Mitigating large background with subtracted cumulants

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arXiv:1803.04413 Daekyoung Kang, YM, Thomas Mehen

arXiv:1812.06977 Yang-Ting Chien, Daekyoung Kang, Kyle Lee, and YM

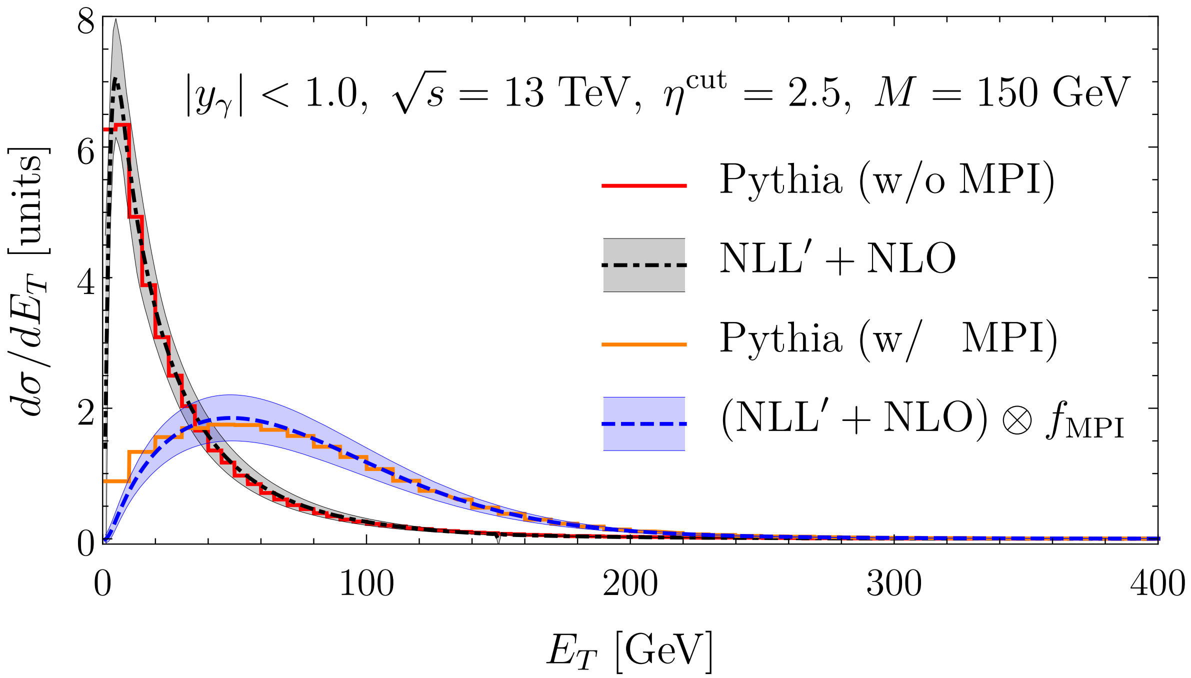
Outline

proton-proton :

background = MPI

Part 1:
Subtracted Cumulants in transverse energy

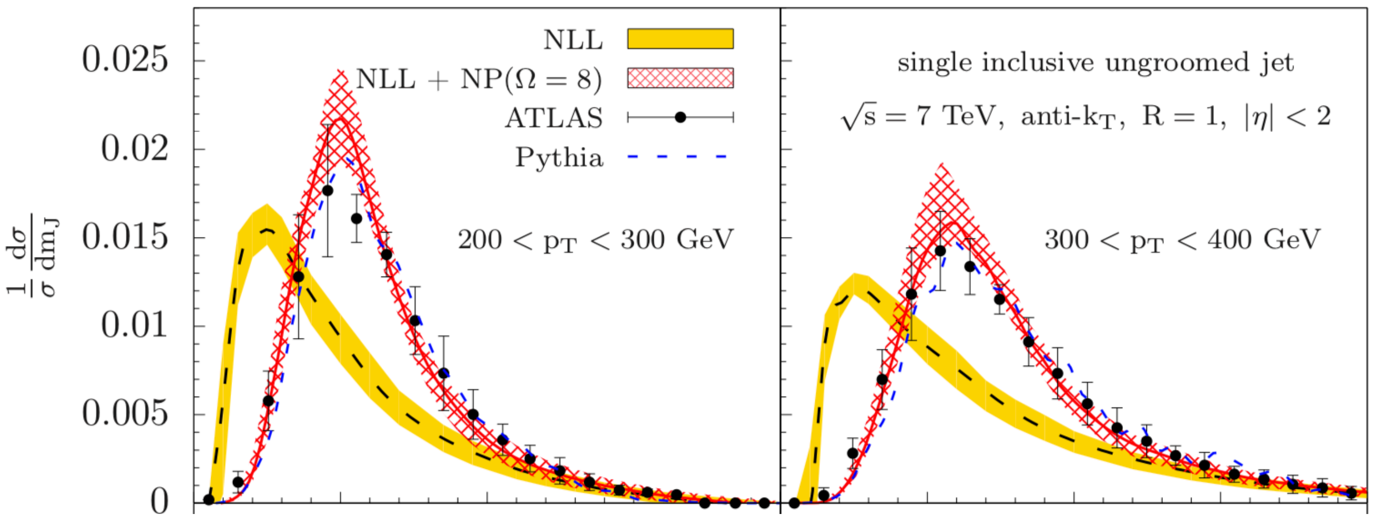
arXiv:1803.04413 Daekyoung Kang, YM, Thomas Mehen



Part 2:
Subtracted Cumulants in jet mass

arXiv:1203.4606: ATLAS

arXiv:1803.03645 Z.B. Kang, K. Lee, X. Liu, and F. Ringer



Introduction-Cumulants

Cumulants: $\kappa_n[X]$

Cumulant-generating function: $K(t) = \ln(\mathrm{E}[e^{tX}]) \longrightarrow \kappa_n[X] = \partial_t^{(n)} K(t)$

$$\kappa_1[X] = E[X]$$

$$\kappa_2[X] = E[X^2] - E[X]^2$$

$\boxed{n \leq 3 \quad \text{cumulants = central moments}}$

$$\vdots$$

Independent additive observables: $Z = X + Y$

$$P_z = P_x \otimes P_y$$

$$\boxed{\kappa_n[Z] = \kappa_n[X] + \kappa_n[Y]}$$

Additive observables in Physics

Additive observables:

- event shapes e.g. transverse energy,… $E_T(\eta_{cut}) = \sum_i |p_T^i| \theta(\eta_{cut} - |\eta_i|)$

- transverse momentum vector e.g. dijet imbalance, hadron-in-jet

Not Additive observables:

- jet shapes e.g. jet-mass (almost additive ?) $m_J^2 = \left(\sum_i \vec{p}_i^{\,+}\right)\left(\sum_i \vec{p}_i^{\,-}\right)$

- transverse momentum scalar e.g. dijet imbalance, hadron-in-jet $|\vec{p}_T| = \left|\sum_i \vec{p}_T^{\,i}\right|$

Transverse energy - Introduction

$$E_T(\eta_{cut}) = \sum_i |p_T^i| \theta(\eta_{cut} - |\eta_i|)$$

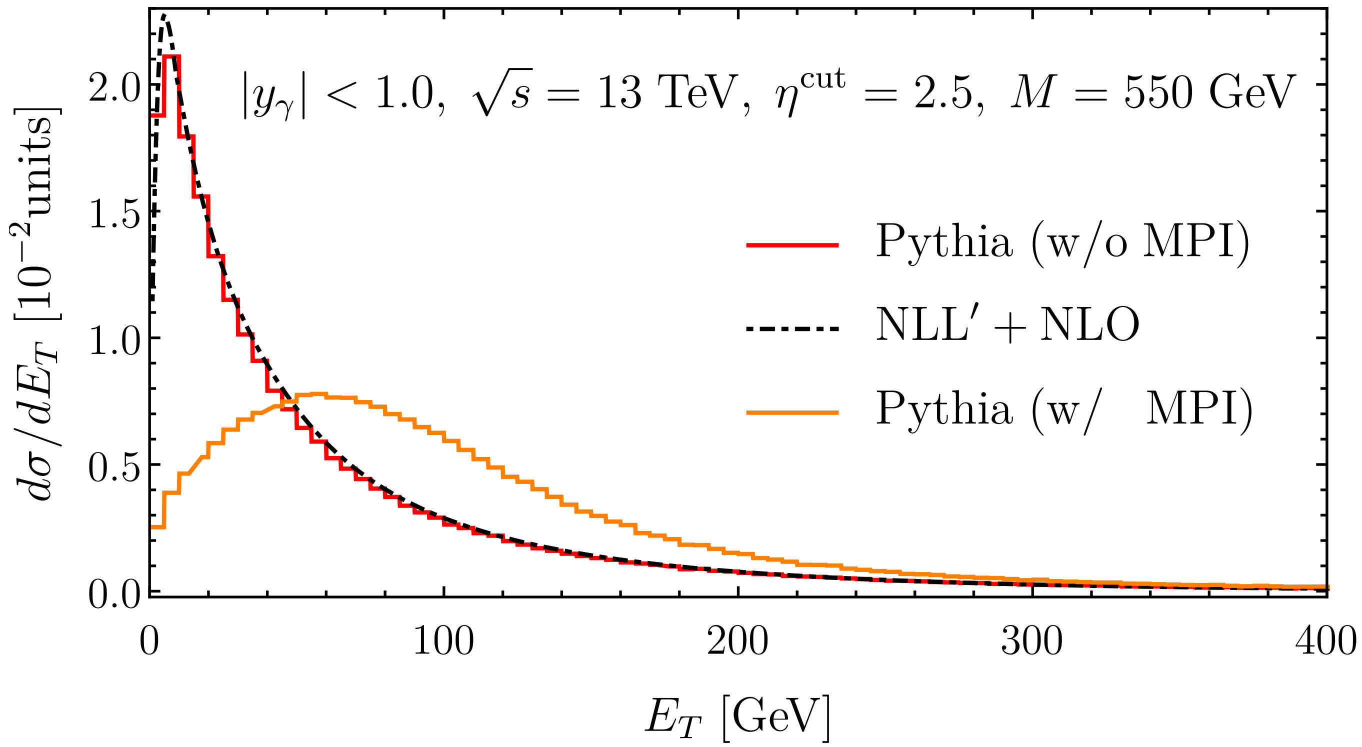
"UE sensitive observables" are useful for studying MPI contributions

Introduce a phasespace constraints

Theoretically challenging. QCD factorization breakdown.

