


NLL' resummation for jet mass

Thomas Becher
University of Bern

1901.09038 and JHEP 1808 (2018) with Marcel Balsiger and Dingyu Shao

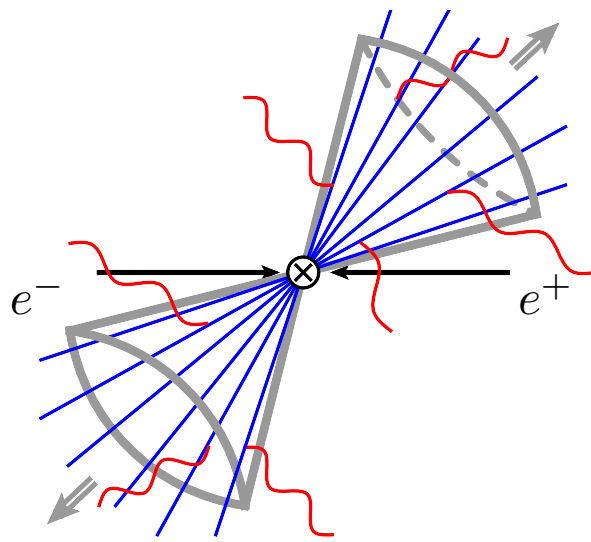
today!



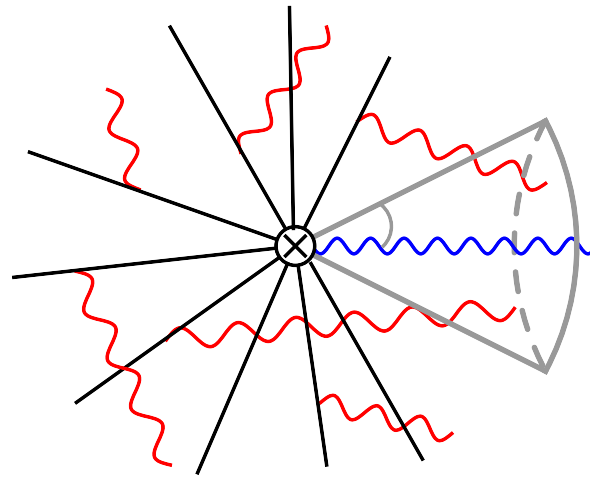
UCLA Santa Fe Jets and Heavy Flavor Workshop,
UCLA, January 28-30, 2019

There has been a lot of progress concerning non-global jet observables over the past few years

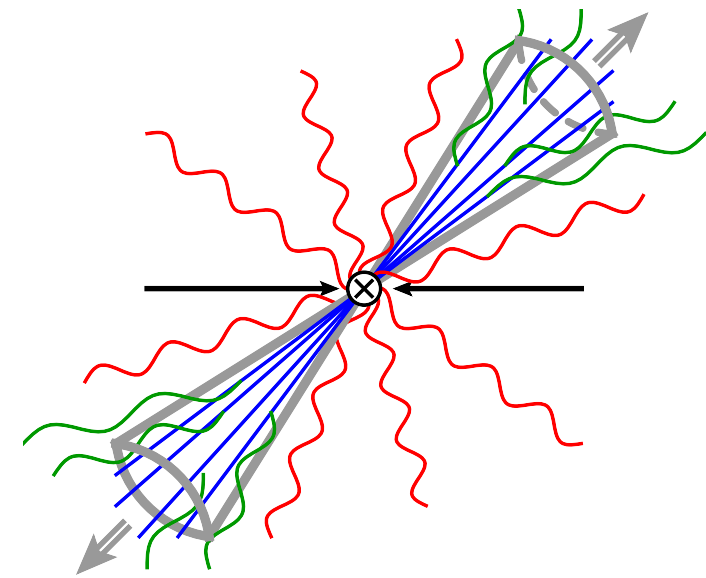
- Color density matrix, [Caron-Huot, JHEP 1803, 036 \(2018\) \[1501.03754\], ...](#)
- Dressed gluon exponentiation, [Larkoski, Moult and Neill, JHEP 1509, 143 \(2015\), ...](#)
- Jet Effective Theory, [TB, Neubert, Rothen and Shao, PRL 116,192001 \(2016\), ...](#)
- Reduced density matrix, [Neill, Vaidya 1803.02372](#)
- Finite- N_c , [Hagiwara, Hatta and Ueda, PPB 756, 254 \(2016\); Angeles Martinez, De Angelis, Forshaw, Plätzer and Seymour, JHEP 1805, 044 \(2018\)](#)
- Double non-global logs, [Hatta, Iancu, Mueller, Triantafyllopoulos JHEP 1802, 075 \(2018\)](#)
- ...



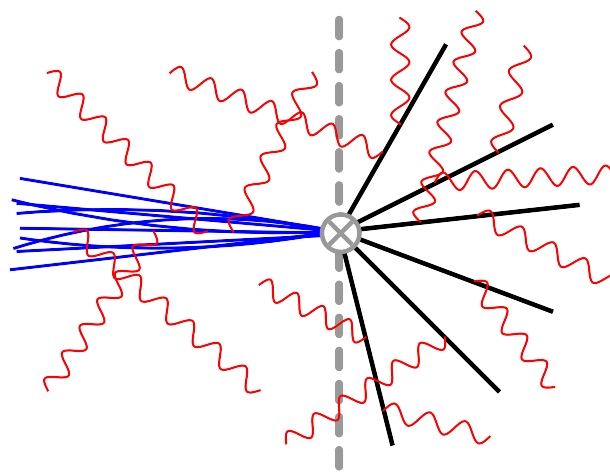
1.) 2.) cone jets,
gaps between jets



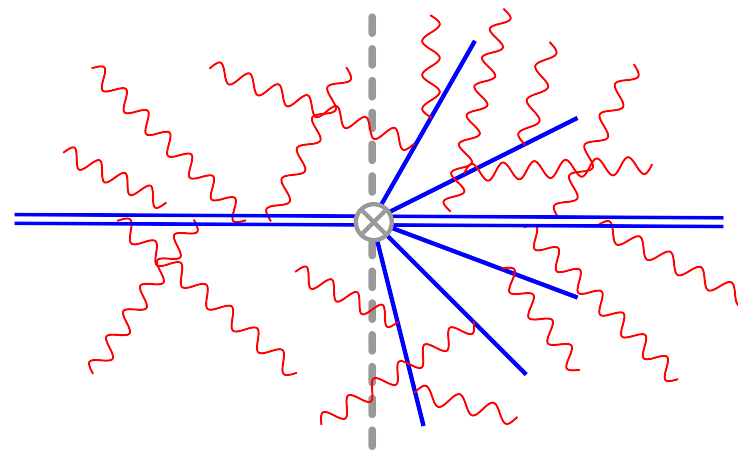
5.) isolation cones



1.) narrow cone jets



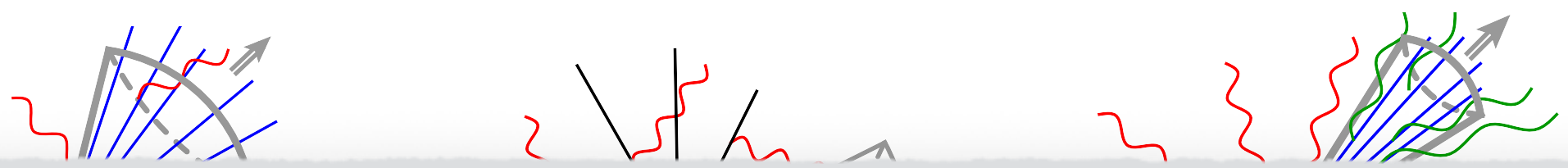
3.) light-jet mass
4.) narrow broadening



3.) hemisphere
soft function

- 1.) 2.) TB, Neubert,
Rothen, Shao '15 '16
- 3.) TB, Pecjak, Shao '16
- 4.) TB, Rahn, Shao '17
- 5.) Balsiger, TB, Shao, '18

Effective field theory for (non-global) jet observables!

- 
- We have obtained factorization theorems for a variety of non-global observables, for both single- and double-log problems.
 - Achieve full scale separation
 - Allow for resummation using RG methods, not restricted to leading logs
 - Verified in several cases, that we reproduce the full logarithmic structure at the two-loop level
 - Implemented leading-log resummation

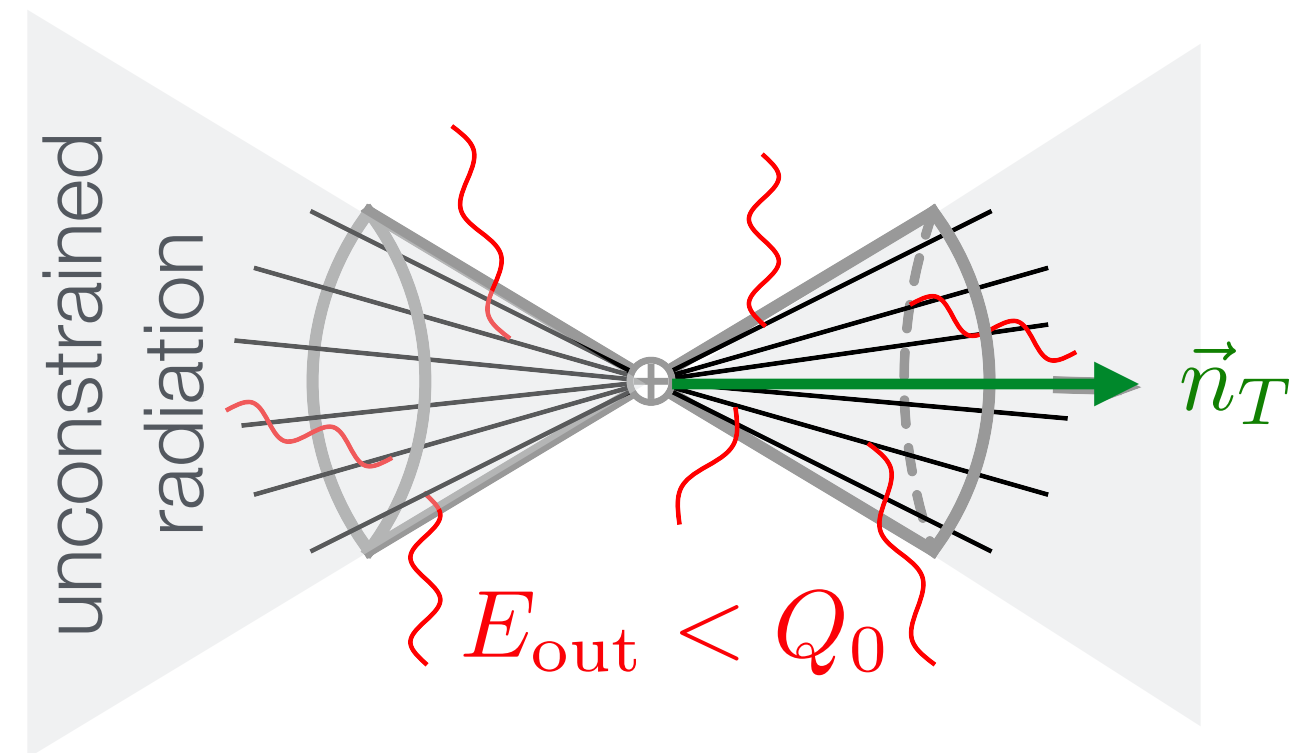
Now: resummation beyond leading NGLs

4.

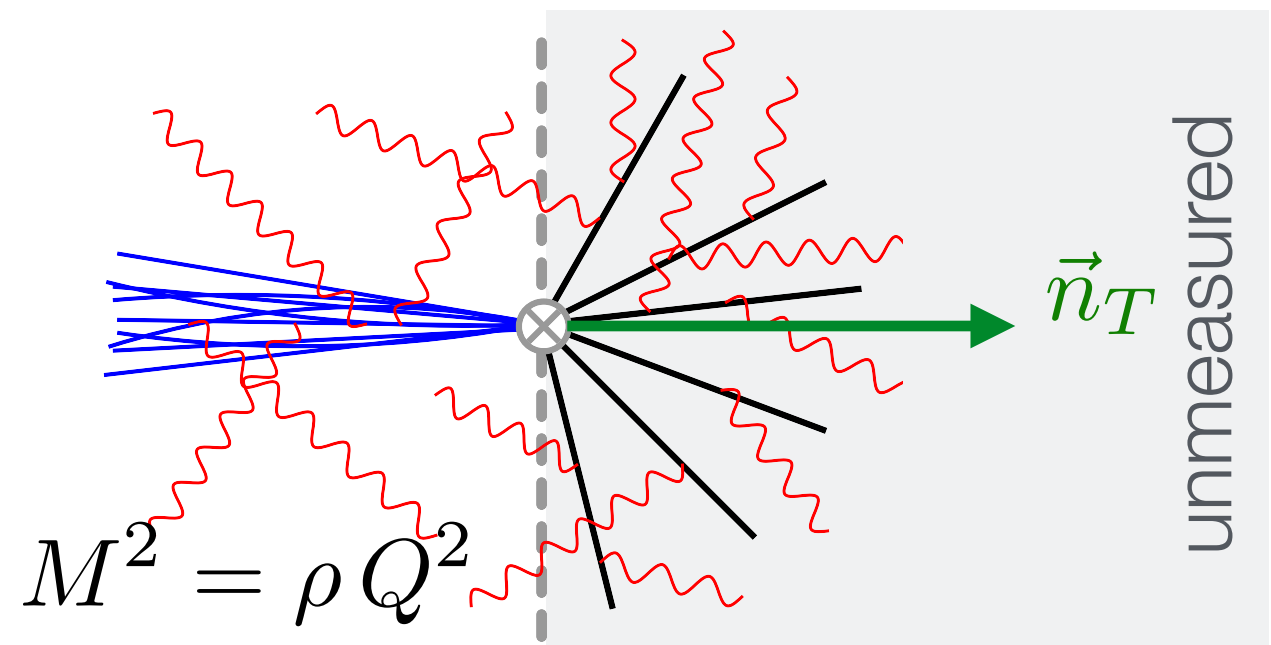
1901.09038 with Marcel Balsiger and Dingyu Shao

Effective field theory for (non-global) jet observables!

