Nuclear Matrix Elements for Neutrinoless Double Beta Decay from Lattice QCD

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BNL High Energy Theory Seminar

Background

$2\nu\beta\beta$ from Lattice QCD [arXiv:1702.02929 (2017)]

$0\nu\beta\beta$ from Lattice QCD [arXiv:1811.05554 (2018)]

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Neutrinos and Neutrino Oscillations

- Oscillation experiments have taught us a lot about neutrinos
- PMNS matrix $U_{\alpha i}$ relates flavor $|\nu_\alpha\rangle$ and mass $|\nu_j\rangle$ eigenstates

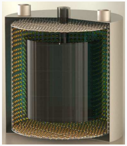
$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle$$

- Oscillations:

$$|\nu_i(L)\rangle = e^{\frac{-i m_i^2 L}{2E}} |\nu_i(0)\rangle , \qquad P_{\alpha\to\beta} = \left| \sum_j U^*_{\alpha j} U_{\beta j} e^{-\frac{i m_j^2 L}{2E}} \right|^2$$

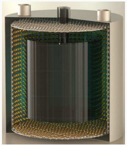
(a) Reactor

$|\nu_\alpha\rangle$ →



(c) Near det.

$|\nu_\alpha\rangle$ → L



(e) Far det.

Neutrinos and Neutrino Oscillations

Δm^2_{12} [eV^2]	$7.5(2) \times 10^{-5}$	2.7%
Δm^2_{13} [eV^2] (NH)	$2.50(3) \times 10^{-3}$	1.2%
Δm^2_{13} [eV^2] (IH)	$2.42(^{+3}_{-4}) \times 10^{-3}$	1.4%
$\sin^2\theta_{12}$	$3.2(2) \times 10^{-1}$	5.5%
$\sin^2\theta_{13}$ (NH)	$2.2(^{+8}_{-7}) \times 10^{-2}$	3.5%
$\sin^2\theta_{13}$ (IH)	$2.2(^{+7}_{-8}) \times 10^{-2}$	
$\sin^2\theta_{23}$ (NH)	$5.5(^{+2}_{-3}) \times 10^{-1}$	4.7%
$\sin^2\theta_{23}$ (IH)		4.4%
δ/π (NH)	1.2(2)	10%
δ/π (IH)	$1.6(^{+1}_{-2})$	9%

Table 1: PMNS [arXiv:1708.01186]

$\sin^2\theta_{12}$	$5.09(4) \times 10^{-2}$	0.8%
$\sin^2\theta_{13}$	$1.2(1) \times 10^{-5}$	8.3%
$\sin^2\theta_{23}$	$1.72(9) \times 10^{-3}$	5.2%
δ/π	0.38(3)	7.9%

Table 2: CKM [PDG]

- Evidence of rich and interesting physics in neutrino sector...
  - Larger mixing angles
  - Potential *CP* violation ($\propto \sin\delta$)
- ...but interesting questions remain:
  - Absolute neutrino masses / mixing angles?
  - Are neutrinos **Dirac** or **Majorana**?
  - Is **lepton number conserved** in nature? (baryogenesis)
- Need other sources of information!

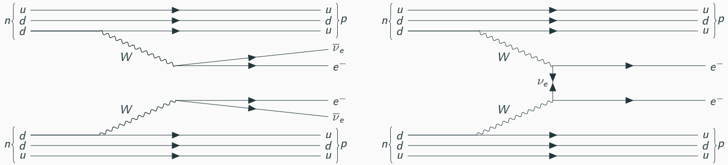
Double Beta Decay

$2\nu\beta\beta$ (neutrinoful):

- ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+2} X + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e$
- $T^{2\nu}_{1/2} \sim 10^{21}$ yrs rarest observed Standard Model process

$0\nu\beta\beta$ (neutrinoless):

- Also allowed if neutrino is Majorana: ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+2} X + e^- + e^-$
- **Only viable method for probing Majorana vs. Dirac!**



(a) $2\nu\beta\beta$ **(b)** $0\nu\beta\beta$

Challenging to observe: requires systems with no β decay background

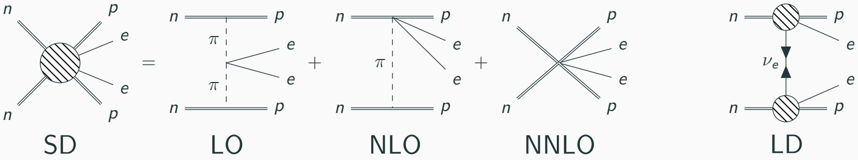
$0\nu\beta\beta$ in the Standard Model EFT and Chiral EFT

SM EFT: $\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \ldots, \quad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)}$

- Unique dim. 5 operator: $\mathcal{L}^{(5)} \sim \frac{1}{\Lambda} HH\ell\ell \sim \frac{v^2}{\Lambda} \nu_L^T C \nu_L$ (!)
- New SD operators at dimensions 7 ($\sim \frac{1}{\Lambda^3}\ell\ell qq$), 9 ($\sim \frac{1}{\Lambda^5}\ell\ell qqqq$), ...

