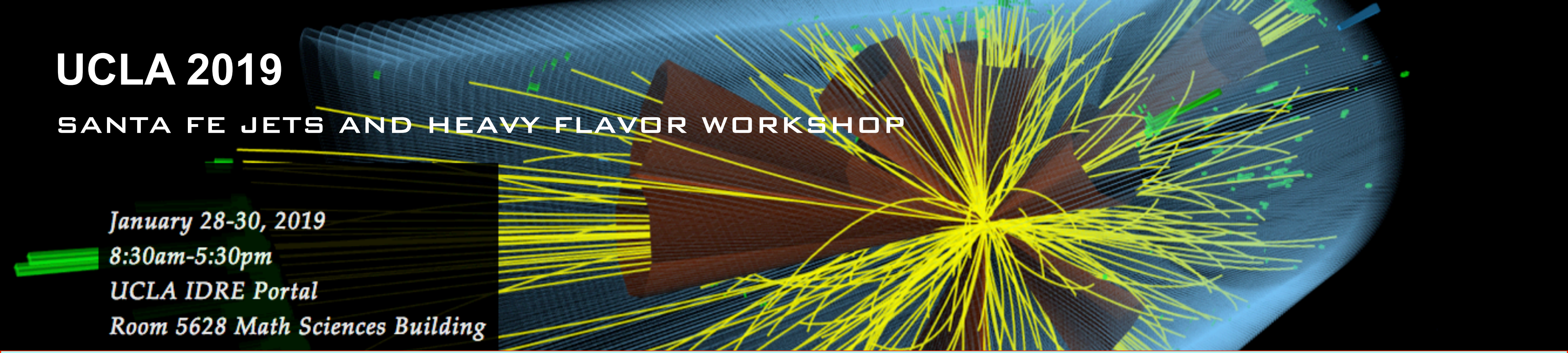


Hadrons' Partonic Structure from ab initio Lattice QCD Calculations

A status report

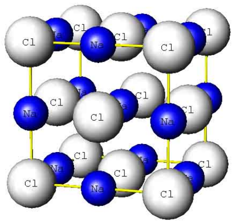
Jianwei Qiu

Theory Center, Jefferson Lab

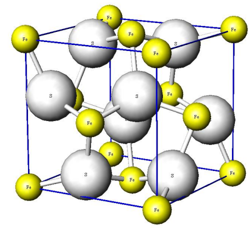
Hadron's partonic structure in QCD

- Structure – "a still picture"

Crystal Structure:

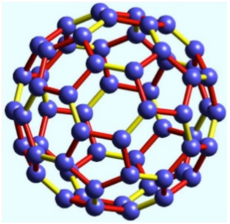


NaCl,
B1 type structure



FeS2,
C2, pyrite type structure

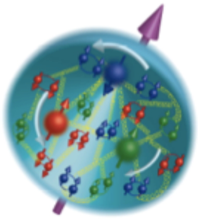
Nano-material:



Fullerene, C60

Motion of nuclei is much slower than the speed of light!

- No "still picture" for hadron's partonic structure!

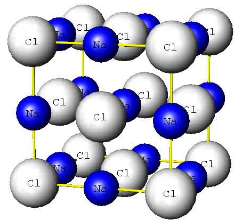


Motion of quarks/gluons is relativistic, color is fully entangled!

Hadron’s partonic structure in QCD

- **Structure – “a still picture”**

Crystal Structure:



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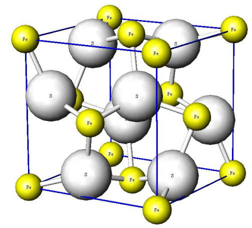

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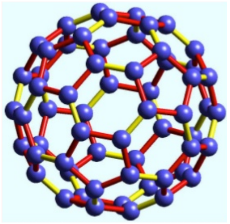

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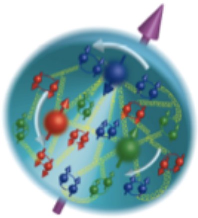
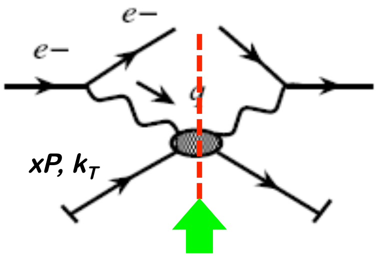


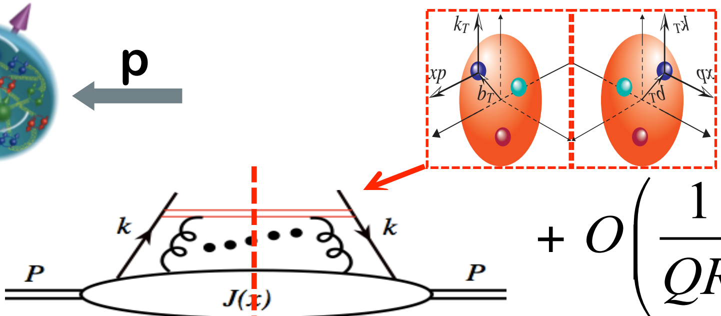
Motion of quarks/gluons is relativistic, color is fully entangled!

- **Asymptotic Freedom → Facotrization → localized “probe”**

$$\sigma_{\text{tot}}^{\text{DIS}} :$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Factorization

Hard-part
Probe

Parton-distribution
Structure

Power corrections
Approximation

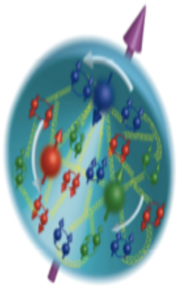
Hadron's partonic structure in QCD

- **Quantifying the partonic structure:**

  = Matrix elements of quarks and/or gluons

  = Quantum "probabilities" $\langle P, S | \mathcal{O}(\overline{\psi}, \psi, A^{\mu}) | P, S \rangle$

  None of these matrix elements is a direct physical observable in QCD – color confinement!



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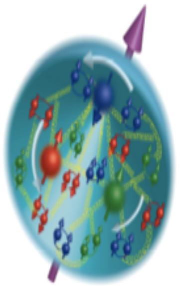
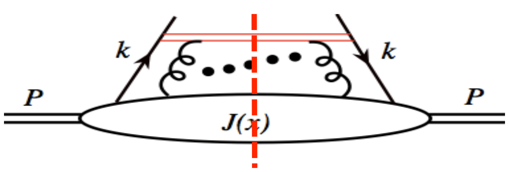
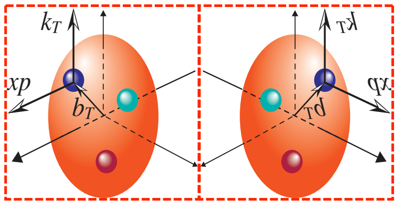


- **Parton distribution functions (PDFs):**



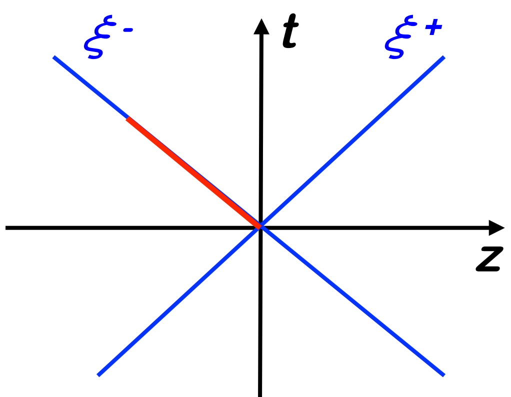


$$f_q(x,\mu^2) \equiv \int \frac{dP^+\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle P|\overline{\psi}(\xi^-)\frac{\gamma^+}{2P^+}\exp\left\{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right\}\psi(0)|P\rangle$$

Dominated by the region:

$$\xi^- \lesssim 1/(xP^+) \sim 1/Q$$

Quantum correlation of quark fields along ξ^- direction! (Conjugated to the large *P*$^+$)



Interpreted as:

Probability density to find a quark with a momentum fraction x

Hadron's partonic structure in QCD

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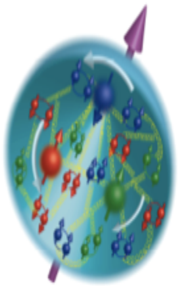
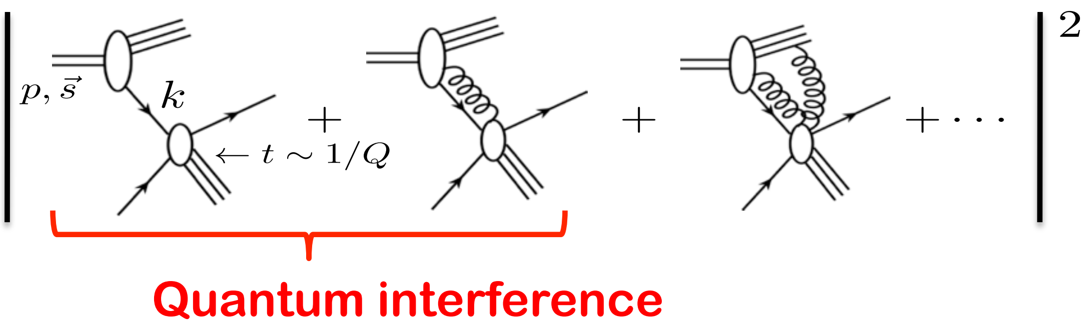


- **Multi-parton quantum correlations:**

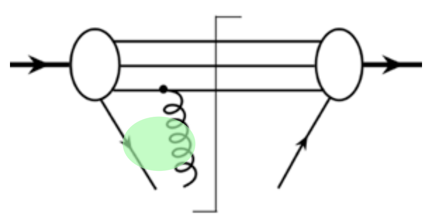
  ✧ **Spin-dependent cross section (one hard scale):**

$\sigma(Q, \vec{s}) \propto$



  ✧ **Spin-Asymmetry:**

$\sigma(s) - \sigma(-s) \;\Rightarrow\; T^{(3)}(x, x) \propto$



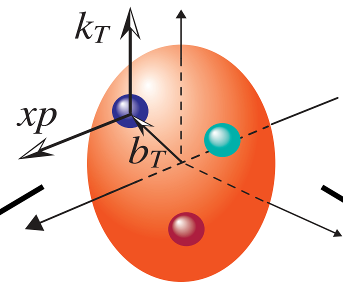
$\Rightarrow$ ***Quantum interference between a quark state and a quark/gluon composite state – Twist-3 operators!***

Hadron's partonic structure in QCD

❑ **Beyond collinear PDFs – 3D confined motion and spatial imaging:**

Momentum Space

Coordinate Space

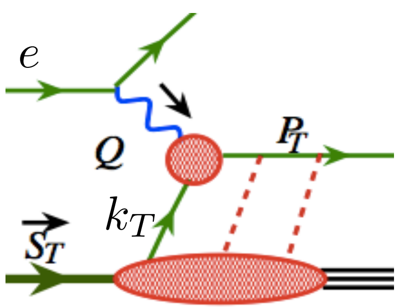


TMDs $\int d^2b_T$ $\int d^2k_T$ *GPDs*

Confined motion $f(x,k_T)$ $f(x,b_T)$ Confined spatial distribution

Two-scale observables!

✧ **Semi-inclusive process (SIDIS):**



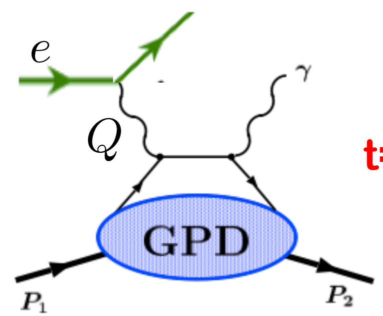
Observe a final-state Hadron (Control P_T)

Two scales: $Q >> P_T$

Initial-state proton is broken!

Probe "P_T" gives the "k_T"

✧ **Exclusive process (DVCS):**



$t=(p_1-p_2)^2$

Two scales: $Q^2 >> |t|^2$

Initial-state proton is NOT broken!

