Medium-Induced Parton Splitting Functions at Second Order in Opacity

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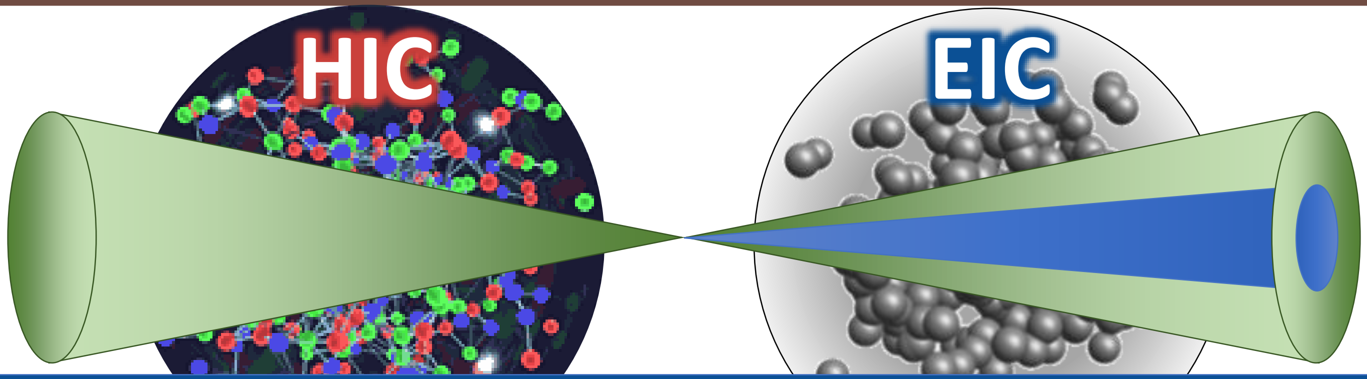
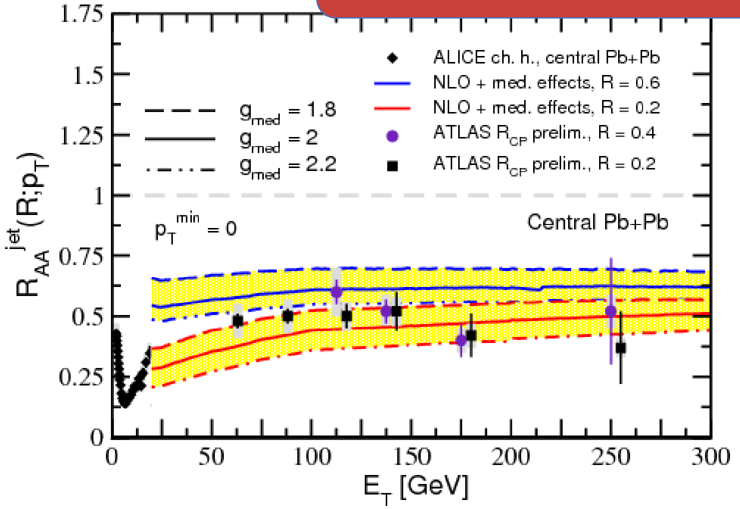
Jet Modification in Hot and Cold Nuclear Matter

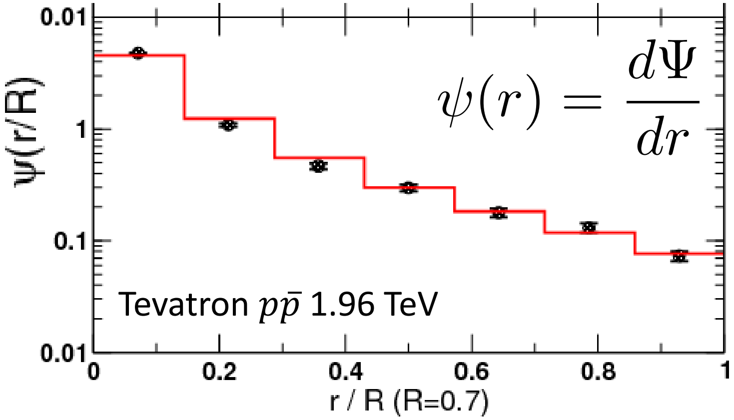


Jets at the EIC and HIC: **Complementarity** and **Universality**

Inclusive Jets vs. Substructure: **Greater Sensitivity** to Medium



He, Vitev, Zhang, Phys. Lett. ***B713*** *(2012)*



Vitev, Wicks, Zhang, JHEP 0811 (2008) 093

Collisional vs. Radiative Energy Loss

- At leading power, jets have only vacuum substructure $\mathcal{O}\left(\frac{\perp^0}{Q^0}\right)$
- Medium modification is always power suppressed:
  - Collisional: $\frac{1}{p_N^- l_f^+} \sim \mathcal{O}\left(\frac{\perp^2}{Q^2}\right)$
  - Radiative: $\begin{cases} \cos \Delta\phi \\ \Delta\phi \sim \frac{\lambda^+}{\ell_f^+} \sim \left(\frac{\perp^2}{Q^2}\right) \frac{A^{1/3}}{\chi} \end{cases}$

Formation length l_f^+

Mean free path λ^+

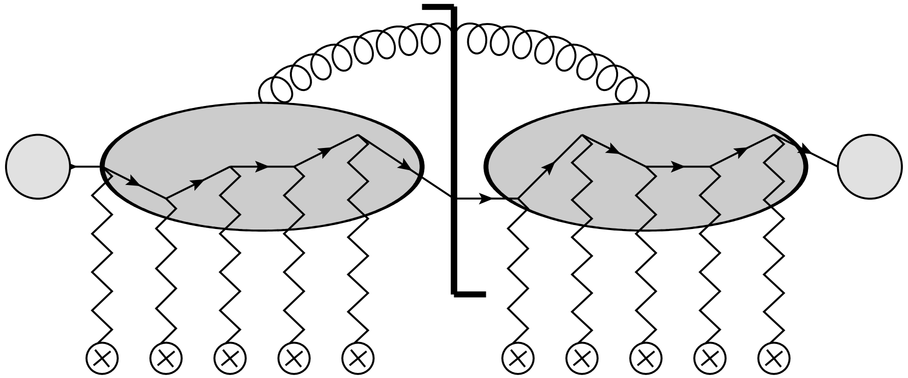
- Coupling to the medium is controlled by the opacity $\chi \equiv \frac{L^+}{\lambda^+} = \langle n \rangle$

Medium length L^+

$$\frac{1}{p_N^-} \ll l_f^+ \sim \lambda^+ \sim L^+$$

MS, *I. Vitev, Phys. Rev.* ***D98*** *(2018)*

The LPM Effect: Medium-Induced Decoherence



- The **Landau-Pomeranchuk-Migdal Effect** in QED:
  - Changes in the emitter's trajectory during the formation time **destroys the perfect coherence**

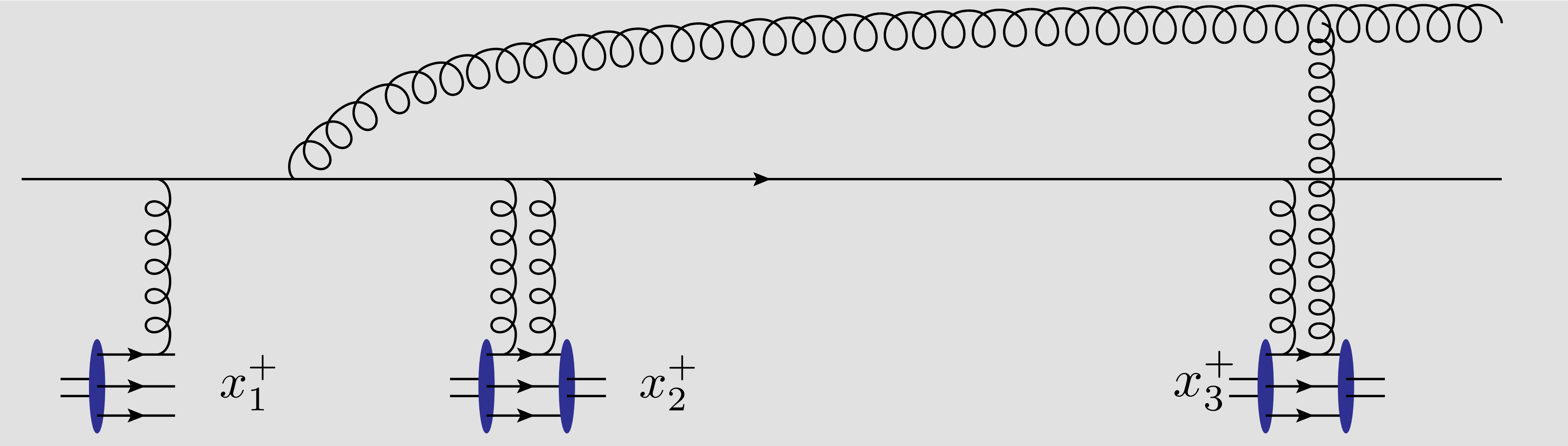
Landau, Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz 92 (1953)
Migdal, Phys. Rev. 103 (1956)
See also: Perl, SLAC-PUB-6514

- The **high-energy limit is trivial**: a return to pointlike scattering
  - The LPM effect (and medium modification in general) is a **power-suppressed effect** compared to vacuum emission

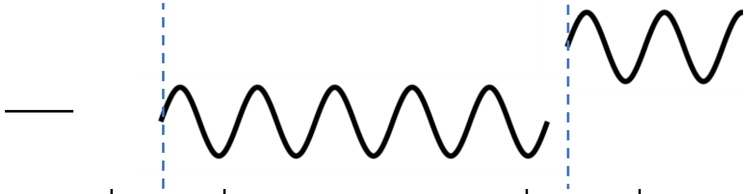
Types of Medium-Induced Phases

- Interactions with the medium **stimulate a different pattern of radiation** through the generation of various **phases**:

__MS__, I. Vitev, Phys. Rev. __D98__ (2018)

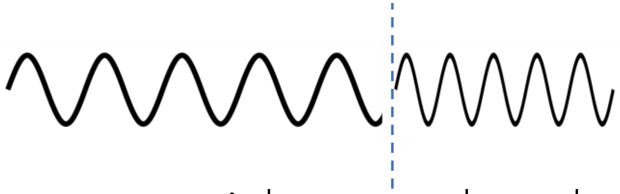


- ➢ Phases from **bounded gluon emission times**:



$$e^{i(x_2^+/\,l_f^+)} - e^{i(x_1^+/\,l_f^+)}$$

- ➢ Phases from **changing the virtuality** of the system:



$$e^{-i(1/\,l_f'^+ - 1/\,l_f^+)x_3^+}$$

A Universal Framework for Jet Modification

- The effects of medium modification due to an **eikonal external potential** are **universal** in any QCD medium
  - Can be formulated at the **Lagrangian level** (e.g. SCET) *Ovanesyan, Vitev, JHEP 06 (2011)*

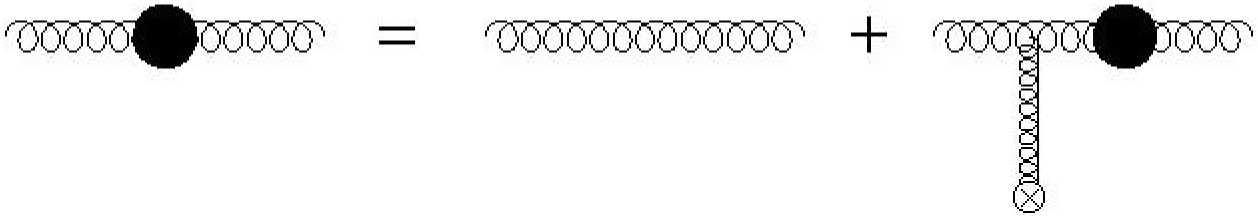
$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ext}^{qG} + \mathcal{L}_{ext}^{3G} + \mathcal{L}_{ext}^{4G} \qquad \frac{d\sigma^{el}}{d^2q} = \frac{1}{(2\pi)^2}\frac{C_F}{2N_c}[v(q_T^2)]^2$$

$$gA_{ext}^{\mu a}(x) = \sum_i \int \frac{d^4q}{(2\pi)^4}\, e^{-iq\cdot(x-x_i)}\, g^{\mu +}\,(t^a)_i\,[2\pi\,\delta(q^+)]\,v(q_T^2)$$

  - Use **Gaussian averaging** of the external fields (leading in α_s for local color neutrality) *Blaizot et al. JHEP 01 (2013)*

$$\left\langle gA_{ext}^{\mu a}(x)\,(gA_{ext}^{\nu b}(y))^*\right\rangle_{med} = g^{\mu +}g^{\nu +}\delta^{ab}\delta(x^+ - y^+)\left[\frac{1}{\lambda_{mfp}^+ C_F}\int\frac{d^2q}{(2\pi)^2}\,e^{i\vec{q}_\perp\cdot(\vec{x}_\perp - \vec{y}_\perp)}\,\frac{(2\pi)^2}{\sigma_{el}}\frac{d\sigma^{el}}{d^2q}\right]$$

A Simple Example: The Dressed Gluon Propagator



Blaizot et al. JHEP 01 (2013)

- **Eikonal scattering** does not disturb the **numerator algebra**

$$G^{\mu\nu}(p',p) = \left(g^{\mu i} - \frac{p_\perp'^{\,i}}{p^+} g^{\mu +} \right) G(p',p) \left(g^{i\nu} - \frac{p_\perp^i}{p^+} g^{+\nu} \right)$$

- One possible approach: a **recursion relation**

$$G(p',p) = \frac{-i}{p^2}(2\pi)^4 \delta^4(p'-p) + \int \frac{d^4p''}{(2\pi)^4} \left[2p^+ \, g_{eff} A_{ext}^-(p''-p) \right] \, G(p'',p)$$

  - Kernel (for the amplitude squared): the **"Reaction Operator"**
  - Basis of the **Gyulassy-Levai-Vitev** approach

Gyulassy, Levai, Vitev, Nucl. Phys. B594 (2001)

The Opacity Expansion

- The **longitudinal averaging** over the scattering centers generates factors of the **opacity**:

$$\int_{0^+}^{L^+} \frac{dz^+}{\lambda^+} = \frac{L^+}{\lambda^+} = \chi$$

- Each **correlated rescattering** generates **higher powers** of the opacity

$$\frac{d\sigma^{(jet)+X}}{d^2p\,dy} = \frac{d\sigma^{(jet)+X}}{d^2p\,dy}\bigg|_{vac} + \mathcal{O}(\chi) + \mathcal{O}(\chi^2) + \cdots$$

  - For **fairly small opacities**, the series can be **truncated** at finite opacity $\chi < \text{few}$
  - For **very large opacities**, the series must be **resummed**. $\chi \gg 1$

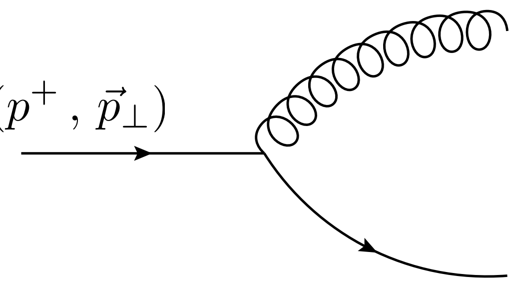
- E.g.) At RHIC:

$$\begin{cases} \chi \sim 2.4 & \text{Min bias} \\ \chi \sim 3.1 & \text{Most central} \end{cases}$$

Vitev, Zhang, Phys. Lett. B669 (2008)

- Light-front **wave functions** as in time-ordered perturbation theory

Lepage, Brodsky, Phys. Rev. D22 (1980)
Brodsky, Pauli, Pinsky, Phys. Rept. 301 (1998)



$$\psi(k,p) = \langle G_{(k)} \, q_{(p-k)} | \, \mathcal{U}(0, -\infty) \, | q_{(p)} \rangle$$

$$\Delta E^- = E_i^- - E_f^- = -\frac{(k-xp)_T^2}{2x(1-x)p^+}$$

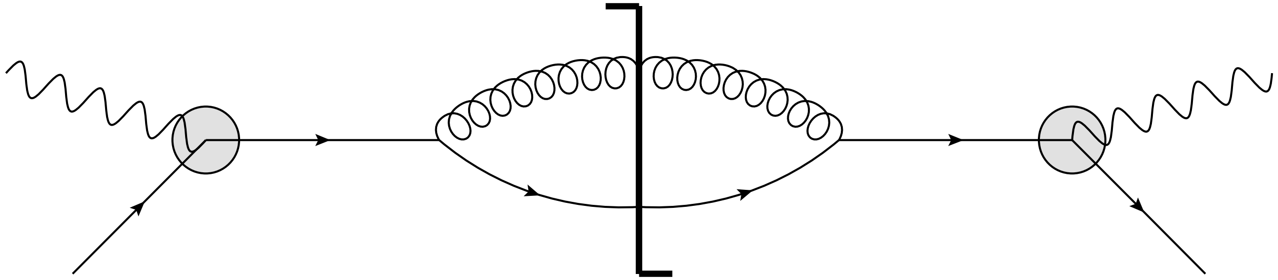
- **Energy denominators** set the lifetime of a fluctuation

$$\ell_f^+ = \frac{1}{|\Delta E^-|}$$

- Depend only on the **intrinsic transverse momentum**
  (2D Galilean symmetry)

$$\psi = \psi(x, \vec{\kappa}_\perp)$$
$$\Delta E^- = \Delta E^-(x, \vec{\kappa}_\perp)$$
$$\vec{\kappa}_\perp = \vec{k}_\perp - x\vec{p}_\perp$$

The Vacuum Splitting Functions



e.g.) DIS in the Breit frame

- **Pointlike scattering**: the branching is just given by LFWF:

$$xp^+ \frac{dN}{d^2k\,dx\,d^2p\,dp^+}\bigg|_{\mathcal{O}(\chi^0)} = \frac{1}{2(2\pi)^3}\frac{C_F}{1-x}\left|\psi(x, \underline{k-xp})\right|^2 \times \left(p^+ \frac{dN_0}{d^2p\,dp^+}\right)$$

