat{q}(\vec{k}_\perp)$   $\phi(x, \vec{k}_\perp)$  depend on the parton energy  $l^-$  and medium color source energy  $p^+$  ( $=\langle k' \rangle$ ) via  $x$  energy transfer via  $x$



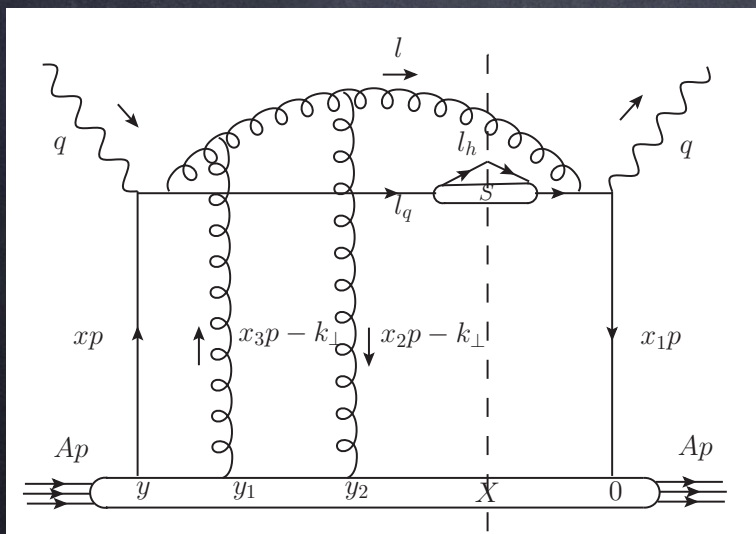