F.T. of "t" gives the "b_T"

Global QCD analyses – A successful story

- World data with “Q” > 2 GeV

  + QCD Factorization:

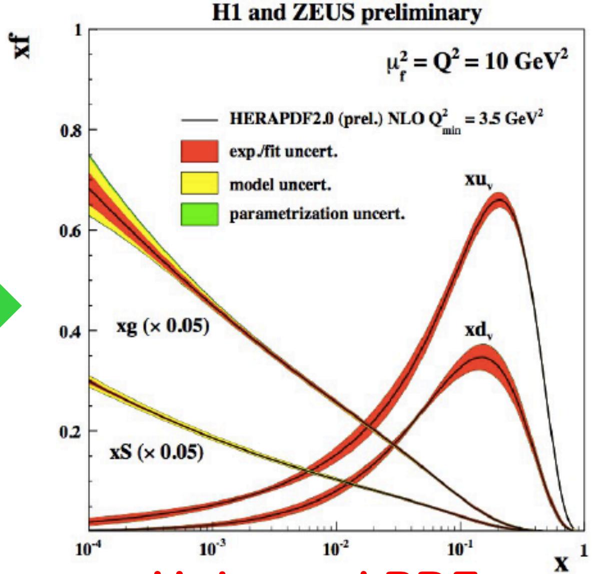
e-H: $F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$

H-H: $\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$

  + DGLAP Evolution:

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

See also Petriello’s talk



Universal PDFs

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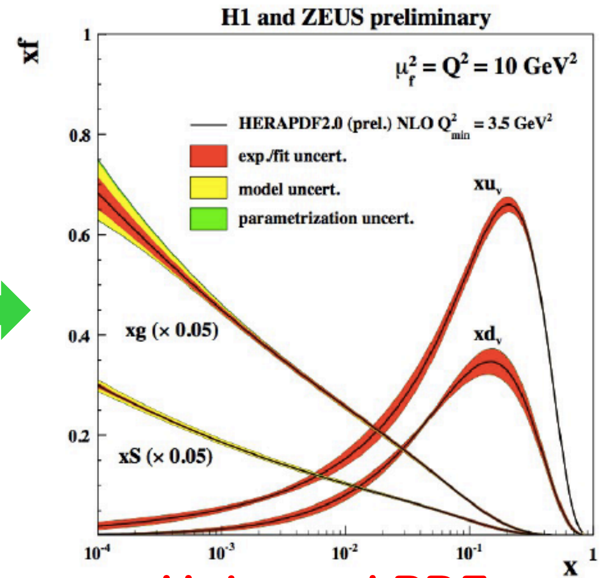

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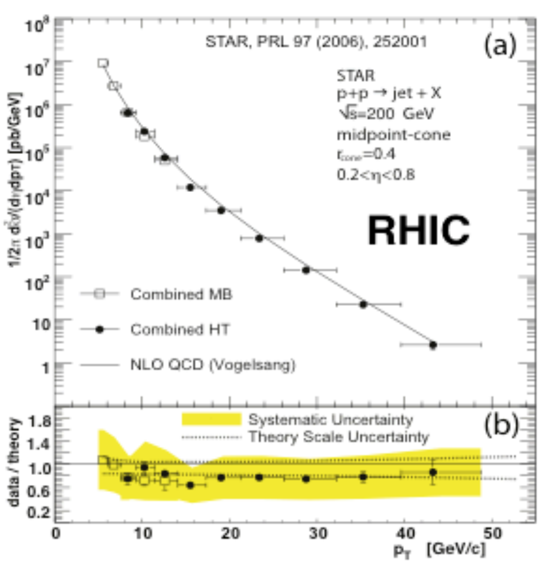
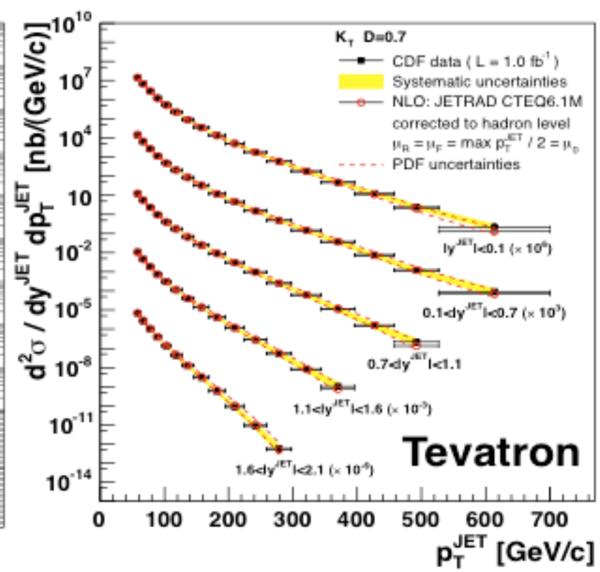
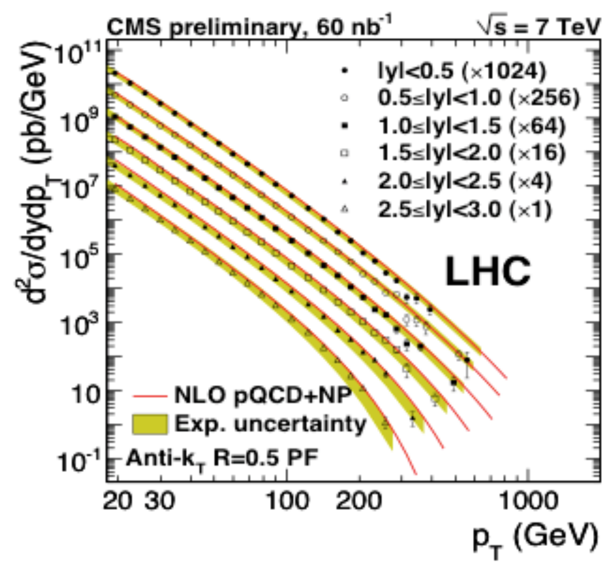
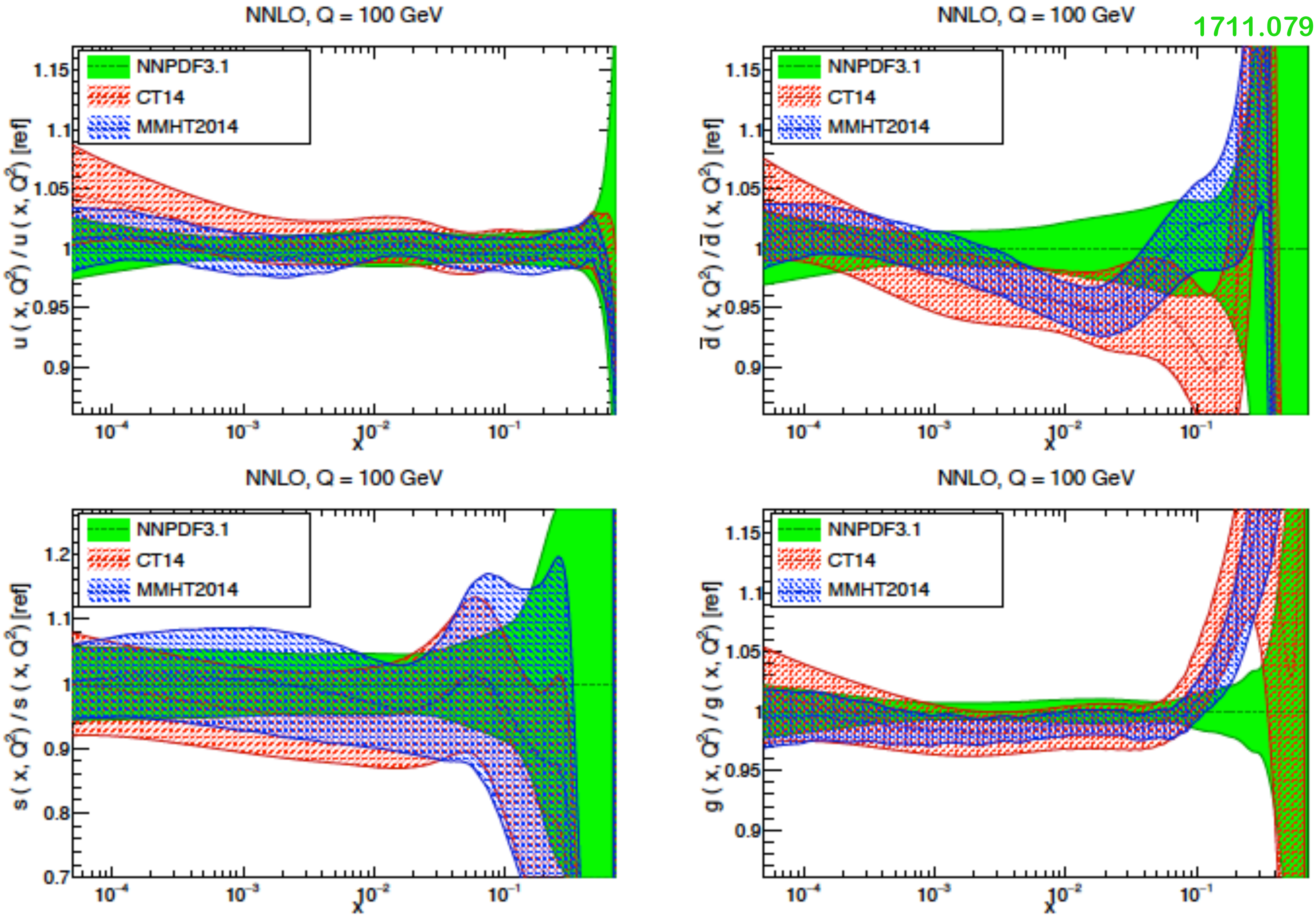






