The iS3D Particlization Module for Heavy-Ion Collisions

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BEST COLLABORATION

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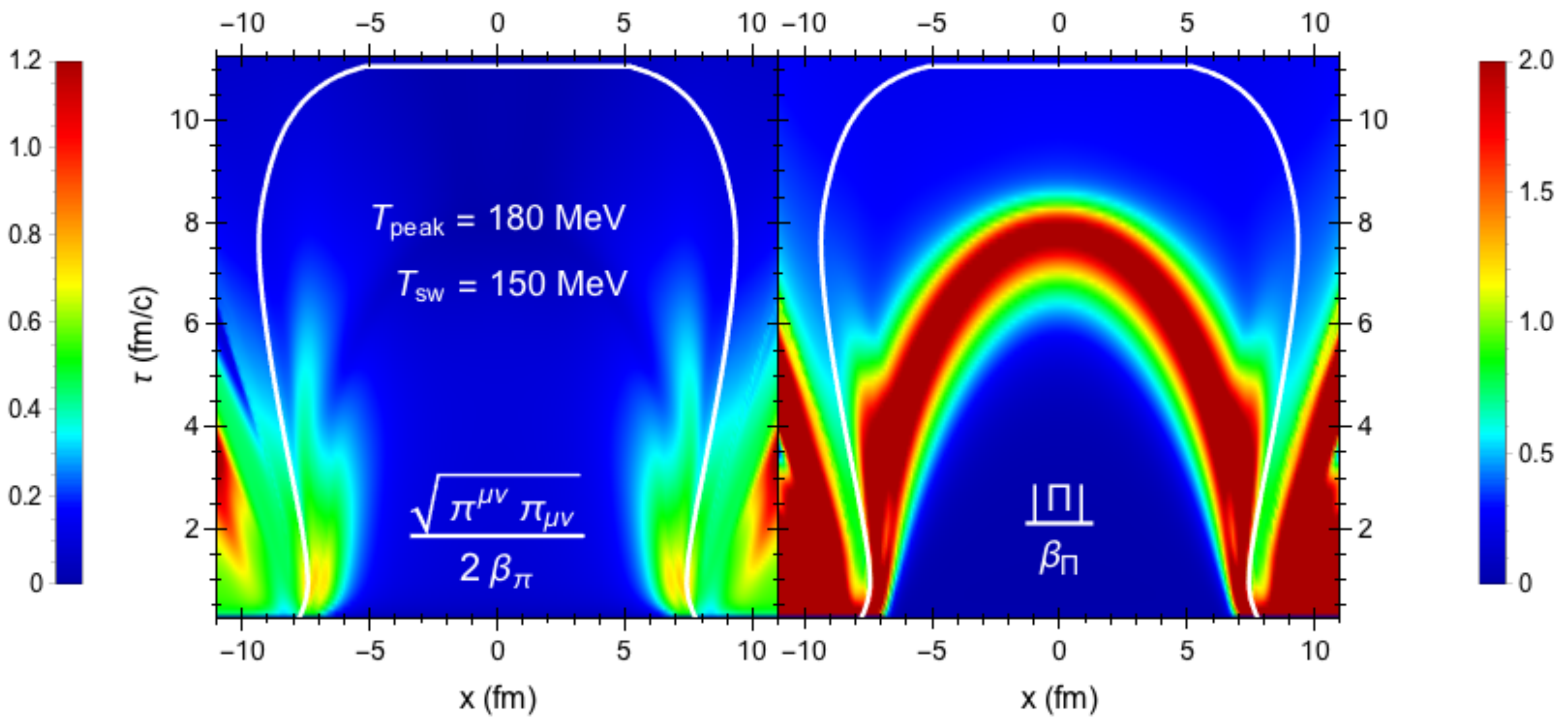
JETSCAPE

Overview

- **What is the best choice for δf?**
- **Linearized δf**
  - 14 moment approximation
  - Chapman Enskog expansion
- **Modified equilibrium distribution**
  - McNelis (OSU)
  - Bernhard (Duke)
- **JETSCAPE will compare these viscous corrections to the particle spectra using iS3D**
- **Discuss particle sampler in iS3D and test its performance**



$$E_p \frac{dN_n}{d^3p} = h^{-3} \int_\Sigma p \cdot d^3\sigma \, f_n \qquad f_n = f_{eq,n} + \delta f_n$$

14 moment approximation

- Moments expansion truncated to the hydrodynamic moments

$$\delta f_n = f_{eq,n}\bar{f}_{eq,n} c_{\mu\upsilon} p^\mu p^\upsilon$$

$$f_{eq,n} = \frac{g_n}{\exp\left(\frac{u\cdot p}{T}\right) \pm 1}$$

$$\bar{f}_{eq,n} = 1 \pm g_n^{-1} f_{eq,n}$$

- Assume irreducible coefficients are species independent
  - Fix to satisfy Landau matching conditions and reproduce viscous part of $T^{\mu\nu}$

$$\boxed{\delta f_{14,n} = f_{eq,n}\bar{f}_{eq,n}\left(c_T m_n^2 + c_E (u\cdot p)^2 + c_\pi^{\langle\mu\nu\rangle} p_{\langle\mu} p_{\nu\rangle}\right)}$$

$$c_T = A_T(T)\Pi \qquad c_E = A_E(T)\Pi \qquad c_\pi^{\langle\mu\nu\rangle} = \frac{\pi^{\mu\nu}}{2(\mathcal{E}+P)T^2}$$

