Medium motion effects in the GLV approach

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LANL, T-2

UCLA, January, 2019

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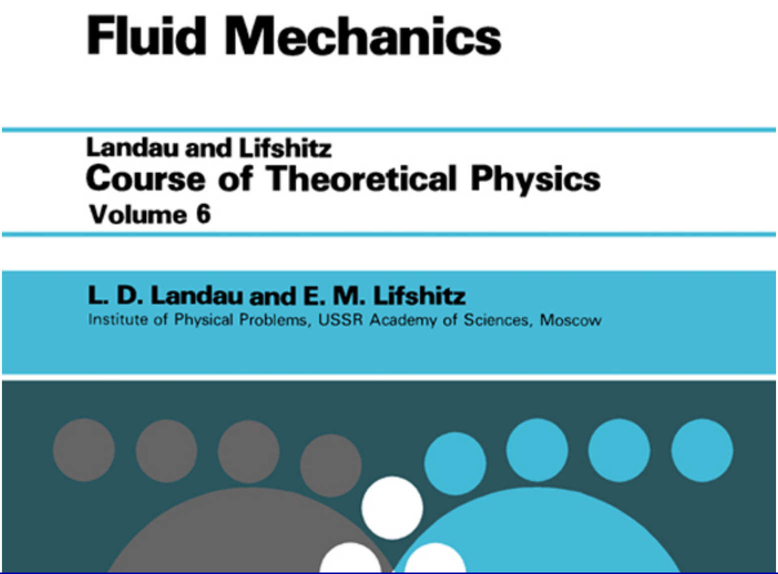

Fluid Mechanics
Landau and Lifshitz
Course of Theoretical Physics
Volume 6
L. D. Landau and E. M. Lifshitz
Institute of Physical Problems, USSR Academy of Sciences, Moscow

The low-energy/large distance dynamics of a system can be described with a minimal set of degrees of freedom. One can start with the charge and stress energy tensor conservations

$$\partial_\mu T^{\mu\nu} = 0 \ , \ \ \partial_\mu J^\mu = 0$$

In the equilibrium the general expectations for the form of the currents are

$$T^{\mu\nu} = w u^\mu u^\nu + P g^{\mu\nu}$$
$$J^\mu = n u^\mu \, ,$$

where $w = \epsilon + P$ is the enthalpy and n is the charge density. The vector $u^\mu = (1, 0, 0, 0)$ defining the rest frame of the system is introduced to keep the transformation properties of these objects explicit.

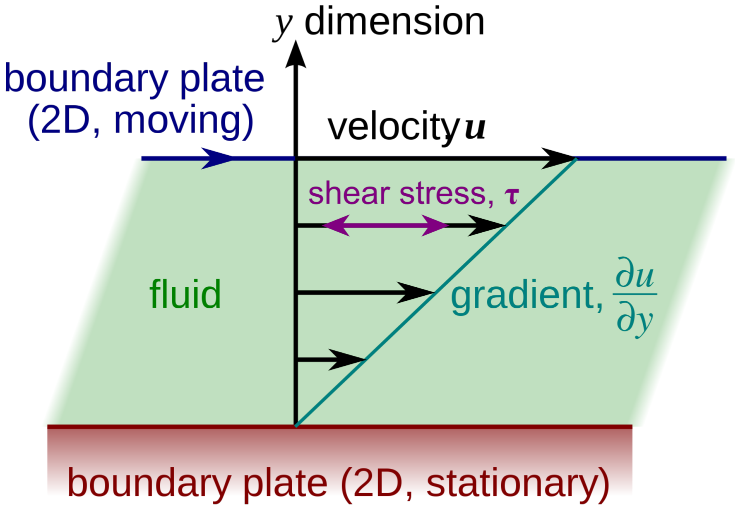
One can check that if the exact equilibrium limit is relaxed, the 4-velocity and the thermodynamic quantities are allowed to vary in space and time, the conservations laws