Example:

$p + p \to \gamma^* + X$



Transverse energy - Factorization conjecture

$Q \ll E_{c.m.}^{(hadr.)}$

Assumption: MPI contribute to the measurement roughly uncorrelated to the hard process.

$$\frac{d\sigma}{dE_T} = \frac{d\sigma^{\text{pert.}}}{dE_T} \otimes f(E_T)$$

The first moment: $\langle E_T \rangle_{\text{exp.}} \simeq \langle E_T \rangle_{\text{pert.}} + \langle E_T \rangle_f$

Process independent:
- Hard scale
- Drell-Yan vs Higgs vs …
- Could depend on experimental configuration/analysis

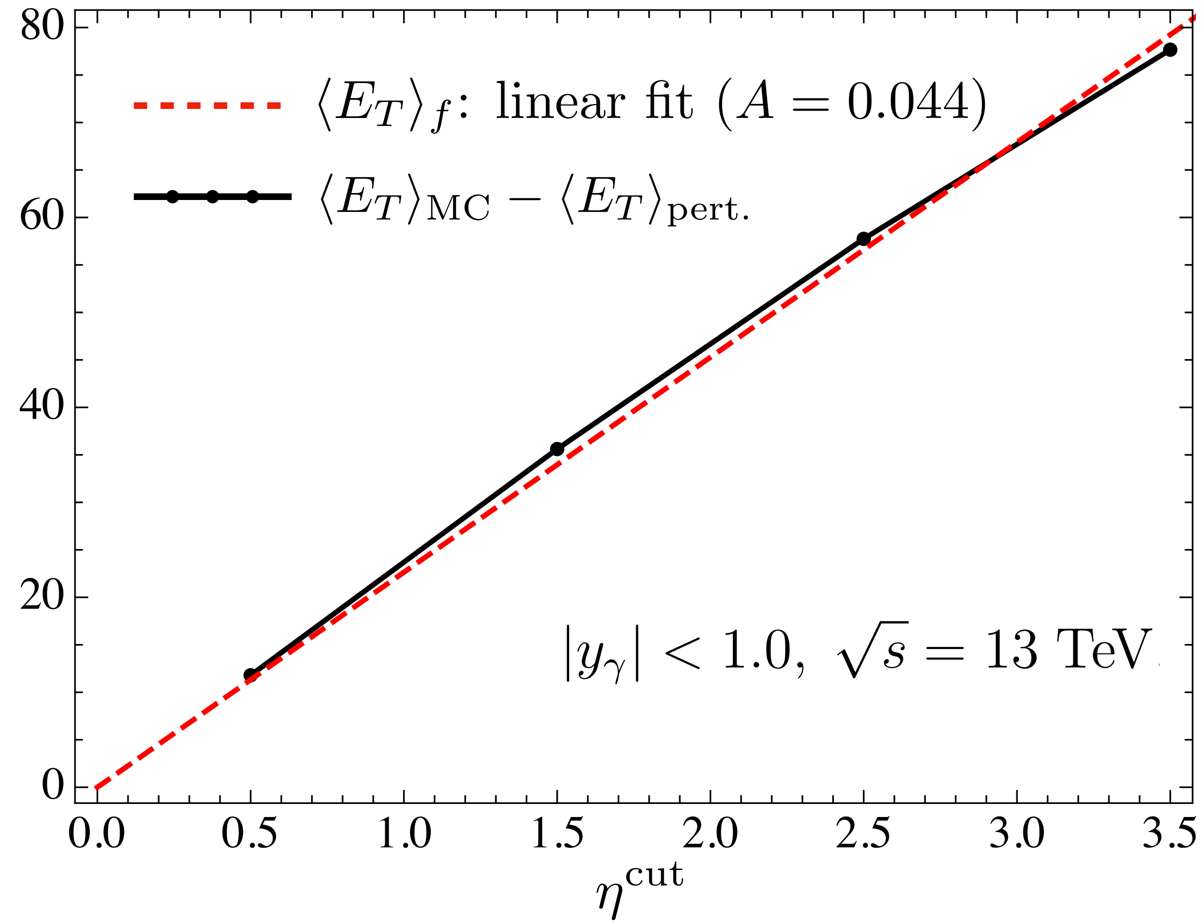
Similarly we can calculate higher moments and get a more complete picture of the model function

See also for same approach used in jet-mass spectrum:

arXiv:1405.6722 I. W. Stewart, F. J. Tackmann, and W. J. Waalewijn (vs Pythia)

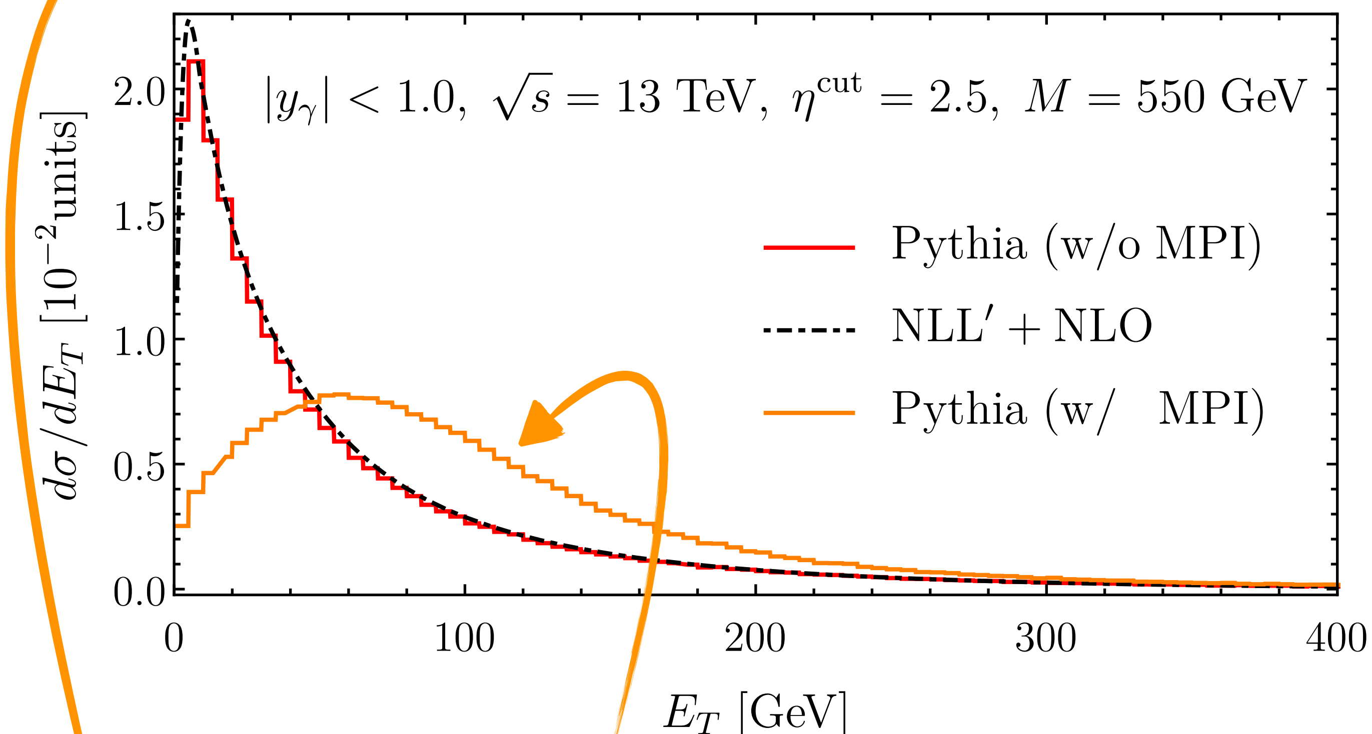
Transverse energy - Modeling MPI

$$\langle E_T\rangle_f \simeq \langle E_T\rangle_{\rm MC} - \langle E_T\rangle_{\rm pert.}$$



For this region of rapidity cutoff the first moment of the model function increase linearly with the cutoff parameter.