Uncertainties of PDFs

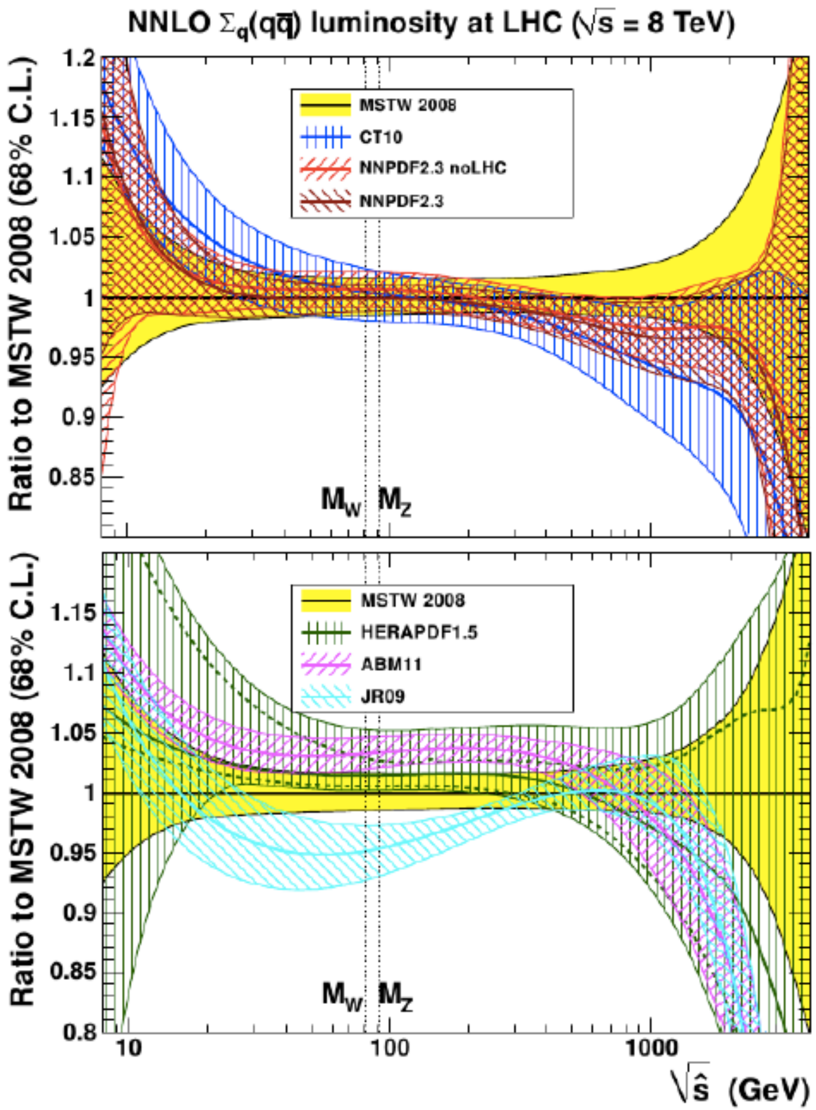
1711.07916



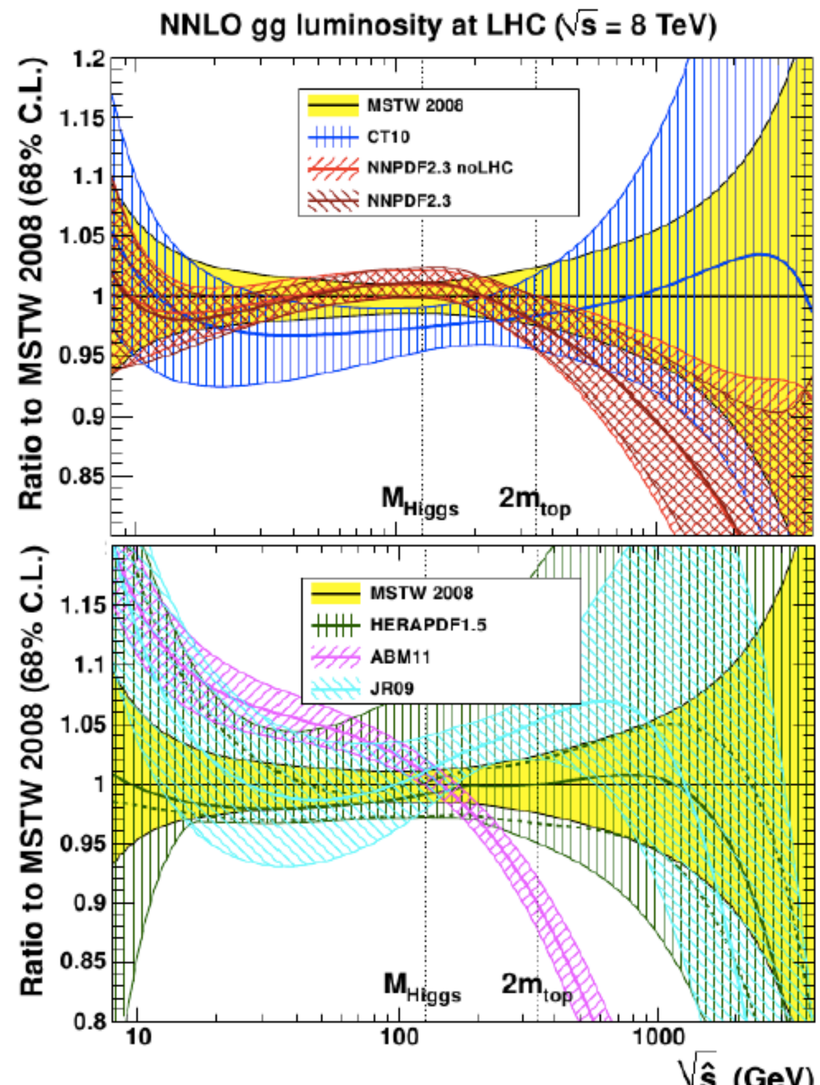
Partonic luminosities – discovery potential

See also Petriello's talk

q - qbar



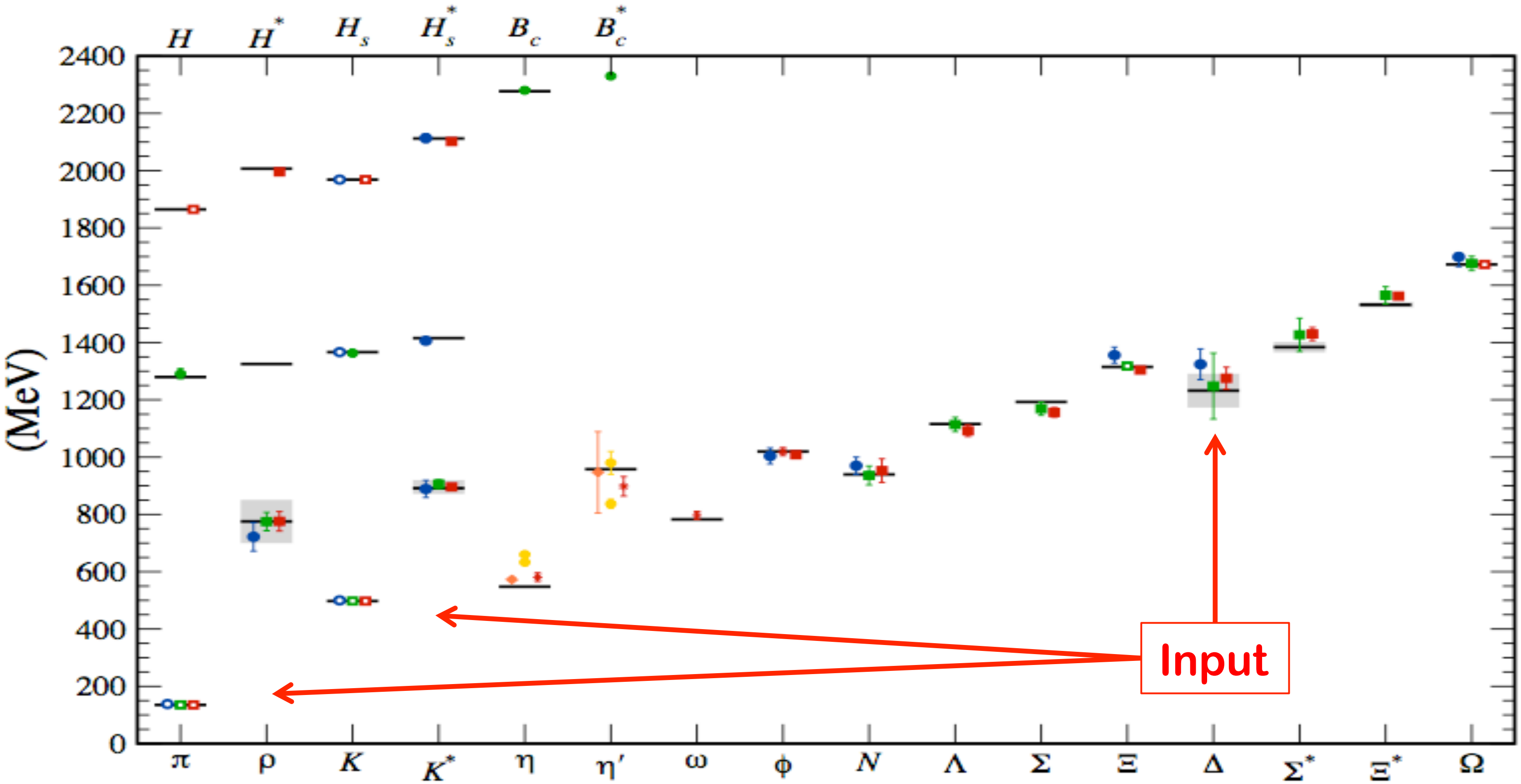
g - g



Lattice QCD

- Hadron masses:

Predictions with limited inputs



Lattice QCD

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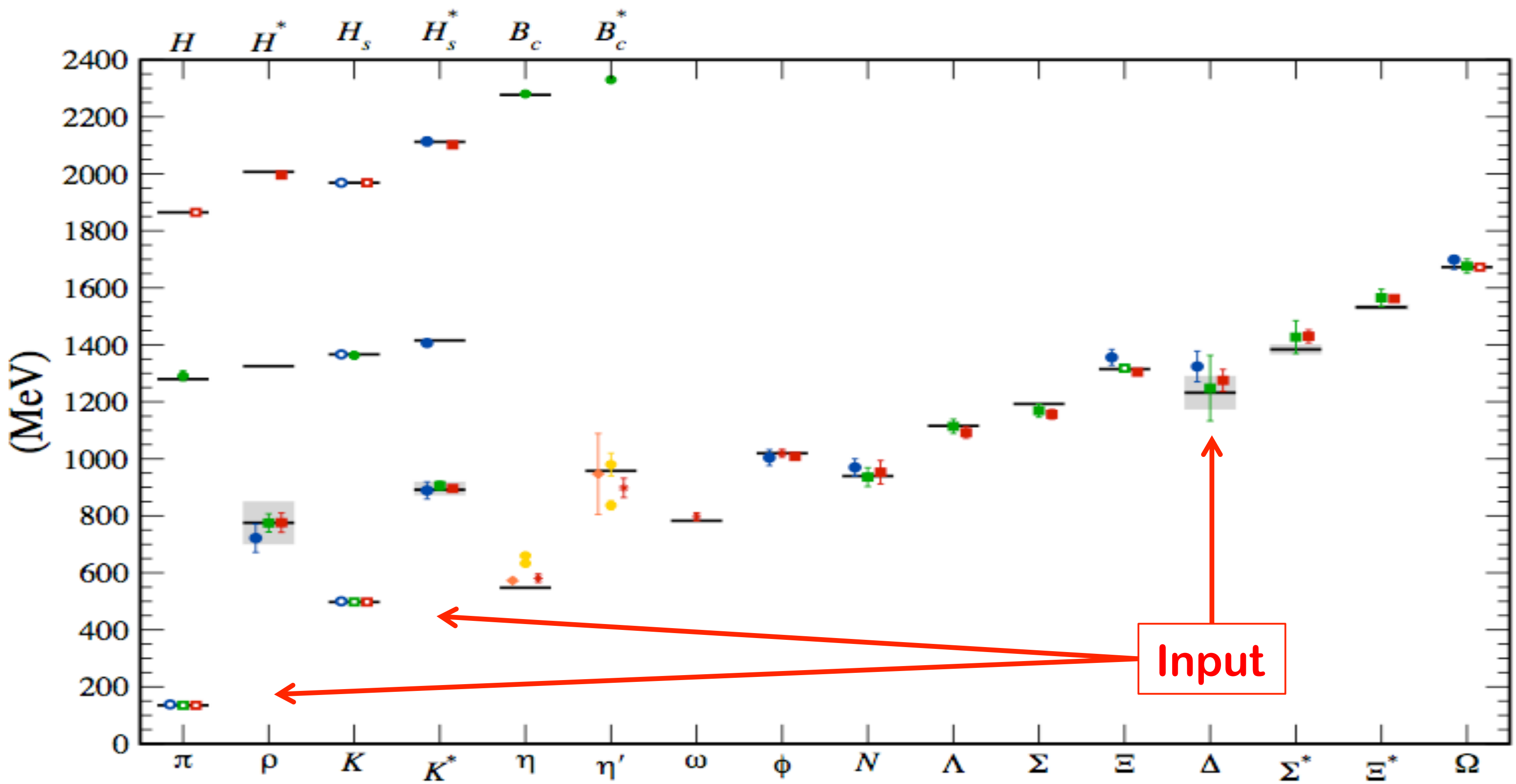


- ❑ **Lattice "time" is Euclidean:** $\tau = i\,t$

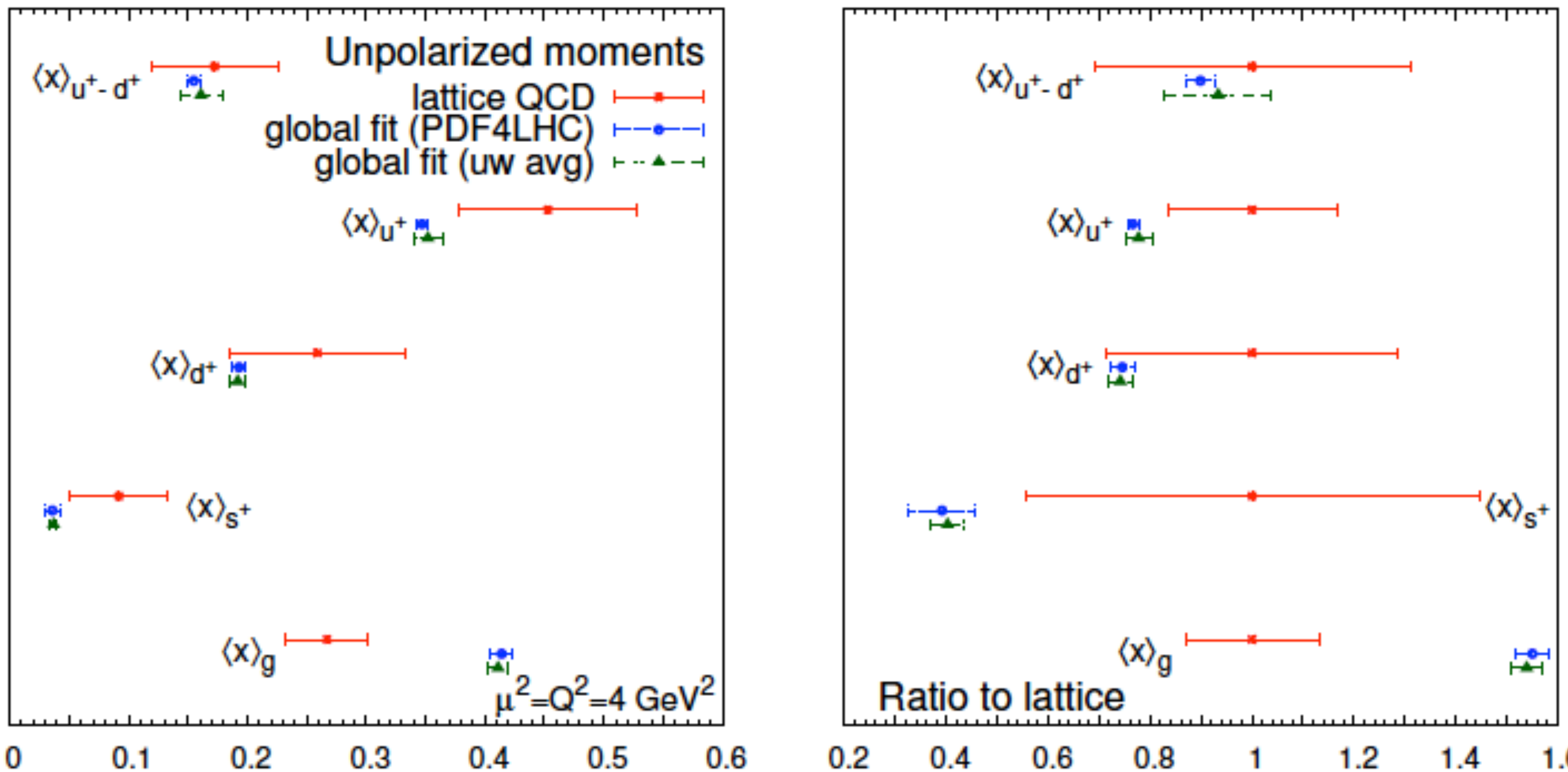
Cannot calculate PDFs, TMDs, …, directly, whose operators are time-dependent!

