BROOKHAVEN NATIONAL LABORATORY

The LPM effect in QCD revisited

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(in preparation)

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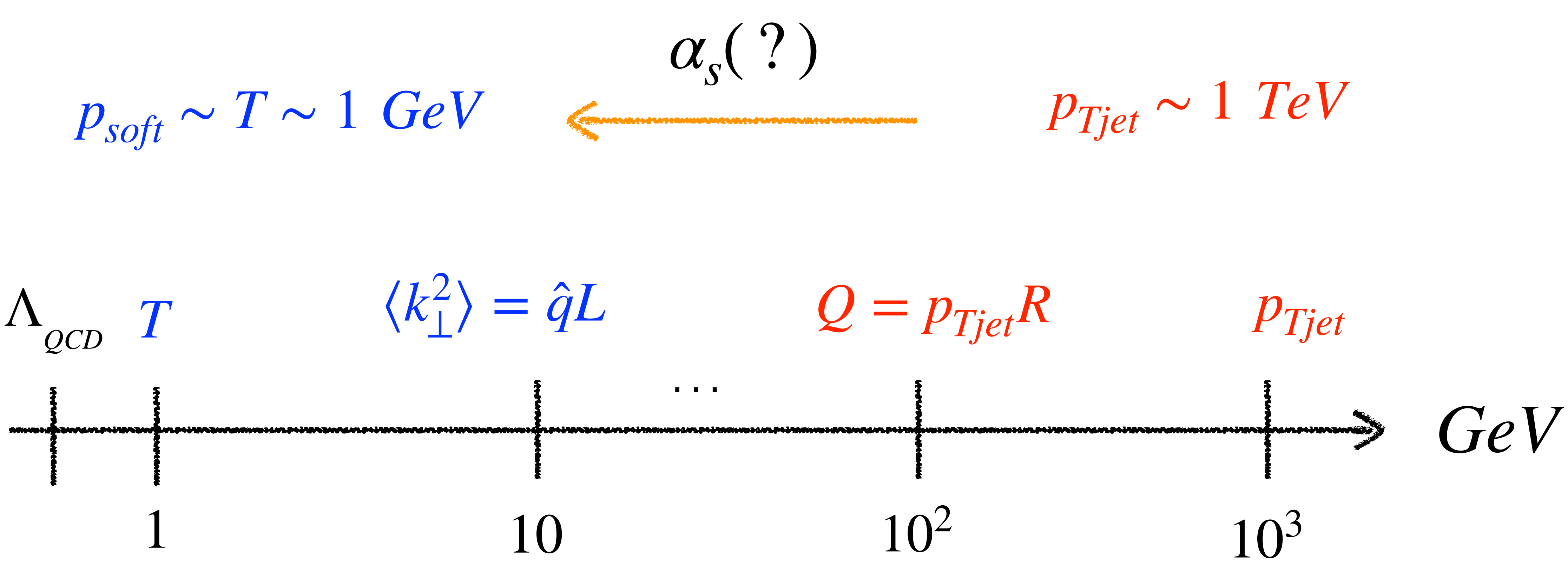
Motivation

- **Physics question:** How is the jet coupled to the quark gluon plasma? Is perturbation theory applicable?



Motivation

- Resummation of multiple emissions + multiple scattering

See talks by E. Iancu and K. Tywoniuk

- Elementary process: **medium-induced radiation**

- Two main analytic approximations in the literature (implemented in various MC event generators)

1. **Dilute medium:** single-hard scattering (Opacity expansion, Higher-Twist)

   Gyulassy-Levai-Vitev (2000) Guo, Wang (2000)

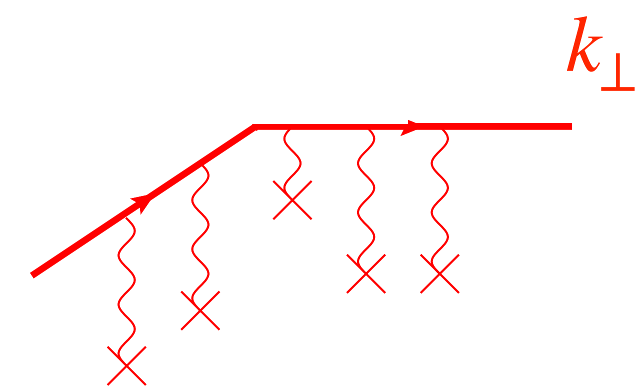
2. **Dense medium:** multiple-soft scattering

   Baier, Dokshitzer, Mueller, Peigné, Schiff (1996) Zakharov (1997)

Momentum broadening and $\hat{q}$

- Jet constituent traversing the plasma may suffer frequent soft elastic collisions. To leading order the diffusion coefficient that characterized the jet-plasma coupling reads

$$\hat{q} \equiv \frac{\mathrm{d}\langle k_T^2 \rangle_{typ}}{\mathrm{d}t} \sim \alpha_s^2 \, C_R \, n \, \ln \frac{Q^2}{m_D^2} \sim \alpha_s^2 T^3$$



soft multiple interactions

- At weak coupling $\alpha_s \ll 1$ (kinetic description):

$$1/m_D \ll \ell_{mfp} \ll L$$

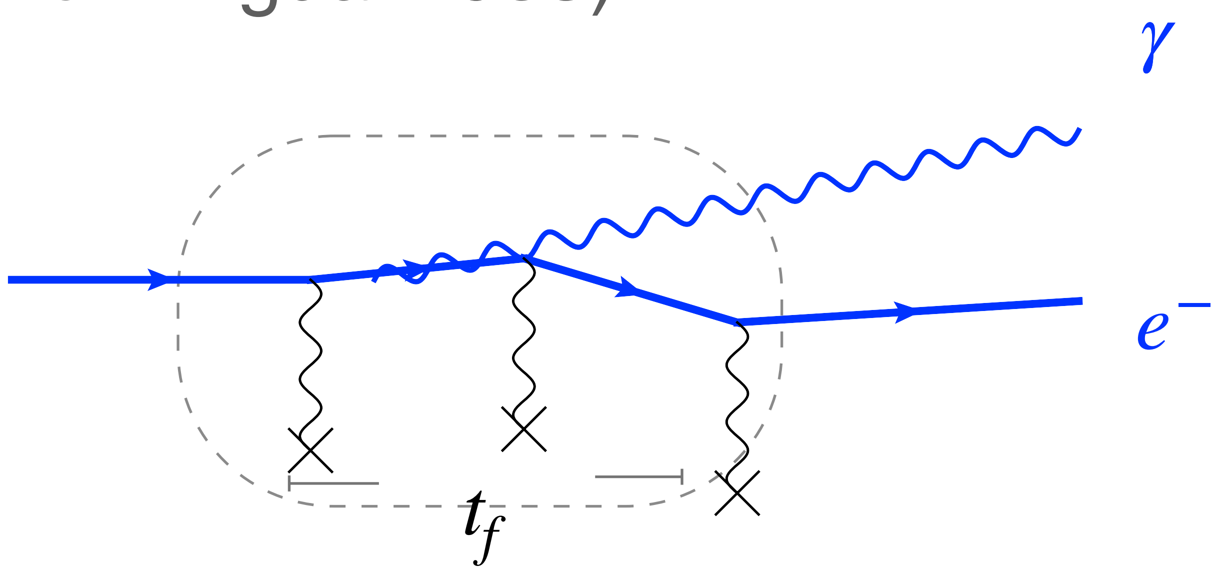
- Note that $Q^2 \sim \hat{q} L \gg \Lambda_{QCD}^2$ which implies that the jet quenching parameter stays under perturbative control for large medium length L