A. Monnai and T. Hirano, Phys. Rev. C80 (2009) 054906

Chapman Enskog expansion

- Gradient expansion of the RTA Boltzmann equation

$$p \cdot \partial f_n = -\frac{(u\cdot p)(f_n - f_{eq,n})}{\tau_R} \quad \rightarrow \quad \delta f_n \approx -\frac{\tau_R p\cdot \partial f_{eq,n}}{u\cdot p}$$

- Assume relaxation time τ_R is momentum and species independent

$$\boxed{\delta f_{CE,n} = f_{eq,n}\bar{f}_{eq,n}\left(\frac{\Pi}{\beta_\Pi}\left(\frac{(u\cdot p)F}{T^2} + \frac{(u\cdot p)^2 - m_n^2}{3(u\cdot p)T}\right) + \frac{\pi_{\mu\upsilon} p^{\langle\mu} p^{\upsilon\rangle}}{2\beta_\pi (u\cdot p)T}\right)}$$

$$F = -\frac{(\mathcal{E}+P)T^2}{J_{30}(T)}$$

$$\beta_\Pi = \frac{F(T)(\mathcal{E}+P)}{T} + \frac{5J_{32}(T)}{3T}$$

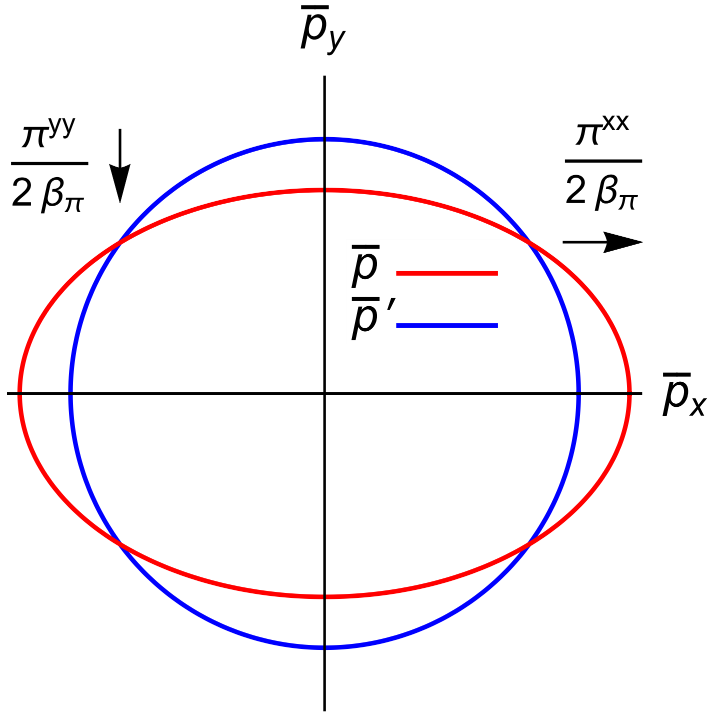
$$\beta_\pi = \frac{J_{32}(T)}{T}$$

- (β_π, β_Π) = shear / bulk viscosity : relaxation time ratios
- $F = \dot{T}/\theta$

A. Jaiswal, R. Ryblewski and M. Strickland, Phys. Rev. C90 (2014) 044908

Pratt McNelis Distribution

$$\boxed{f_{PM,n} = \frac{Z_n\, g_n}{\exp\left(\frac{\sqrt{m_n^2 + (p')^2}}{T + \beta_\Pi^{-1}\Pi F}\right) \pm 1}}$$



- Construct a map $p = M(p')$ in the LRF
  - Momentum space deformations ∝ viscous corrections

$$p_i = A_{ij} p'_j \qquad A_{ij} = \left(1 + \frac{\Pi}{3\beta_\Pi}\right)\delta_{ij} + \frac{\pi_{ij}}{2\beta_\pi}$$

- Effective temperature increases with negative Π
- Renormalize particle density to the Chapman Enskog one