$$\langle E_T\rangle_f(\eta^{\rm cut}) = \frac{1}{A}\eta^{\rm cut}$$



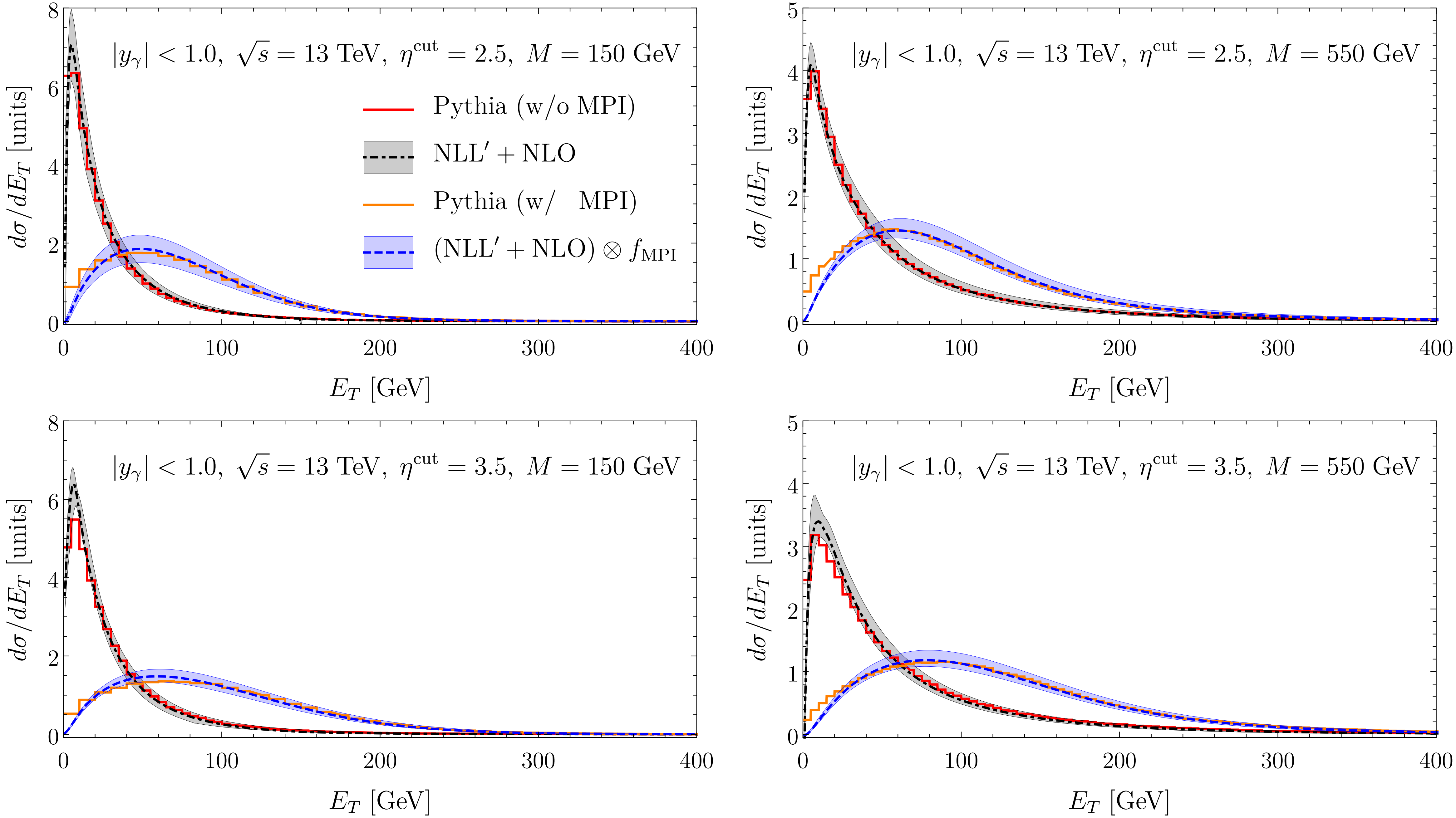
$$f(E_T, \eta^{\rm cut}, A) = \mathcal{N} \exp\left[-E_T^2\Big(\frac{A}{\eta^{\rm cut}\sqrt{\pi}}\Big)^2\right]$$

Transverse energy - Modeling MPI

$Q = M = 150$ GeV | $Q = M = 550$ GeV

$\eta^{\mathrm{cut}} = 3.5$

$\eta^{\mathrm{cut}} = 2.5$



Transverse energy - Subtracted cumulants

Additive observables: $e = \sum_{i \in \text{event}} \hat{e}_i = \sum_{i \in \text{signal}} \hat{e}_i + \sum_{i \in \text{SUEs}} \hat{e}_i = e_{\text{S}} + e_{\text{B}}$

SUEs = "Soft" Uncorrelated Emissions (soft emissions statistically independent from signal with probability distribution independent from the details of hard process)

$$\frac{d\sigma}{dedQdy}(e, Q, y) = \int de_{\text{B}} \frac{d\sigma_{\text{S}}}{dedQdy}(e - e_{\text{B}}, Q, y) f(e_{\text{B}})$$

"Soft" : not necessary for truly additive observables. Becomes relevant for jet substructure

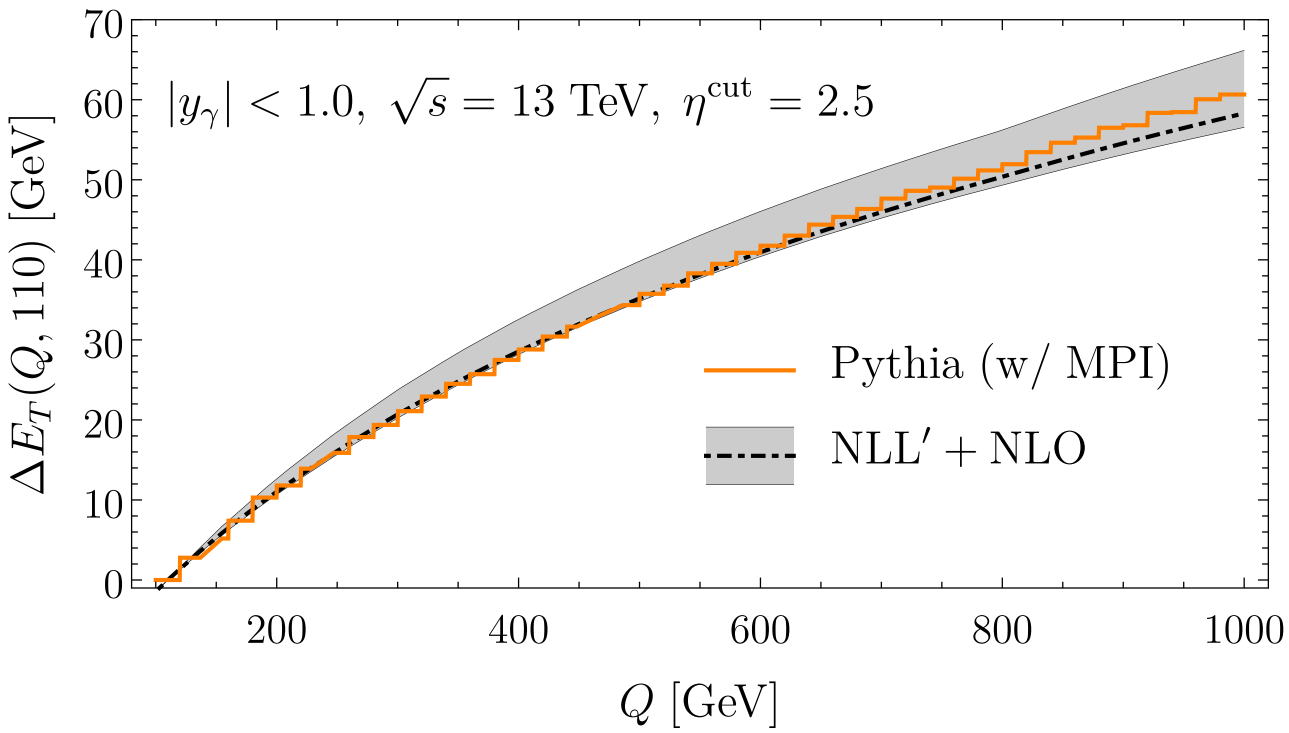
$\rightarrow \quad \kappa_n[d\sigma] = \kappa_n[d\sigma_S] + \kappa_n[f]$

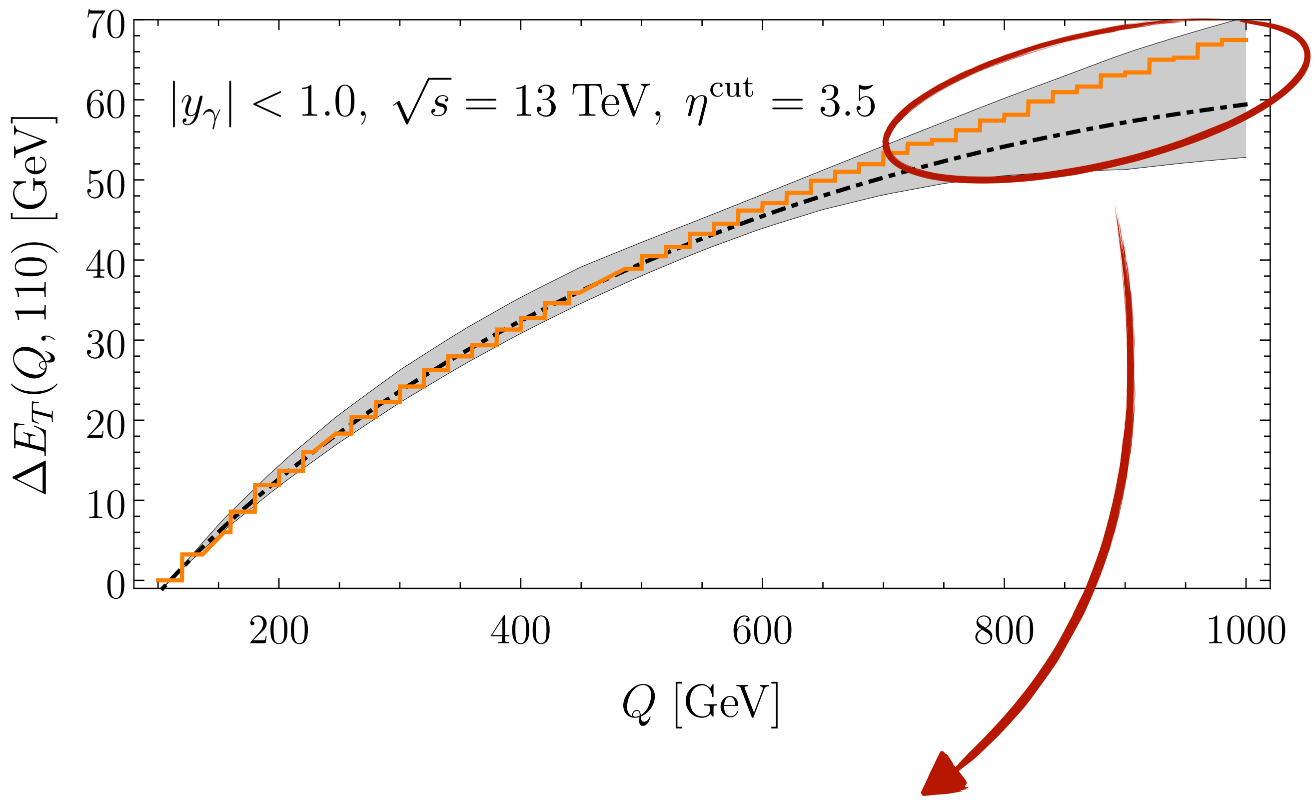
Subtracted cumulants independent of model function

$$\rightarrow \quad \boxed{\Delta_e^{(n)}(Q_1, Q_2) = \kappa_n[d\sigma]\Big|_{Q_1} - \kappa_n[d\sigma]\Big|_{Q_2} = \kappa_n[d\sigma_S]\Big|_{Q_1} - \kappa_n[d\sigma_S]\Big|_{Q_2}}$$

Transverse energy - Subtracted cumulants

$n = 1$





Beam saturation effects?