Will discuss two simple jet observables in e^+e^- collisions at center-of-mass energy Q



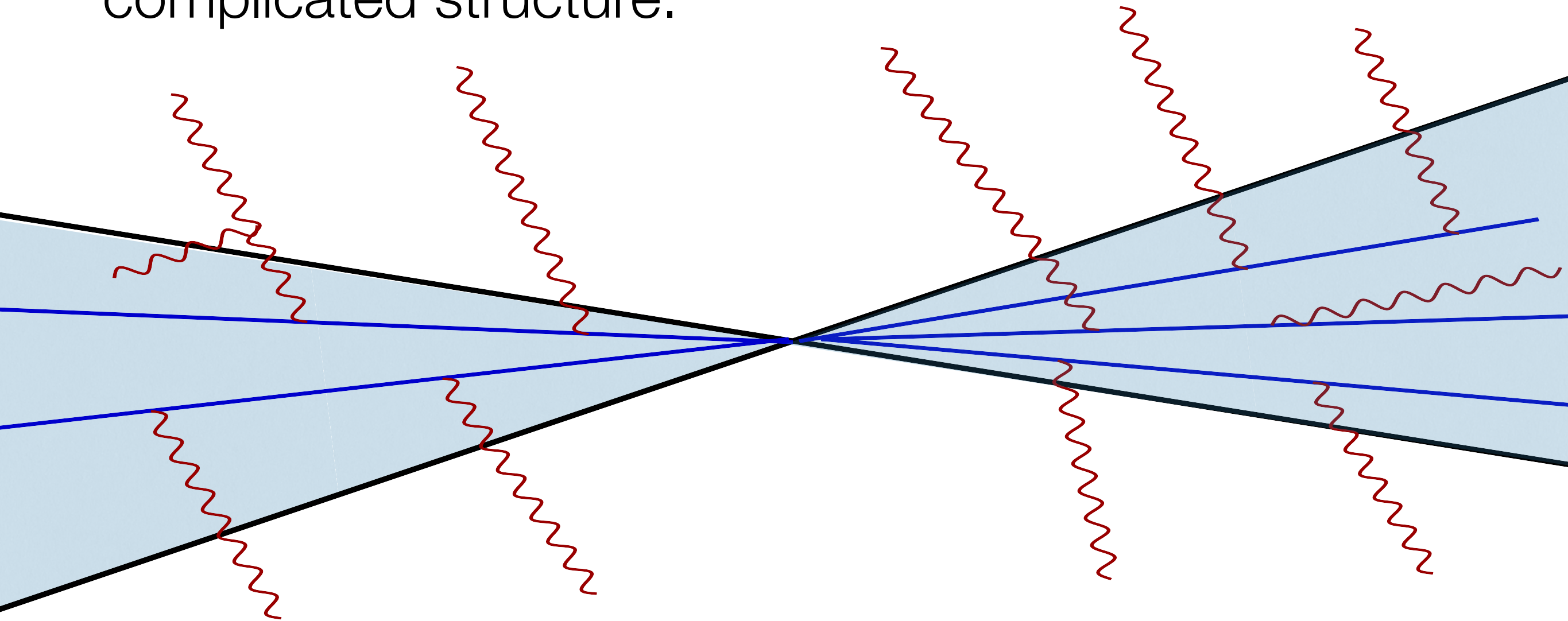
interjet energy flow
gaps between jets
single logarithmic \rightarrow LL'



jet mass
double logarithmic \rightarrow NLL'

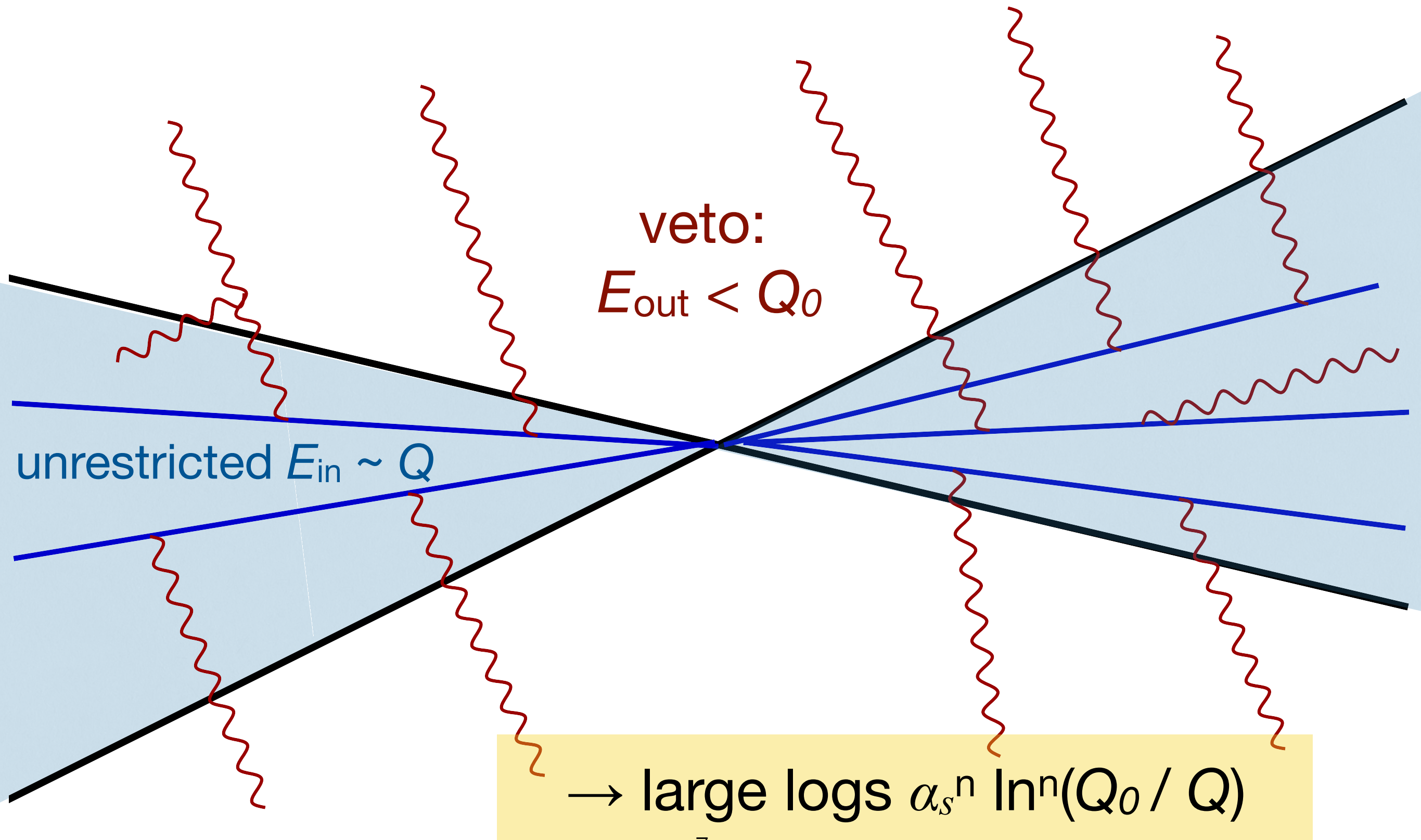
We now include full one-loop hard, jet and soft functions into MC framework for resummation. I will first illustrate our method for simplest case, interjet energy flow.

Soft radiation in jet processes has in general a very complicated structure.



Hard partons inside jets act as sources: **soft radiation** pattern depends on **color-charges** and **directions of all hard partons!**

Interjet energy flow displays complicated pattern of
“non-global logarithms” (NGLs) Dasgupta Salam ‘01



Soft emissions in process with m energetic particles are obtained from the matrix elements of the operator

$$\mathbf{S}_1(n_1) \mathbf{S}_2(n_2) \dots \mathbf{S}_m(n_m) |\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions
of the energetic particles / jets
(color matrices)

hard scattering amplitude
with m particles
(vector in color space)

To get the amplitudes with additional soft partons, one takes the matrix element of the multi-Wilson-line operators:

$$\langle X_s | \mathbf{S}_1(n_1) \dots \mathbf{S}_m(n_m) | 0 \rangle$$

Factorization for interjet energy flow

TB, Neubert, Rothen, Shao '15 '16, see also Caron-Huot '15

Hard function

m hard partons along
fixed directions $\{n_1, \dots, n_m\}$

$$\mathcal{H}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$$

Soft function

squared amplitude with
with m Wilson lines

$$\sigma(Q, Q_0) = \sum_{m=2}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

color trace integration over directions

Achieves scale separation! Can resum logs by solving RG.

“global” vs “non-global” logs

Much of the SCET literature has been reluctant to deal with complications from multi-Wilson-line operators.

Work with “factorization formula”

$$\sigma(Q, Q_0) = \mathcal{H}_2(Q, \mu) \mathcal{S}_2(Q_0, \mu) \mathcal{S}^{\text{non-global}}(Q/Q_0, \mu)$$

and then resum “global” logs by solving equation

$$\frac{d}{d \ln \mu} \mathcal{S}_2(Q_0, \mu) = \gamma_s \mathcal{S}_2(Q_0, \mu)$$

and evolving this from $\mu \sim Q_0$ up to scale $\mu \sim Q$.

- Non-global logs (NGLs) first arise at two-loops.

Not (even) wrong, but

- neglected NGLs are LL and parametrically of the same size as the global logs (in double logarithmic problems NGLs are NLL).
- RG equations are not consistent: \mathcal{H}_2 is double logarithmic, S_2 single logarithmic.
- correct equations have operator mixing!
- Standard parton shower will have a better description of soft radiation!

Same difficulties are present in double logarithmic problems, though less immediately visible.

Time to deal with multi-Wilson line structure of soft emissions!

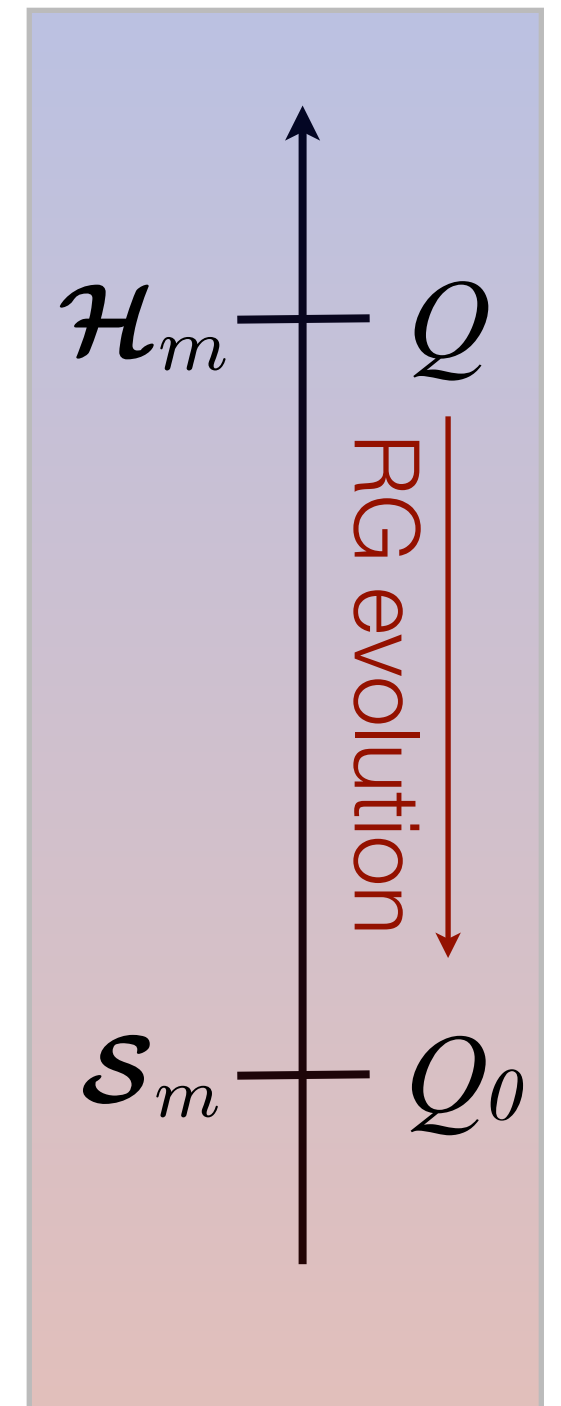
Resummation by RG evolution

Wilson coefficients fulfill renormalization group (RG) equations

$$\frac{d}{d \ln \mu} \mathcal{H}_m(Q, \mu) = - \sum_{l=2}^m \mathcal{H}_l(Q, \mu) \Gamma_{lm}^H(Q, \mu)$$

1. Compute \mathcal{H}_m at a characteristic high scale $\mu_h \sim Q$
2. Evolve \mathcal{H}_m to the scale of low energy physics $\mu_s \sim Q_0$
3. Evaluate S_m at low scale $\mu_s \sim Q_0$

Avoids large logarithms $\alpha_s^n \ln^n(Q/Q_0)$ of scale ratios which can spoil convergence of



RG = Parton Shower

- Ingredients for LL

$$\mathcal{H}_2(\mu = Q) = \sigma_0$$

$$\mathcal{H}_m(\mu = Q) = 0 \text{ for } m > 2$$

$$\mathcal{S}_m(\mu = Q_0) = 1$$

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} \mathbf{V}_2 & \mathbf{R}_2 & 0 & 0 & \dots \\ 0 & \mathbf{V}_3 & \mathbf{R}_3 & 0 & \dots \\ 0 & 0 & \mathbf{V}_4 & \mathbf{R}_4 & \dots \\ 0 & 0 & 0 & \mathbf{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

$$\frac{d}{dt} \mathcal{H}_m(t) = \mathcal{H}_m(t) \mathbf{V}_m + \mathcal{H}_{m-1}(t) \mathbf{R}_{m-1}.$$

shower evolution time

$$t \equiv t(\mu_h, \mu_s) = \int_{\alpha_s(\mu_s)}^{\alpha_s(\mu_h)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{4\pi}$$

- equivalent to parton shower equation

$$\mathcal{H}_m(t) = \mathcal{H}_m(t_1) e^{(t-t_1) \mathbf{V}_n} + \int_{t_1}^t dt' \mathcal{H}_{m-1}(t') \mathbf{R}_{m-1} e^{(t-t') \mathbf{V}_n}$$

1-loop anomalous dimension

$$V_m = 2 \sum_{(ij)} (\mathbf{T}_{i,L} \cdot \mathbf{T}_{j,L} + \mathbf{T}_{i,R} \cdot \mathbf{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi} W_{ij}^k$$

$$R_m = -4 \sum_{(ij)} \mathbf{T}_{i,L} \cdot \mathbf{T}_{j,R} W_{ij}^{m+1} \Theta_{\text{in}}(n_{m+1})$$

$$\mathcal{H}_m \propto |\mathcal{M}_m\rangle \langle \mathcal{M}_m|$$

$\mathbf{T}_{i,L}$: acts on $|\mathcal{M}_m\rangle$

$\mathbf{T}_{i,R}$: acts on $\langle \mathcal{M}_m|$

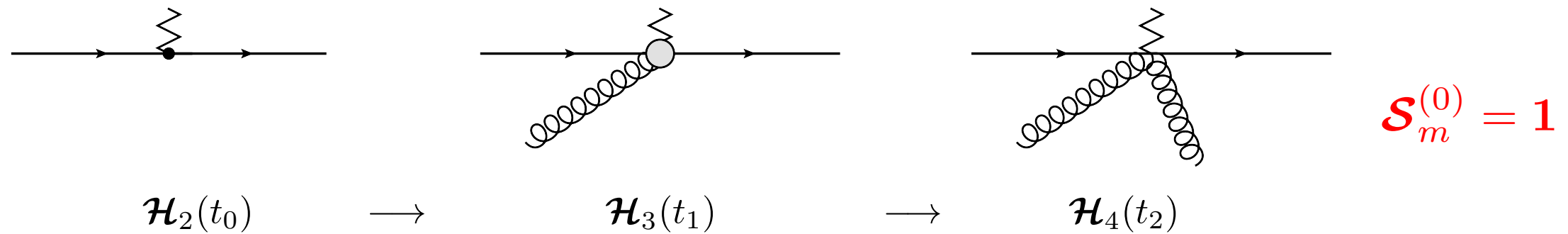
- Dipoles \rightarrow dipole shower

$$W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k n_j \cdot n_k}$$

product of two eikonal factors

- RG has full color information, form as described by Nagy and Soper '07,... Will work at large N_c :

$$\mathbf{T}_i \cdot \mathbf{T}_j \rightarrow -\frac{N_c}{2} \delta_{j,i\pm 1}$$



$$\sigma_{\text{LL}}(Q, Q_0) = \sum_{m=2}^{\infty} \langle \mathcal{H}_2^{(0)} \otimes U_{2m} \hat{\otimes} \mathcal{S}_m^{(0)} \rangle$$

$$= \langle \mathcal{H}_2^{(0)}(t) + \int \frac{d\Omega_3}{4\pi} \mathcal{H}_3^{\text{LL}} + \int \frac{d\Omega_3}{4\pi} \int \frac{d\Omega_4}{4\pi} \mathcal{H}_4^{\text{LL}} + \dots \rangle$$