$$Z_n = \frac{n_{\mathrm{eq},n} + \delta n_{\Pi,n}}{n_n^{(mod)}}$$

- In small (Π, π_{ij}) limit → Chapman Enskog expansion

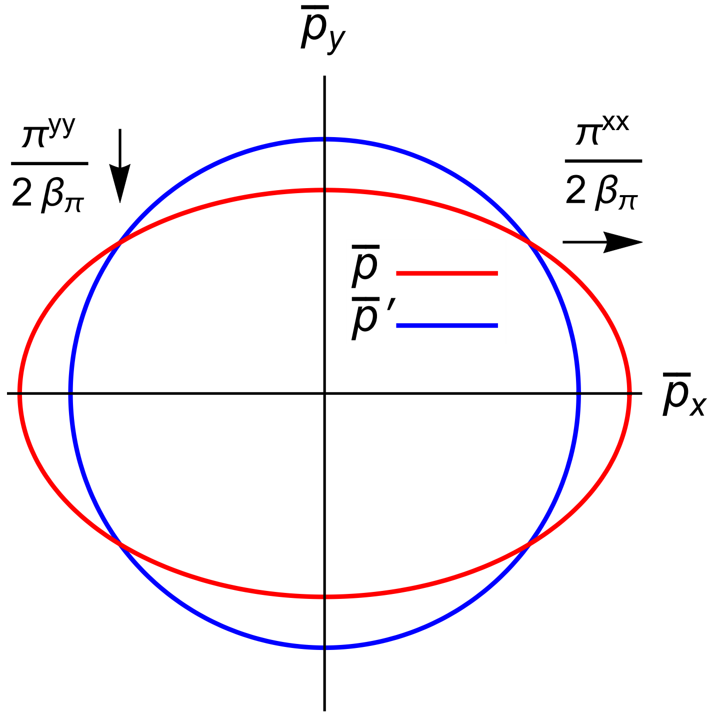
$p_i = -X_i \cdot p$ (LRF momentum)

$\pi_{ij} = X_i \cdot \pi \cdot X_j$ (LRF shear stress)

S. Pratt and G. Torrieri, Phys. Rev. C82 (2010) 044901

Jonah Bernhard's Distribution

$$f_{JB,n} = \boxed{\frac{Z g_n}{\exp\left(\frac{\sqrt{m_n^2 + (p')^2}}{T}\right) \pm 1}}$$



- Momentum rescaling: $p_i = \Lambda_{ij} p'_j \qquad \Lambda_{ij} = (1 + \lambda_\Pi)\delta_{ij} + \frac{\pi_{ij}}{2\beta_\pi}$
- Renormalization: $Z = \frac{z_\Pi}{\det\Lambda}$
- Same shear stress modification, no modified temperature
- (λ_Π, z_Π) parameterized to reproduce bulk pressure without changing the energy density (exactly, in absence of π_{ij})
- Particle abundance ratios do not change with bulk pressure

$p_i = -X_i \cdot p$ (LRF momentum)

$\pi_{ij} = X_i \cdot \pi \cdot X_j$ (LRF shear stress)

J.E. Bernhard, arxiv:1804.06469 (2018)

Sampling Particles from Cooper Frye

$$\boxed{E_p \frac{dN_n}{d^3p} = h^{-3} \int_\Sigma p \cdot d^3\sigma \; f_n \; \Theta(p \cdot d^3\sigma) \; \Theta(f_n)}$$

- Need to enforce positivity with $\Theta(p \cdot d^3\sigma) \; \Theta(f_n)$
  - Sampled particle spectra will slightly deviate from the physical ones
- Methodology:
  1) Sample the number of hadrons emitted from each freezeout cell
  2) For each hadron, sample its type and momentum

S. Pratt and G. Torrieri, Phys. Rev. C 82, 044901 (2010)

C. Shen et al., Comput. Phys. Commun. 199 (2016) 61-85

L.G. Pang, H. Peterson, and X.N. Wang, Phys. Rev. C97 (2018) 064918

J.E. Bernhard, arxiv:1804.06469 (2018)

Number of Hadrons

- Assume hadron number of each type follow Poisson statistics

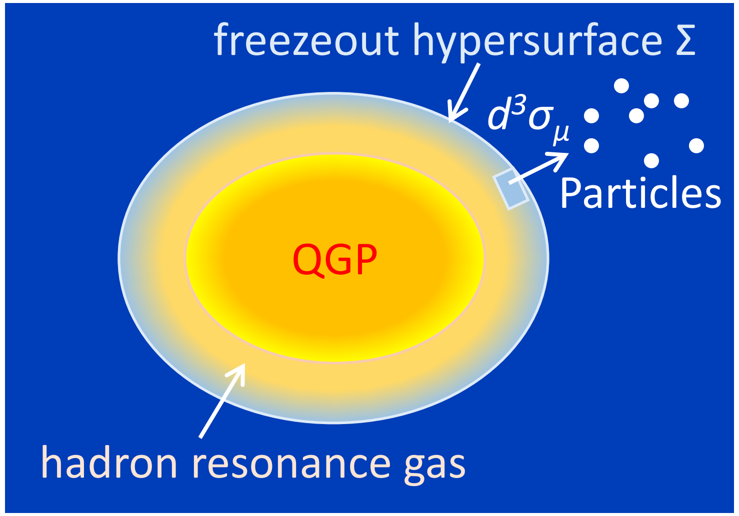
$$\mathrm{P}(N) = \frac{\exp(-\Delta N)(\Delta N)^N}{N!}$$ (hadron number distribution)

$$\Delta N = \sum_n \Delta N_n = \int_p p \cdot d^3\sigma\, f_n\, \Theta(p \cdot d^3\sigma)\, \Theta(f_n)$$ (mean number of hadrons emitted from freezeout cell)

$$D_n = \frac{\Delta N_n}{\Delta N}$$ (hadron type distribution)

