Nonperturbative Renormalization of the Quasi-PDF and Its Matching

Yong Zhao
Massachusetts Institute of Technology
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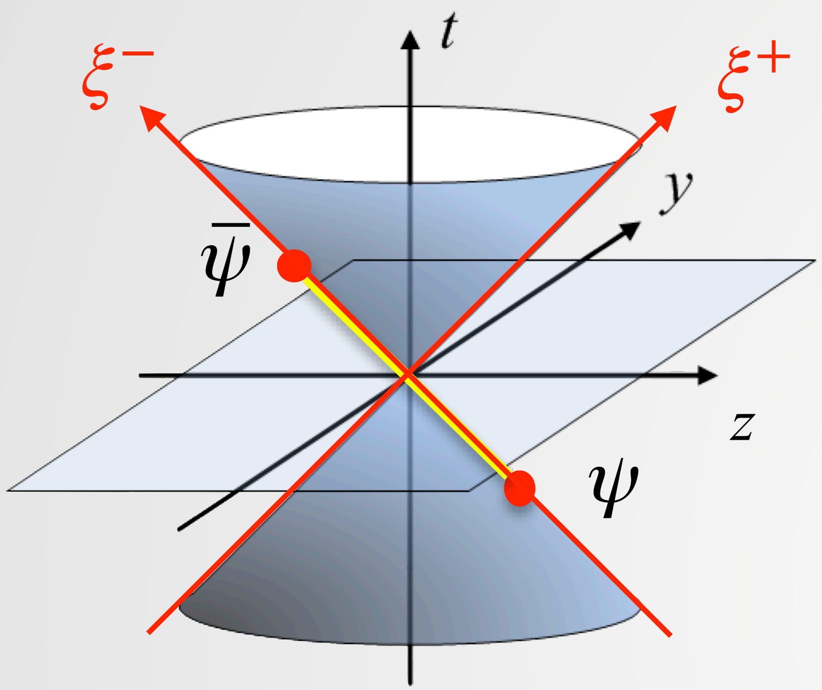
Outline

- Renormalization
  - Renormalizability of the quasi-PDF
  - Nonperturbative renormalization on lattice
- Perturbative matching
  - One-step matching
  - Two-step matching: “Ratio” and “Modified-MSbar” schemes
  - Comparison

Large-momentum effective theory (LaMET)

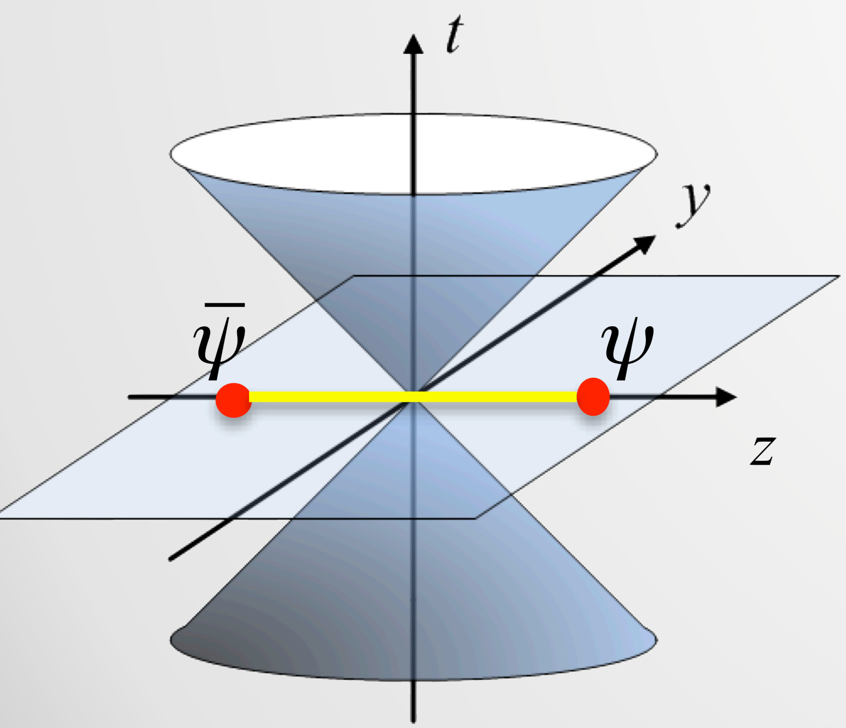


- Ji, PRL110 (2013), SCPMA57 (2014).

Light-cone PDF:

$$q(x,\mu) = \int \frac{d\xi^-}{2\pi} e^{-i\xi^-(xP^+)} \langle P | \bar\psi(\xi^-) \frac{\gamma^+}{2} W[\xi^-,0]\psi(0) | P\rangle$$

$$\xi^\pm = \frac{t \mp z}{\sqrt{2}}$$



Quasi-PDF:

$$\tilde q(x, P^z, \mu) = \int \frac{dz}{2\pi} e^{iz(xP^z)} \langle P | \bar\psi(z) \frac{\Gamma}{2} W[z,0]\psi(0) | P\rangle$$

$$\Gamma = \gamma^z \text{ or } \gamma^t$$

Systematic procedure of calculating the PDFs

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

Systematic procedure of calculating the PDFs

1. Simulation of the quasi PDF in lattice QCD

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
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2. Renormalization of the lattice quasi PDF, and then taking the continuum limit

Nonperturbative renormalization on the lattice:

- I. Stewart and Y.Z., PRD97 (2018);
- J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).

- Constantinou and Panagopoulos, PRD96 (2017);
- C. Alexandrou et al., ETM Collaboration, NPB923 (2017).

Systematic procedure of calculating the PDFs

3. Subtraction of power corrections

- O Nachtmann, NPB63 (1973);
- J.W. Chen et al. (LP3), NPB911 (2016).

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$$q(x) \cdot O\left(\frac{\Lambda_{\rm QCD}^2}{x^2(1-x)P_z^2}\right)$$

Renormalon contribution to the power correction:

Braun, Vladimirov, and Zhang, PRD99 (2019).

Systematic procedure of calculating the PDFs

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

4. Matching to the PDF.

- Matching for the quasi-PDF:
  - X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
  - I. Stewart and Y.Z., PRD97 (2018);
  - Y.-S. Liu, Y.Z. et al. (LP3), arXiv:1807.06566;
  - T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
  - Y.-S. Liu, Y.Z. et al., arXiv:1810.10879;
  - Y.Z., Int.J.Mod.Phys. A33 (2019);
  - C. Alexandrou et al. (ETMC), arXiv:1902.00587.

- The gluon case:
  - W. Wang, S. Zhao, and R. Zhu, EPJC 78 (2018);
  - W. Wang and S. Zhao, JHEP 05 (2018);
  - W. Wang, J.-H. Zhang, S. Zhao, and R. Zhu, arXiv:1904.00978.

Systematic procedure of calculating the PDFs

5. Extract $q(y)$

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

4. Matching to the PDF.

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- Y.-S. Liu, Y.Z. et al., arXiv:1810.10879;
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- W. Wang and S. Zhao, JHEP 05 (2018);
- W. Wang, J.-H. Zhang, S. Zhao, and R. Zhu, arXiv: 1904.00978.

Renormalization of the Wilson-line operator

Multiplicative renormalizability in coordinate space:

$$\tilde{O}(z,\mu) = Z_{j_1}^{-1} Z_{j_2}^{-1} e^{-\delta m|z|} \tilde{O}(z,\epsilon)$$

- **δm renormalizes linear divergence in Wilson line self energy (under a gauge invariant UV regularization);**
- **Z_j renormalizes logarithmic divergences.**

- X. Ji, J.-H. Zhang, and Y.Z., PRL120 (2018);
- J. Green et al., PRL121 (2018);
- T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, PRD96 (2017).

Gluon case:

- Zhang et al., PRL 122 (2019);
- Li et al., PRL122 (2019);

Renormalization on lattice

- Lattice perturbation theory;
  - Xiong, Luu, and Meißner, arXiv:1705.00246;
  - Ishikawa, Ma, Qiu and Yoshida, arXiv:1609.02018;
  - Constantinou and Panagopoulos, PRD96 (2017);
- Static quark-antiquark potential;
  - B. Musch et al., PRD 83 (2011);
  - Ishikawa, Ma, Qiu and Yoshida, arXiv:1609.02018;
  - J.-H. Zhang et al. (LP3), PRD 95 (2017).
- Smeared quasi-PDF in the gradient flow method;
  - C. Monahan and K. Orginos, JHEP 1703 (2017);
  - C. Monahan, PRD 97 (2018)
- **Regularization-independent momentum subtraction (RI/MOM) scheme;**
  - I. Stewart and Y.Z., PRD97 (2018);
  - J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).
  - Constantinou and Panagopoulos, PRD96 (2017);
  - C. Alexandrou et al., ETM Collaboration, NPB923 (2017).
- Reduced Ioffe-time distribution.
  - can be regarded as a nonperturbative renormalization at short distances.