LL shower equivalent to [Dasgupta Salam '01](#). Have flexible implementation for general k -jet processes

- uses LHE event files from Madgraph for LO \mathcal{H}_k
- used different forms of collinear cutoff
- studied gap fractions and photon isolation cones, both in e^+e^- and pp collisions

Ingredients for NLL

1. One-loop matching corrections

- Hard functions

TB, Neubert, Rothen, Shao '15

$$\mathcal{H}_2 = \sigma_0 \left(\mathcal{H}_2^{(0)} + \frac{\alpha_s}{4\pi} \mathcal{H}_2^{(1)} + \dots \right), \quad \mathcal{H}_3 = \sigma_0 \left(\frac{\alpha_s}{4\pi} \mathcal{H}_3^{(1)} + \dots \right)$$

- Soft functions

$$\mathcal{S}_m = \mathbf{1} + \frac{\alpha_s}{4\pi} \mathcal{S}_m^{(1)} + \dots$$

2. Two-loop anomalous dimension

$$\Gamma^{(2)} = \begin{pmatrix} v_2 & r_2 & d_2 & 0 & \dots \\ 0 & v_3 & r_3 & d_2 & \dots \\ 0 & 0 & v_4 & r_4 & \dots \\ 0 & 0 & 0 & v_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

see
Caron-Huot '15

Ingredients for LL'

1. One-loop matching corrections

- Hard functions

TB, Neubert, Rothen, Shao '15

$$\mathcal{H}_2 = \sigma_0 \left(\mathcal{H}_2^{(0)} + \frac{\alpha_s}{4\pi} \mathcal{H}_2^{(1)} + \dots \right), \quad \mathcal{H}_3 = \sigma_0 \left(\frac{\alpha_s}{4\pi} \mathcal{H}_3^{(1)} + \dots \right)$$

- Soft functions

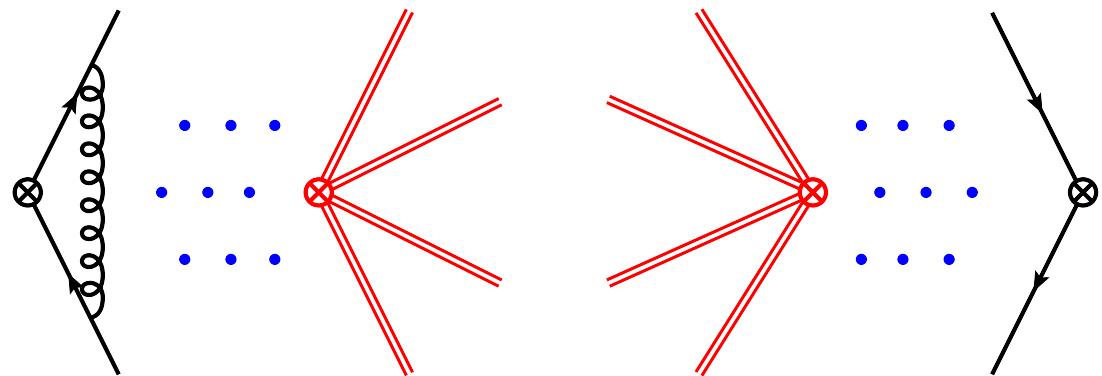
$$\mathcal{S}_m = \mathbf{1} + \frac{\alpha_s}{4\pi} \mathcal{S}_m^{(1)} + \dots$$

2. Two-loop anomalous dimension

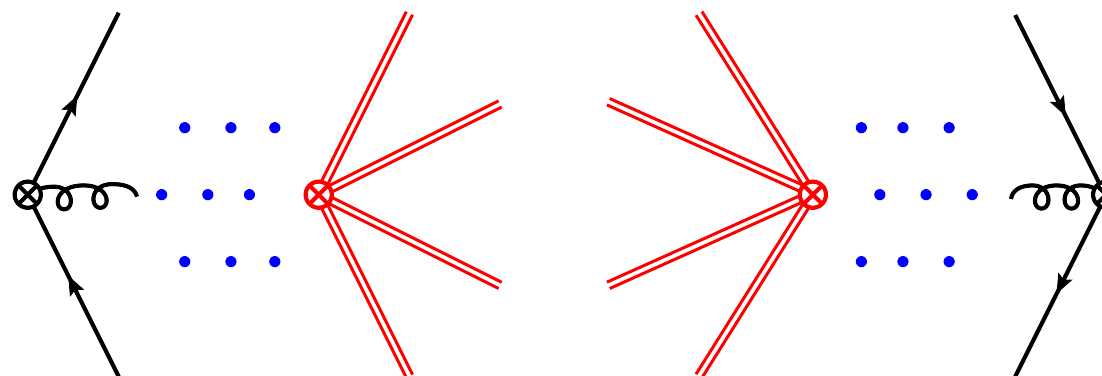
$$\Gamma^{(2)} = \begin{pmatrix} v_2 & r_2 & d_2 & 0 & \dots \\ 0 & v_3 & r_3 & d_3 & \dots \\ 0 & 0 & v_4 & r_4 & \dots \\ 0 & 0 & 0 & v_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

see
Caron-Huot '15

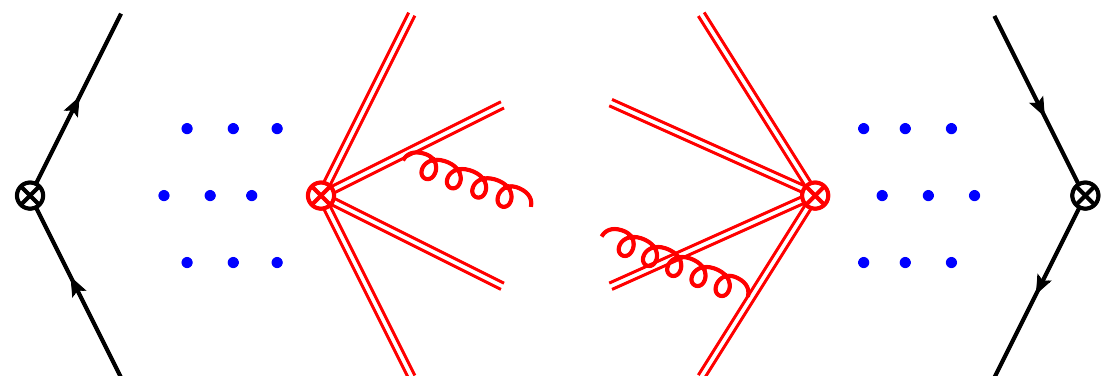
$O(\alpha_s)$ corrections at LL'



$$\sim \mathcal{H}_2^{(1)} \otimes U_{2m} \hat{\otimes} \mathcal{S}_m^{(0)}$$

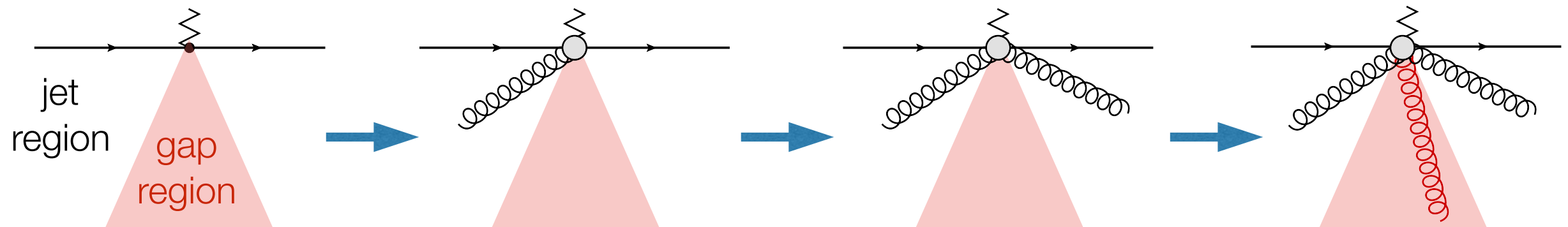


$$\sim \mathcal{H}_3^{(1)} \otimes U_{3m} \hat{\otimes} \mathcal{S}_m^{(0)}$$



$$\sim \mathcal{H}_2^{(0)} \otimes U_{2m} \hat{\otimes} \mathcal{S}_m^{(1)}$$

Soft corrections



- Shower stops when emission hits **interjet region** (evolution produces hard partons)
- Use the last emission for NLO soft function

$$\mathcal{S}_m^{(1)}(\{\underline{n}\}, Q_0, \mu) = \frac{N_c}{2} \sum_{i,j} \delta_{i,j\pm 1} \int d\hat{y} \int_0^{2\pi} \frac{d\hat{\phi}}{2\pi} \left[-4 \ln \frac{\mu}{Q_0} + 4 \ln \frac{2 |\sin \hat{\phi}|}{f_{ij}(\hat{\phi}, \hat{y})} \right] \Theta_{\text{out}}^{\text{lab}}(\hat{y}, \hat{\phi})$$

in dipole rest frame

weight factor

- efficient and general method to get $\mathcal{S}_m^{(1)}$

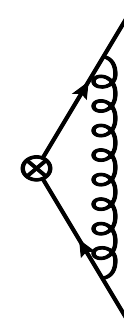
Hard corrections

Virtual corrections to \mathcal{H}_2 give trivial prefactor

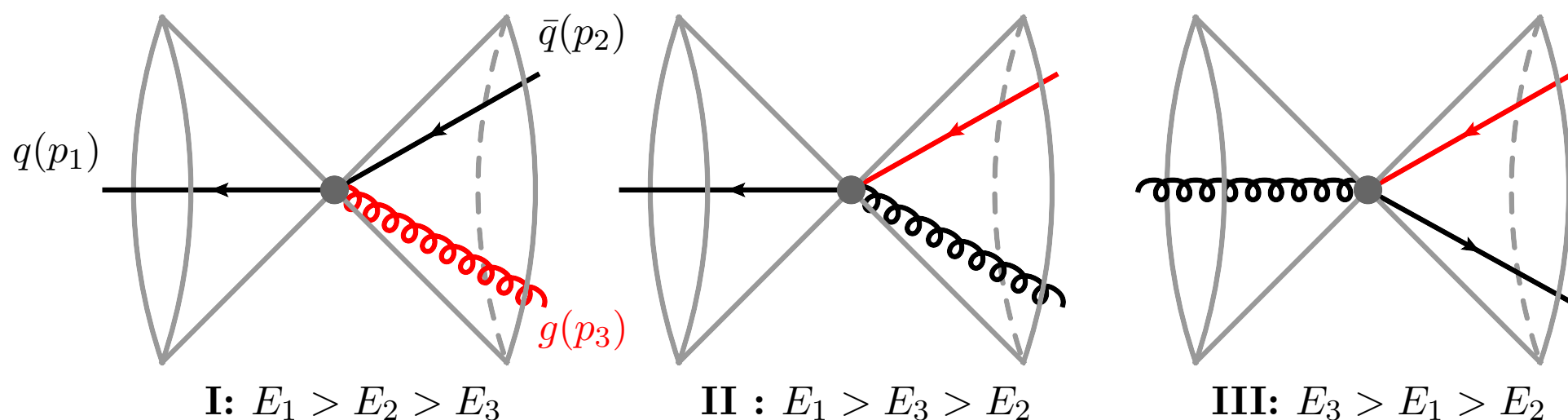
$$\langle \mathcal{H}_2(Q, \mu) \otimes \mathcal{S}_2(Q_0, \mu) \rangle = \sigma_0 H_2(Q^2, \mu) \langle \hat{\mathcal{S}}_2(Q_0, \mu) \rangle$$

standard SCET
2-jet hard function

LL shower



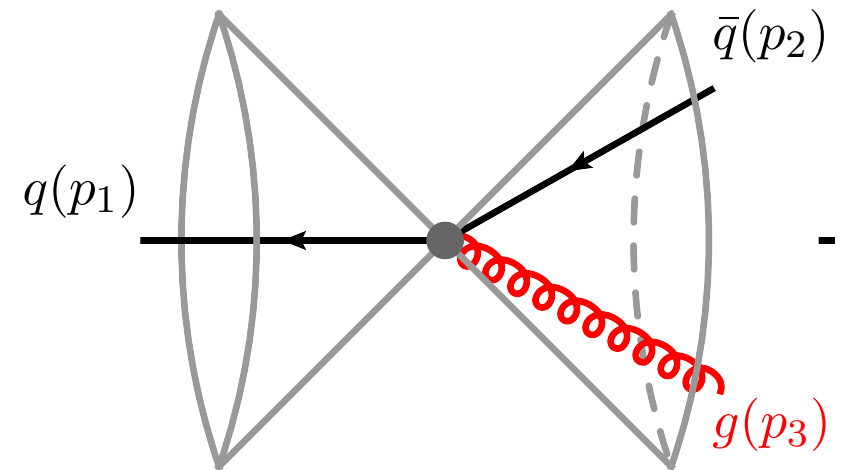
but $\mathcal{H}_3^{(1)}$ is a function of two angles



Hard corrections

In region I, we parameterize

$$v = \tan \frac{\theta_g}{2}, \quad u v = \tan \frac{\theta_{\bar{q}}}{2}$$



Write angular convolution as

$$\langle \mathcal{H}_3^{(1)}(\{\underline{n}\}, Q, \mu_h) \otimes \hat{\mathcal{S}}_3(\{\underline{n}\}, \mu_h) \rangle = \int_0^1 du \int_0^1 dv \langle \mathcal{H}_3^{(1)}(u, v, Q, \mu_h) \hat{\mathcal{S}}_3(u, v, \mu_h) \rangle$$

LL shower

MC over u and v , shower 3-parton configuration.