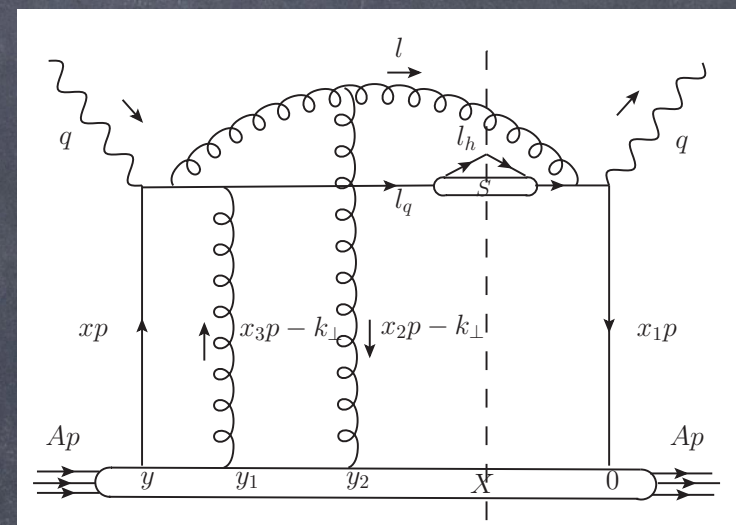
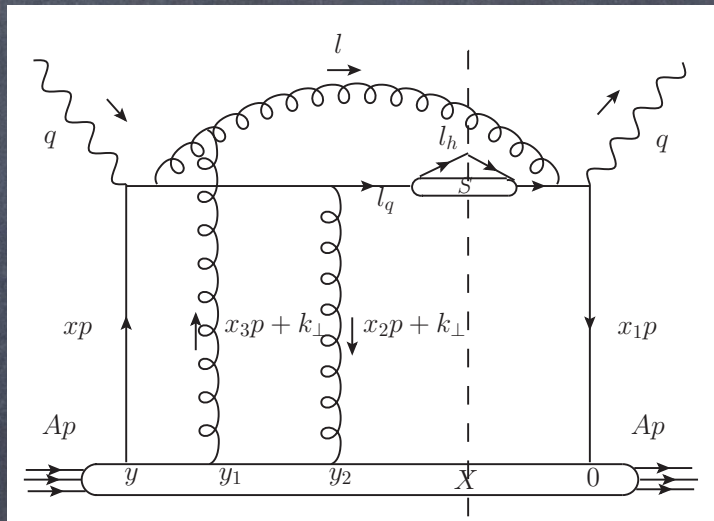
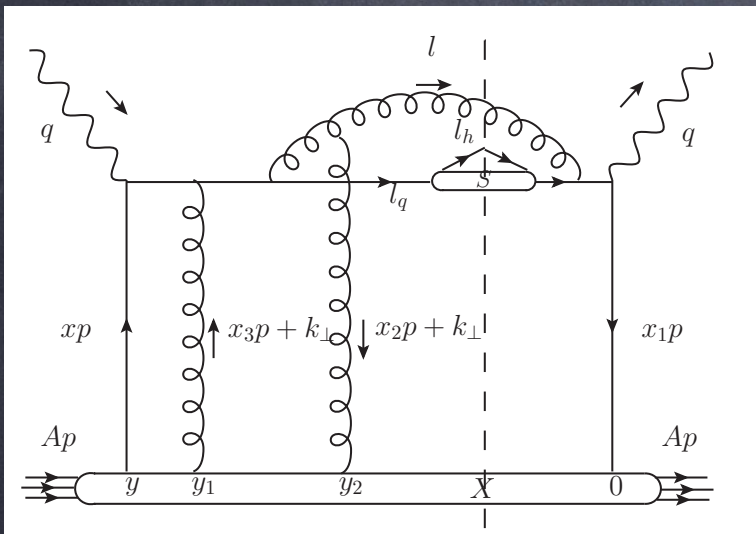
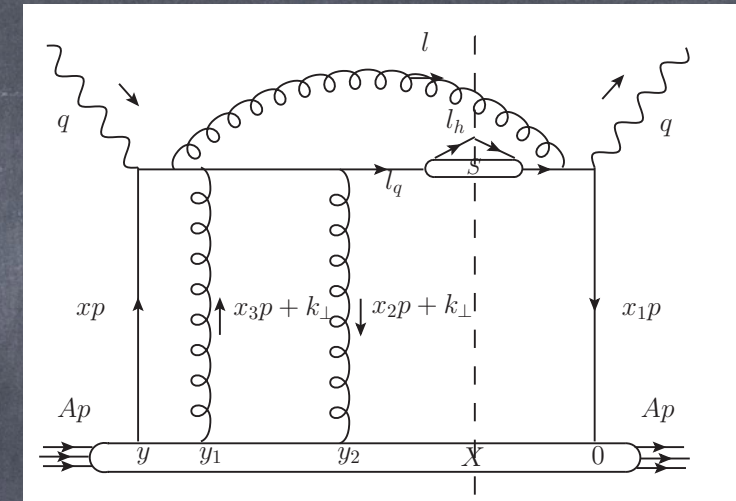
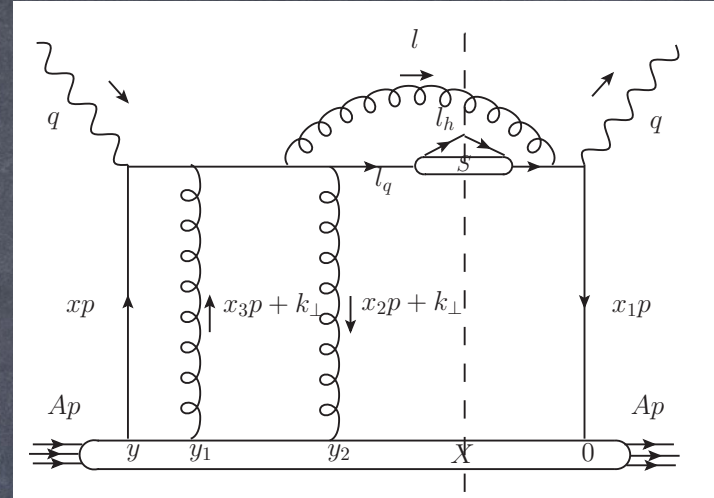
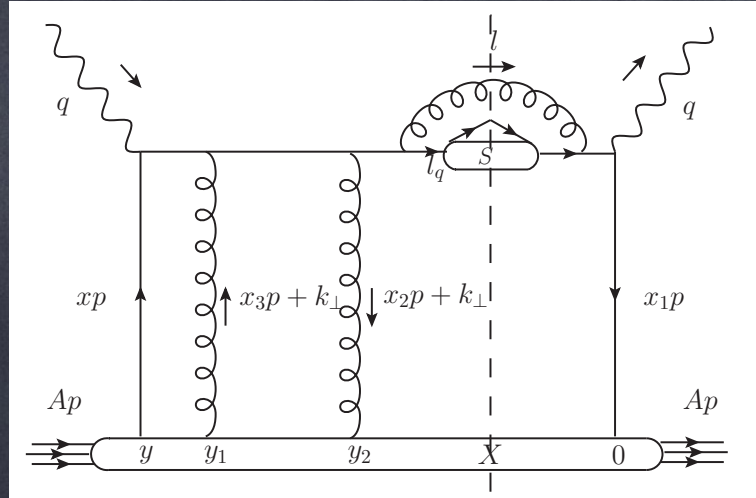
# Gluon Spectrum Result