Chiral EFT: Matching to (low-energy EFT of hadrons) is understood:

- Left: short distance [arXiv:hep-ph/0303205]
- Right: long distance, light Majorana exchange [arXiv:1710.01729]



- Generically need lattice QCD input to determine SM rates

Neutrinoless Double Beta Decay

$$\frac{1}{T_{1/2}^{0\nu}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{0\nu} \left|M^{0\nu}\right|^2$$

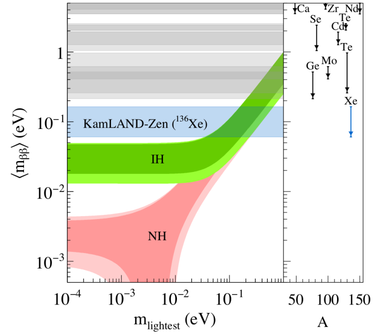
- $T_{1/2}^{0\nu}$: $0\nu\beta\beta$ half-life (expt.)
- $m_{\beta\beta} = \left|\sum_i m_i U_{ei}^2\right|$: effective Majorana neutrino mass
- $G^{0\nu}$: phase space factor (theory)
- $M^{0\nu}$: nuclear matrix element (theory) $\leftarrow$ **dominant uncertainty**

Table 1
Some $\beta\beta(0\nu)$-decay isotopes of experimental interest that are discussed in this paper, shown with most recent half-life limits. Natural abundances and Q-values taken from [28].

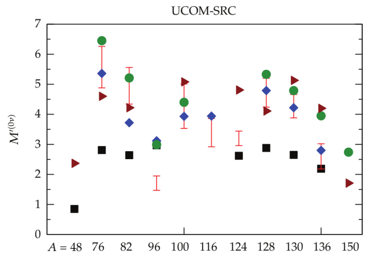
Isotope	$\beta\beta(0\nu)$ Half-life limit (years)	Natural Abundance [%]	Q-value (MeV)
^{48}Ca	$>1.4 \times 10^{22}$ [31]	0.187	4.2737
^{76}Ge	$>3.0 \times 10^{25}$ [32]	7.8	2.0391
^{82}Se	$>1.0 \times 10^{23}$ [33]	9.2	2.9551
^{100}Mo	$>1.1 \times 10^{24}$ [34]	9.6	3.0350
^{130}Te	$>4.0 \times 10^{24}$ [35]	34.5	2.5303
^{136}Xe	$>1.1 \times 10^{25}$ [36]	8.9	2.4578
^{150}Nd	$>1.8 \times 10^{22}$ [37]	5.6	3.3673

[Henning, Rev. Phys. 1, 29-35 (2016)]

Constraints on $m_{\beta\beta}$ from $T^{0\nu}_{1/2}$ Limits



(a) KamLAND-Zen [arXiv:1605.02889]



(b) Giuliani et al. [Adv. High Energy Phys. 857016 (2012)]

- $\sim 2-3\times$ spread in different model calculations of same NME
- Interpreting $T^{0\nu}_{1/2}$ as constraint on neutrino properties / models of $0\nu\beta\beta$ will require improving this situation...
- Realistically, lattice QCD could:
  1. Compute inputs to EFT ($\pi\pi \to ee$, $n\pi \to pee$, $nn \to ppee$ vertices)
  2. Directly test nuclear models for small nuclei

Background

$2\nu\beta\beta$ from Lattice QCD [arXiv:1702.02929 (2017)]

$0\nu\beta\beta$ from Lattice QCD [arXiv:1811.05554 (2018)]

Setup

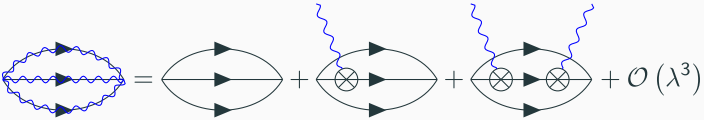
- Lattice calculation:
  - $nn \to pp$ matrix element on $32^3 \times 48$ Wilson-clover ensemble
  - $m_u = m_d = m_s^{\rm phys}$, giving $m_\pi \approx 800$ MeV, $m_N \approx 1600$ MeV
- $2\nu\beta\beta$ decay mechanism is well-understood: $(T_{1/2}^{2\nu})^{-1} = G^{2\nu} \left| M_{GT}^{2\nu} \right|^2$, with

$$\frac{1}{6} M_{GT}^{2\nu} = \sum_{n=0}^{\infty} \frac{\langle f | A_3 | n \rangle \langle n | A_3 | i \rangle}{E_n - (E_i + E_f)/2} = \beta_A^{(2)} - \frac{|\langle pp | A_3 | d \rangle|^2}{E_{nn} - E_d},$$