A. Radyushkin, PRD96 (2017); K. Orginos et al., PRD96 (2017).

RI/MOM Scheme

Green's function:

$$G(z,p) = \sum_x \left\langle \gamma_5 S^\dagger(p, z+x)\gamma_5 U(z+x, x)\frac{\Gamma}{2}S(p,x)\right\rangle$$

Amputated Green's function (or vertex function):

$$\Lambda(z,p) = \left(\gamma_5 \left[S^{-1}(p)\right]^\dagger\right) G(z,p) S^{-1}(p)$$

Momentum subtraction condition:

$$Z_{\mathcal{O}}^{-1}(z, p_z^R, \mu_R)\; G(z,p)\Big|_{p_\mu = p_\mu^R} = G^{\text{tree}}(z,p) = e^{ip\cdot z}\Gamma$$

RI/MOM:

- I. Stewart and Y.Z., PRD97 (2018);
- J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).

$$Z_{\mathcal{O}}^{-1}(z, p_z^R, \mu_R) Z_q(\mu_R)\; G(z,p)\Big|_{p_\mu = p_\mu^R} = G^{\text{tree}}(z,p)\,, \quad Z_q(\mu_R) = \frac{1}{12}\mathrm{Tr}\left[S^{-1}(p)S^{\text{tree}}(p)\right]\Big|_{p^2=\mu_R^2}$$

RI/MOM':

- Constantinou and Panagopoulos, PRD96 (2017);
- C. Alexandrou et al., ETM Collaboration, NPB923 (2017).

In continuum theoy, there is no difference at one-loop order in the Landau gauge, because the quark wavefunction renormalization is zero.

Choice of projection operator

- Parametrization of amputated Green's functions:

$$\Lambda_{\gamma^t}(z,p) = \tilde{F}_t\gamma^t \;+\; \tilde{F}_z\frac{p^t\gamma^z}{p^z} \;+\; \tilde{F}_{\not p}\frac{p^t\not p}{p^2}$$

$$\Lambda_{\gamma^z}(z,p) = \tilde{F}_z\gamma^z \;+\; \tilde{F}_{\not p}\frac{p^z\not p}{p^2}$$

  - Red terms which are proportional to the tree-level Green's functions include all the UV divergences;
  - Choice of projection must include the red terms.

Minimal projection: $\mathrm{Tr}\left[\Lambda_{\gamma^t}(z,p)P_{mp}\right] = \tilde{F}_t$

- Y.-S. Liu, Y.Z. et al. (LP3), arXiv:1807.06566.

P-slash projection: $\mathrm{Tr}\left[\Lambda_{\gamma^t}(z,p)\not p\right]/(4p^t) = \tilde{F}_t \;+\; \tilde{F}_z \;+\; \tilde{F}_{\not p}$

γ^t projection: $\mathrm{Tr}\left[\Lambda_{\gamma^t}(z,p)\gamma^t\right]/4 = \tilde{F}_t \;+\; \tilde{F}_{\not p}\frac{p_t^2}{p^2}$

Operator mixing on lattice

- Due to chiral symmetry breaking, the vector-like nonlocal Wilson-line operator could also mix with the scalar operator

$$\bar{\psi}(z)\mathbf{1}P\exp\left[-ig\int^{z}dz'A^{z}(z')\right]\psi(0)$$

  - For $\Gamma=\gamma^z$, mixing starts at $O(a^1)$;
  - For $\Gamma=\gamma^t$, mixing starts at $O(a^0)$.

- Constantinou and Panagopoulos, PRD96 (2017);
- J. Green et al., PRL121 (2018);
- J. W. Chen et al. (LP3), arXiv:1710.01089.

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Regularization-independence in the renormalized quasi-PDF

- Continuum limit of the renormalized matrix element:

$$\lim_{a\to 0} \frac{\langle P|\tilde{O}(z,a)|P\rangle}{Z_{\mathcal{O}}(z,p_\mu^R,\mu_R,a)} = \frac{\langle P|\tilde{O}(z,\epsilon)|P\rangle}{Z_{\mathcal{O}}(z,p_\mu^R,\mu_R,\epsilon)}$$

$$D = 4 - 2\epsilon$$

- Regularization-independence allows the matching to be done in continuum perturbation theory (with dimensional regularization.)

Two matching strategies

- Constantinou and Panagopoulos, PRD96 (2017);
- C. Alexandrou et al., ETM Collaboration, NPB923 (2017).

$$
\begin{tikzcd}
\dfrac{\langle P|\tilde{O}(z,\epsilon)|P\rangle}{Z_{\mathcal{O}}(z,p^R_\mu,\mu_R,\epsilon)} \arrow[r] \arrow[dd, "\text{F.T.}"'] & \times\dfrac{Z_{\mathcal{O}}(z,p^R_z,\mu_R,\epsilon)}{Z^{\overline{\text{MS}}}_{\mathcal{O}}(\epsilon,\mu)} = \dfrac{\langle P|\tilde{O}(z,\epsilon)|P\rangle}{Z^{\overline{\text{MS}}}_{\mathcal{O}}(\epsilon,\mu)} \arrow[d, "\text{F.T.}"] \\
 & \tilde{q}(x,P^z,\mu) \arrow[d, "\text{Matching}"] \\
\tilde{q}(x,P^z,\mu_R,p^R_z) \arrow[r, "\text{Matching}"] & \tilde{q}(x,\mu)
\end{tikzcd}
$$

- I. Stewart and Y.Z., PRD97 (2018);
- J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).

One-step matching

- Matching formula:

• I. Stewart and Y.Z., PRD97 (2018);

$$\tilde{q}(x, P^z, \mu_R, p_z^R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_z^R}\right) q(y, \mu) + O(1/P_z^2)$$

$$r = \frac{\mu_R^2}{(p_z^R)^2}$$

- Matching kernel:

$$C\left(\xi, r, \frac{p^z}{\mu}, \frac{p^z}{p_z^R}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{2\pi}\left[C_B\left(\xi, \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_z^R}\right| h\left(1 + \frac{p^z}{p_z^R}(\xi - 1), r\right)\right]_+^{(-\infty,\infty)}$$

$\xi = \frac{x}{y}, \quad p^z = yP^z$

$$\left[f(x)\right]_+^{(-\infty,\infty)} = f(x) - \delta(x-1)\int_{-\infty}^{\infty} dy\, f(y)$$

😄Formally satisfying vector current (or particle number conservation): $\int_{-\infty}^{\infty} d\xi\; C(\xi) = 1$

😒Renormalization scale dependence to be cancelled in the final result, making systematics analysis more complicated.

One-step matching

For $\Gamma = \gamma^z$, Feynman Gauge

- I. Stewart and Y.Z., PRD97 (2018);

$$\left[C_B\left(\xi, \frac{p^z}{\mu}\right)\right]_+^{-\infty,\infty} = \begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1\right]_\oplus & \xi>1 \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{4(p^z)^2}{\mu^2}+\frac{1+\xi^2}{1-\xi}\ln\left[\xi(1-\xi)\right]+(2-\xi)-\frac{2\xi}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}-1\right]_\ominus & \xi<0 \end{cases}$$

$$h(x,\rho) \equiv \begin{cases} \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x}-\frac{\rho}{2(1-x)}\right]\ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}}-\frac{\rho}{4x(x-1)+\rho}+1 & x>1 \\ \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x}-\frac{\rho}{2(1-x)}\right]\ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}}-\frac{2x}{1-x} & 0<x<1 \\ \frac{1}{\sqrt{1-\rho}}\left[\frac{1+x^2}{1-x}-\frac{\rho}{2(1-x)}\right]\ln\frac{2x-1-\sqrt{1-\rho}}{2x-1+\sqrt{1-\rho}}+\frac{\rho}{4x(x-1)+\rho}-1 & x<0 \end{cases}$$

Well-behaving matching kernel

$$C\left(\xi, r, \frac{p^z}{\mu}, \frac{p^z}{p_z^R}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{2\pi}\left[C_B\left(\xi, \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_z^R}\right| h\left(1 + \frac{p^z}{p_z^R}(\xi - 1), r\right)\right]_+^{(-\infty,\infty)}$$

Singularity at ξ=1is regulated by plus function.

Asymptotic region:

$$\lim_{\xi\to\infty} C_B\left(\xi, \frac{p^z}{\mu}\right) = -\frac{3}{2|\xi|} \qquad \lim_{\xi\to\infty}\left|\frac{p^z}{p_z^R}\right| h\left(1 + \frac{p^z}{p_z^R}(\xi - 1), r\right) = -\frac{3}{2|\xi|}$$

$$\lim_{\xi\to\infty}\left[C_B\left(\xi, \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_z^R}\right| h\left(1 + \frac{p^z}{p_z^R}(\xi - 1), r\right)\right] \sim \frac{1}{\xi^2}$$

$$\int_{-\infty}^{\infty} d\xi' \left[C_B\left(\xi', \frac{p^z}{\mu}\right) - \left|\frac{p^z}{p_z^R}\right| h\left(1 + \frac{p^z}{p_z^R}(\xi' - 1), r\right)\right] = \text{UV finite}$$

Two-step matching

- Scheme conversion:

- Constantinou and Panagopoulos, PRD96 (2017)

$$C(z, p_z^R, \mu_R, \mu) = \frac{Z_{\mathcal{O}}(z, p_z^R, \mu_R, \epsilon)}{Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}(\epsilon, \mu)}$$

Renormalization scale dependence to be cancelled in the first step, useful for the systematics analysis.