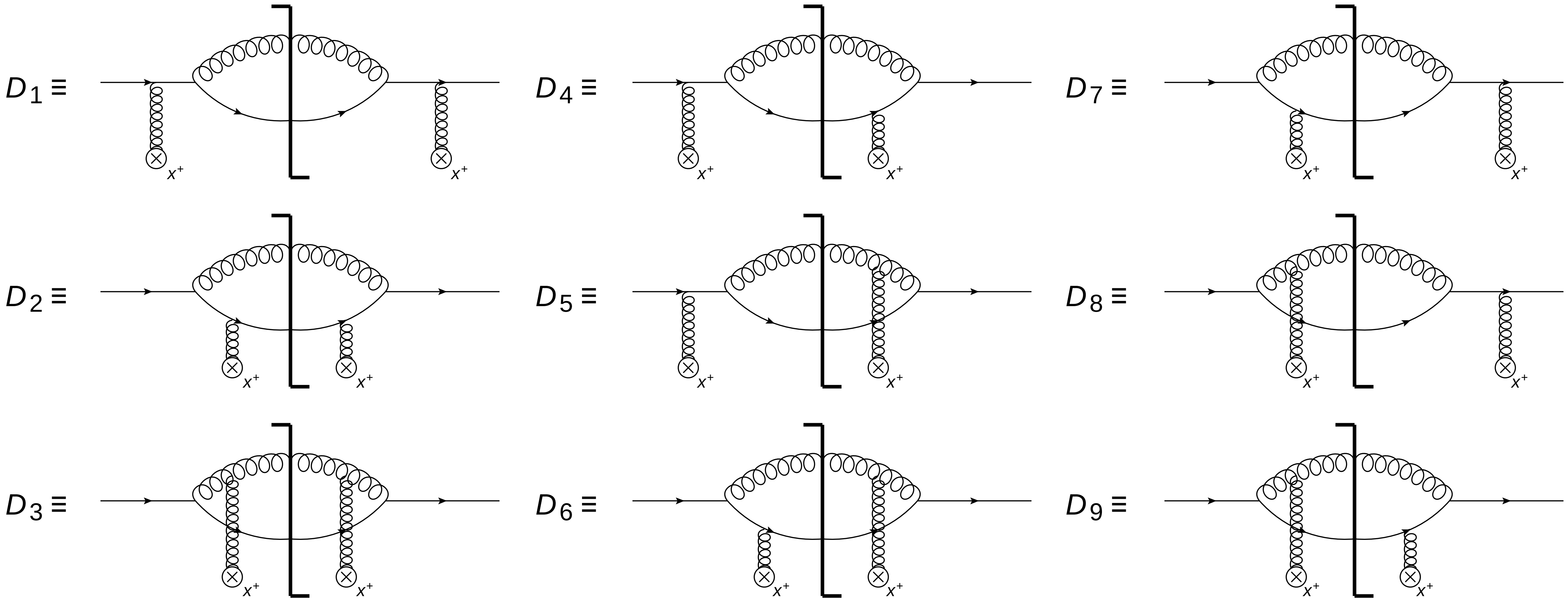
- Squares are simply proportional to **DGLAP splitting kernels**

$$P_{G/q}(x) = \frac{C_F}{2g^2}\frac{\kappa_T^2}{x(1-x)}|\psi(x,\vec{\kappa}_\perp)|^2 = C_F\frac{1+(1-x)^2}{x}$$

- Gluon is emitted **coherently** over the whole **formation time**

$$\ell_f^+ = \frac{2x(1-x)p^+}{(k-xp)_T^2}$$

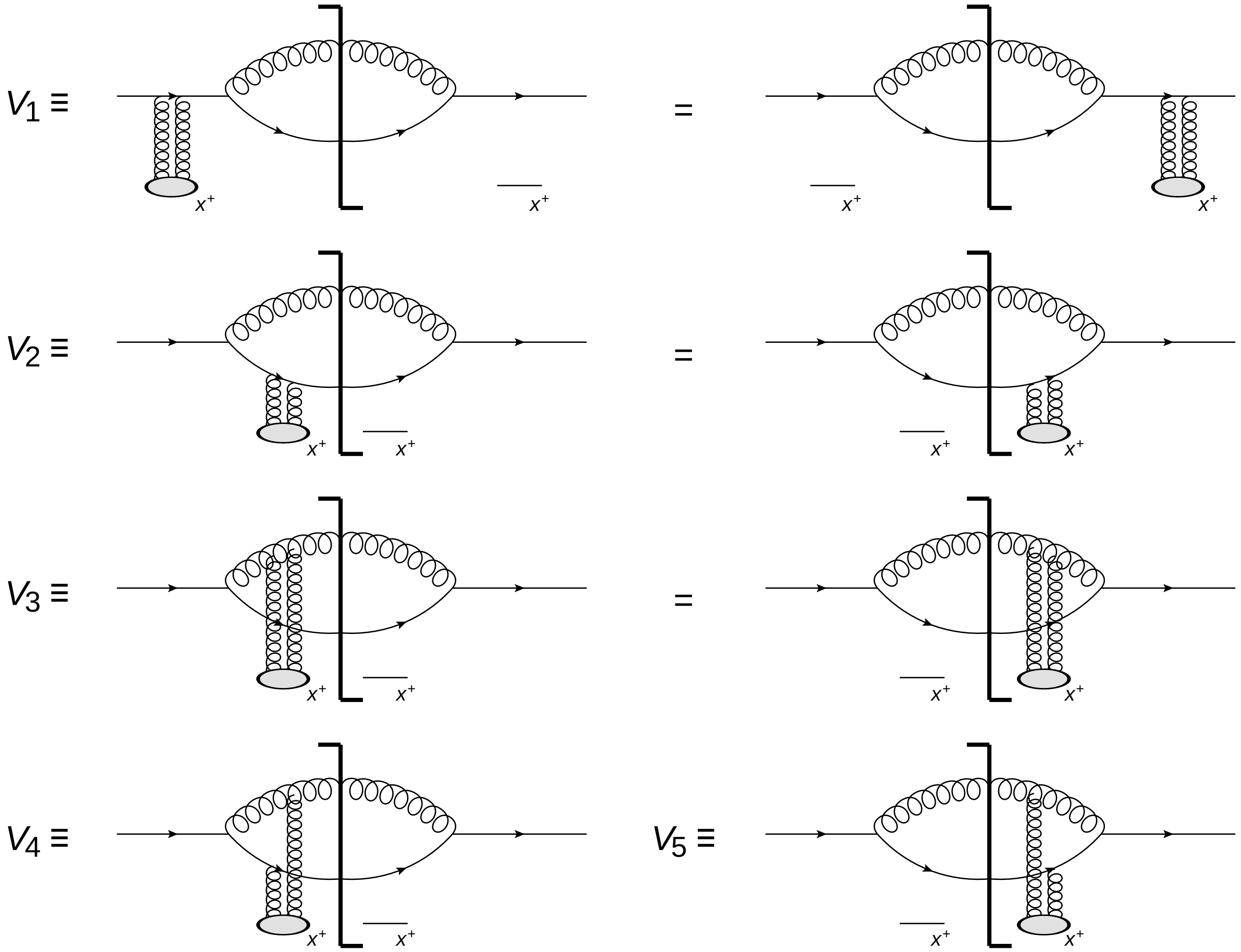
"Direct"



MS**, I. Vitev, Phys. Rev.* ***D98 *(2018)*

Medium Modification at LOpacity

“Virtual”



MS**, I. Vitev, Phys. Rev.* ***D98 *(2018)*

Medium Modification at LOpacity

$$x\,p^+ \frac{dN}{d^2k\,dx\,dp^+\,d^2p}\bigg|_{N=1} = \frac{C_F}{2(2\pi)^3(1-x)} \int_{0^+}^{L^+} \frac{dz_1^+}{\lambda^+} \int \frac{d^2q}{(2\pi)^2} \left(\frac{(2\pi)^2}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \right)$$

"Direct"

$$\times \Bigg\{ \left(p^+ \frac{dN_0}{d^2(p-q)\,dp^+} \right) \Bigg[\left|\psi(\underline{k}-x\underline{p})\right|^2 + 2\left(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z_1^+\right]\right) \left|\psi(\underline{k}-x\underline{p}+x\underline{q})\right|^2$$

$$+ 2\frac{N_c}{C_F}\left(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})z_1^+\right]\right) \left|\psi(\underline{k}-x\underline{p}-(1-x)\underline{q})\right|^2$$

$$+ \frac{1}{N_c C_F}\left(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z_1^+\right]\right) \psi(\underline{k}-x\underline{p})\psi^*(\underline{k}-x\underline{p}+x\underline{q})$$

$$- \frac{N_c}{C_F}\left(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})z_1^+\right]\right) \psi(\underline{k}-x\underline{p})\psi^*(\underline{k}-x\underline{p}-(1-x)\underline{q})$$

$$- \frac{N_c}{C_F}\Bigg(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z_1^+\right] - \cos\left[\Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})z_1^+\right]$$

$$+ \cos\left[\left(\Delta E^-(\underline{k}-x\underline{p}+x\underline{q}) - \Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})\right)z_1^+\right]\Bigg) \psi(\underline{k}-x\underline{p}+x\underline{q})\psi^*(\underline{k}-x\underline{p}-(1-x)\underline{q}) \Bigg]$$

"Virtual"

$$+ \left(p^+ \frac{dN_0}{d^2p\,dp^+} \right) \Bigg[-\left|\psi(\underline{k}-x\underline{p})\right|^2 - \frac{N_c}{C_F}\left(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p})z_1^+\right]\right) \left|\psi(\underline{k}-x\underline{p})\right|^2$$

$$+ \frac{N_c}{C_F}\left(\cos\left[\left(\Delta E^-(\underline{k}-x\underline{p}) - \Delta E^-(\underline{k}-x\underline{p}-\underline{q})\right)z_1^+\right] - \cos\left[\Delta E^-(\underline{k}-x\underline{p})z_1^+\right]\right) \psi(\underline{k}-x\underline{p})\psi^*(\underline{k}-x\underline{p}-\underline{q}) \Bigg] \Bigg\}$$

***MS**, I. Vitev, Phys. Rev. **D98** (2018)*

LOpacity: Explicit Structure and Heavy Quarks

Masses enter naturally on the same footing

$$x p^+ \frac{dN}{d^2k\, dx\, d^2p\, dp^+}\Big|_{\mathcal{O}(\chi^1)} = \frac{\alpha_s C_F}{2\pi^2} \int_0^{L^+} \frac{d(\delta z^+)}{\lambda_g^+} \int \frac{d^2q}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \times \frac{1}{\mu^2} \left(p^+ \frac{dN_0}{dp^+} \right)$$

$$\times \Bigg\{ \Bigg(\Bigg[\frac{B_T^2\, [1+(1-x)^2] + x^4 m^2}{[B_T^2 + x^2m^2]^2} - \frac{(\underline{B}\cdot\underline{C})\, [1+(1-x)^2] + x^4m^2}{[B_T^2+x^2m^2]\,[C_T^2+x^2m^2]} \Bigg]$$

$$+ \frac{1}{N_c^2} \Bigg[\frac{(\underline{B}\cdot\underline{A})\, [1+(1-x)^2] + x^4m^2}{[B_T^2+x^2m^2]\,[A_T^2+x^2m^2]} - \frac{B_T^2\, [1+(1-x)^2]+x^4m^2}{[B_T^2+x^2m^2]^2} \Bigg] \Bigg) \Big(1 - \cos\big((\Omega_1 - \Omega_2)\delta z^+ \big) \Big)$$

$$+ \Bigg(2 \frac{C_T^2\, [1+(1-x)^2] + x^4m^2}{[C_T^2+x^2m^2]^2} - \frac{(\underline{C}\cdot\underline{A})\, [1+(1-x)^2]+x^4m^2}{[C_T^2+x^2m^2]\,[A_T^2+x^2m^2]} - \frac{(\underline{C}\cdot\underline{B})\, [1+(1-x)^2]+x^4m^2}{[C_T^2+x^2m^2]\,[B_T^2+x^2m^2]} \Bigg)$$

$$\times \Big(1 - \cos\big((\Omega_1 - \Omega_3)\delta z^+ \big) \Big)$$

$$+ \frac{(\underline{B}\cdot\underline{C})\, [1+(1-x)^2]+x^4m^2}{[B_T^2+x^2m^2]\,[C_T^2+x^2m^2]} \Big(1 - \cos\big((\Omega_2 - \Omega_3)\delta z^+ \big) \Big)$$

$$+ \Bigg(\frac{(\underline{A}\cdot\underline{D})\, [1+(1-x)^2]+x^4m^2}{[A_T^2+x^2m^2]\,[D_T^2+x^2m^2]} - \frac{A_T^2\, [1+(1-x)^2]+x^4m^2}{[A_T^2+x^2m^2]^2} \Bigg) \Big(1 - \cos\big(\Omega_4 \delta z^+ \big) \Big)$$

$$- \frac{(\underline{A}\cdot\underline{D})\, [1+(1-x)^2]+x^4m^2}{[A_T^2+x^2m^2]\,[D_T^2+x^2m^2]} \Big(1 - \cos\big(\Omega_5 \delta z^+ \big) \Big) \Bigg\},$$

__MS__, I. Vitev, Phys. Rev. __D98__ (2018)

The Broad Source and Medium-Induced Splitting Functions

Different momenta in general…

But if the hard jet production vertex is insensitive to shifts from the medium…

$$\frac{dN}{d^2(p-q)} \approx \frac{dN}{d^2p}$$

__MS__, I. Vitev, Phys. Rev. __D98__ (2018)

$$
\begin{aligned}
x\,p^+ \frac{dN}{d^2k\,dx\,dp^+\,d^2p}\bigg|_{N=1} &= \frac{C_F}{2(2\pi)^3(1-x)} \int_{0^+}^{L^+} \frac{dz_1^+}{\lambda^+} \int \frac{d^2q}{(2\pi)^2} \left(\frac{(2\pi)^2}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \right) \\
&\times \Bigg\{ \left(p^+ \frac{dN_0}{d^2(p-q)\,dp^+} \right) \Bigg[\left|\psi(\underline{k}-x\underline{p})\right|^2 + 2\Big(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z_1^+\right]\Big) \left|\psi(\underline{k}-x\underline{p}+x\underline{q})\right|^2 \\
&\quad + 2\frac{N_c}{C_F}\Big(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})z_1^+\right]\Big) \left|\psi(\underline{k}-x\underline{p}-(1-x)\underline{q})\right|^2 \\
&\quad + \frac{1}{N_c C_F}\Big(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z_1^+\right]\Big) \psi(\underline{k}-x\underline{p})\psi^*(\underline{k}-x\underline{p}+x\underline{q}) \\
&\quad - \frac{N_c}{C_F}\Big(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})z_1^+\right]\Big) \psi(\underline{k}-x\underline{p})\psi^*(\underline{k}-x\underline{p}-(1-x)\underline{q}) \\
&\quad - \frac{N_c}{C_F}\Big(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z_1^+\right] - \cos\left[\Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})z_1^+\right] \\
&\qquad + \cos\left[\Big(\Delta E^-(\underline{k}-x\underline{p}+x\underline{q}) - \Delta E^-(\underline{k}-x\underline{p}-(1-x)\underline{q})\Big)z_1^+\right]\Big) \psi(\underline{k}-x\underline{p}+x\underline{q})\psi^*(\underline{k}-x\underline{p}-(1-x)\underline{q}) \Bigg] \\
&+ \left(p^+ \frac{dN_0}{d^2p\,dp^+} \right) \Bigg[-\left|\psi(\underline{k}-x\underline{p})\right|^2 - \frac{N_c}{C_F}\Big(1-\cos\left[\Delta E^-(\underline{k}-x\underline{p})z_1^+\right]\Big) \left|\psi(\underline{k}-x\underline{p})\right|^2 \\
&\quad + \frac{N_c}{C_F}\Big(\cos\left[\Big(\Delta E^-(\underline{k}-x\underline{p}) - \Delta E^-(\underline{k}-x\underline{p}-\underline{q})\Big)z_1^+\right] - \cos\left[\Delta E^-(\underline{k}-x\underline{p})z_1^+\right] \Big) \psi(\underline{k}-x\underline{p})\psi^*(\underline{k}-x\underline{p}-\underline{q}) \Bigg] \Bigg\}
\end{aligned}
$$

The Broad Source and Medium-Induced Splitting Functions

...the medium effects decouple from the hard jet production vertex

$\propto P_{i/j}$

Vacuum:

$$xp^+ \frac{dN}{d^2k\, dx\, d^2p\, dp^+}\bigg|_{\mathcal{O}(\chi^0)} = \frac{1}{2(2\pi)^3} \frac{C_F}{1-x} \left|\psi(x, \underline{k - xp})\right|^2 \times \left(p^+ \frac{dN_0}{d^2p\, dp^+}\right)$$

Medium:

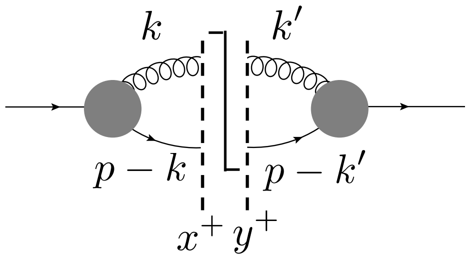
$$xp^+ \frac{dN}{d^2k\, dx\, d^2p\, dp^+}\bigg|_{\mathcal{O}(\chi^1)} = \frac{1}{2(2\pi)^3} \frac{C_F}{1-x}\, f(x, \underline{k}, \underline{p}) \times \left(p^+ \frac{dN_0}{d^2p\, dp^+}\right)$$

$\equiv P_{i/j}^{med}$

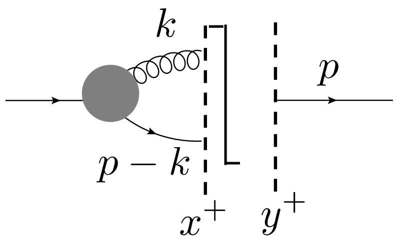
Beyond LOpacity: Recursion Relations for Higher Orders

- Full **recursion relations** for the **amplitude squared**:

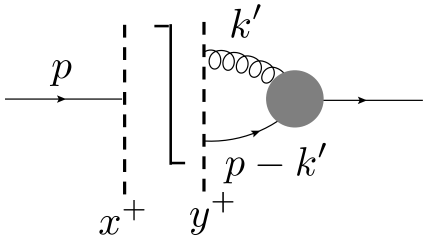
$f^{(N)}_{F/F}(\underline{k},\underline{k}',\underline{p}\,;\,x^+,y^+)$



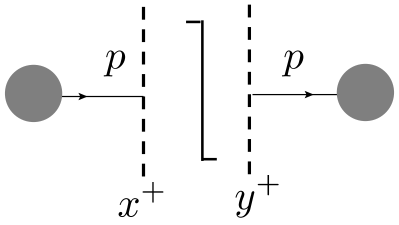
$f^{(N)}_{F/I}(\underline{k},\underline{p}\,;\,x^+,y^+)$



$f^{(N)}_{I/F}(\underline{k}',\underline{p}\,;\,x^+,y^+)$



$f^{(N)}_{I/I}(\underline{p}\,;\,x^+,y^+)$



- The **reaction operator** is a matrix relating **4 (3) functions**

$$\begin{bmatrix} f^{(N)}_{F/F}(\underline{k},\underline{k}',\underline{p}\,;\,x^+,y^+) \\ f^{(N)}_{I/F}(\underline{k}',\underline{p}\,;\,x^+,y^+) \\ f^{(N)}_{F/I}(\underline{k},\underline{p}\,;\,x^+,y^+) \\ f^{(N)}_{I/I}(\underline{p}\,;\,x^+,y^+) \end{bmatrix} = \int\limits_{x_0^+}^{\min[x^+,y^+,R^+]} \frac{dz^+}{\lambda^+} \int \frac{d^2q}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \begin{bmatrix} \mathcal{K}_1 & \mathcal{K}_2 & \mathcal{K}_3 & \mathcal{K}_4 \\ 0 & \mathcal{K}_5 & 0 & \mathcal{K}_6 \\ 0 & 0 & \mathcal{K}_7 & \mathcal{K}_8 \\ 0 & 0 & 0 & \mathcal{K}_9 \end{bmatrix} \begin{bmatrix} f^{(N-1)}_{F/F}(\underline{k},\underline{k}',\underline{p}\,;\,x^+,y^+) \\ f^{(N-1)}_{I/F}(\underline{k}',\underline{p}\,;\,x^+,y^+) \\ f^{(N-1)}_{F/I}(\underline{k},\underline{p}\,;\,x^+,y^+) \\ f^{(N-1)}_{I/I}(\underline{p}\,;\,x^+,y^+) \end{bmatrix}$$