- $\beta_A^{(2)}$: isotensor axial polarizability
- Trick: compute *compound propagators* in background axial field $\propto \lambda$

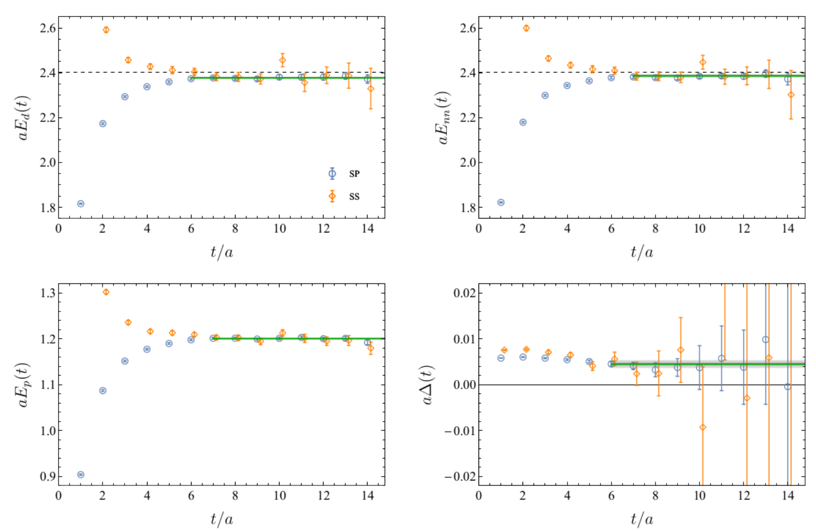
$$S_\lambda(x,y) = S(x,y) + \lambda \int d^4 z \, S(x,z) A_3(z) S(z,y) + \mathcal{O}(\lambda^2)$$

- $\mathcal{O}(\lambda^n)$ compound two-point function has n axial current insertions:

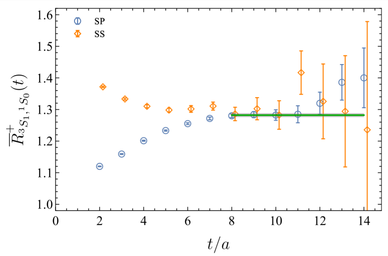


Deuteron and Dinucleon Masses at $m_\pi \approx 800$ MeV

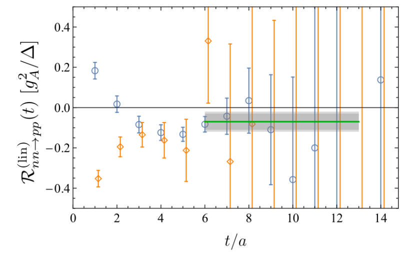
$|pp\rangle$, $|d\rangle = |pn\rangle$, and $|nn\rangle$ are bound states $\Rightarrow$ FV corrections $\sim e^{-ML}$

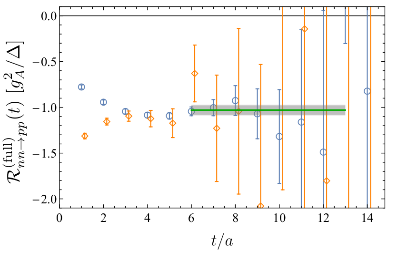


$\langle pp|A_3|d\rangle$, $\beta_A^{(2)}$, and $M_{GT}^{2\nu}$



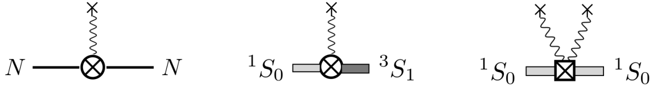
- Top: $|\langle pp|A_3|d\rangle|^2$
- Bottom left: $\beta_A^{(2)}$
  ($^1S_0$ isotensor axial polarizability)
- Bottom right: $M_{GT}^{2\nu}$



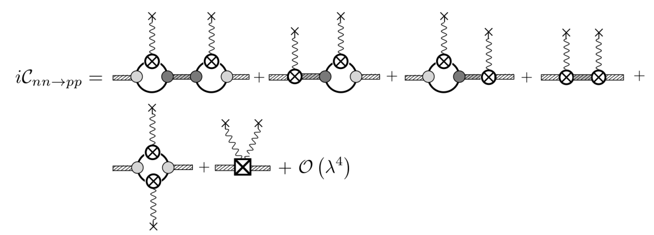


Matching to Pionless EFT

- Match to pionless EFT in background axial field
- Basic vertices (left: g_A, middle: $\mathbb{L}_{1,A}$, right: $\mathbb{H}_{2,S}$):



- $C_{nn\to pp}$ at $\mathcal{O}(\lambda^2)$:



- Result: $\mathbb{H}_{2,S} = 4.7(1.3)(1.8)$ fm
- Future calculations in EFT framework could relate to ME of larger nuclei

Summary of Results

- Successful first calculation with unphysical quark masses...
- Key results ($\Delta \equiv E_{nn} - E_d$):

$$\boxed{\begin{aligned} \frac{\Delta}{g_A^2}\frac{M_{GT}^{2\nu}}{6} &= -1.04(4)(4) \\ \frac{\Delta}{g_A^2}\frac{|\langle pp|A_3|d\rangle|^2}{\Delta} &= 1.00(3)(1) \\ \mathbb{H}_{2,S} &= 4.7(1.3)(1.8)\ \mathrm{fm} \end{aligned}}$$

- Moving toward physical point introduces new challenges:
  - $|nn\rangle$ and $|pp\rangle$ unbound $\Rightarrow$ power law finite volume effects
  - Other intermediate states may become important
  - At some point light pions must be included in EFT
- ...however, compound propagators do not easily generalize to $0\nu\beta\beta$
  - Current insertions are connected by internal Majorana neutrino propagator
  - **New methods needed!**

Background

$2\nu\beta\beta$ from Lattice QCD [arXiv:1702.02929 (2017)]

$0\nu\beta\beta$ from Lattice QCD [arXiv:1811.05554 (2018)]

SM Long-Distance, Light Majorana Exchange Mechanism

- For lattice scales $a^{-1} \ll m_W$ suffices to work in Fermi effective theory

$$H_W = 2\sqrt{2} G_F V_{ud} \left(\overline{u}_L \gamma_\mu d_L \right) \left(\overline{e}_L \gamma_\mu \nu_{eL} \right)$$

- Treat H_W as perturbation to H_{QCD}: $0\nu\beta\beta$ induced at second order
- Matrix element decomposes into **leptonic** and **hadronic** pieces

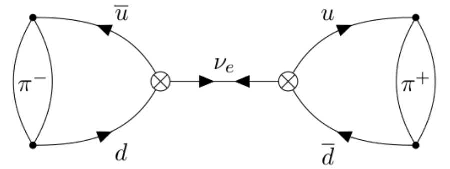
$$\int d^4x\, d^4y \left\langle fee \right| T\{H_W(x) H_W(y)\} \left| i \right\rangle = 4 m_{\beta\beta} G_F^2 V_{ud}^2 \int d^4x\, d^4y\, \boldsymbol{L_{\alpha\beta}(x,y) H^{\alpha\beta}(x,y)}$$

$$\boldsymbol{L_{\alpha\beta}(x,y) \equiv \overline{e}_L(p_1) \gamma_\alpha \gamma_\beta e_L^C(p_2) S_\nu(x,y) e^{-ip_1 \cdot x} e^{-ip_2 \cdot y}}$$

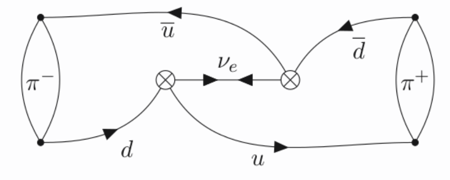
$$\boldsymbol{H_{\alpha\beta}(x,y) \equiv \langle f | T \left\{ \overline{u}_L(x) \gamma_\alpha d_L(x) \overline{u}_L(y) \gamma_\beta d_L(y) \right\} | i \rangle}$$

- Develop lattice methods by first computing $\pi^- \to \pi^+ e^- e^-$ amplitude
  - Simple Wick contractions
  - Clean lattice signals: for nuclear systems $SNR(t) \sim \exp(-A(m_n - \frac{3}{2} m_\pi) t)$
- Related calculations:
  1. Long-distance $\pi^- \pi^- \to e^- e^-$ by Feng et al. [arXiv:1809.10511]
  2. Short-distance $\pi^- \to \pi^+ e^- e^-$ by Nicholson et al. [arXiv:1805.02634]

Wick Contractions for $\pi^- \to \pi^+ e^- e^-$ Transition



(a) Type ①



(b) Type ②

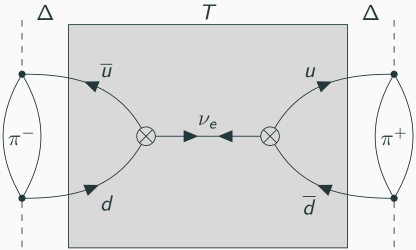
$$① = \mathrm{Tr}\left[S_u^\dagger(t_- \to x)\gamma_\alpha (1-\gamma_5) S_d(t_- \to x)\right] \cdot \mathrm{Tr}\left[S_u^\dagger(t_+ \to y)\gamma_\beta (1-\gamma_5) S_d(t_+ \to y)\right]$$

$$② = \mathrm{Tr}\left[S_u^\dagger(t_+ \to x)\gamma_\alpha (1-\gamma_5) S_d(t_- \to x) S_u^\dagger(t_- \to y)\gamma_\beta (1-\gamma_5) S_d(t_+ \to y)\right]$$