Jet substructure - Subtracted cumulants

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Subtracted cumulants are independent of the model function: ideal for situations with large background contamination such as high-luminosity or QGP

- Alternative way to compare theoretical/model calculation of signal effects directly with experiment for well studied observables i.e. jet-mass.

- For AA or pA collisions compared against pp result can be used to probe nuclear effects

Jet substructure - Subtracted cumulants

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pp: signal = partonic, background = MPI

Working in pp: are MPI part of SUEs or signal?

Based on Pythia, yes*

* Under certain conditions: $p_T \ll E^{c.m.}_{hadr.} \sim 10\ \text{TeV}, \eta \lesssim 2$

Jet substructure - Subtracted cumulants

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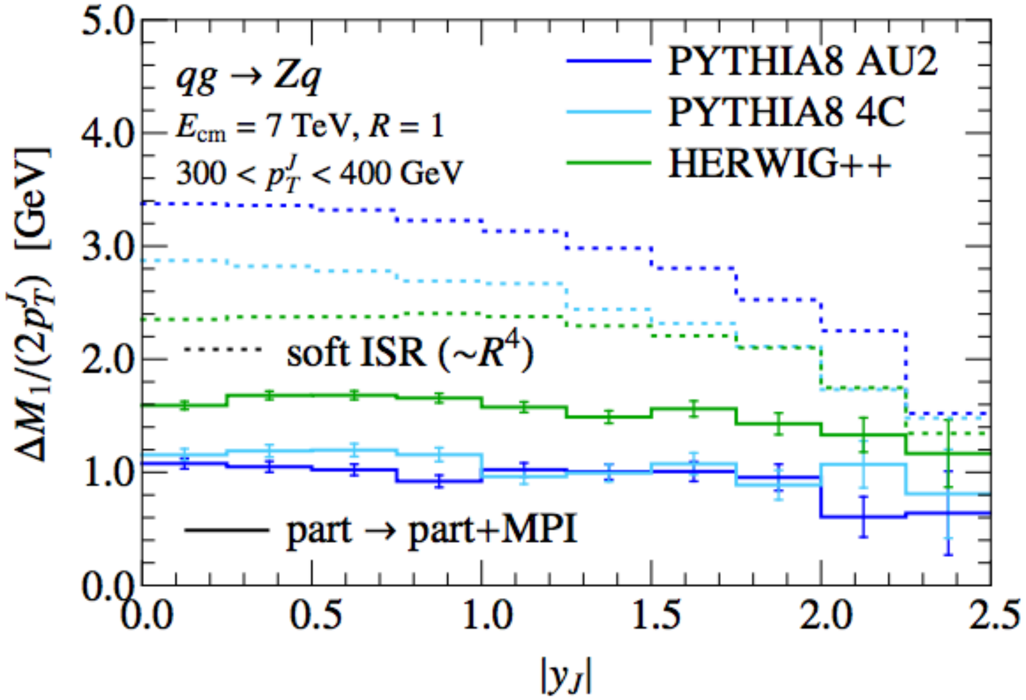
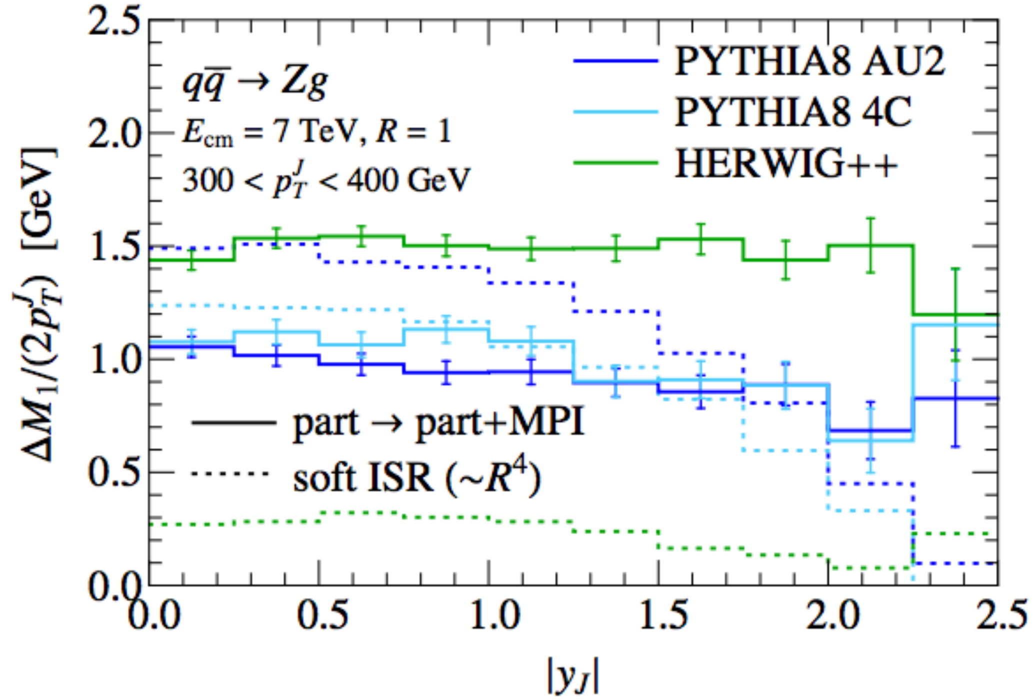
$$\Delta M_1 = \langle m^2 \rangle \Big|_{w/\ \ MPI} - \langle m^2 \rangle \Big|_{w/o\ MPI}$$

arXiv:1405.6722 I. W. Stewart, F. J. Tackmann, and W. J. Waalewijn (vs Pythia)





Jet substructure - Subtracted cumulants

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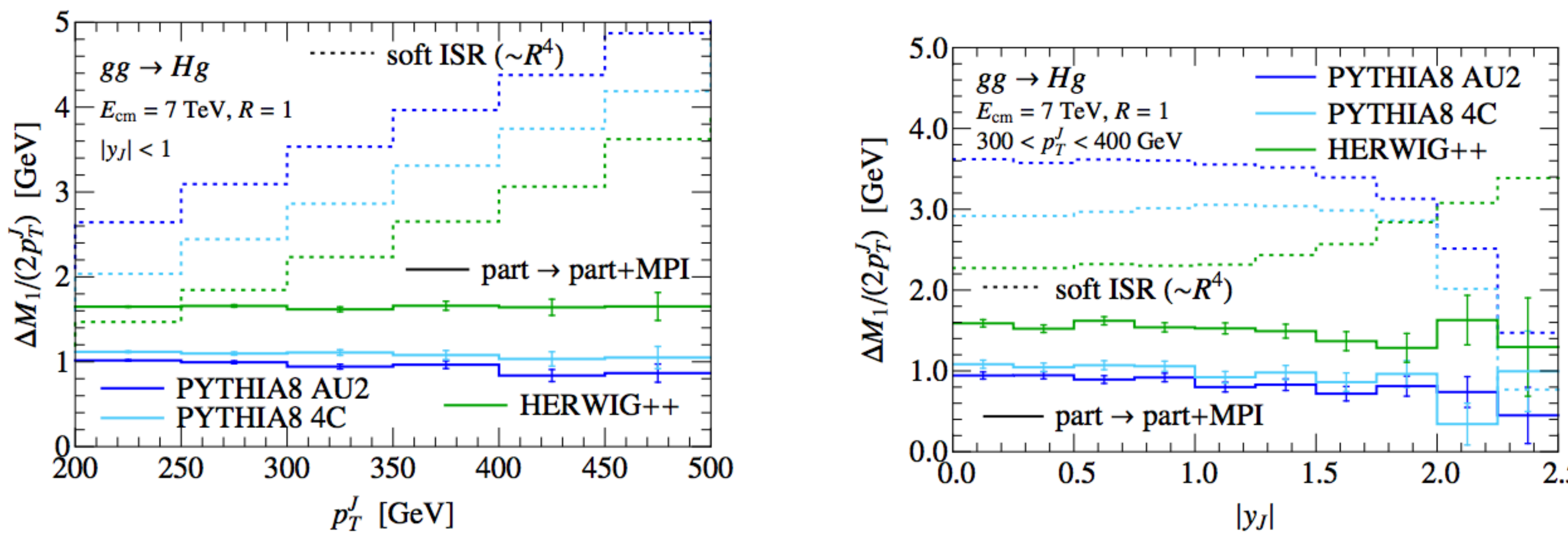

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Additive observable: $e = m^2/p_T$

Jet-mass based observables: $\langle m^2 \rangle = \langle m^2 \rangle_S + p_T \times \langle e_B \rangle_f \, (1 + \mathcal{O}(E_\text{SUEs}/E_J))$

$$\Delta_s(p_T^1, p_T^2) = \kappa_n[d\sigma]\Big|_{p_T^1} - \kappa_n[d\sigma]\Big|_{p_T^2} \frac{p_T^1}{p_T^2}$$