***MS**, I. Vitev, Phys. Rev. **D98** (2018)*

A Standard Prescription for all Partonic Channels

- The LFWF framework is very general: easy to extend to all channels

$$\langle \psi(x, \underline{\kappa})\, \psi^*(x, \underline{\kappa}') \rangle = \frac{8\pi\alpha_s\, f(x)}{[\kappa_T^2 + \nu^2 m^2]\,[\kappa_T'^2 + \nu^2 m^2]} \left[g(x)\, (\underline{\kappa} \cdot \underline{\kappa}') + \nu^4 m^2 \right]$$

$$\Delta E^-(x, \underline{\kappa}) = -\frac{\kappa_T^2 + \nu^2 m^2}{2x(1-x)p^+}$$

***MS**, I. Vitev, B. Yoon, In Preparation*

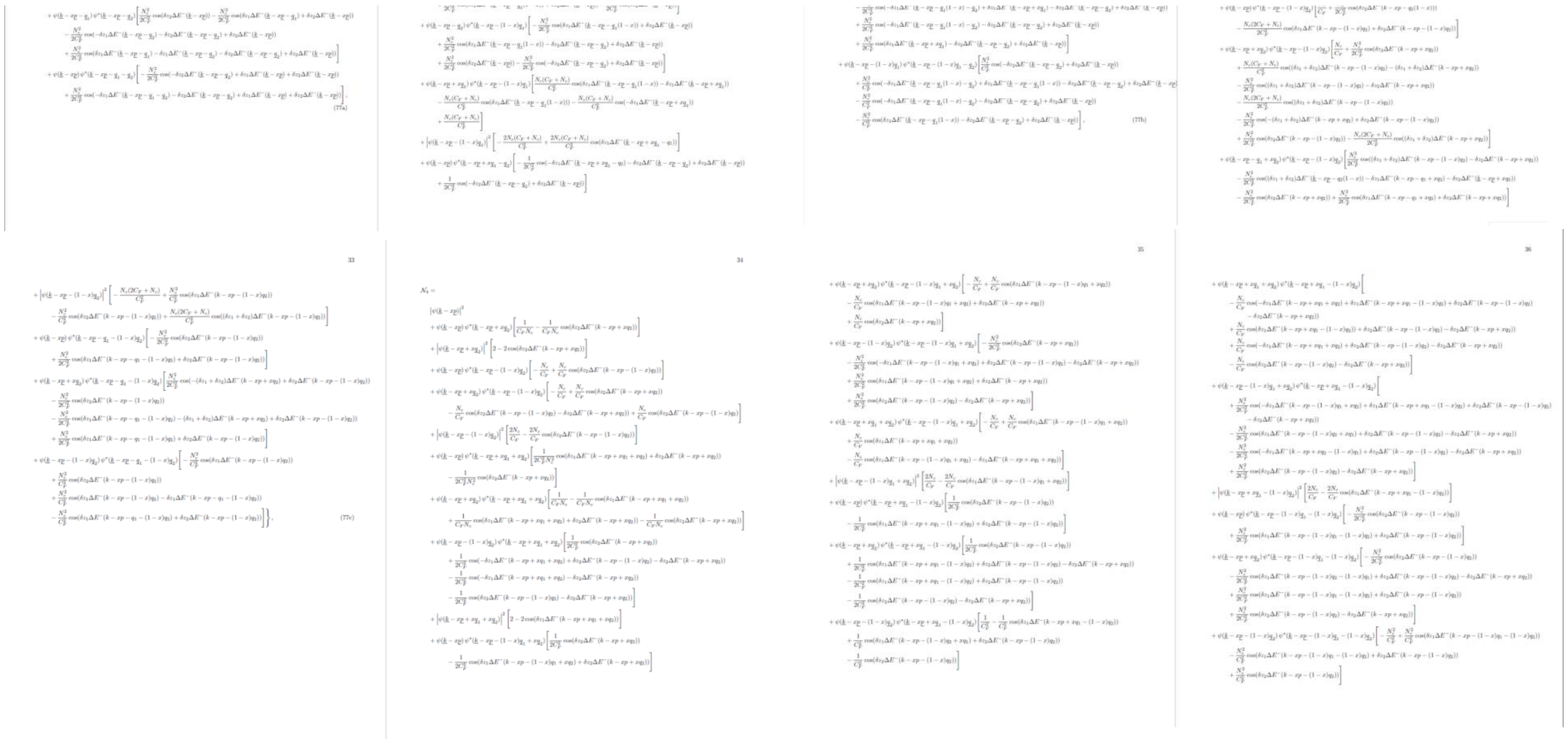
$$p^+ \frac{dN}{d^2k\, dx\, d^2p\, dp^+}\bigg|_{\mathcal{O}(\chi^0)} = \frac{\mathcal{C}_0}{2(2\pi)^3\, x\,(1-x)} \langle \psi(x, \underline{k - xp})\, \psi^*(x, \underline{k - xp}) \rangle \times \left(p^+ \frac{dN_0}{d^2p\, dp^+} \right)$$

	d_1	d_2	d_3	d_4	d_5	d_6	v_1	v_3	v_5	v_7	λ_R^+	$\mathcal{C}_0$	ν	$f(x)$	$g(x)$
$q \to gq$	1	1	$\frac{N_c}{C_F}$	$\frac{-1}{2N_c C_F}$	$\frac{N_c}{2C_F}$	$\frac{-N_c}{2C_F}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-N_c}{2C_F}$	$\frac{N_c}{2C_F}$	λ_q^+	C_F	x	$1-x$	$1+(1-x)^2$
$q \to qg$	1	1	$\frac{N_c}{C_F}$	$\frac{-1}{2N_c C_F}$	$\frac{N_c}{2C_F}$	$\frac{-N_c}{2C_F}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-N_c}{2C_F}$	$\frac{N_c}{2C_F}$	λ_q^+	C_F	$1-x$	x	$1+x^2$
$g \to q\bar{q}$	1	$\frac{C_F}{N_c}$	$\frac{C_F}{N_c}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2N_c^2}$	$-\frac{1}{2}$	$-\frac{C_F}{2N_c}$	$\frac{-C_F}{2N_c}$	$\frac{-1}{2N_c^2}$	λ_G^+	$\frac{1}{2}$	1	$x(1-x)$	$x^2+(1-x)^2$
$g \to gg$	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	λ_G^+	N_c	0	$1+x^4+(1-x)^4$	1

A First Calculation of NLOpacity

The calculation to second order in opacity is straightforward…

…but awful to write out the answer.



A First Calculation of NLOpacity

The calculation to second order in opacity is straightforward…

…but awful to write out the answer.

…But easy to code directly!