Outline

- Laudau-Pomeranchuk-Migdal (LPM) effect in QCD
- Medium-Induced gluon spectrum: multiple soft scattering and single hard (N=1 opacity expansion) approximations
- Medium-induced radiation beyond multiple-soft scattering approximation
- Numerical results

The LPM effect on the back of the envelop

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects (Landau-Pomeranchuk Migdal 1953)

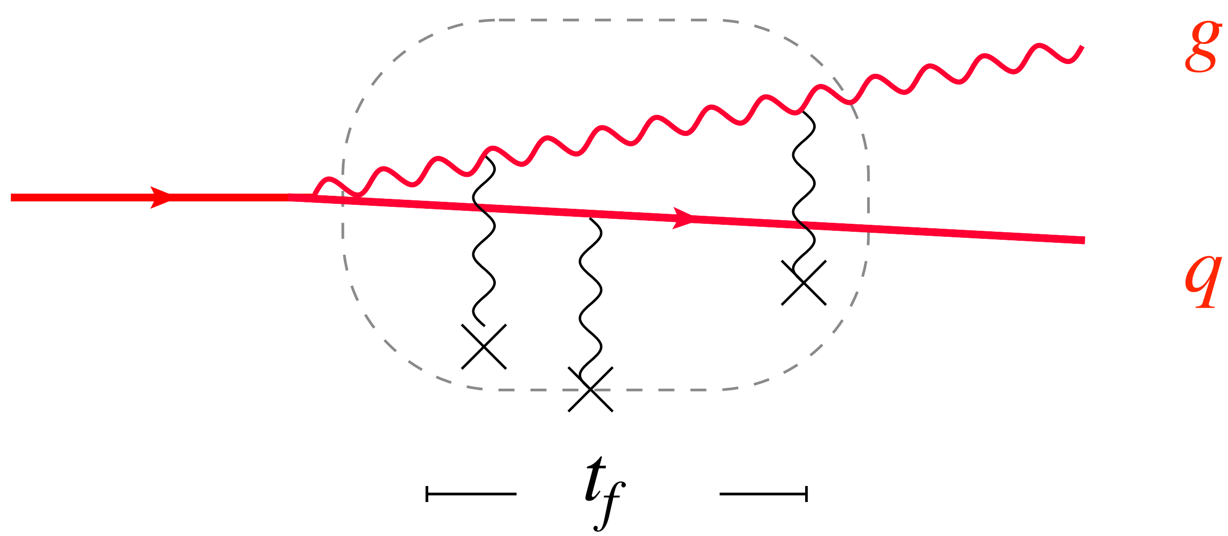


- Coherence length: during the quantum mechanical formation time N_{coh} scattering centers act coherently reducing the radiation spectrum

$$\omega \frac{dI^{LPM}}{d\omega} \sim \alpha_e N_{eff} \sim \alpha_e \frac{N_{scatt}}{N_{coh}} \sim \alpha_e \frac{L}{t_f(\omega)}$$

The LPM effect on the back of the envelop

- Same effect in QCD except the gluon interacts with the plasma and suffers "brownian kicks"



$$t_f(\omega) = \frac{\omega}{k_\perp^2}$$

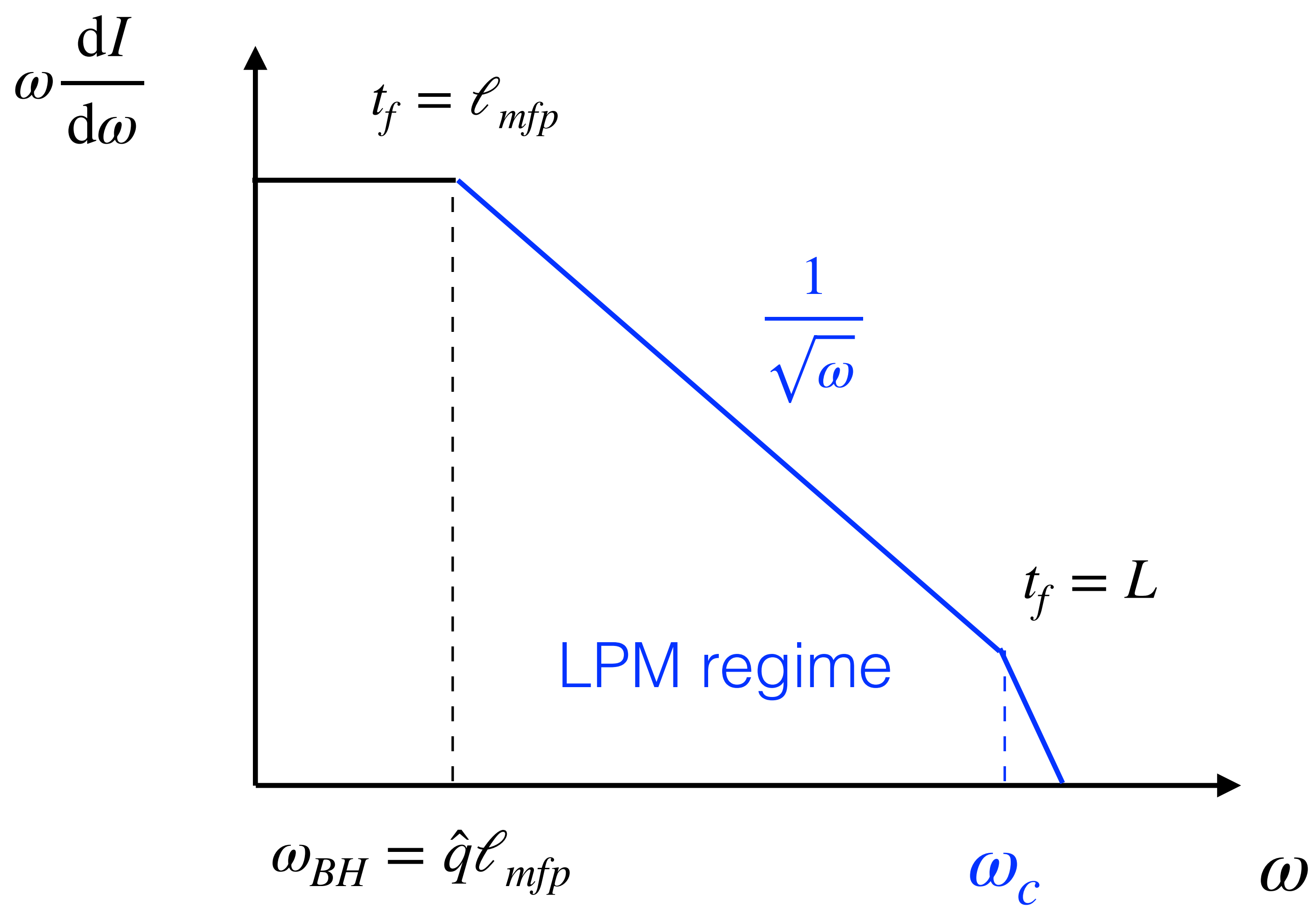
$$k_\perp^2 \sim \hat{q}\, t_f$$

- In QCD the spectrum is suppressed in the UV

$$t_f(\omega) = \sqrt{\frac{\omega}{\hat{q}}}$$ and $$\omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_s \sqrt{\frac{\omega}{\hat{q}}}\, L \propto \frac{1}{\sqrt{\omega}}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