Long-Distance $0\nu\beta\beta$ Pilot Calculation: Formalism

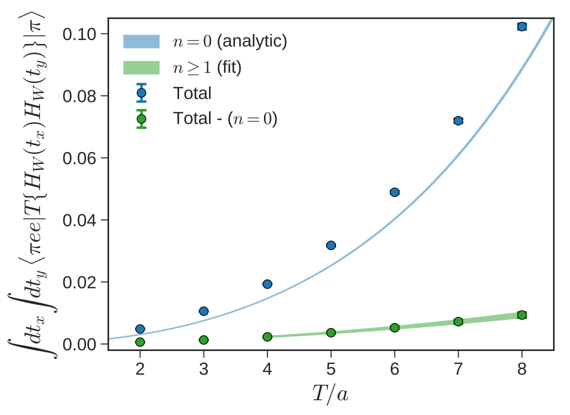
- Pilot calculation on $16^3 \times 32 \times 16$ DWF ensemble [arXiv:hep-lat/0701013]
  - $m_\pi = 400$ MeV, $a^{-1} = 1.6$ GeV, $L = 2$ fm
  - (Free) overlap fermion propagator for neutrino
  - Coulomb gauge-fixed wall source propagators for quarks
  - Asymmetric treatment of weak current insertions
- Similar to ΔM_K [arXiv:1406.0916], rare Kaon decays [arXiv:1806.11520]

$$\mathcal{M}^{0\nu}(T) = |Z_\pi|^2 \, e^{-m_\pi(T+2\Delta)} \sum_n \frac{\langle \pi ee | H_W | n \rangle \langle n | H_W | \pi \rangle}{E_n - m_\pi} \left[T + \frac{e^{-(E_n - m_\pi)T} - 1}{E_n - m_\pi} \right]$$



- Choose Δ to suppress excited states
- To see $\mathcal{M}^{0\nu} \sim T$, must remove:
  - Source/sink dependence
  - $|e\overline{\nu}_e\rangle \propto e^{(m_\pi - (m_{\beta\beta}+m_e))T}$
  - $|\pi^0 e\overline{\nu}_e\rangle \propto \frac{1}{2} T^2$
- **Strategy:** remove $|e\overline{\nu}_e\rangle$ state, then extract **ME** from quadratic fit

Long-Distance $0\nu\beta\beta$ Pilot Calculation: Results



- $am_\nu = 0.0001$
- Remove $n = 0$ analytically

$$\propto f_\pi^2 e^{(m_\pi - (m_{\beta\beta} + m_e))T}$$

- Fit quadratic to $n \geq 1$
- Reconstruct ME

$$M^{0\nu} = \sum_{n=0}^{\infty} \frac{\langle \pi e e | H_W | n \rangle \langle n | H_W | \pi \rangle}{E_n - m_\pi}$$

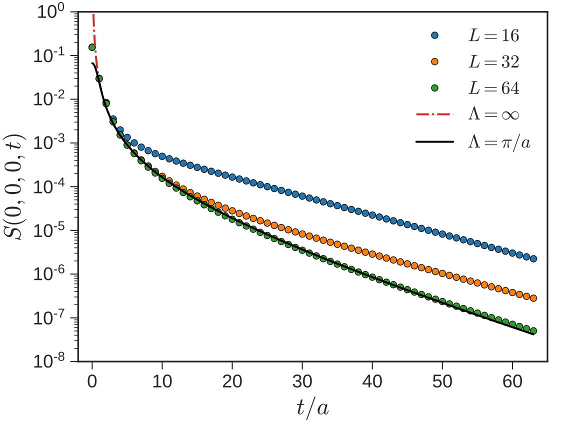
from fit

	$\lvert e\overline{\nu}_e \rangle$ $(n = 0)$	$\lvert \pi^0 e\overline{\nu}_e \rangle$ $(n = 1)$	$n \geq 2$
$\left[\frac{\langle \pi e e \lvert H_W \rvert n \rangle \langle n \lvert H_W \rvert \pi \rangle}{E_n - m_\pi}\right] / \left[\sum_{n=0}^{\infty} \frac{\langle \pi e e \lvert H_W \rvert n \rangle \langle n \lvert H_W \rvert \pi \rangle}{E_n - m_\pi}\right]$	-0.0082(15)	1.0082(13)	0.00009(26)

- $n = 0$ $(n \geq 2)$ correction is $\mathcal{O}(1\%)$ $(\mathcal{O}(0.01\%))$

Improved Methods I: Continuum Neutrino Propagator

Replace lattice S_ν with continuum S_ν to reduce FV effects ($am_\nu = 0.1$):