- Time(null)-like cells: $p \cdot d^3\sigma > 0 \;\; \rightarrow \;\; \Delta N_n = n_n d^3\sigma_t$
- Space-like cells: momentum regions with $p \cdot d^3\sigma < 0$ cut out
  - Enforcing outflow $\Theta(p \cdot d^3\sigma)$ increases particle yield

$$\boxed{\Delta N_n = \Delta N_{t,n} + \Delta N_{x,n} = n_{t,n} d^3\sigma_t + n_{x,n} |d^3\sigma_i| \geq n_n d^3\sigma_t}$$



https://slideplayer.com/slide/14963596/

$$d^3\sigma_t = u \cdot d^3\sigma$$

$$d^3\sigma_i = -X_i \cdot d^3\sigma$$

$$\int_p = \int \frac{d^3p}{h^3 E_p}$$

Momentum

- Sample local-rest-frame momentum p_i from the distribution $Q_n(p_i)$
  - Here only show method for linear δf_n

$$Q_n(p_i)d^3p_{LRF} \sim \frac{d^3p}{E_p}\, p\cdot d^3\sigma\,\left(f_{eq,n} + \delta f_n\right)\Theta(p\cdot d^3\sigma)\,\Theta\left(f_{eq,n} + \delta f_n\right)$$

$$= R_n(p_i)d^3p_{LRF} \times \frac{f_{eq,n}}{R_n(p_i)}\left(d^3\sigma_t - \frac{p_i d^3\sigma_i}{E}\right)\Theta\left(d^3\sigma_t - \frac{p_i d^3\sigma_i}{E}\right)\left(1 + \frac{\delta f_n}{f_{eq,n}}\right)\Theta\left(f_{eq,n} + \delta f_n\right)$$

$$\sim R_n(p_i)d^3p_{LRF} \times w_n(p_i)$$

- Draw p_i from distribution $R_n(p_i)d^3p_{LRF}$ and accept sample with probability $w_n(p_i)$
  - Choose $R_n(p_i)$ such that $0 \leq w_n \leq 1$

$$\boxed{w_n(p_i) = w_{eq} \times w_{d\sigma} \times w_{\delta f}}$$

Acceptance-Rejection Method

$$R_n(p_i)d^3p_{LRF} = \begin{pmatrix}\exp(-p/T)\\ \exp(-E/T)\end{pmatrix} \times p^2\, dp\, d\cos\theta\, d\phi \qquad \begin{pmatrix}\text{pions}\\ \text{heavy hadrons}\end{pmatrix}$$

$$\boxed{w_n(p_i) = w_{eq} \times w_{d\sigma} \times w_{\delta f}}$$

$$w_{eq,\pi} = \frac{\exp(p/T)}{\exp(E/T) - 1}$$

($w_{eq,\pi} \leq 1$ for $\mathrm{m}/T \geq 0.8554$ or $T \leq 157.8$ MeV)

$$w_{eq,heavy} = \frac{p}{E}\frac{\exp(E/T)}{\exp(E/T) \pm 1}$$

($w_{eq,heavy} \leq 1$ for $\mathrm{m}/T \geq 1.008$)

$$w_{d\sigma} = |d^3\sigma|^{-1}\left(d^3\sigma_t - \frac{p_i d^3\sigma_i}{E}\right)\Theta\left(d^3\sigma_t - \frac{p_i d^3\sigma_i}{E}\right)$$

$$d^3\sigma_t - \frac{p_i d^3\sigma_i}{E} \leq d^3\sigma_t + |d^3\sigma_i| = |d^3\sigma|$$

$$w_{\delta f} = \left(1 + \frac{\delta f_n}{f_{eq,n}}\right)\Theta(f_{eq,n} + \delta f_n) \to \frac{1}{2}\left(1 + \frac{\delta f_{reg,n}}{f_{eq,n}}\right)$$

$$|\delta f_{reg,n}| \leq f_{eq,n}$$

(lower bound from Θ function)

