Bayesian Estimation of Jet Parameter with Multi-stage Jet Evolution Model

JETSCAPE Workshop 2019

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Jet transport coefficient

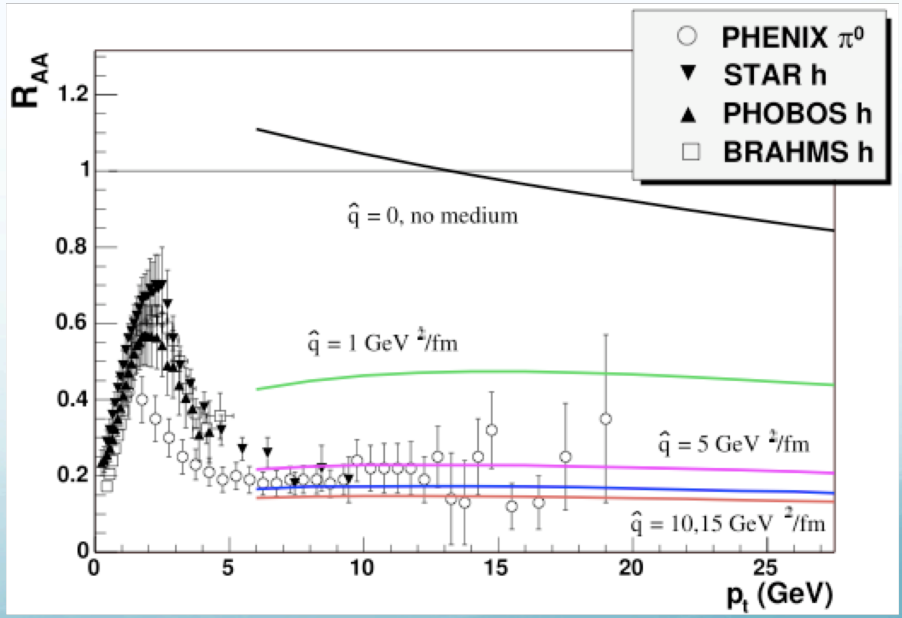
$$\hat{q} \equiv \frac{d\left\langle k_\perp^2 \right\rangle}{dL}$$

Average transverse momentum broadening square per unit length/time due to elastic scatterings with medium

$$\hat{q}_a = \sum_{b,(cd)} \int_{\mu_D^2}^{s/4} dk_\perp^2 \frac{d\sigma_{ab\to cd}}{dk_\perp^2} \rho_b k_\perp^2$$

$\frac{d\sigma_{ab\to cd}}{dk_\perp^2}$ jet property and jet-medium interaction ρ_b medium property

- Crucial parameter input in many theory calculations
- Probe the medium property with jets



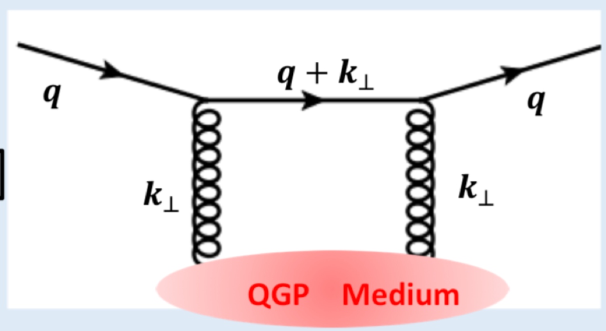
Lattice QCD calculation

From Amit Kumar's talk (1/11)

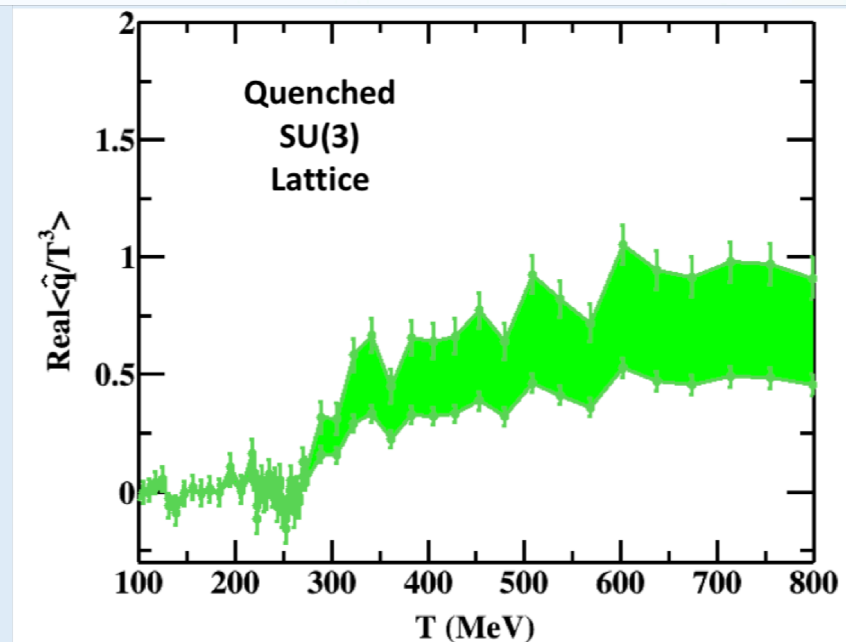
- Relate the jet coefficient to the field correlator on lattice
- Provide the first result on a quenched SU(3) lattice
- Challenges remain in extending to unquenched plasma (QGP)

$$\hat{Q}(q^+) = \frac{16\sqrt{2}\pi\alpha_s}{N_c}\int \frac{d^4y\, d^4k}{(2\pi)^4} e^{iky}\, q^- \frac{\langle M|F_\perp^{+\mu}(0)\, F^+_{\perp\mu}(y)|M\rangle}{(q+k)^2+i\epsilon}$$

$\boxed{\hat{Q}(q^+)}$



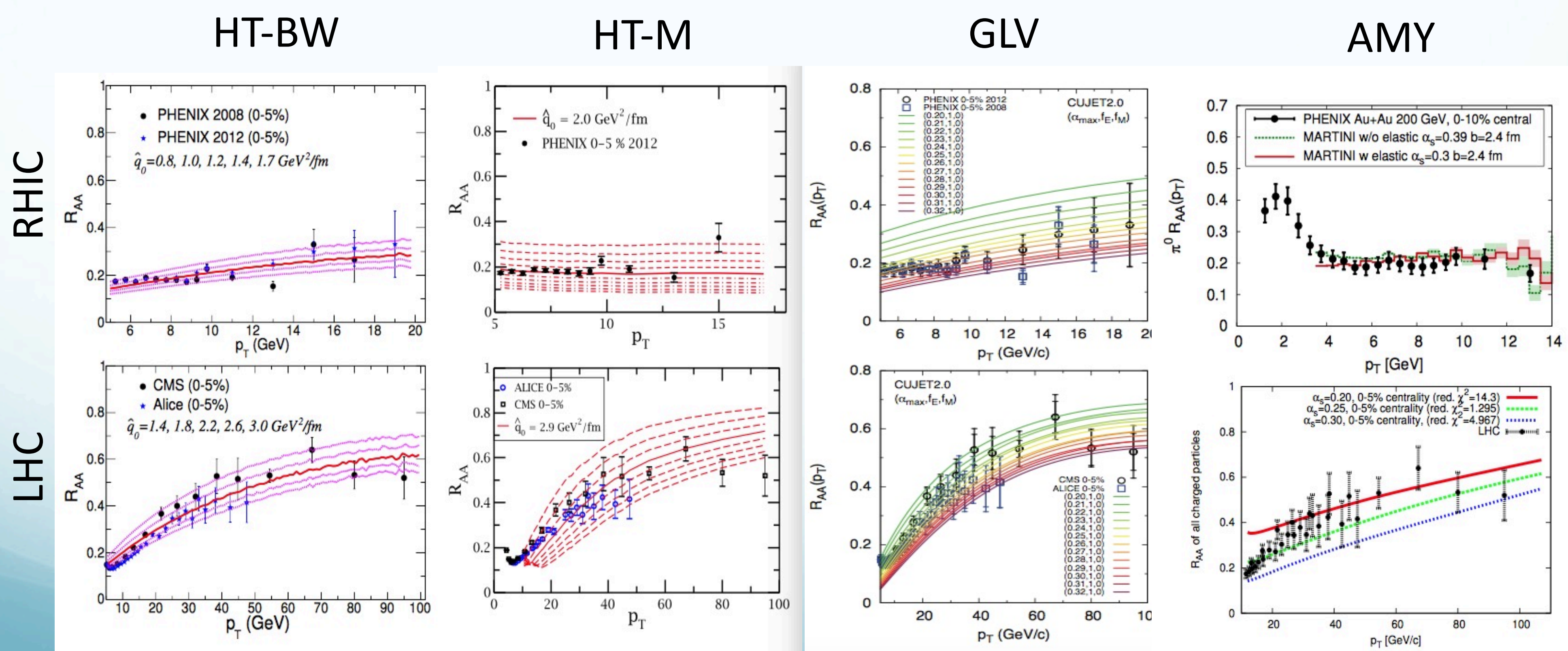
When $q^+ \sim T$ $\quad \mathrm{Disc}[\hat{Q}(q^+)]_{at\, q^+\sim T} = \hat{q}$