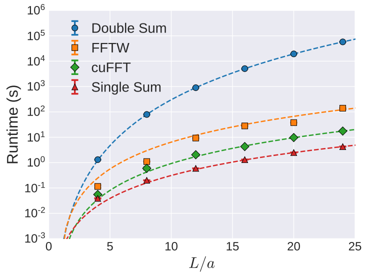
1. Lattice with $T=\infty$: $S(x) = \frac{1}{V}\sum_{\hat{\vec{q}}}\left[\int_{-\pi}^{\pi}\frac{d\hat{q}_3}{2\pi}\frac{1}{\hat{q}_3^2+\hat{\vec{q}}^2+m^2}e^{i\hat{q}_3\cdot t}\right]e^{i\hat{\vec{q}}\cdot\vec{x}}$
2. Continuum: $S(x) = \int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m^2}e^{iq\cdot x}$
3. UV-regulated continuum: $S_\Lambda(x) = \int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m^2}e^{iq\cdot x}e^{-q^2/\Lambda^2}$

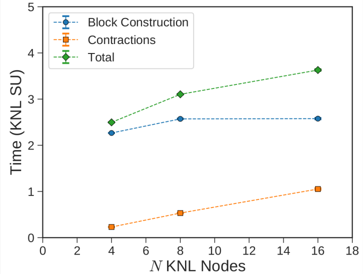
Improved Methods II: Exact Double Integration

- Integrate currents in $\mathcal{O}(V \log V)$ via 1D FFTs [Microw. Opt. Technol. Lett. 31, 28 (2001)] :
  1. Construct spin-color matrix valued "blocks" describing quark/neutrino propagation through one (integrated) current insertion

$$B_\alpha(x; t_1, t_2) = \int d^3y\, L_{\alpha\beta}(x-y) \left[S_u^\dagger(t_1 \to y)\, \gamma_\beta (1-\gamma_5)\, S_d(t_2 \to y)\right]$$
$$= \mathscr{F}^{-1}\left[\mathscr{F}(L_{\alpha\beta}) \cdot \mathscr{F}\left(S_u^\dagger \gamma_\beta (1-\gamma_5) S_d\right)\right]$$

  2. Integrate $\int d^3x\, B_\alpha(x) \left[S_u^\dagger(t_+ \to x)\, \gamma_\alpha (1-\gamma_5)\, S_d(t_- \to x)\right]$
- Readily generalizes to more complicated amplitudes such as $nn \to ppee$



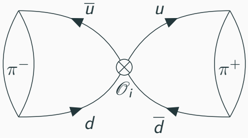


Production Calculation: Setup

We have recently started a production calculation

Ensemble	a (fm)	L	T	am_s	am_{ud}	m_π (MeV)	N_{meas}
24I	0.11	24	64	0.04	0.005	339.6	54
					0.01	432.2	53
32I	0.08	32	64	0.03	0.004	302.0	48
					0.006	359.7	43
					0.008	410.8	33

including SD [arXiv:1606.04549] and LD contributions to $\pi^- \to \pi^+ e^- e^-$



$$\mathcal{O}_1 = \left(\overline{q}_L \tau^+ \gamma^\mu q_L\right) \left[\overline{q}_R \tau^+ \gamma_\mu q_R\right]$$

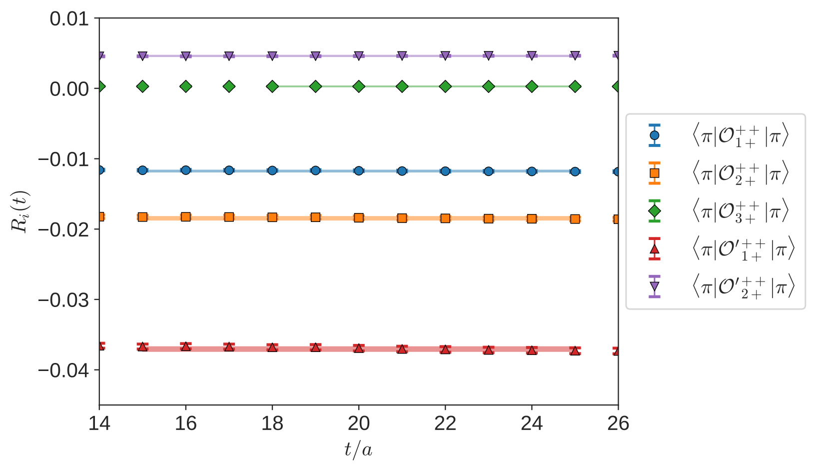
$$\mathcal{O}_2 = \left(\overline{q}_R \tau^+ q_L\right) \left[\overline{q}_R \tau^+ q_L\right] + \left(\overline{q}_L \tau^+ q_R\right) \left[\overline{q}_L \tau^+ q_R\right]$$

$$\mathcal{O}_3 = \left(\overline{q}_L \tau^+ \gamma^\mu q_L\right) \left[\overline{q}_L \tau^+ \gamma_\mu q_L\right] + \left(\overline{q}_R \tau^+ \gamma^\mu q_R\right) \left[\overline{q}_R \tau^+ \gamma_\mu q_R\right]$$

$$\mathcal{O}_1' = \left(\overline{q}_L \tau^+ \gamma^\mu q_L\right] \left[\overline{q}_R \tau^+ \gamma_\mu q_R\right)$$