- Matching in the MSbar scheme:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018)

Two-step matching

$$C_{V_\nu(A_\nu)} = 1 - \frac{g^2\,C_f}{16\,\pi^2}\Bigg(-7-4\gamma_E+\log(16)+4F_2+\frac{4\bar{q}_\nu^2|z|}{\bar{q}}F_5+(\beta-4)\log\left(\frac{\bar{\mu}^2}{\bar{q}^2}\right)-(\beta+2)\log(\bar{q}^2z^2)$$
$$+\beta\Big[3-2\gamma_E+\log(4)-2\left(\frac{\bar{q}_\nu^2|z|}{\bar{q}}F_4+\left(\bar{q}^2+\bar{q}_\mu^2\right)G_3\right)-2F_1+z^2\left(\bar{q}^2\left(F_3-\frac{F_1-F_2}{2}\right)+\bar{q}_\nu^2(F_1-F_2-F_3)\right)\Big]$$
$$+i\left\{4\bar{q}_\mu G_1+\beta\bar{q}_\mu\Big[\bar{q}(z|z|F_5+2(G_4-2G_5))-2G_1+2G_2\Big]\right\}\Bigg) \qquad (36)$$

- Constantinou and Panagopoulos, PRD96 (2017)

$$C^{\overline{\mathrm{MS}}}\left(\xi,\frac{\mu}{|y|P^z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_sC_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{3}{2\xi}\right)_{+(1)}^{[1,\infty]}-\frac{3}{2\xi} & \xi>1\\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2}+\ln\left(4\xi(1-\xi)\right)\right]-\frac{\xi(1+\xi)}{1-\xi}\right)_{+(1)}^{[0,1]} & 0<\xi<1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi}-1+\frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]}-\frac{3}{2(1-\xi)} & \xi<0\end{cases}$$
$$+\frac{\alpha_sC_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{\mu^2}{4y^2P_z^2}+\frac{5}{2}\right).$$

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018)

Problems with MSbar scheme

$$C_{V_\nu(A_\nu)} = 1 - \frac{g^2\, C_f}{16\, \pi^2}\Bigg(-7-4\gamma_E+\log(16)+4F_2+\frac{4\bar{q}_\nu^2|z|}{\bar{q}}F_5+(\beta-4)\log\left(\frac{\bar{\mu}^2}{\bar{q}^2}\right)-(\beta+2)\log(\bar{q}^2z^2)$$
$$+\beta\left[3-2\gamma_E+\log(4)-2\left(\frac{\bar{q}_\nu^2|z|}{\bar{q}}F_4+\left(\bar{q}^2+\bar{q}_\mu^2\right)G_3\right)-2F_1+z^2\left(\bar{q}^2\left(F_3-\frac{F_1-F_2}{2}\right)+\bar{q}_\nu^2(F_1-F_2-F_3)\right)\right]$$
$$+i\left\{4\bar{q}_\mu G_1+\beta\bar{q}_\mu\left[\bar{q}(z|z|F_5+2(G_4-2G_5))-2G_1+2G_2\right]\right\}\Bigg) \tag{36}$$

For minimal projection

$$\lim_{z\to 0} C(z, p_z^R, \mu_R, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right]$$

- Independent of p_z^R and μ_R !
- Gauge-invariant, but the constant part depends on the quark wave function renormalization scheme;
- Breaks down vector current conservation in the MSbar quasi-PDF;
- Logarithmic divergence in the local limit could be an issue when implemented numerically.

Problems with MSbar scheme

- Breaks down vector current conservation;
- This effect could cancel that from the scheme conversion so that the final PDF satisfies vector current conservation. However, such cancellation is nontrivial when implemented numerically.

$$
C^{\overline{\mathrm{MS}}}\left(\xi,\frac{\mu}{|y|P^z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1+\frac{3}{2\xi}\right)_{+(1)}^{[1,\infty]}-\frac{3}{2\xi} & \xi>1\\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2}+\ln\big(4\xi(1-\xi)\big)\right]-\frac{\xi(1+\xi)}{1-\xi}\right)_{+(1)}^{[0,1]} & 0<\xi<1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi}-1+\frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]}-\frac{3}{2(1-\xi)} & \xi<0\end{cases}
$$

$$
+\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{\mu^2}{4y^2P_z^2}+\frac{5}{2}\right).
$$

Ratio scheme

$$C^{\rm ratio}(z, p_z^R, \mu_R, \mu) = \frac{C(z, p_z^R, \mu_R, \mu)}{1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right]}$$

$$\lim_{z\to 0} C(z, p_z^R, \mu_R, \mu) = 1$$

- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
- Y.Z., Int.J.Mod.Phys. A33 (2019);
- A. Radyushkin, PLB781 (2018).

$$C^{\rm ratio}\left(\xi, \frac{\mu}{|y|P^z}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)}^{[1,\infty]} & \xi > 1\\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2} + \ln\left(4\xi(1-\xi)\right) - 1\right] + 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[0,1]} & 0 < \xi < 1\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}^{[-\infty,0]} & \xi < 0\end{cases}$$

- Satisfying vector current conservation at each step;
- Well-behaving matching kernel; $\lim_{\xi\to\infty} C^{\rm ratio}\left(\xi, \frac{p^z}{\mu}\right) \sim \frac{1}{\xi^2}$
- Actually,

$$\mathrm{F.T.}\left\{\frac{\alpha_s C_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right]\right\} = C^{\overline{\rm MS}}(\xi, \frac{\mu}{p^z}) - C^{\rm Ratio}(\xi, \frac{\mu}{p^z})$$

Modified MSbar scheme

$$C^{\mathrm{MMS}}(z,p_z^R,\mu_R,\mu)=\frac{C(z,p_z^R,\mu_R,\mu)}{Z^{\mathrm{M}\overline{\mathrm{MS}}}_{\Gamma_{\gamma^0}}(z\bar{\mu})}$$

$$\begin{aligned}Z^{\mathrm{M}\overline{\mathrm{MS}}}_{\Gamma_{\gamma^0}}(z\bar{\mu}) &= 1-\frac{\alpha_s}{2\pi}C_F\left(\frac{3}{2}\ln\left(\frac{1}{4}\right)+\frac{5}{2}\right)\\ &+\frac{3}{2}\frac{\alpha_s}{2\pi}C_F\left(i\pi\frac{|z\bar{\mu}|}{2z\bar{\mu}}-\mathrm{Ci}(z\bar{\mu})+\ln(z\bar{\mu})-\ln(|z\bar{\mu}|)-i\mathrm{Si}(z\bar{\mu})\right)\\ &-\frac{3}{2}\frac{\alpha_s}{2\pi}C_F e^{iz\bar{\mu}}\left(\frac{2\mathrm{Ei}(-iz\bar{\mu})-\ln(-iz\bar{\mu})+\ln(iz\bar{\mu})+i\pi\mathrm{sgn}(z\bar{\mu})}{2}\right)\end{aligned}$$

$$\lim_{z\to 0}C(z,p_z^R,\mu_R,\mu)=1$$

$$C^{\mathrm{M}\overline{\mathrm{MS}}}_{\gamma^0,\gamma^3,\gamma^3\gamma^5}\left(\xi,\frac{\bar{\mu}}{p_3}\right)=\delta(1-\xi)$$

$$+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right)+1+\frac{3}{2\xi}\right)_{+(1)}, & \xi>1,\\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\left(\frac{p_3^2}{\bar{\mu}^2}\right)+\ln\left(4\xi(1-\xi)\right)\right]-\frac{\xi(1+\xi)}{1-\xi}+2\iota(1-\xi)\right)_{+(1)}, & 0<\xi<1,\\ \left(-\frac{1+\xi^2}{1-\xi}\ln\left(\frac{-\xi}{1-\xi}\right)-1+\frac{3}{2(1-\xi)}\right)_{+(1)}, & \xi<0,\end{cases}$$

- C. Alexandrou et al. (ETMC), arXiv:1902.00587.