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```
In[ ]:= N22 = N2[[2]] /. {ψ[a_] :> ψ[Expand[a]], ΔE[a_] :> ΔE[Expand[a]]} /. {k → 0, p → 0};
N22clean = N22 /. {
    ΔE[a_] :> En2[a /. {q1 :> 1, q2 :> 0}, a /. {q1 :> 0, q2 :> 1}],
    ψ[b_] :> ψ[b /. {q1 :> 1, q2 :> 0}, b /. {q1 :> 0, q2 :> 1}],
    N0[c_] :> N02[c /. {q1 :> 1, q2 :> 0}, c /. {q1 :> 0, q2 :> 1}]
   } /. {
   ψ[a1_, a2_] ψ[b1_, b2_] → Phi2[a1, a2, b1, b2],
   ψ[a1_, a2_]^2 → Phi2[a1, a2, a1, a2]} /. {
  Cos[a_] → cos[a]
  }
Export["C:\\Users\\sieve\\OneDrive\\Desktop\\LANL Projects\\Vitev_Group\\Reaction_Generalized\\NLO_Term2.txt", CForm@N22clean]
```

Two inputs for any medium model: $\frac{d\sigma}{d^2q}$ $\lambda = \frac{1}{\rho\sigma}$

In the following:

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- QGP simulated using 2+1D **iEBE-VISHNU viscous hydrodynamics**
- Monte Carlo-Glauber initial conditions for PbPb at 2.76 TeV at **0-10% centrality**
- **S95-1 Equation of State**
- **Gyulassy-Wang potential**

$$\frac{d\sigma^{el}}{d^2q}(z) = \frac{9}{32\pi^2}\frac{g^4}{\left(q_T^2 + g^2T^2(z)\right)^2}$$

- Thermal densities of quarks and gluons

$$\rho(z) = \frac{16\,\zeta(3)}{\pi^2}T^3(z) \quad \text{(gluons)}$$

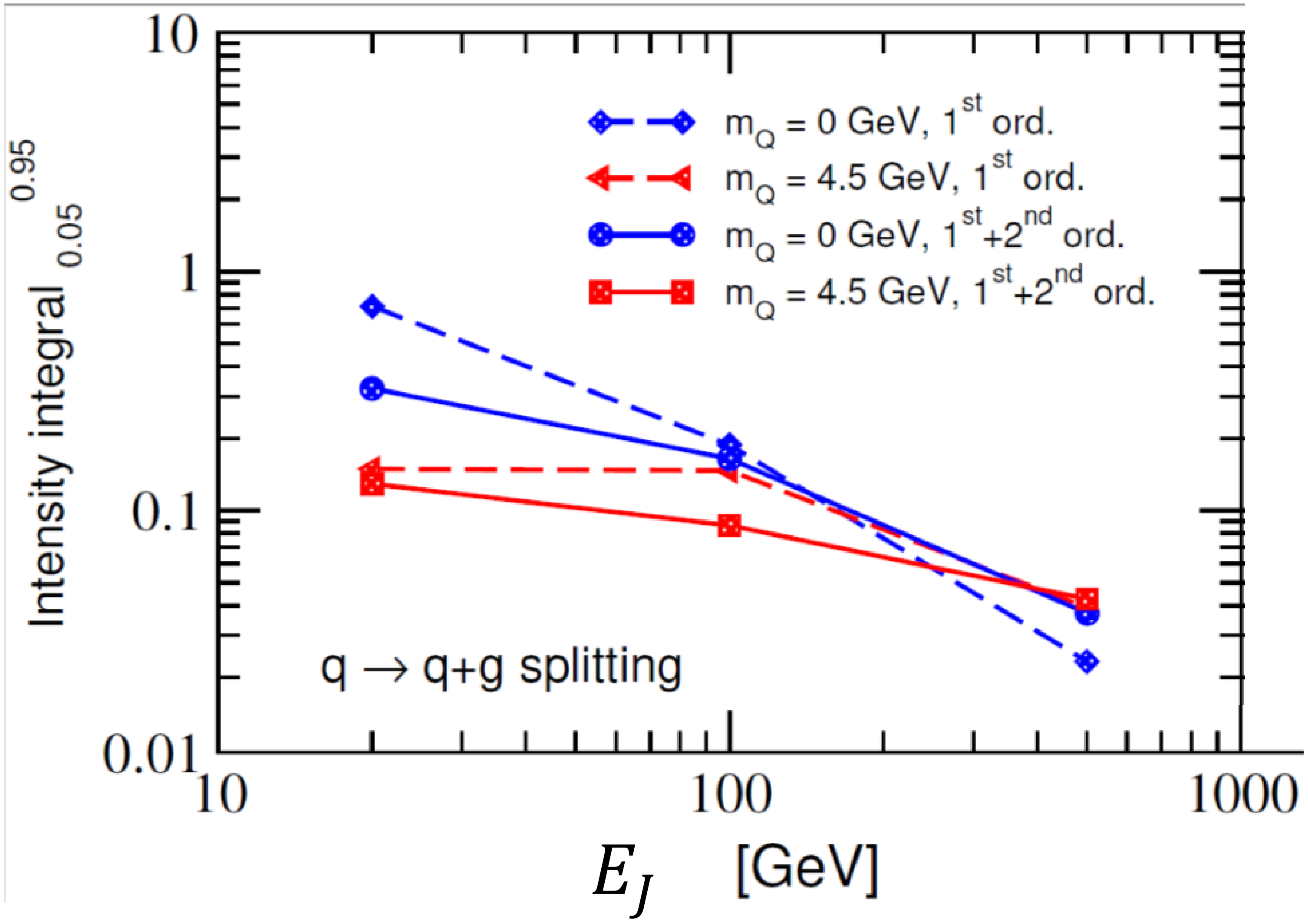
- Jet production **weighted by N_{bin}**
- Integrals computed using adaptive **VEGAS algorithm**

*P. Lepage, J. Comput. Phys. **27** (1978)*

Preliminary Results: The Integrated Spectrum

$$\mathcal{I}_{x_{\min}}^{x_{\max}} = \int_{x_{\min}}^{x_{\max}} dx \int d^2k \; x \frac{dN}{d^2k\, dx}$$

- Total energy carried away by radiated parton
- Not necessarily soft!
- Endpoint cutoffs for validity of approximations
- Only a loose interpretation as “parton energy loss”



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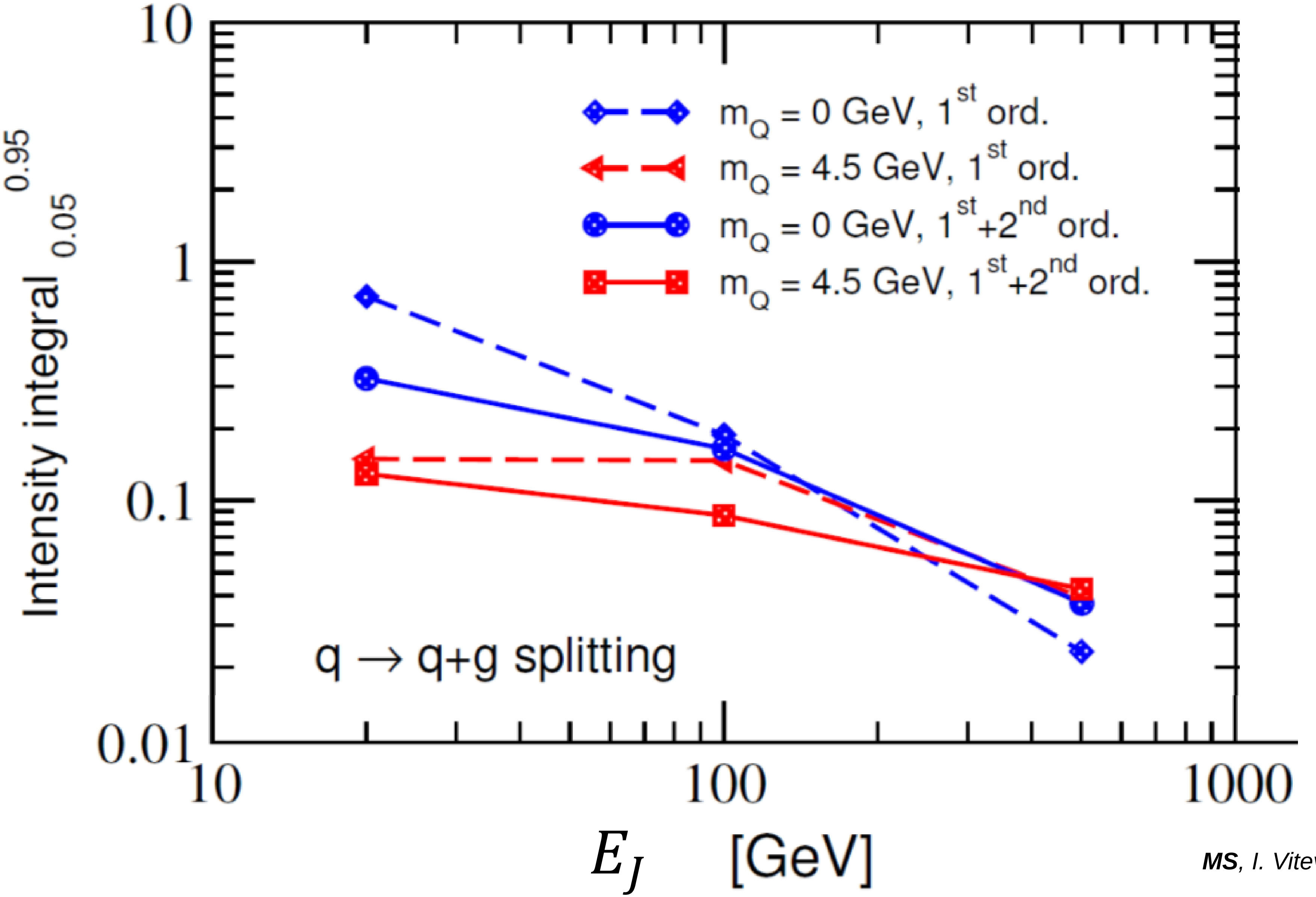