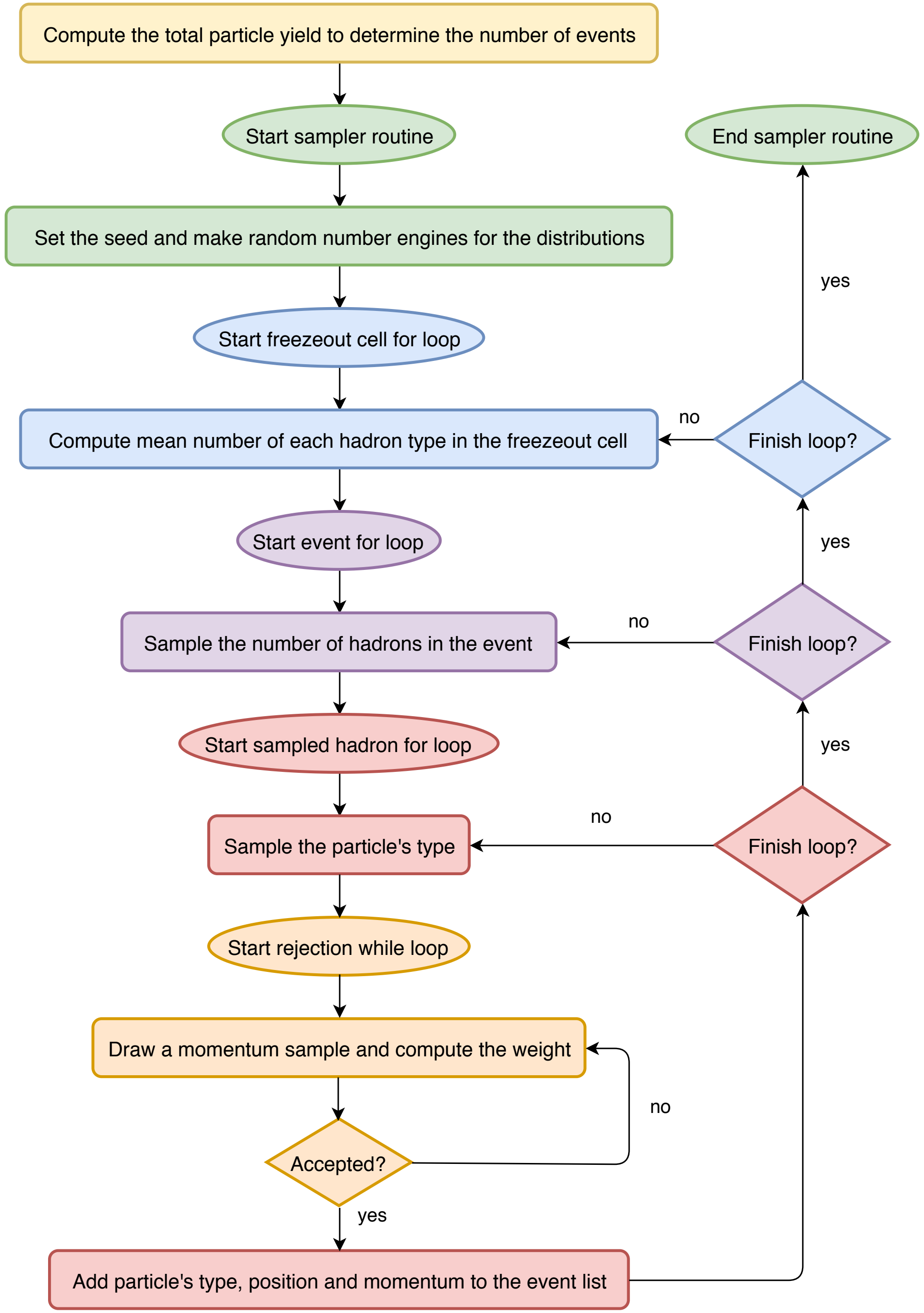
(place additional upper bound)

Program Flow Chart

$$N_{events} \sim \frac{N_{sampled}}{N_{mean}}$$

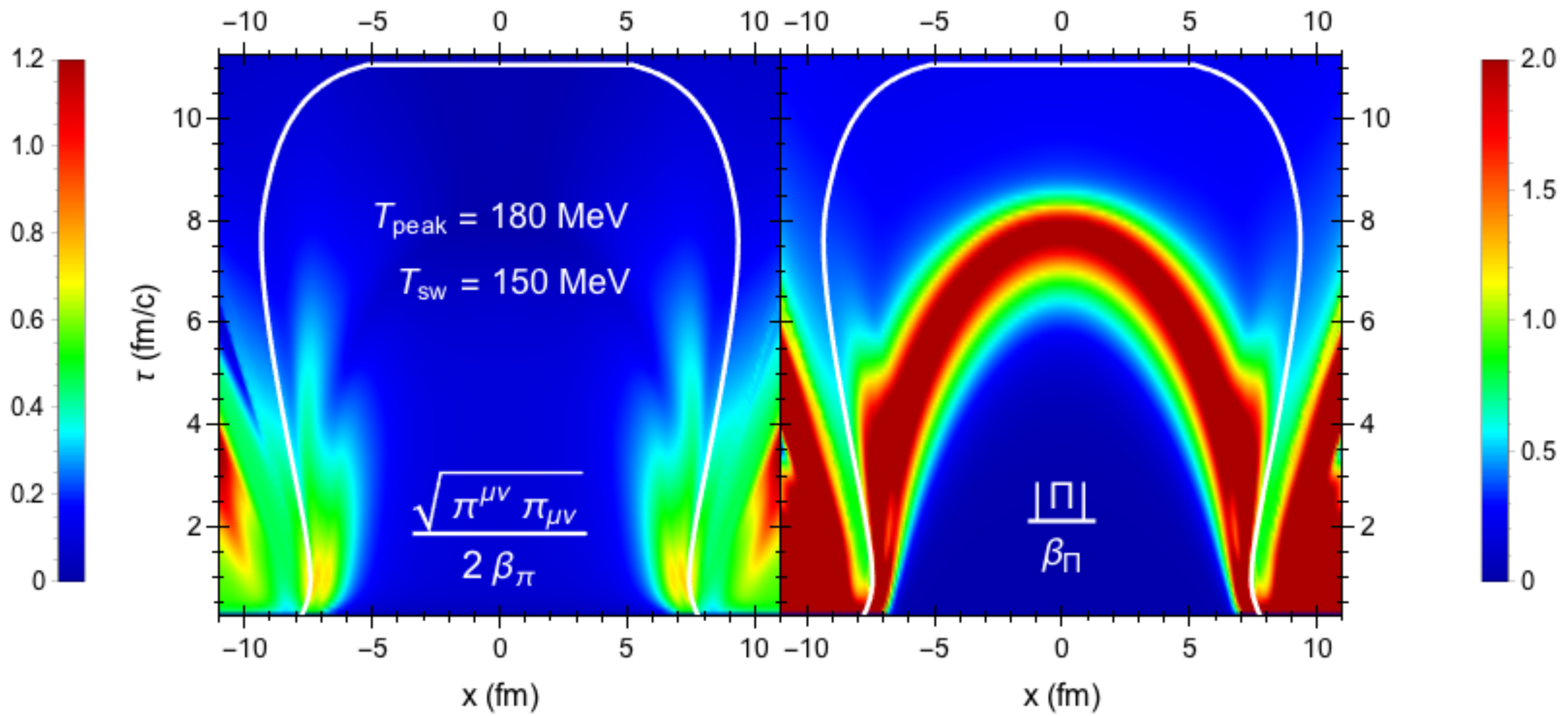
- $N_{sampled}$ = statistical ensemble
  - user input parameter