Extraction from model-to-data comparison

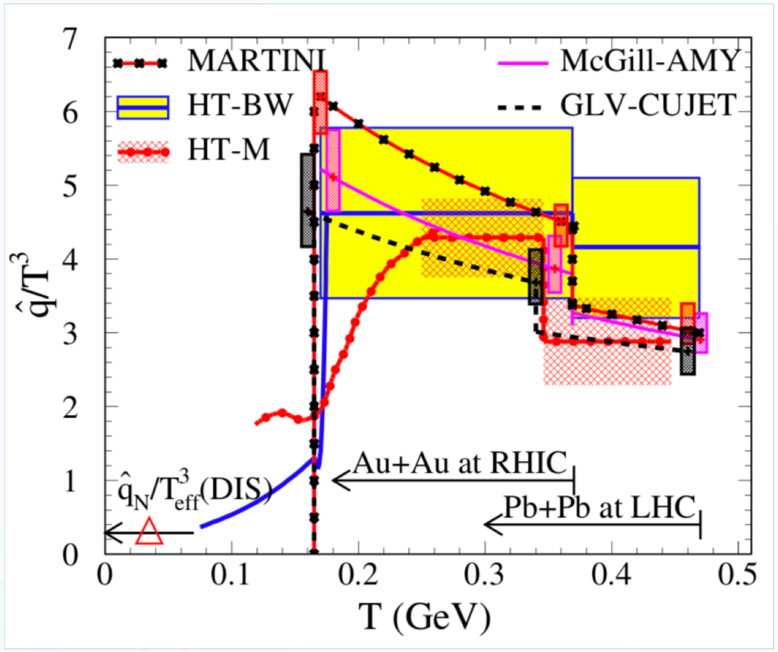
From the JET Collaboration work [PRC 90 (2014) 014909]

- Compare different energy loss formalisms to data
- Use χ^2 fit to constrain the value of jet coefficient

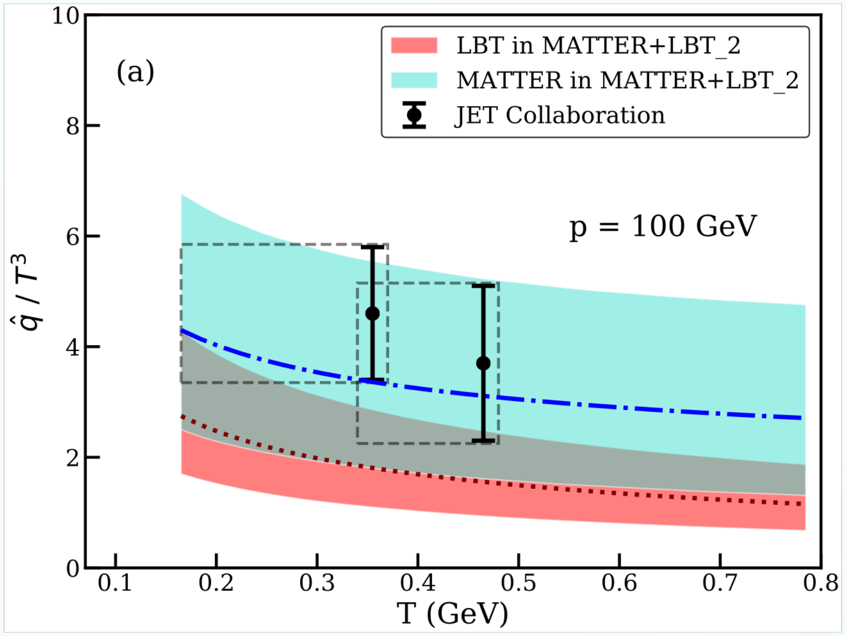


Paradigm shift of model to data comparison

[**JET**: PRC 90 (2014) 014909]



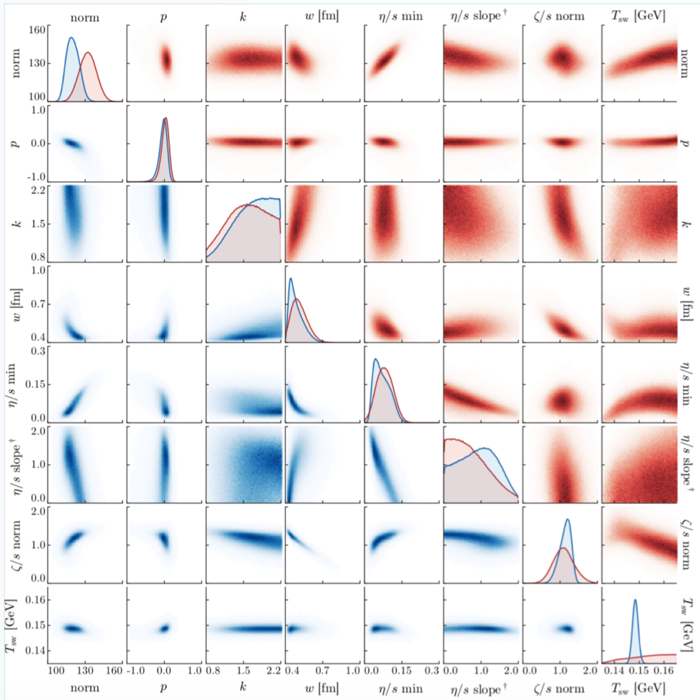
[**JESCAPE preliminary**]

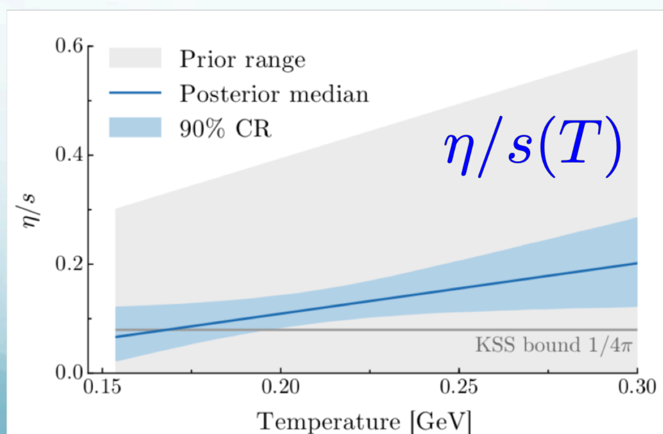


- Single parameter fit to single data set → Simultaneous calibration of multiple parameters on multiple data set
- Two separate values of $\hat{q}$ at RHIC and LHC → $\hat{q}$ as a smooth function of the medium T and jet p
- Single energy loss approach, semi-analytical calculation → Multi-stage jet evolution approach with full Monte-Carlo simulation

Application of Bayesian calibration in HIC

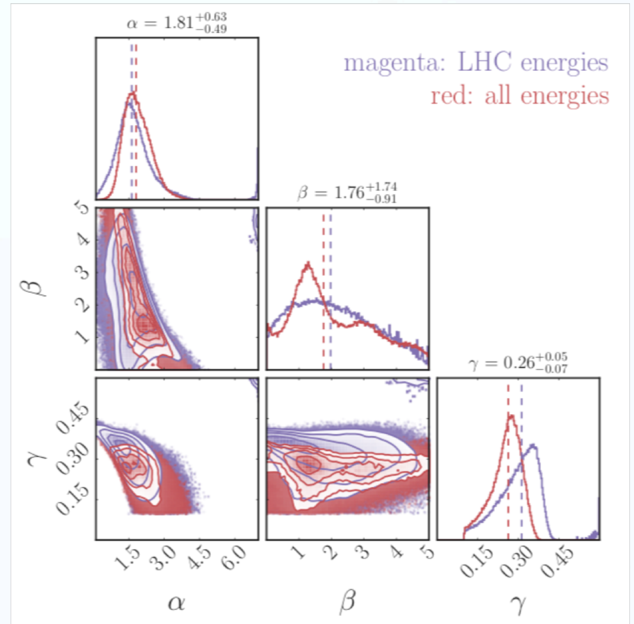
Calibration on bulk data

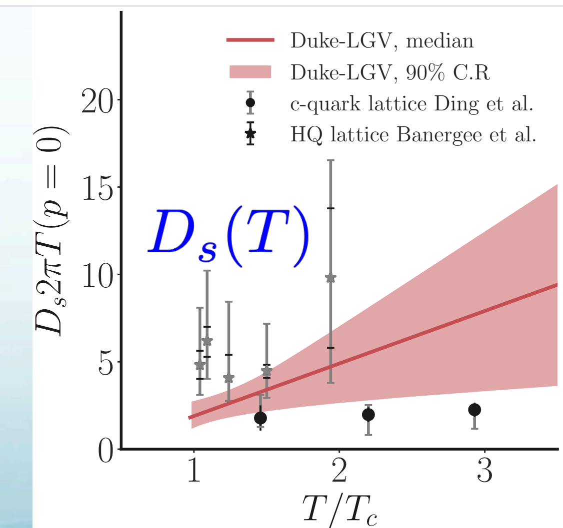




Bernhard, et., al, PRC 94 (2016)

