Emberi Erőforrások Minisztériuma
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Non-extensive Motivated Parton Fragmentation Functions

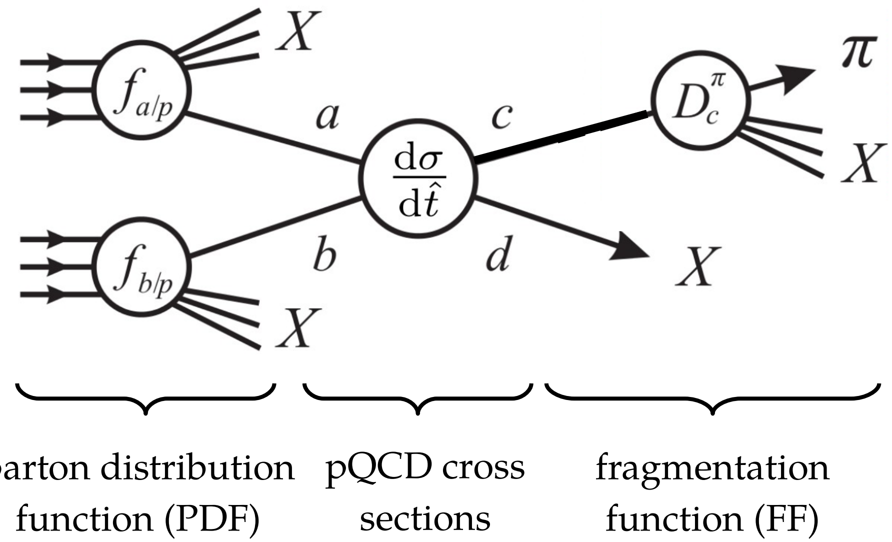
[arXiv:1811.01974]

Collaborators: Gergely G. Barnafoldi, Gabor Papp, Tamas S. Biro

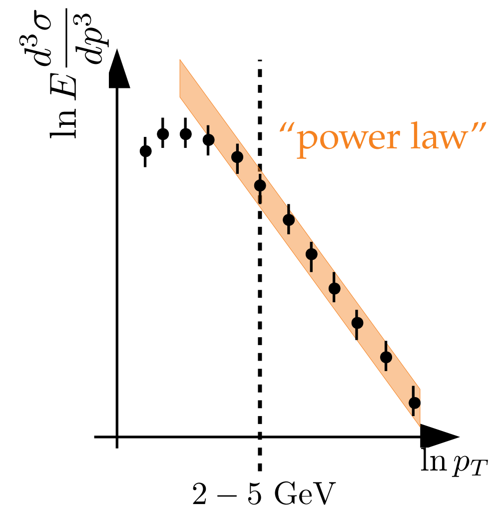
01/13/2019 – JETSCAPE Workshop, Texas A&M University, US

Hadron Spectrum in the pQCD Parton Model

$$E_h \frac{\mathrm{d}^3\sigma_h^{pp}}{\mathrm{d}^3p_h} \sim$$



$\sim$



- Investigate the uncertainties of the hadron spectrum.
- Uncertainties at low p_T.
- Can we improve the predictions?
- We're interested in FF. [A. Takacs et al Entropy **19** 3, 88 (2017), R. Ichou and D. d'Enterria Phys.Rev. D**82**: 014015 (2010)]

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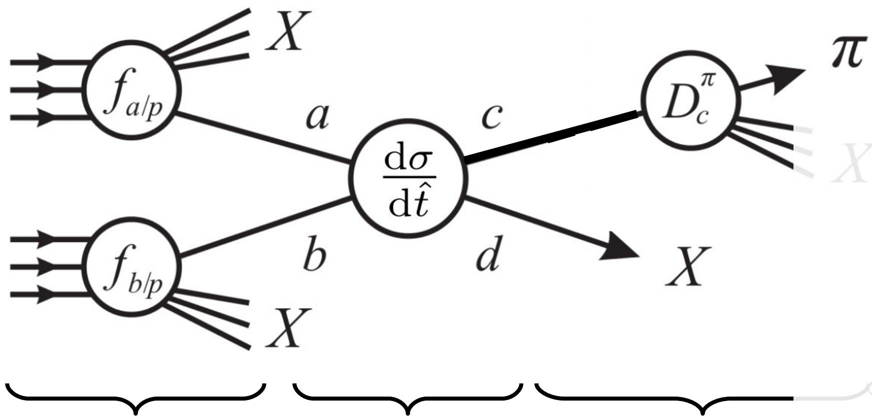
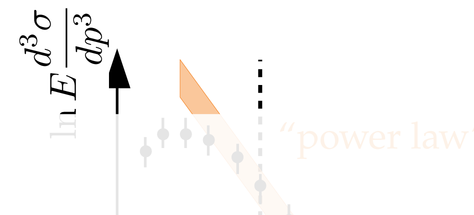
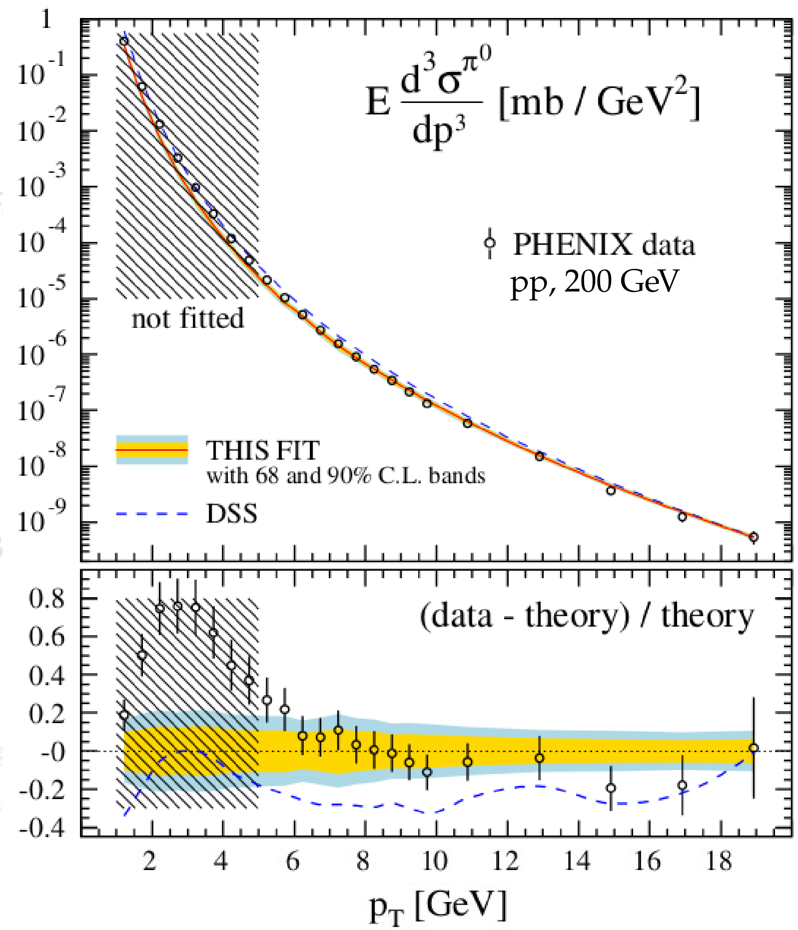


parton distribution function (PDF) | pQCD cross sections | fragmentation function (FF)