For discrete pT bins

$$\boxed{\Delta_s^{ij} = \kappa_1^{[i]} - \kappa_1^{[j]} \frac{\langle p_T \rangle^{[i]}}{\langle p_T \rangle^{[j]}}}$$

Jet substructure - Subtracted cumulants

$$\tau = \frac{m^2}{p_T^2}$$

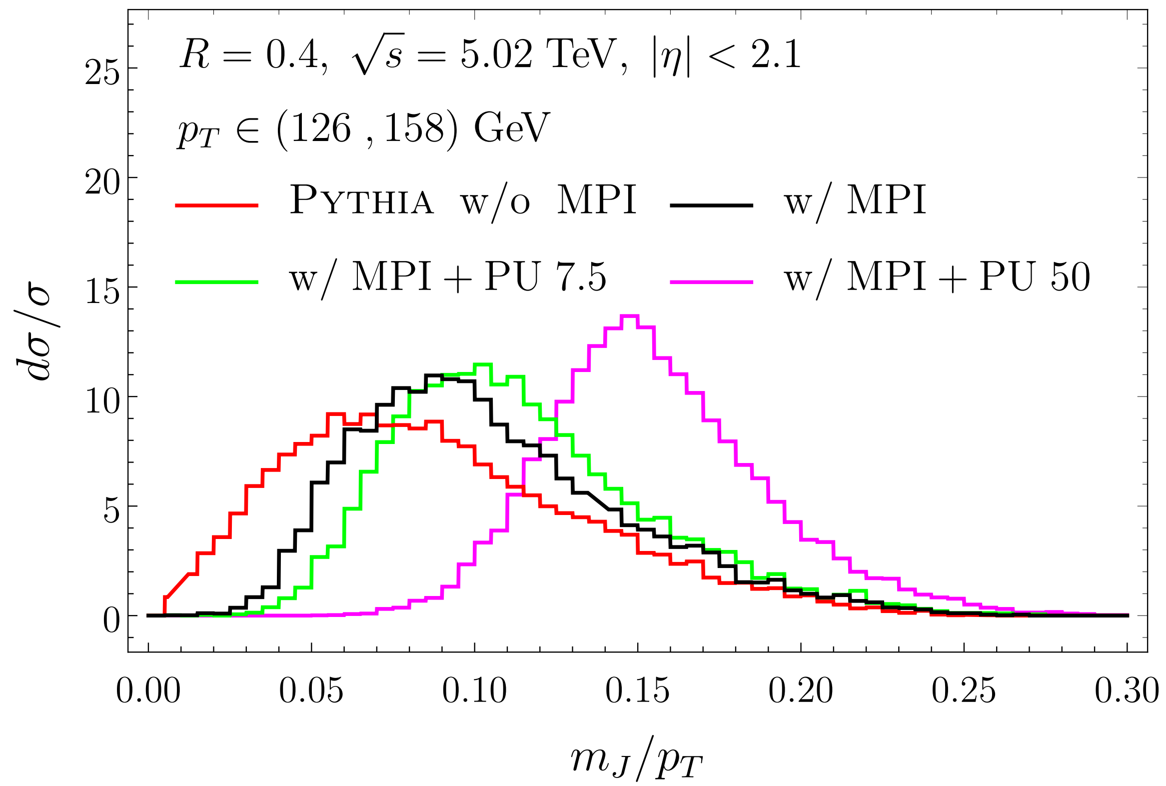
$$d\sigma(NLL + NLO) = d\sigma(NLL) + d\sigma(NLO) - d\sigma(singular)$$

NLL': **arXiv:1803.03645** Z.B. Kang, K. Lee, X. Liu, and F. Ringer

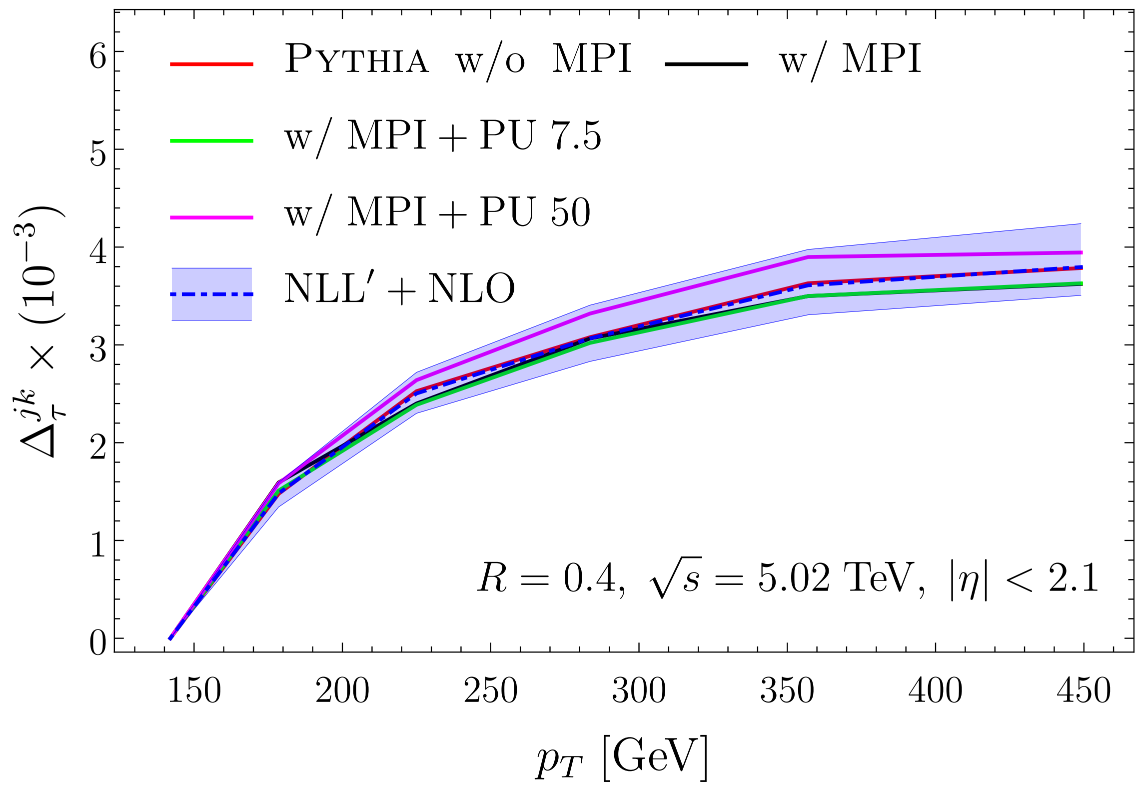
Binning and scaling requires modification for cancelation of the model dependence

$$\boxed{\Delta_\tau^{ij} = \kappa_1^{[i]} - \kappa_1^{[j]} \frac{\langle p_T^{-1} \rangle^{[i]}}{\langle p_T^{-1} \rangle^{[j]}}}$$

SUE - Sensitive

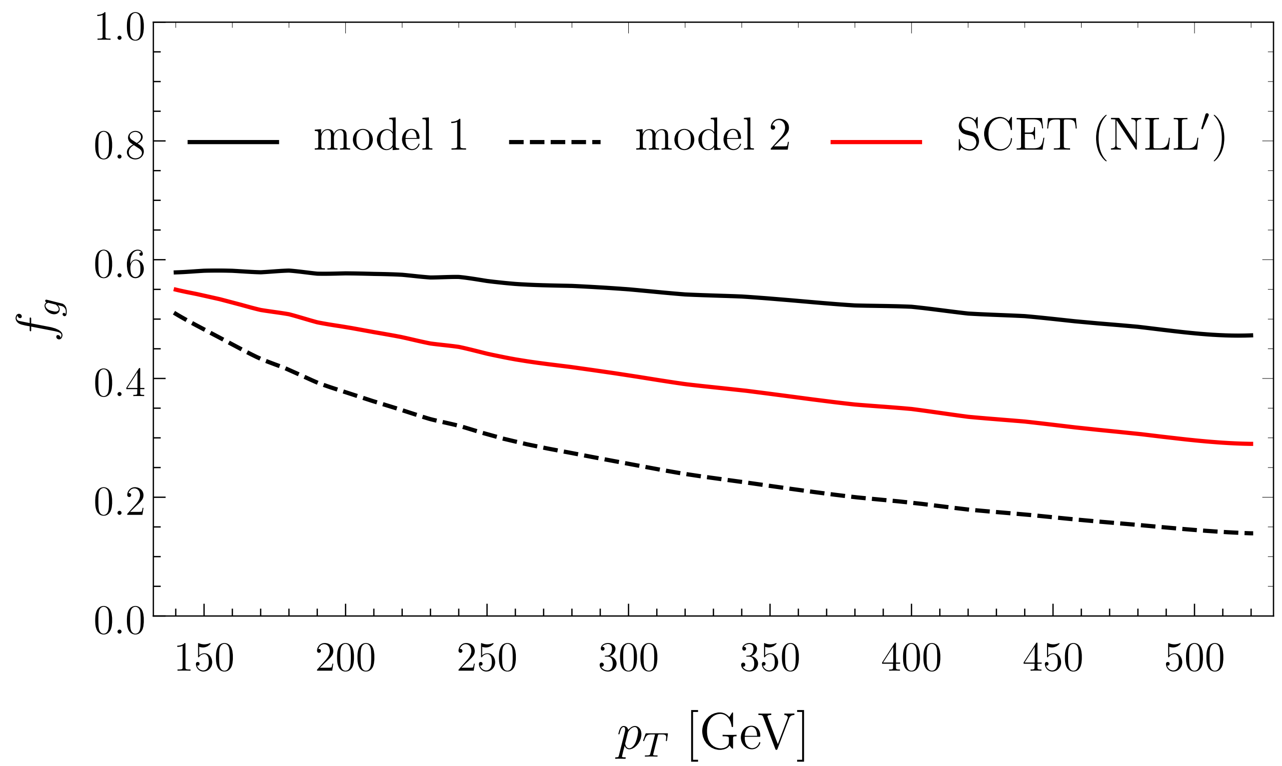


SUE - Insensitive



Jet substructure - Subtracted cumulants

Sensitivity to quark/gluon jet fraction: Quark and gluon jets are affected/quenched differently