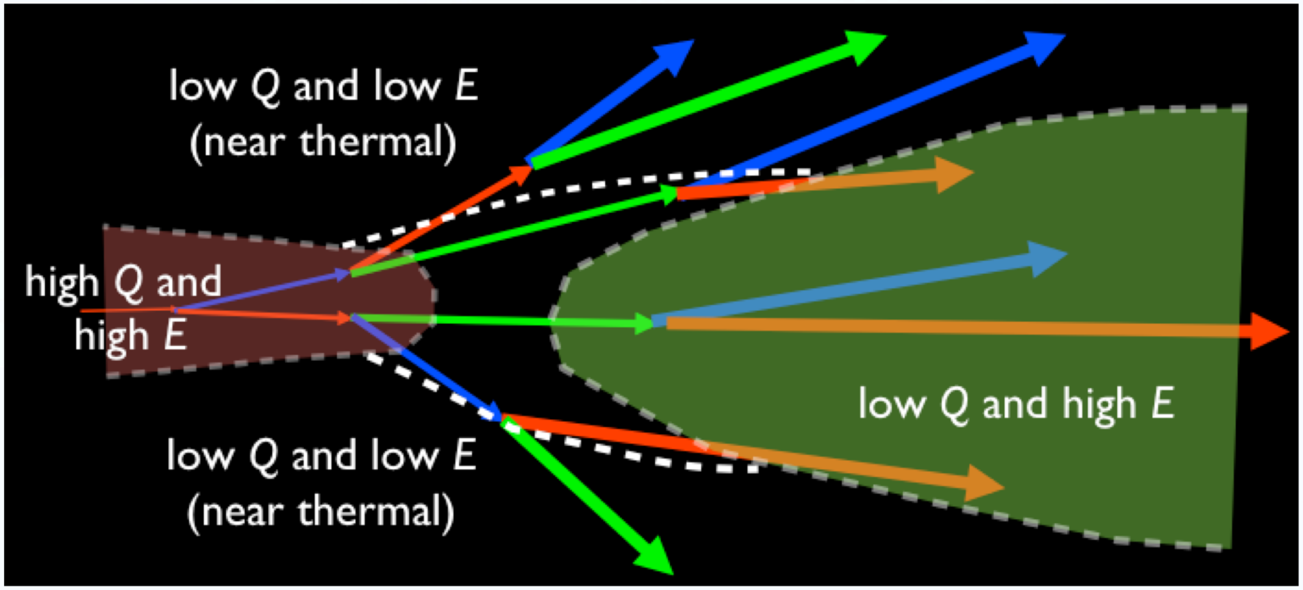
Calibration on heavy quark data



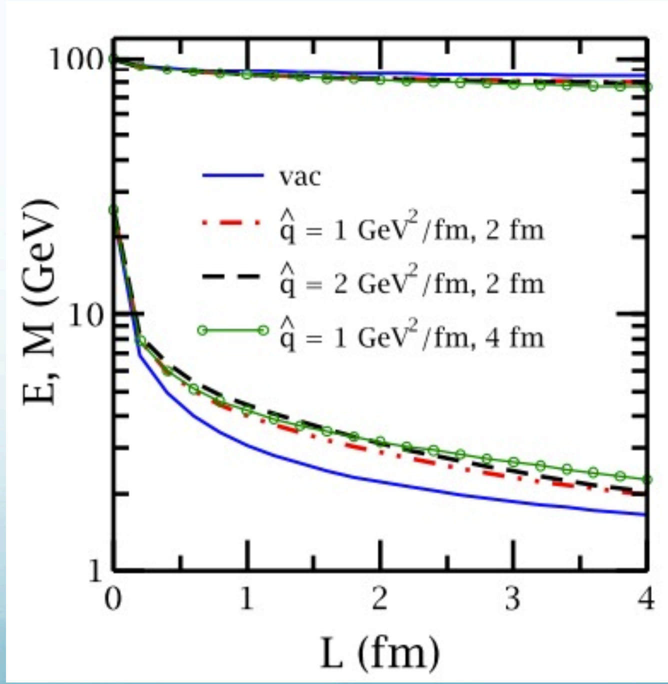


Xu, et., al, PRC 97 (2018)

Multistage jet evolution in heavy-ion collisions



- Stage 1: High Q and high E (**DGLAP, higher-twist**); lose Q faster than E [Majumder and Putschke, PRC 93 (2016)]
- Stage 2: low Q and high E (**Transport, higher-twist, AMY**)
- Stage 3: low Q and low E (near thermal) (**strongly coupled approach**)
- However, one usually applies a single theory through all stages



Jet model (MATTER) at high *Q* and high *E*

DGLAP evolution for parton fragmentation function at high *Q*:

$$\frac{\partial}{\partial Q^2} D(z, Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} \int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}, Q^2\right)$$

Sudakov form factor (probability of NO splitting in [*Q* , Q_{max}]):

$$\Delta(Q_{\text{max}}, Q) = \exp\left[-\frac{\alpha_s}{2\pi} \int_{Q^2}^{Q^2_{\text{max}}} \frac{dQ^2}{Q^2} \int_{z_c}^{1-z_c} \frac{dy}{y} P(y)\right]$$

Splitting function:

$$P(y) = P_{\text{vac}}(y) + P_{\text{med}}(y)$$

$$P_{\text{med}}(y, k_\perp^2) = \frac{2 C_A \alpha_s}{\pi k_\perp^4} P_{\text{vac}}(y) \int_{t_i}^{t_{\text{max}}} dt\, \hat{q}(t) \sin^2\left(\frac{t - t_i}{2\tau_f}\right)$$

[*higher-twist* energy loss formalism: Guo and Wang (2000), Majumder (2012)]

MATTER: a MC simulation of virtuality-ordered parton showers from initial Q_{max} to Q_0 [Wayne: PRC 88, 014909, arXiv:1712.10055]

Jet model (LBT) at low *Q* and high *E*

Linear Boltzmann Transport: a time-ordered transport model simulates parton showers at (or below) Q_0 with on-shell approximation

[LBL-CCNU: PRC 94 (2016) 014909, PLB 777 (2018) 255, PLB 782 (2018) 707]

Evolution of jet parton "1": $p_1 \cdot \partial f_1(x_1, p_1) = E_1(\mathcal{C}_{\rm el} + \mathcal{C}_{\rm inel})$

Elastic Scattering rate:

$$\Gamma^{\rm el}_{12\to34}(\vec{p}_1) = \frac{\gamma_2}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2)S_2(s,t,u)(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)|\mathcal{M}_{12\to34}|^2$$

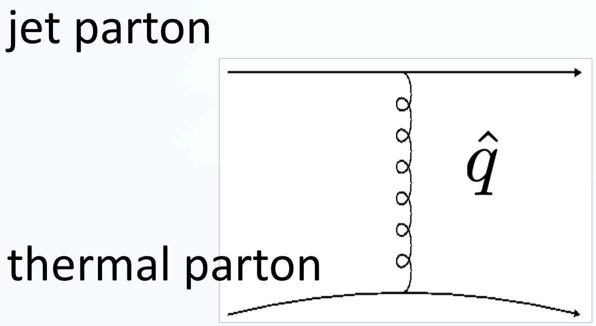
Inelastic scattering rate (average gluon number per *Δt*):

$$\Gamma^{\rm inel} = \langle N_g\rangle(E,T,t,\Delta t)/\Delta t = \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}$$