- N_{mean} = total mean particle yield



Performance

- Central Pb+Pb collision
  - Smooth Glauber initial conditions
  - T_{cent} = 0.6 GeV
  - τ_0 = 0.25 fm/c
- 2+1d hydrodynamics (GPU-VH)
  - η/s = 0.2
  - T_{peak} = 180 MeV (ζ/s peak)
- Cooper Frye Formula (iS3D)
  - T_{sw} = 150 MeV
  - Sample (π^+, K^+, p) separately
    - $N_{sampled}$ = 10^7 particles
    - Δp_T = 0.03 GeV
    - Average over (y, ϕ_p)



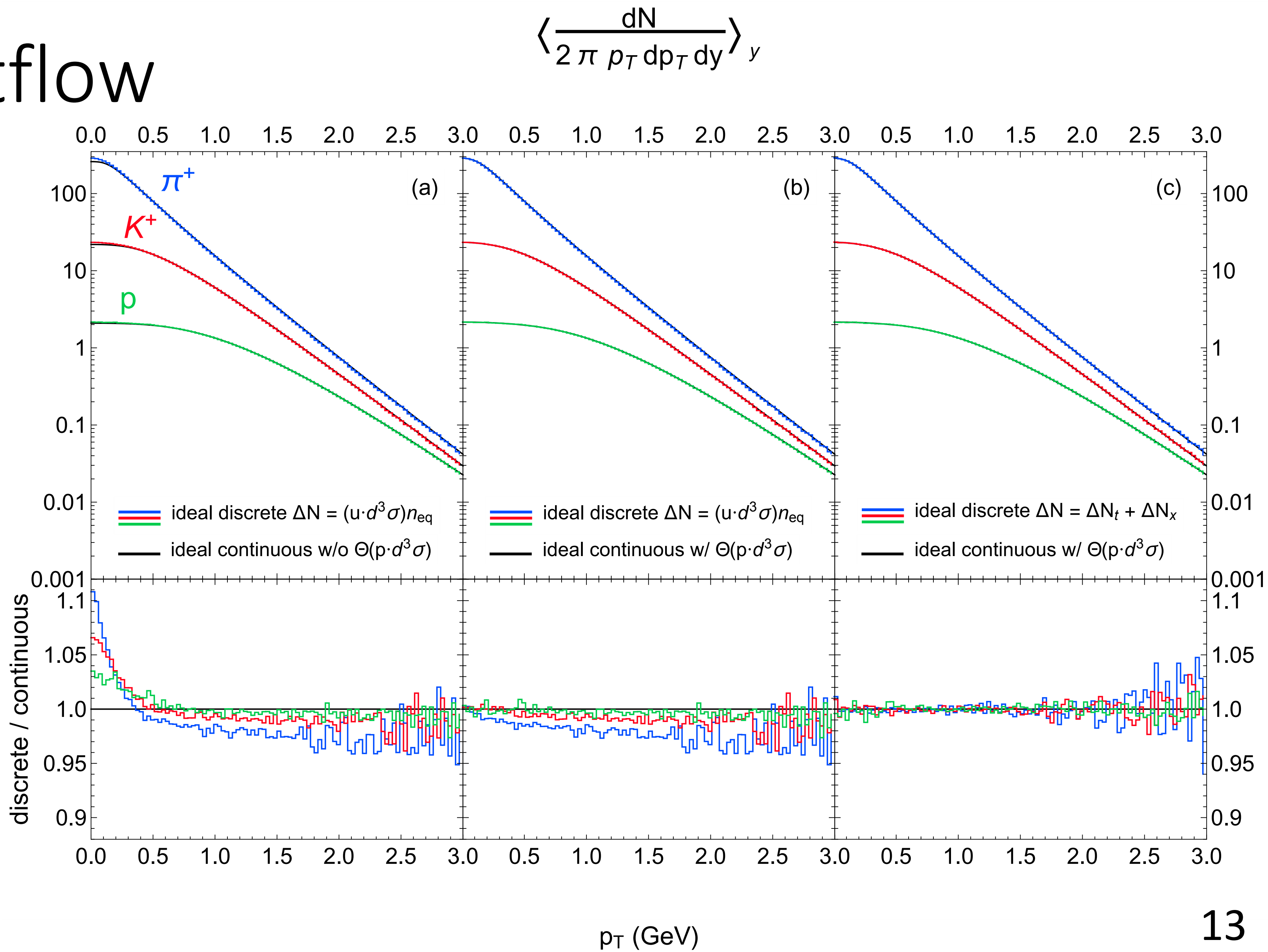
D. Bazow, U. Heinz and M. Strickland, Comput. Phys. Commum. 225 (2018) 92-113