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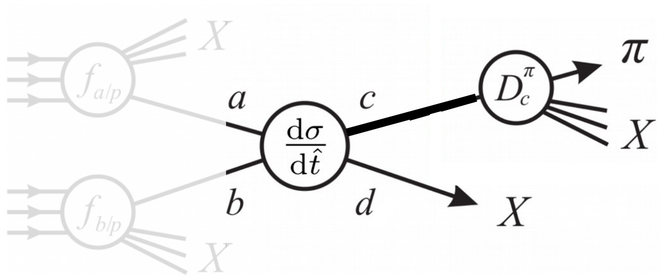
Partonic Fragmentation Functions

Electron-positron annihilation: no PDF!



Hadronic cross section:

$$\frac{d\sigma^h}{dx} = \sum_i^{q,\bar{q},g} \int_x^1 \frac{dz}{z} \underbrace{C_{P,i}(z, \alpha_s(\mu), Q, \mu)}_{\text{known from pQCD}} \underbrace{D_i^h(\frac{x}{z}, \mu)}_{\text{arbitrary function!}}$$

DGLAP evolution:

$$\frac{dD_j^h(x,\mu)}{d\ln\mu^2} = \sum_i^{q,\bar{q},g} \int_x^1 \frac{dz}{z} \overbrace{P_{ij}(\frac{x}{z}, \mu)} \overbrace{D_i^h(z,\mu)}$$

Probability interpretation (from energy conservation):

$$1 = \sum_h \int_0^1 dz\, z D_i^h(z, Q)$$

Parametrizing Fragmentation Functions

1. Make an **ansatz** for the FF functional form at a low Q_0

   Most used assumption (AKK, KKP, HKN, DSS) has 3 parameters/channel

$$D_i^h(z, Q_0^2) = N_i^h(Q_0^2) z^{\alpha_i^h(Q_0^2)} (1-z)^{\beta_i^h(Q_0^2)}$$

2. Using DGLAP, evolve the scale from $Q_0 \to Q_{\text{exp}}$

$$D_i^h(z, Q_0^2) \longrightarrow D_i^h(z, Q_{exp}^2) = N_i^h(Q_{exp}^2) z^{\alpha_i^h(Q_{exp}^2)} (1-z)^{\beta_i^h(Q_{exp}^2)}$$

3. Calculate the spectrum

$$\frac{d\sigma^h}{dx} = \sum_i^{q,\bar{q},g} \int_x^1 \frac{dz}{z} C_{P,i}(z, \alpha_s(\mu), Q, \mu) D_i^h(\frac{x}{z}, Q_{exp}^2)$$

4. Compare the spectrum to the measured data and change the parameters

$$\alpha_i^h(Q_0^2 \approx 1\,\text{GeV}^2) = [-3, -1]$$
$$\beta_i^h(Q_0^2 \approx 1\,\text{GeV}^2) = [4, 7]$$

5. It works!

Parametrizing Fragmentation Functions

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2. Using DGLAP, evolv

$D_i^h(z, Q_0^2)$ ... $z)^{\beta_i^h(Q_{exp}^2)}$

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$\frac{d\sigma^h}{dx}$ ... $_{xp})$

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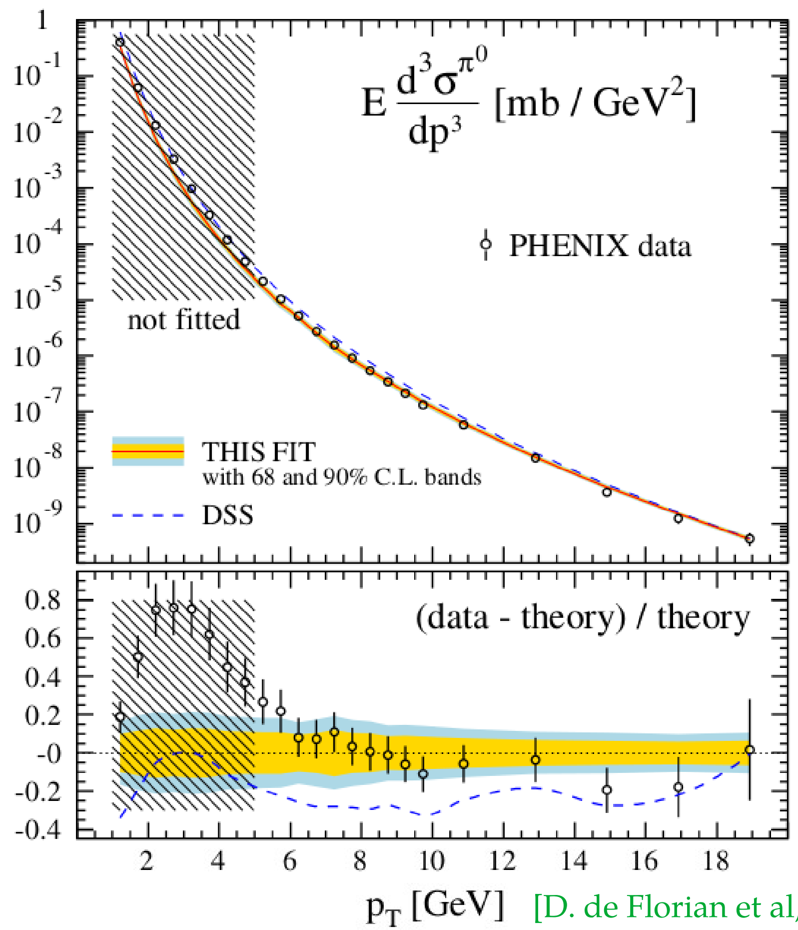