Preliminary Results: The Integrated Spectrum



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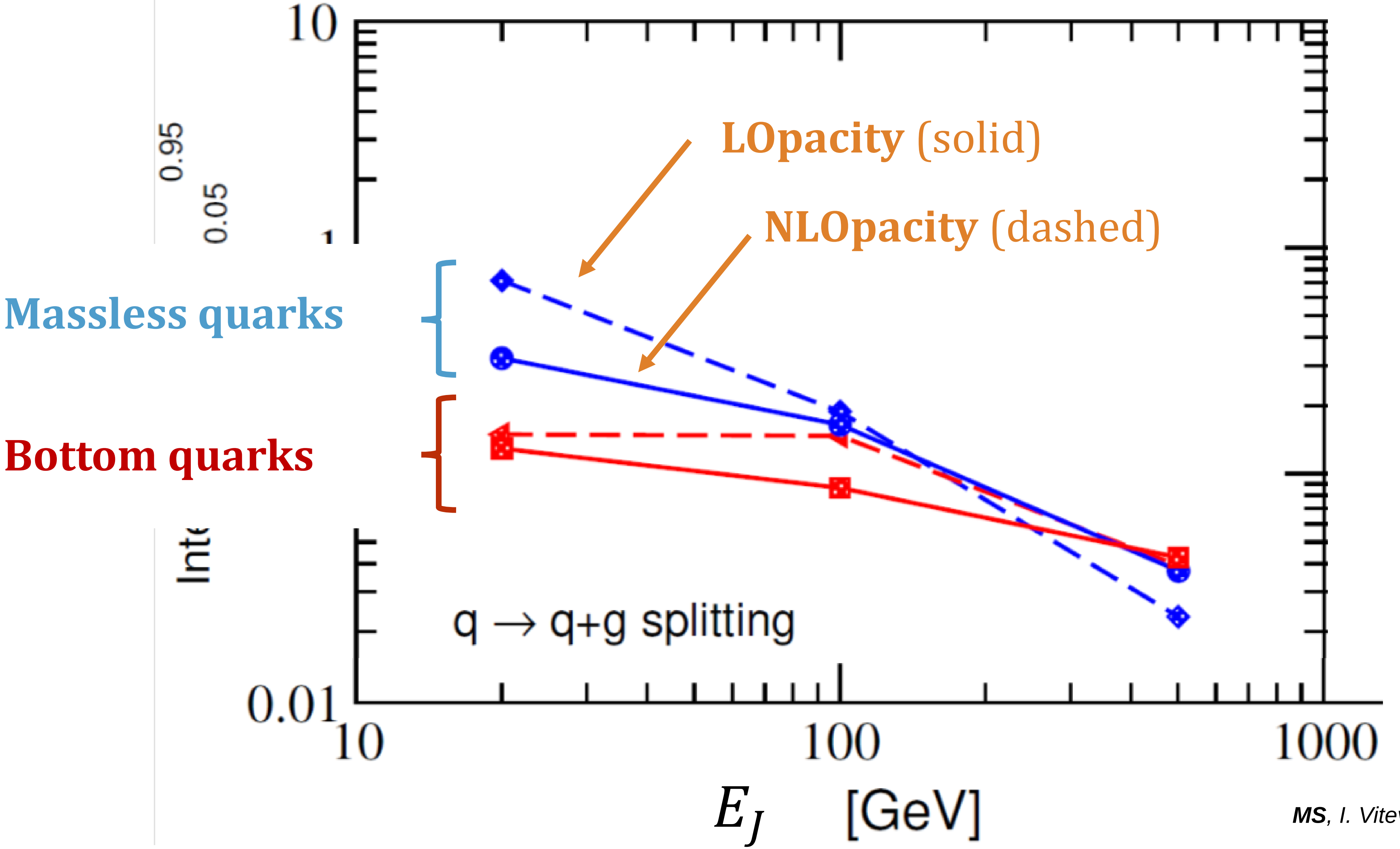
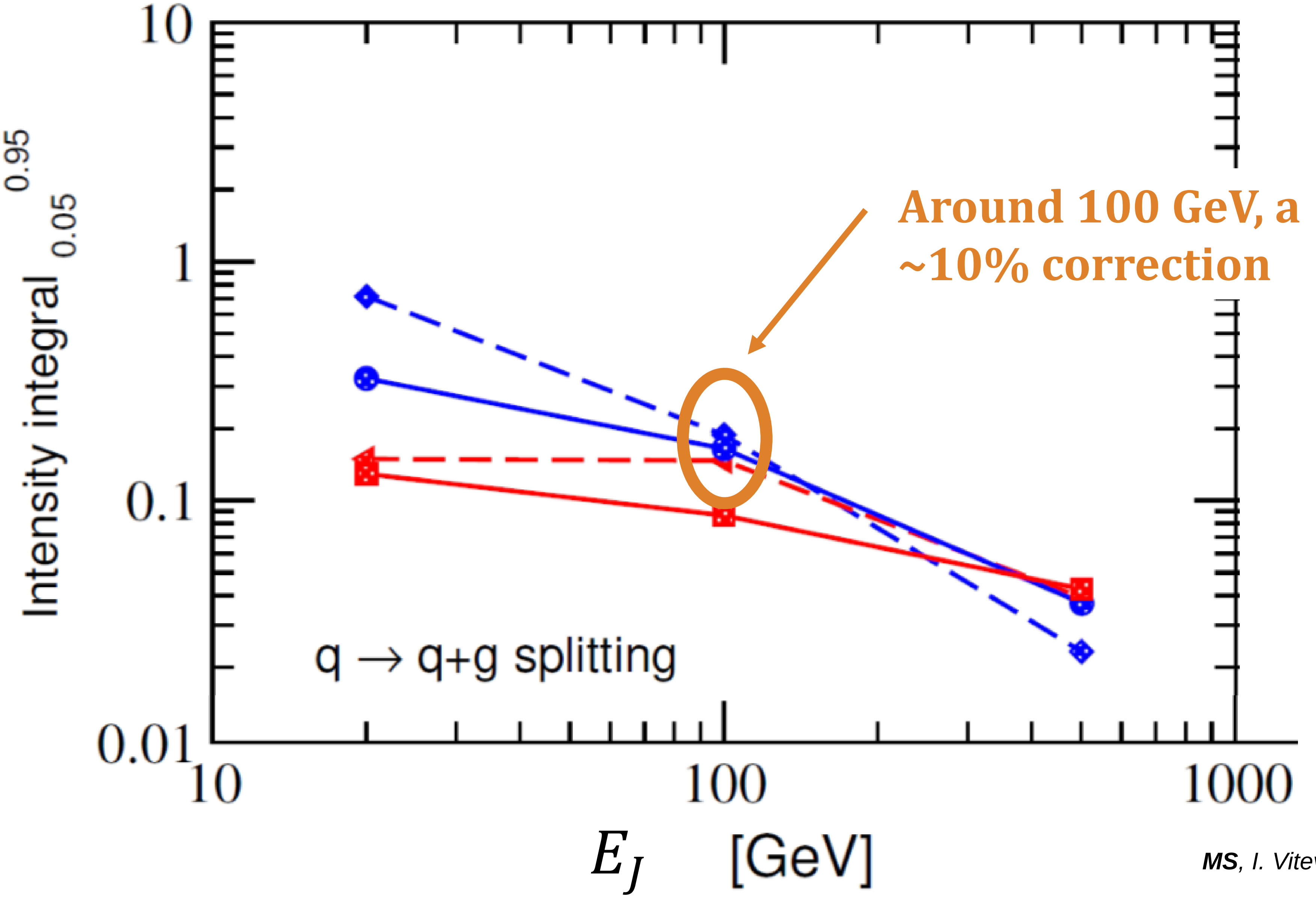


***MS**, I. Vitev, B. Yoon, In Preparation*



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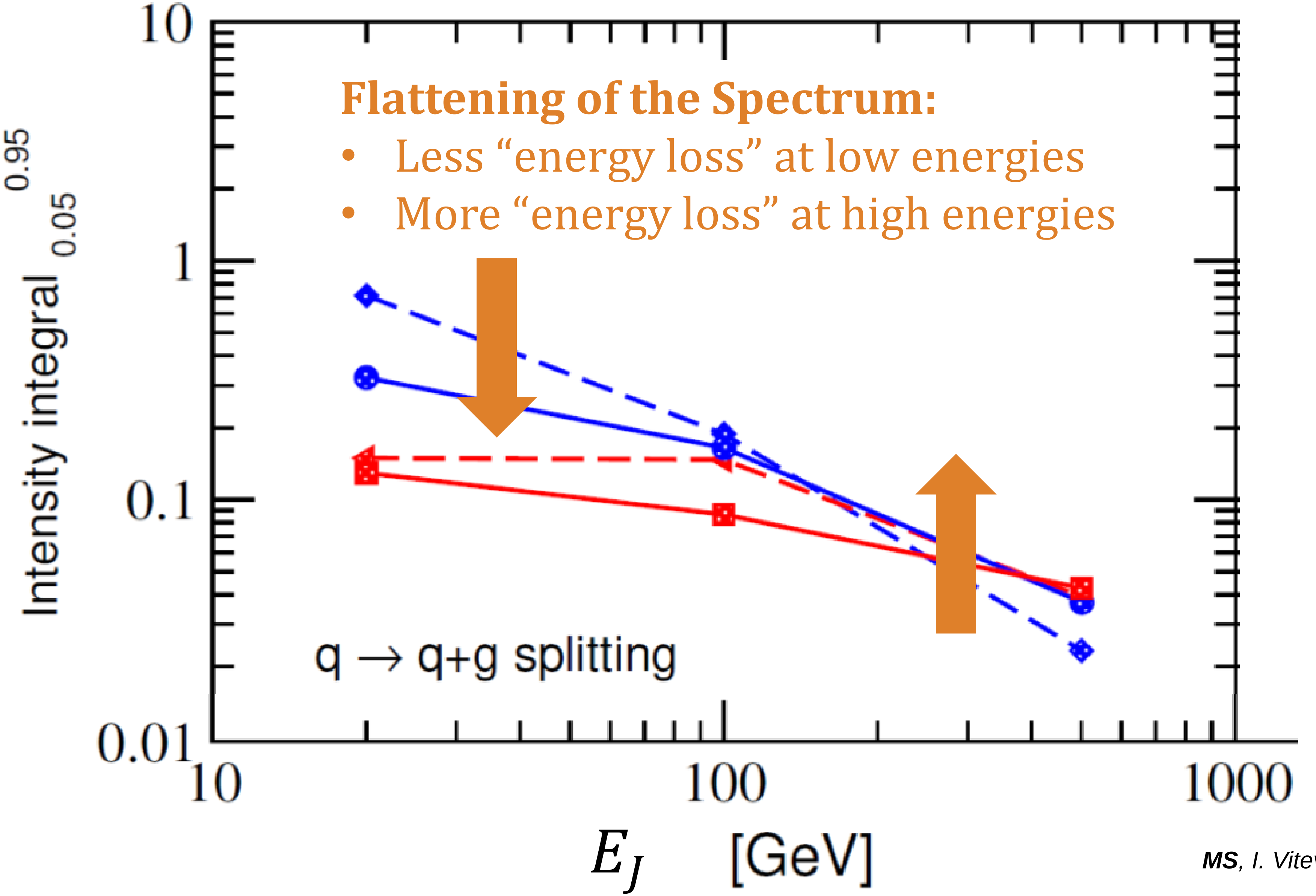
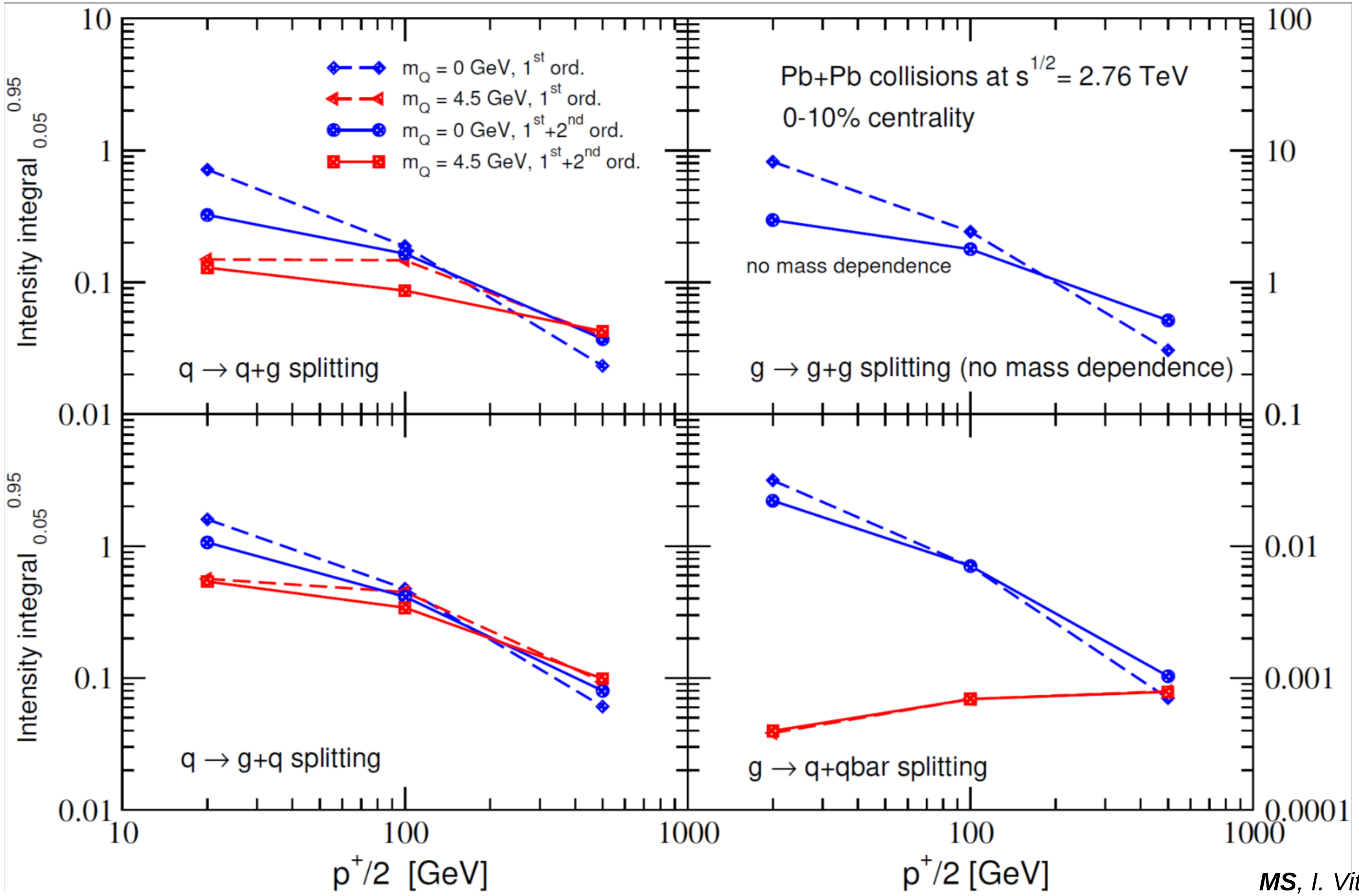
Preliminary Results: The Integrated Spectrum



***MS**, I. Vitev, B. Yoon, In Preparation*

Preliminary Results: The Integrated Spectrum



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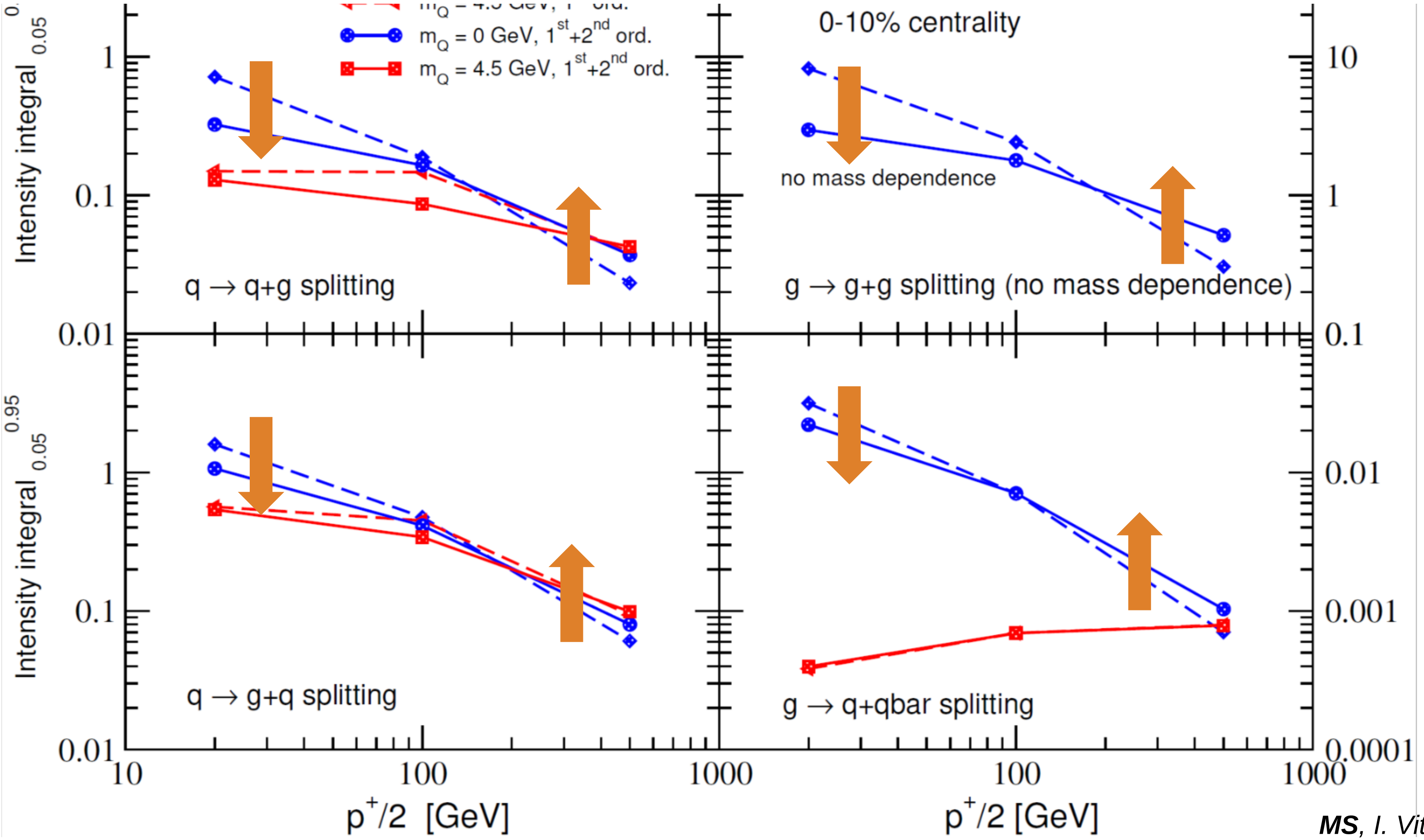
Similar flattening seen in all channels



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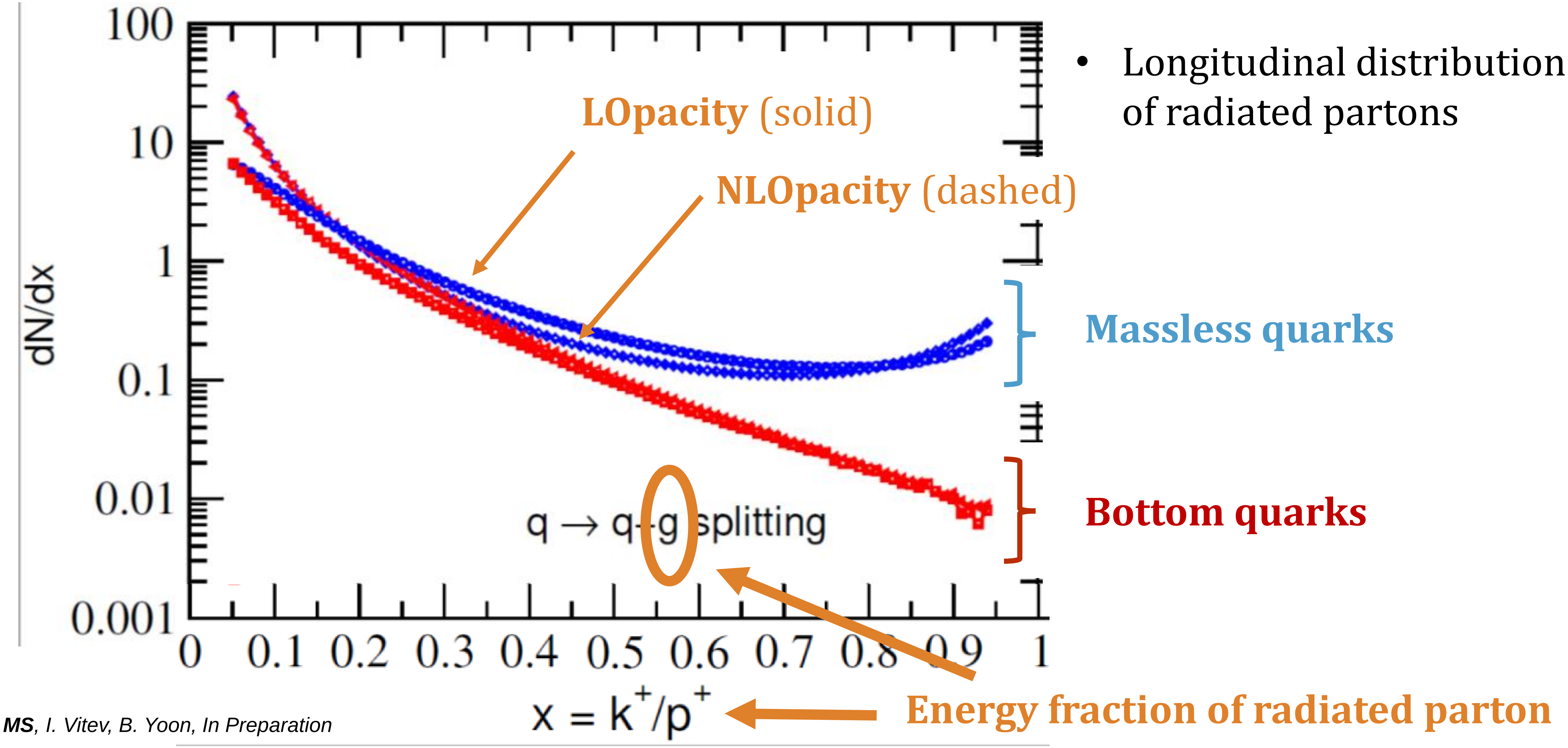

- Longitudinal distribution of radiated partons

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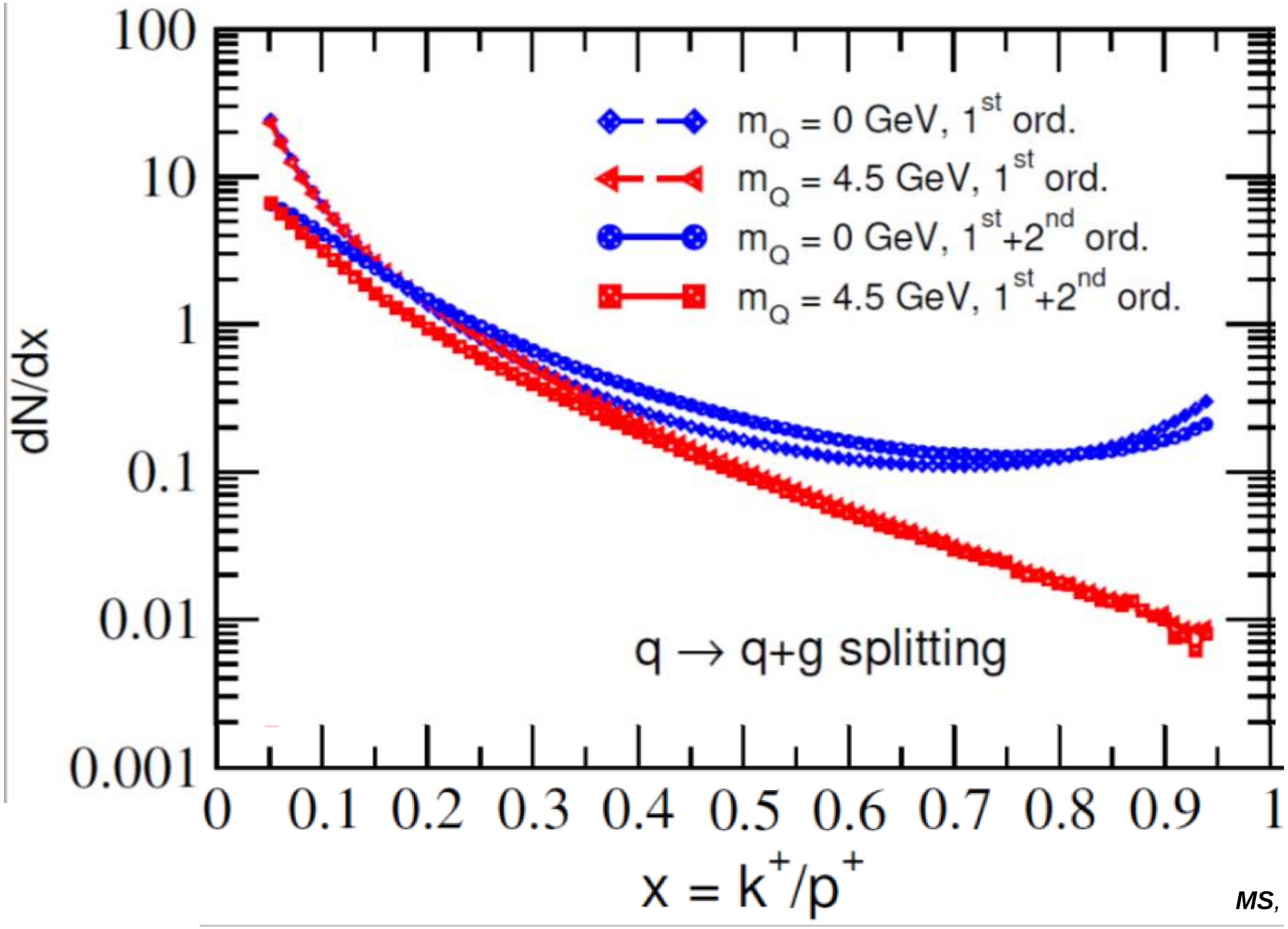
Preliminary Results: The Longitudinal Spectrum



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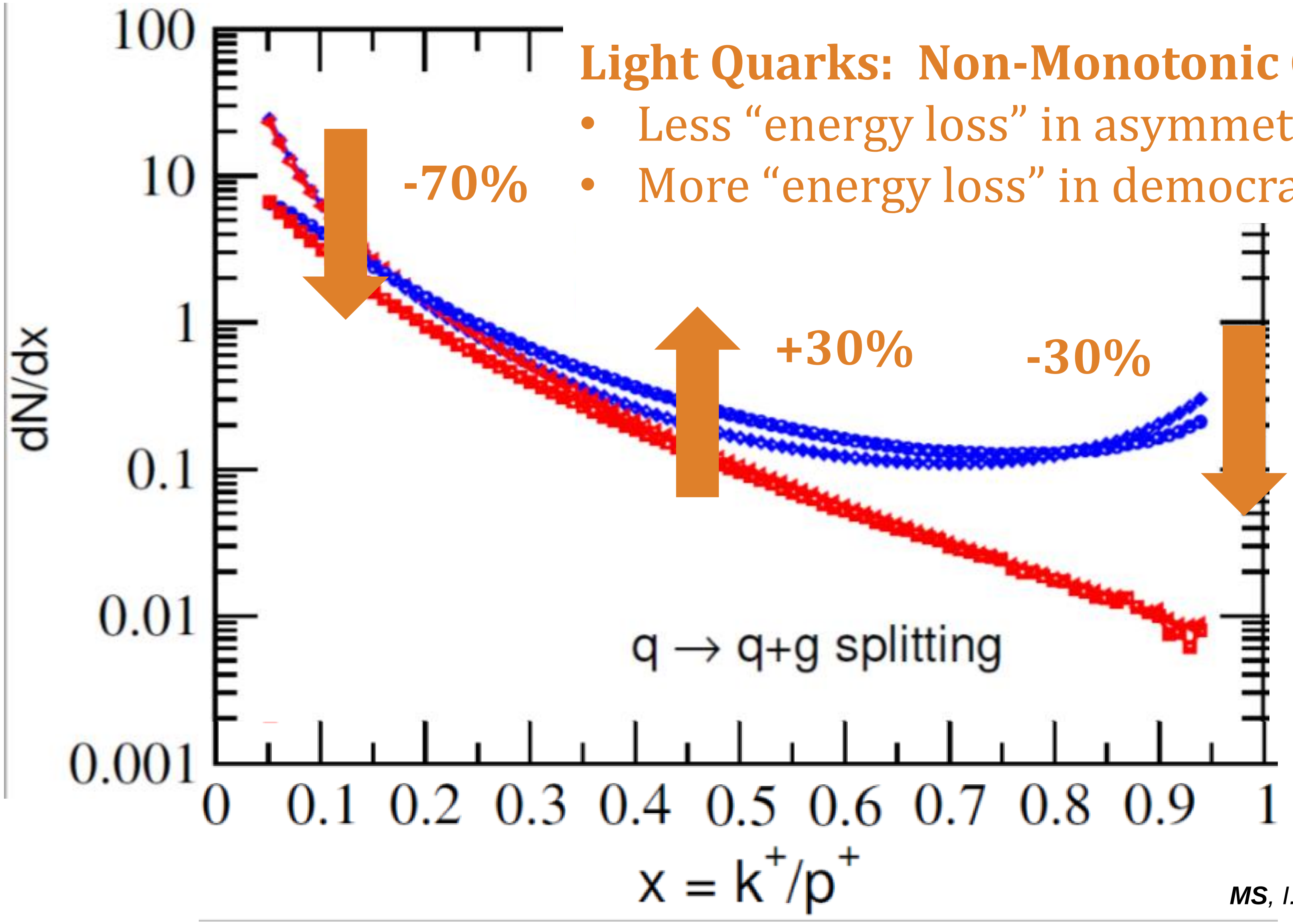
Light Quarks: Non-Monotonic Corrections
- Less "energy loss" in asymmetric splittings
- More "energy loss" in democratic splittings



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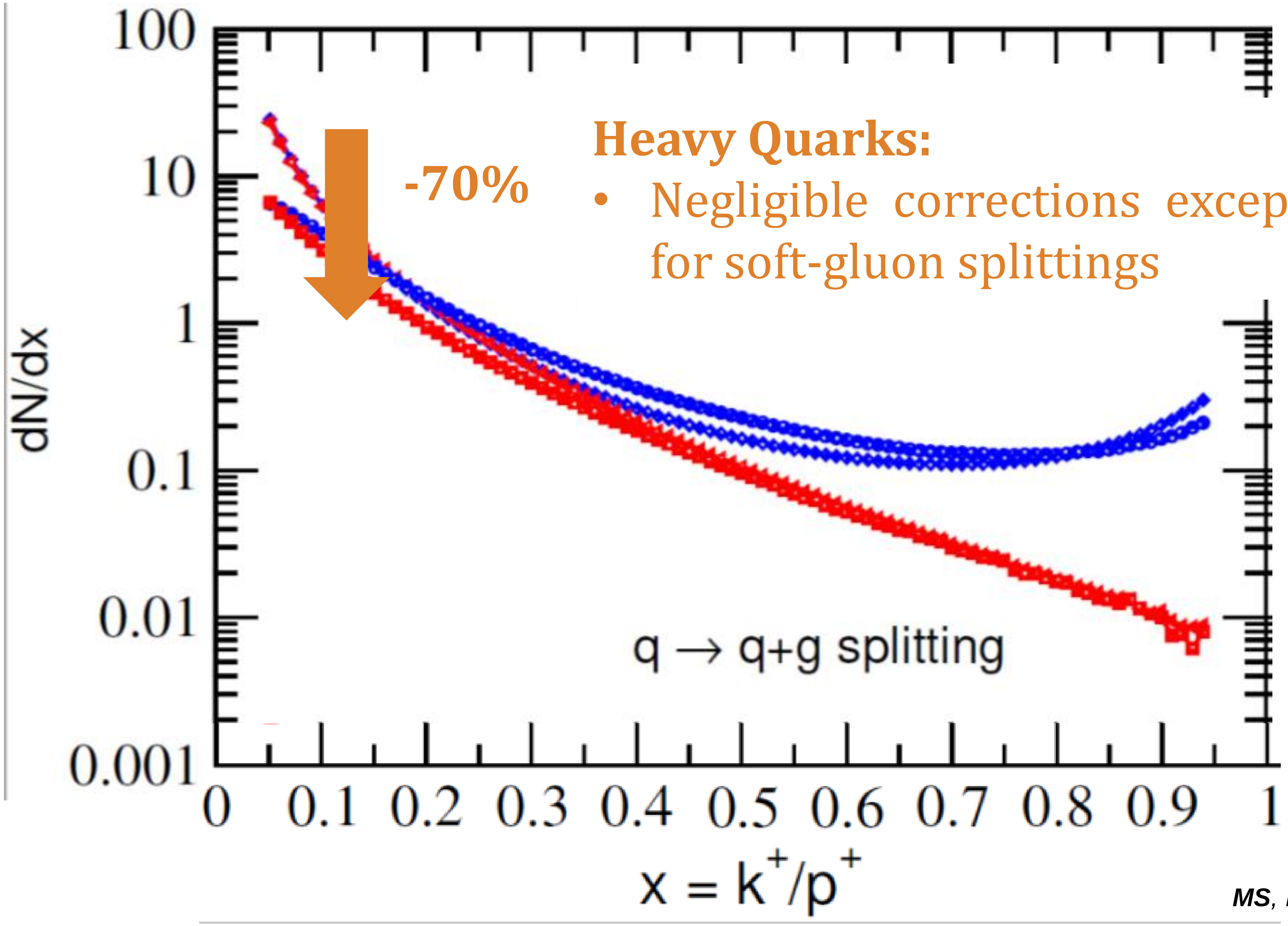

Heavy Quarks:

- Negligible corrections except for soft-gluon splittings

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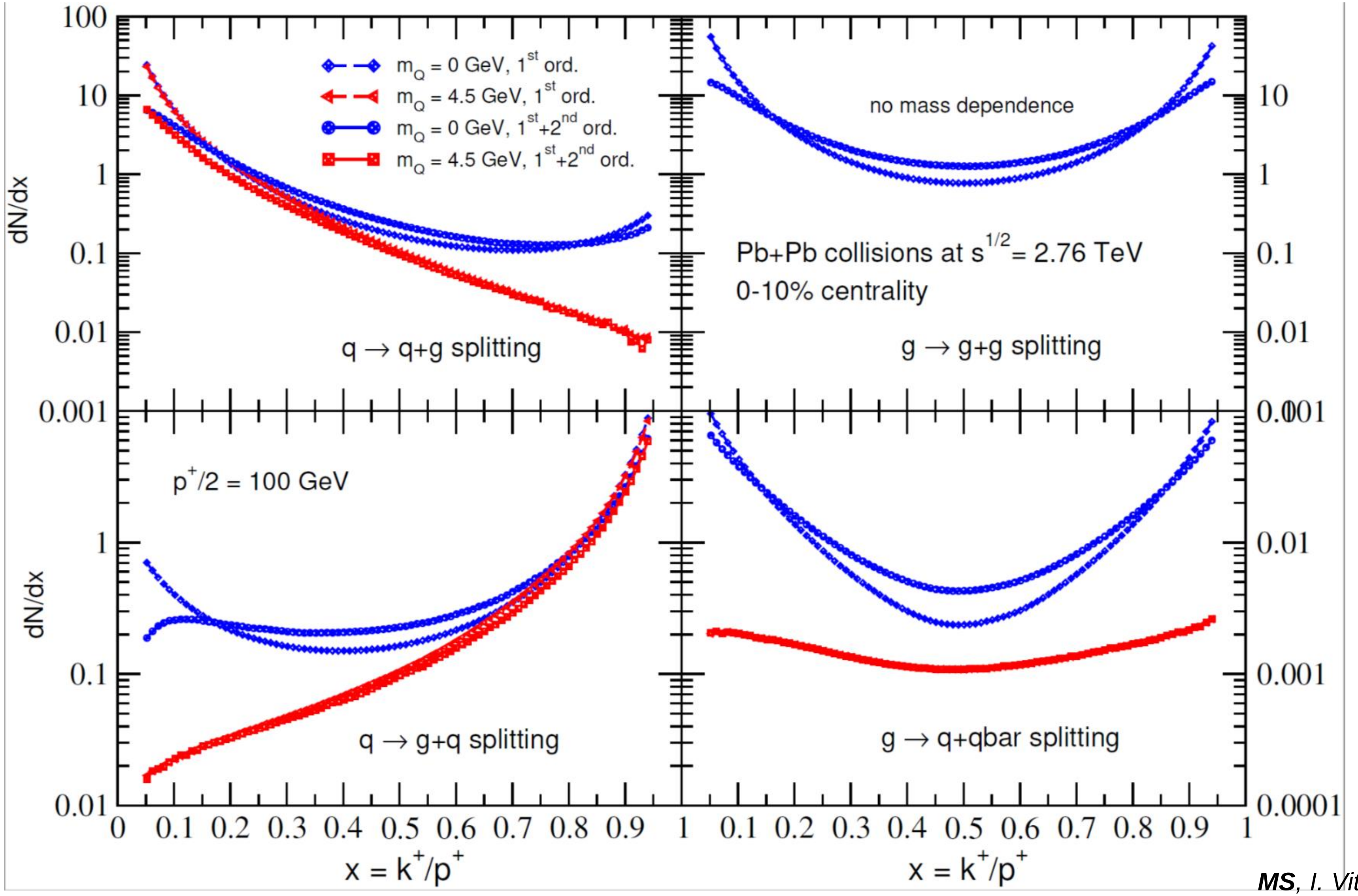
Preliminary Results: The Longitudinal Spectrum



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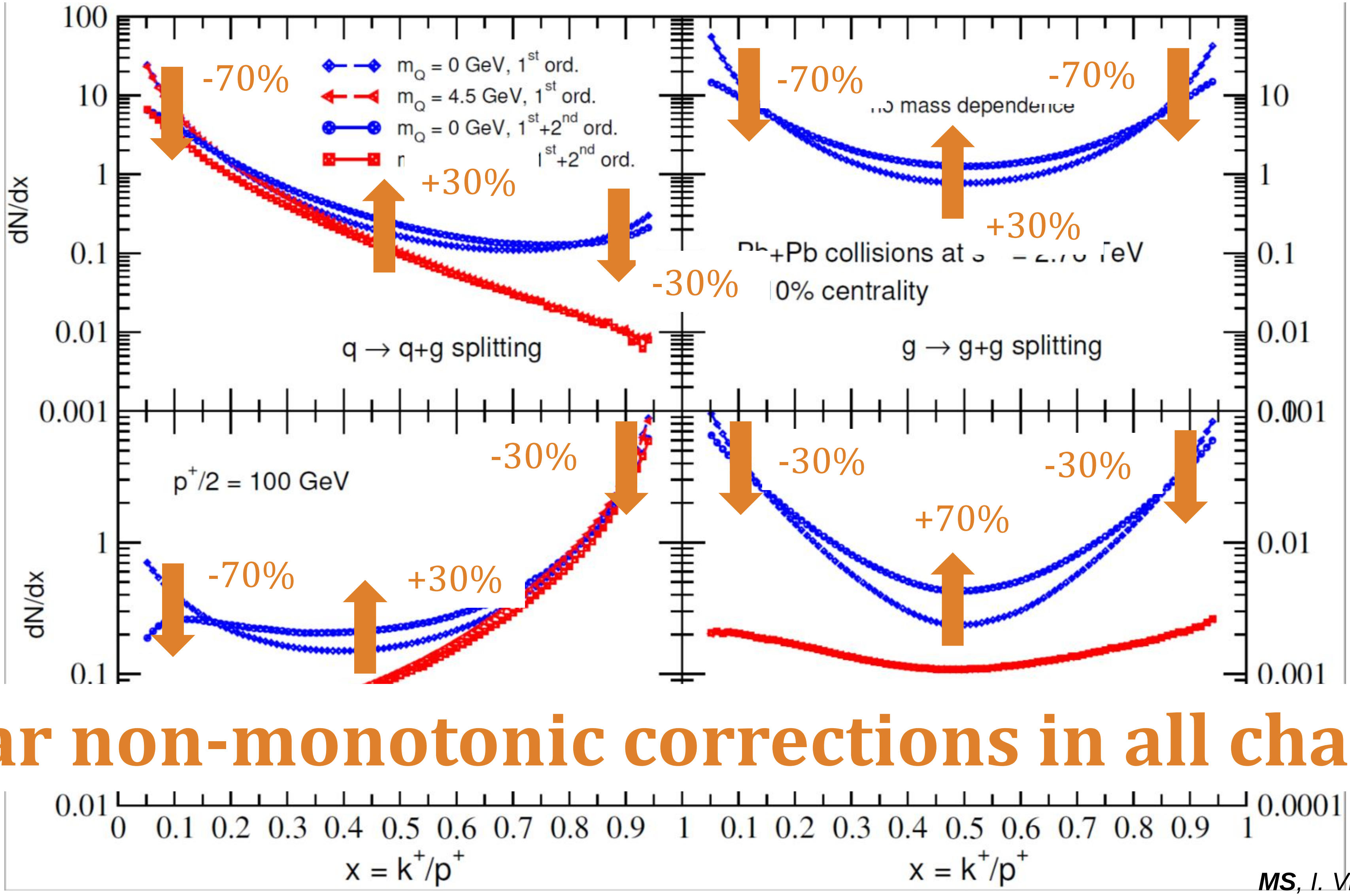

Similar non-monotonic corrections in all channels

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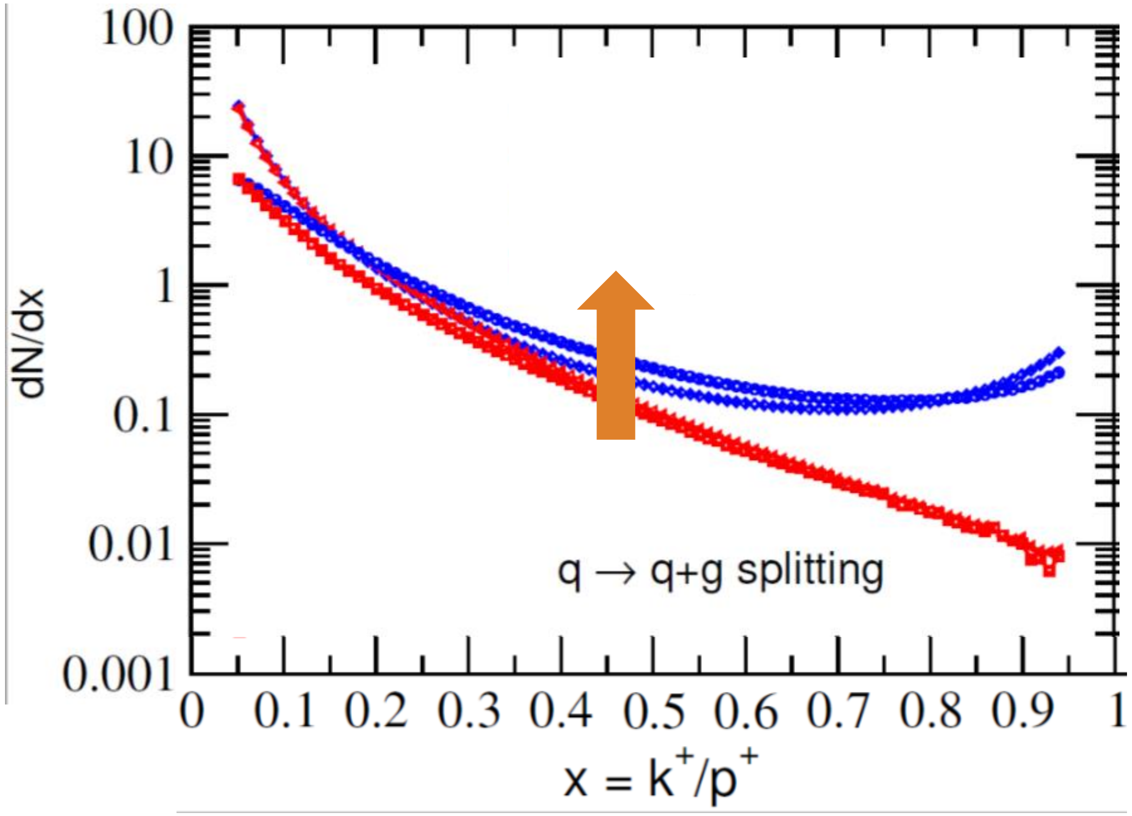
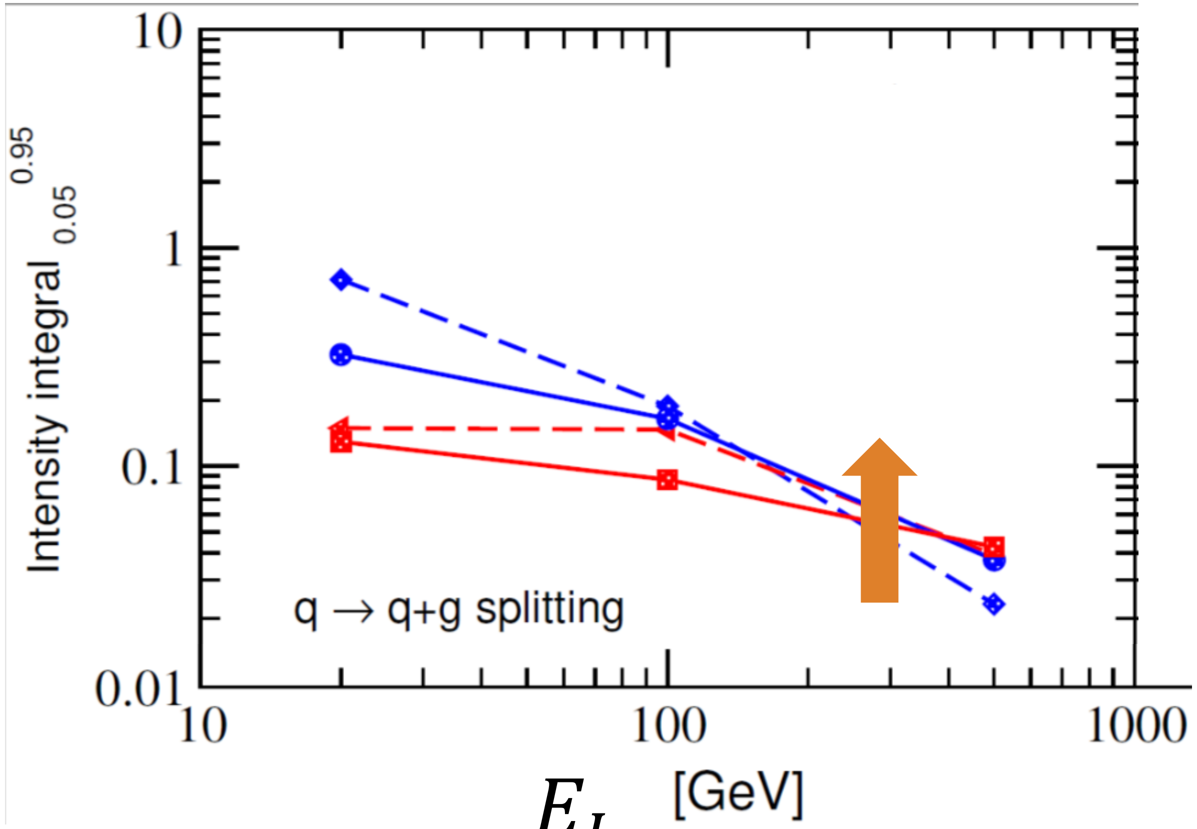
Rescattering of democratic branchings enhances “energy loss” at high energies