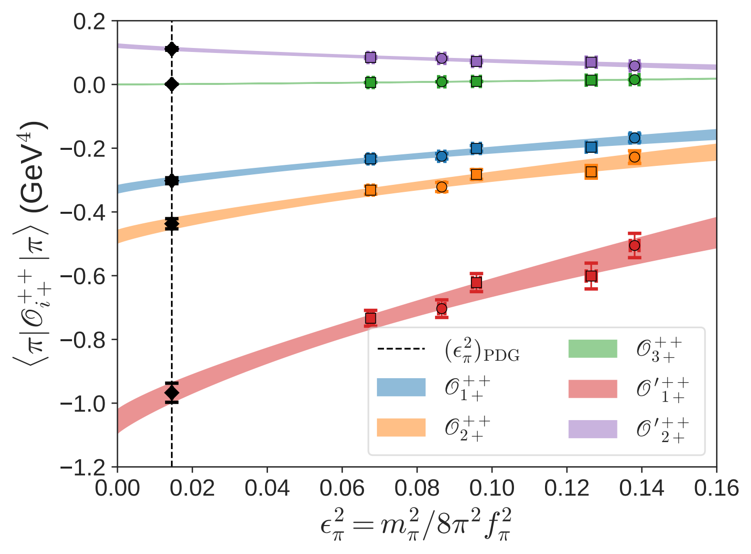
$$\mathcal{O}_2' = \left(\overline{q}_R \tau^+ q_L\right] \left[\overline{q}_R \tau^+ q_L\right) + \left(\overline{q}_L \tau^+ q_R\right] \left[\overline{q}_L \tau^+ q_R\right)$$

SD Preliminary Lattice Results (32I, $m_\pi \approx 300$ MeV)



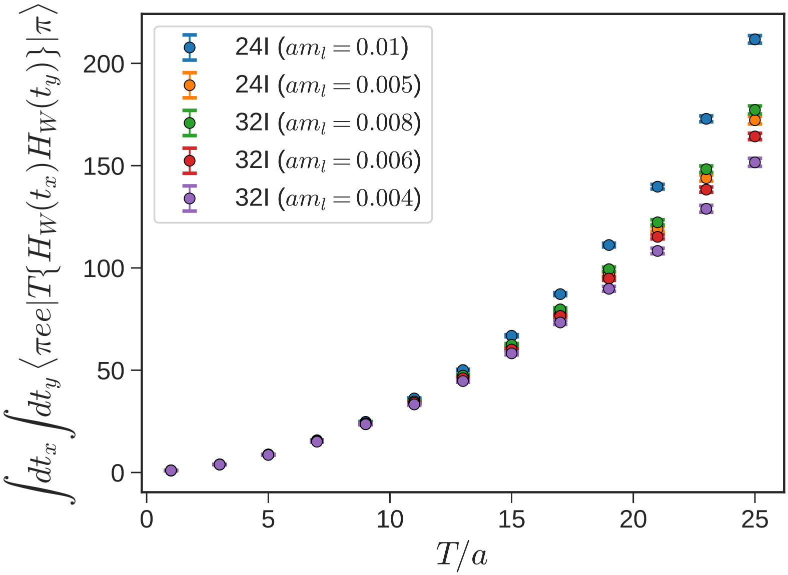
$$R_i(t) = \frac{C_{\pi\mathscr{O}_i\pi}(0,t,2t)}{C_\pi(2t)} \overset{t\gg 1}{\simeq} \frac{\langle\pi|\,\mathscr{O}_i\,|\pi\rangle}{2m_\pi}$$

SD Preliminary Physical Point Extrapolation



Fit ansatz: (continuum NLO χPT) + (NLO χPTFV) + ($c_a a^2$)

LD Preliminary Lattice Results



Next Step: Few-Body Nucleon Systems

- $n\pi \to pee$ excluded by parity in most experimental searches ($0^+ \to 0^+$)
  - Challenging on lattice due to disconnected diagrams
- Working now to implement $nn \to ppee$
  - ~ 600 Wick contractions, expect signal-to-noise problem
  - Baryon block algorithms can reduce contraction costs from exponential to polynomial in number of quarks [arXiv:1207.1452]