Moments of PDFs from lattice QCD

- Moments of PDFs – matrix elements of local operators

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2) \qquad q^\pm \equiv q \pm \bar{q} \quad \text{and} \quad \Delta q^\pm \equiv \Delta q \pm \Delta\bar{q}$$

- Moments of unpolarized PDFs:



$\mu^2 = 4\ \mathrm{GeV}^2$

arXiv:1711.07916

Moments of PDFs from lattice QCD

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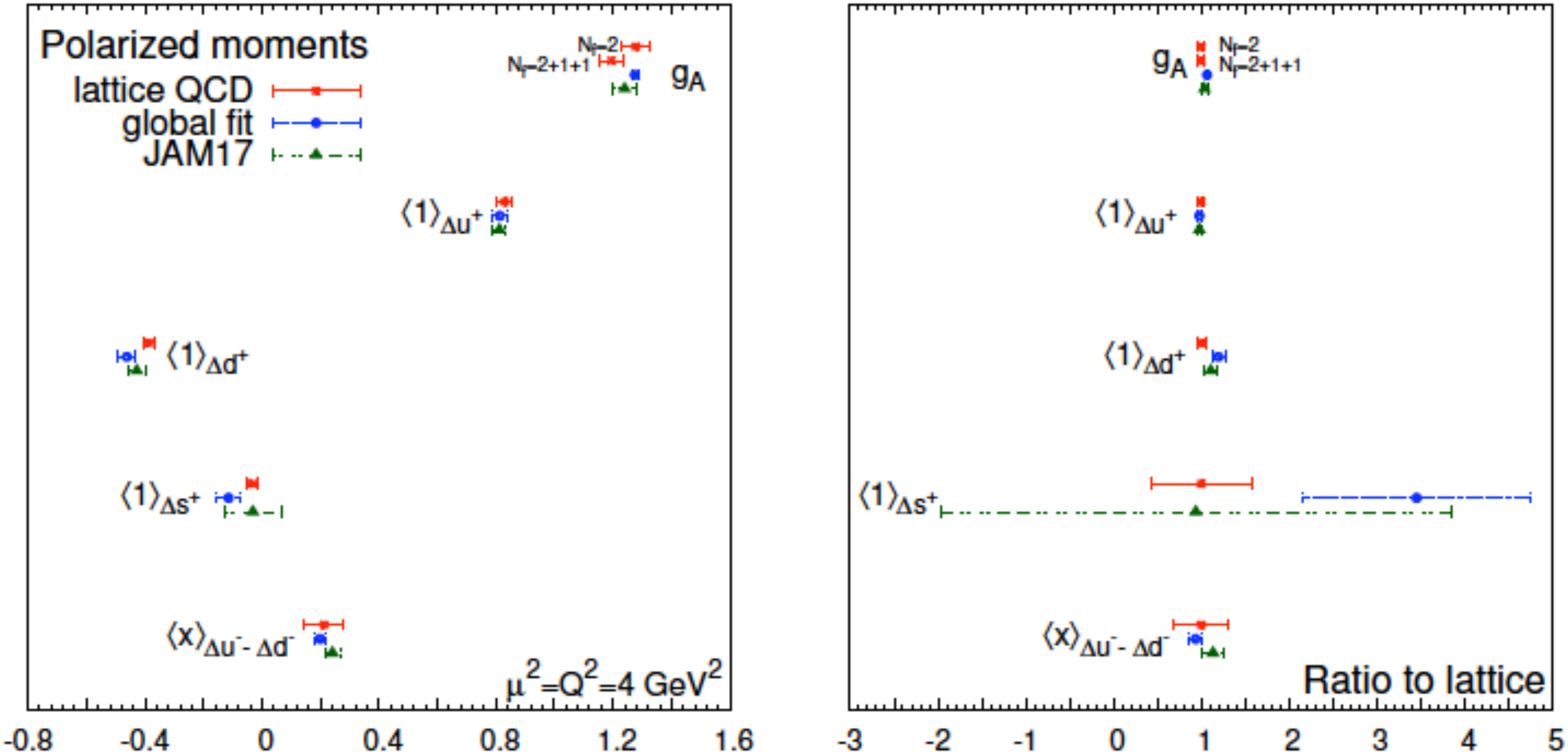
$\mu^2 = 4\ \text{GeV}^2$



- Axial charge:

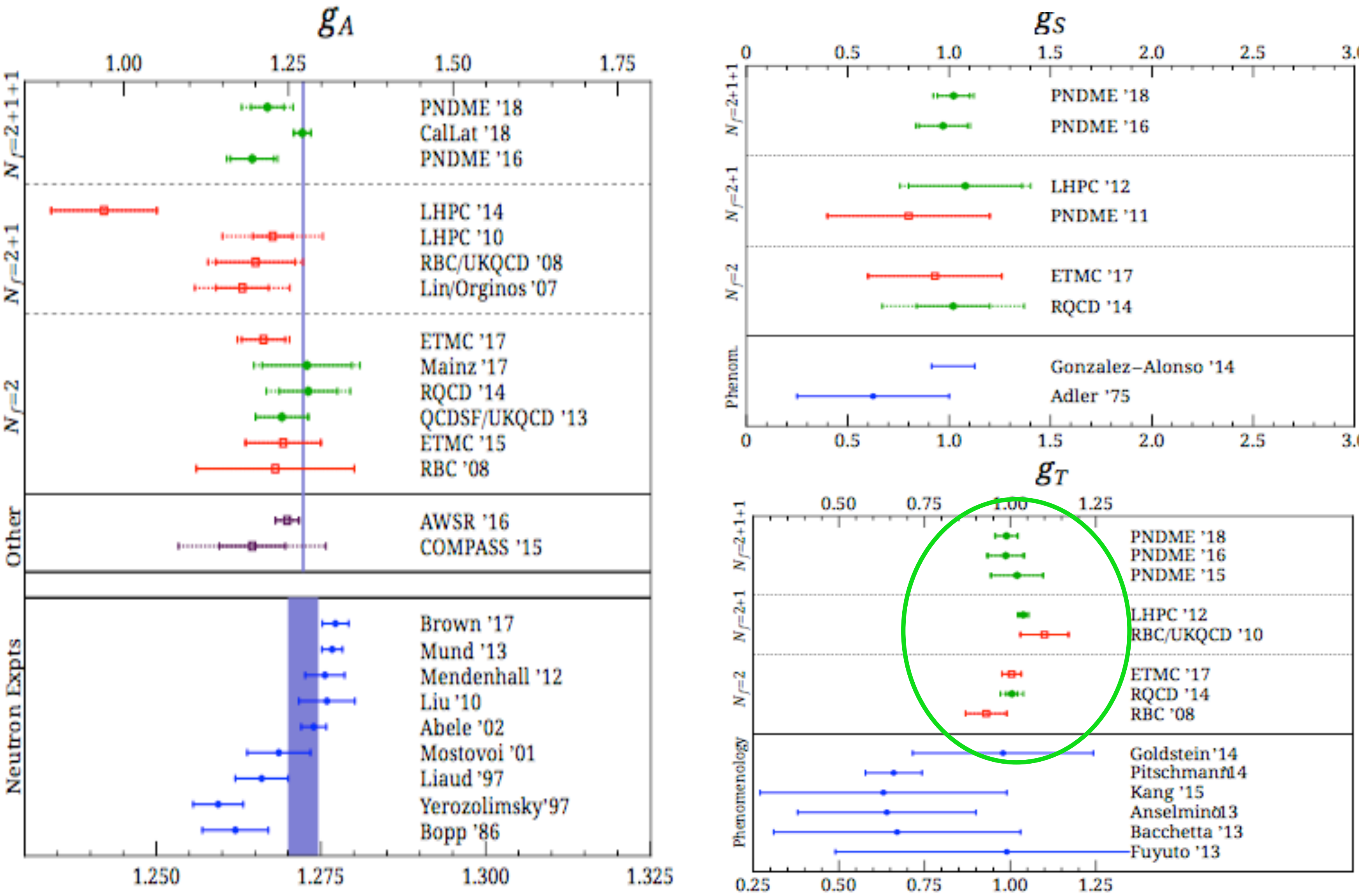
	Lattice QCD	Global Fit
$g_A \equiv \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$	$1.195(39)\ (N_f = 2+1+1)$ $1.279(50)\ (N_f = 2)$	$1.275(12)$

arXiv:1711.07916

Moments of PDFs from lattice QCD

arXiv:1806.09006

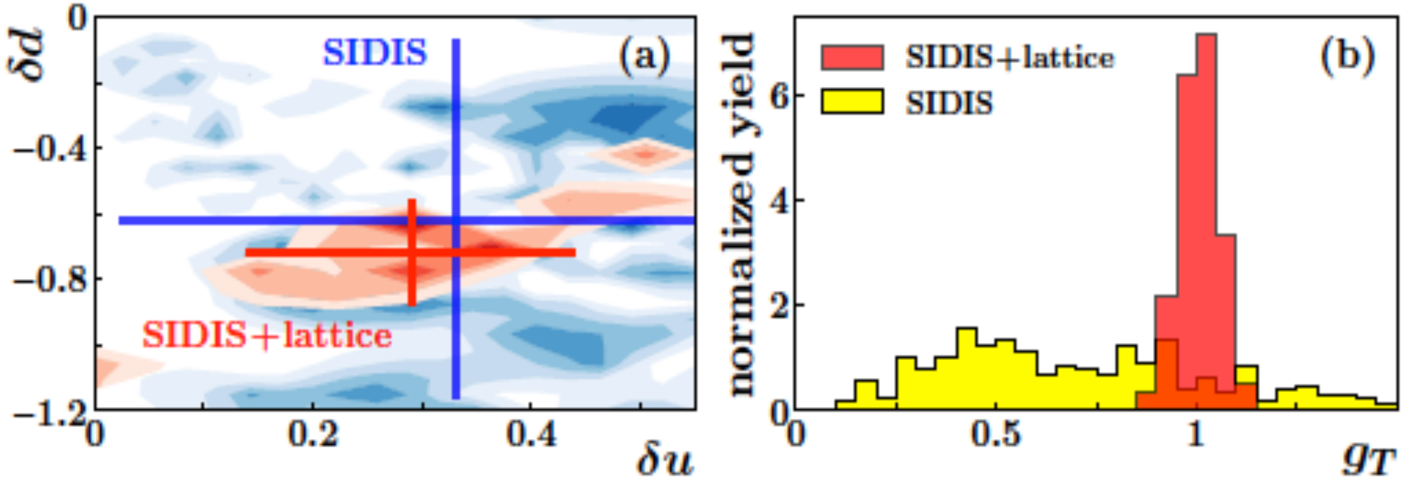
- **Isovector charges of nucleon:** $g_A^{u-d}, g_S^{u-d}, g_T^{u-d}$