- Medium-induced gluon spectrum is taken from HT (same as MATTER)
- Multiple gluon emission in *Δt* is allowed – Poisson distribution of $\langle N_g \rangle$

Parameter space design #1

- Recall: jet parton scattering with a thermal parton



$$\frac{\hat{q}}{T^3} \approx C_R \frac{42\zeta(3)}{\pi} \alpha_s^2 \ln\left(\frac{6ET}{4\mu_D^2}\right)$$

$$\mu_D^2 = 6\pi\alpha_s T^2$$

$$\alpha_s = \frac{4\pi}{9\ln\left(Q^2/\Lambda^2\right)}$$

- A more general ansatz

$$\frac{\hat{q}}{T^3} = 42 C_R \frac{\zeta(3)}{\pi}\left(\frac{4\pi}{9}\right)^2 \left\{ \frac{A\left[\ln\left(\frac{E}{\Lambda}\right) - \ln(B)\right]}{\left[\ln\left(\frac{E}{\Lambda}\right)\right]^2} + \frac{C\left[\ln\left(\frac{E}{T}\right) - \ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2} \right\}$$

- 5D space (add Q_0) for MATTER+LBT approach

Pure dependence on the scale of jet when Q >> T
(preferred by MATTER)

Perturbative scattering with quasi-particles from a thermal medium (*T*)
(preferred by LBT)

Model to data comparison

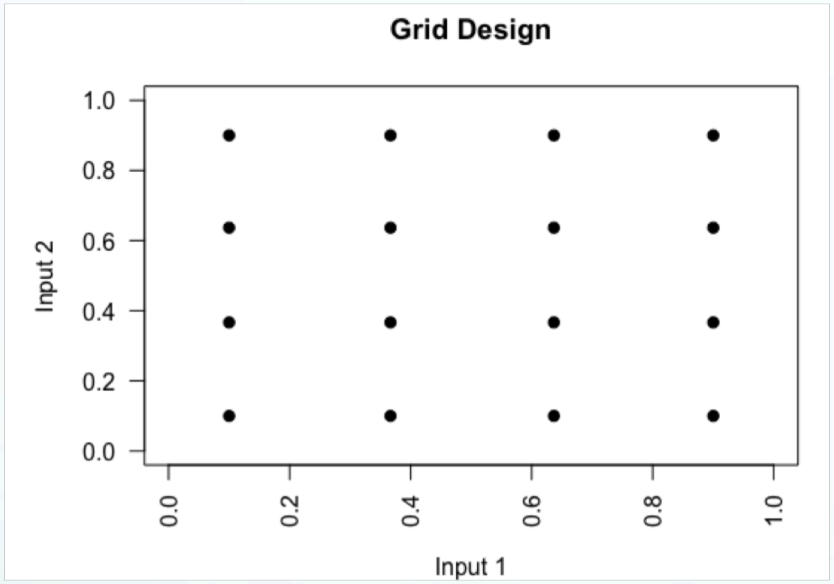
- Model: MATTER, LBT, MATTER+LBT
- Parameter space: 4D for MATTER, LBT, 5D for MATTER+LBT
- Data: hadron RAA from different centrality bins at RHIC and LHC
- Global calibration for $\hat{q}$

Computational cost with traditional fitting method

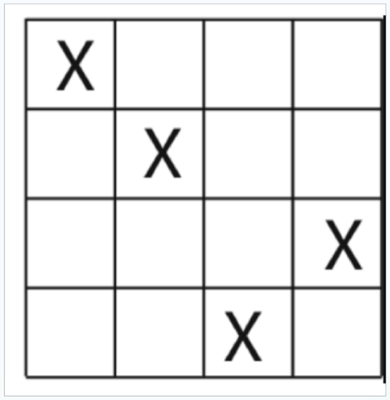
- Full model simulation on each (A, B, C, D, Q): 10k CPU hrs
- Scan through 5D parameter space with usual grid design: $n^5 \times 10$k CPU hrs (1 billion CPU hrs for $n = 10$)
- Needs advanced computational and statistical tools! [Details in Winter School Session 1/11 by Coleman and Ke]

Gaussian Process Emulator

Train Gaussian Process Emulator (GP) on a limited number of parameter points – ***Latin hypercube*** – with real model calculation



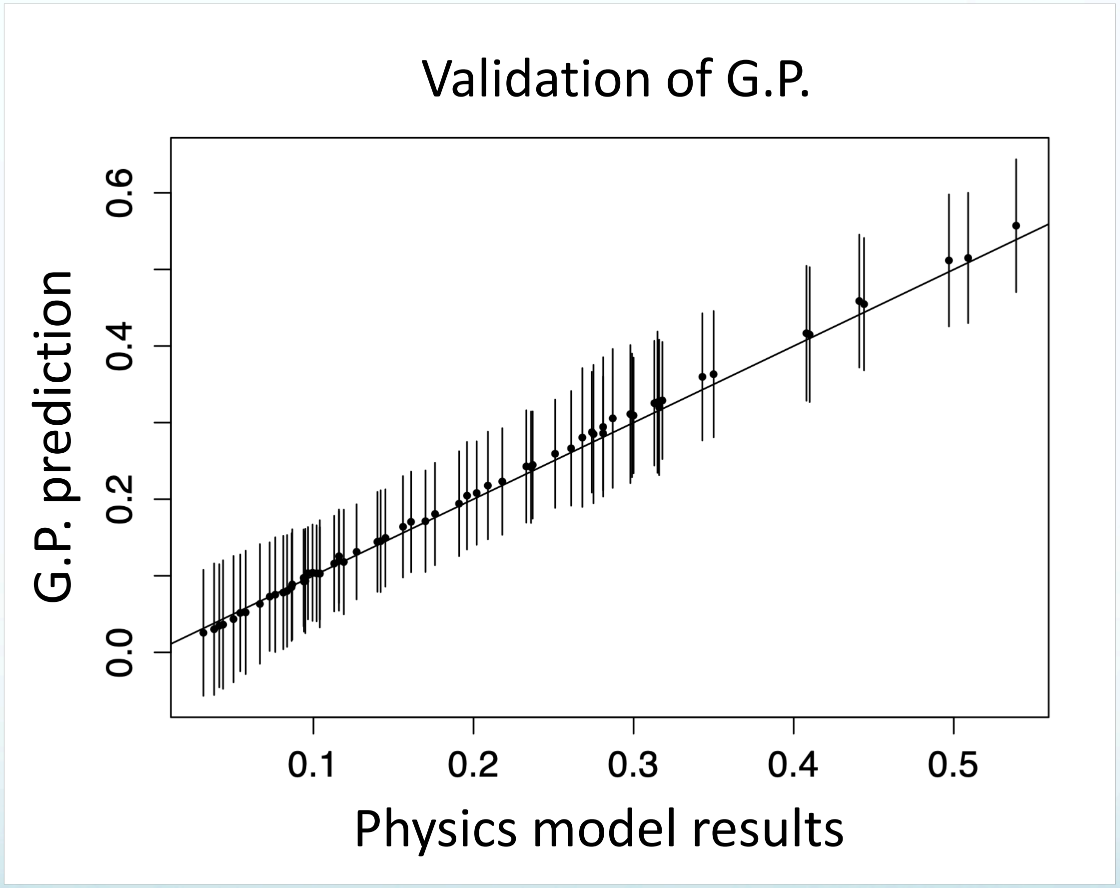
Latin hypercube design



- A grid is inefficient – n^d total pts for only *n* different marginal pts
- Latin hypercube ensures only one design point for each row **AND** column – every design point is in exactly one “bin” for **EACH** dimension
- Real calculations on 10*d* pts are sufficient for training the GP

Gaussian Process Emulator

After being trained, G.P. serves as fast surrogate of real physics model in model-to-data comparison.



Reduce computation from billions of CPU hrs to less than a million, finish within 2 weeks via ***Open Science Grid*** (free of charge).