# summing up all the diagrams





# Gluon Spectrum Result



- vacuum + medium induced radiation interference
- left cut diagrams symmetric to right cut diagrams



# Gluon Spectrum Result: Full Result

We get radiated gluon spectrum from the hadronic tensor

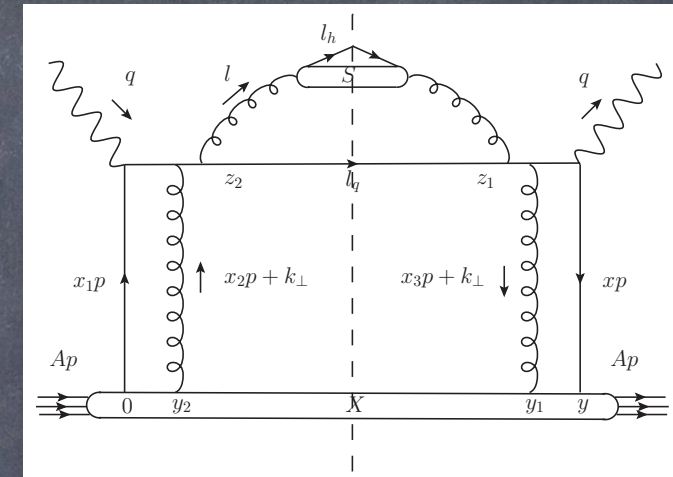
$$\frac{dW_{D(1)g}^{\mu\nu}}{dz_h} = \int dx H_{(0)}^{\mu\nu}(x) \int \frac{dz}{z} D_{g \rightarrow h}(z_h/z) \int dl_{\perp}^2 T_{qg}(x, z, l_{\perp}^2)$$

Quark-gluon correlation function  $T_{qg}(x, z, l_{\perp}^2)$

nuclear enhancement

$$T_{qg} = \pi \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_{\perp}}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp})$$

$$[H_C^D + \frac{1}{2}H_L^D + \frac{1}{2}H_R^D]$$



Gluon spectrum : probability of quark to radiate one gluon with longitudinal momentum  $zq^-$  transverse momentum  $\vec{l}_{\perp}$

$$\frac{dN}{dl_{\perp}^2 dz} \sim \frac{T_{qg}}{f_q^A(x)}$$



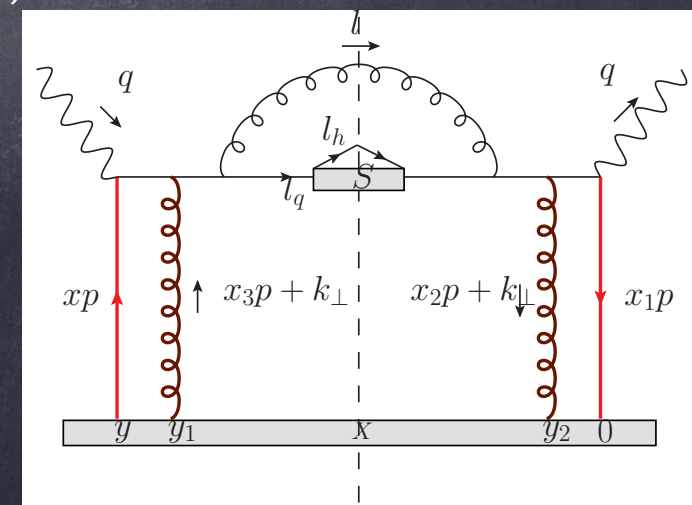
# Gluon Spectrum Result: Full Result

$$\begin{aligned}
 H_C^D = & \left\{ \left[ \frac{C_A}{(l_\perp - k_\perp)^2} \frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L + \frac{x_T}{z}) - \frac{C_A}{l_\perp^2} \frac{\phi(\frac{1-z}{z}x_T, -\vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L) \right] \right. \\
 & + \left[ \left( \frac{C_F}{l_\perp^2} + C_A \frac{\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_F}{[\vec{l}_\perp - z\vec{k}_\perp]^2} + \frac{1}{N} \frac{\vec{l}_\perp \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{l_\perp^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right. \right. \\
 & \left. \left. - C_A \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \frac{\phi(x_L + x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x) \right] \\
 & \left[ \left( -\frac{C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \right. \\
 & \left. \frac{\phi(x_L + x_T, \vec{k}_\perp)}{k_\perp^2} e^{-i(x_L + \frac{x_T}{z})p^+ y_1^-} f_q^A(x + x_L + \frac{x_T}{z}) \right] \\
 & \left[ \left( -\frac{C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \right. \\
 & \left. \frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} e^{i(x_L + \frac{x_T}{z})p^+ y_1^-} f_q^A(x) \right] \left. \right\}
 \end{aligned}$$

**x : energy transfer**

$$H_L^D = \dots$$

$$H_R^D = \dots$$





# Gluon Spectrum Result: Contact Terms

There are contact terms, which are negligible. One kind of them

$$\frac{dN_{\text{contact}}}{dl_{\perp}^2 dz} = \frac{\pi}{f_q^A(x)} \frac{\alpha_s}{2\pi} \frac{1 + (1 - z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_{\perp}}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \\ H_{\text{contact}} [\theta(y^- - y_1^-) \theta(-y_2^-) - \theta(y_1^- - y_2^-) \theta(y^- - y_1^-) - \theta(y_2^- - y_1^-) \theta(-y_2^-)]$$

The  $\theta$  functions constrain the integration region

$$\int dy_1^- dy_2^- [\theta(y^- - y_1^-) \theta(-y_2^-) - \theta(y_1^- - y_2^-) \theta(y^- - y_1^-) - \theta(y_2^- - y_1^-) \theta(-y_2^-)] \\ = - \int_0^{y^-} dy_1^- \int_0^{y_1^-} dy_2^-$$

gives  $0 < y_2^- < y_1^- < y^-$ , quark and gluon comes from same nucleon, no nuclear enhancement.