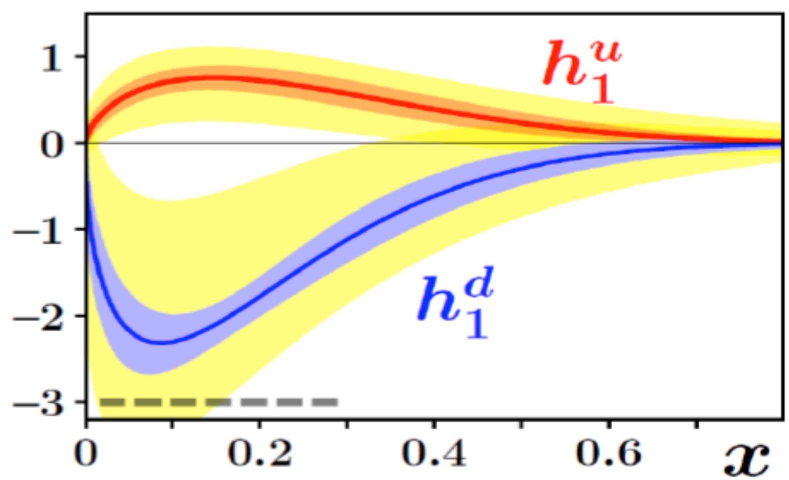


Lattice QCD helps QCD global analyses

- Improved transversity distribution with LQCD g_T:

$$h_1(x,\mu^2) = \int \frac{dP^+\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle P|\overline{\psi}(\xi^-)\frac{\gamma^+}{2P^+}\gamma_\perp\gamma_5 \exp\left\{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right\}\psi(0)|P\rangle$$





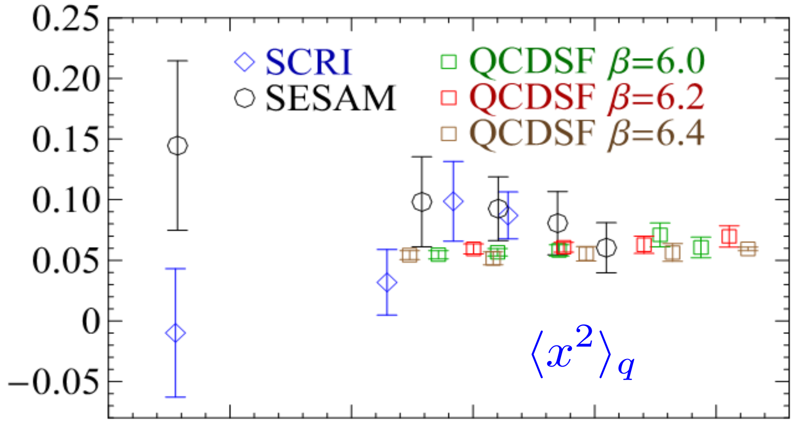
Much smaller error band

H.W. Lin et al, PRL 120, 152502 (2018)

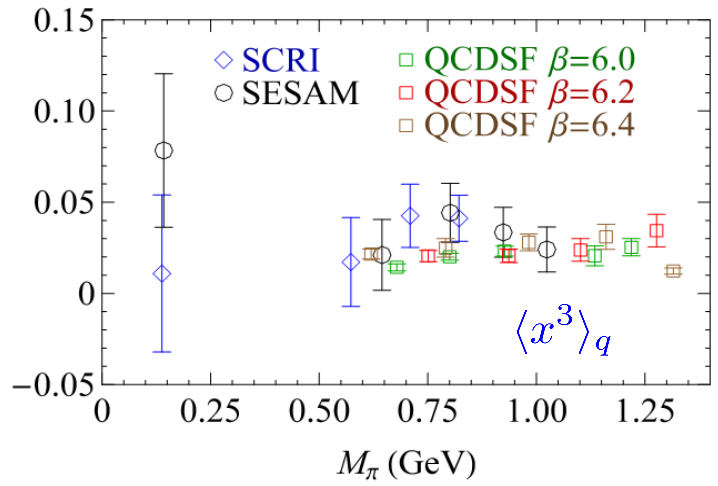
X-dependent PDFs from lattice QCD

- Limited moments:

Dolgov et al., hep-lat/0201021



Gockeler et al., hep-ph/0410187



Limited moments – hard to get the full x-dependent distributions!

X-dependent PDFs from lattice QCD

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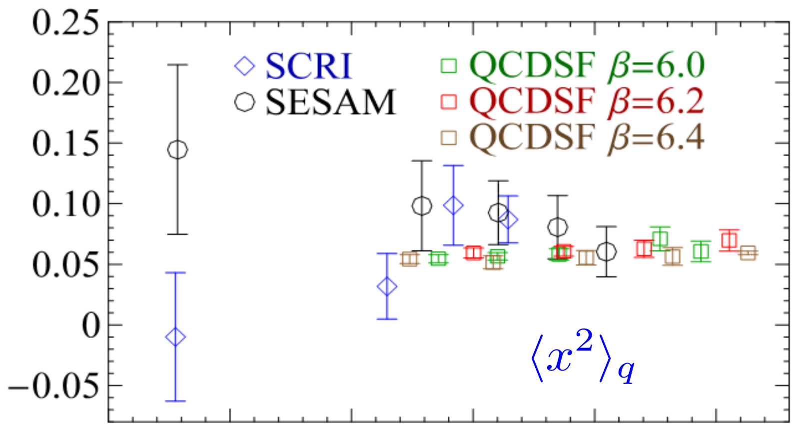

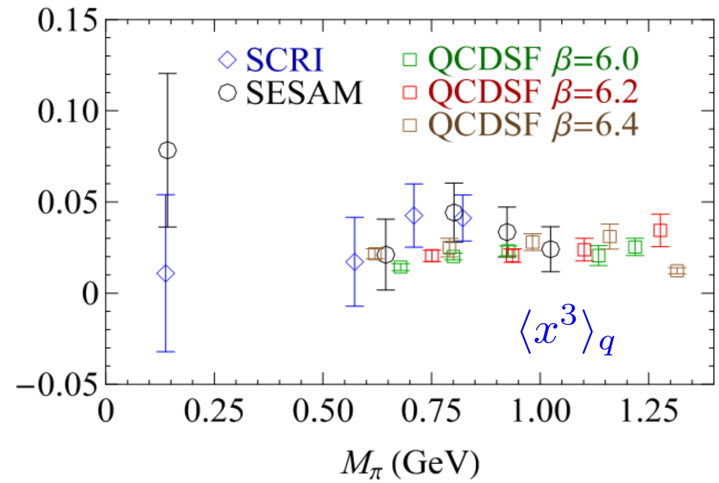

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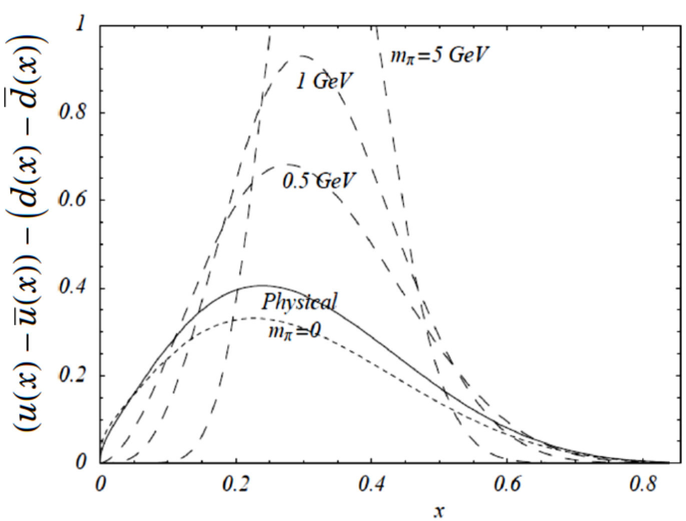
❑ Early efforts:



✧ **Assume a smooth functional form:**

$$xq(x) = a\,x^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma\,x)$$

✧ **Fix parameters with LQCD moments:**

W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

Cannot distinguish valence quark contribution from sea quarks

New ideas

- **Quasi-PDFs:**

Ji, arXiv:1305.1539

$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \equiv \int \frac{d\xi_z}{4\pi}\, e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(\frac{-\xi_z}{2})|P\rangle$$

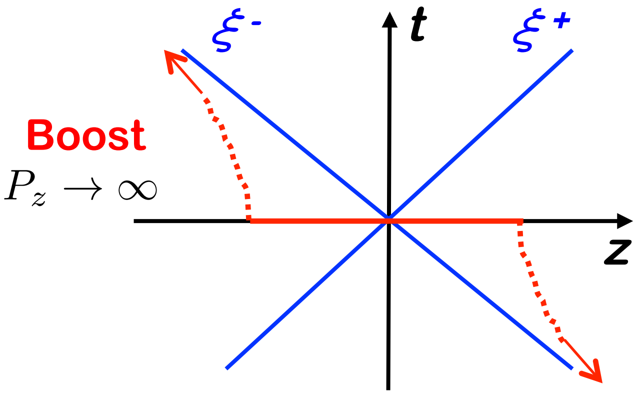
Idea:

Quasi-PDFs are not boost invariant

$\tilde{q}(\tilde{x},\mu_R^2,P_z) \longrightarrow q(x,\mu^2)$ when $P_z \to \infty$

Note:

In Lattice QCD calculation, difficult to take $P_z \to \infty$ limit



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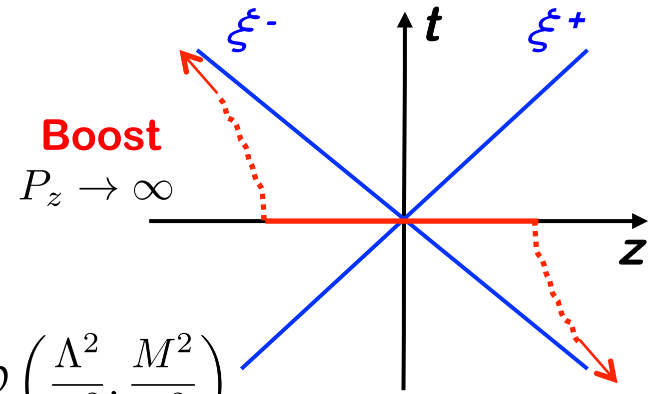

Note:

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Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

In terms of Large Momentum Effective Theory (LaMET)

Caution:

Braun, Vladimirov and Zhang, 1810.00048

Power corrections could be of the form: $\dfrac{\Lambda_{QCD}^2 R}{x^2(1-x)P_z^2}$

Mixing with gluon and other flavor contribution beyond LO

Power UV divergence - μ_R does not obey DGALP

New ideas

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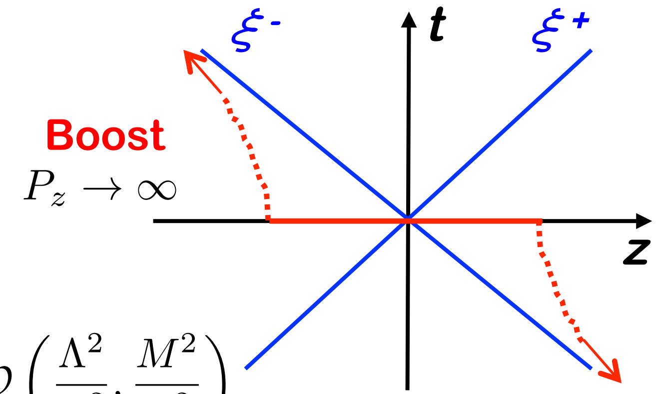

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Power UV divergence - μ_R does not obey DGALP

Progress:

Quark: 1706.08962, 1707.03107, 1707.07152 Gluon: 1808.10824, 1809.01836

Power UV divergences are multiplicatively renormalizable

Renormalized Q-PDFs are collinearly factorizable to PDFs 01404.6860

New ideas

- **Pseudo-PDFs:**
  - **Lattice calculation with** $\alpha = 0$**:**

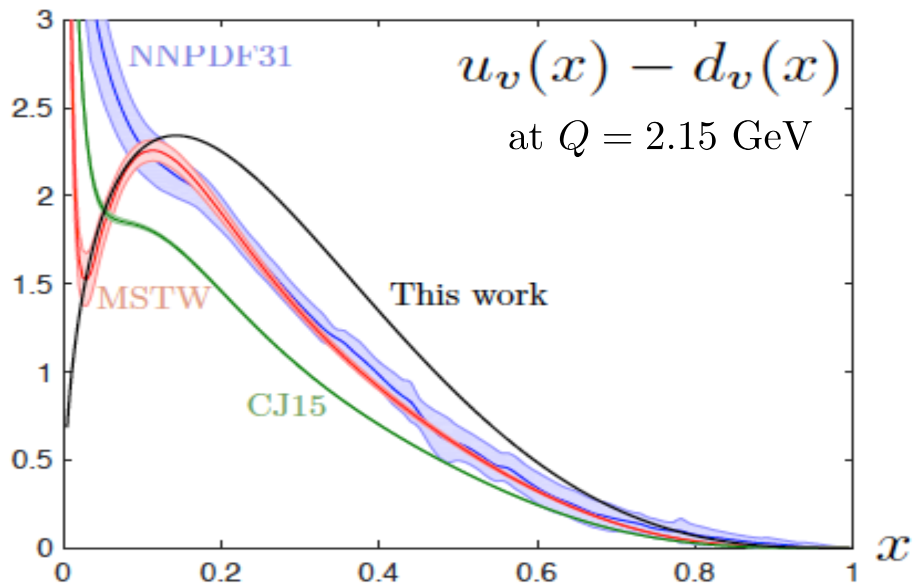
$$\mathcal{M}^{\alpha}(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^{\alpha}\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$
$$\equiv 2p^{\alpha}\mathcal{M}_p(\nu,\xi^2) + \xi^{\alpha}(p^2/\nu)\mathcal{M}_{\xi}(\nu,\xi^2) \approx 2p^{\alpha}\mathcal{M}_p(\nu,\xi^2)$$

$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi}\, e^{ix\,\nu}\mathcal{M}_{p=p^0}(\nu,\xi^2)/\mathcal{M}_{p=p^0}(0,\xi^2)$$

Remove UV!

  - **Model quasi-PDFs:**

- **First numerical results:**



Orginos, et al,
PRD96, 094503 (2017)

One-loop matching recently Completed!

A. Radyushkin, arXiv:1801.02427

New ideas

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- Good "Lattic cross sections":

  = Single hadron matrix element:

  $\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$ with $\omega \equiv P \cdot \xi,\ \xi^2 \neq 0,\ \text{and}\ \xi_0 = 0;$ and

  1) can be calculated in lattice QCD with precision,
     has a well-defined continuum limit (UV+IR safe perturbatively), and
  2) can be factorized into universal matrix elements of quarks and gluons
     with controllable approximation

Collaboration between lattice QCD and perturbative QCD!

New ideas

Ma and Qiu, arXiv:1404.6860
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Collaboration between lattice QCD and perturbative QCD!

- **Current-current correlators:**

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}^{-1}\, Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

with

d_j : Dimension of the current

Z_j : Renormalization constant of the current

Sample currents:

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q \gamma\cdot\xi\psi_q](\xi),$$

$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q \gamma\cdot\xi\psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4}F_{\mu\nu}^c F_{\mu\nu}^c](\xi), \ldots$$

- **Quasi- and pseudo-PDFs:**

$$\mathcal{O}_q(\xi) = Z_q^{-1}(\xi^2)\overline{\psi}_q(\xi)\,\gamma\cdot\xi\,\Phi(\xi,0)\,\psi_q(0) \qquad \Phi(\xi,0) = \mathcal{P}e^{-ig\int_0^1 \xi\cdot A(\lambda\xi)\,d\lambda}$$

Exploratory effort and its success

❑ **Two lattice collaborations did pioneering work for quark-qPDFs:**

Both LP3 and ETMC obtained their results at physical pion mass!

✧ **Calculate quasi-quark distribution:**

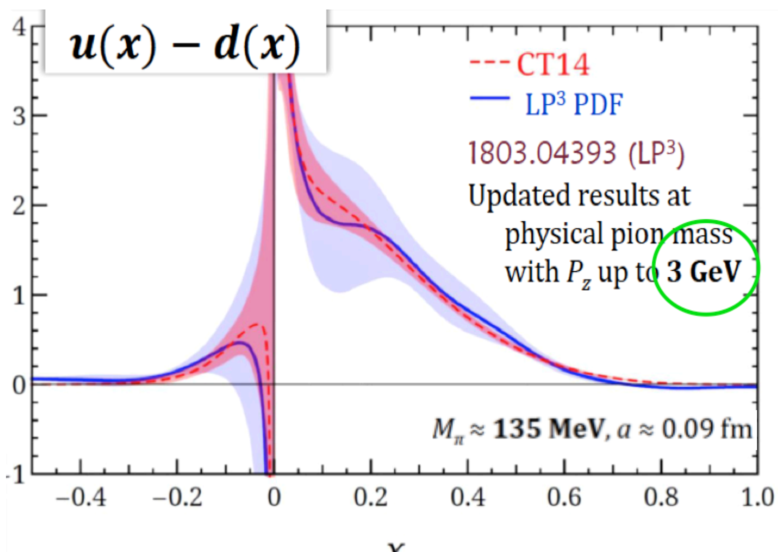
$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\, \Gamma \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

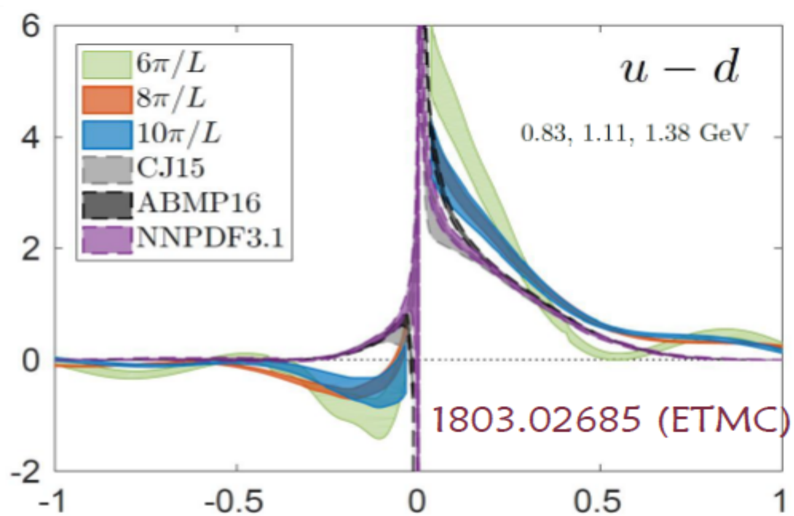
✧ **Extract quark distribution:**

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Inverse to get the PDFs

✧ **The state of art:**



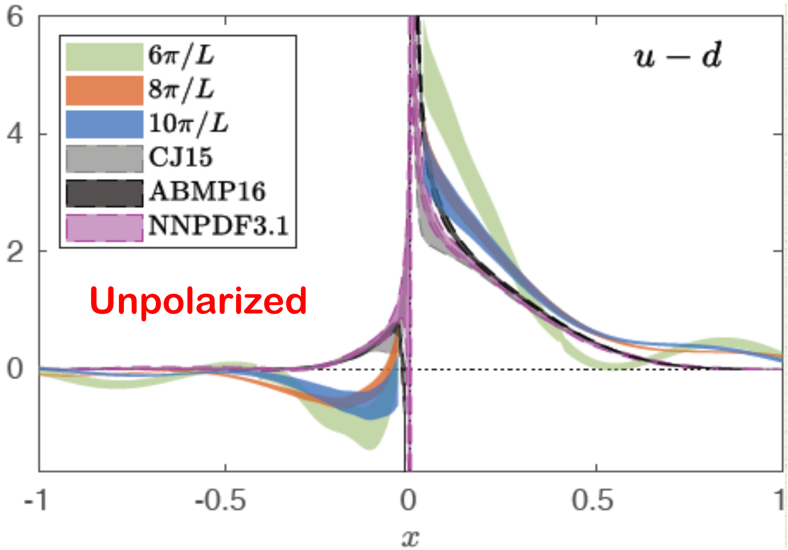


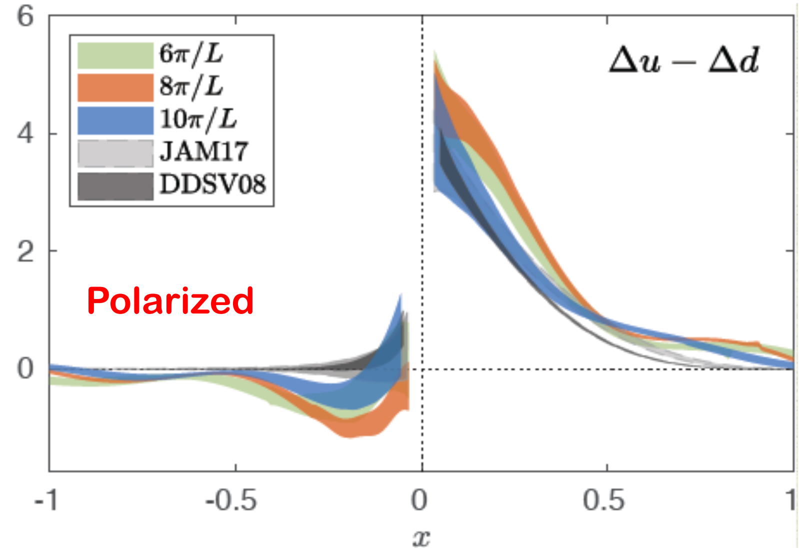
With one-loop perturbative matching and target mass correction

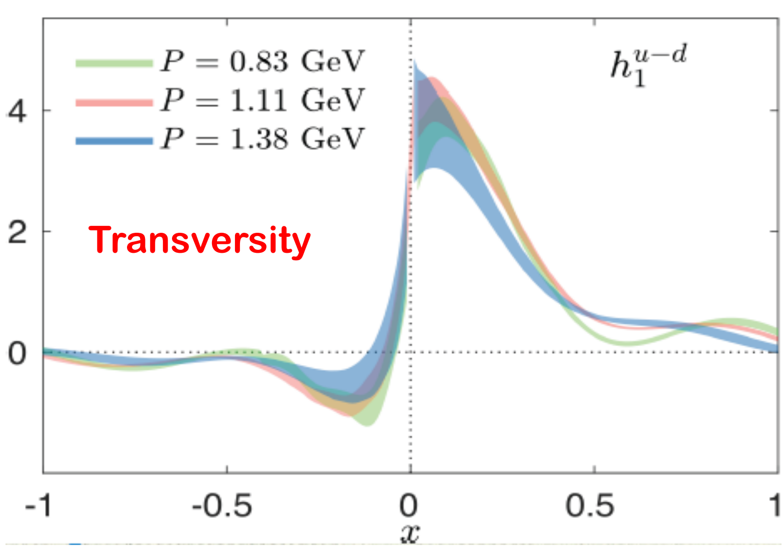
Exploratory effort and its success

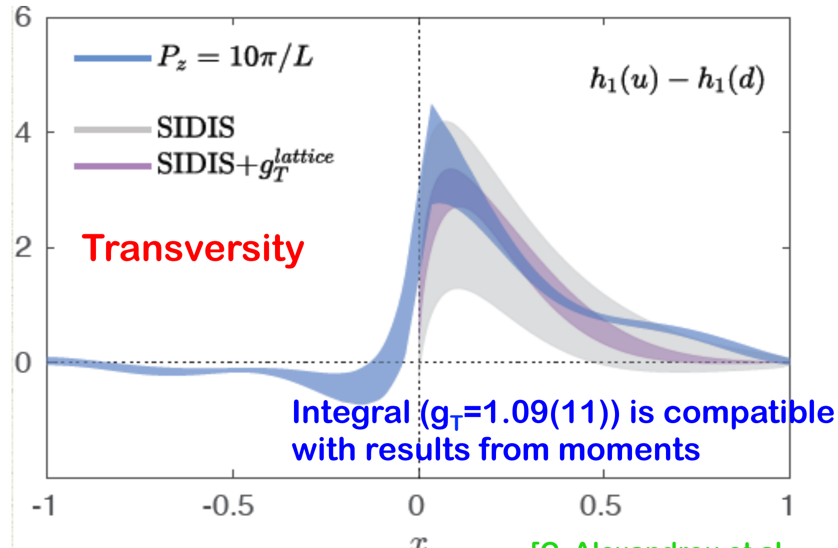
M. Constantinou at CTEQ meeting

❑ Light-cone PDFs - ETMC:









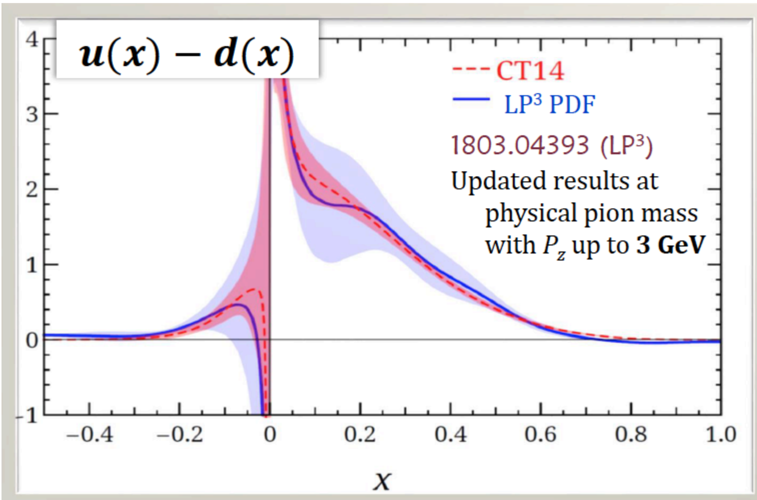
All with one-loop perturbative matching!