<u>Consider the following toy models:</u>

$$f_g(p_T; a, b) = f_g^{\rm NLL}(p_T)\left(\frac{p_T}{a}\right)^b$$

$$\text{model 1:} \quad a = 120 \text{ GeV}, \; b = +1/3$$

$$\text{model 2:} \quad a = 120 \text{ GeV}, \; b = -1/2$$

$$\langle\tau\rangle^{[j]} = f_g^{[j]} \, \langle\tau\rangle_g^{[j]} + (1 - f_g^{[j]}) \, \langle\tau\rangle_q^{[j]}$$

Jet substructure - Subtracted cumulants

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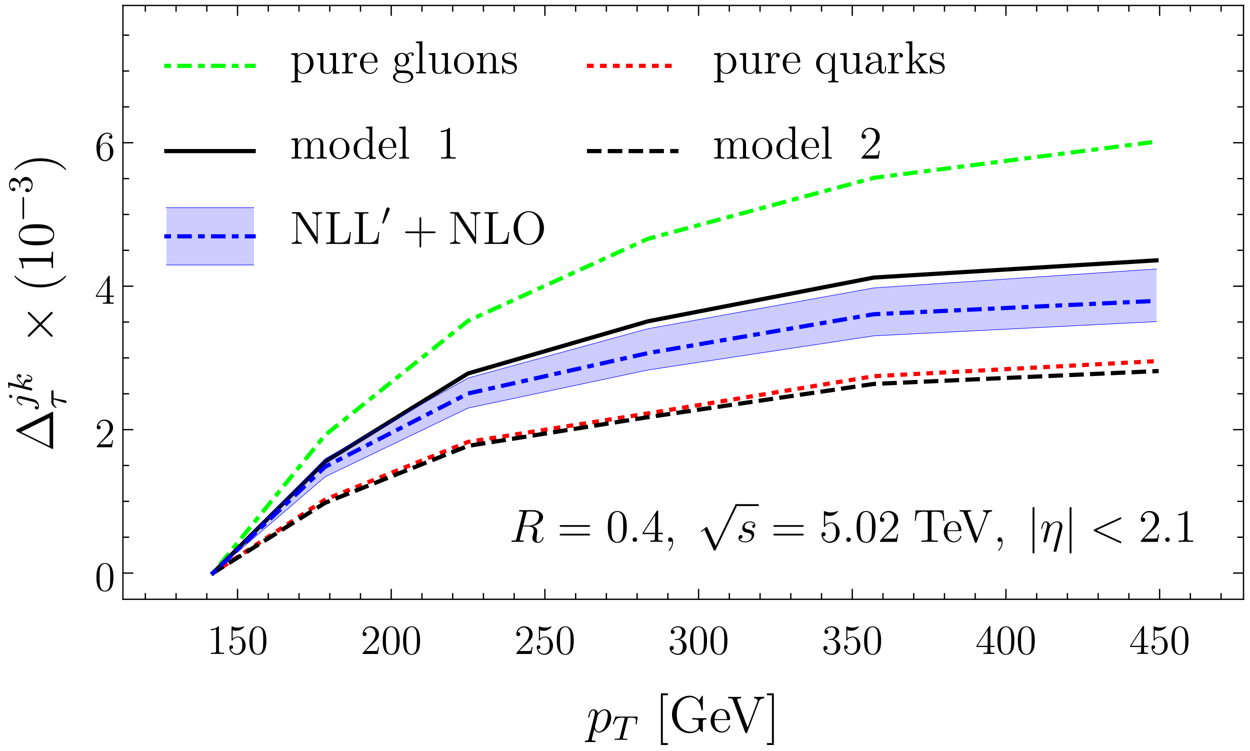

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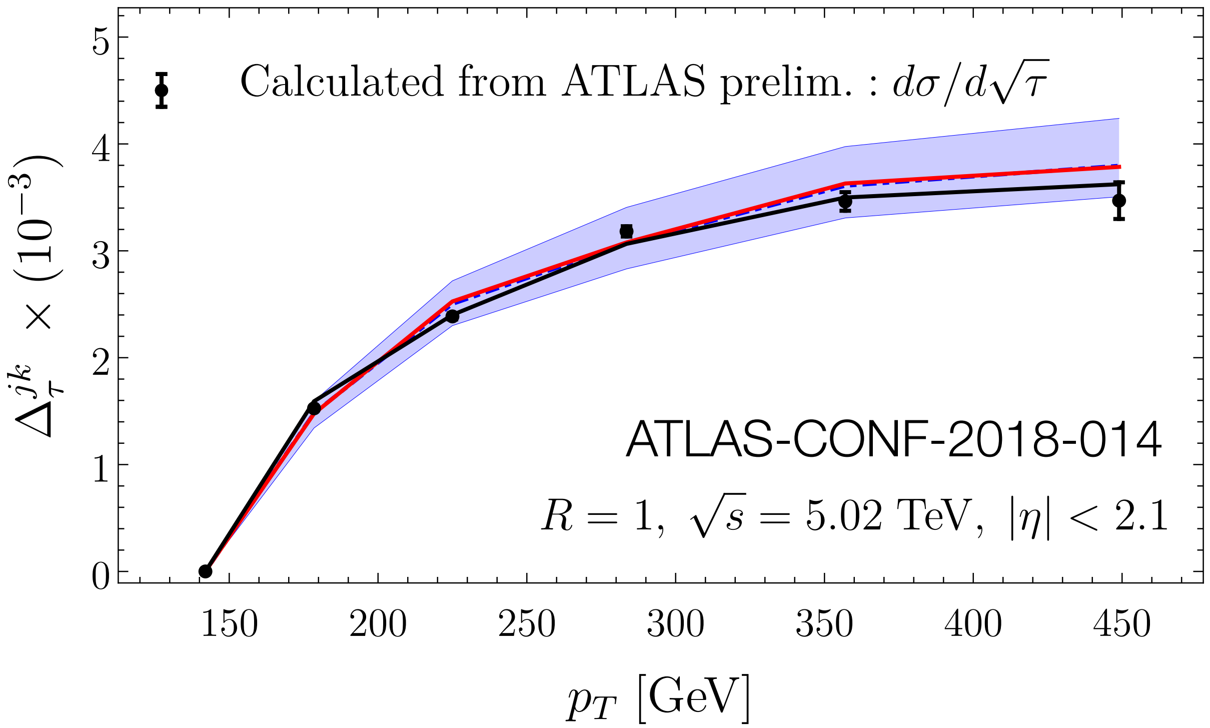
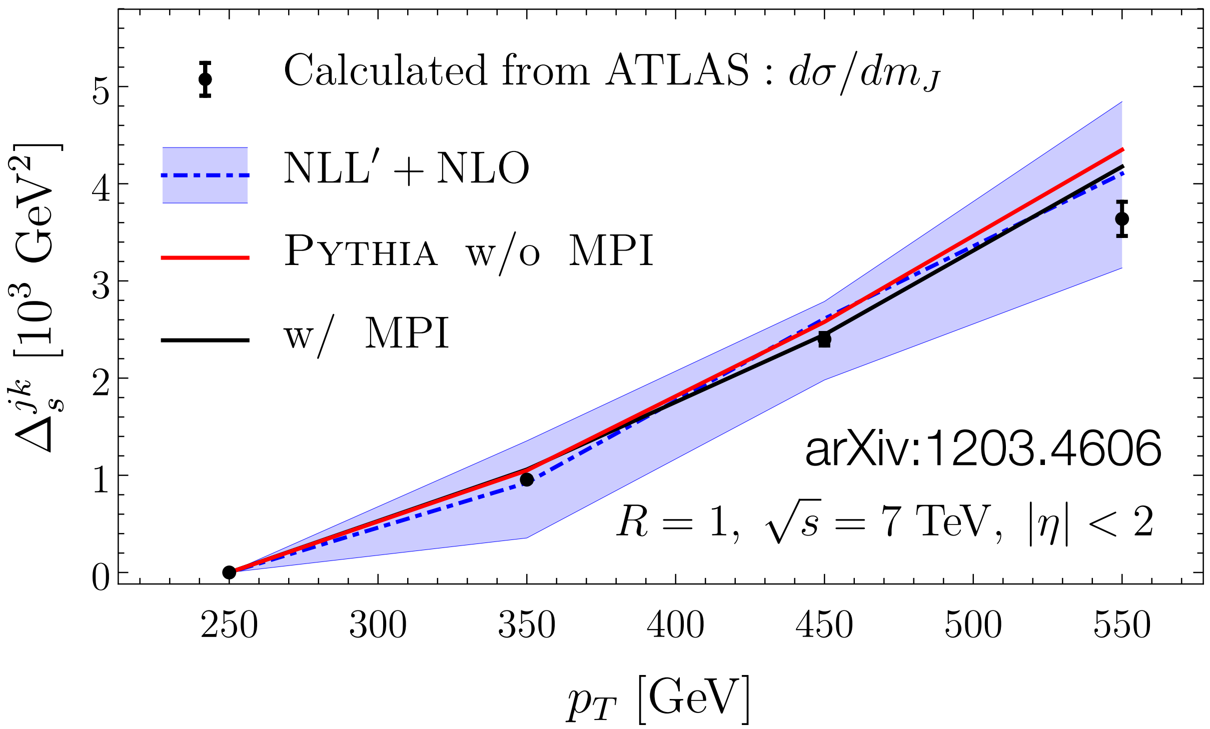
Jet substructure - Subtracted cumulants