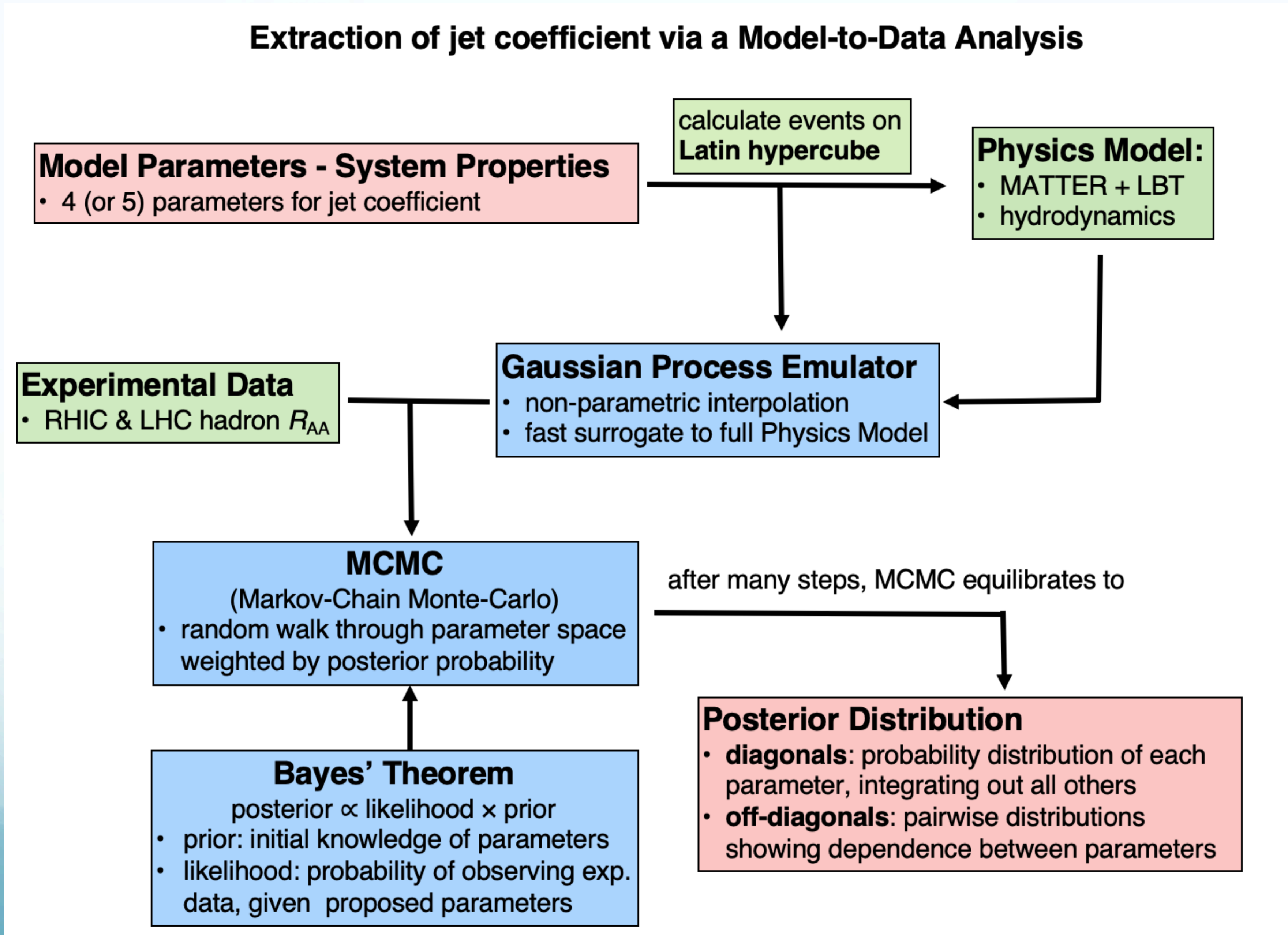
Bayesian Calibration

Use the Gaussian process emulator to sweep over the parameter space, compare to experimental data, and compute the posterior probability of each set of parameters based on the **Bayes' Theorem**

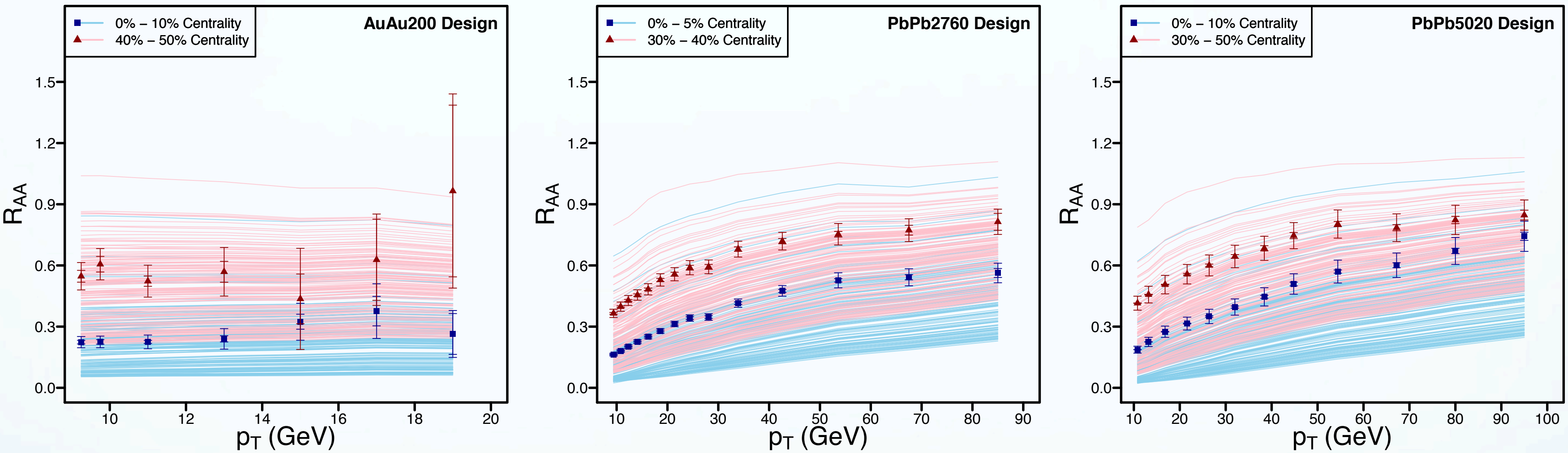
$$P_{\text{post}}(x_*|X,Y,y_{exp}) \propto P_{\text{likelihood}}(X,Y,y_{exp}|x_*)P_{\text{prior}}(x_*)$$

Substitute P_{prior} with P_{post} and iterate the process with a **Markov-Chain Monte-Carlo (MCMC)** method until probability distribution equilibrates.

Flow chart of statistics analysis

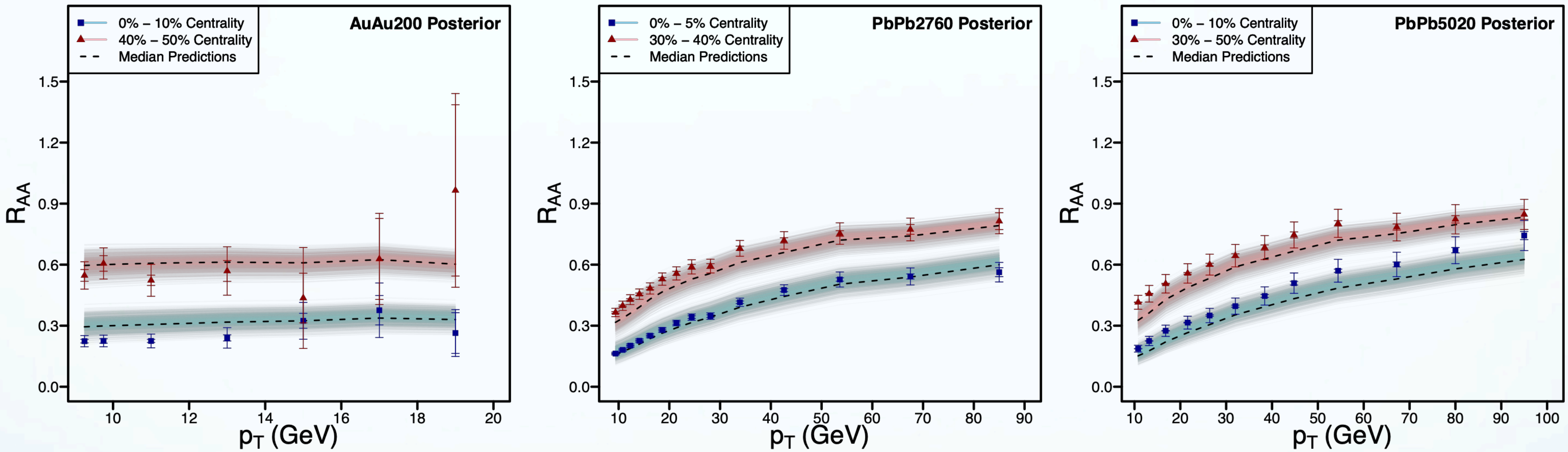


Calibration for LBT (Prior)