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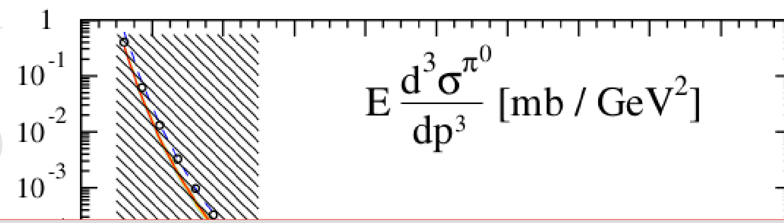

$D_i^h(z, Q_0^2)$ … $z)^{\beta_i^h(Q^2_{exp})}$

Complications:

- The $z^\alpha (1-z)^\beta$ form of FF is an assumption
- Number of parameters ~ 15 for a hadron
- DGLAP evolution for points and not for the function: the evolved points do not necessarily follow the assumed form: there is no $N_i^h(Q^2), \alpha_i^h(Q^2), \beta_i^h(Q^2)$
- Many fits use *pp* data → FF correlates with PDF
- Fits do not consider the energy conservation

$$1 \neq \sum_h \int_0^1 dz\, z D_i^h(z, Q)$$

- The resulted parameters violate the probability interpretation

$$\alpha_i^h < -2$$

Parametrizing Fragmentation Functions

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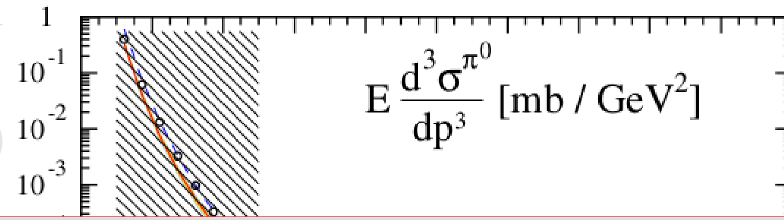


$D_i^h(z,Q_0^2)$ … $z)^{\beta_i^h(Q^2_{exp})}$

Idea: find a FF form, which is more suitable for the analysis (maybe has some physics)

Assumption 1: Tsallis distribution

- 3 parameters
- Starts in exponential
- Ends in power law

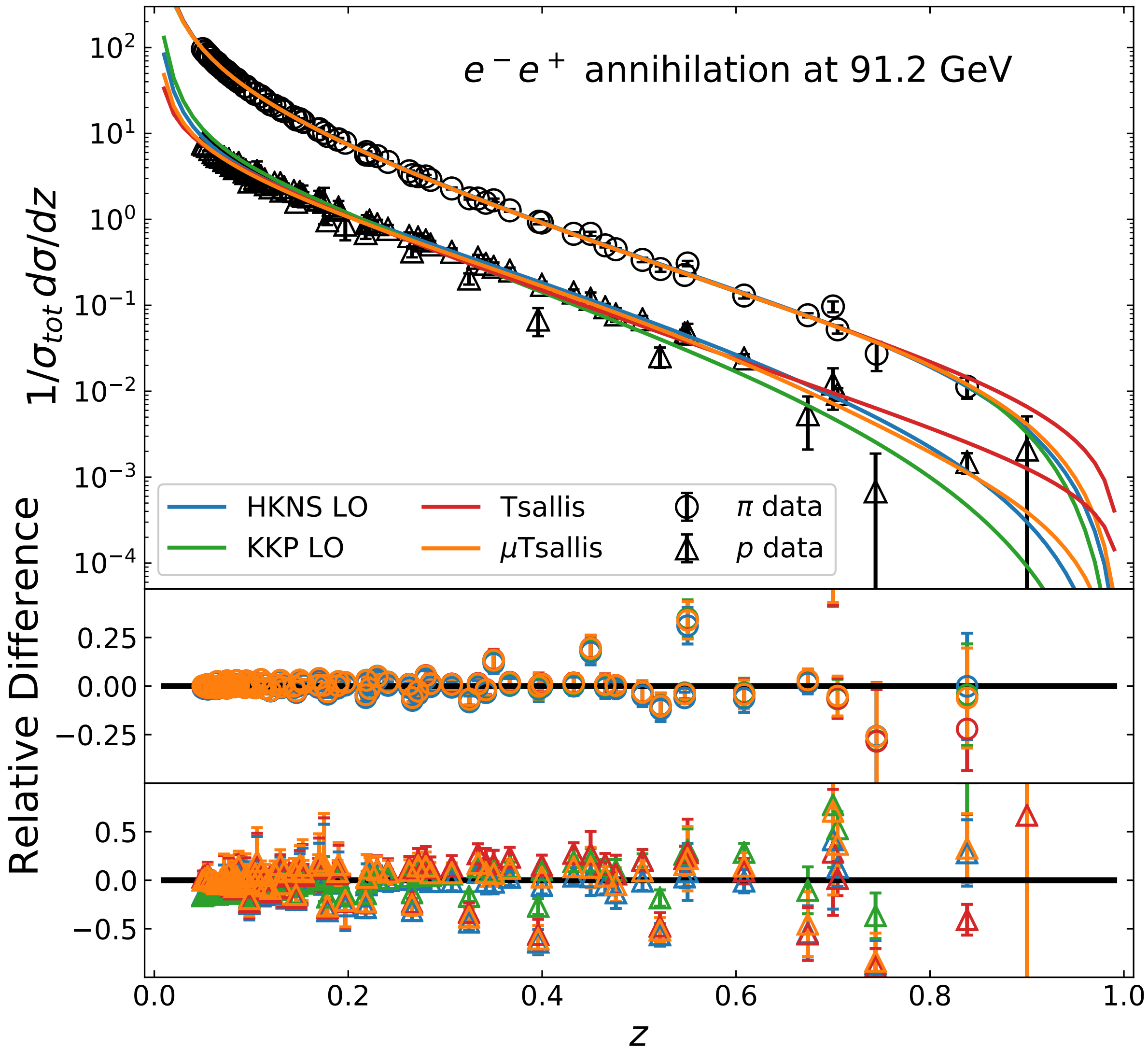
$$D_i^h(z,Q^2) = A_i^h(Q^2)\left[1 + \frac{q_i^h(Q^2)-1}{T_i^h(Q^2)} z\right]^{-\frac{1}{q_i^h(Q^2)-1}}$$

Assumption 2: Microcalonical Tsallis distribution from jet studies [K. Urmossy, G.G. Barnafoldi, T.S. Biro Phys.Lett. B701 111 (2011)]