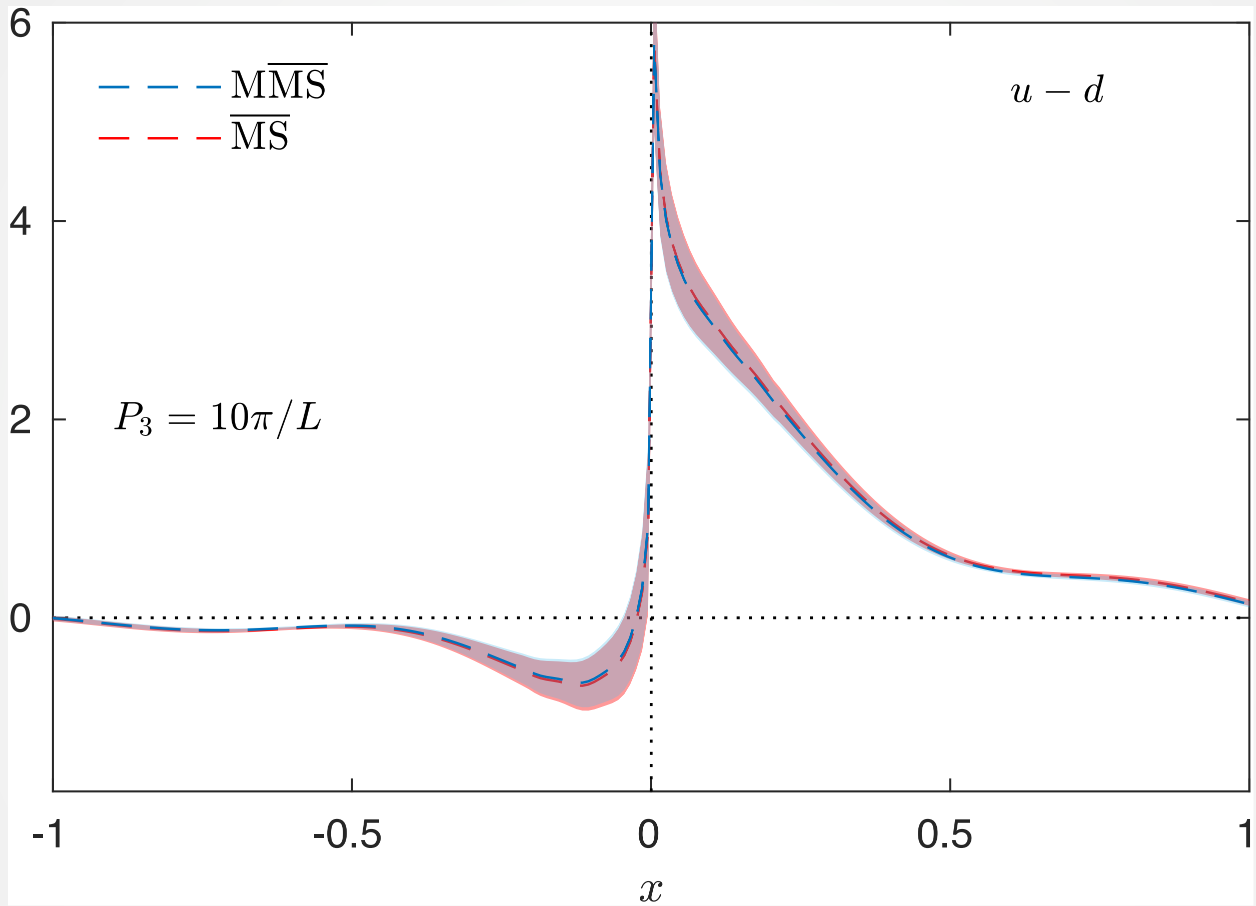
$$\mathrm{F.T.}\left\{Z^{\mathrm{MMS}}_{\Gamma\gamma^0}(z\mu)\right\}=C^{\overline{\mathrm{MS}}}(\xi,\frac{\mu}{p^z})-C^{\mathrm{MMS}}(\xi,\frac{\mu}{p^z})$$

Comparison among different schemes

- C. Alexandrou et al. (ETMC), arXiv:1902.00587.

- There is still discrepancy among different schemes for the same lattice input data.

Comparison among different schemes



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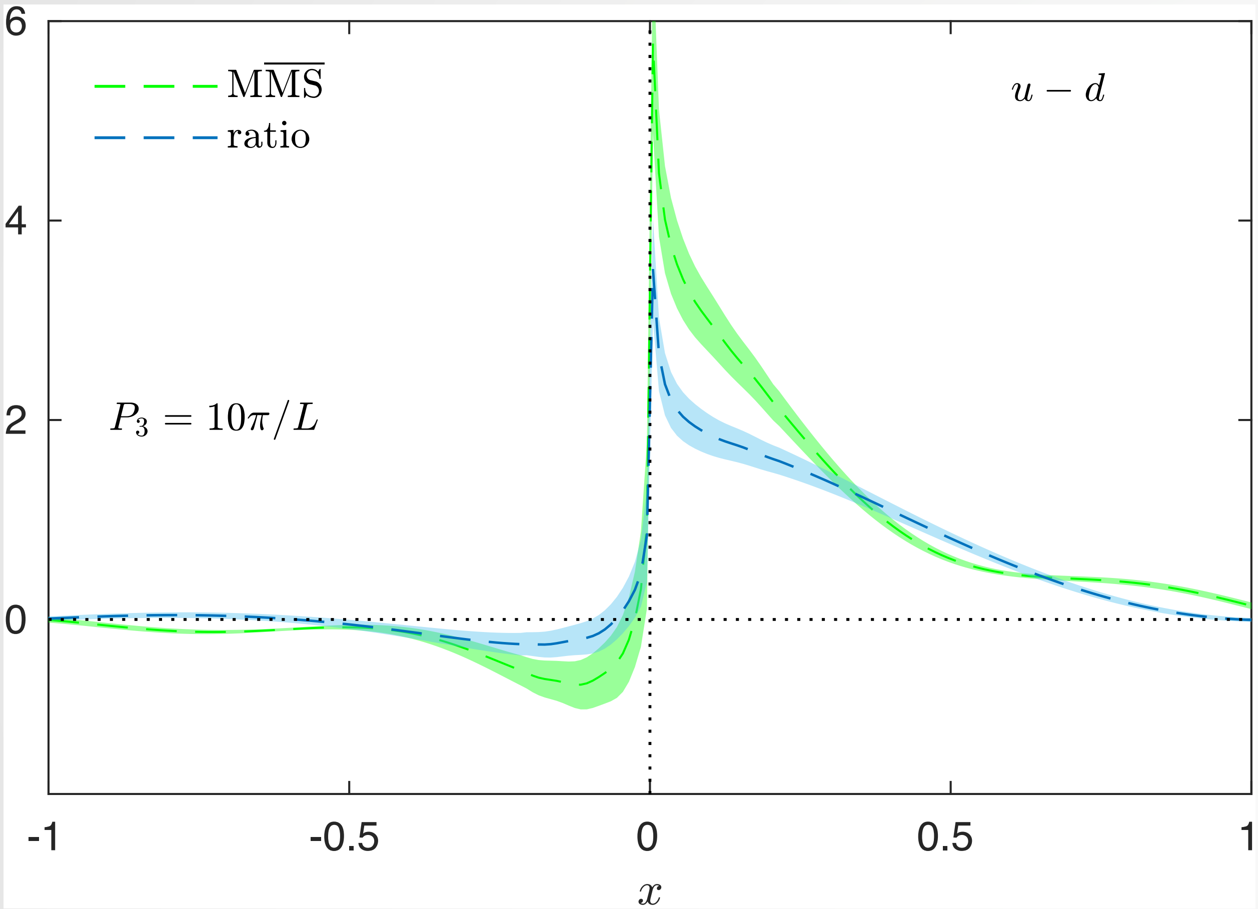
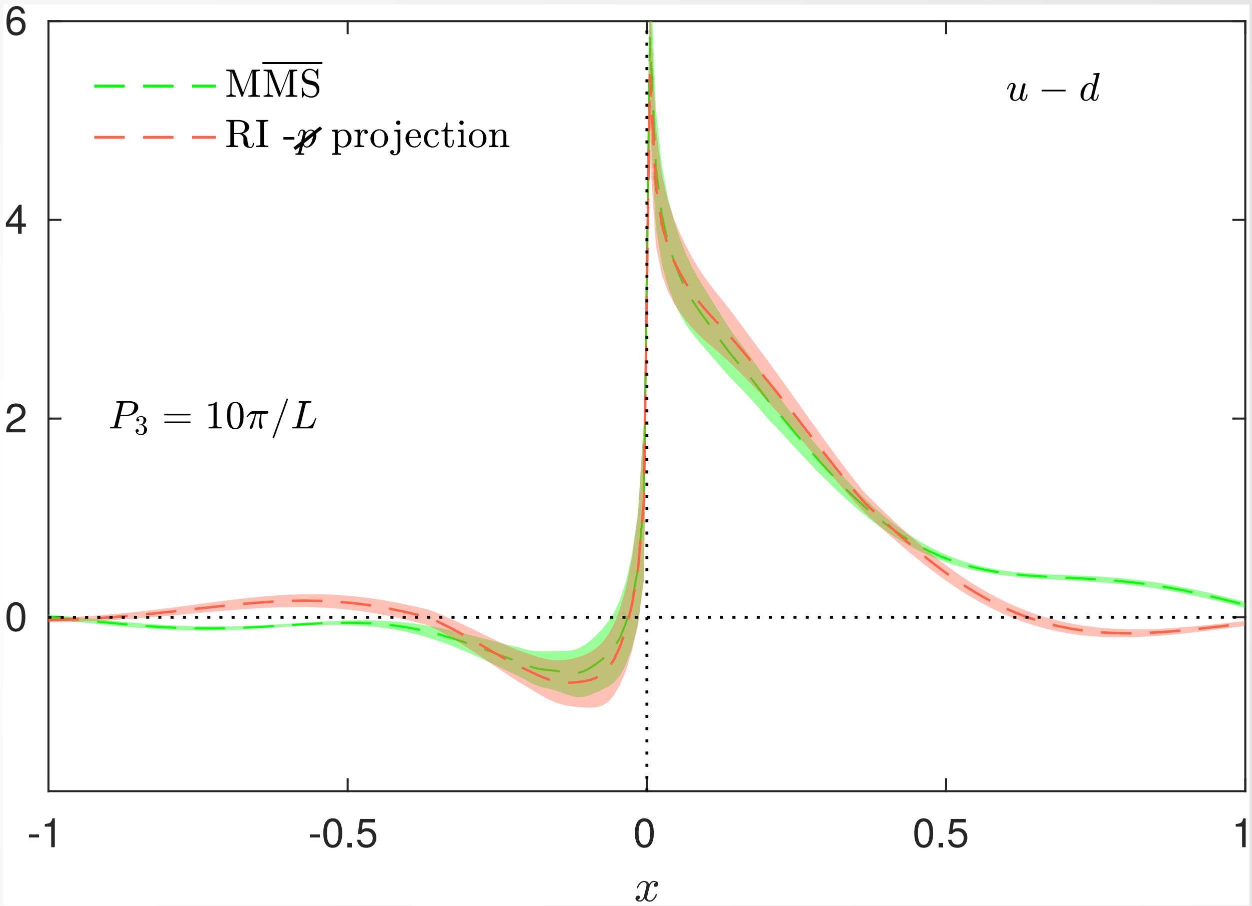




- C. Alexandrou et al. (ETMC), arXiv:1902.00587.