The other kind comes from right cut diagram, same integration region  $0 < y_2^- < y_1^- < y^-$ .



# Gluon Spectrum Approximations

## 1. Static scattering center approximation:

no energy transfer for medium scattering

no  $x$  dependence in TMD gluon distribution function

$$\left\{ \begin{array}{l} \phi(x_L + \frac{1-z}{z}x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \dots \approx \phi(0, \vec{k}_\perp) \\ f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B) \end{array} \right.$$

equivalent to  $Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)}$  or  $x_B \gg x_L, \frac{x_T}{z}$

$$\begin{aligned} \frac{dN_{static}}{dl_\perp^2 dz} = & \pi \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \left[ \frac{C_A}{(l_\perp - k_\perp)^2} - \frac{C_A}{l_\perp^2} + \frac{C_F}{l_\perp^2} \right. \\ & + C_A \frac{\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_F}{[\vec{l}_\perp - z\vec{k}_\perp]^2} + \frac{1}{N} \frac{\vec{l}_\perp \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{l_\perp^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} - C_A \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \\ & + \left( -\frac{2C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + C_A \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + C_A \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \right) \cos[(x_L + \frac{x_T}{z})p^+ y_1^-] \\ & + \left( -2C_F \frac{1}{l_\perp^2} - \frac{1}{N} \frac{\vec{l}_\perp \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{l_\perp^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} + \frac{C_A}{l_\perp^2} \right) \cos[x_L p^+ y_1^-] \Big] \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2} \end{aligned}$$



# Gluon Spectrum Approximations

## 2. Soft radiated gluon approximation:

the radiated gluon momentum fraction  $z \rightarrow 0$

$$\frac{dN_{\text{soft}}}{dl_{\perp}^2 dz} = \frac{\pi}{f_q^A(x)} \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_{\perp}}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp})$$

$$\left[ (H_C^D)_{\text{soft}} + \frac{1}{2} (H_L^D)_{\text{soft}} + \frac{1}{2} (H_R^D)_{\text{soft}} \right]$$

$$(H_C^D)_{\text{soft}} = \left\{ \left[ \frac{C_A}{(l_{\perp} - k_{\perp})^2} \frac{\phi(\frac{1-z}{z} x_T, \vec{k}_{\perp})}{k_{\perp}^2} \left( f_q^A(x + x_L + \frac{x_T}{z}) - f_q^A(x + x_L) \right) \right] \right.$$

$$+ \left[ C_A \frac{k_{\perp}^2}{l_{\perp}^2 (\vec{l}_{\perp} - \vec{k}_{\perp})^2} \frac{\phi(x_L + x_T, \vec{k}_{\perp})}{k_{\perp}^2} f_q^A(x) \right]$$

$$+ \left[ -C_A \frac{\vec{k}_{\perp} \cdot \vec{l}_{\perp}}{l_{\perp}^2 (\vec{l}_{\perp} - \vec{k}_{\perp})^2} \frac{\phi(x_L + x_T, \vec{k}_{\perp})}{k_{\perp}^2} e^{I(x_L + \frac{x_T}{z}) p^+ y_1^-} f_q^A(x + x_L + \frac{x_T}{z}) \right]$$

$$\left. + \left[ -C_A \frac{\vec{k}_{\perp} \cdot \vec{l}_{\perp}}{l_{\perp}^2 (\vec{l}_{\perp} - \vec{k}_{\perp})^2} \frac{\phi(\frac{1-z}{z} x_T, \vec{k}_{\perp})}{k_{\perp}^2} e^{-I(x_L + \frac{x_T}{z}) p^+ y_1^-} f_q^A(x) \right] \right\}$$

$$(H_L^D)_{\text{soft}} = \cdots \quad (\tilde{H}_L^D)_{\text{soft}} = \cdots$$

no dependence on  $z$  except splitting function,  $f_q^A$  and  $\phi$



# Glueon Spectrum Approximations

## 3. Static scattering + Soft radiated gluon approximation:

$$\left\{ \begin{array}{l} \phi(x_L + \frac{1-z}{z}x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \dots \approx \phi(0, \vec{k}_\perp) \\ f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B) \end{array} \right. \quad \text{and } z \rightarrow 0$$

equivalent to  $Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)}$ , or  $x_B \gg x_L, \frac{x_T}{z}$  as  $z \rightarrow 0$

the gluon spectrum is

$$\frac{dN_{static+soft}}{dl_\perp^2 dz} = \pi \frac{\alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \int d^2 \vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp})$$

$$C_A \frac{2\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \left( 1 - \cos[(x_L + \frac{x_T}{z})p^+ y_1^-] \right) \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2}$$