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad \partial_\mu J^\mu = 0$$

supplemented by the constituent relations

$$T^{\mu\nu} = w(x^\alpha)u^\mu(x^\alpha)u^\nu(x^\alpha) + P(x^\alpha)g^{\mu\nu}$$
$$J^\mu = n(x^\alpha)u^\mu(x^\alpha)$$

are equivalent to the Euler equation in the non-relativistic limit which describes dynamics of an ideal fluid (no dissipation).



y dimension
boundary plate (2D, moving)
velocity $\boldsymbol{u}$
shear stress, $\boldsymbol{\tau}$
fluid
gradient, $\frac{\partial u}{\partial y}$
boundary plate (2D, stationary)

The low-energy dynamics of a dissipative system involves the same degrees of freedom and is described by the same conservations laws

$$\partial_\mu T^{\mu\nu} = 0 \quad , \quad \partial_\mu J^\mu = 0$$

which now should be supplemented by generalized constituent relations

$$\begin{aligned} T^{\mu\nu} &= w u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} \\ J^\mu &= n u^\mu + \nu^\mu \, , \end{aligned}$$

where ν^μ and $\tau^{\mu\nu}$ should take care of dissipation (and some other non-trivial contributions not seen in the ideal hydrodynamics) and commonly considered as a series in derivatives – derivative expansion.

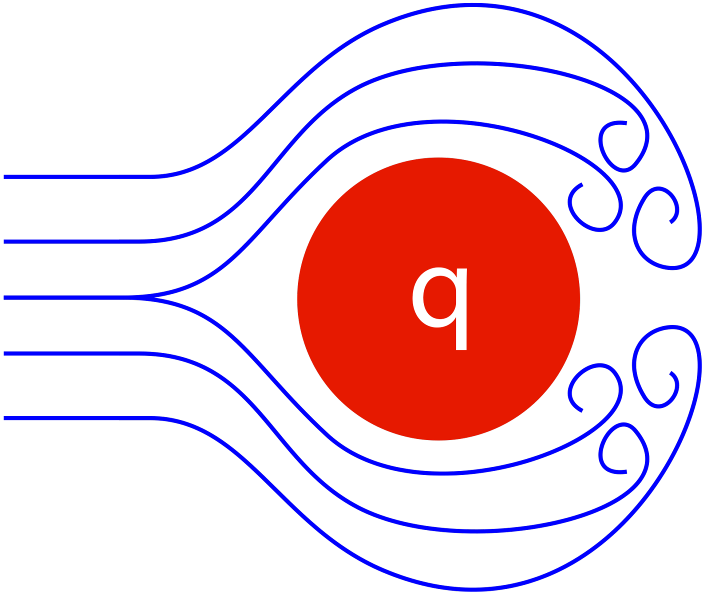
Following the standard textbook discussion one can write the most general form of these corrections in a “normal fluid” at the leading order in gradients as

$$\tau^{\mu\nu} = -\eta P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \left(\zeta - \frac{2}{3}\eta\right) P^{\mu\nu} \partial \cdot u$$

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \frac{\mu}{T} + \sigma E^\mu \,,$$

where the velocity is defined in a such way that $u^\alpha \nu_\alpha = u^\alpha \tau_{\alpha\beta} = 0$. These relations ensure that the entropy current has non-negative divergence required by the second law of thermodynamics.