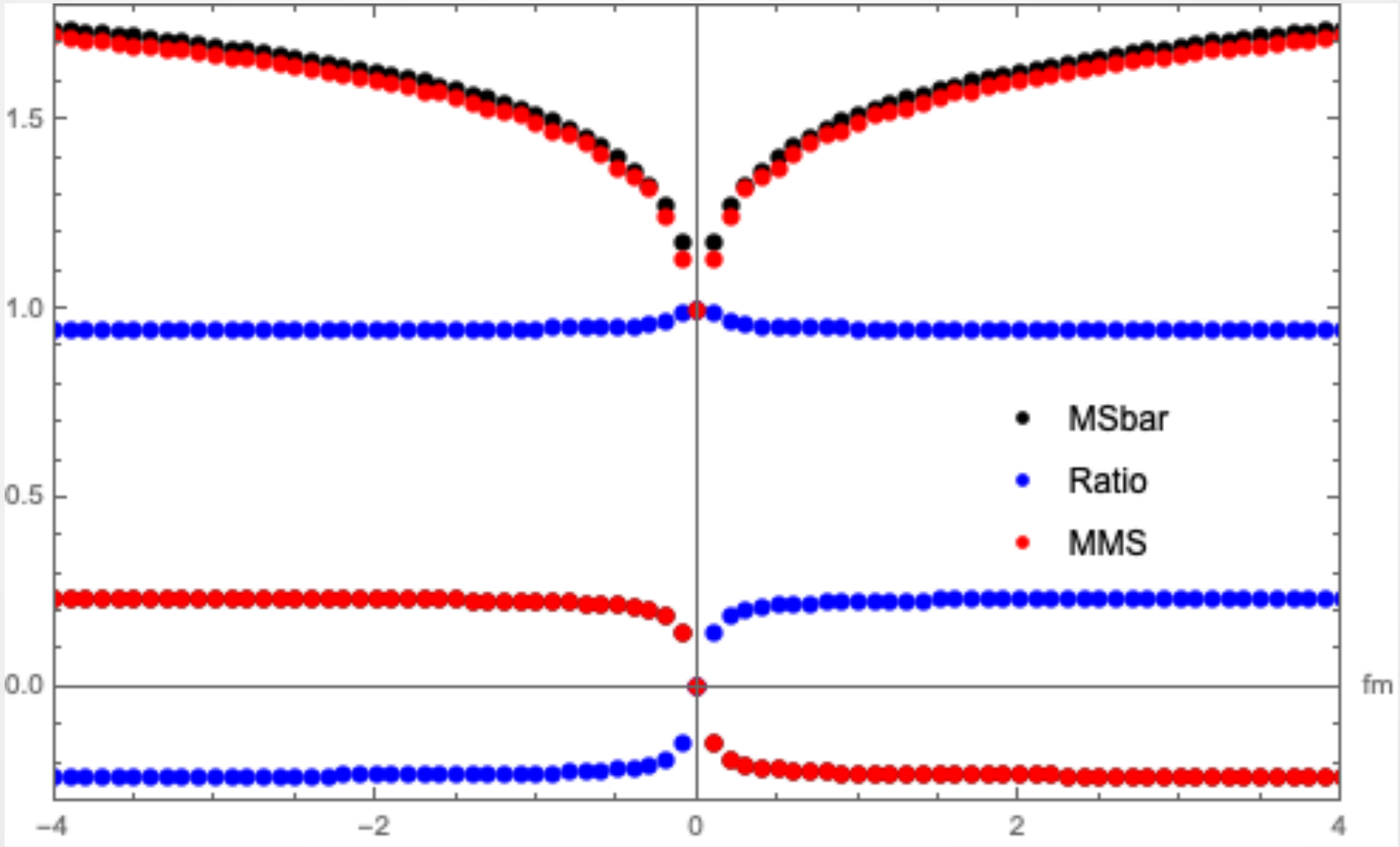
- There is still discrepancy among different schemes for the same lattice input data.

Re-examine the differences among various strategies and schemes

- Conversion factors:

$$P^z = 3.0\ \mathrm{GeV};\ \mu = 3.0\ \mathrm{GeV};\ p_z^R = 2.2\ \mathrm{GeV};\ \mu_R = 3.7\ \mathrm{GeV};\ \alpha_s = 0.258.$$

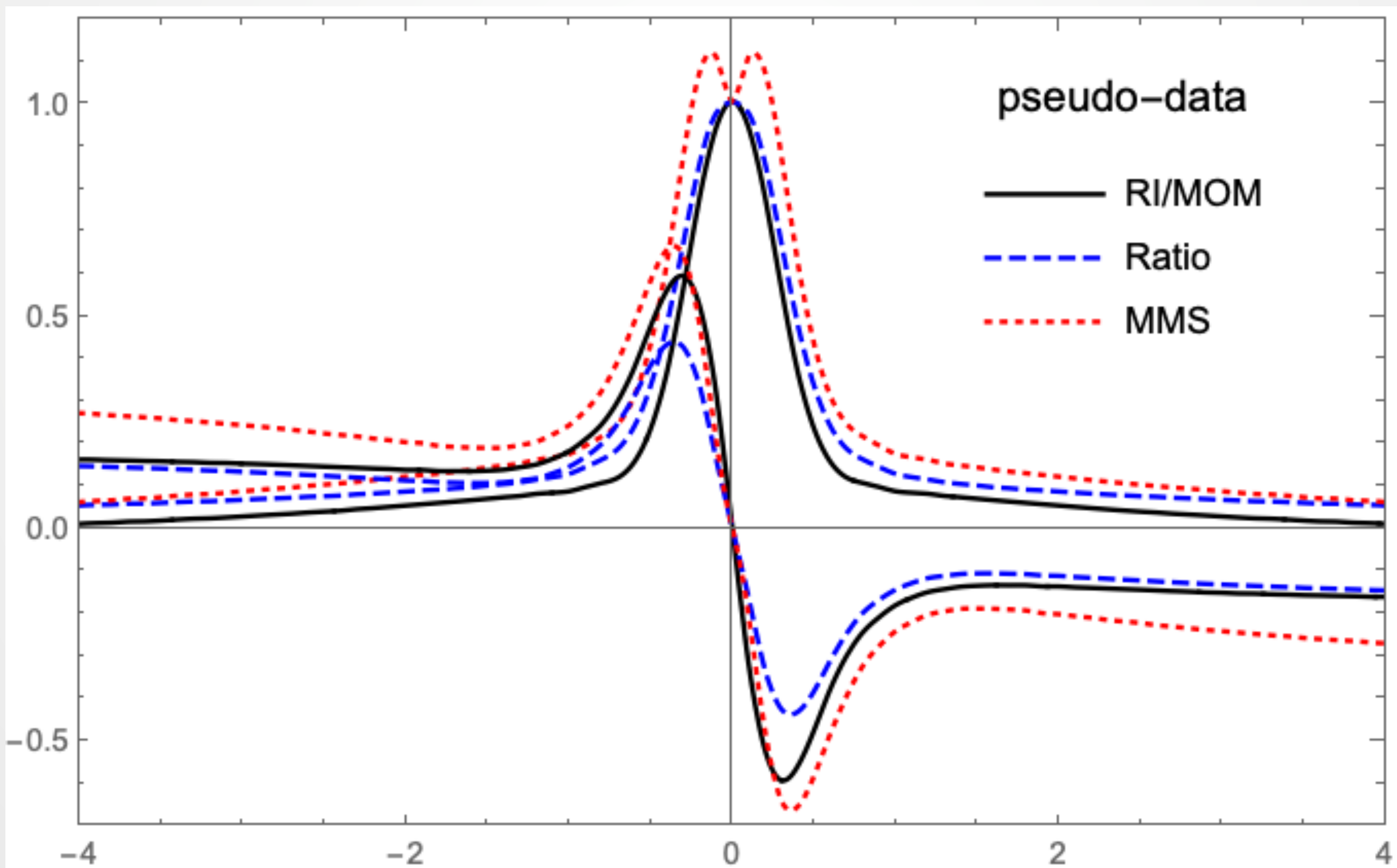
In collaboration with Yu-Sheng Liu



$$\int_{-z_{\max}}^{z_{\max}} \frac{d(zp^z)}{2\pi} e^{i\xi z p^z}\, Z_{\Gamma\gamma^0}^{\mathrm{MMS/Ratio}}(z\mu) \neq C^{\overline{\mathrm{MS}}}(\xi, \frac{\mu}{p^z}) - C^{\mathrm{MMS/Ratio}}(\xi, \frac{\mu}{p^z})$$