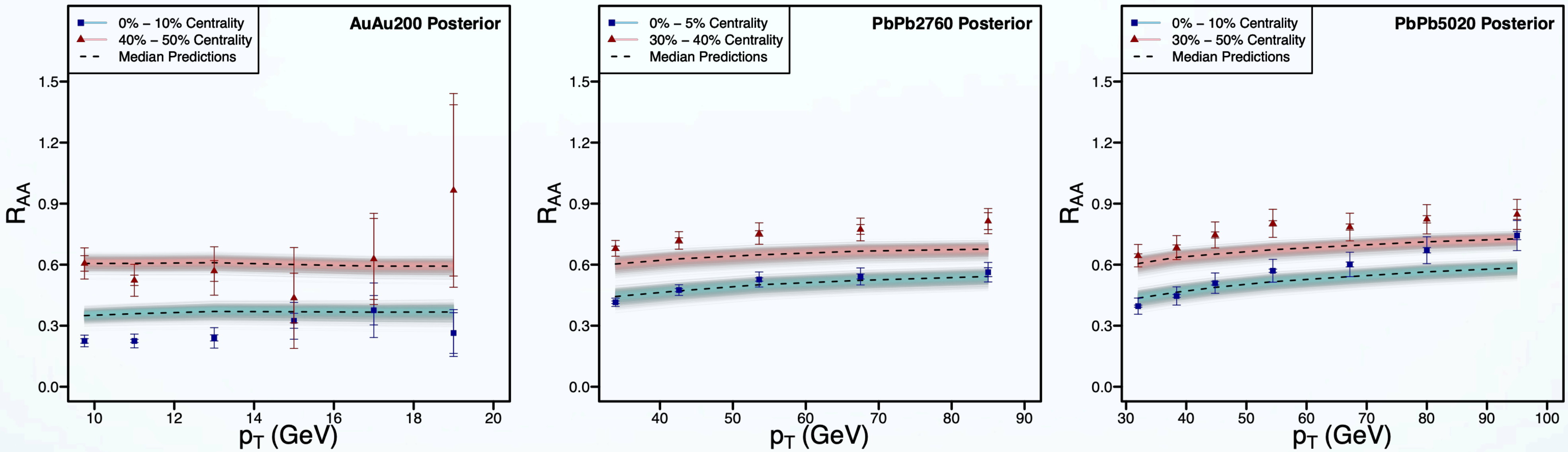
- Calculation prior to calibration (no knowledge of parameter space)
- Wide enough prior range to cover all the data.

Calibration for LBT (Posterior)



- Calculation after Bayesian calibration on data.

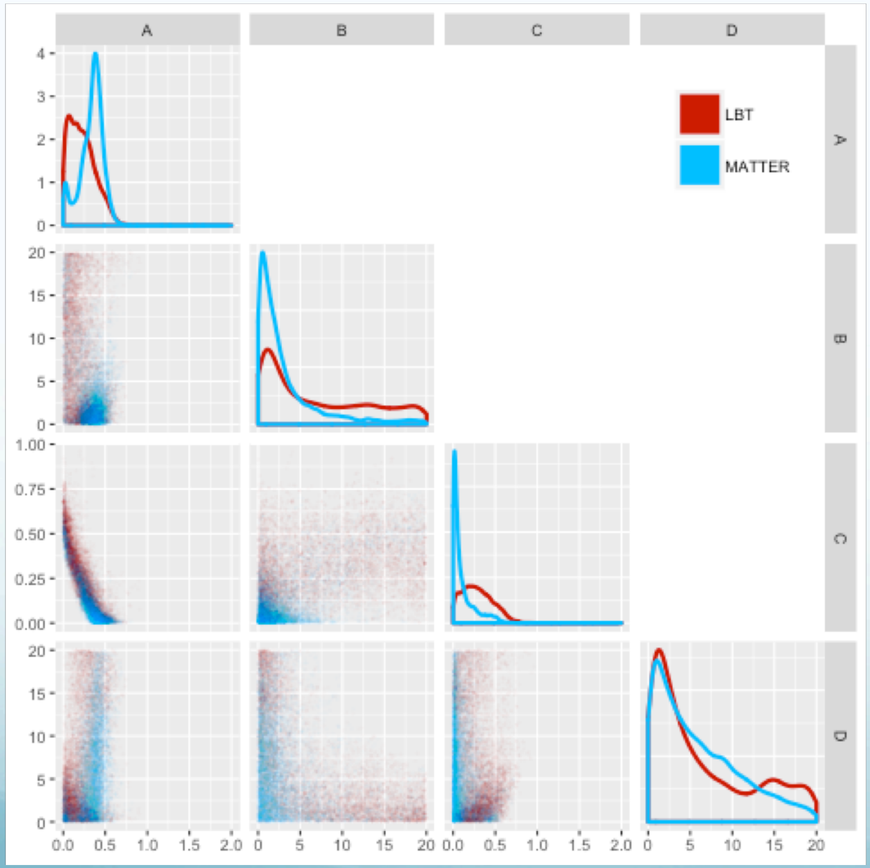
Calibration for MATTER



- Calculation after Bayesian calibration on data.

Extracted parameter space distribution

$$\frac{\hat{q}}{T^3} = 42C_R\frac{\zeta(3)}{\pi}\left(\frac{4\pi}{9}\right)^2\left\{\frac{A\left[\ln\left(\frac{E}{\Lambda}\right)-\ln(B)\right]}{\left[\ln\left(\frac{E}{\Lambda}\right)\right]^2}+\frac{C\left[\ln\left(\frac{E}{T}\right)-\ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2}\right\}$$

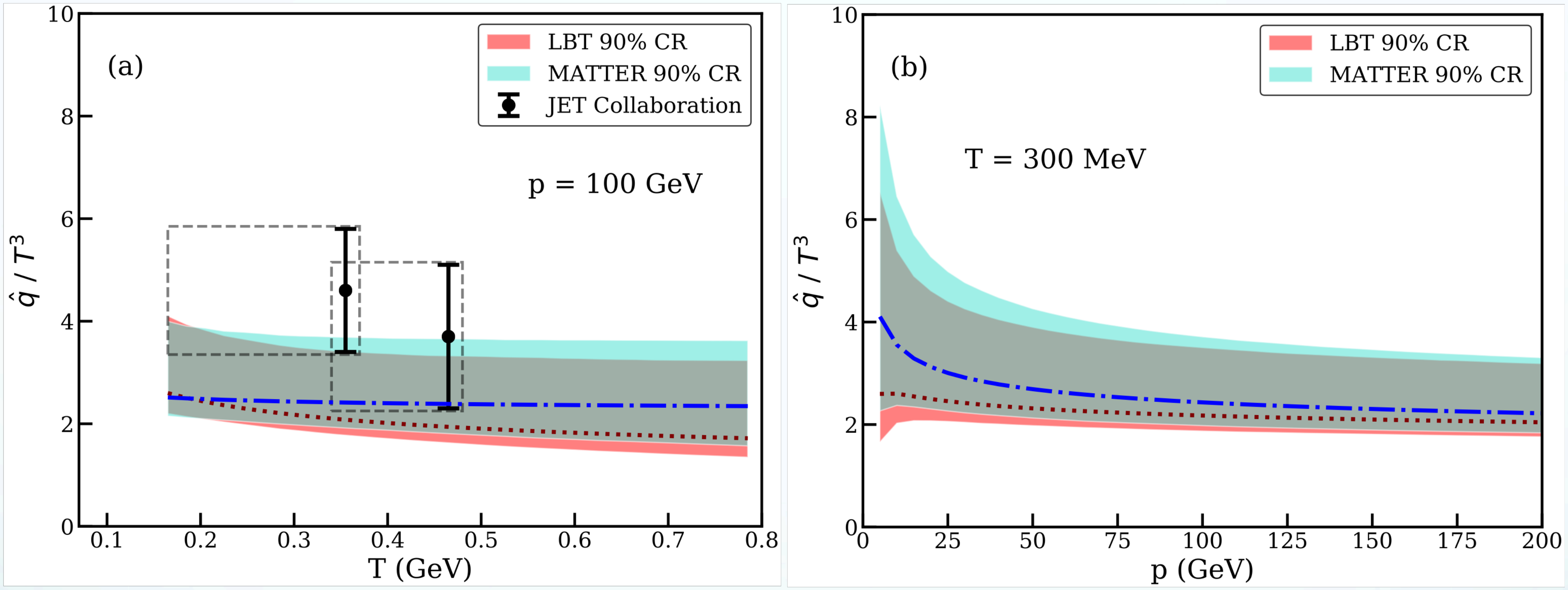


MATTER
prefers the *A* term

LBT
prefers the *C* term

Consistent with the model assumption

Extracted jet transport coefficient



- Smooth functions of $\hat{q}$ (mean value together with a 90% C.R.)
- Stronger *T* dependence extracted from LBT than from MATTER
- Consistent with by smaller than the previous JET results