The LPM effect on the back of the envelop



- Maximum radiation frequency: $\omega_c = \hat{q}L^2$
- Minimum radiation angle (no mass singularity): $\theta_c = \frac{1}{\sqrt{\hat{q}L^3}}$

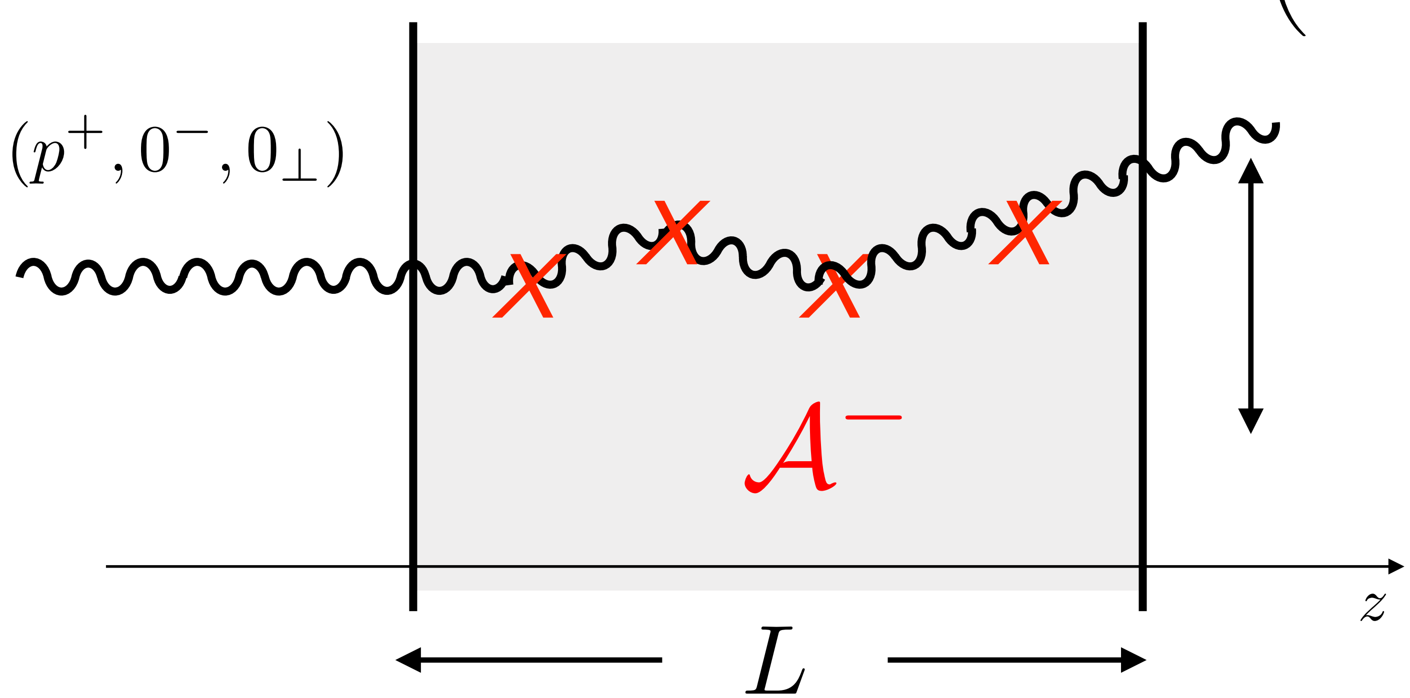
General formalism

- Working assumption: neglect power corrections of the small momentum transfer $q^+ \ll p^+$

$$\text{eikonal vertex} \sim \delta(q^+)\, p^\mu \;\Leftrightarrow\; \mathcal{A}^-(x^+, x_\perp)$$

- Large medium: allow the gluon to explore the transverse plan between two scatterings

$$p^- L^+ \sim (p_\perp^2/p^+)\, L^+ \sim 1 \qquad \left(p^+, \frac{q_\perp^2}{2p^+}, q_\perp\right)$$



Wilson line

$$\sim U[\mathcal{A}^-(x(t), t)]$$

A model for the medium

- Medium average: assume **Gaussian random variable**

$$\langle \mathcal{A}_a^-(q_\perp, t)\, \mathcal{A}_b^-('q_\perp, t') \rangle \equiv \delta^{ab}\, \delta(t - t')\, \delta(q_\perp - q'_\perp)\, \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

- Static scattering centers

Gyulassy-Wang (1992)
Gyulassy-Levai-Vitev (2000)

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp} \equiv \frac{g^4 n}{(q_\perp^2 + \mu^2)^2}$$

- Thermal medium (HTL)

Aurenche-Gelis-Zakaret (2000)

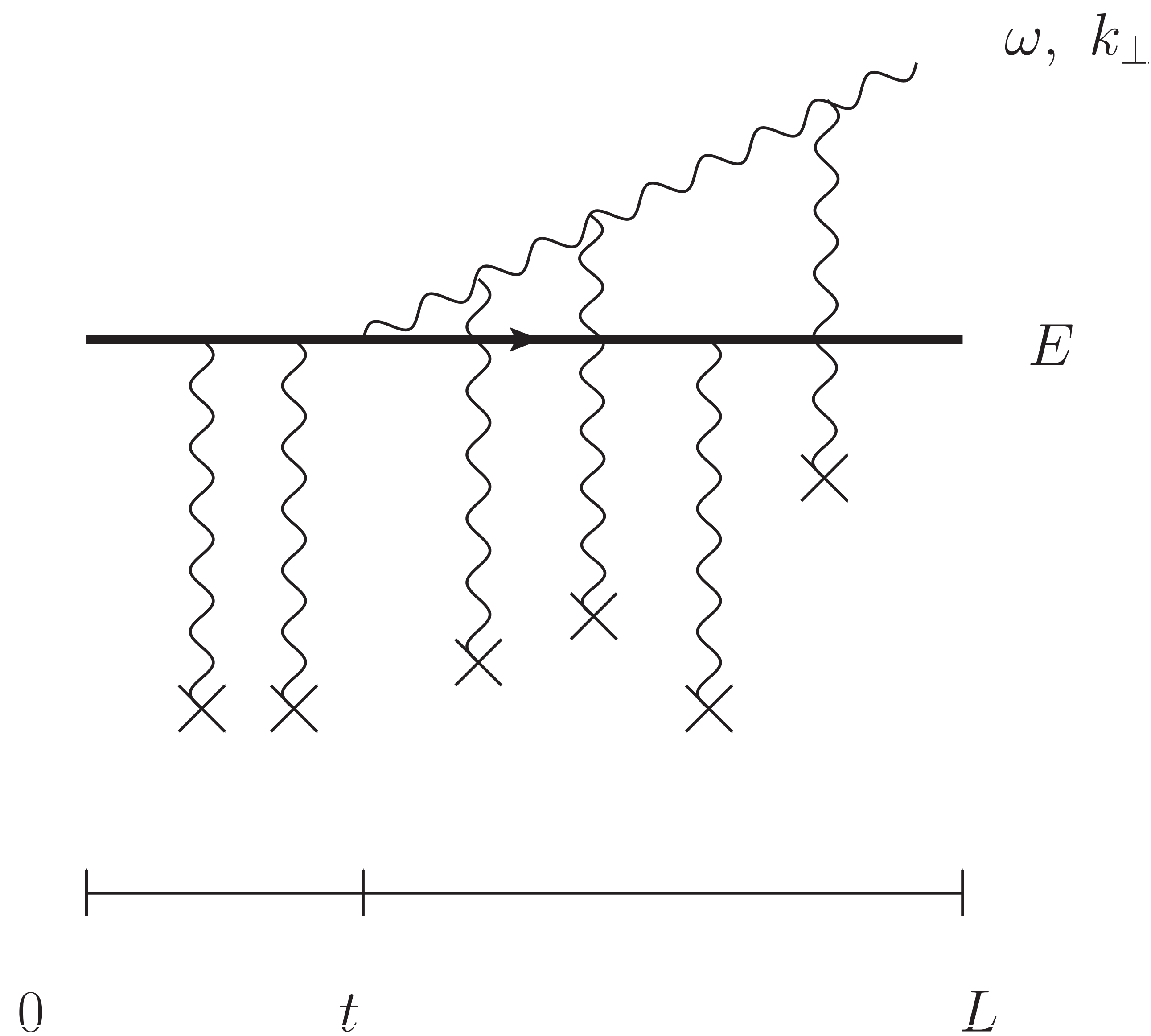
$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp} \equiv \frac{g^2 m_D^2 T}{q_\perp^2 (q_\perp^2 + \mu^2)}$$