Complication: $\mathcal{H}_3^{(1)}(u, v, Q, \mu_h)$ is a distribution.

$$\mathcal{H}_{3,I}^{(1)}(u, v, Q, \mu) = C_F \left\{ \left[4 \ln^2 \frac{\mu}{Q} - \frac{\pi^2}{6} \right] \delta(u) \delta(v) - 8 \ln \frac{\mu}{Q} \delta(u) \left(\frac{1}{v} \right)_+ + 8 \delta(u) \left(\frac{\ln v}{v} \right)_+ + \left[- \ln \frac{\mu}{Q} F(u, 0) + \frac{2u^2}{(1+u)^3} - F(u, 0) \ln(1+u) \right] \delta(v) \left(\frac{1}{u} \right)_+ + F(u, 0) \delta(v) \left(\frac{\ln u}{u} \right)_+ + \boxed{F(u, v)} \left(\frac{1}{u} \right)_+ \left(\frac{1}{v} \right)_+ \right\} \Theta_{\text{in}}(v).$$

regular function

$$\hat{\mathcal{S}}_3(u, v=0) = \hat{\mathcal{S}}_2$$

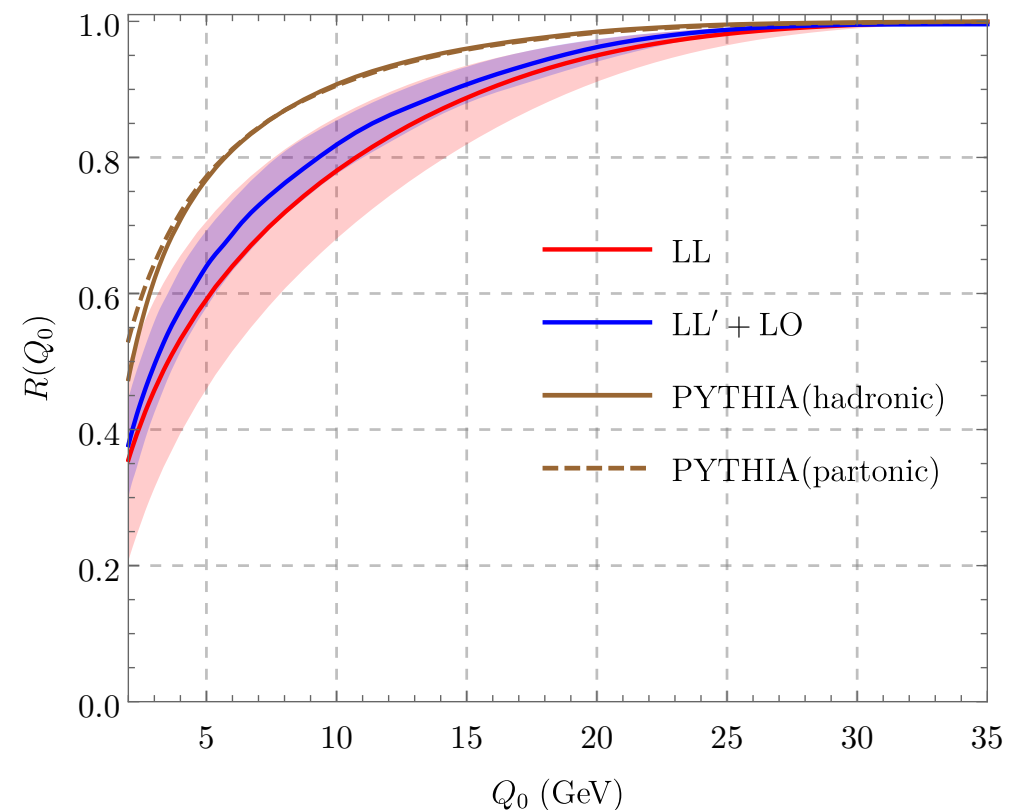
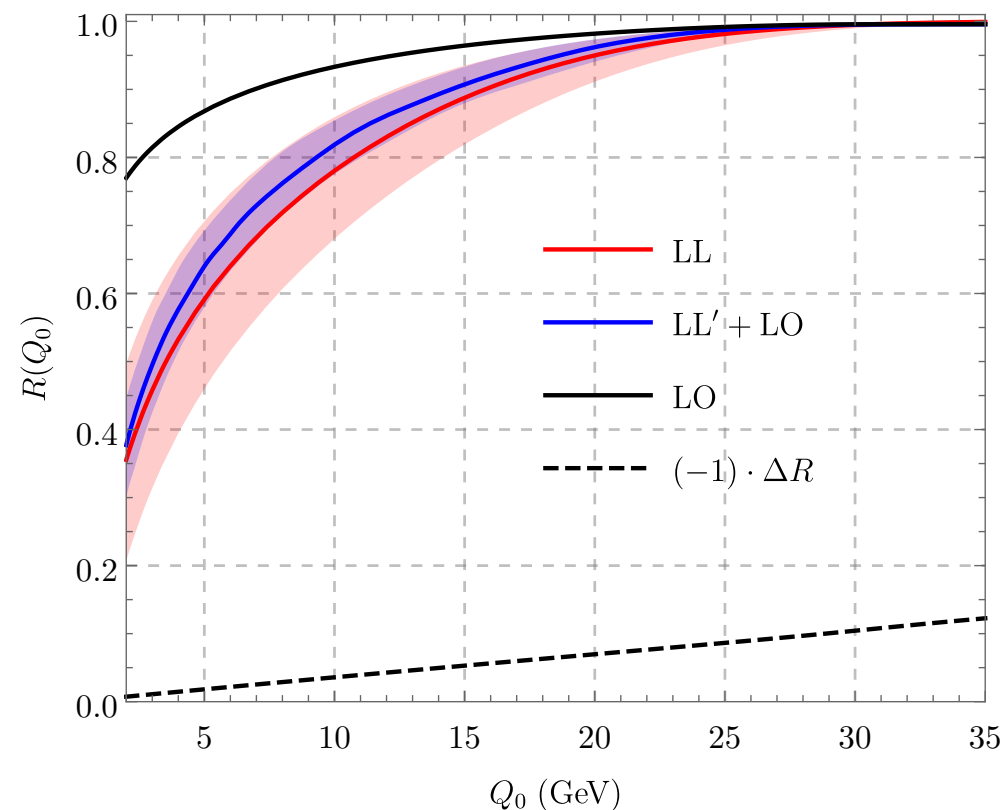
zero angle between
Wilson lines

Use simple slicing method, e.g.

$$\int_0^1 dv \left[\frac{1}{v} \right]_+ \hat{\mathcal{S}}_3(u, v) = \int_0^1 \frac{dv}{v} \left[\hat{\mathcal{S}}_3(u, v) - \boxed{\hat{\mathcal{S}}_2} \right] = \int_{v_0}^1 \frac{dv}{v} \hat{\mathcal{S}}_3(u, v) + \ln v_0 \hat{\mathcal{S}}_2 + \mathcal{O}(v_0)$$

Works well for the simple case we consider. Checked independence on cutoff v_0 using alternate scheme based on interpolating $S_3(u, v)$.

Gap fraction $R(Q_0)$ at $Q=M_Z$

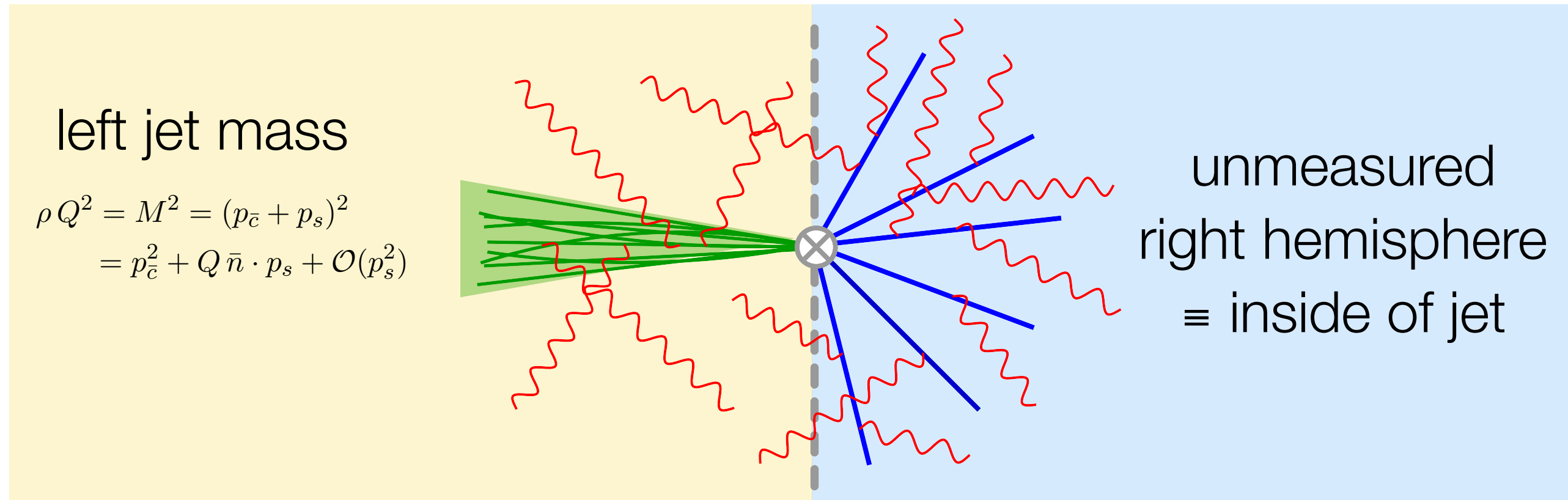


- Bands from variation of hard and soft scales by factor 2.
- By construction $R(Q_0) = 1$ at end-point $Q_0=Q/2$. we match to fixed order and use a profile function function to switch off resummation.
- Unfortunately there is no exp. data.

$$R(Q_0) = \int_0^{Q_0} dE_s \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dE_s}$$

$E_s = \text{energy in gap}$

Jet mass $M^2 = \rho Q^2$



- **Soft radiation** resolves directions of **hard partons** on the right: multi Wilson-line operators
- New: **jet function** for branching of energetic parton on the left.

Factorization theorem for jet mass

TB, Pecjak, Shao '16

Hard function
 m hard partons along
 fixed directions $\{\bar{n}, n_1, \dots, n_m\}$

Soft function
 with $m + 1$ Wilson lines

$$\frac{d\sigma}{dM^2} = \sum_{i=q,\bar{q},g} \int_0^\infty d\omega_L J_i(M^2 - Q\omega_L) \sum_{m=1}^\infty \langle \mathcal{H}_m^i(\{\underline{n}\}, Q) \otimes \mathcal{S}_m(\{\underline{n}\}, \omega_L) \rangle$$

Jet function
 standard inclusive jet function

integration over the m
 directions

$\omega_L \equiv$ (light-cone) energy of
 soft radiation on the left

Jet mass $M^2 = \rho Q^2$ is a **double logarithmic** variable

- Factorization in Laplace space $\rho \rightarrow \tau$. Laplace inversion can be done analytically. TB, Neubert '09
- Due to RG invariance

$$\Gamma_{lm}^{H_i}(\{\underline{n}\}, Q, \mu) = \Gamma_{lm}^{S_i}(\{\underline{n}\}, \tau, \mu) + \Gamma^{J_i}(\tau Q, \mu) \delta_{lm}$$

double logs are tied to jet function, have simple structure

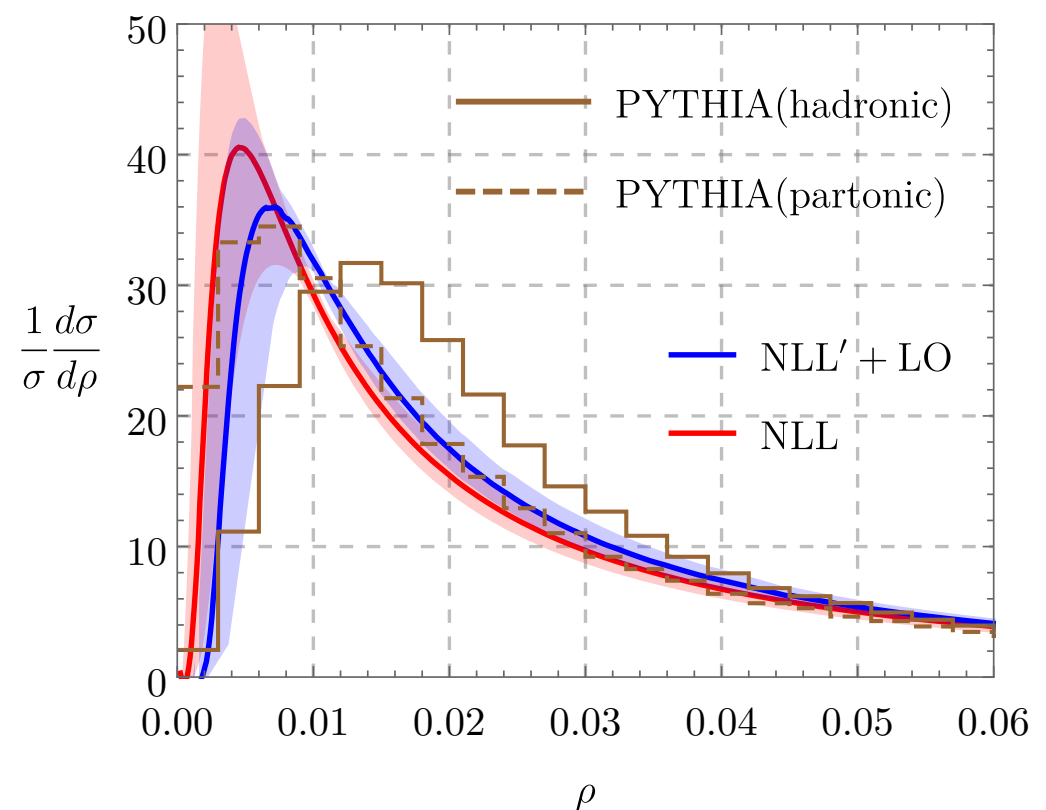
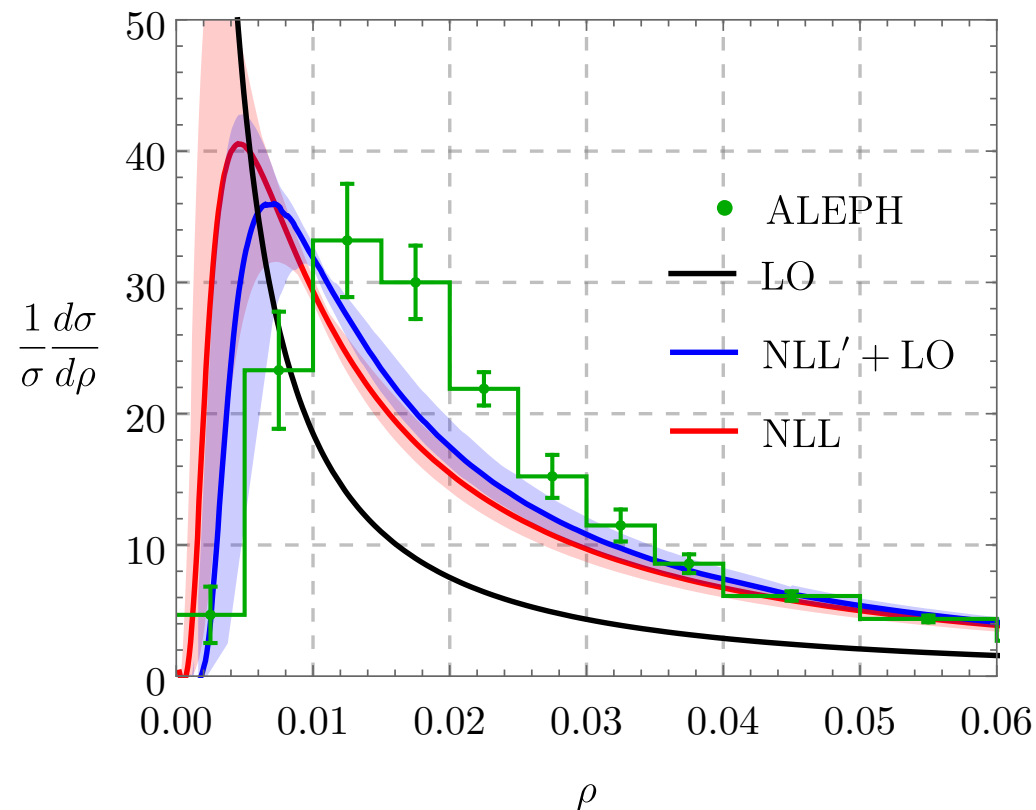
$$\Gamma_{lm}^S(\{\underline{n}\}, \tau, \mu) = 2C_i \gamma_{\text{cusp}} \ln\left(\frac{\tau}{\mu}\right) \delta_{lm} + \hat{\Gamma}_{lm}(\{\underline{n}\})$$