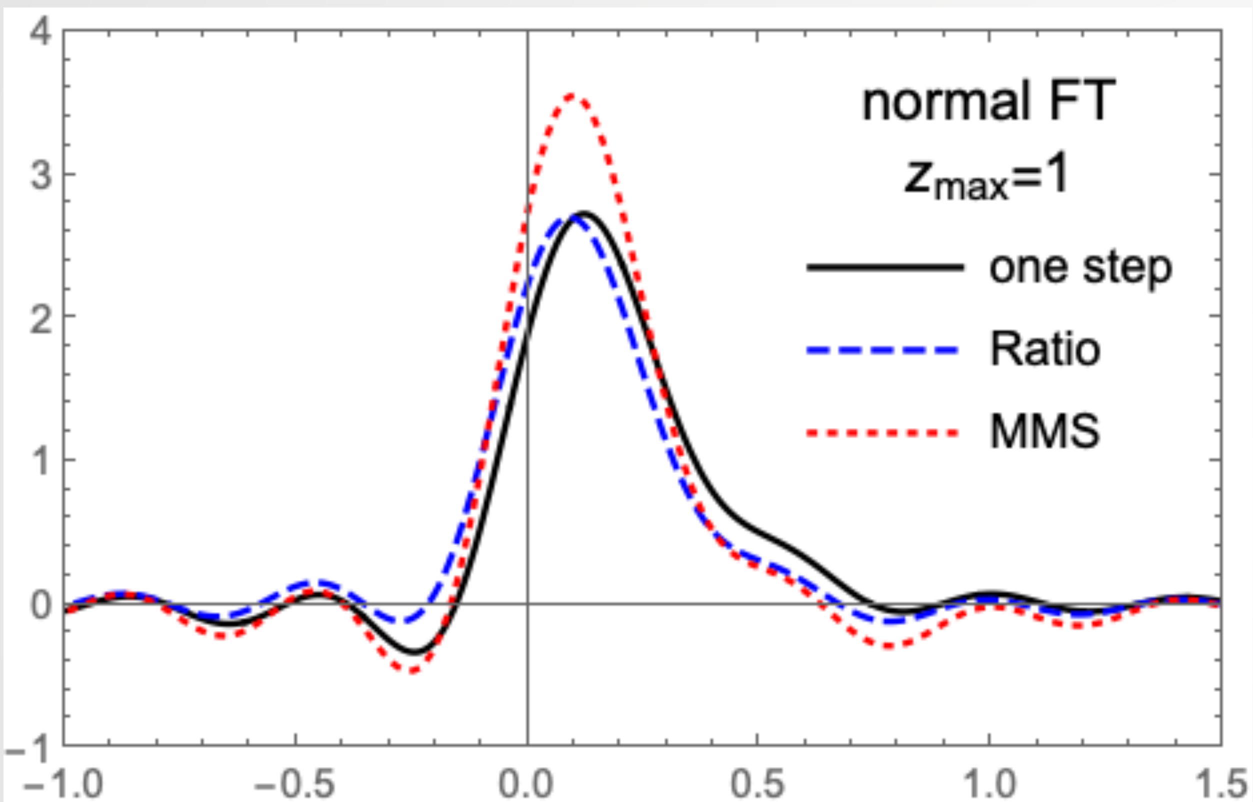
- **Conversion of pseudo data for the RI/MOM matrix element:**

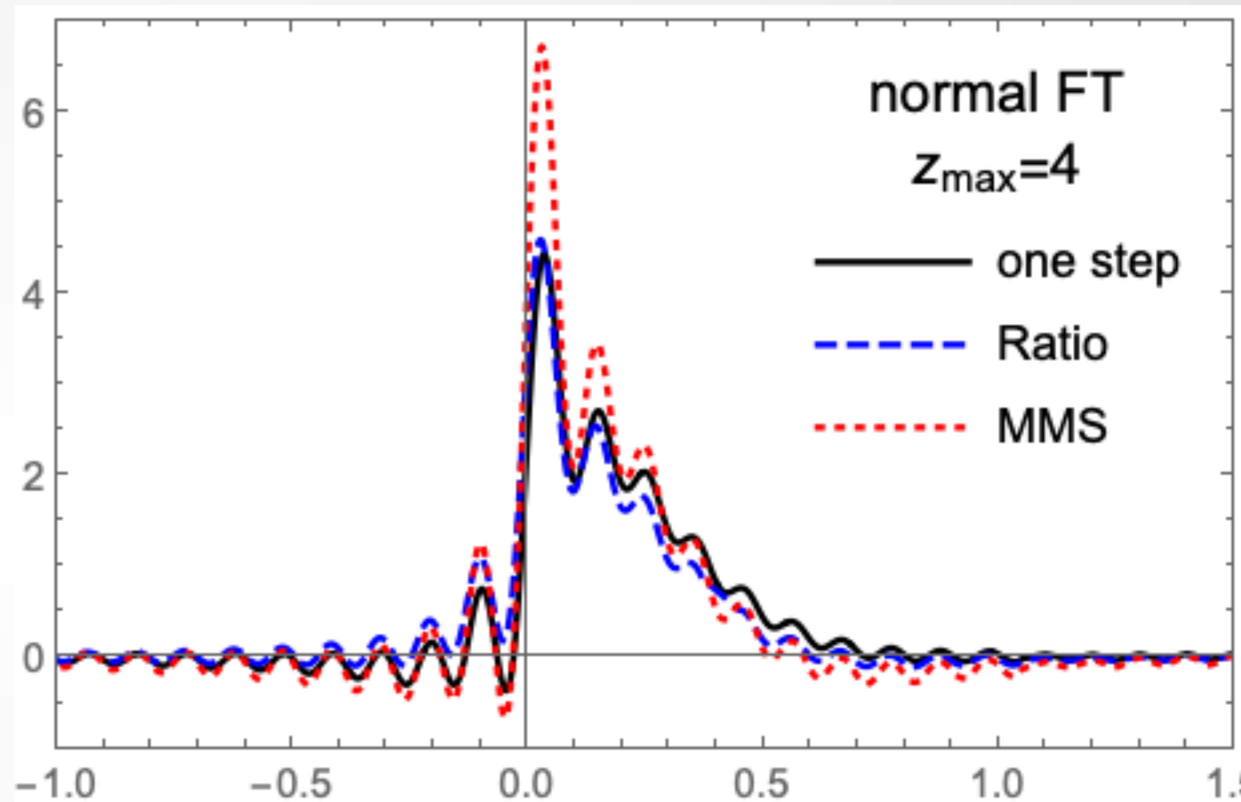
$$P^z = 3.0 \text{ GeV}; \mu = 3.0 \text{ GeV}; p_z^R = 2.2 \text{ GeV}; \mu_R = 3.7 \text{ GeV}; \alpha_s = 0.258.$$



fm

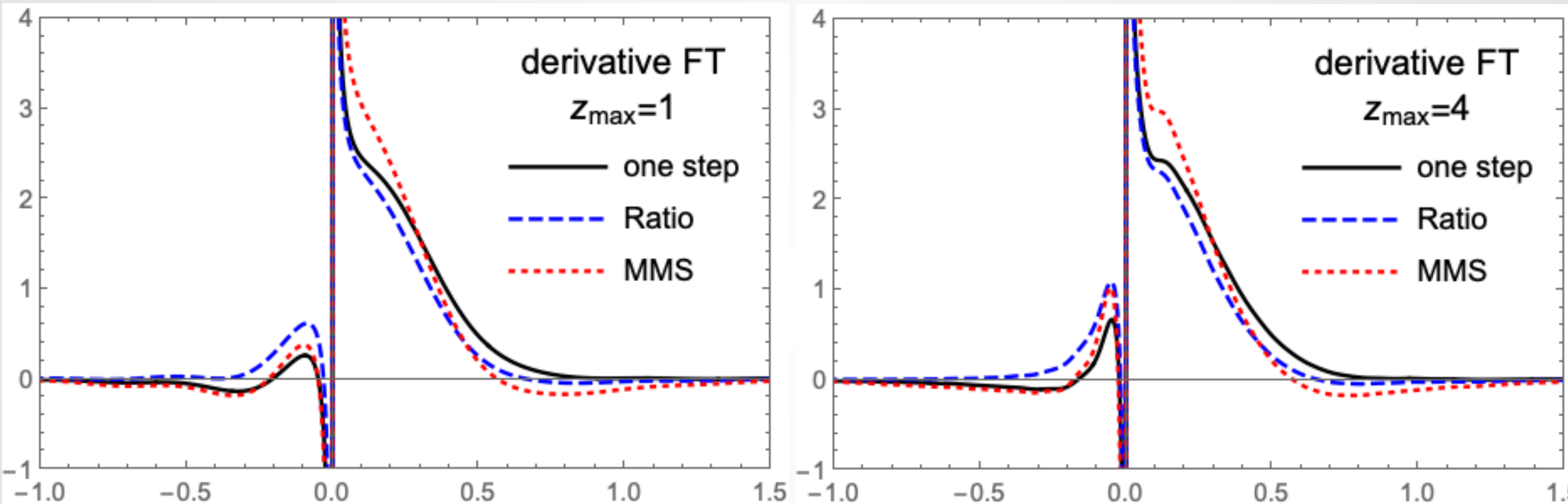
Fourier transform





“One step” standards for the pseudo RI/MOM matrix elements.