- Large momentum transfer is given by 2 to 2 QCD matrix element:

$$1/q_\perp^4 \qquad \text{for} \qquad q_\perp \gg \mu$$

Medium-induced gluon spectrum

All orders in opacity:



High energy limit → 2-D non-relativistic quantum mechanics

Medium-induced gluon spectrum

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_R}{\omega^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1$$
$$\times\, \boldsymbol{\partial}_x \cdot \boldsymbol{\partial}_y \left[\mathcal{K}(\boldsymbol{x}, t_2 | \boldsymbol{y}, t_1) - \mathcal{K}_0(\boldsymbol{x}, t_2 | \boldsymbol{y}, t_1) \right]_{\boldsymbol{x}=\boldsymbol{y}=\boldsymbol{0}}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

- The Green's function K obeys a Schrödinger equation

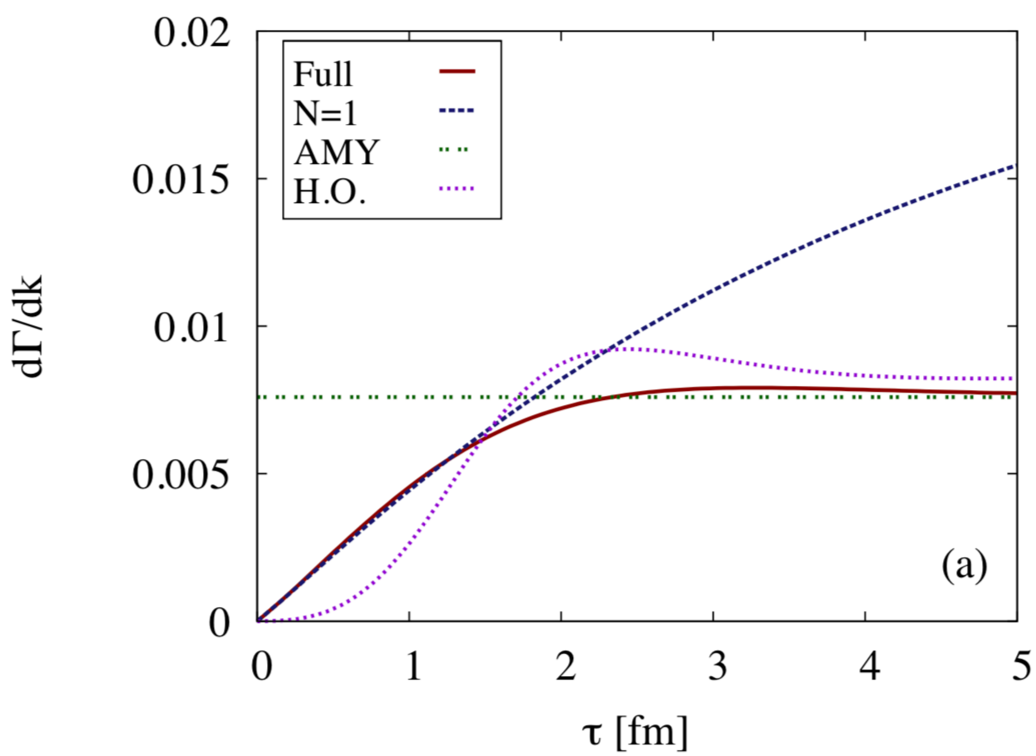
$$\left[i \frac{\partial}{\partial t} + \frac{\boldsymbol{\partial}^2}{2\omega} + i\sigma(\boldsymbol{x}) \right] \mathcal{K}(\boldsymbol{x}, t | \boldsymbol{y}, t_1) = i\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - t_1)$$

- Where the imaginary potential is given by

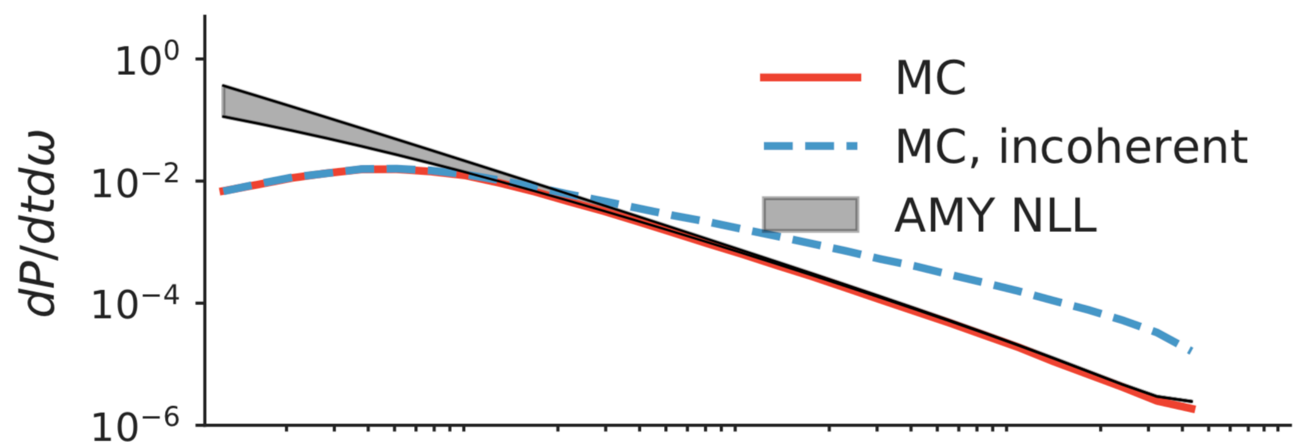
$$\sigma(\boldsymbol{x}, t) = N_c \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2\boldsymbol{q}} \left(1 - \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right) \quad \sim x_\perp^2 \left(\ln \frac{1}{x_\perp^2 \mu^2} + O(x_\perp^2 \mu^2) \right)$$

Medium-induced gluon spectrum

- Difficult to solve. Numerical solutions



Caron-Huot and Gale (2010)