$\phi(x, \vec{k}_\perp)$

non-perturbative



# Gluon Spectrum Approximations

## 3. Static scattering + Soft radiated gluon approximation:

TMD gluon pdf relation to transport parameter  $\hat{q}$

$$\hat{q} = \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1} \rho(y) \int \frac{d^2 k_\perp}{(2\pi)^2} \phi(0, \vec{k}_\perp) \quad \star$$

One method : Static potential model to calculate  $\hat{q}$

$$\langle d\sigma \rangle = \frac{C_2(R)C_2(T)}{d_A} \frac{4\pi\alpha_s^2}{t^2} dt$$

Casimir  $C_2(R)$   $C_2(T)$

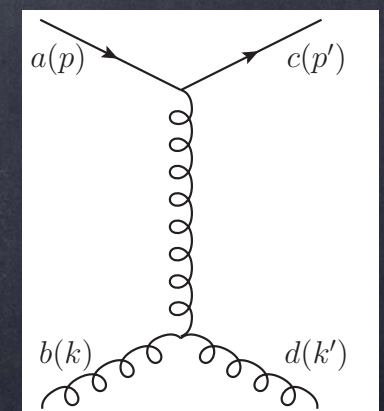
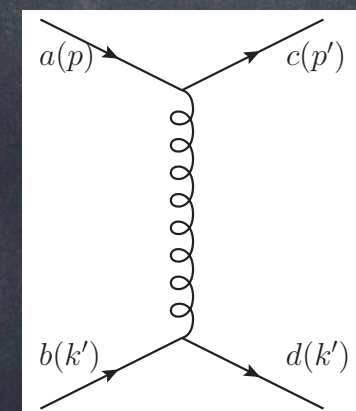
Mandelstam variable  $t$

$$\hat{q} = \rho \int dk_\perp^2 \frac{C_2(R)C_2(T)}{d_A} \frac{4\pi\alpha_s^2}{k_\perp^2 + \mu_0^2} \quad \star$$

$$t = k_\perp^2 + \mu_0^2$$

compare two  $\hat{q}$

$$\phi(0, \vec{k}_\perp) = C_2(T) \frac{4\alpha_s}{k_\perp^2 + \mu_0^2}$$





# Gluon Spectrum Approximations

## 3. Static scattering + Soft radiated gluon approximation:

substitute  $\phi(0, \vec{k}_\perp)$  into gluon spectrum, one get

$$\frac{dN_{static+soft}}{dl_\perp^2 dz} = 8\pi\alpha_s^3 \frac{C_2(T)C_A}{N_c} \frac{1 + (1-z)^2}{z} \int \frac{d^2 k_\perp}{(2\pi)^2} \int dy_1^- \rho(y_1^-) \frac{\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \left( 1 - \cos\left[\left(x_L + \frac{x_T}{z}\right)p^+ y_1^-\right] \right) \frac{1}{(k_\perp^2 + \mu_0^2)^2}$$

agrees with of GLV formalism at first order in opacity

arguments in Cosine function  $\left\{ \begin{array}{l} \omega_1 \approx \sqrt{2}\left(x_L + \frac{1}{z}x_T\right)p^+ \\ y_{10} = y_1 - y_0 \approx \frac{y_1^-}{\sqrt{2}} \end{array} \right.$



# Gluon Spectrum Approximations

## Numerical Results

Plot the integration kernel of gluon spectrum, for GLV (or GHT static+soft) and GHT static

$$\frac{dN_{static+soft}}{dl_{\perp}^2 dz} = \pi \frac{\alpha_s}{2\pi} \frac{1 + (1 - z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int dy_1^- \int d^2\vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \int \frac{dk_{\perp}^2}{2(2\pi)^2} \frac{C_A}{l_{\perp}^2} \int d\theta \frac{2\vec{k}_{\perp} \cdot \vec{l}_{\perp}}{(\vec{l}_{\perp} - \vec{k}_{\perp})^2} \left(1 - \cos\left[\left(x_L + \frac{x_T}{z}\right)p^+ y_1^-\right]\right) \frac{\phi(0, \vec{k}_{\perp})}{k_{\perp}^2}$$