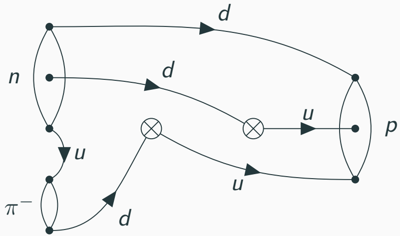


Figure 1: $n\pi \to pee$

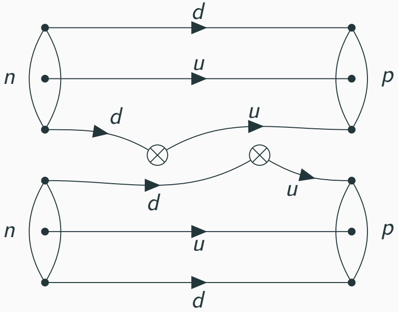


Figure 2: $nn \to ppee$

Conclusions

- We have previously computed the $2\nu\beta\beta$ decay amplitude for the process $nn \to ppee\overline{\nu}_e\overline{\nu}_e$
- We have also recently explored techniques for computing long-distance contributions to $0\nu\beta\beta$ decays
- In some cases, we have improved on these techniques:
  1. UV-regulated continuum neutrino propagator
  2. Explicit integration over locations of both current insertions via FFTs
- We are currently analyzing data from a complete calculation of the LD (light Majorana exchange) and SD contributions to $\pi^- \to \pi^+ e^- e^-$ on a series of DWF ensembles
  - Continuum / chiral / infinite volume limit
  - To-do (LD): Matching to χPT $\to$ extract $g_\nu^{\pi\pi}$
  - To-do (SD): Renormalization of $\mathscr{O}_i$ MEs in $\overline{\mathrm{MS}}$
- Working to prepare for calculation of $nn \to ppee$ amplitudes on $m_\pi \approx 800$ MeV Wilson-clover ensemble targeting $\sim 5\%$ uncertainty

Thank you!

Backup slides

Lattice QCD in One Slide

Basic idea: compute discretized (Euclidean) PI using a computer

$$\langle \mathscr{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathscr{D}U \mathscr{D}\psi \mathscr{D}\overline{\psi} \mathscr{O}(U, \overline{\psi}, \psi) e^{-S_G[U] - \overline{\psi} \not{D}(U) \psi}$$

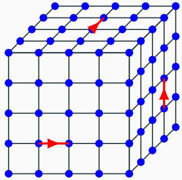
A typical lattice calculation proceeds in two steps:

1. Sample gauge field $\{U_i\}_{i=1}^{N}$ according to $P(U) \propto e^{-S(U)}$
2. Compute $\langle \mathscr{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathscr{O}(U_i) + \mathcal{O}(1/\sqrt{N})$

Use Wick's theorem to relate correlation functions to lattice propagators

$$\left\langle \overline{d}\gamma_5 u(y) \overline{u}\gamma_5 d(x) \right\rangle = \mathrm{Tr}\left[\gamma_5 S_u\left(x \to y\right) \gamma_5 S_d\left(y \to x\right)\right],$$

which are obtained by numerically solving the Dirac equation $\not{D}\psi = \phi$



ψ_x on sites, $U_{x,\mu}$ on links

- Solving Dirac equation is expensive (large, sparse matrix inversion)
- Freedom to tune $\mathscr{O}$ and source ϕ
- Cost (naively) grows exponentially in number of quarks

Second Order Weak Processes on the Lattice

- $2\nu\beta\beta$ and $0\nu\beta\beta$ are both second-order weak processes
  - Compute $\mathcal{M} = \int d^4x\, d^4y \, \langle f | \, T\{j_\mu(x) j_\mu(y)\} \, | i \rangle$ non-perturbatively
  - Extract $M = \sum_n \frac{\langle f | j_\mu | n \rangle \langle n | j_\mu | i \rangle}{E_n - (E_i + E_f)/2}$ by fitting to lattice data
- Similar to ΔM_K [arXiv:1406.0916] and rare Kaon decay [arXiv:1806.11520] calculations
- Can show integrated lattice correlation function behaves like

$$C_{i\to f}(t) = \sum_{\vec{x},\vec{y},\vec{z}} \sum_{t_1=0}^{t} \sum_{t_2=0}^{t} \langle 0 | \, \mathscr{O}_f(\vec{z}, t) \, T\left\{ j_\mu(\vec{y}, t_2) j_\mu(\vec{x}, t_1) \right\} \mathscr{O}_i^\dagger(0) \, | 0 \rangle$$

$$\simeq \frac{2}{a^2} Z_i^\dagger Z_f e^{-E_f t} \sum_n \frac{\langle f | \, j_\mu \, | n \rangle \langle n | \, j_\mu \, | i \rangle}{E_n - E_i} \left(\frac{e^{-(E_n - E_f)t} - 1}{E_n - E_f} + \frac{e^{(E_f - E_i)t} - 1}{E_f - E_i} \right)$$

- Form ratios which cancel t-dependence $R_{i\to f}(t) \overset{t\to\infty}{\simeq} M$