Fourier transform



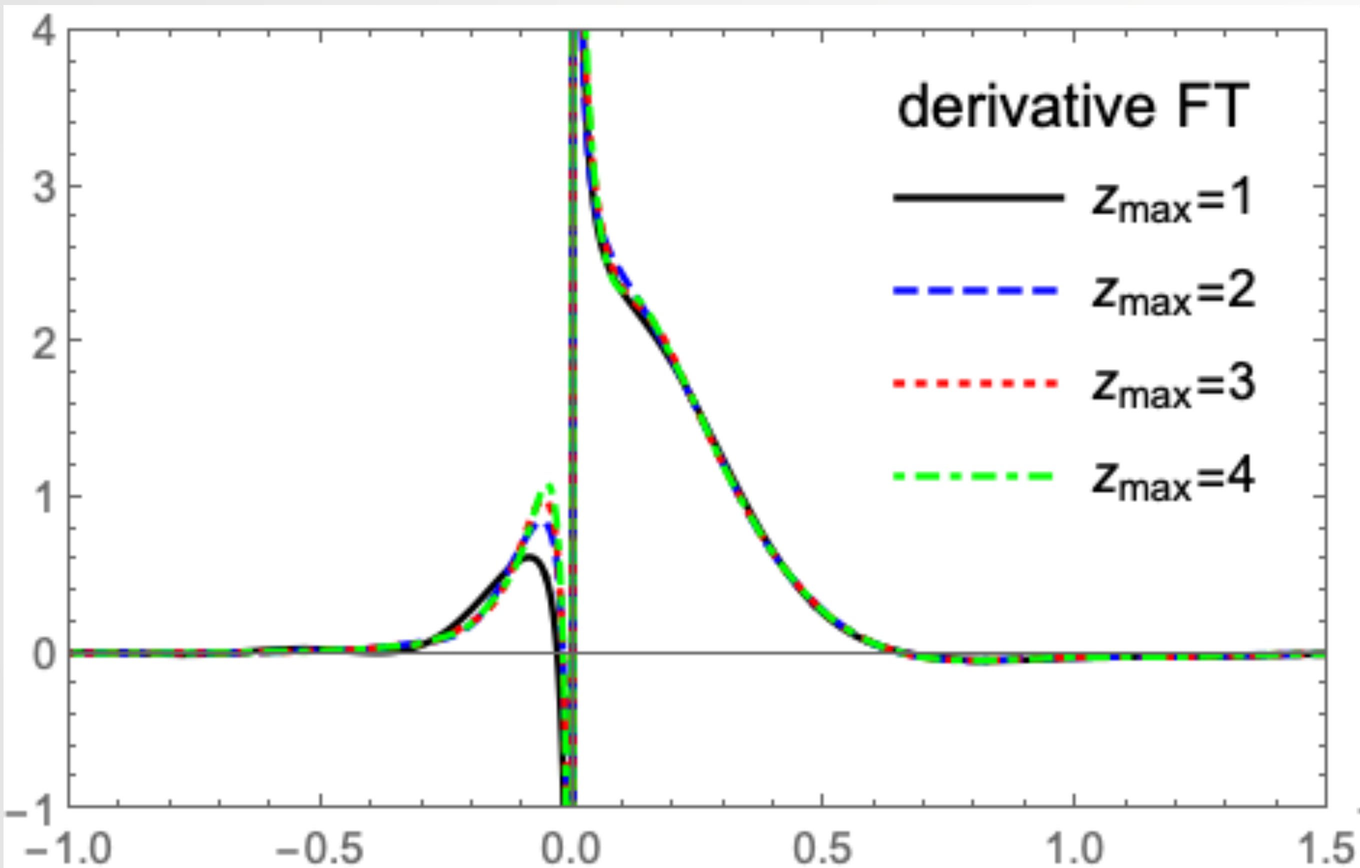
"One step" standards for the pseudo RI/MOM matrix elements.

Derivative method:

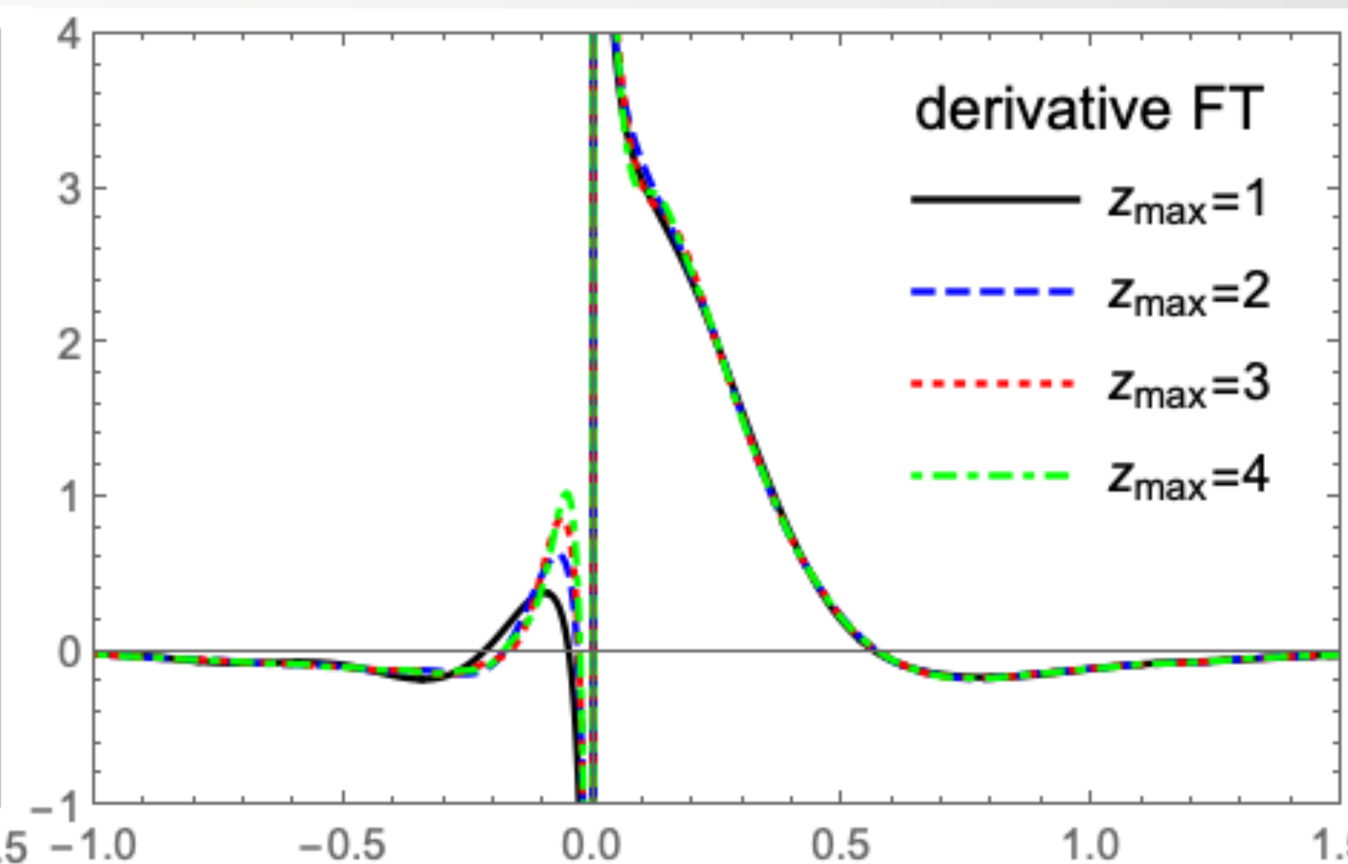
$$\tilde{q}(x) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixzP^z} \frac{ih'(z, P^z)}{x}$$