- Also soft function has double logs, separate and exponentiate analytically

$$\tilde{\mathcal{S}}_m^i(\{\underline{n}\}, \tau, \mu_s) = \tilde{S}_G^i(\tau, \mu_s) \hat{\mathcal{S}}_m^i(\{\underline{n}\}, \tau, \mu_s)$$

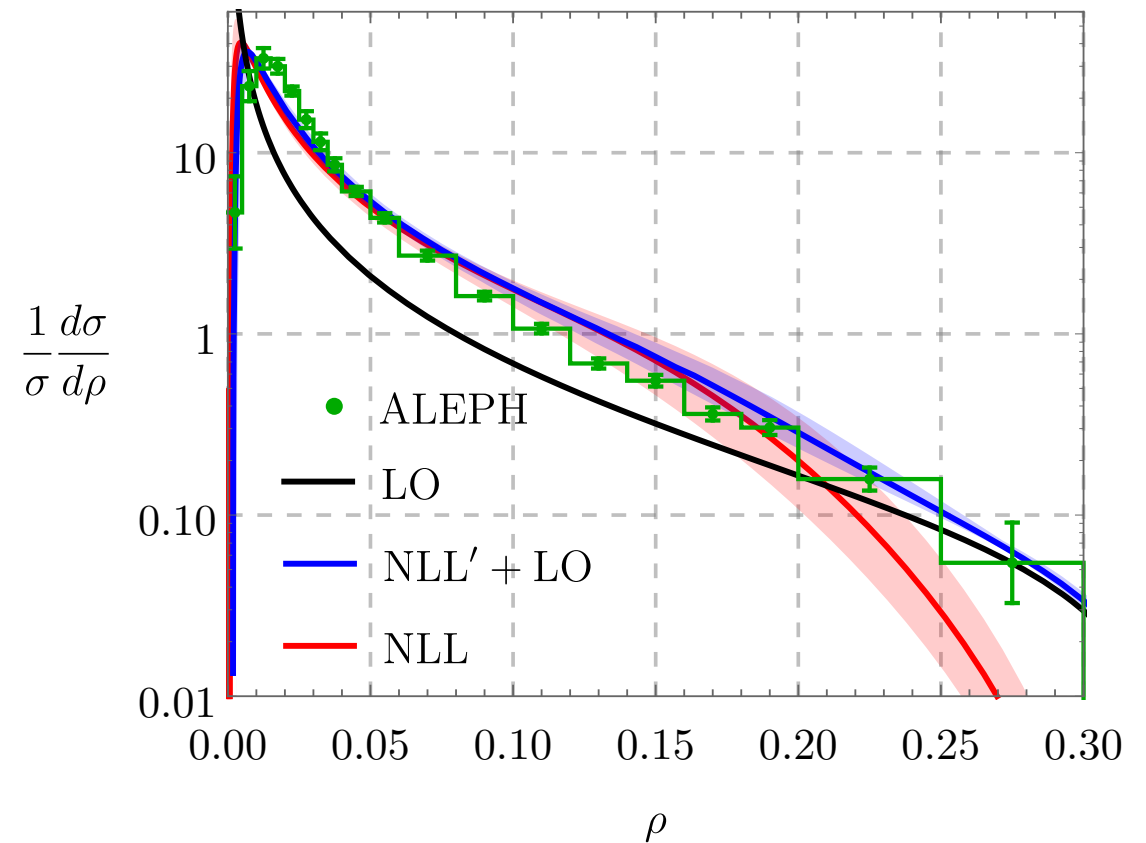
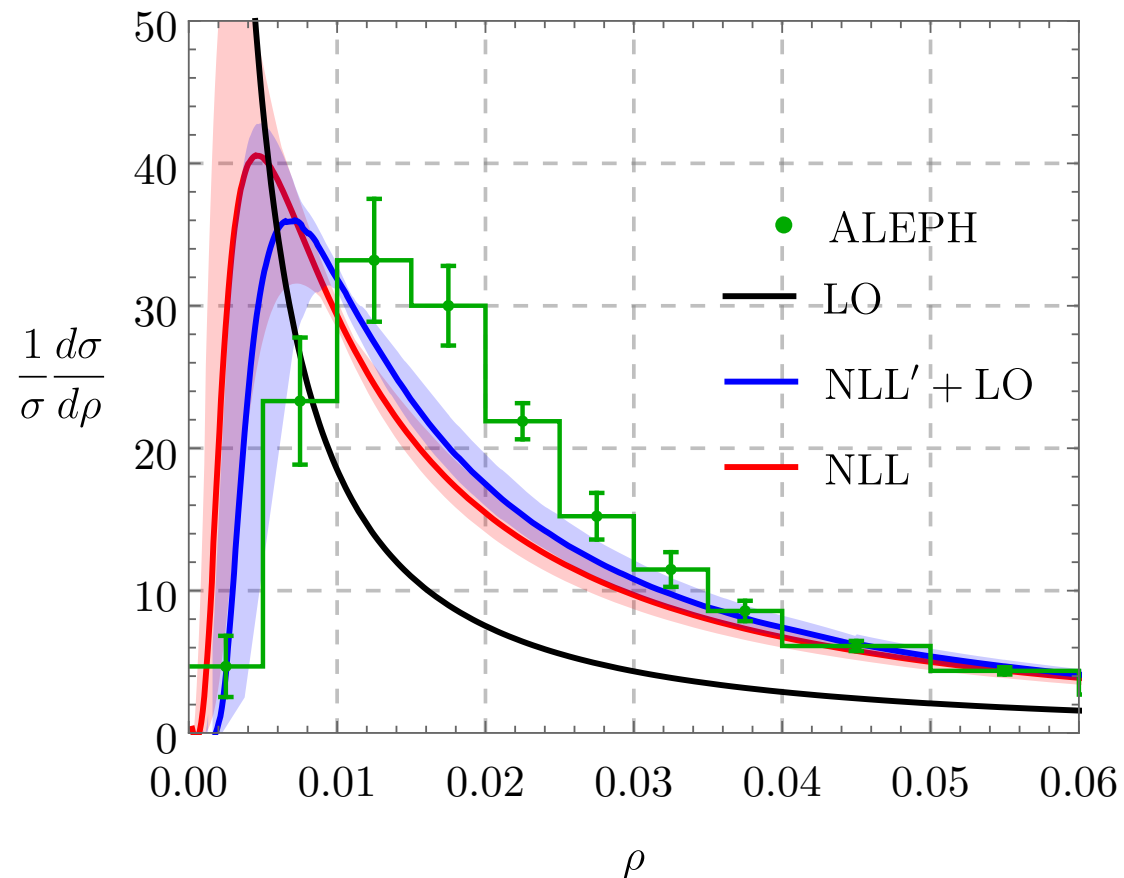
single-logarithmic,
implemented in MC

Numerical results for jet mass



- Exp. data from combining ALEPH light- and heavy-jet mass.
- Peak at $\rho \approx 0.006$ corresponds to $\mu_s \approx 0.5$ GeV. Non-perturbative effects are important and shift the peak, see PYTHIA.
- Bands from varying μ_h and μ_s . Keep $\mu_j = \sqrt{\mu_s \mu_h}$. Bands are narrow since soft variation changes sign after peak.
- Partonic PYTHIA is close to NLL'

Numerical results for jet mass



- Match to LO and use profile function $\mu_s(\rho)$ to switch off resummation at end-LO end-point $\rho \approx 1/3$.
- Replace $\mu_s(\rho) \rightarrow \mu_s(\rho) + \Lambda$ to shift Landau pole to $\rho = 0$, exponentiate NLO correction. The Landau pole is at $\Lambda=230$ MeV.
- Cosmetics, but avoids negative cross sections near $\rho = 0$...

Conclusions

Presented first results for non-global observables which go beyond a simple resummation of the leading logarithms

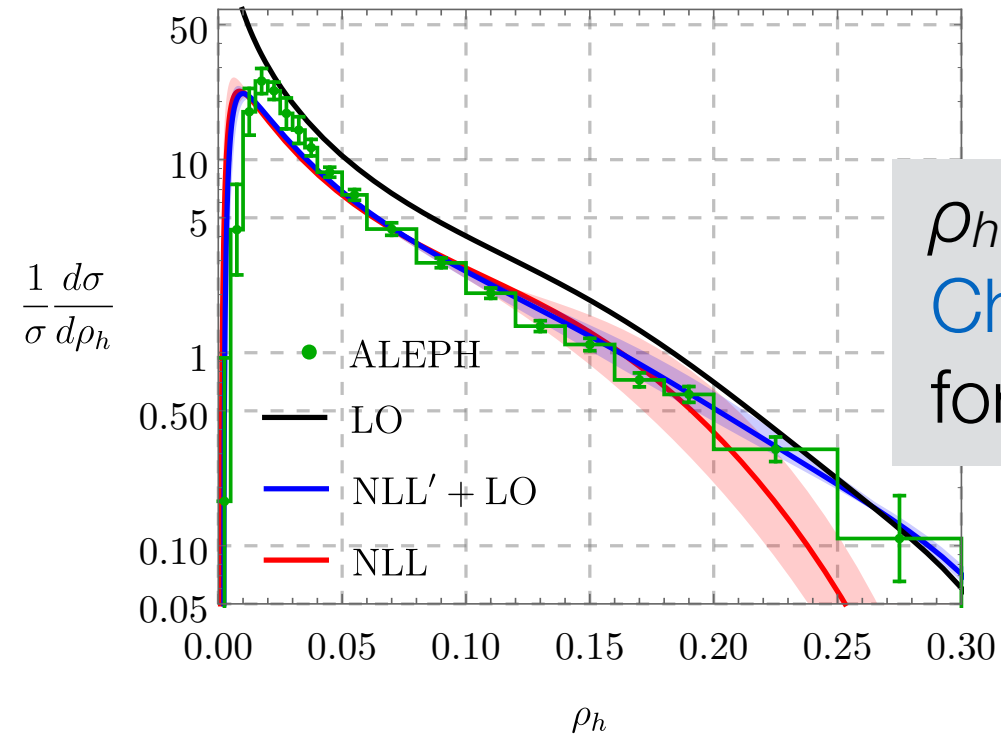
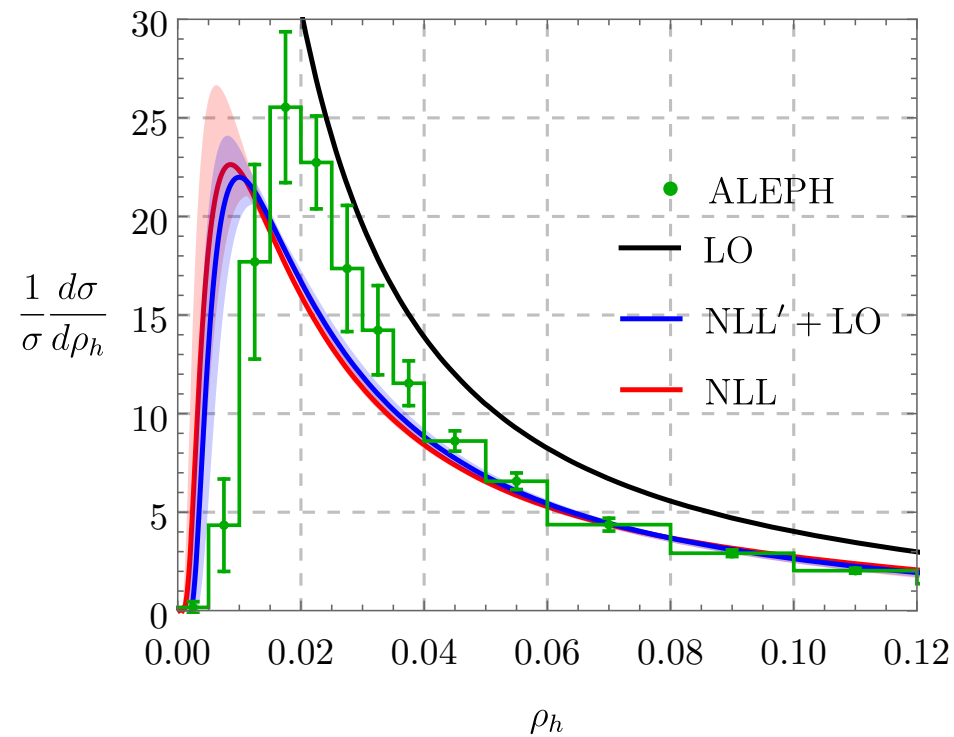
- full one-loop corrections to hard, jet and soft functions: NLL' for jet mass, LL' for interjet energy flow
- implemented corrections in MC framework (an example of a systematically improved shower!)
- NLO corrections significantly improve results