Effect of Outflow

(a) Both discrete and continuous spectra have same yield but different momentum distributions.

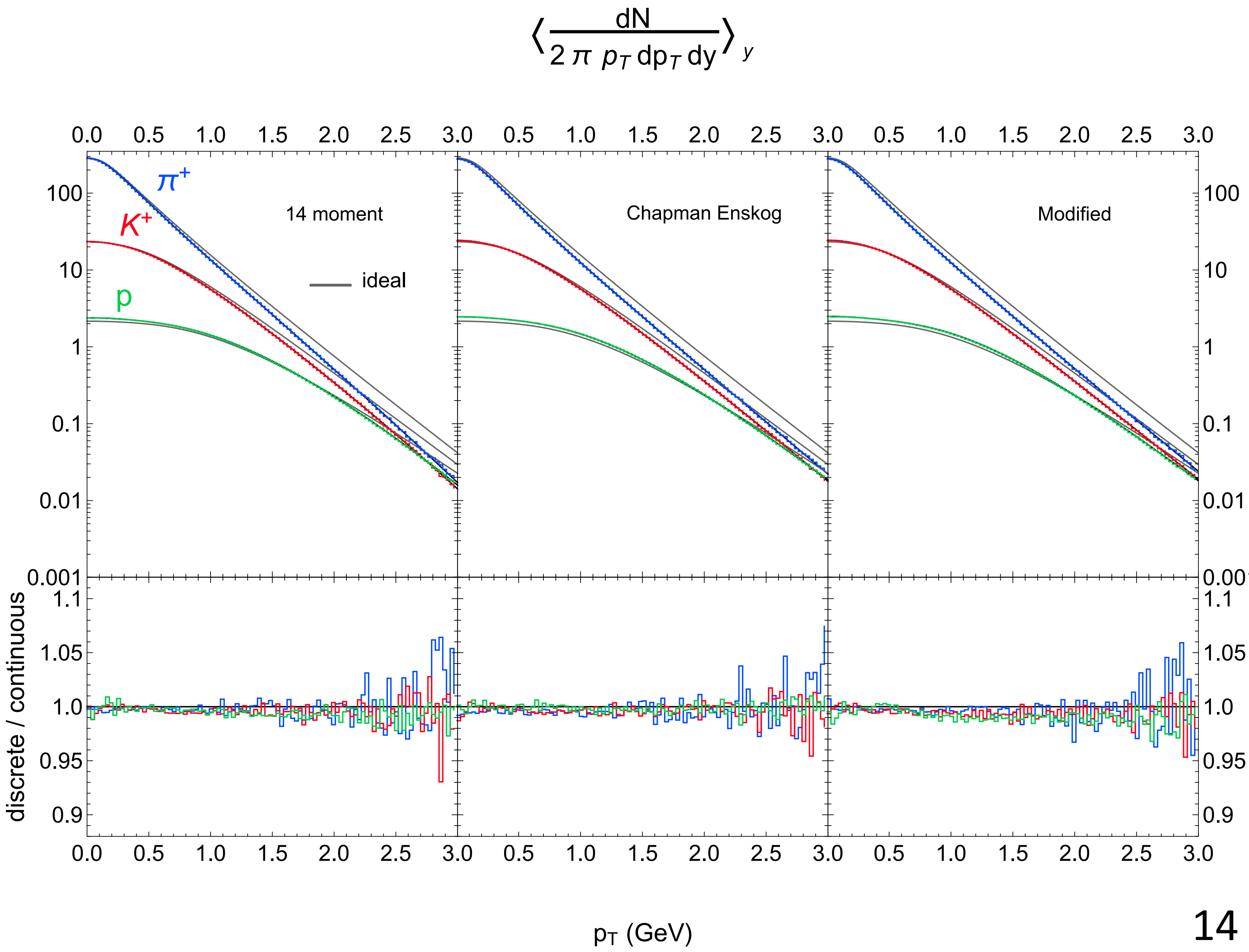
(b) Better agreement but sampler underestimates yield by a few percent.