E_J

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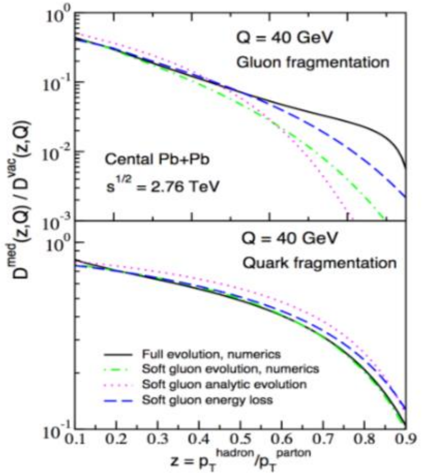
Preliminary Results: The Transverse Spectrum

Conclusions

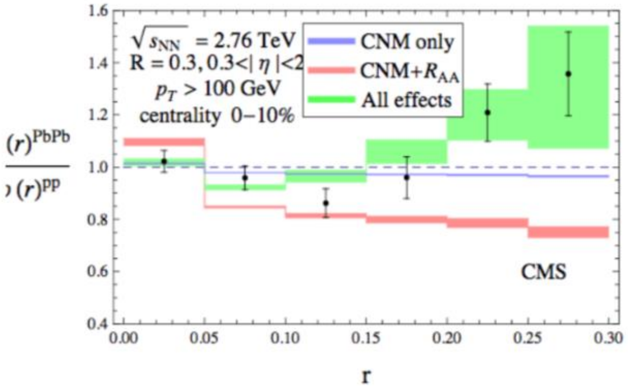
- We have computed the **medium-induced splitting functions** to **NLOpacity**
  - Evaluated using a **realistic 2+1 event-by-event hydro** simulation of the QGP
  - While interesting, the real meaning is as an **ingredient** to full jet calculations
  - Needs a **sensitivity analysis** to model parameters

Take the **phenomenology** of real jet (and hadron-level) observables to **NLOpacity**

Applications

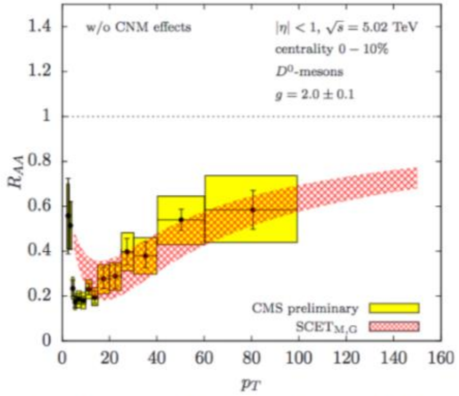


Modification of fragmentation functions for gluon and quark Kang et al 2014



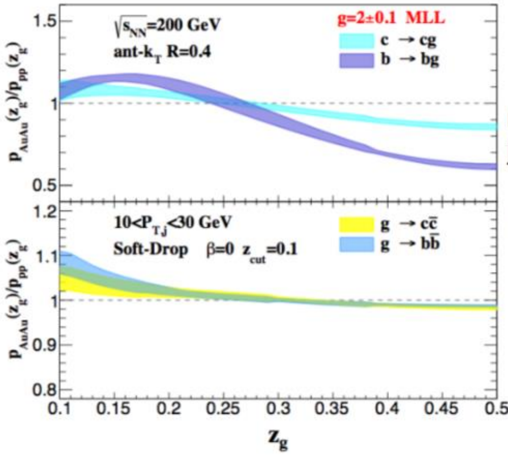
Modification of jet shape

Chien et al 2015



Nuclear modification factor RAA for D0 meson (massive)

Kang et al 2016



modifications of the jet splitting functions

HTL, Vitev 2018

Many other applications ……

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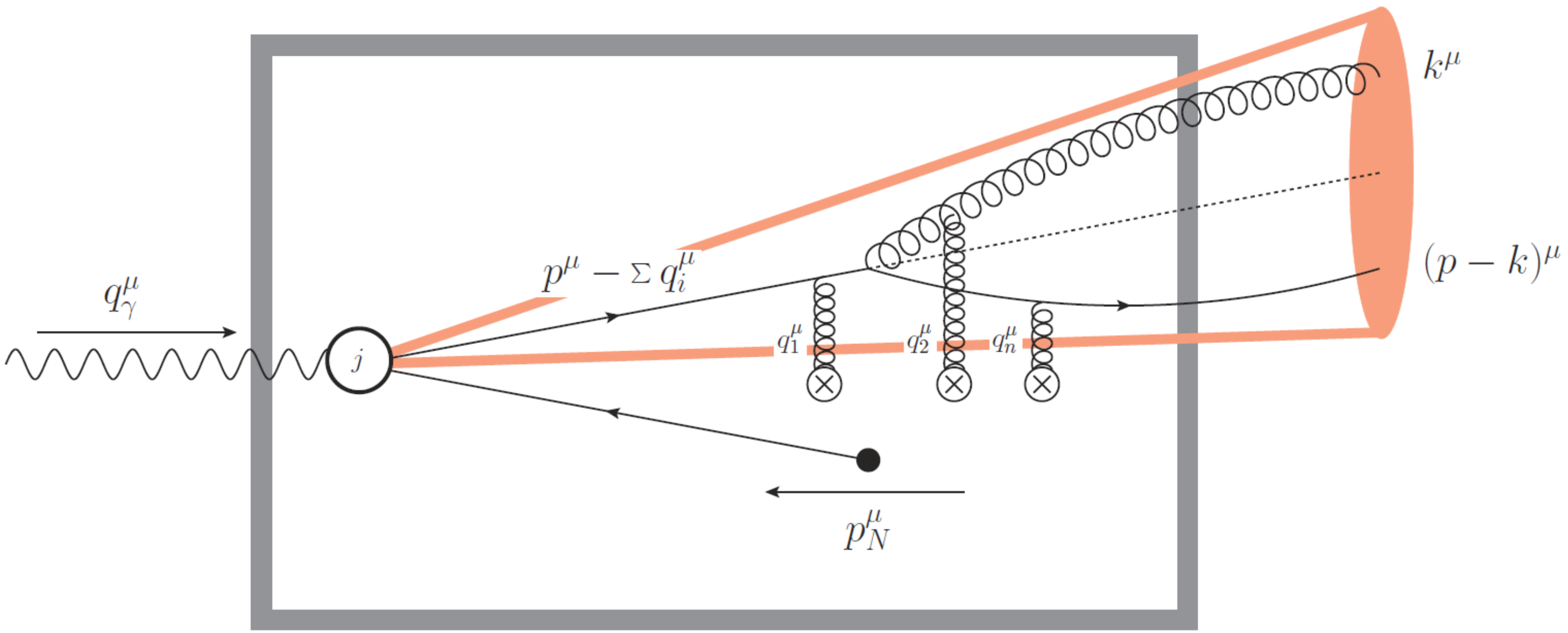
[Slide by Hai Tao Li]



- Direct contact with **factorization theorems** for DIS / SIDIS in inclusive jet production
- Detailed coupling to hard parton production through **PDFs and TMDs**
  - Impact at low vs high pT jets (Breit frame)

Outlook: Apples to Apples Theory Comparisons

- Different theoretical frameworks are best applied in **opposite regimes of opacity**

  - Opacity expansion $\chi \ll 1$
  - "Reality" $\chi \approx 2 - 3$
  - Harmonic approximation to BDMPS-Z $\chi \gg 1$

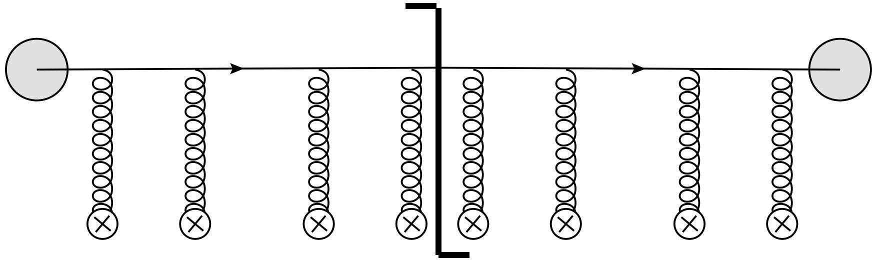
- By pushing the opacity expansion to higher orders, we can test and quantify the **convergence** and **discrepancies** between the approaches

  - Where does **opacity resummation become mandatory** for a given observable?