This pretty universal description needs microscopic input for the unknown coefficients which can be identified with the shear and bulk viscosities and the electric conductivity.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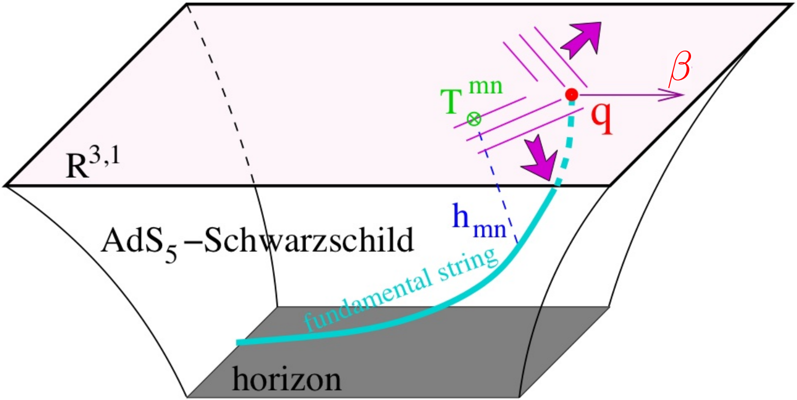
- In what follows, we will be interested in the jet-medium interaction. It is instructive to ask what general properties of this interaction can be understood from the basic hydrodynamic intuition.

- For this purpose it is useful to consider a simpler problem – the energy loss of a heavy impurity moving through a hydrodynamic medium.

- The dissipative force acting on it is expected to behave as $\boldsymbol{f} = \varkappa T^2 \boldsymbol{v} \propto \frac{\boldsymbol{p}}{M}$, where $\varkappa$ depends on the details of the interaction.

- This leading order answer is expected to gain further (gradient) corrections of the form

$$f^i = \varkappa T^2 v^i + A^{ijk} \partial_j v_k$$



S. Gubser et al., PRD, 2017

- The DF is not a Lorentz 4-vector but it has particular transformation properties allowing to restrict the form of the 0th order to

$$f^{\mu}_{(0)} = \varkappa \frac{T^2}{\gamma_n} \left(s n^{\mu} + u^{\mu} \right) \,,$$

where n is the impurity velocity, γ_n is the gamma factor, and $s \equiv u \cdot n$.

- The leading gradient correction can be directly calculated in a microscopic theory. For instance, in the holographic plasma it reads

$$f^{\mu}_{(1)} = \varkappa \Big[c_1 \left(u^{\mu} w^{\alpha} \partial_{\alpha} s - s \partial^{\mu} s - s (s u^{\alpha} + w^{\alpha}) \partial_{\alpha} U^{\mu} \right) + \\ + c_2 U^{\mu} \partial_{\alpha} u^{\alpha} - \sqrt{-s} u^{\alpha} \partial_{\alpha} U^{\mu} \Big] \,,$$

where $c_{1,2}$ are known functions of s not important for our discussion.

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see e.g. M. Lekaveckas, K. Rajagopal, JHEP, 2014

This pretty general picture leads to a variety of questions

- Could we study medium motion effects in jet-medium interaction in a similar way?
- Could one do so in a QCD based first principle description?
- What is the simplest way to include local velocities to a presumably QFT based calculation?
- How sensitive these corrections to the details of the underlying theory?
- etc...

The GLV approach is based on a particular choice of the medium potential

$$H_{int} = \sum_i \int d^3x\, v(x - x_i)(T_a)_i \phi^\dagger(x,t)\, T_a(R)(i\overleftrightarrow{\partial_t})\phi(x,t)\,,$$

which corresponds to a superposition of color fields of a large number of heavy scattering centers.

To take the medium motion into account, the first step is to consider the potential of moving sources

$$A^\mu = 2\pi \sum_i e^{iq\cdot x_i} \frac{ig_{eff}(T_a)_i}{q^2+\mu^2}(2p_{S,i} - q)^\mu \delta\left((p_{S,i} - q)^2 - M^2\right)$$

where μ is the thermal mass and $p_{S,i}$ is the momenta of the heavy source particles.