Next steps

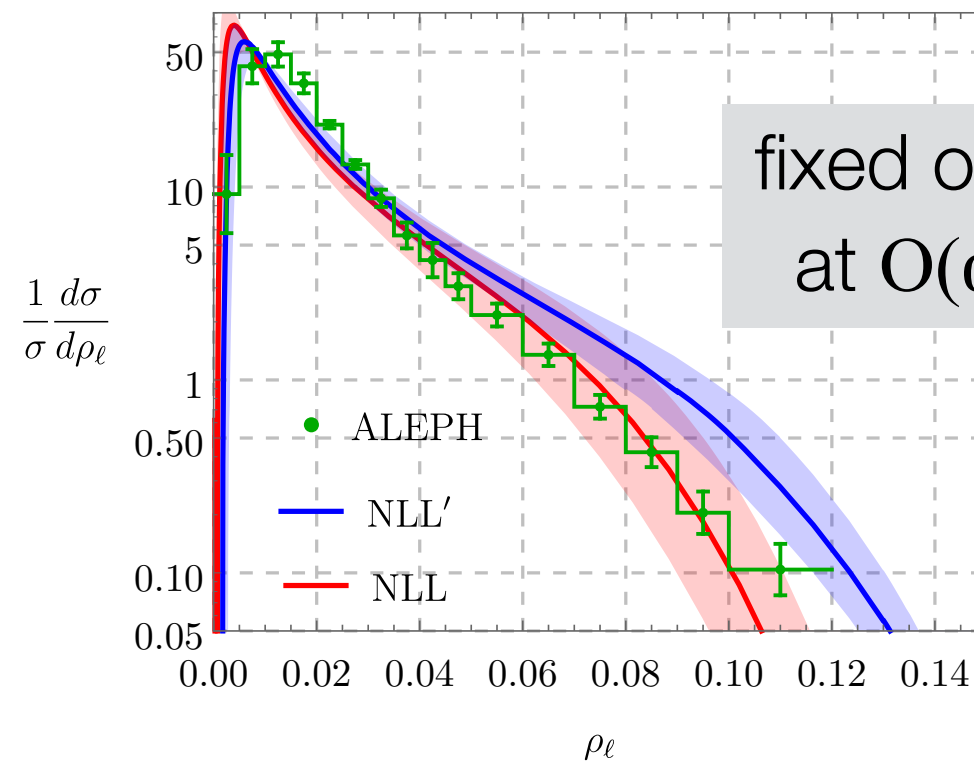
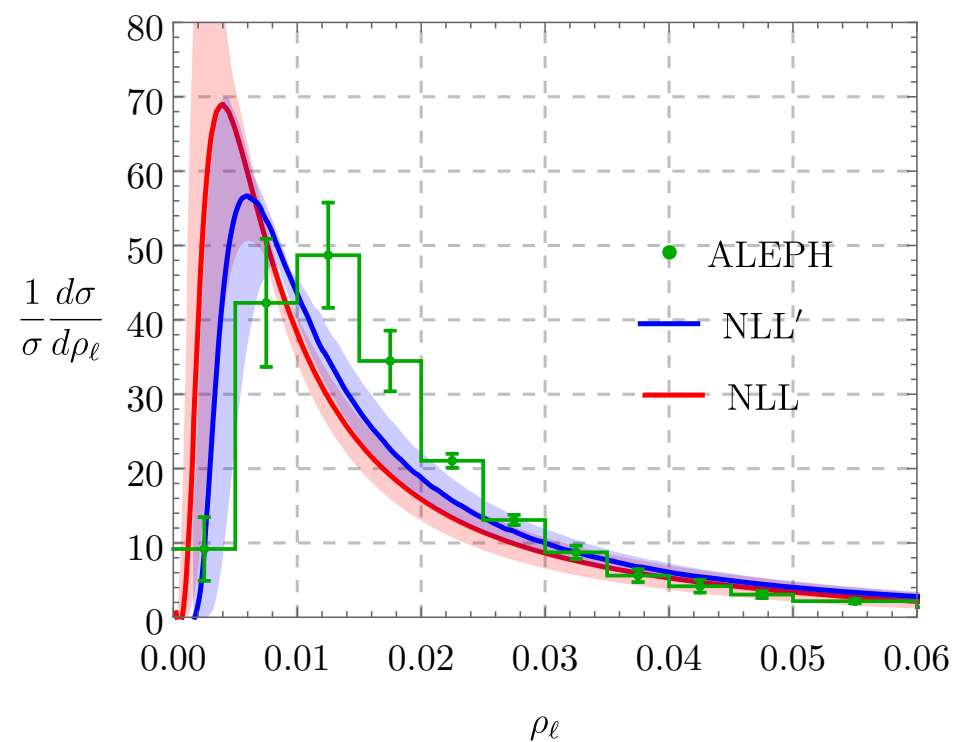
- determination and implementation of two-loop anomalous dimension for full higher-log resummation
- more complicated processes, hadronic collisions

Extra slides

Heavy and light jet mass: $\frac{d\sigma}{d\rho} = \frac{1}{2} \left(\frac{d\sigma}{d\rho_\ell} + \frac{d\sigma}{d\rho_h} \right)$

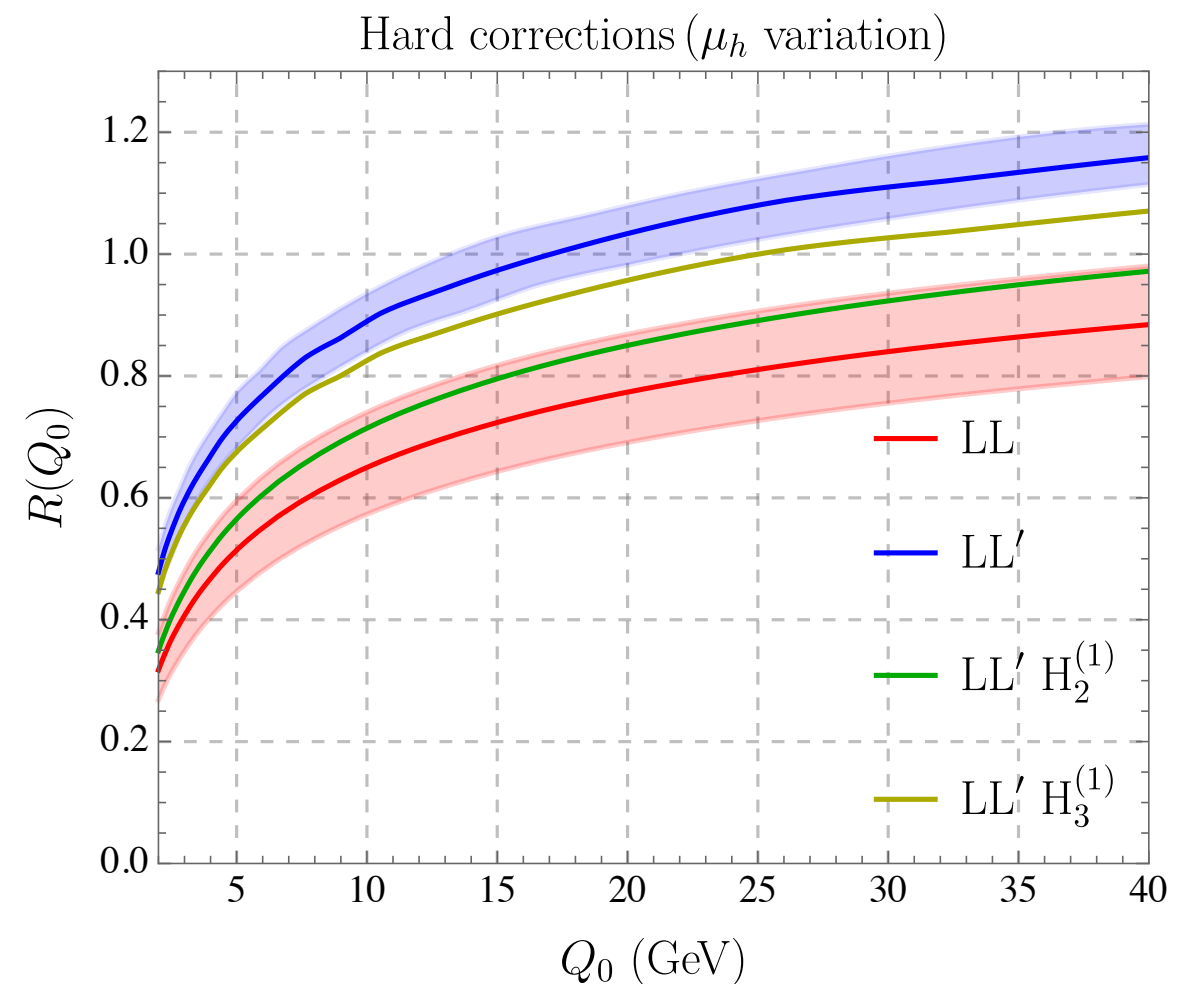
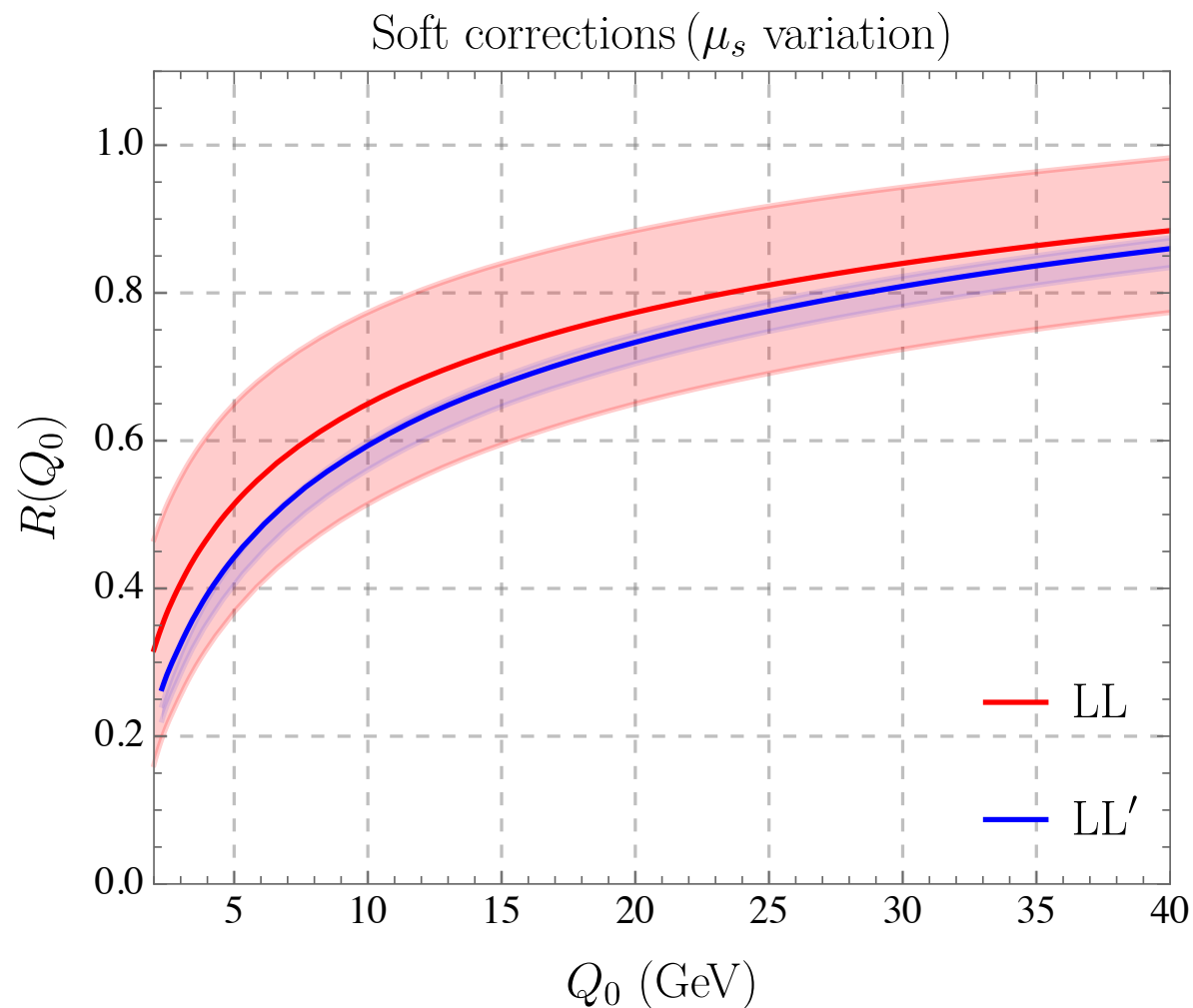


ρ_h is global, see
Chien, Schwartz '10
for N³LL.