  - How good is the **quality of the harmonic approximation** for a given observable?

Backup Slides

An Example: Jet Broadening

- **Jet broadening** is a simple test case for which the **exact solution** is known:



➢ The **recursion relation** is particularly simple in **coordinate space**:

$$\frac{dN_n}{d^2p\,dy} = \int d^2r\, e^{-i\vec{p}_\perp\cdot\vec{r}_\perp}\,\frac{dN_n}{d^2r\,dy}$$

$$\frac{dN_n}{d^2r\,dy} = \frac{\chi}{n}\left(\frac{(2\pi)^2}{\sigma_{el}}\sigma(\underline{r}) - 1\right)\frac{dN_{n-1}}{d^2r\,dy}$$

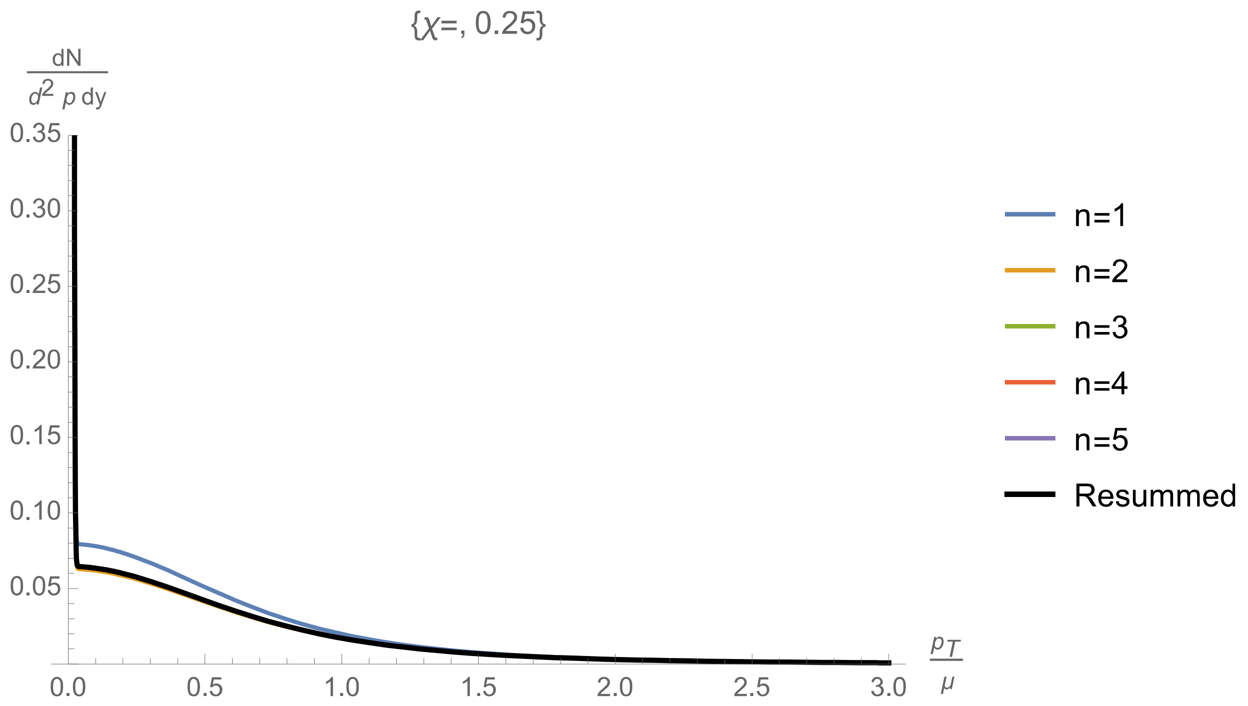
➢ The opacity series can be **resummed exactly**:

$$\frac{dN}{d^2p\,dy} = \int d^2r\,\frac{d^2p'}{(2\pi)^2}\,e^{-i(\vec{p}_\perp-\vec{p}\,'_\perp)\cdot\vec{r}_\perp}\,\exp\left[\chi\left(\frac{(2\pi)^2}{\sigma_{el}}\sigma(\vec{r}_\perp) - 1\right)\right]\frac{dN_0}{d^2p'\,dy}$$

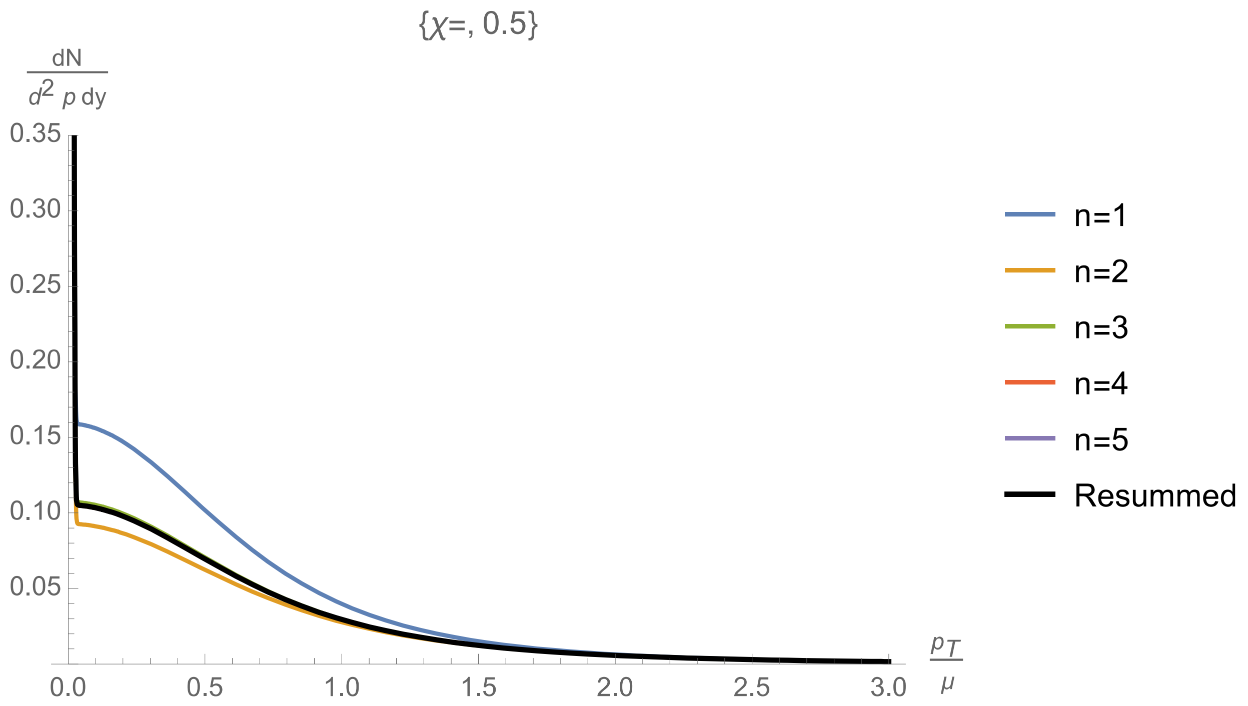
➢ The kernel for a **massive gluon propagator**:

$$\frac{(2\pi)^2}{\sigma_{el}}\sigma(\vec{r}_\perp) = \mu r_T\,K_1(\mu r_T)$$

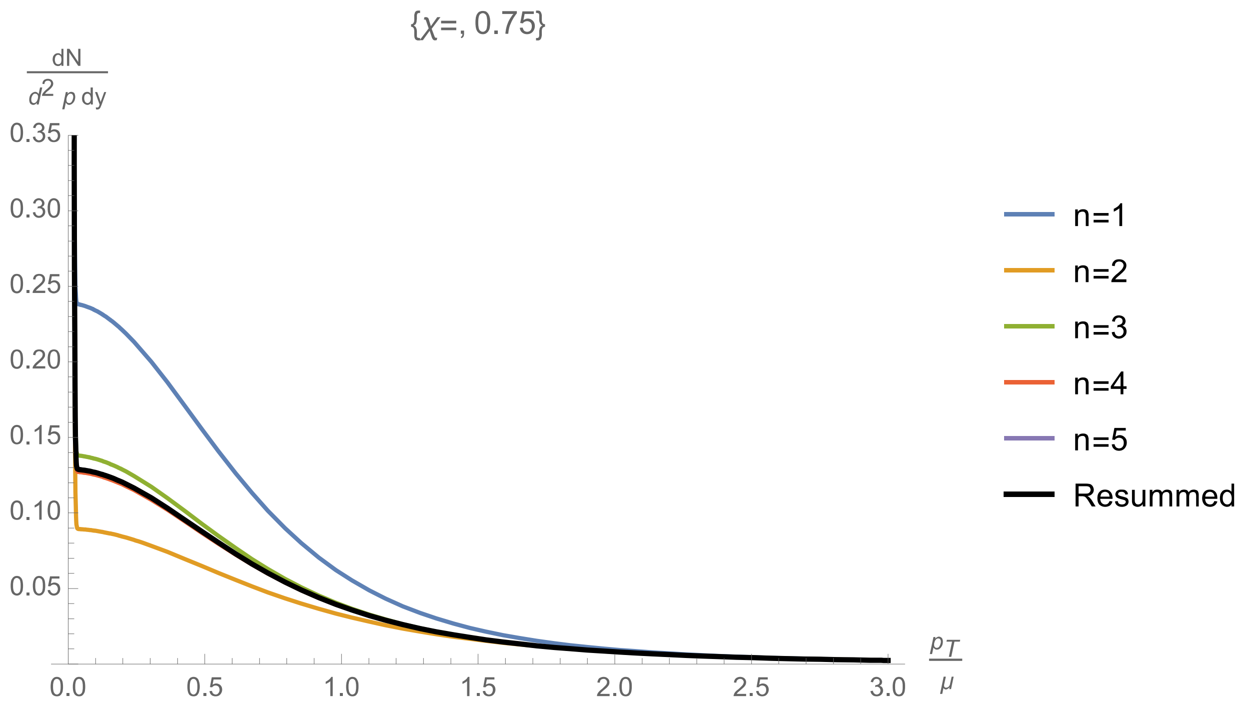
Convergence of the Opacity Expansion



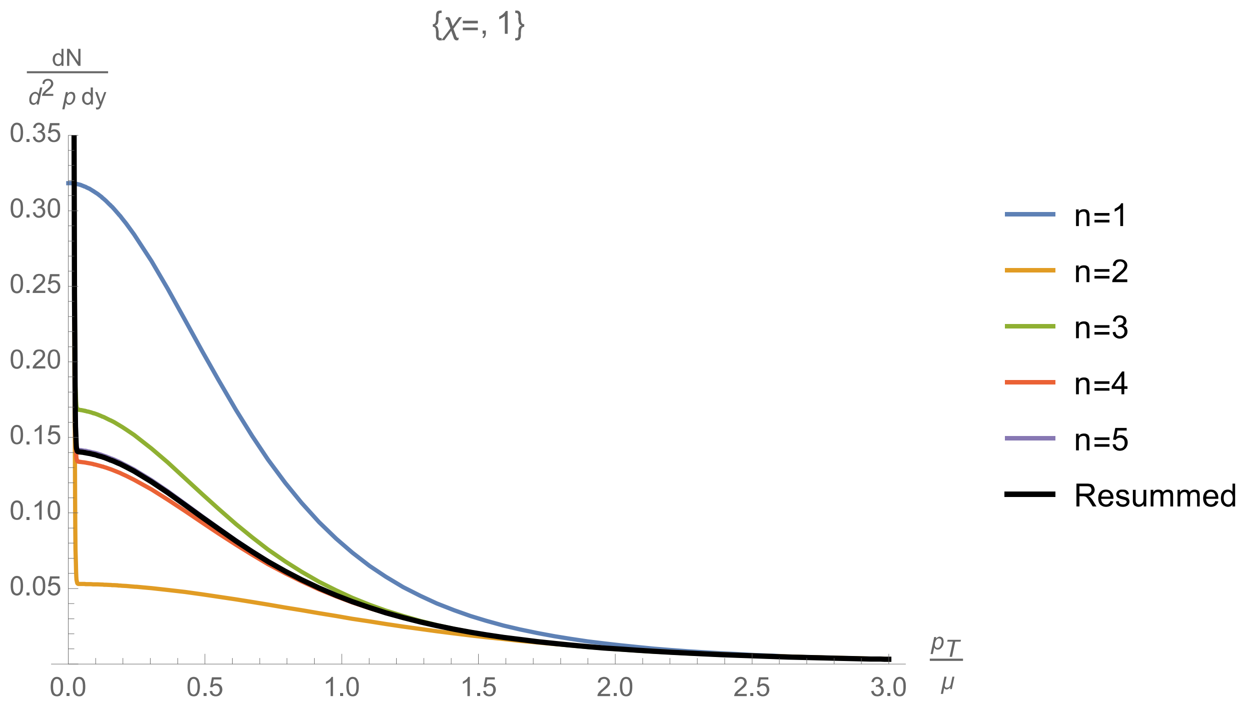
Convergence of the Opacity Expansion



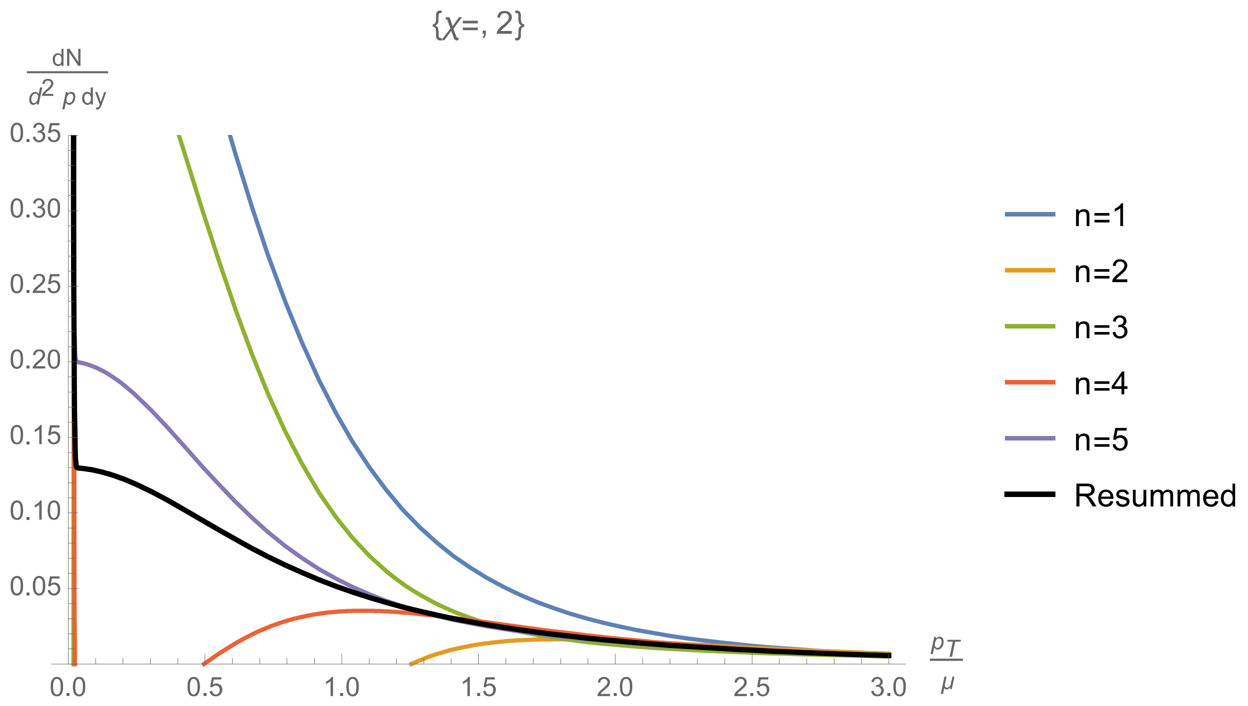
Convergence of the Opacity Expansion



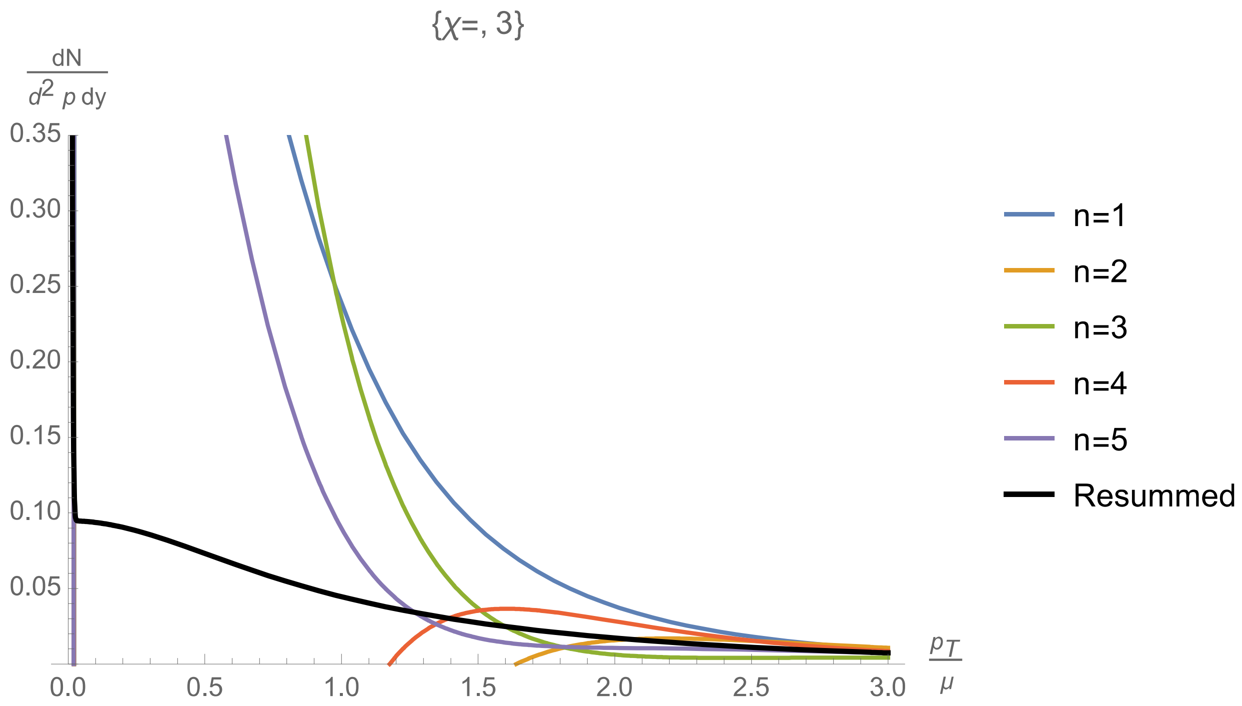
Convergence of the Opacity Expansion



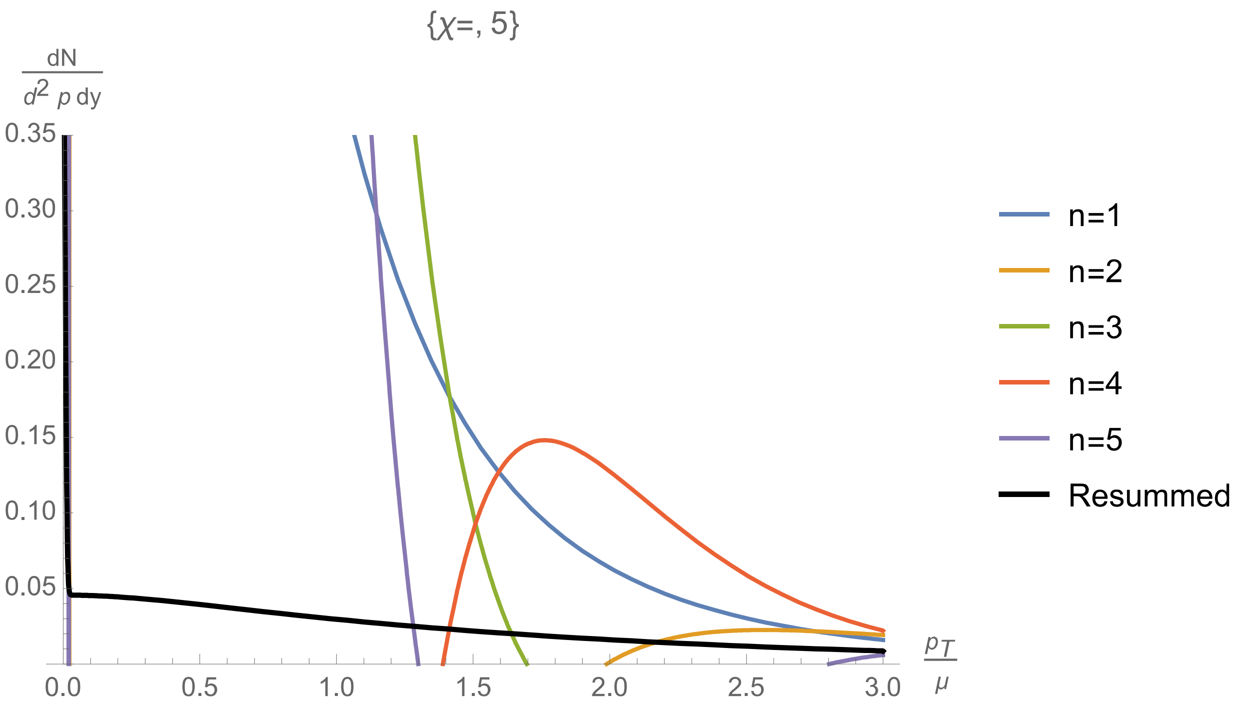
Convergence of the Opacity Expansion



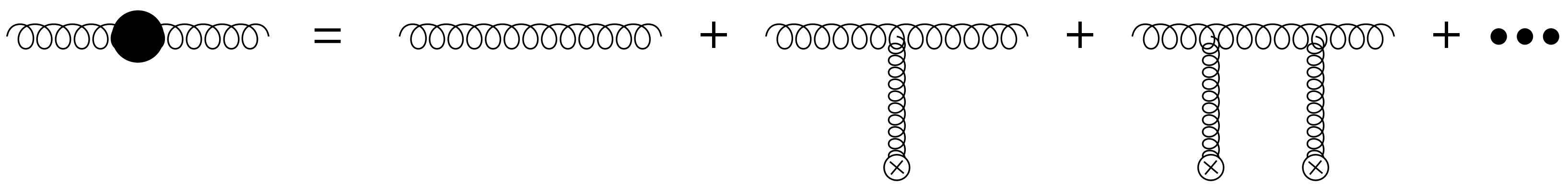
Convergence of the Opacity Expansion



Convergence of the Opacity Expansion



Alternatively: A Schrodinger Equation



- The integral recursion relation can instead be recast as a **differential equation**
  - **Klein-Gordon** equation with **minimal coupling**:

$$\left[2\partial_x^+ \mathcal{D}_x^- - \nabla_{x\perp}^2\right] G(x,y) = \delta^4(x-y)$$