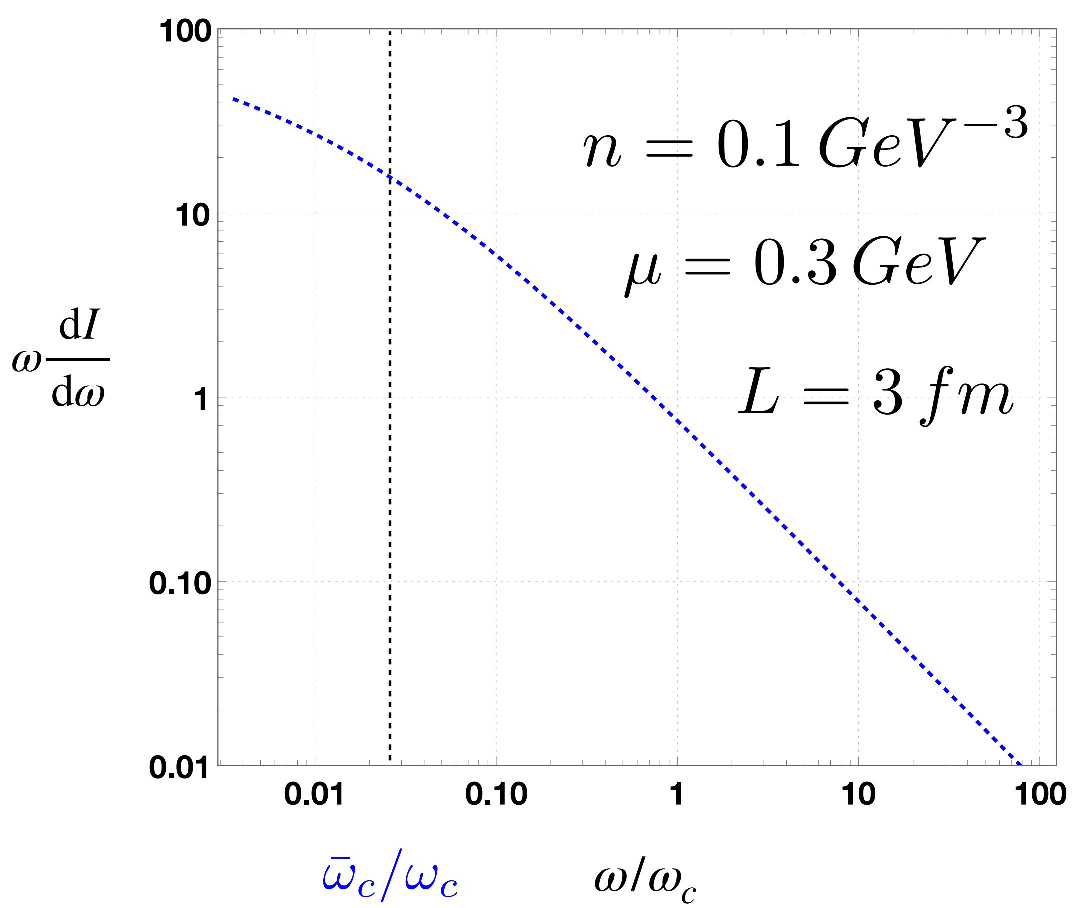
Ke , Xu, Bass (2018)

- Analytic limits:
  - N=1 opacity (GLV) dilute medium or hard radiation
  - Harmonic Oscillator (HO) (BDMPS) dense medium $\sigma(x_\perp) \sim x_\perp^2$
- This talk: opacity expansion around HO to account for both regimes

N=1 Opacity (Gyulassy-Levai-Vitev (2000))

- Assuming a dilute medium and expand to leader order in $\sigma(x_\perp)$

$$\omega \frac{\mathrm{d}I_{\mathrm{GLV}}}{\mathrm{d}\omega} \simeq 2\bar{\alpha} n\, L \begin{cases} \ln \frac{\bar{\omega}_c}{\omega} & \text{for} \quad \omega \ll \bar{\omega}_c \\ \frac{\pi}{4}\left(\frac{\bar{\omega}_c}{\omega}\right) & \text{for} \quad \omega \gg \bar{\omega}_c \end{cases}$$



$$\bar{\omega}_c = \frac{1}{2}\mu^2 L \simeq \ 0.7\, GeV$$

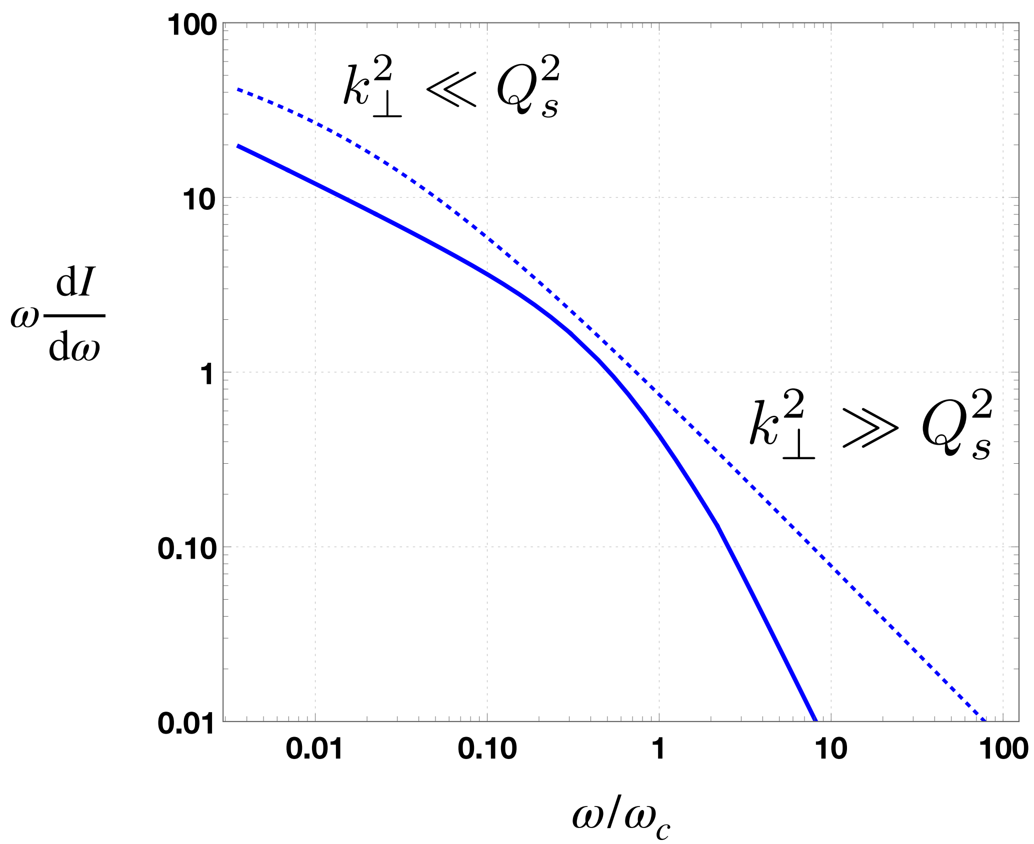
$$\omega_c = nL^2 \simeq \ 22.5 GeV$$

Multiple-soft scattering (BDMPS (1997))

- Strong LPM suppression due to multiple soft scattering

$$\omega \frac{\mathrm{d}I_{\mathrm{HO}}}{\mathrm{d}\omega} = 2\bar{\alpha} \ln \left| \cos\left(\frac{1-i}{2} \sqrt{\frac{\omega_c}{\omega}} \right) \right|$$

$$\simeq 2\bar{\alpha} \begin{cases} \sqrt{\frac{\omega_c}{2\omega}} & \text{for} \quad \omega \ll \omega_c \\ \frac{1}{12}\left(\frac{\omega_c}{\omega}\right)^2 & \text{for} \quad \omega \gg \omega_c \end{cases}$$



Validity of approximations:

- Single hard scattering $> \omega_c$
- Multiple-soft scattering $< \omega_c$

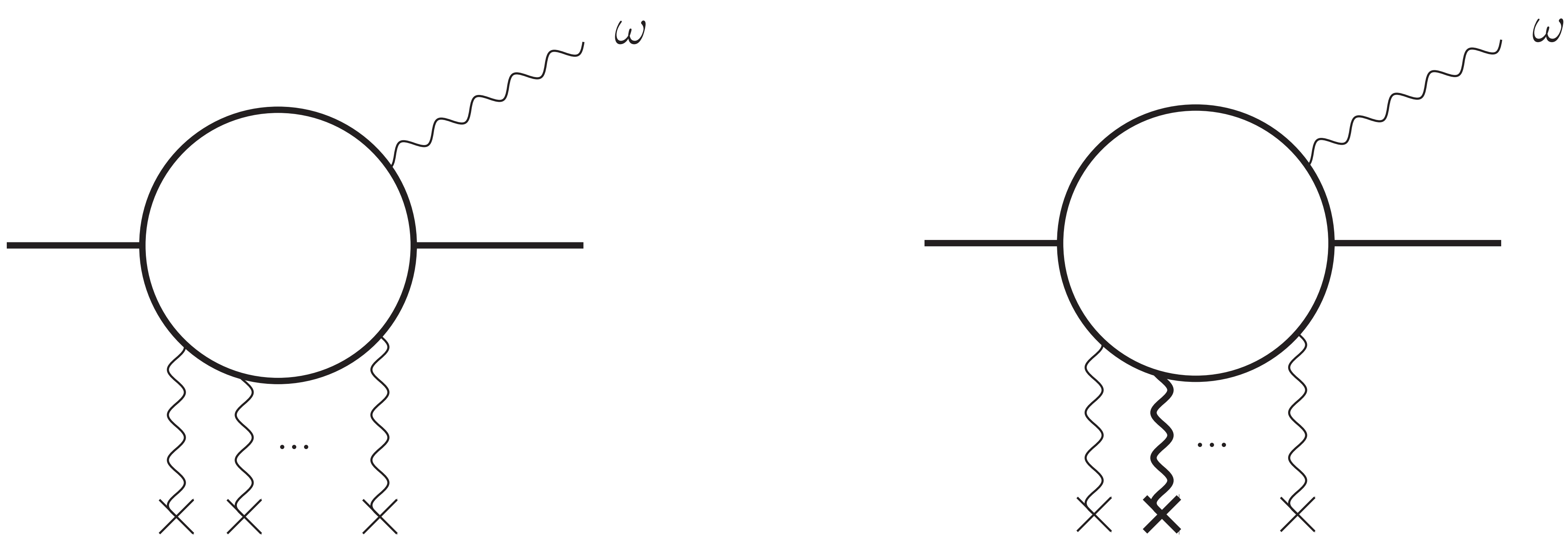
$$Q_s^2 \equiv \hat{q}L \sim 5 - 10\, GeV^2$$

Medium-induced gluon spectrum

Since: $Q^2 \sim \langle x_\perp^2 \rangle^{-1} \simeq \sqrt{\omega \hat{q}} \sim \sqrt{\omega n \ln(Q^2/\mu^2)}$

We can extract a large log from the dipole cross-section

$$\begin{aligned} \sigma(t, \boldsymbol{x}) &= n(t)\, \boldsymbol{x}^2 \left(\ln \frac{Q^2}{m_D^2} + \ln \frac{1}{\boldsymbol{x}^2 Q^2} \right) \\ &\equiv \sigma_{\mathrm{HO}}(t, \boldsymbol{x}) + \sigma_{\mathrm{pert}}(t, \boldsymbol{x}), \end{aligned}$$



Molière (1948)

Medium-induced gluon spectrum

Correction to the Harmonic oscillator:

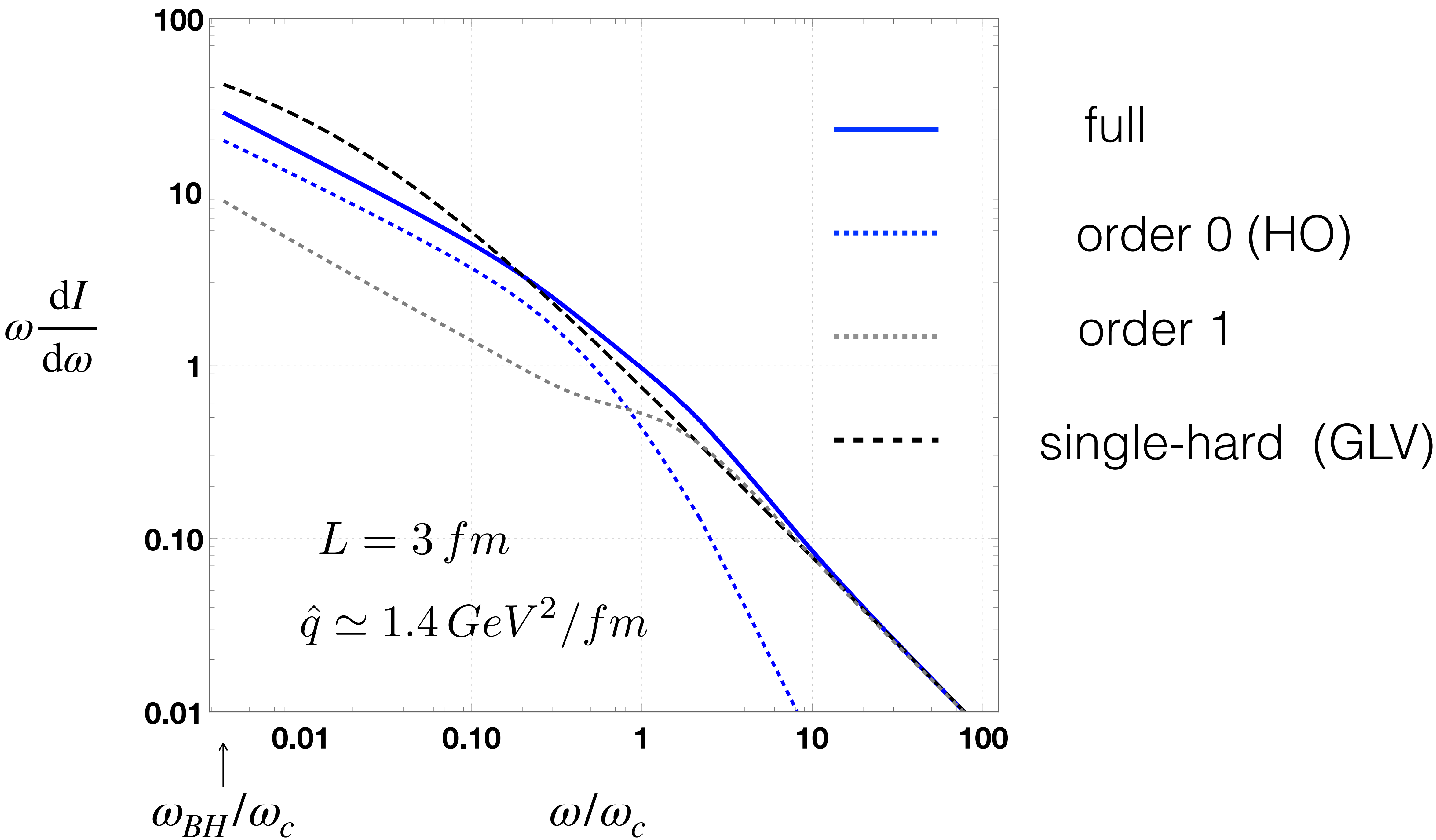
$$\omega \frac{\mathrm{d}I^{(1)}}{\mathrm{d}\omega} = \frac{\alpha_s C_R\, n}{2\pi} \,\mathrm{Re} \int_0^L \mathrm{d}s \,\frac{1}{k^2(s)} \left[\ln \frac{k^2(s)}{Q^2} + \gamma \right]$$

$$k^2(s) = i \frac{\omega \Omega}{2} \left(\cot(\Omega s) - \tan(\Omega(L-s)) \right) \qquad \Omega \equiv \frac{1-i}{2} \sqrt{\frac{\hat{q}}{\omega}}$$