(c) $\Theta(p \cdot d^3\sigma)$ correction to sampled yield necessary for self-consistency and precision testing of the sampler.

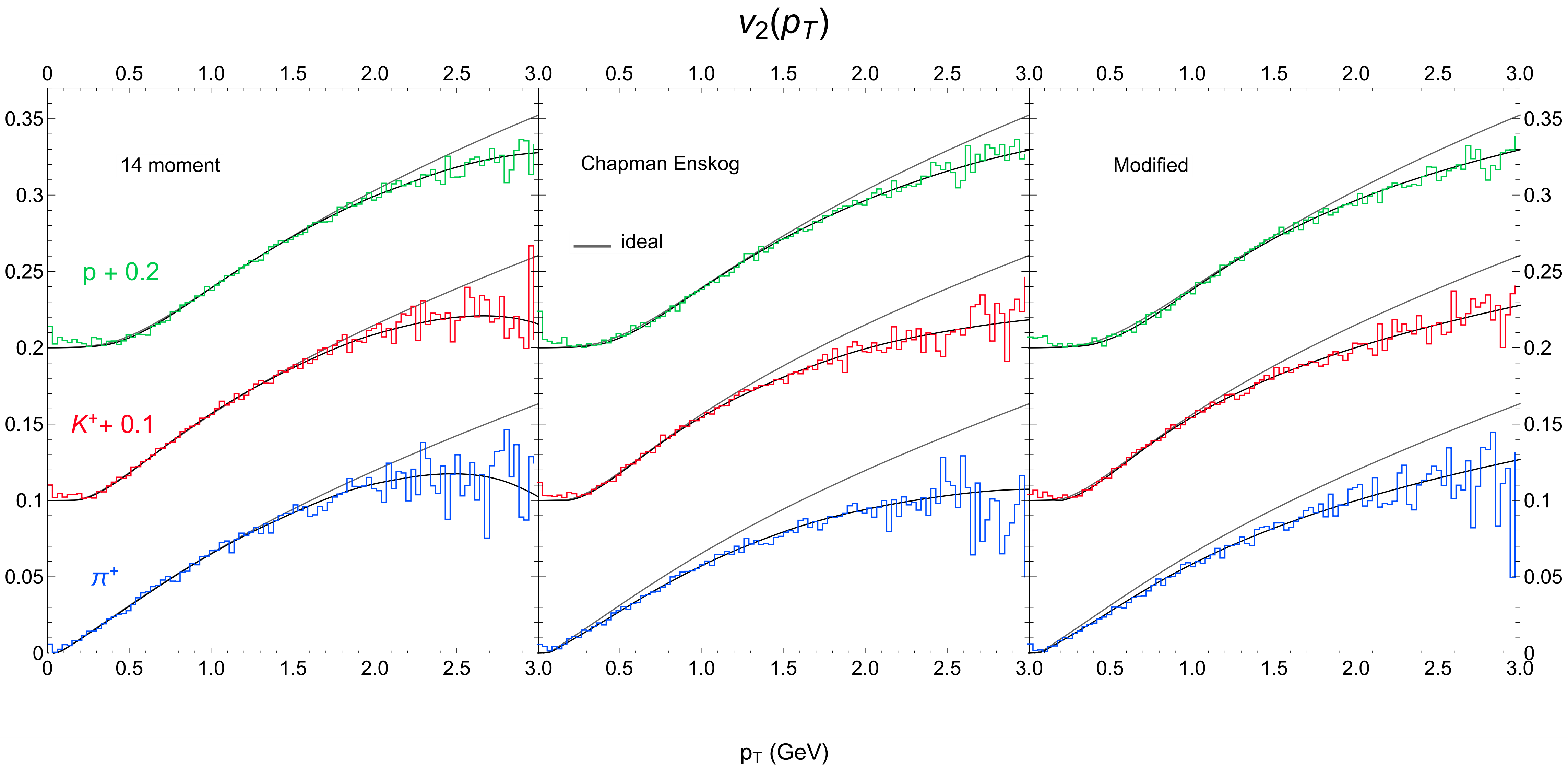


Viscous Corrections

- Shear + bulk viscous corrections to the spectra
  - 14 moment
  - Chapman Enskog
  - Modified (McNelis)
- Overall, good agreement but sampled particle yield slightly underestimated
- Difficulties in calculating ΔN_n with outflow and viscous corrections
  - Especially modified distribution



Non-central collision (b = 5 fm)



Achievements and Outlook

- Incorporated a variety of modes in iS3D:
    - Continuous and sampled spectra (boost invariant or non-boost invariant)
    - Viscous δf corrections (linear and modified)

- Tested the particle sampler for central and non-central collisions
    - Event-averaged p_T spectra and v_2 in good agreement with the Cooper Frye formula

- To do list:
    - Test the sampled spacetime distributions
    - Continue testing particle sampler integration in JETSCAPE
    - Add Jonah's modified distribution to iS3D