  - Equivalent to a **time-dependent Schrodinger equation:**

$$\left[i\mathcal{D}_x^- + \frac{\nabla_{x\perp}^2}{2p^+}\right] G(x,y;p^+) = i\delta^2(\vec{x}_\perp - \vec{y}_\perp)\delta(x^+ - y^+)$$

  - Basis of the **Baier-Dokshitzer-Mueller-Peigné-Schiff** approach

Baier et al, Nucl. Phys. B531 (1998)

Alternatively: A Path Integral

- The **nonrelativistic Schrodinger equation** reflects the **Galilean symmetry** of the light-front Hamiltonian

$$\mathcal{H}^- = -\frac{\nabla^2_{x\perp}}{2p^+} - gA^-_{ext}(x)$$

Blaizot et al. JHEP 01 (2013)

  - ➢ And the solution in non-relativistic QM is a **path integral**:

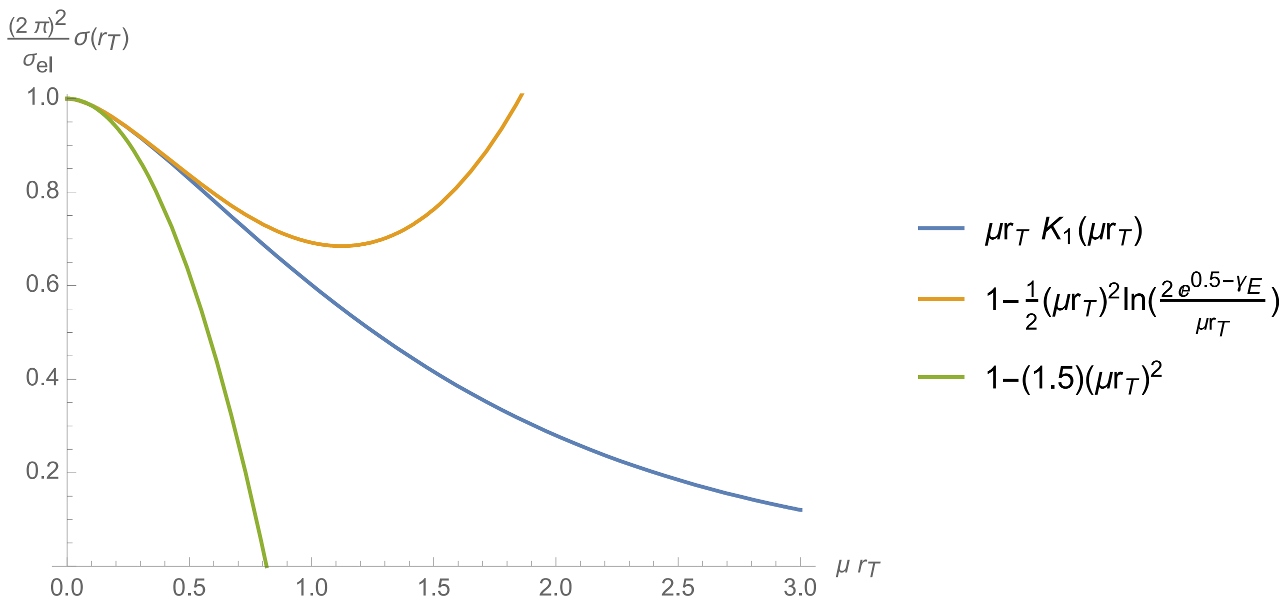
$$G(t_f, t_i) = \int [\mathcal{D}r(t)] \exp\left[i\frac{m}{2} \int_{t_i}^{t_f} dt\, [\dot{r}(t)]^2 \right] \exp\left[-i \int_{t_i}^{t_f} dt\, V[r(t)] \right]$$

$$G(x, y; p^+) = \int [\mathcal{D}r_\perp(z^+)] \exp\left[i\frac{p^+}{2} \int_x^y dz^+ \left(\frac{d\vec{r}_\perp}{dz^+} \right)_T^2 \right] \mathcal{P}\exp\left[ig \int_x^y dz^+\, A^-[r(z^+)] \right]$$

  - ➢ Basis of the **Zakharov** approach

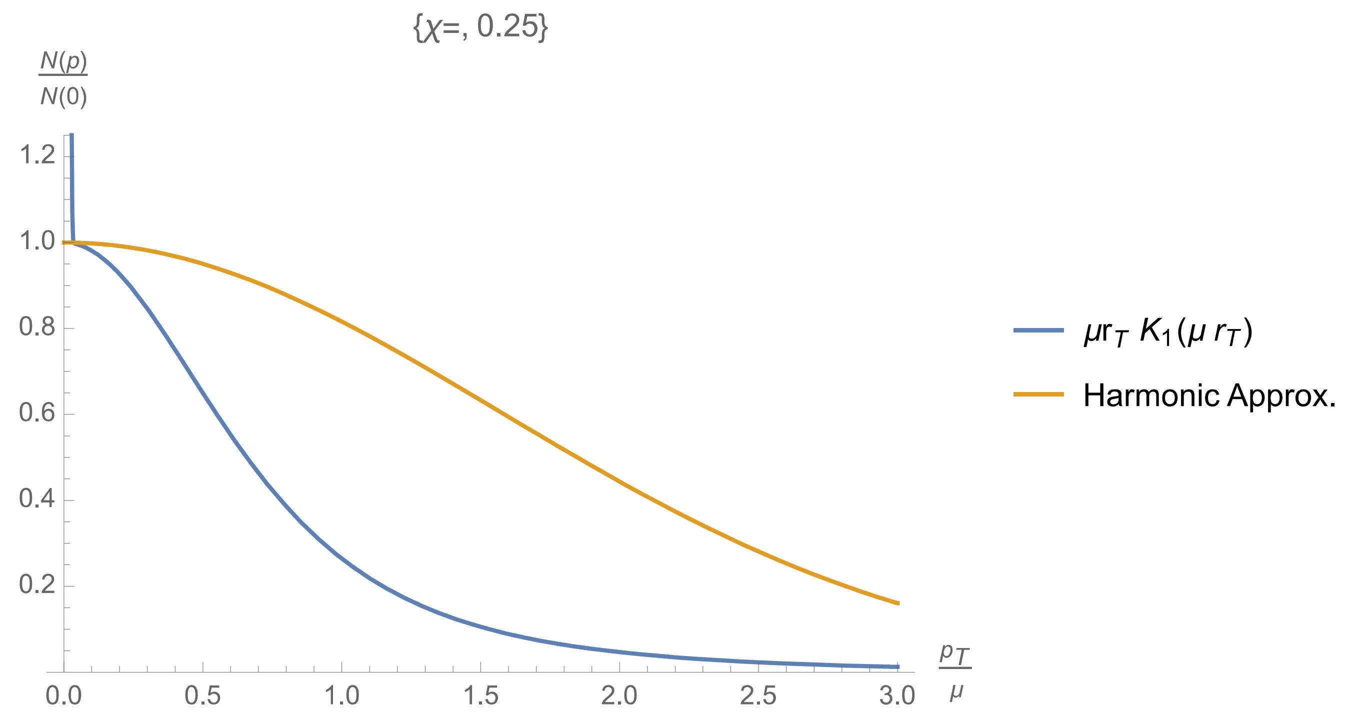
Zakharov, JETP Letters 63 (1996)

How Good is the Harmonic Approximation?

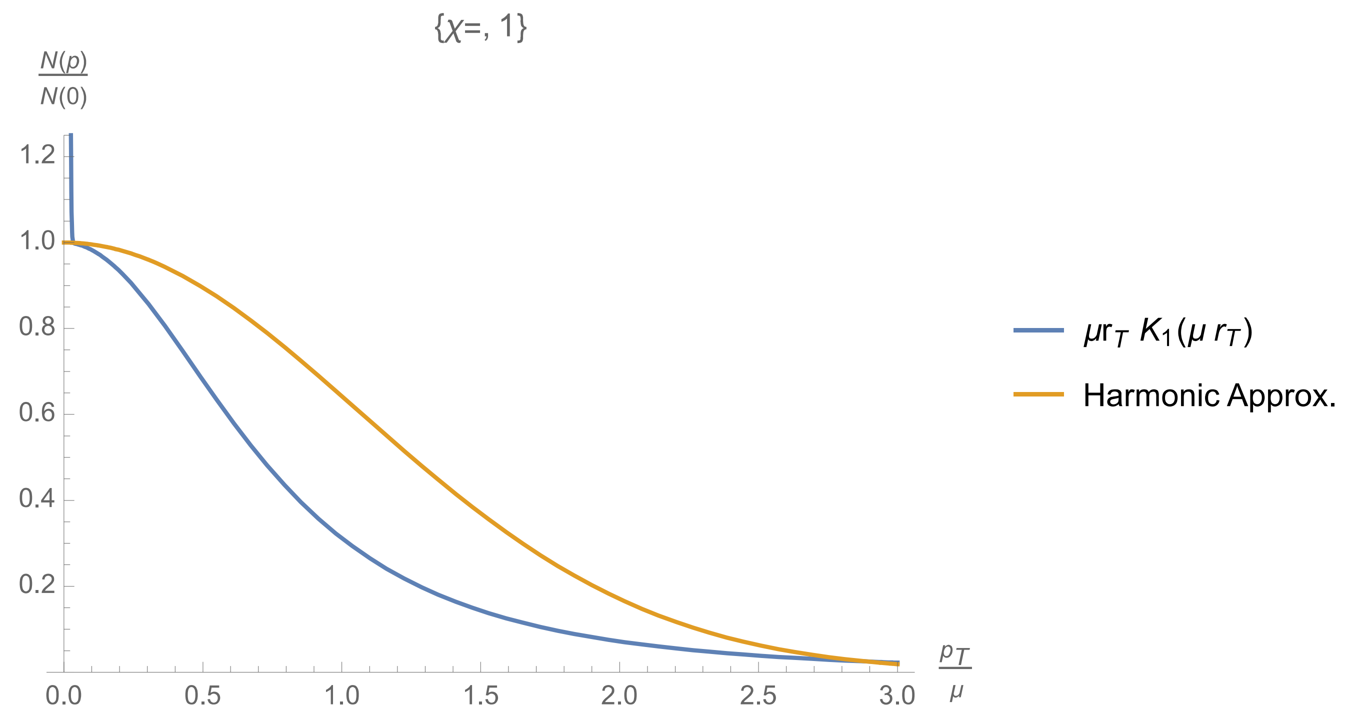


- Loosely corresponds to a **small-argument expansion** of the coordinate-space cross section **$\mu\, r_T \ll 1$**
  - In momentum space: $p_T / \mu \gg 1$
  - But **multiple scattering selects for small sizes**…
  - **Better at high opacities** with more screening.

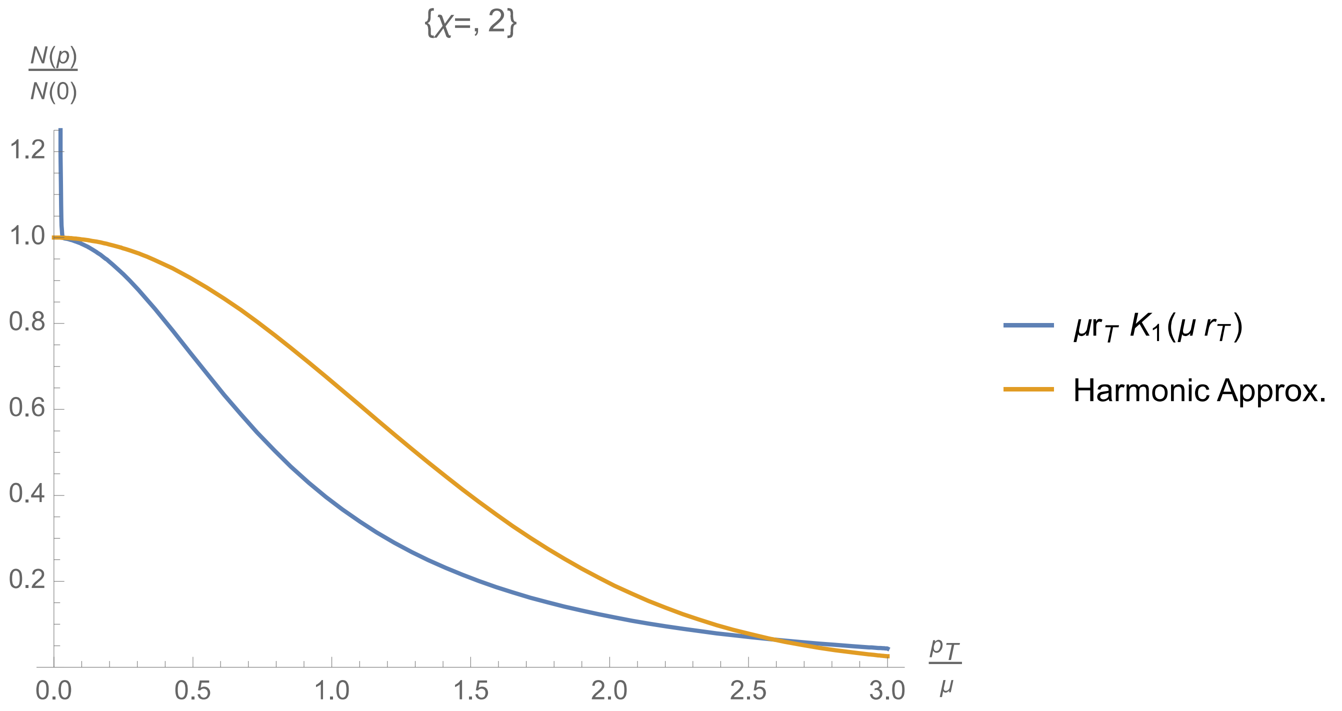
The Harmonic Approximation: Jet Broadening



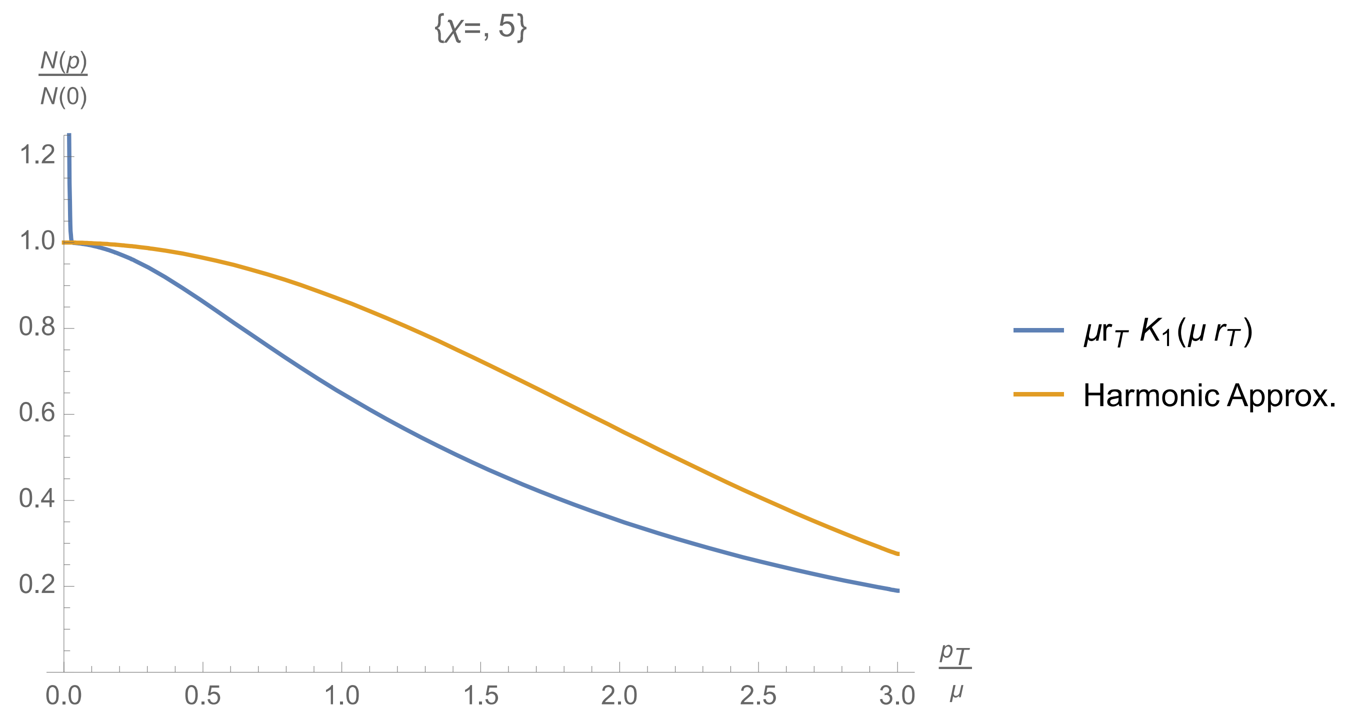
The Harmonic Approximation: Jet Broadening



The Harmonic Approximation: Jet Broadening



The Harmonic Approximation: Jet Broadening



The Harmonic Approximation: Jet Broadening