- 3 parameters
- Starts in power law
- Ends in power law
- Normalized

$$D_i^h(z,Q^2) = A_i^h(Q^2)(1-z)\left[1 - \frac{q_i^h(Q^2)-1}{T_i^h(Q^2)} \log(1-z)\right]^{-\frac{1}{q_i^h(Q^2)-1}}$$

Resulted Parametrization



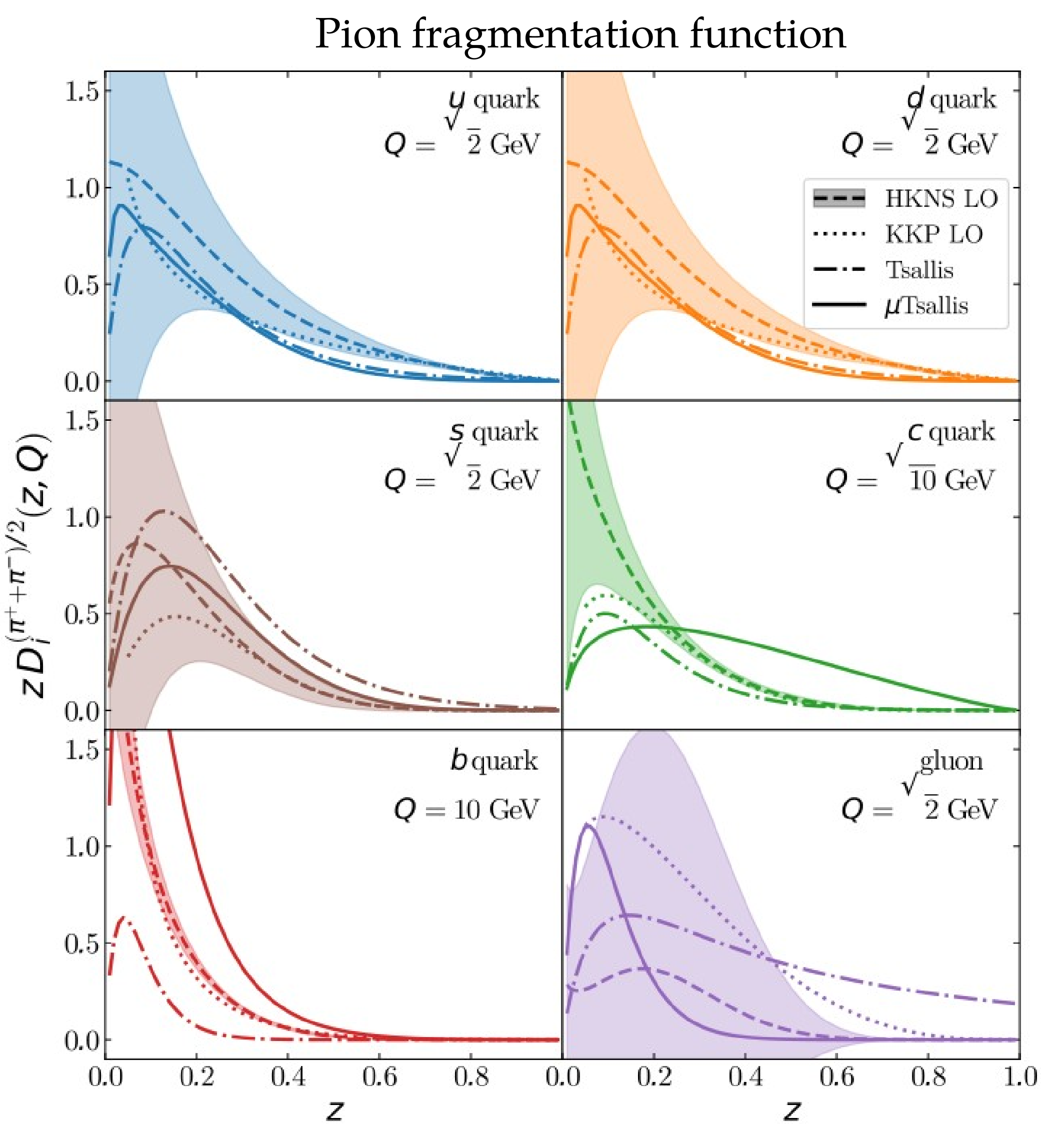
About the(very simple) fit:

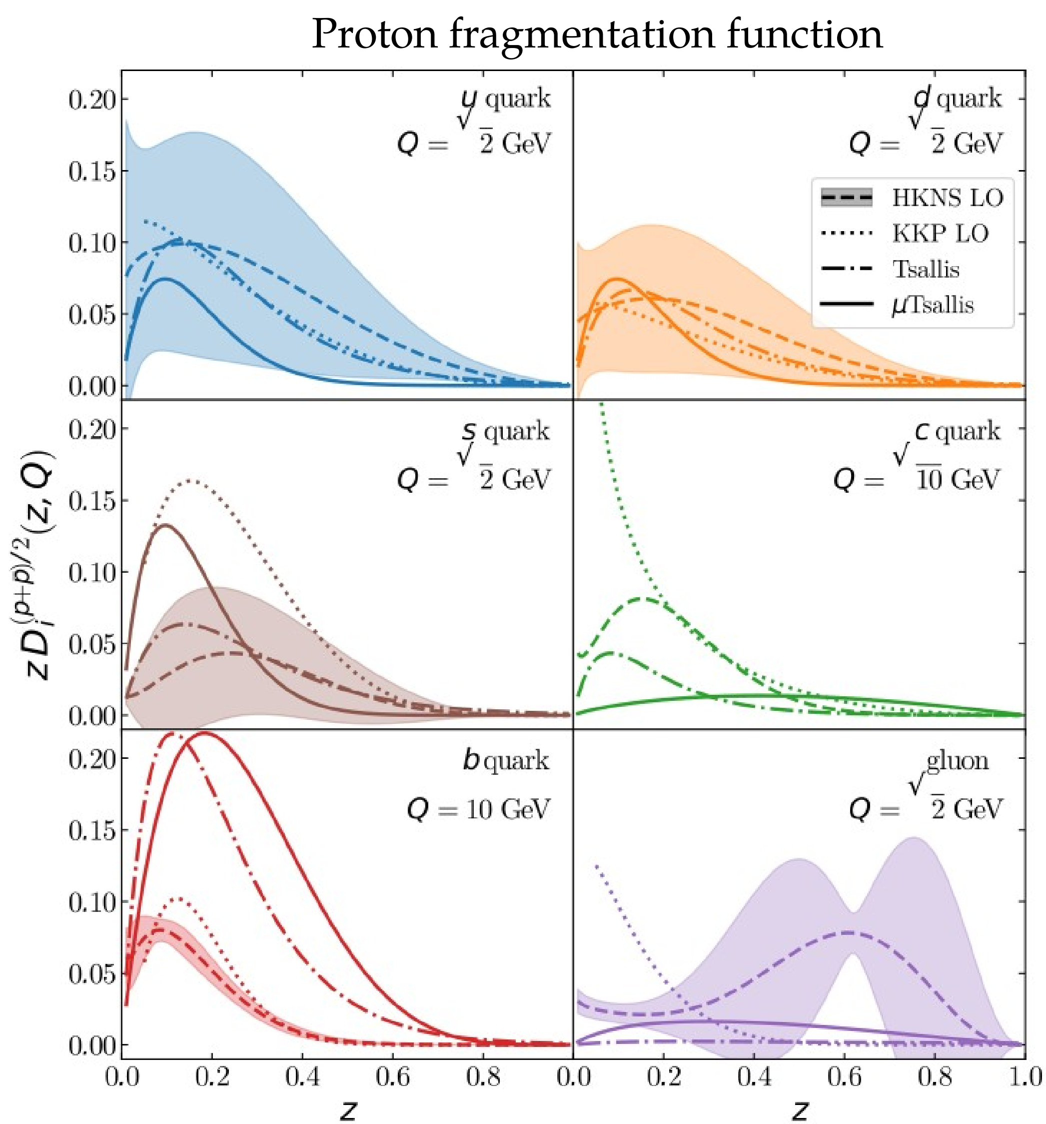
- Chi2 fit, without parameter constraint
- Leading order pQCD, from Q_0=1 GeV
- Only energy: Q_{exp}=91.2 GeV