$$\begin{aligned} \frac{dN_{static}}{dl_{\perp}^2 dz} = & \pi \frac{\alpha_s}{2\pi} \frac{1 + (1 - z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int dy_1^- \int d^2\vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \int \frac{dk_{\perp}^2}{2(2\pi)^2} \frac{C_A}{l_{\perp}^2} \int d\theta \frac{l_{\perp}^2}{C_A} \left[ \frac{C_A}{(l_{\perp} - k_{\perp})^2} - \frac{C_A}{l_{\perp}^2} \right. \\ & + \frac{C_F}{l_{\perp}^2} + C_A \frac{\vec{k}_{\perp} \cdot \vec{l}_{\perp}}{l_{\perp}^2 (\vec{l}_{\perp} - \vec{k}_{\perp})^2} + \frac{C_F}{[\vec{l}_{\perp} - z\vec{k}_{\perp}]^2} + \frac{1}{N} \frac{\vec{l}_{\perp} \cdot [\vec{l}_{\perp} - z\vec{k}_{\perp}]}{l_{\perp}^2 [\vec{l}_{\perp} - z\vec{k}_{\perp}]^2} - C_A \frac{(\vec{l}_{\perp} - \vec{k}_{\perp}) \cdot [\vec{l}_{\perp} - z\vec{k}_{\perp}]}{(l_{\perp} - k_{\perp})^2 [\vec{l}_{\perp} - z\vec{k}_{\perp}]^2} \\ & + \left( -\frac{2C_A}{(\vec{l}_{\perp} - \vec{k}_{\perp})^2} + C_A \frac{\vec{l}_{\perp} \cdot (\vec{l}_{\perp} - \vec{k}_{\perp})}{l_{\perp}^2 (\vec{l}_{\perp} - \vec{k}_{\perp})^2} + C_A \frac{(\vec{l}_{\perp} - \vec{k}_{\perp}) \cdot [\vec{l}_{\perp} - z\vec{k}_{\perp}]}{(l_{\perp} - k_{\perp})^2 [\vec{l}_{\perp} - z\vec{k}_{\perp}]^2} \right) \cos\left[\left(x_L + \frac{x_T}{z}\right)p^+ y_1^-\right] \\ & \left. + \left( -2C_F \frac{1}{l_{\perp}^2} - \frac{1}{N} \frac{\vec{l}_{\perp} \cdot [\vec{l}_{\perp} - z\vec{k}_{\perp}]}{l_{\perp}^2 [\vec{l}_{\perp} - z\vec{k}_{\perp}]^2} + \frac{C_A}{l_{\perp}^2} \right) \cos[x_L p^+ y_1^-] \right] \frac{\phi(0, \vec{k}_{\perp})}{k_{\perp}^2} \end{aligned}$$