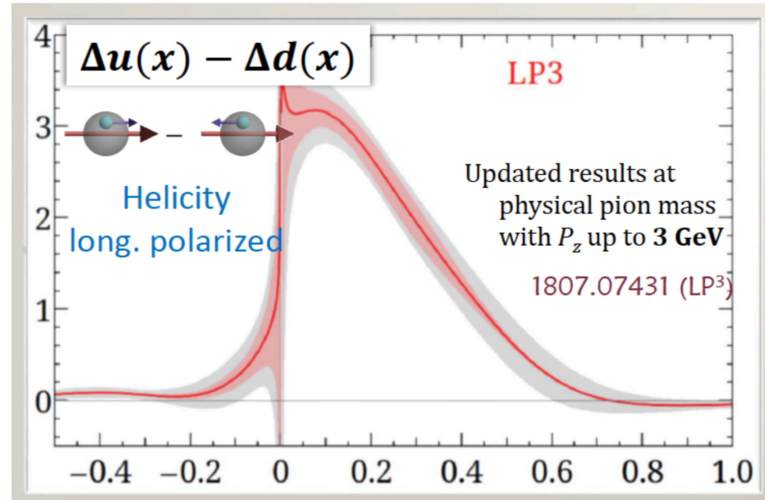
[C. Alexandrou et al., PRD95, 114514 (2017)]

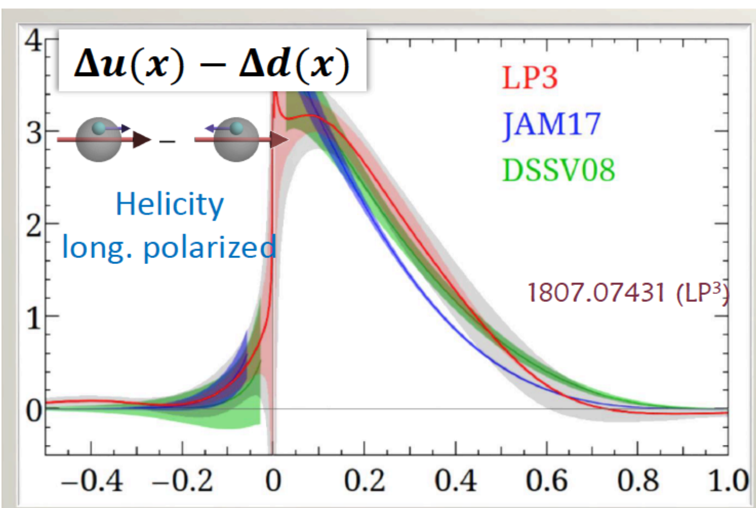
Exploratory effort and its success

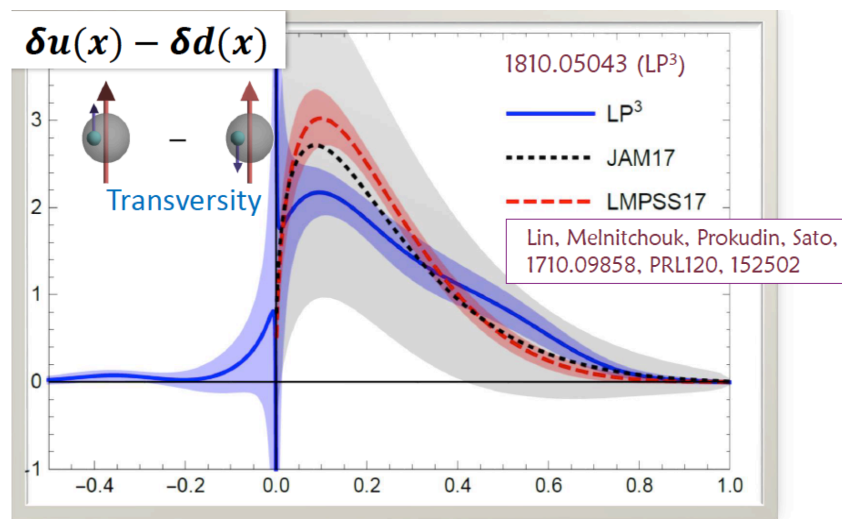
❑ Light-cone PDFs – LP3:

H.W. Lin at CTEQ meeting









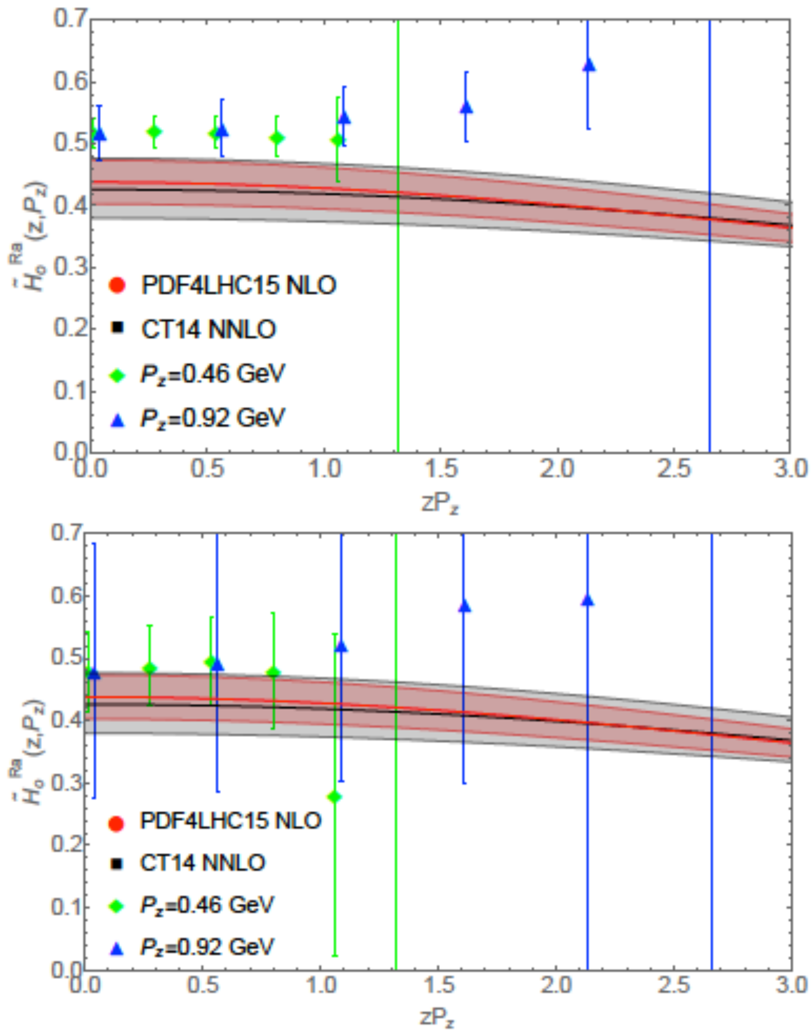
All with one-loop perturbative matching!

Exploratory effort and its success

- **First glimpse into gluon Q-PDFs – LP3:**

H.W. Lin at CTEQ meeting
1808.02077

Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass



Like quark Q-PDFs, gluon Q-PDFs have power UV divergence, and could mix with others under renormalization, …

UV divergence are multiplicatively Renormlizable to all orders in pQCD

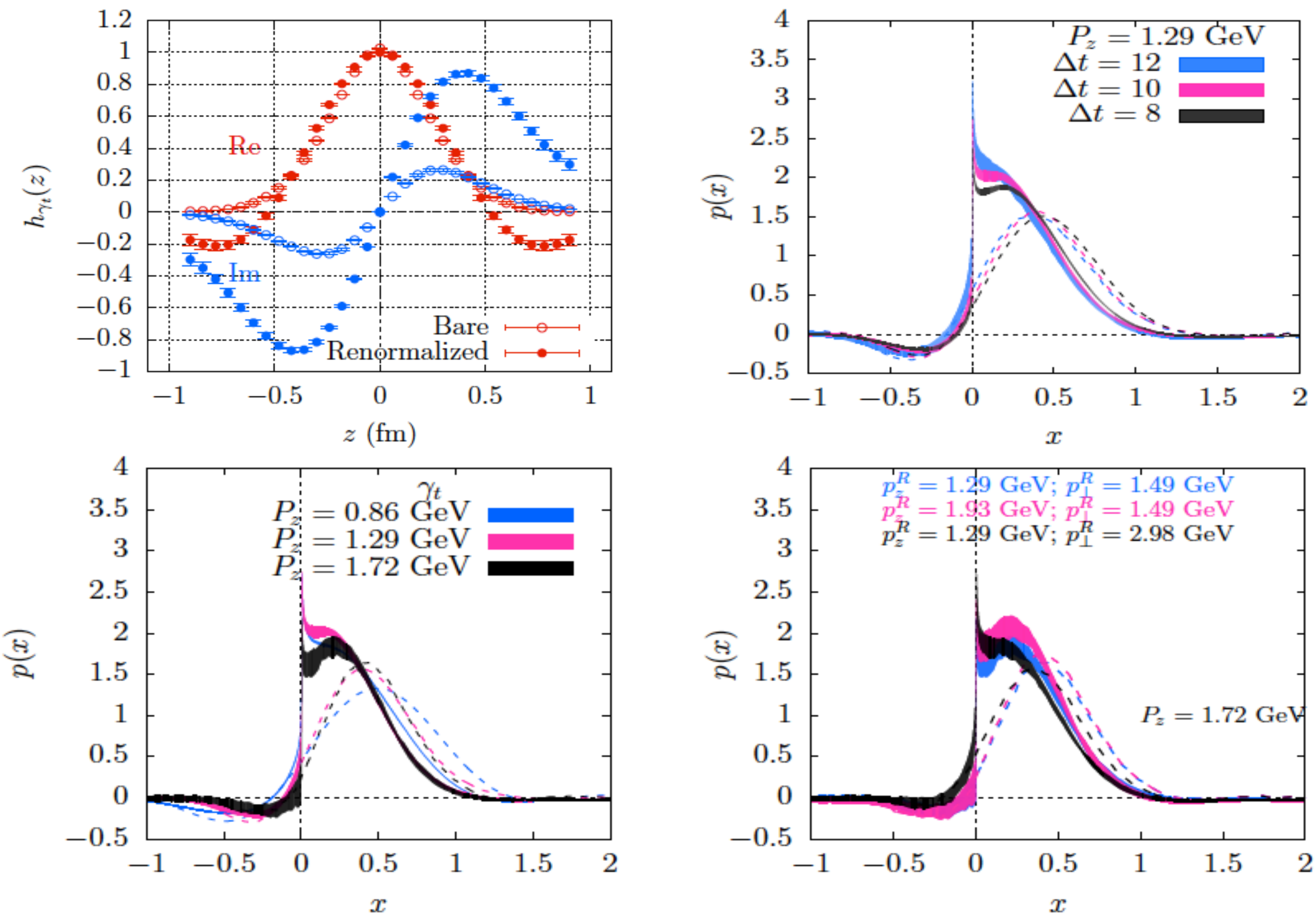
Zhang et al, arXiv:1808.10824
Li et al, arXiv:1809.01836

$$\tilde{H}_0^{Ra}(z,P_z,\mu) = \frac{\tilde{H}_0^{\overline{\rm MS}}(0,0,\mu)}{\tilde{H}_0(z,0)}\tilde{H}_0(z,P_z)$$