About the result

- Good agreement with data
- Good agreement with other calculations
- Differs only at very low and high p_T

Resulted Parametrization

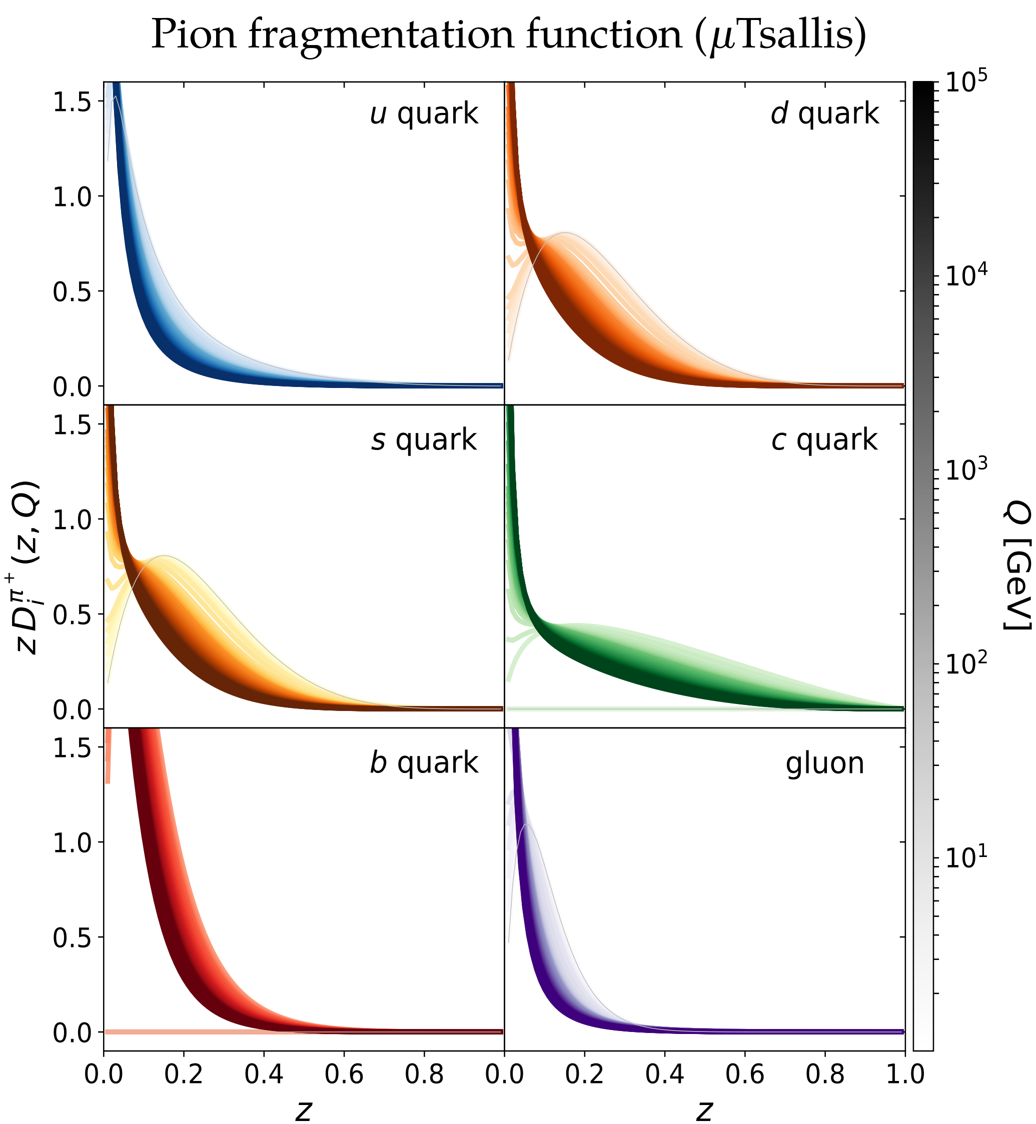


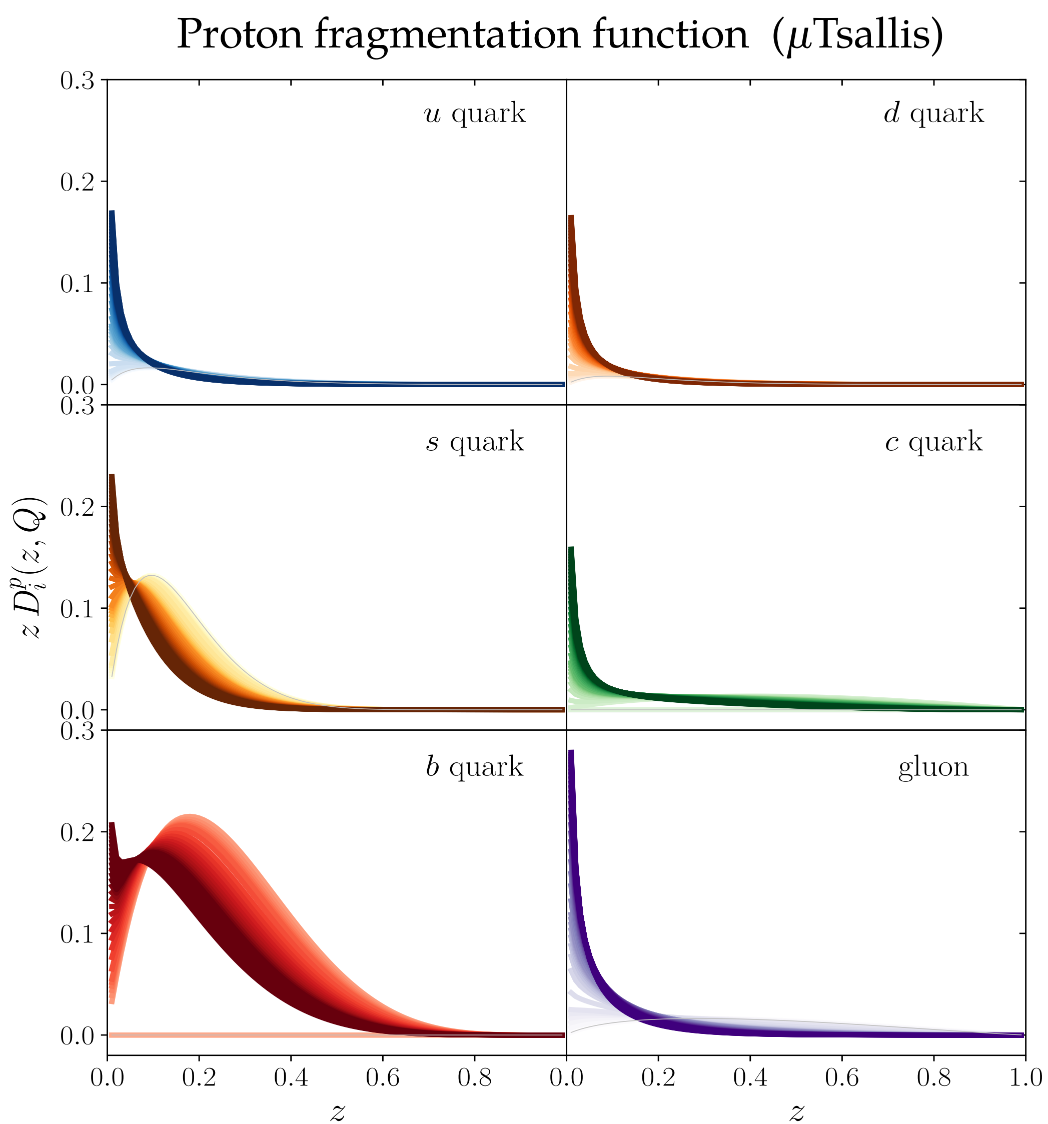


- Similar behaviour
- Differs at low z (peak)
- Differs in c and b-quark: no tagged data

[B. Kniehl, G. Kramer, B. Potter, Nuc.Phys. B**582** 514 (2000)]
[M. Kirai, S. Kumano, T. Nagai, K. Sudoh Phys.Rev. D**75** 094009 (2007)]

Scale Evolution by Solving DGLAP (for point to point)