Indication for the size of statistical uncertainties using: $\sqrt{\frac{\mathrm{Var}[\tau]}{N}}$

From differential distribution of the jet mass measurements from ATLAS we calculated the subtracted cumulants.



$$\boxed{\Delta_s^{ij} = \kappa_1^{[i]} - \kappa_1^{[j]} \frac{\langle p_T \rangle^{[i]}}{\langle p_T \rangle^{[j]}}}$$

Jet substructure - Subtracted cumulants

Extremely large background necessary modifications

$$\langle m^2 \rangle = \langle m^2 \rangle_S + p_T \times \langle e_B \rangle_f (1 + \mathcal{O}(E_{\mathrm{SUEs}}/E_J))$$

1) x-axis: bin migration

2) y-axis: pT re-scale from additivity equation

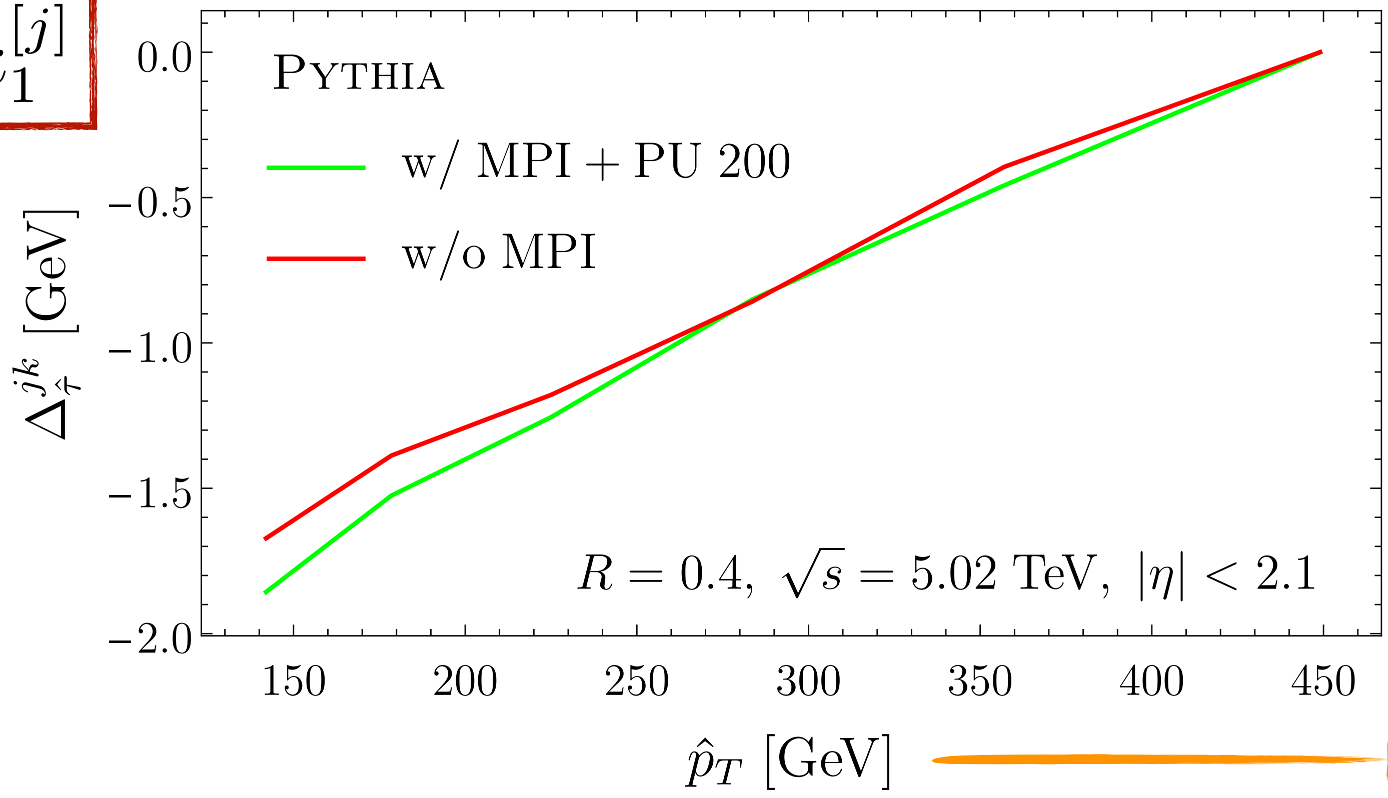
Jet substructure - Subtracted cumulants

Large background will spoil the additivity of the observable

$$\hat{\tau} = 2\cosh(y) \underbrace{\sum_{i\in \mathrm{jet}} p_i^+}_{\text{Solves (2): Additive}} \simeq \frac{m^2}{p_T}\,, \qquad p^+ = p^0 - \vec{n}\cdot\vec{p}$$

Jet mass (or related observables) in large background.

$$\boxed{\Delta_{\hat{\tau}}^{ij} = \kappa_1^{[i]} - \kappa_1^{[j]}}$$



Solves (1):
Bin migration effects need to be considered. We correct the "x-axis" by subtracting average transverse momentum per unit area. (as implemented in `FastJet`)

Subtracted cumulants - Why ?

Comparisons of Th./Exp. vs Th./Exp. : no experiment, jet radius, or jet algorithm dependence.

Test of the factorization (independence) hypothesis

- → Is easy to measure: smaller uncertainties.
- → Probe to the factorization breaking effects

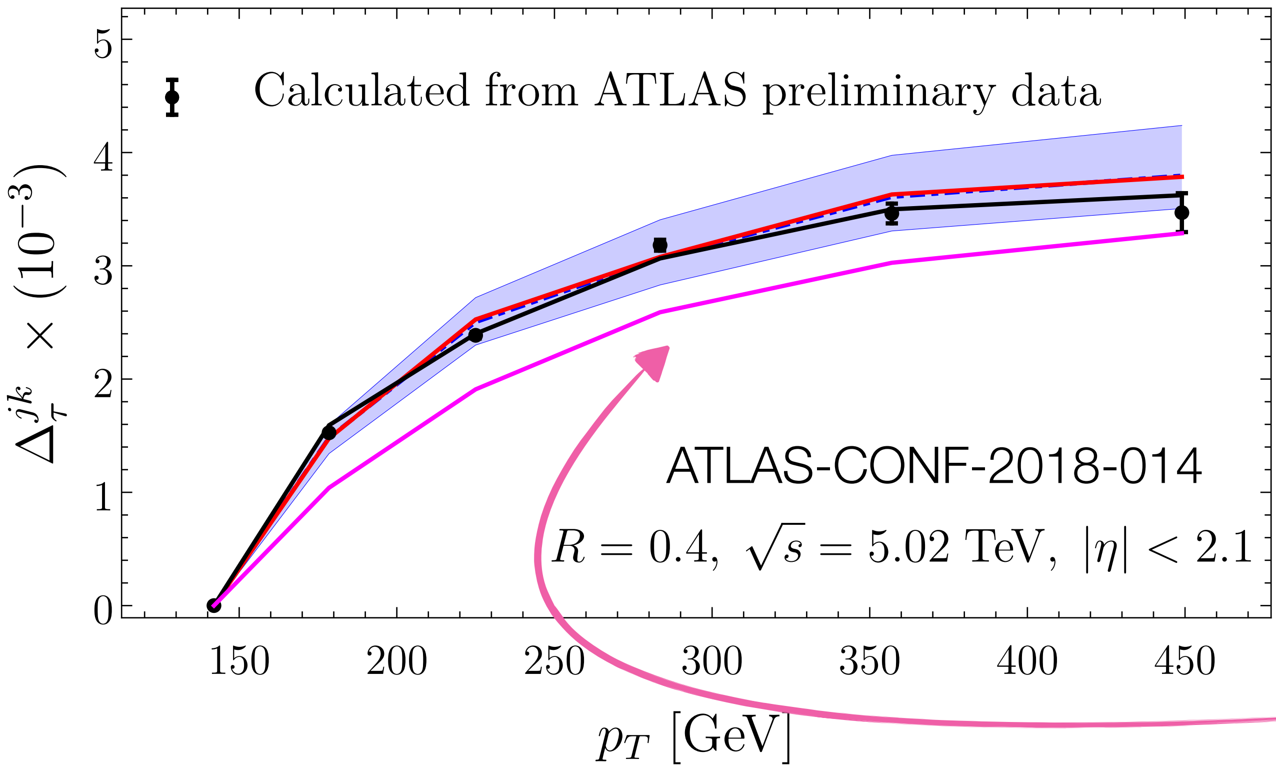
Studies of pseudo rapidity dependence of MPI (UE) using TE.