→ Contains the large frequency limit of N=1 opacity (GLV spectrum)

Numerics

Medium-induced gluon spectrum for $\omega_c = nL^2 = 22.5\, GeV$



Numerics

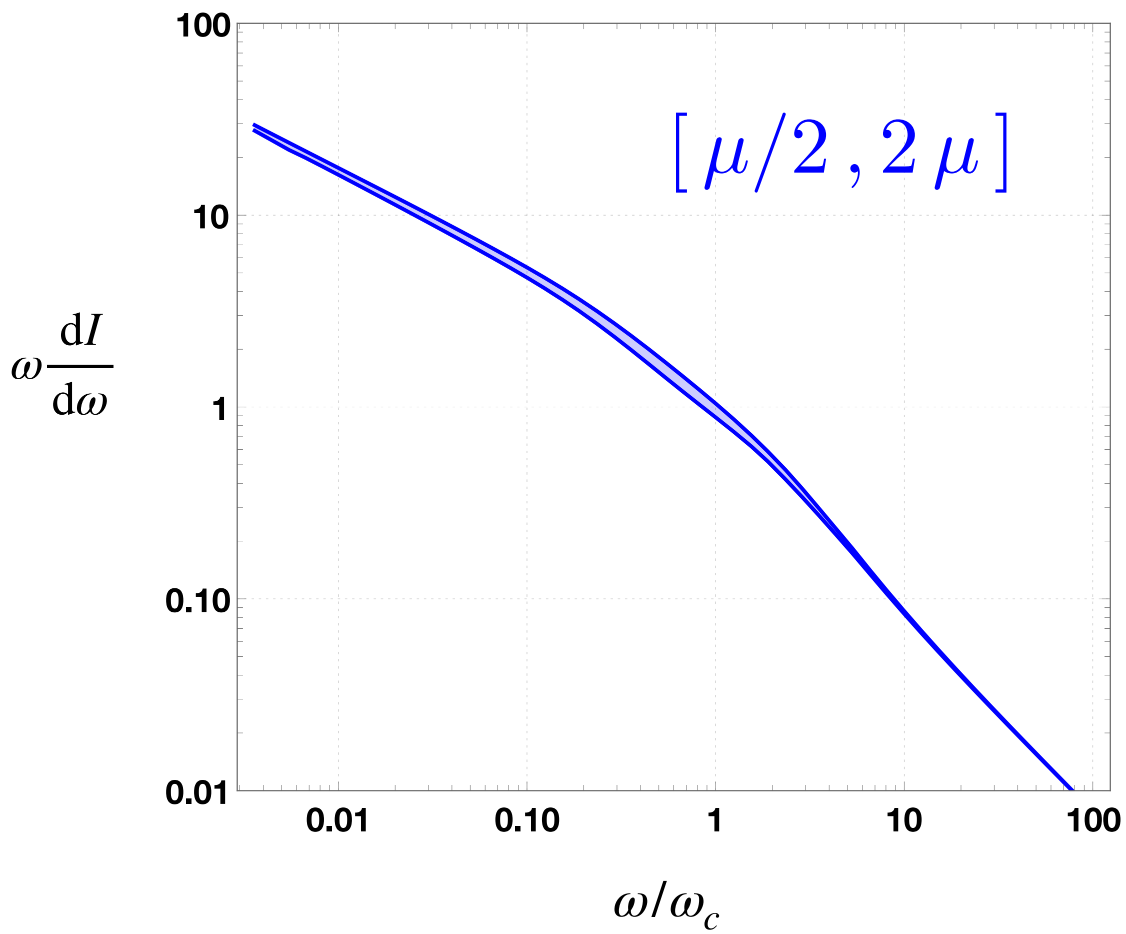
	N=1 (GLV)	full
$\Delta E(\omega < 100\, GeV)$	83 GeV	88 GeV
$N(\omega > 10^{-2}\omega_c)$	40	29

- The mean energy loss is dominated by single hard scattering
- Multiplicity is dominated by multiple soft scattering

IR sensitivity

- Typical transverse momentum scale:

$$Q^2 \sim n\, L\, \ln(nL/\mu^2) \sim 4\, GeV^2 \gg \mu^2 \sim (0.3)^2\, GeV^2$$



$$n = 0.1\, GeV^{-3}$$

$$L = 3\, fm$$

$$\hat{q} \simeq 1.4\, GeV^2/fm$$

$$\omega_c = nL^2 = 22.5\, GeV$$

→ Weak dependence on IR scale for large/dense media: controlled perturbative expansion

Summary and outlook

- We have developed a new systematic method to perform analytic calculation of the medium-induced gluon spectrum beyond multiple-soft scattering approximation by expanding around the harmonic oscillator

- We have calculated the first two orders that encompass multiple-soft and single hard scattering regimes

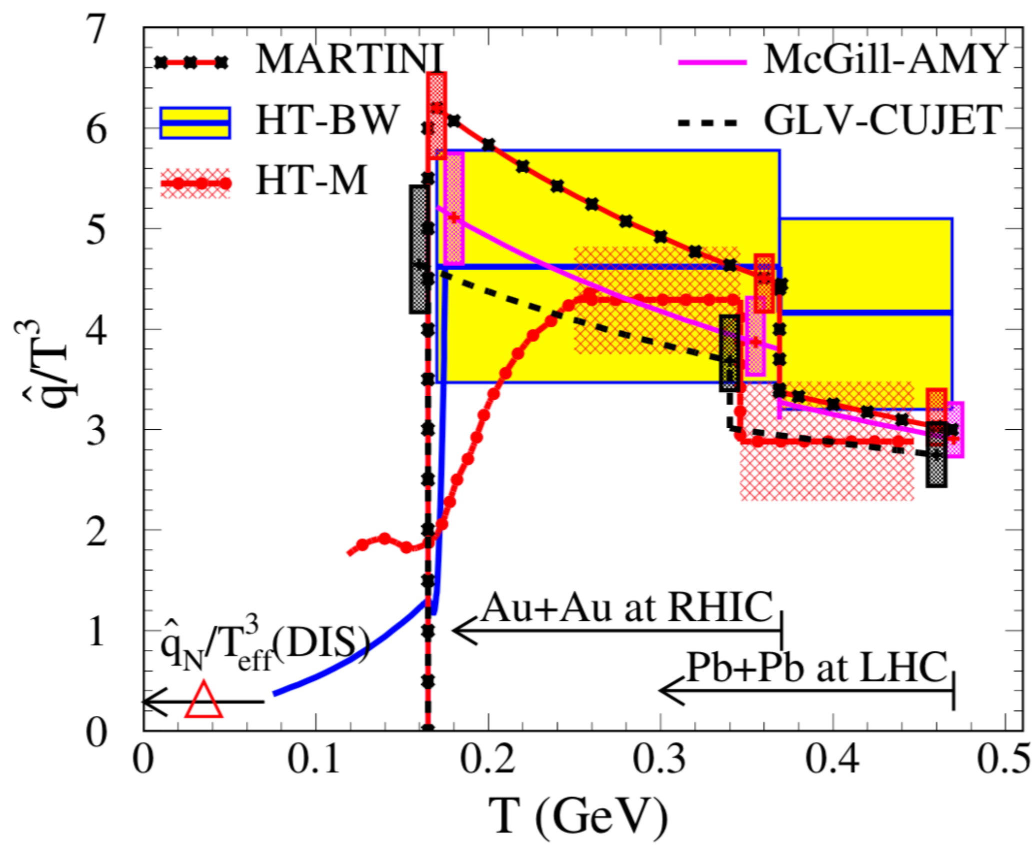
- Under perturbative control for large media