Calibration for MATTER+LBT

Design #1 $$\frac{\hat{q}}{T^3} = 42C_R\frac{\zeta(3)}{\pi}\left(\frac{4\pi}{9}\right)^2\left\{\frac{A\left[\ln\left(\frac{E}{\Lambda}\right)-\ln(B)\right]}{\left[\ln\left(\frac{E}{\Lambda}\right)\right]^2}+\frac{C\left[\ln\left(\frac{E}{T}\right)-\ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2}\right\}$$

with Q_0, 5 parameters in total

can be directly compared to MATTER and LBT

Design #2 $$\frac{\hat{q}}{T^3} = 42C_R\frac{\zeta(3)}{\pi}\left(\frac{4\pi}{9}\right)^2\left\{\frac{A\left[\ln\left(\frac{Q}{\Lambda}\right)-\ln\left(\frac{Q_0}{\Lambda}\right)\right]}{\left[\ln\left(\frac{Q}{\Lambda}\right)\right]^2}\theta(Q-Q_0)+\frac{C\left[\ln\left(\frac{E}{T}\right)-\ln(D)\right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2}\right\}$$

4 parameters in total

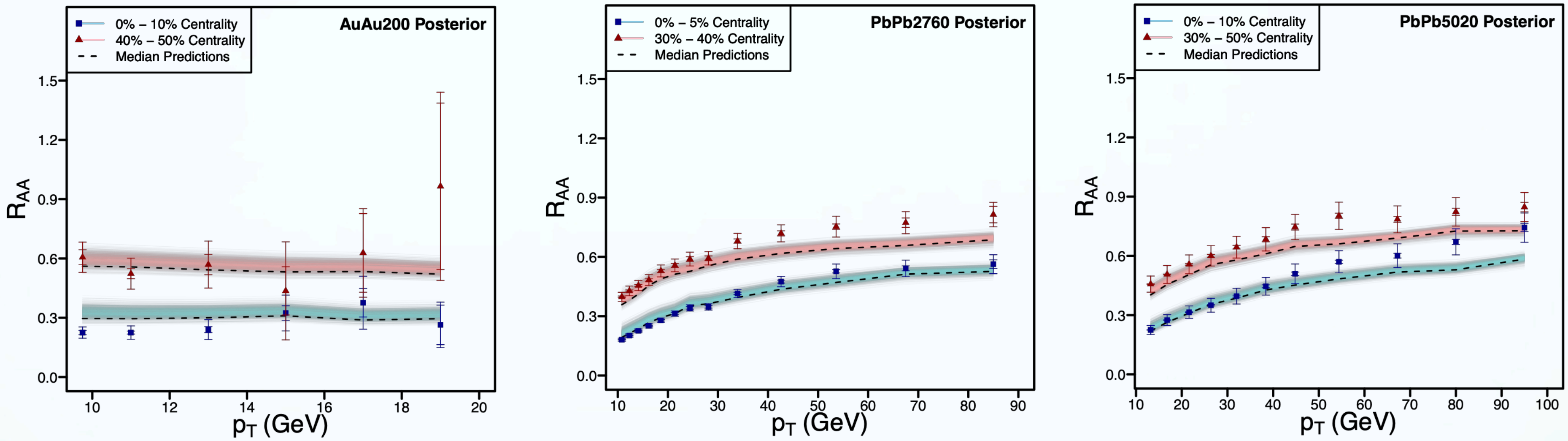
In MATTER stage (above Q_0), both terms contributes

In LBT stage (below Q_0), only the second term contributes

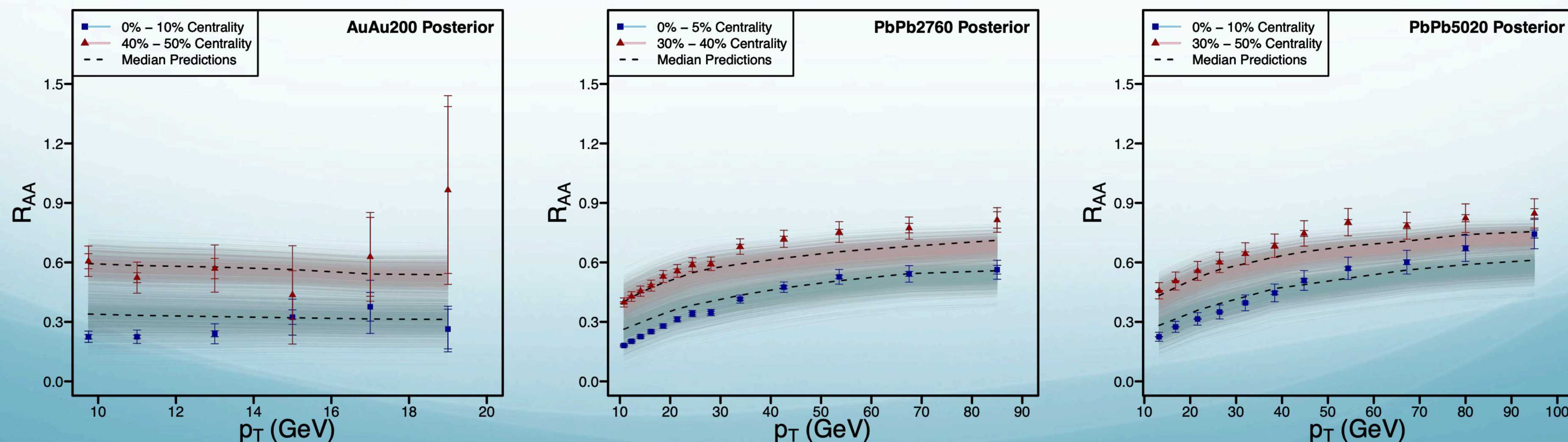
More physical for the multi-stage evolution