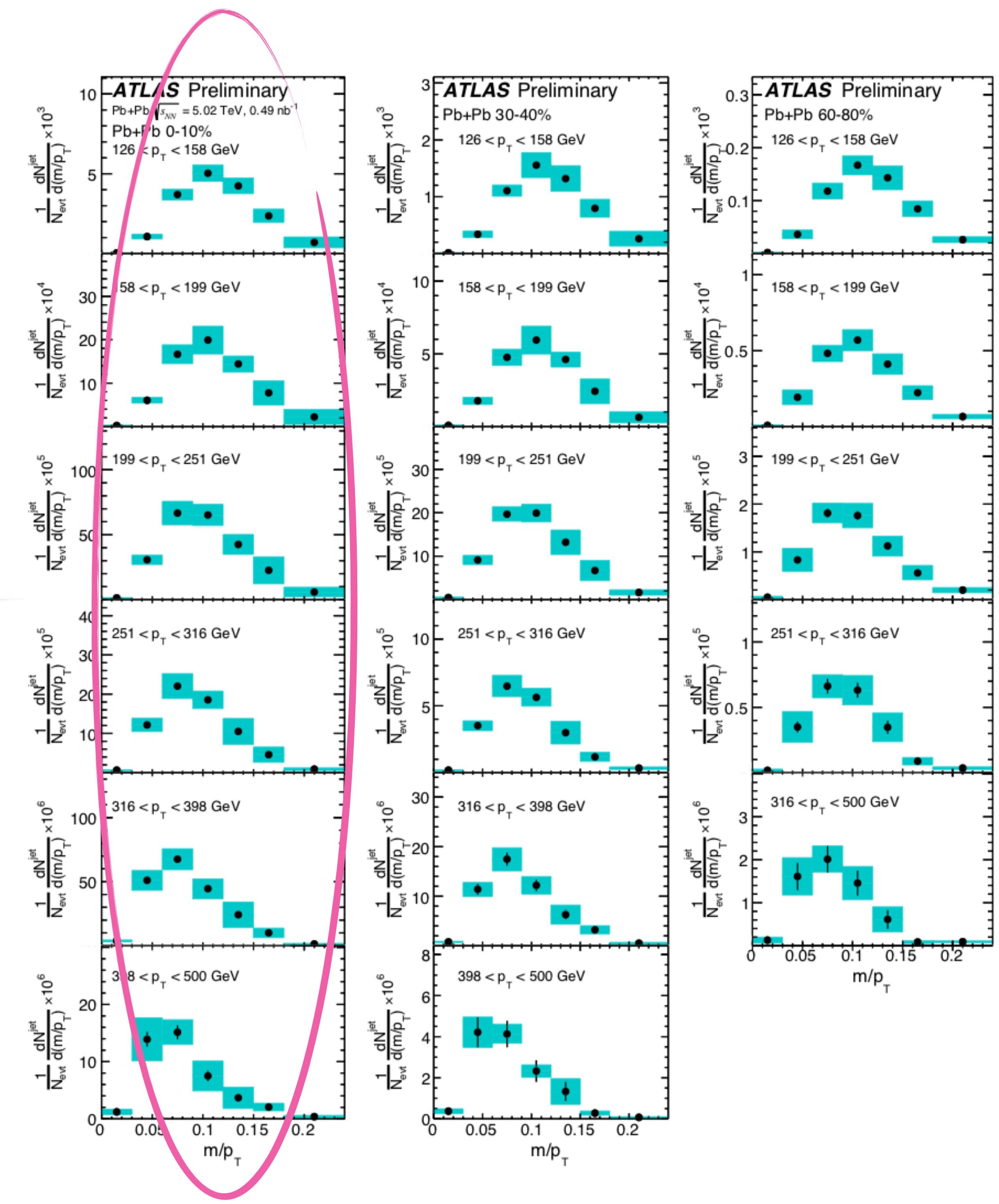
Test for Monte-Carlo models for MPI + ISR + FSR + …

Pile-up subtraction (test + phenomenology)?

Thank you

Subtracted cumulants - Pb+Pb



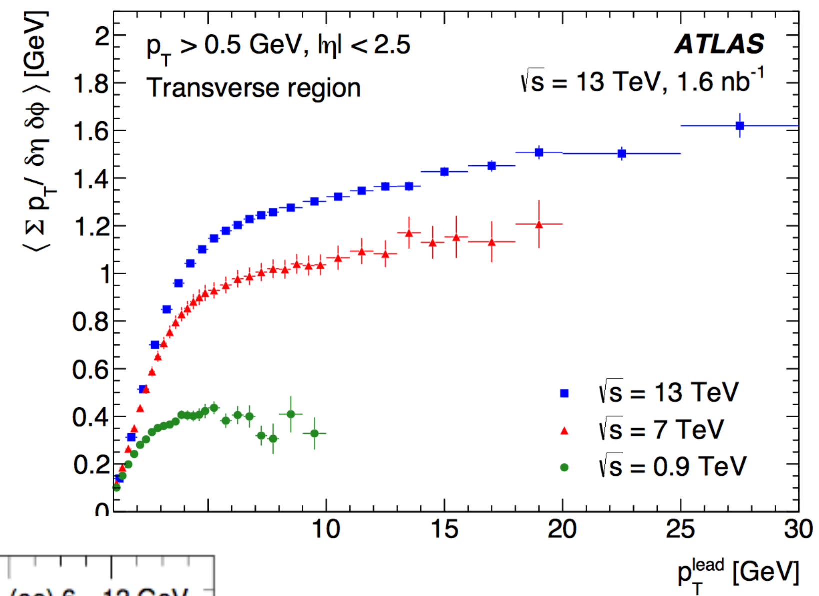


Subtracted cumulants - Transverse energy

Why ?

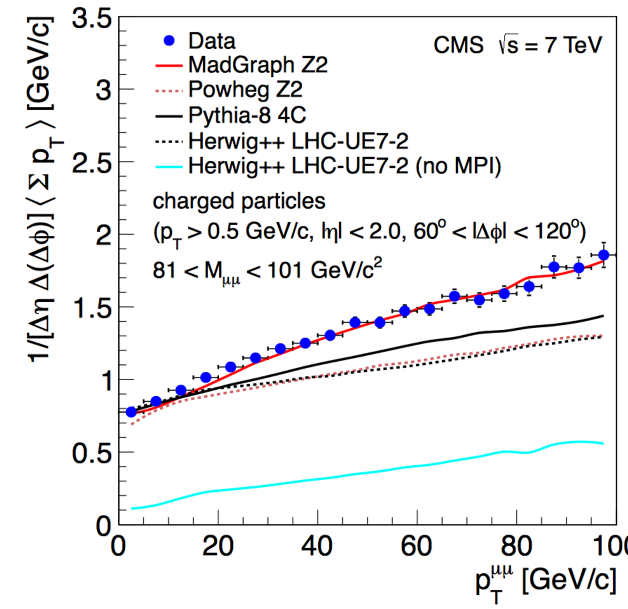
$$pp \to Z(\ell^+\ell^-) + X \qquad \eta^{\rm cut} = 2.5$$

arXiv:1602.08980



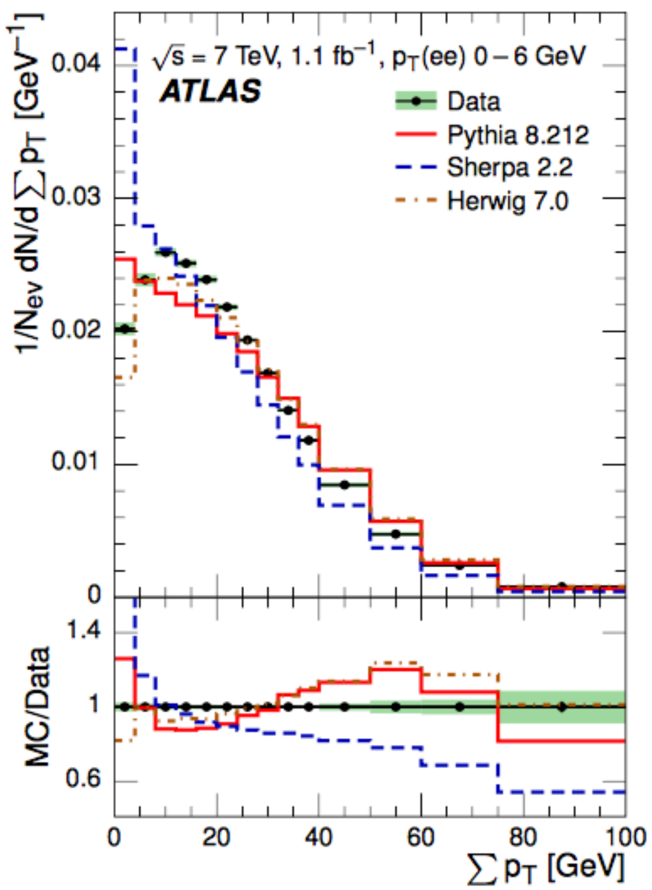
arXiv:1701.05390

$$pp \to X$$

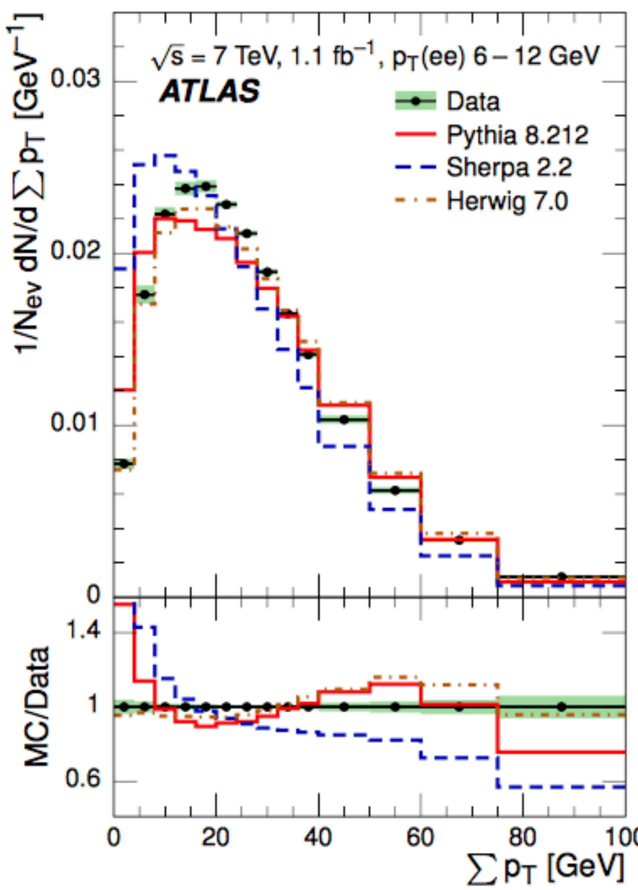


arXiv: 1204.1411(CMS)

$$pp \to Z(\mu^+\mu^-) + X$$



(a) $\sum p_T$, $p_T(e^+e^-)$: 0–6 GeV



(b) $\sum p_T$, $p_T(e^+e^-)$: 6–12 GeV