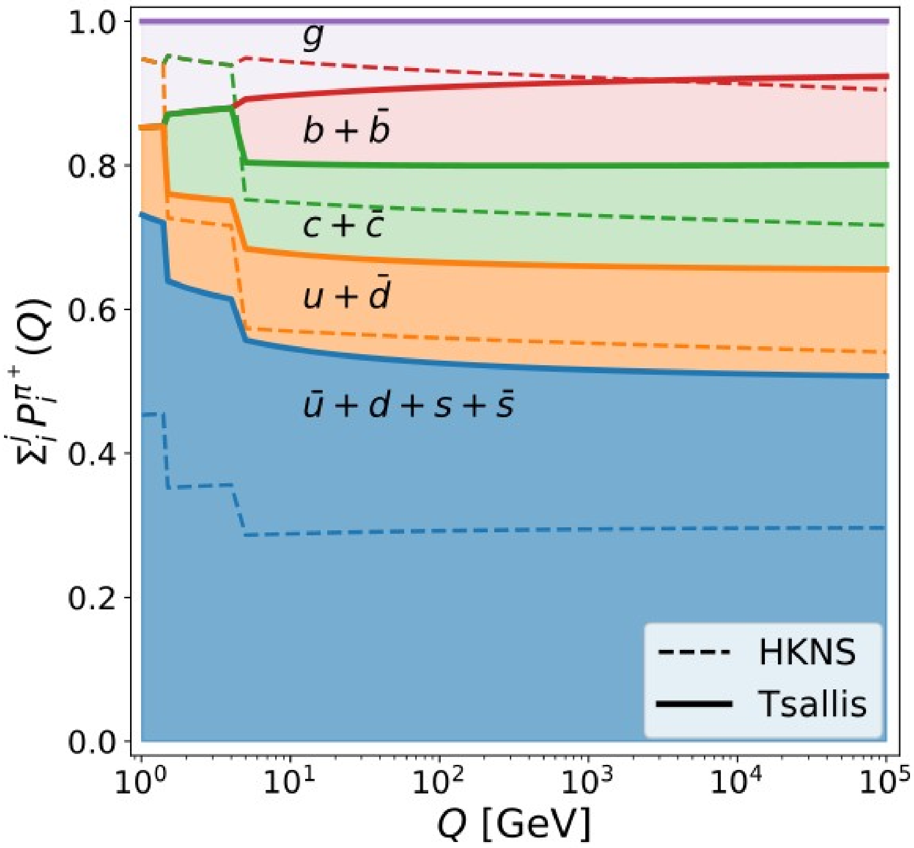




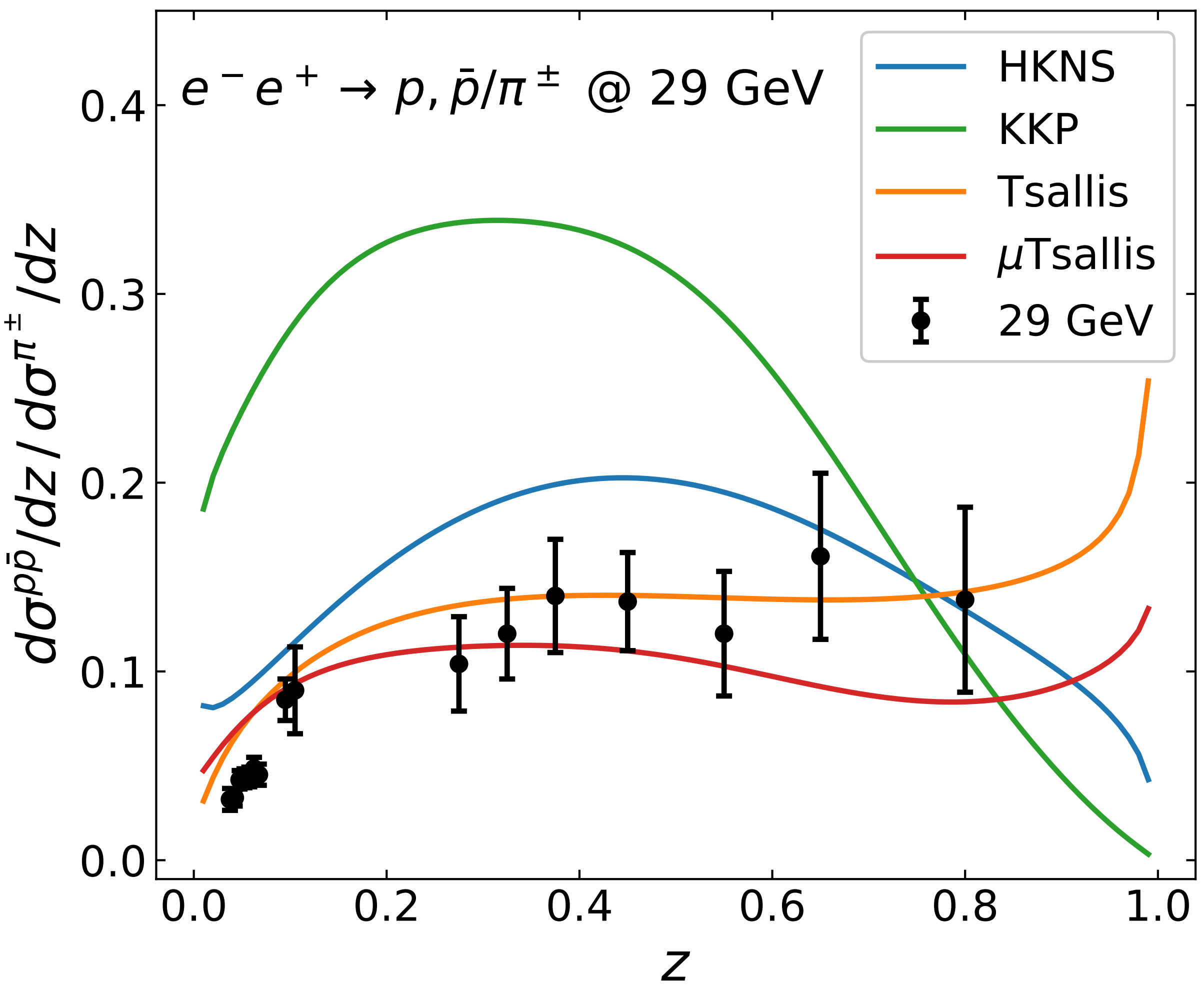
- Shapes become similar for different FF forms → initial differences do not matter at high Q: for e^-e^+: $Q \sim \sqrt{s}$ high energy spectrum, for pp: $Q \sim p_T$ high p_T becomes the same

Scale Evolution of the Yield

Sum Rule $$1 = \sum_h \underbrace{\int_0^1 dz\, z D_i^h(z,Q)}_{P_i^h(Q)}$$



Ratio of produced hadrons



Summary

- pQCD works well, although fragmentation function is arbitrary
- Polynomial based fragmentation works well, but a more suitable fragmentation could exist, we tried Tsallis and Microcanonical Tsallis distribution
- The new formula is tested with exp. data and is compared to other pQCD calculations. It seems more suitable functions could exist

+ More sophisticated parametrization

+ Extract for: photons, heavy flavor, higher orders, ...

+ What is the dynamical description, is there a process, which could lead for these forms?