GLV, NPB, 2001; M. Sievert, I. Vitev, PRD, 2018;

If the sources are static one can put $p_S = (M, 0, 0, 0)$ and, if we ignore the recoil of the source, it lead to

$$A^\mu = 2\pi \sum_i e^{iq\cdot x_i} \frac{ig_{eff}\left(T_a\right)_i}{q^2+\mu^2} \, 2Mg^{\mu 0} \, \delta\left(2Mq^0\right) \; ,$$

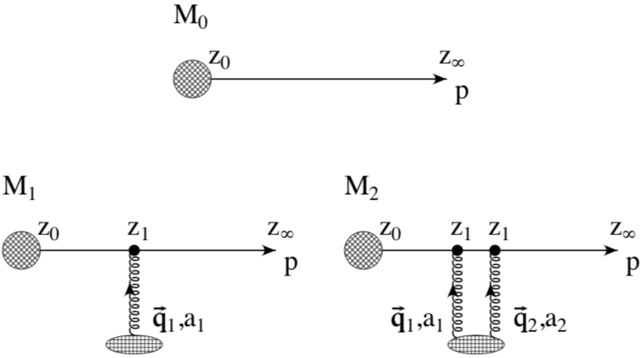
which is equivalent to the same static potential as before.

Thus, in what follows we will take into account only the leading effects of the medium motion:

- $p_S = M(1, \boldsymbol{V})$ with $\boldsymbol{V} = \frac{\boldsymbol{p}_S}{M}$
- keeping the assumption $|q| \ll M$

$$A^\mu = 2\pi \sum_i 2e^{iq\cdot x_i} \frac{ig_{eff}\left(T_a\right)_i}{q^2+\mu^2} p^\mu_{S,i} \delta\left(2p_{S,i}\cdot q\right) \; .$$

It is instructive to start with the broadening of a jet due to in-medium scatterings which corresponds to



The first contribution to study is the single Born diagram which reads

$$M_i = i(T_a)_i T_a(R) e^{ip\cdot x_0} \int \frac{d^4 q}{(2\pi)^3} e^{iq\cdot(x_i - x_0)} v(q) \frac{(2p-q)\cdot p_S}{(p-q)^2 + i\epsilon} \delta\left(p_S \cdot q\right) j(p-q)\,,$$

where the medium motion is encoded into the form of p_S. After some algebra it can be reduced to

$$M_i = i(T_a)_i T_a(R) e^{ip\cdot x_0} \theta(x_i^+ - x_0^+) \int \frac{d^2 q_\perp}{(2\pi)^2} e^{iq\cdot(x_i - x_0)} v(q_\perp) \frac{p\cdot p_S}{p^+ p_S^-} j(p-q)\,,$$

where all effects of the constant medium velocity enters in the phase, as an overall multiple, and to the source:

$$q^+ = V_\perp \cdot q_\perp \quad , \quad \frac{p\cdot p_S}{p^+ p_S^-} = 1 - \frac{p_\perp \cdot V_\perp}{p^+}$$

The color algebra sets the terms in $|M|^2$ involving two different sources to zero. The sum over large number of sources can be replaced by an integral

$$\sum_i f(x_i^+, x_{i,\perp}) = \rho \int d^2 x_\perp \int_{-R^+}^{R^+} dx^+ f(x^+, x_\perp)\,.$$

As long as the medium motion is uniform the integration over the transverse directions results in a cancellation of the phases leading to

$$|M|^2 \sim \rho \int \frac{d^2 q_\perp}{(2\pi)^2} [v(q_\perp)]^2 \left[\frac{p \cdot p_S}{p^+ p_S^-}\right]^2 |j(p-q)|^2\,.$$

Repeating the same steps for the double Born diagram which contributes at the same order and combining all contributions one finds

$$|M|^2_{1+2} = |j(p)|^2 + \rho L \left[\frac{p \cdot p_S}{p^+ p_S^-}\right]^2 \int \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{d\sigma_{el}}{d^2 q_\perp} - \sigma_{el} \delta^{(2)}(q_\perp)\right] |j(p-q)|^2$$

- If the sources are moving with different velocities only the averaging procedure of the GLV framework is modified

$$|M_i|^2 \sim \int \frac{d^2q d^2q'}{(2\pi)^4} e^{i(q_i - q_i')\cdot X_i} v(q)v(q') \left[\frac{p \cdot p_{S,i}}{p^+ p^-_{S,i}}\right]^2 j(p-q) j^\star(p-q'),$$

where subscript i in q_i corresponds to the velocity dependence in q^+.