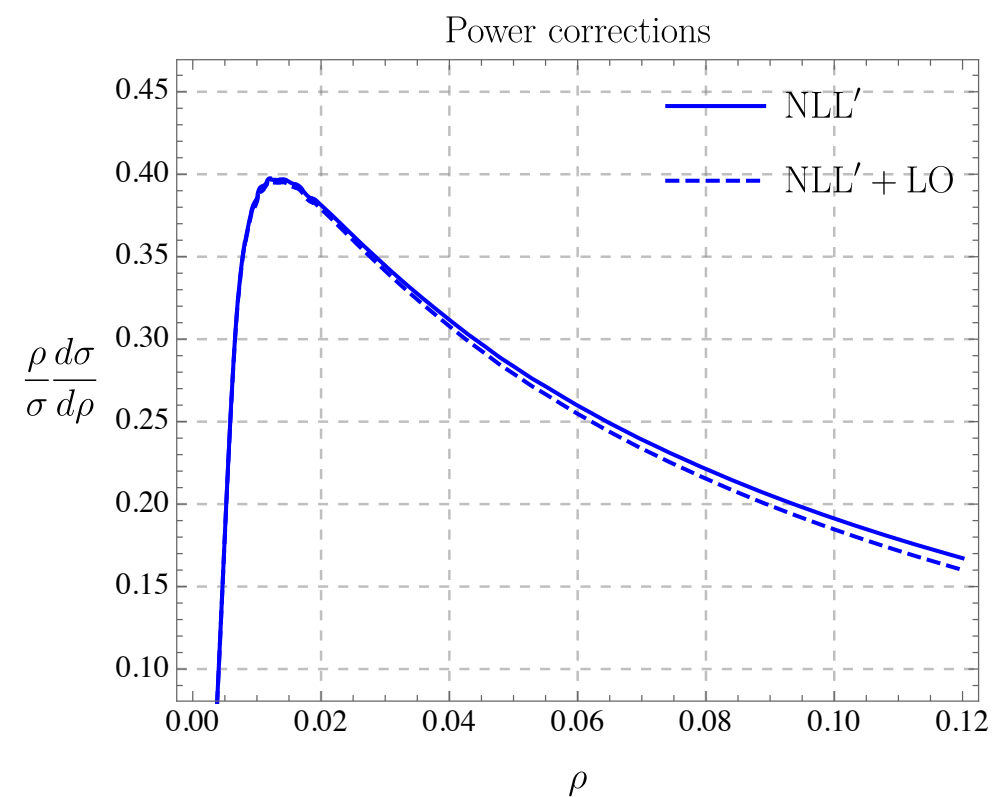
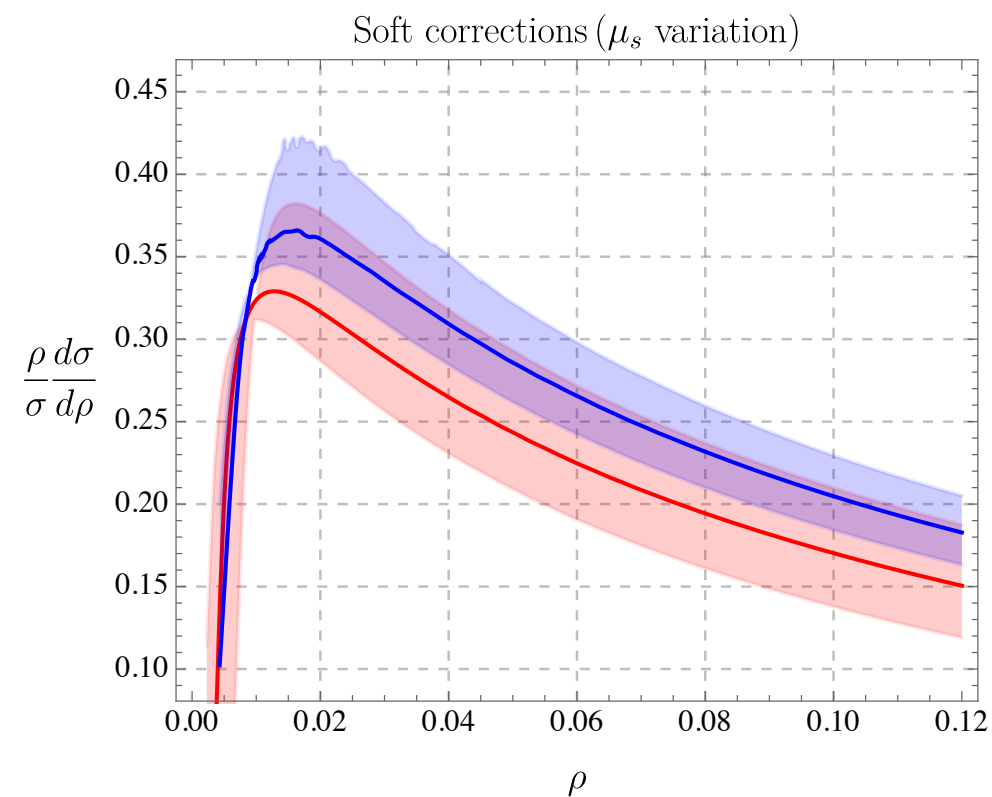
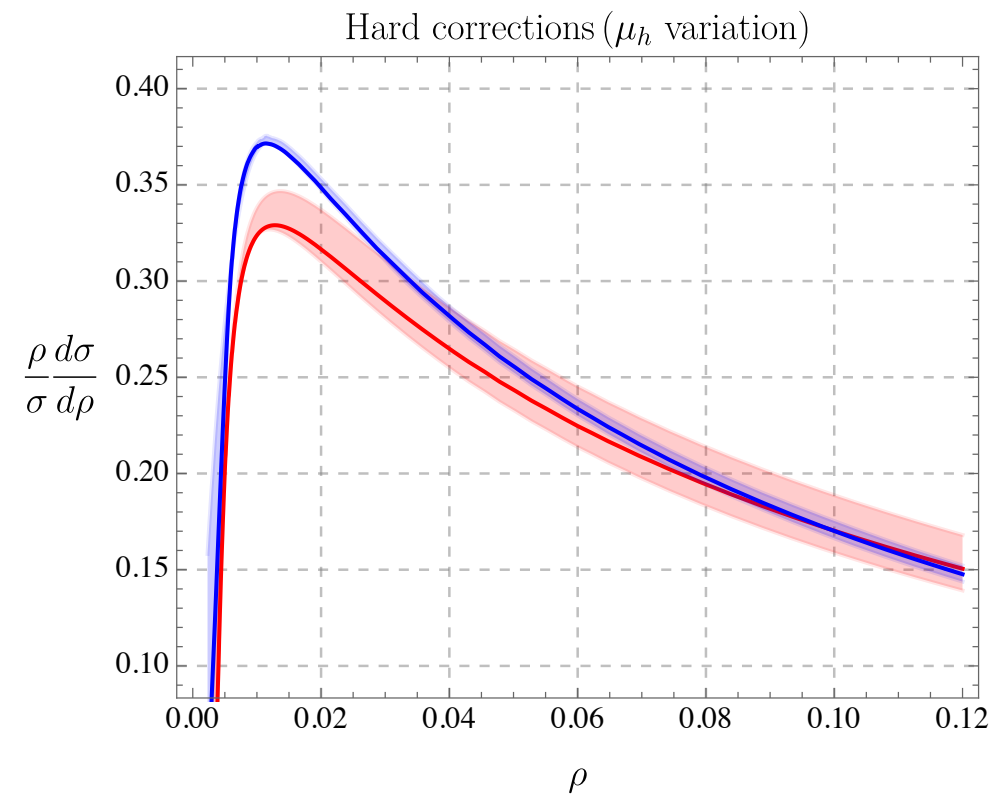
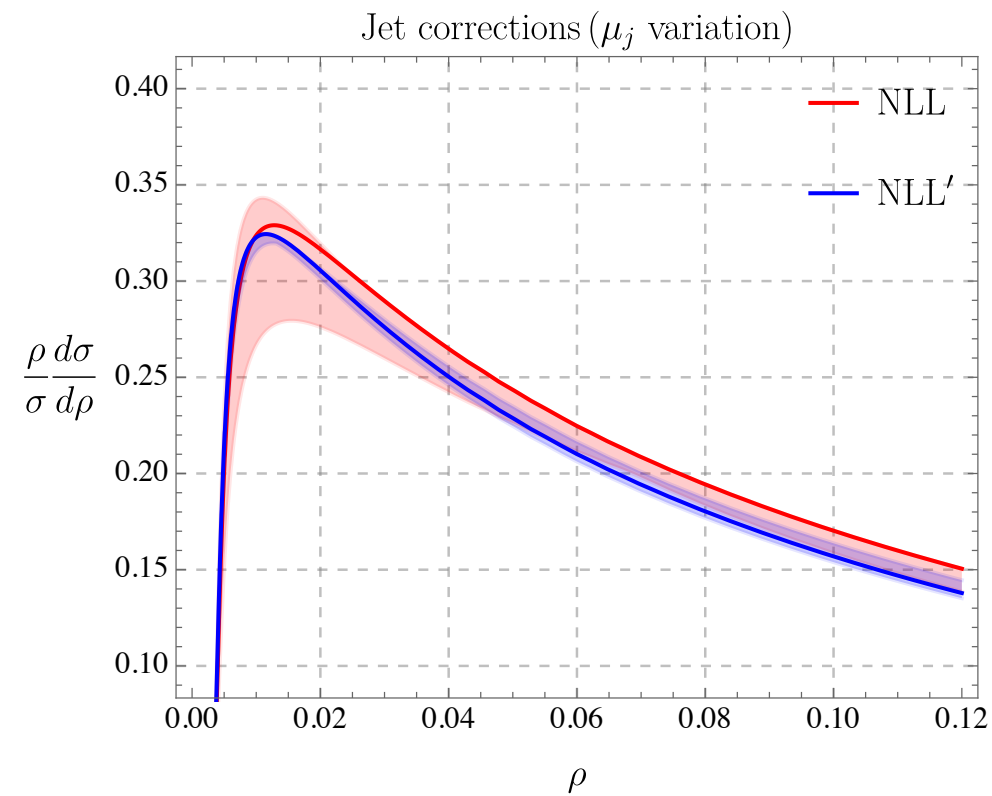
fixed order starts
at $O(\alpha_s^2)$ for ρ_ℓ

Individual corrections, gap fraction

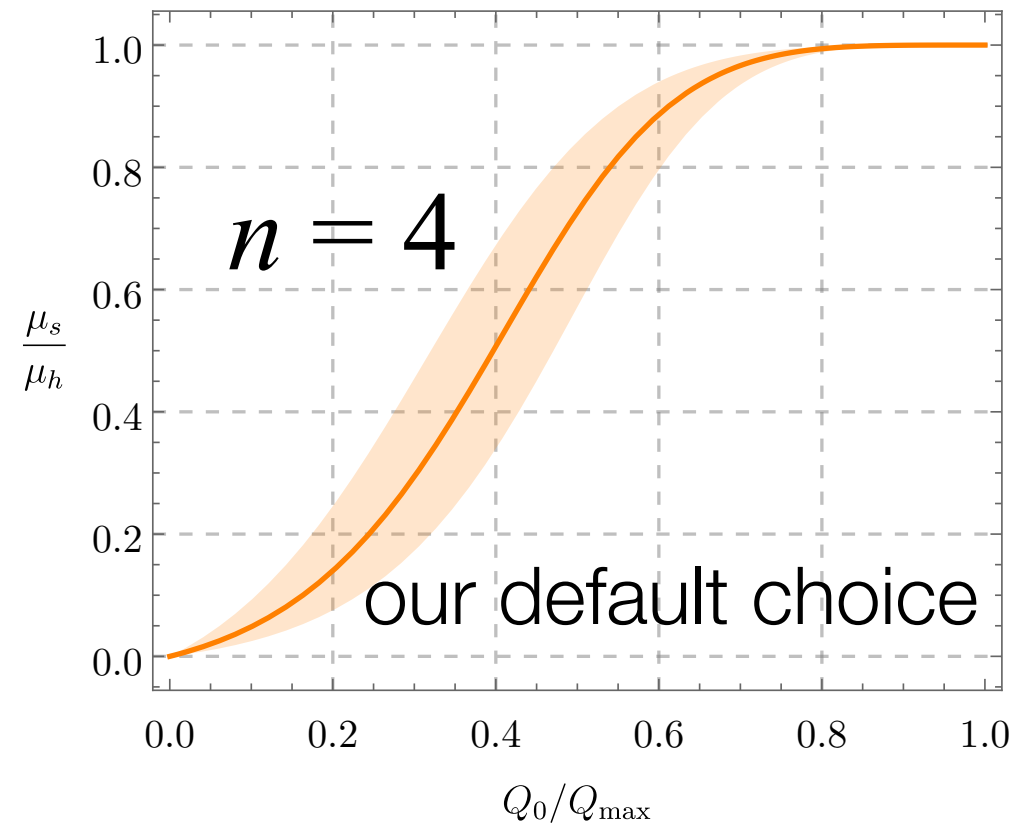
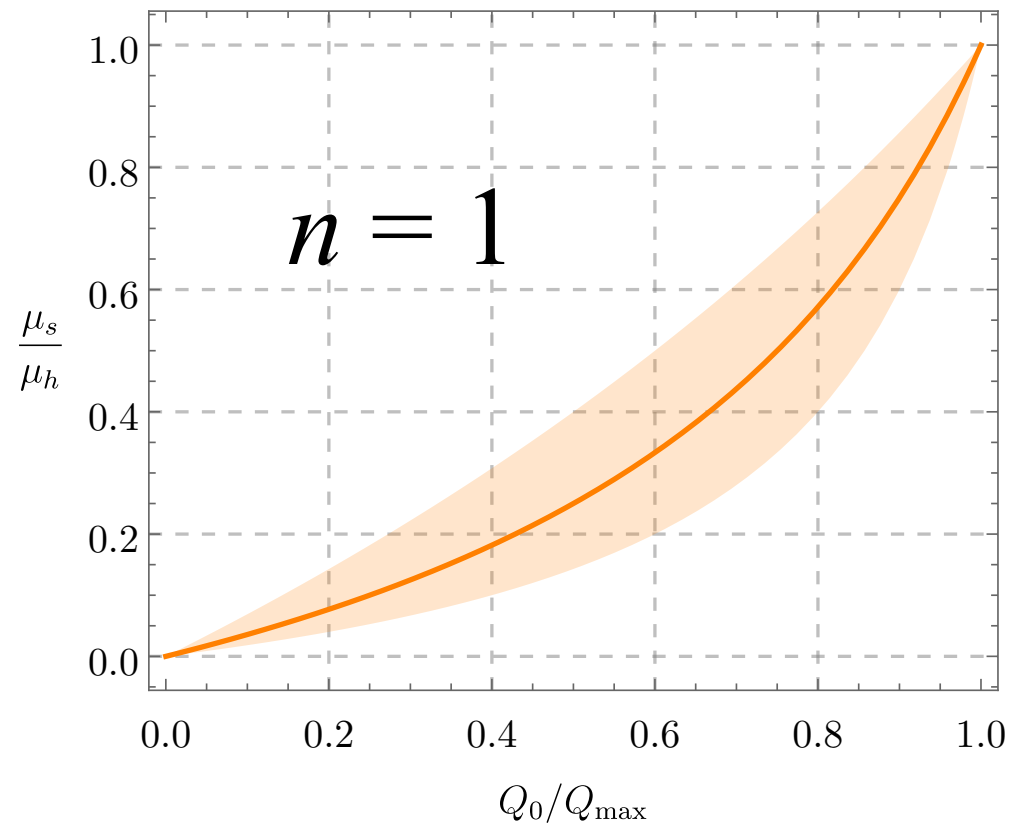


- Only include one correction and vary corresponding scale by factor 2
- Largest correction from \mathcal{H}_3 . Hard corrections at large Q_0 get cancelled by matching to fixed order.
- No profile function etc. for these plots!

Individual corrections for jet mass



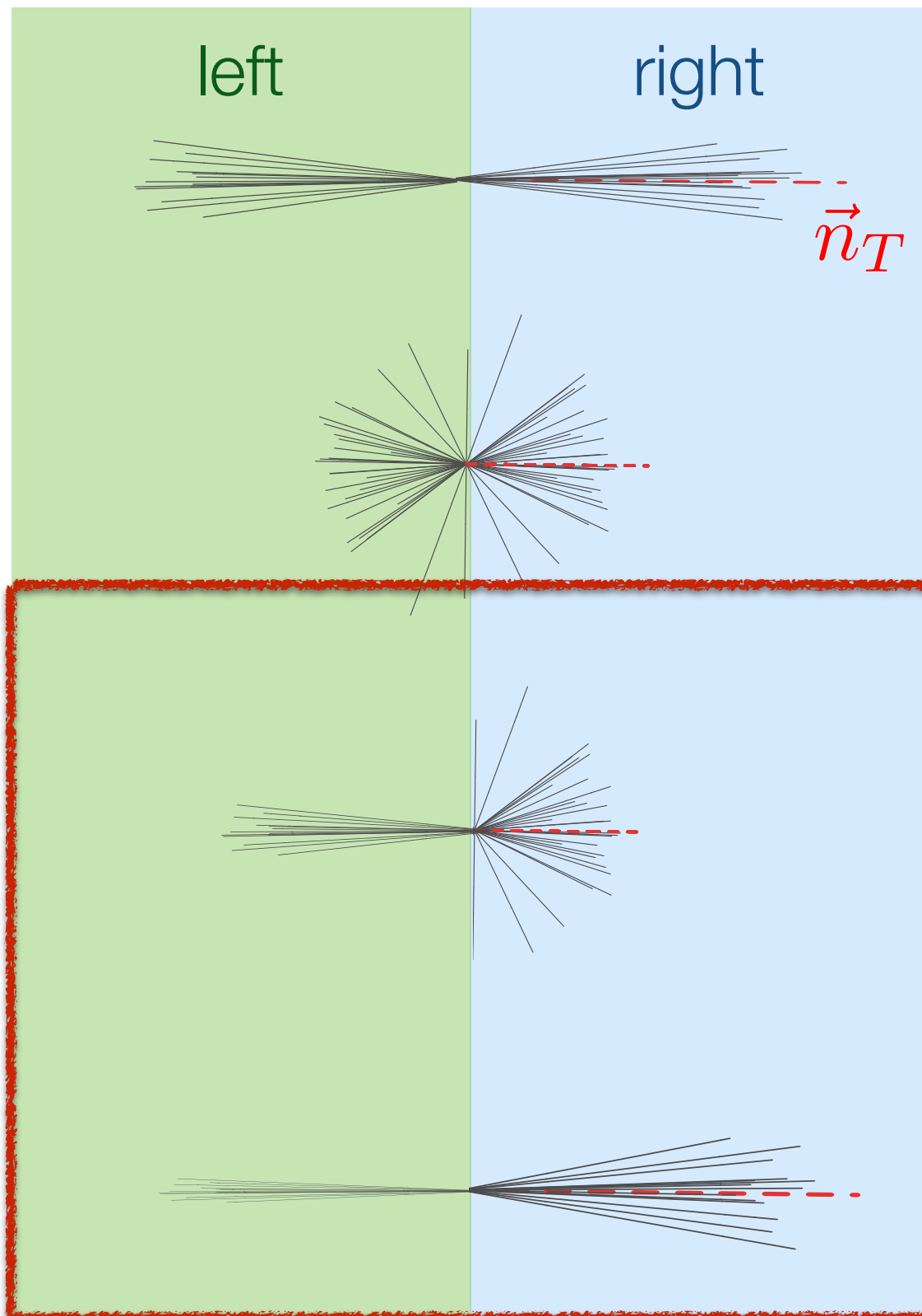
Profile functions



$$\mu_s(Q_0) = \frac{x_s Q_0}{1 + \frac{x_s Q_0}{\mu_h} + \sum_{i=1}^n c_i \left(\frac{Q_0}{Q_{\max}} \right)^i}$$