Exploratory effort and its success

- ❑ **Pion qPDFs & PDFs:**

N. Karthik at Lattice2018
BNL, SB, UConn, …

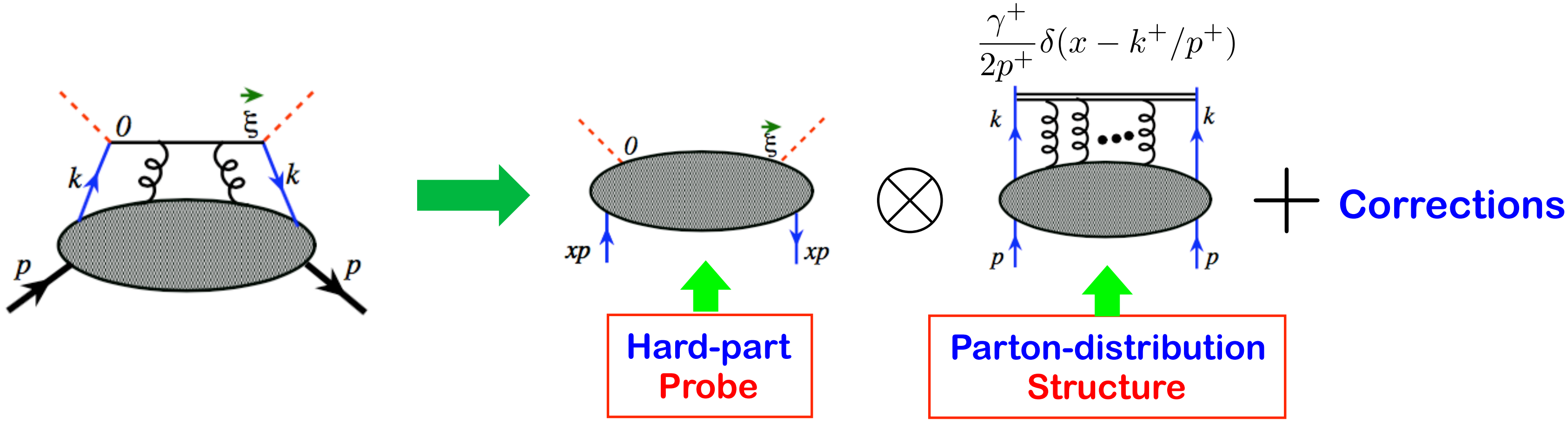


All with one-loop perturbative matching!

Exploratory effort and its success

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- "Lattice cross section":



- QCD Global analysis:

  Need data of "many" good lattice cross sections to be able to extract the x, Q, flavor dependence of the structure, …

- Complementarity and advantages:
  - ✧ Complementary to existing approaches for extracting PDFs,
  - ✧ Quasi-PDFs and pseudo-PDFs are special cases,
  - ✧ Have tremendous potentials:

    Neutron PDFs, … (no free neutron target!)
    Meson PDFs, such as pion, …
    More direct access to gluons – gluonic current, quark flavor, …

Exploratory effort and its success

JLab lattice group
Ma, Qiu, arXiv:1709.03018
PRL (2018)

- **Pion/Keon PDFs:**

  – using a vector-axial-vector correlation as an example

✧ **Parity-Time-reversal invariance:**

$$\sigma_{ij}^{\mu\nu}(\xi, p) = \xi^4 \langle \pi(p) | \mathcal{J}_i^{\mu}(\xi/2)\, \mathcal{J}_j^{\nu}(-\xi/2) | \pi(p) \rangle$$

$$\frac{1}{2}\left[\sigma_{VA}^{\mu\nu}(\xi, p) + \sigma_{AV}^{\mu\nu}(\xi, p)\right] \equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\omega, \xi^2) + (p^\mu \xi^\nu - \xi^\mu p^\nu)\, T_2(\omega, \xi^2)$$

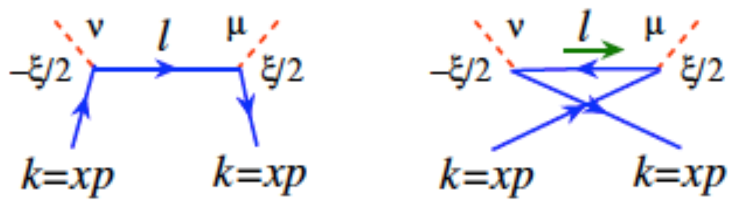
✧ **Collinear factorization:**

$$T_i(\omega, \xi^2) = \sum_{a=q,\bar{q},g} \int_0^1 \frac{dx}{x} f_a(x, \mu^2)\, C_i^a(x\omega, \xi^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

✧ **Lowest order coefficient functions:**

$$C_1^{q(0)}(x\omega, \xi^2) = T_1^{q(0)}(x\omega, \xi^2) = \frac{2x}{\pi^2}\cos(x\omega)$$

$$C_2^{q(0)}(x\omega, \xi^2) = 0$$



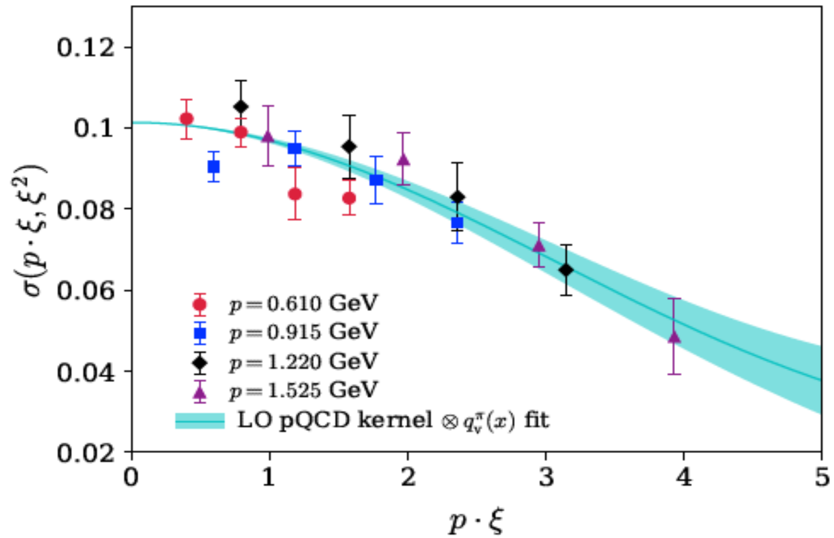
$$\tilde{T}_1(\tilde{x}, \xi^2) \equiv \int \frac{d\omega}{2\pi} e^{-i\tilde{x}\omega}\, T_1(\omega, \xi^2)$$
$$\approx \frac{1}{\pi^2}\{q(\tilde{x}) - \bar{q}(\tilde{x})\} = \frac{1}{\pi^2} q_{\rm v}(\tilde{x})$$

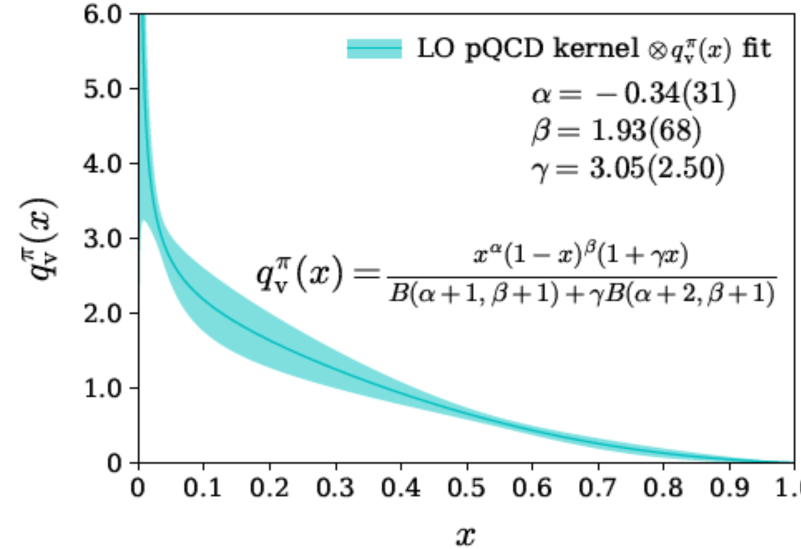
Lattice QCD calculation ⟶ *pion valence quark distribution!*

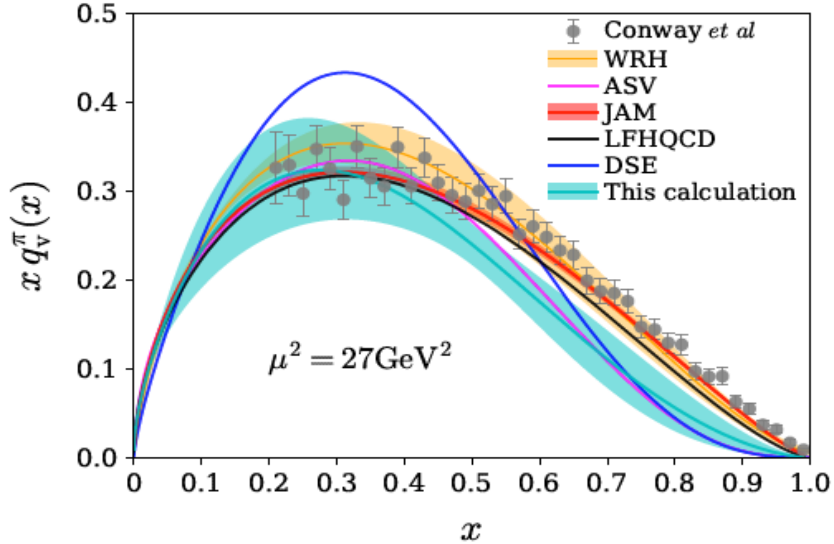
Exploratory effort and its success

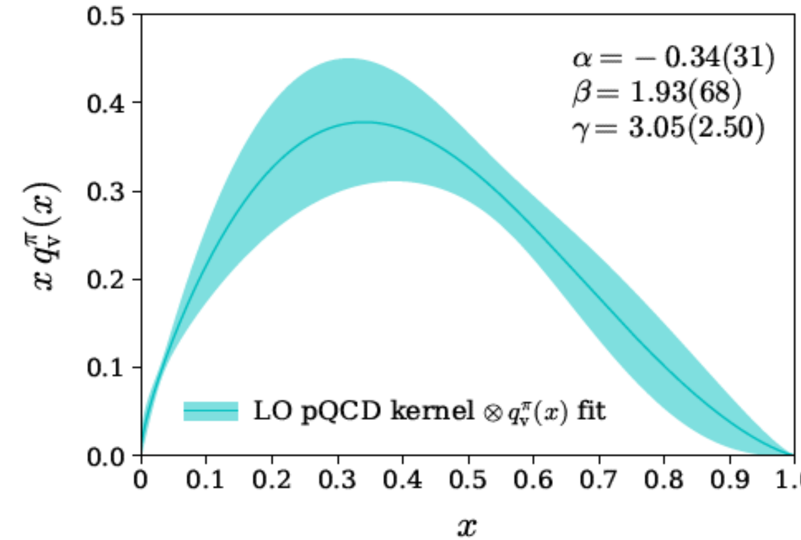
JLab lattice group
arXiv:1901.03921

❑ Pion valence PDFs:









Tree-level (LO) hard coefficient only!

Summary and outlook

- Lattice QCD calculation will provide rich information on hadron's partonic structure:

  - ✧ Quasi-PDFs approach
  - ✧ Pseudo-PDFs approach
  - ✧ "Lattice cross section" approach

  Key: Controllable matching to PDFs, GPDs, TMDs, …

Community White Paper
arXiv:1711.07916

Summary and outlook

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Community White Paper
arXiv:1711.07916

Key: Controllable matching to PDFs, GPDs, TMDs, …

- Results of initial exploratory calculations are encouraging:
  - ✧ Good for relatively large x partonic structure – computing power
  - ✧ Complementary to experimental measurements

$$\tilde{q}(\tilde{x}, \mu_R^2, P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(\frac{-\xi_z}{2})|P\rangle$$

With P_z ~ GeV, ξ_z ~ fm, it is hard for $x < 0.1$ while keeping the exponential to be larger than 1!

- Like its success in calculating the "baryon interaction" and "hadron spectroscopy", lattice QCD can be used to study hadron structure, while more works are needed!

Thank you!