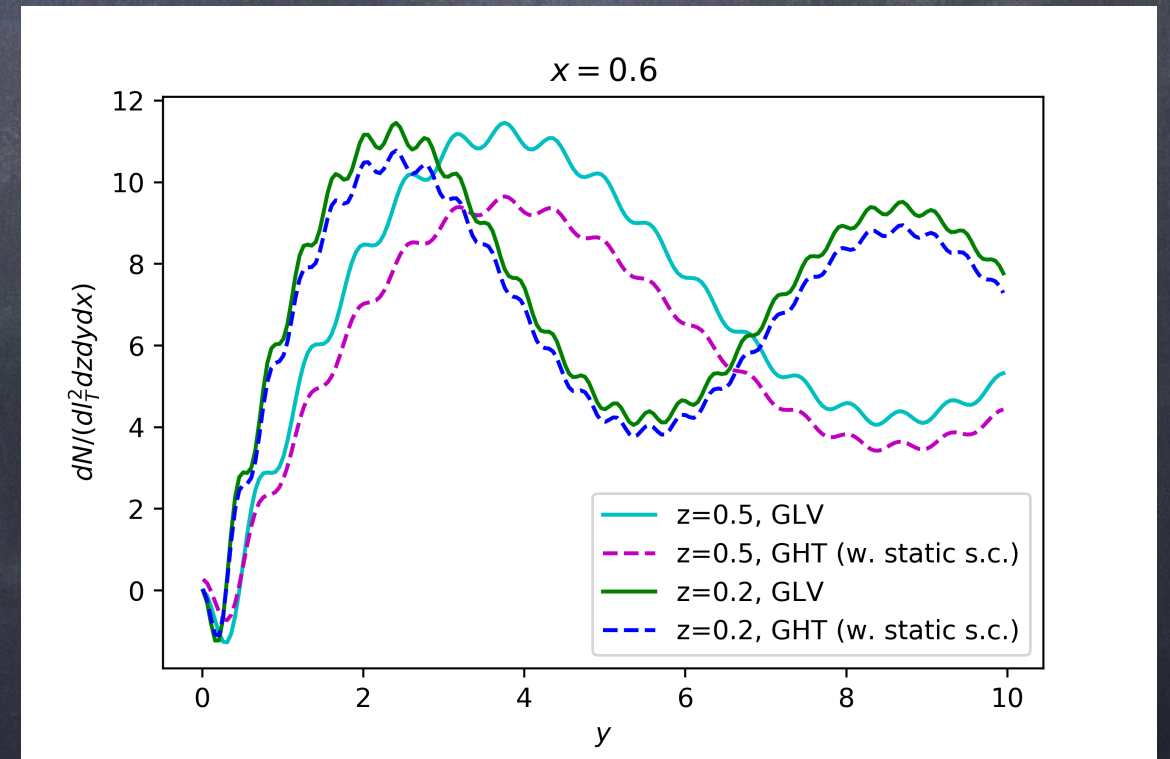
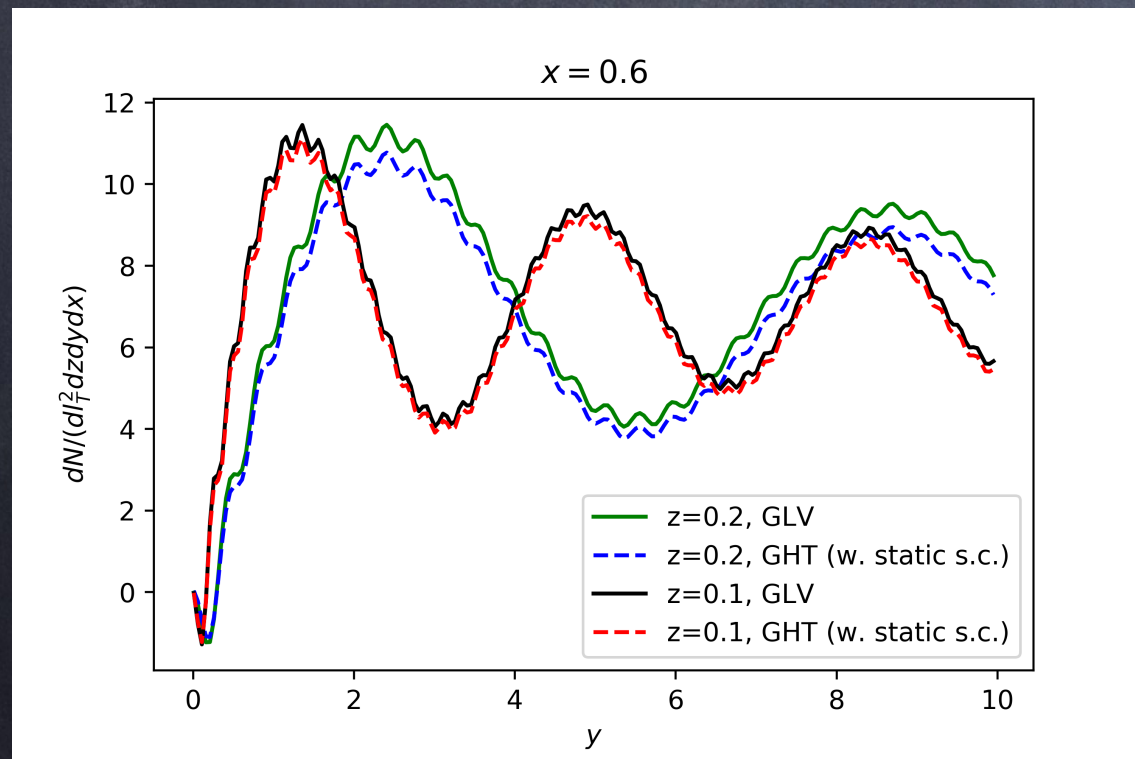
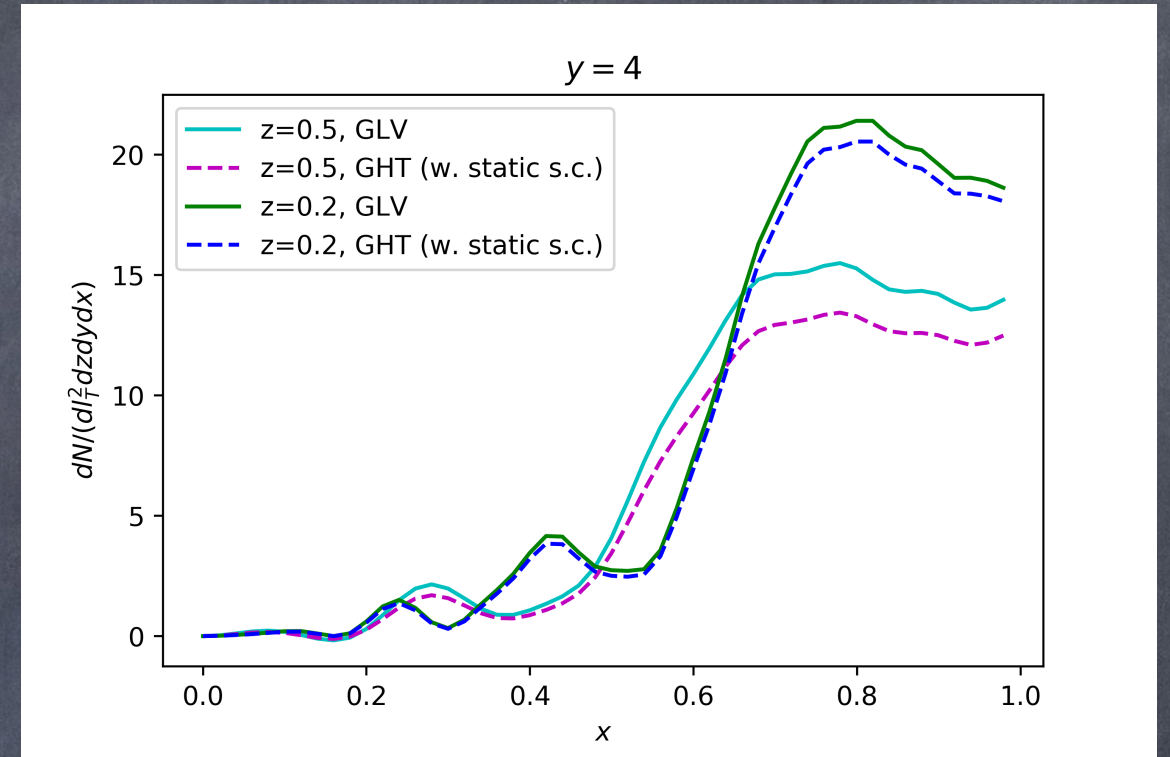
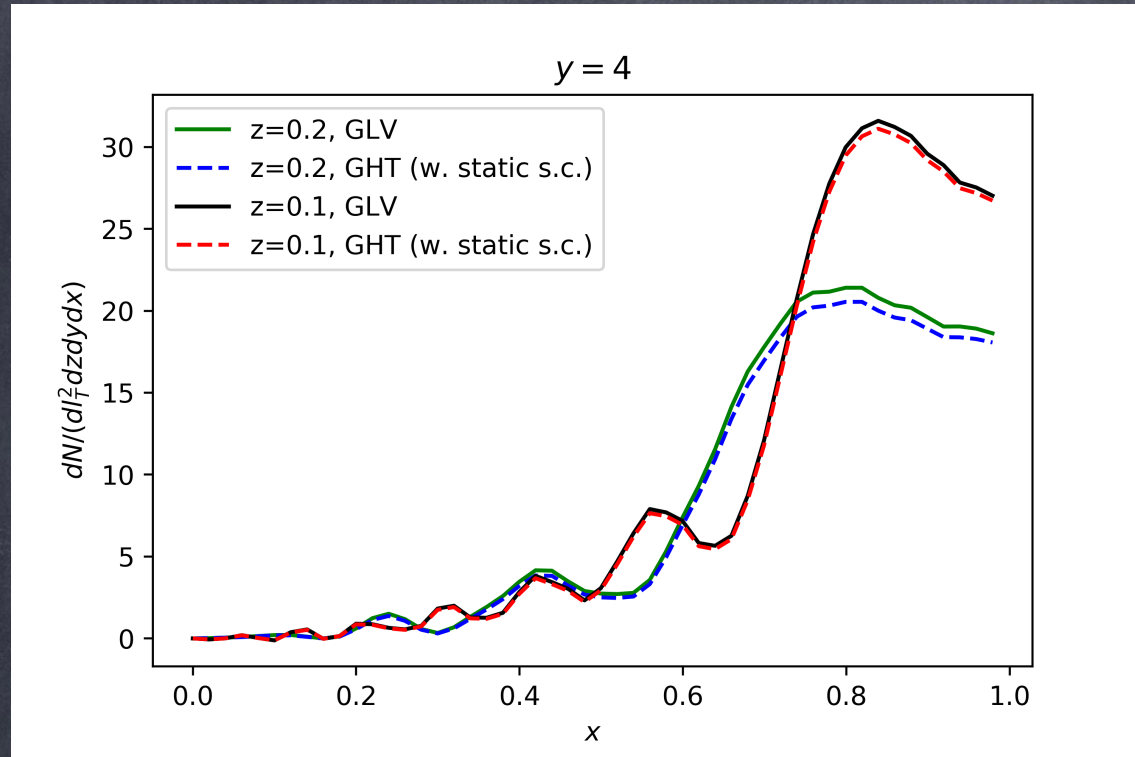


# Gluon Spectrum Approximations

## Numerical Results

$$\frac{dN}{dl_{\perp}^2 dz dy dx}$$

$$x = \frac{k_{\perp}}{l_{\perp}}, y = \frac{y_1^- * l_{\perp}^2}{2q^-}$$





# Summary

- Radiative energy loss in semi-inclusive DIS process
- Generalized High Twist Approach
  - no soft radiated gluon approximation
  - dynamic scattering center (energy transfer)
  - ★ - relax  $k_{\perp} \ll l_{\perp}$  approximation,  $k_{\perp} \sim l_{\perp}$
  - ★ - transverse momentum dependent (TMD) gluon pdf  
TMD transport parameter
- Relation of Generalized High Twist approach result with GLV result

## Further work

- implementation into CoLBT-Hydro Model, JETSCAPE
- Investigate the gauge link (high order effect)



Thanks!