Calibration for MATTER+LBT

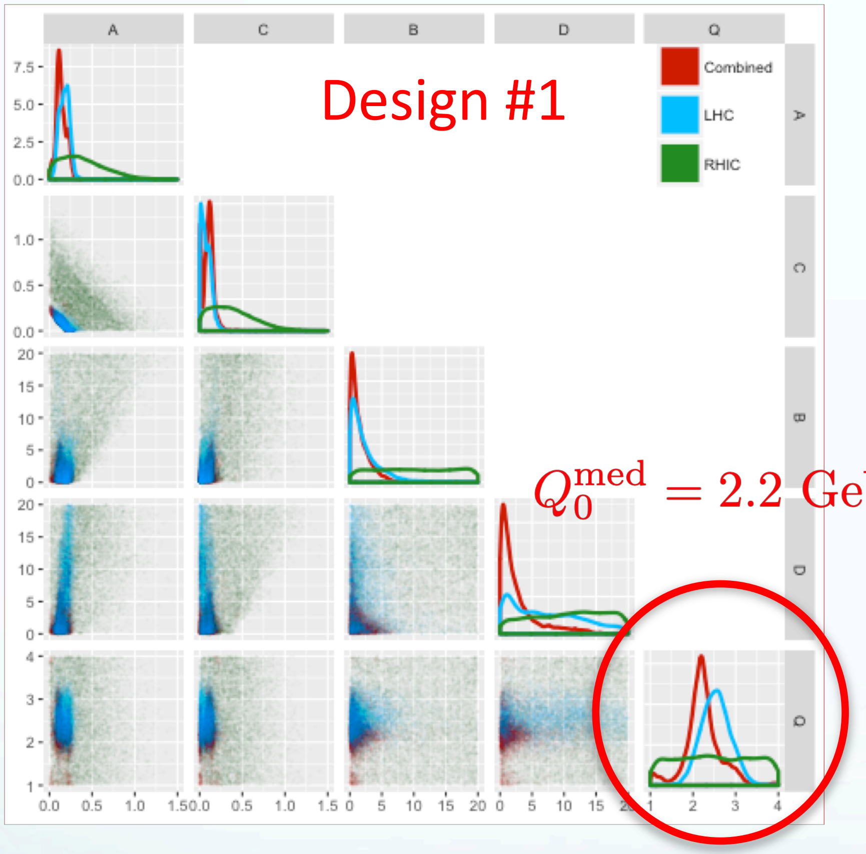
Design #1 – 5D

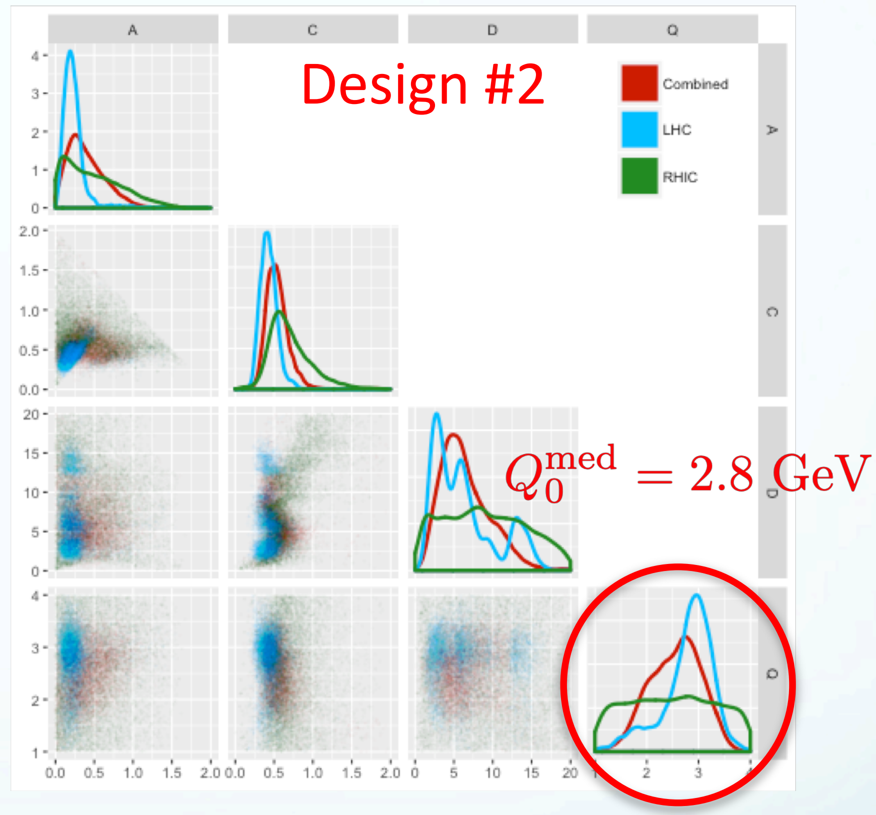


Design #2 – 4D



Extraction of Q_0 in MATTER+LBT

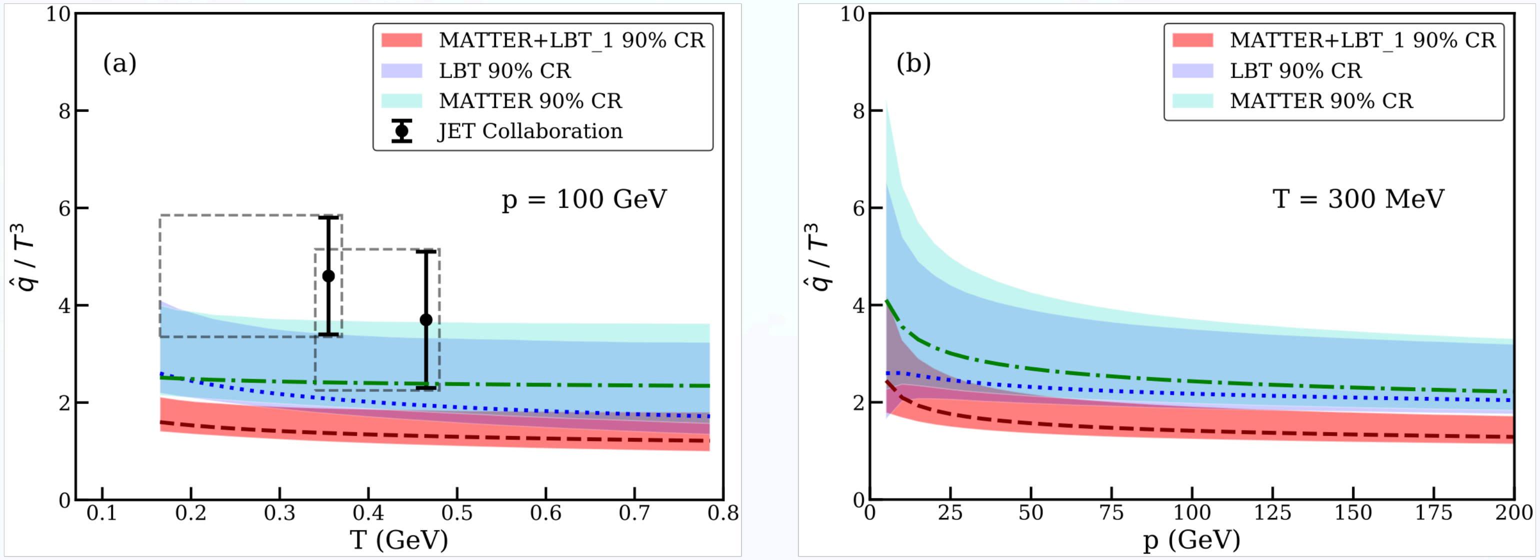




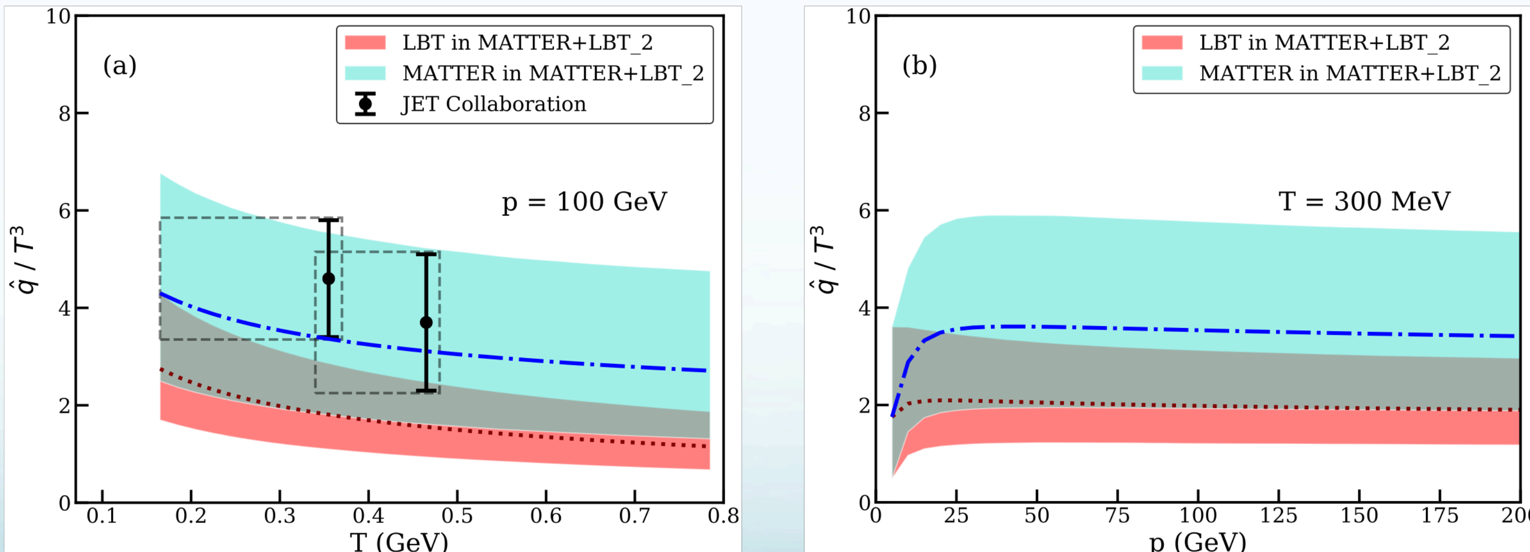
- Better constraint on parameters (shaper peaks) with more data input
- First quantitative constraint on Q_0
- Extracted parameters are model dependent

Extracted jet transport coefficient

Design #1



Design #2



Model dependent; within #1, MATTER+LBT requires smaller jet parameter compared to single model approaches

Summary

- Conducted systematic extraction of jet parameter with the Bayesian method and 4 different model and parameter setups
- Obtained smooth *T* an *p* dependence of $\hat{q}$
- Provided the 1st quantification of the medium virtual scale Q_0

Outlook

- Constant Q_0 -> Q_0 (p, T) with more observables included
- An objective algorithm for model evaluation
- Release of our statistics package within JETSCAPE 2.0

Thank you!