- As in the drag force case we can start with the leading derivative correction taking $V_\perp = \hat{A} x_\perp$ or $V_\perp \sim e^{il_\perp \cdot x_\perp}$ with small momenta and see whether the averaging can be performed analytically.
- The same procedure can be then applied to study gradient corrections to the medium-modified gluon radiation.

If we assume that the source is not very sensitive to the large momentum component there are two main effects at the leading order in gradients:

- One possible effect comes from the phases

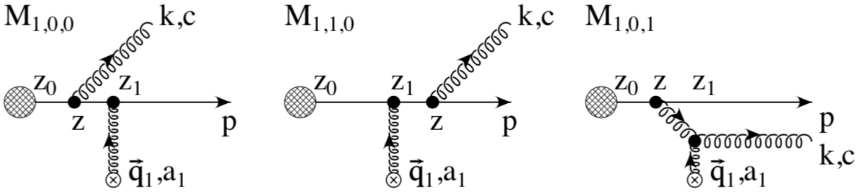
$$\int d^2x_\perp e^{i(q_\perp - q'_\perp)_a(\delta_{ab} - A_{ab}x^-)x_{b,\perp}} = \sqrt{\frac{(2\pi)^2}{\det(1-A)}}\ \delta^{(2)}(q-q')$$

  and corresponds to a rescaling of the density of sources.

- Another effect originates in the overall multiple in the integrand

$$\delta^{(2)}(q_\perp - q'_\perp)\frac{\partial}{\partial(q-q')}e^{i(q^- - q'^{,-})\cdot x^+}v(q)v(q')j(p-q)j^\star(p-q')$$

  and corresponds to a non-trivial modification of $|j(p-q)|^2$


M1,0,0
k,c
z0
z1
z
p
q1,a1
M1,1,0
k,c
z0
z1
z
p
q1,a1
M1,0,1
z0
z
z1
p
k,c
q1,a1

To keep formulas compact they are given in the limit $k^+ \ll p^+$:

$$M_{1,1,0} \sim \int \frac{dq_\perp^2}{(2\pi)^2} e^{iq\cdot X_i} e^{ip\cdot x_0} e^{-i[p^- - (p-k)^- - k^-]x_i^+} \times$$

$$\times\, v(q) \frac{\epsilon_\perp \cdot k_\perp}{k_\perp^2} \frac{p_M^\nu p_\nu}{p_M^-(p^+ - q^+)}\,,$$

where $q^- = p^- - (p-q)^-$ and $a^- = \frac{a_\perp^2}{2a^+}$.

$$M_{1,0,0} \sim \int \frac{dq_\perp^2}{(2\pi)^2} e^{iq\cdot X_i} e^{i(p-q)\cdot x_0} e^{-i[(p-q)^- - k^- - (p-k-q)^-]z^+}\Big|_{x_0^+}^{x_i^+} \times$$

$$\times\, v(q) \frac{\epsilon_\perp \cdot k_\perp}{k_\perp^2} \frac{p_M^\nu (p-k)_\nu}{p_M^-(p^+ - q^+ - k^+)}\,,$$

where $q^- = (p-k)^- - (p-k-q)^-$.

- We have studied how the GLV results are modified in a medium slowly moving in the transverse direction;
- The GLV approach allows to include slow variation of the source velocity;
- It appears to be straightforward in the case of the broadening at the first order in opacity;
- Currently we are finishing to work with the gradient effects in the medium-modified gluon radiation;
- The amplitudes in the ideal hydrodynamic regime can be already used to investigate effects of a realistic flow;

More details in AS, M. Sievert, I. Vitev, 19xx.xxxxx