$$\mu_j = \sqrt{\mu_s \mu_h}$$

Dijet-mass configurations



Symmetric

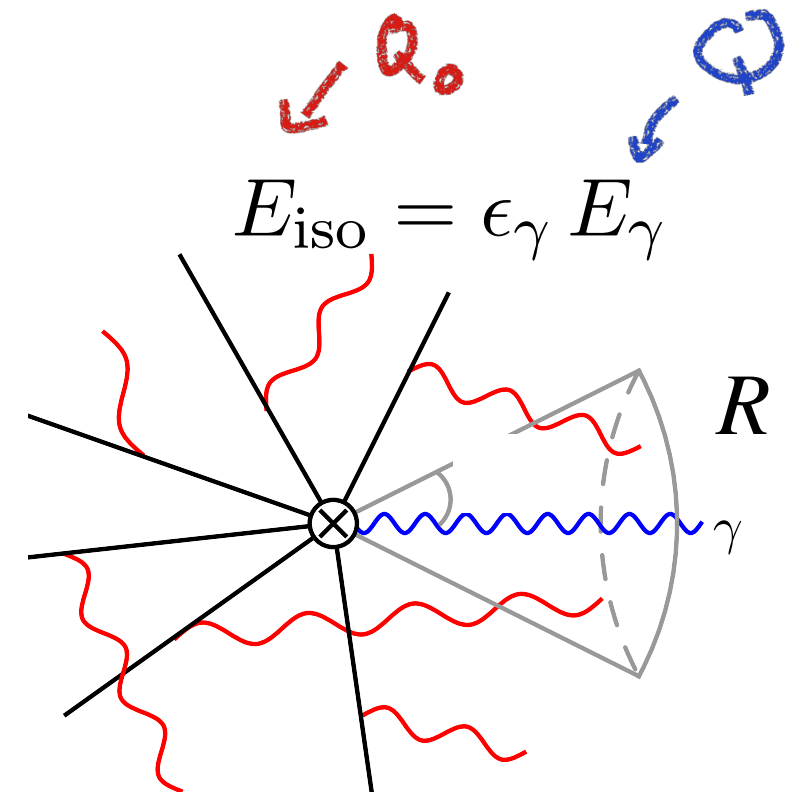
- $M_L \sim M_R \ll Q$
- $M_L \sim M_R \sim Q$

Asymmetric \rightarrow **NGLs**

- $M_L \ll M_R \sim Q$
- $M_L \ll M_R \ll Q$

Isolated photon production

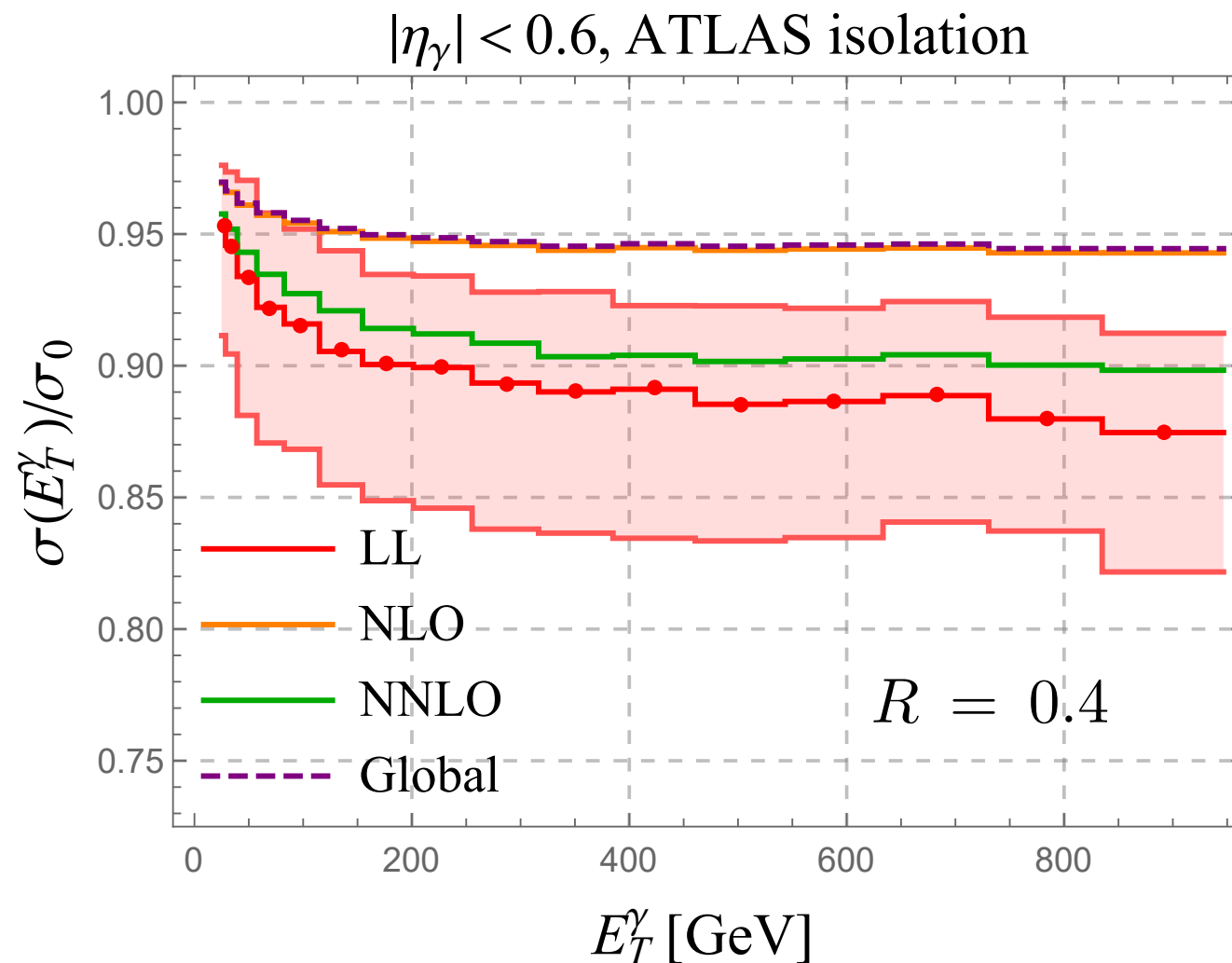
- Exp. needs isolation cones to distinguish photon from hard scattering from hadron decays such as $\pi^0 \rightarrow \gamma\gamma$.



- ATLAS '16 imposes $E_{\text{iso}}^T = 4.8 \text{ GeV} + 0.0042 E_\gamma^T$ on hadronic energy inside cone with $R = 0.4$.
- Large logarithms $\alpha_s^n \ln^n \epsilon_\gamma$ with $\epsilon_\gamma \sim 0.01$
- GLs: $(\alpha_s R^2 \ln \epsilon_\gamma)^n$ NGLs: $R^2 \times \alpha_s^n \ln^n \epsilon_\gamma \ln^{n-1} R$

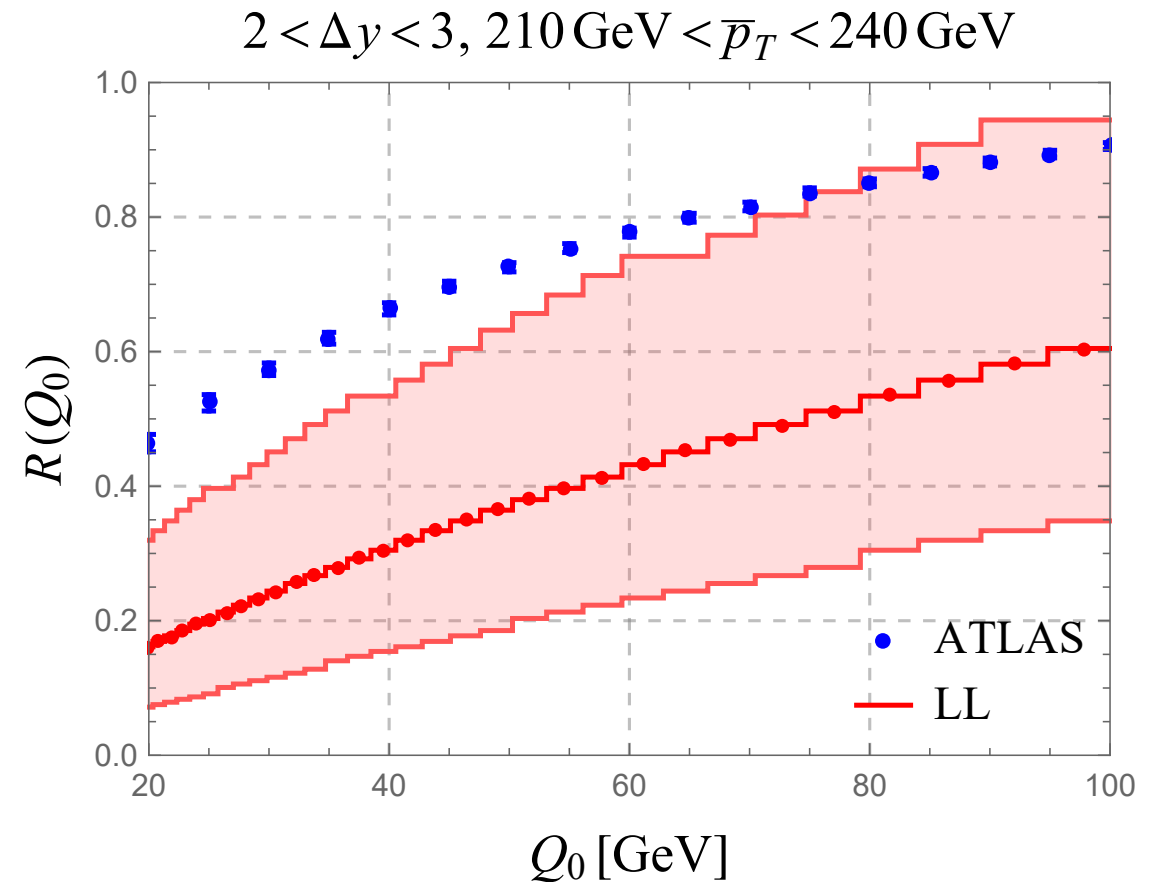
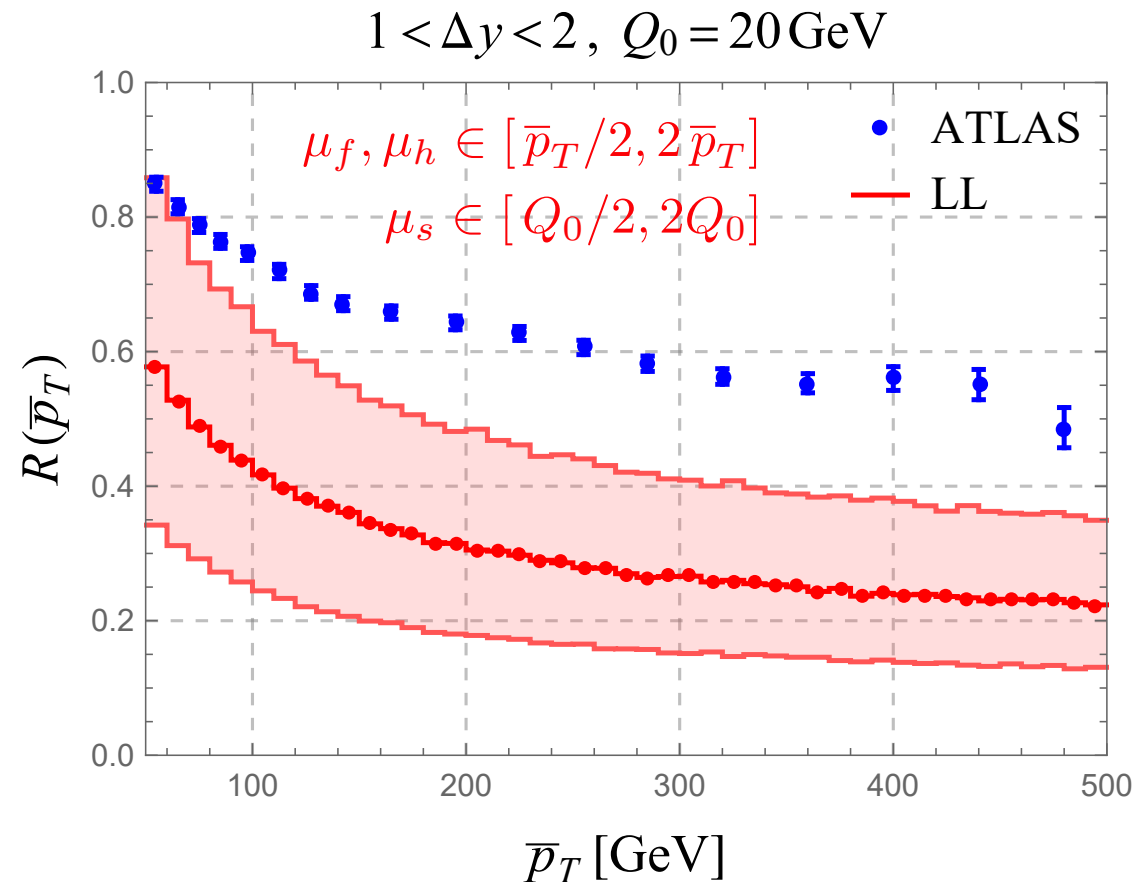
see Hatta et al. 1710.06722

Effects of γ isolation at LHC



- **NLO**: ~5% reduction, **NNLO** ~10%, **resummed** ~ 12%
- NGLs dominate over global contribution since GLs are suppressed by powers of R : naive exponentiation (**dashed**) not appropriate!
- **LL resummation** has large scale **uncertainty band**

LL gap fractions compared to ATLAS



- focus on central jets so that gap size is small, but even so $\ln Q_0/Q \sim \Delta y$ not very large
- LO fixed order gives a better description!
- Note: beyond the large N_c limit Glauber phases induce super-leading logarithms [Forshaw, Kyrieleis and Seymour '06](#)