JET TOOLS 2019

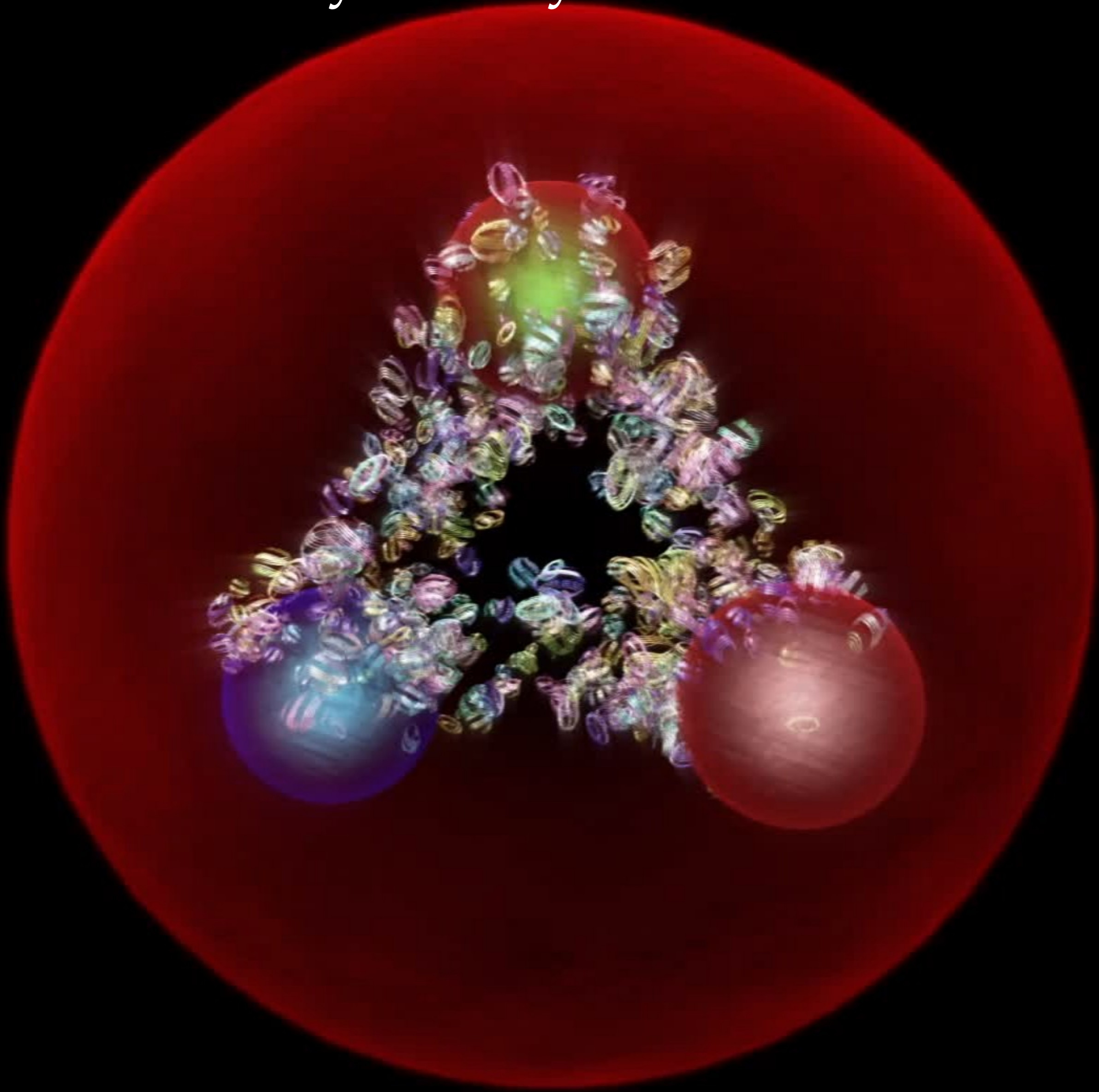
2nd Heavy-Ion Jet Substructure Workshop

13 - 17 May
University of Bergen

- new tools for jet physics at the frontiers
- jet substructure and heavy flavor
- splitting maps of the shower
- jet modifications in small systems
- interplay between jets and underlying event
- statistical & machine-learning techniques

https://jettols.w.uib.no

Thank you for your attention!



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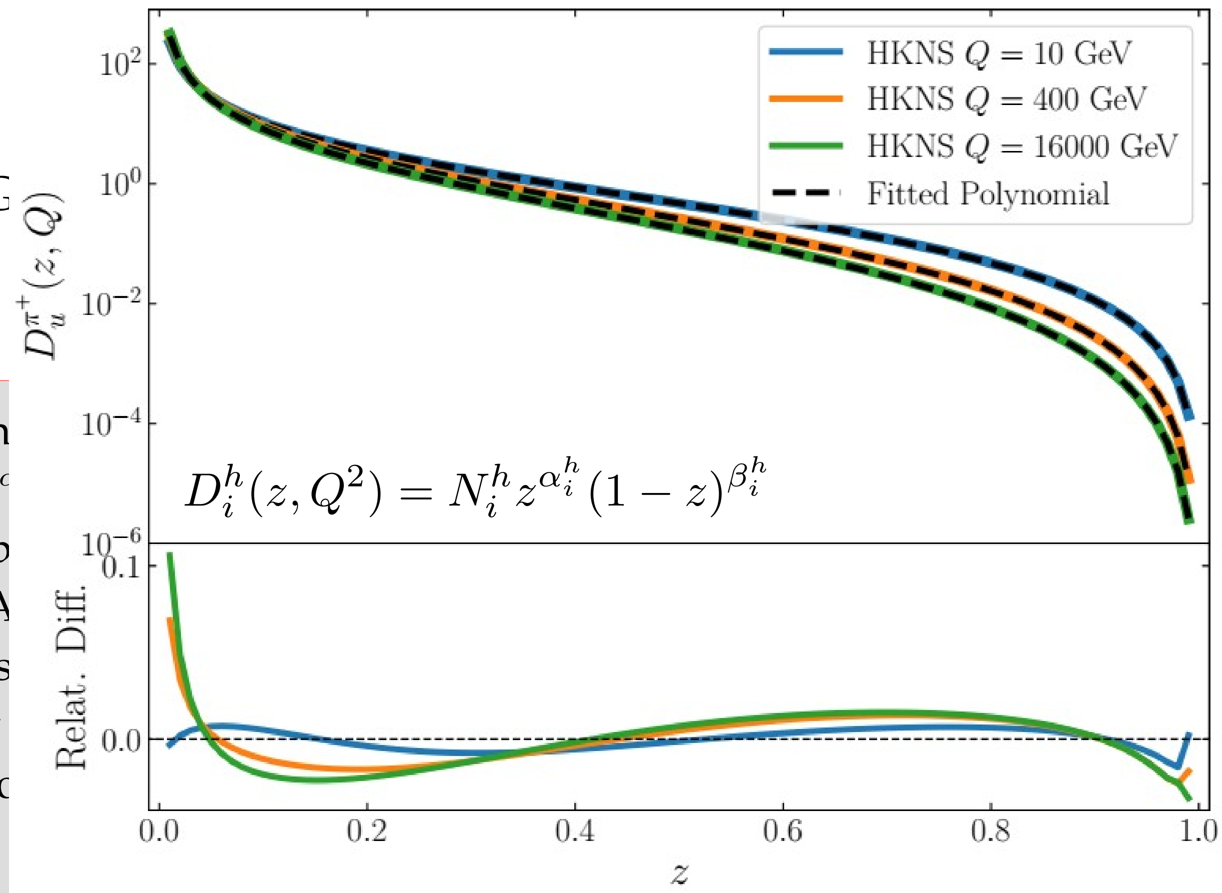

Complication

- The z^{α}
- Numb
- DGLA … s not necess
- Many

  Fits dc

- The resulted parameters violates the probability interpretation

$$\alpha_i^h < -2$$

„Non-Extensive" Statistical Physics

Additive statistics for independent systems:

$$S_B = -\sum_i p_i \ln p_i \longrightarrow S_{12} = S_1 + S_2$$

Non-additive statistics:

$$S_q = \frac{1}{q-1}(1 - \sum_i p_i^q) \longrightarrow S_{q,12} = S_{q,1} + S_{q,2} + (1-q)S_{q,1}S_{q,2}$$

Tsallis-Pareto distribution:

$$f_{TP}(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

Small ε: $\sim e^{-\varepsilon/T}$

High ε: $\sim \varepsilon^{-\frac{1}{q-1}}$