- Outlook: generalize to finite gluon energy and transverse momentum dependence. MC implementation

Momentum broadening and $\hat{q}$

- Extraction of the jet quenching parameter (JET Collaboration):

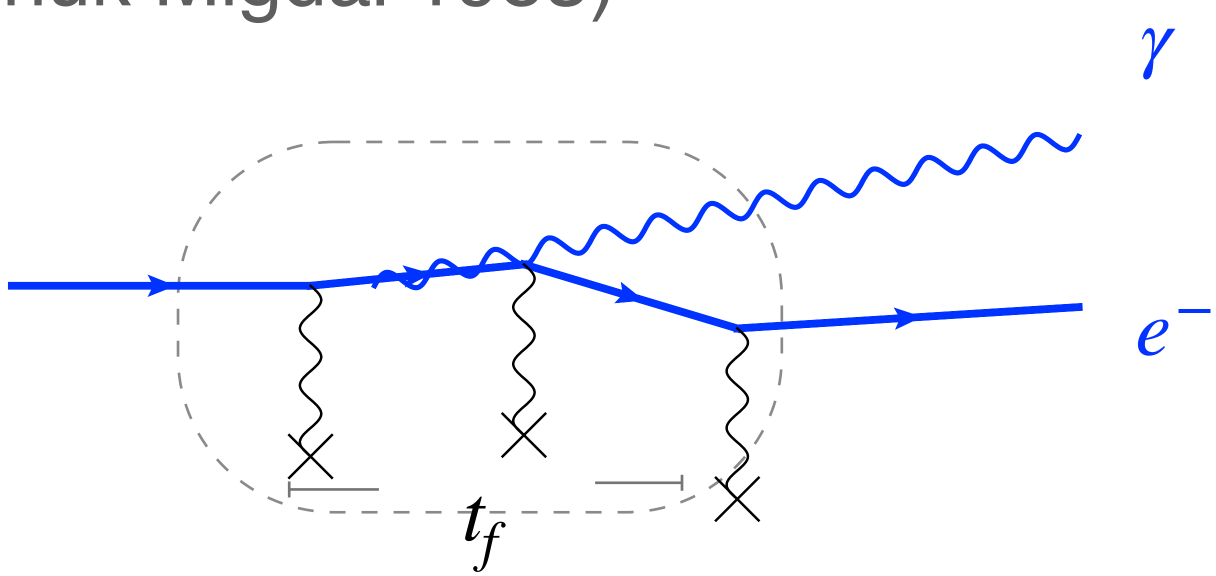
arXiv:1312.5003



$$\hat{q} \approx \begin{cases} 1.2 \pm 0.3 \\ 1.9 \pm 0.7 \end{cases} \text{GeV}^2/\text{fm} \quad \text{at} \quad \begin{matrix} \text{T=370 MeV} \\ \text{T=470 MeV} \end{matrix}$$

The LPM effect in QED (digression)

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects (Landau-Pomeranchuk Migdal 1953)

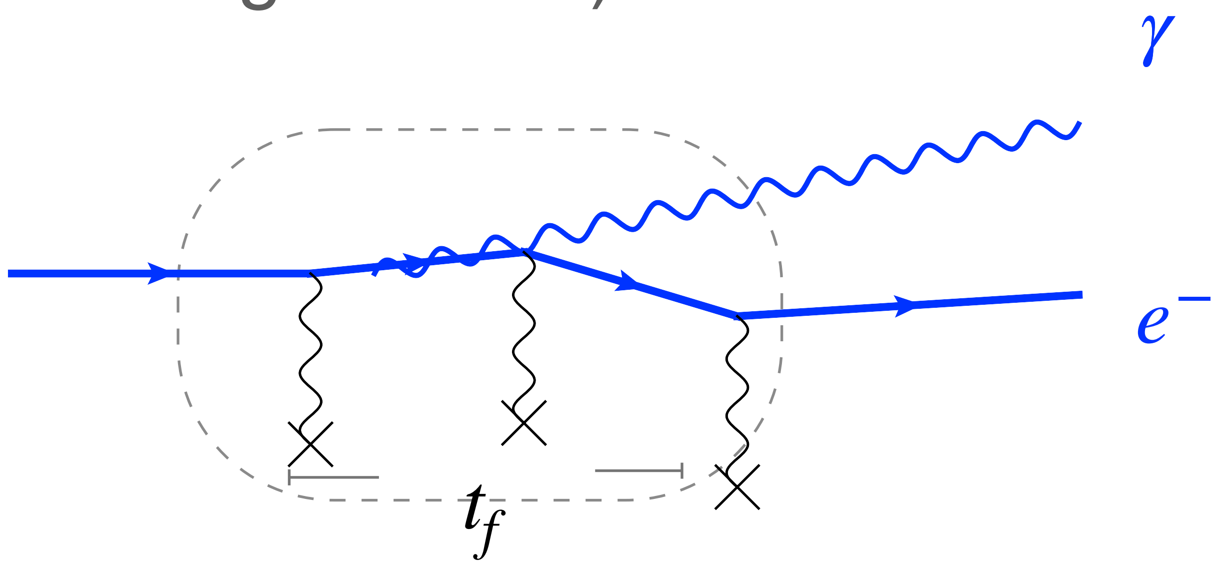


- The Bethe-Heitler spectrum assumes incoherent scatterings center

$$\omega \frac{\mathrm{d}I^{BH}}{\mathrm{d}\omega} \sim \alpha_e N_{scatt} \sim \alpha_e \frac{L}{\ell_{mfp}}$$

The LPM effect on the back of the envelop

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects (Landau-Pomeranchuk Migdal 1953)



- Coherence length: during the quantum mechanical formation time N_{coh} scattering centers act coherently reducing the radiation spectrum

$$\omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_e N_{eff} \sim \alpha_e \frac{N_{scatt}}{N_{coh}} \sim \alpha_e \frac{L}{t_f(\omega)}$$

The LPM effect on the back of the envelop

- From the uncertainty principle we have:

$$t_f(\omega) = \frac{\omega}{k_\perp^2} = \frac{1}{\omega\theta_\gamma^2}$$

- During that time the electron suffers momentum broadening

$$k_\perp^2 \sim \hat{q}\, t_f \qquad\qquad \theta_\gamma^2 \sim \theta_e^2 = \frac{k_\perp^2}{E_e^2}$$

- Solving for t_f one finds

$$t_f(\omega) = \frac{E_e}{\sqrt{\omega\hat{q}}} \qquad \text{and} \qquad \omega\frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_e\frac{\sqrt{\omega\hat{q}}}{E_e}L \propto \sqrt{\omega}$$

LPM suppression in the IR

The LPM effect in QED (digression)

- The LPM effect was observed at SLAC in 1995