Sensitivity to z_{max}

Ratio Scheme

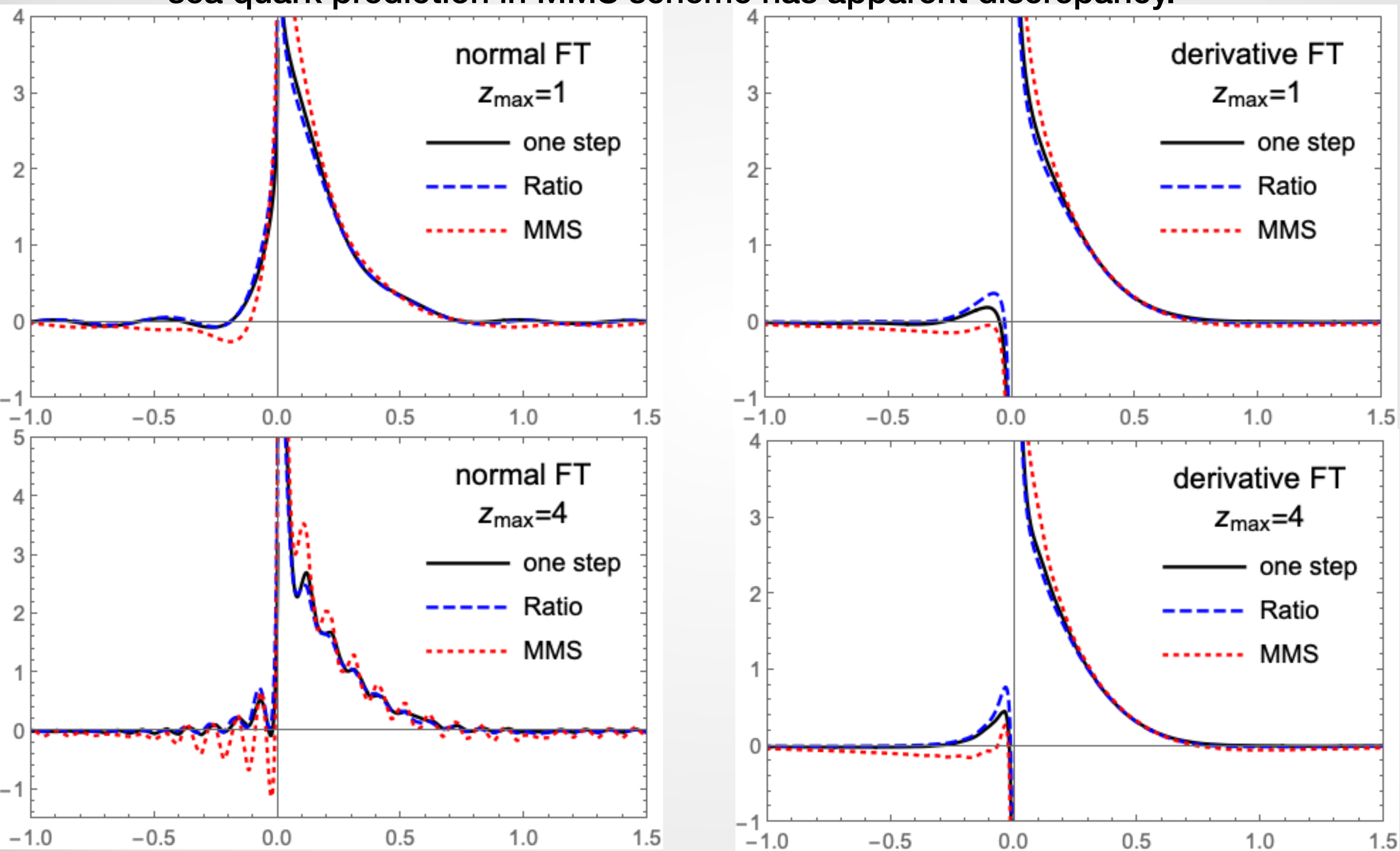


MMS Scheme



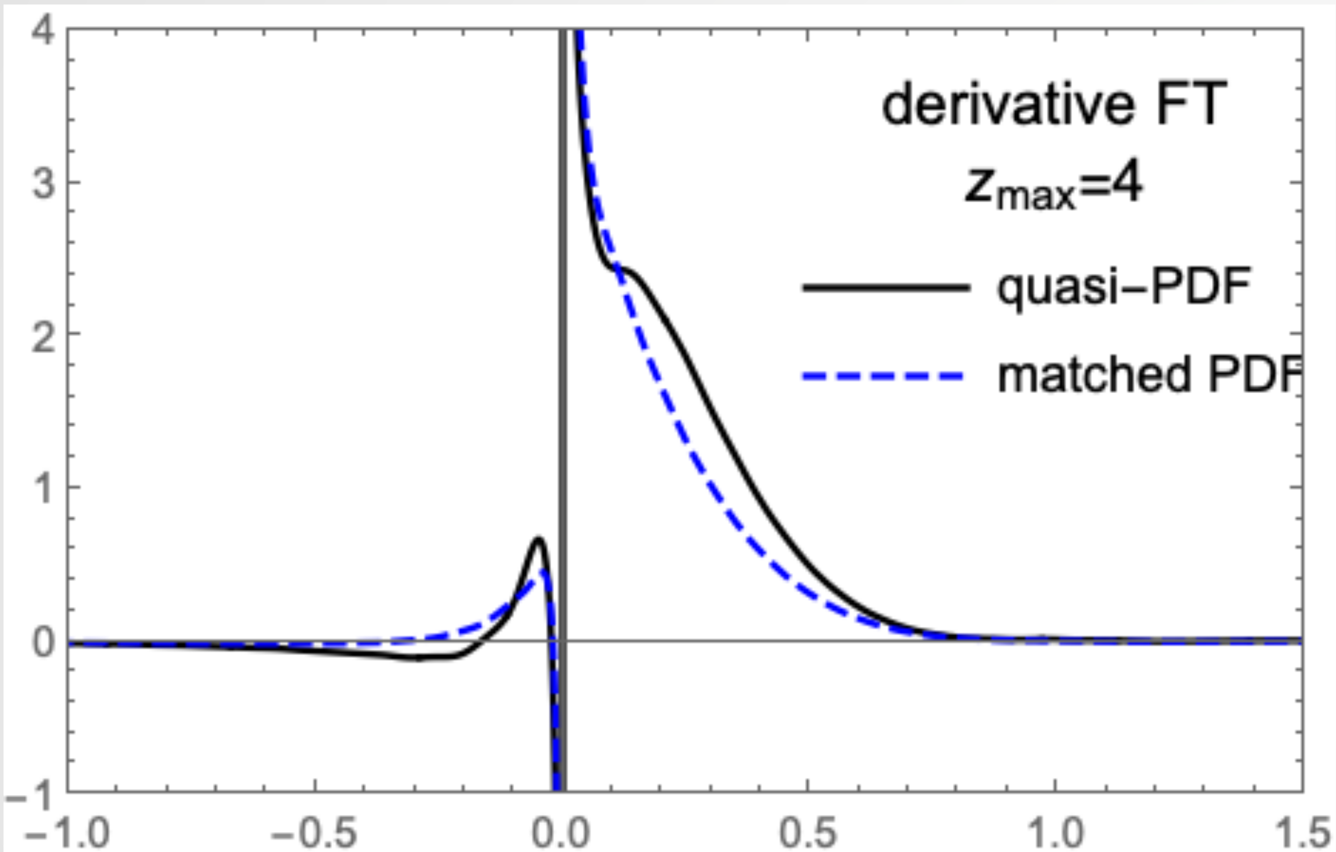
Final Result of the PDF

Ratio scheme shows more consistency with the one-step matching; sea quark prediction in MMS scheme has apparent discrepancy.

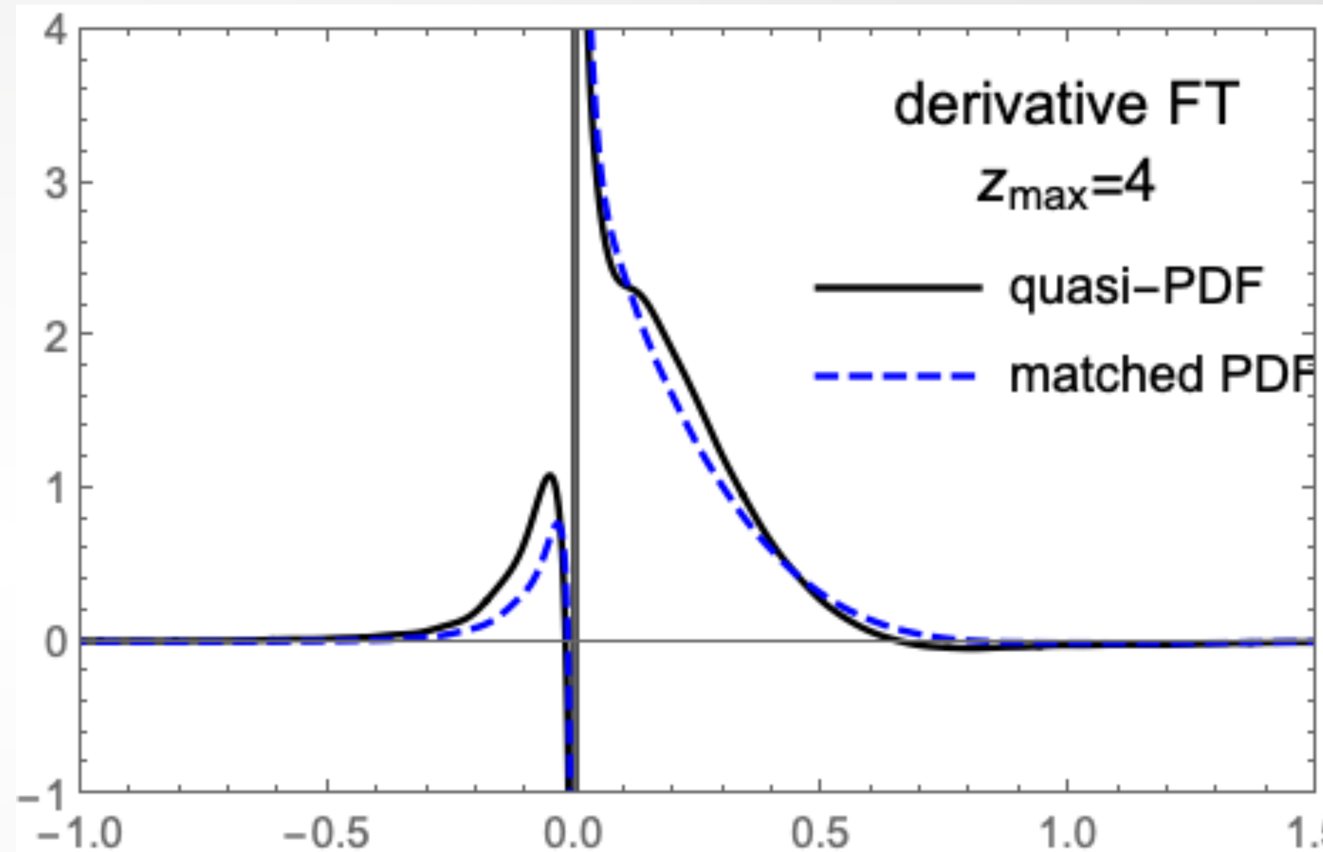


Effect of Matching

One-step matching



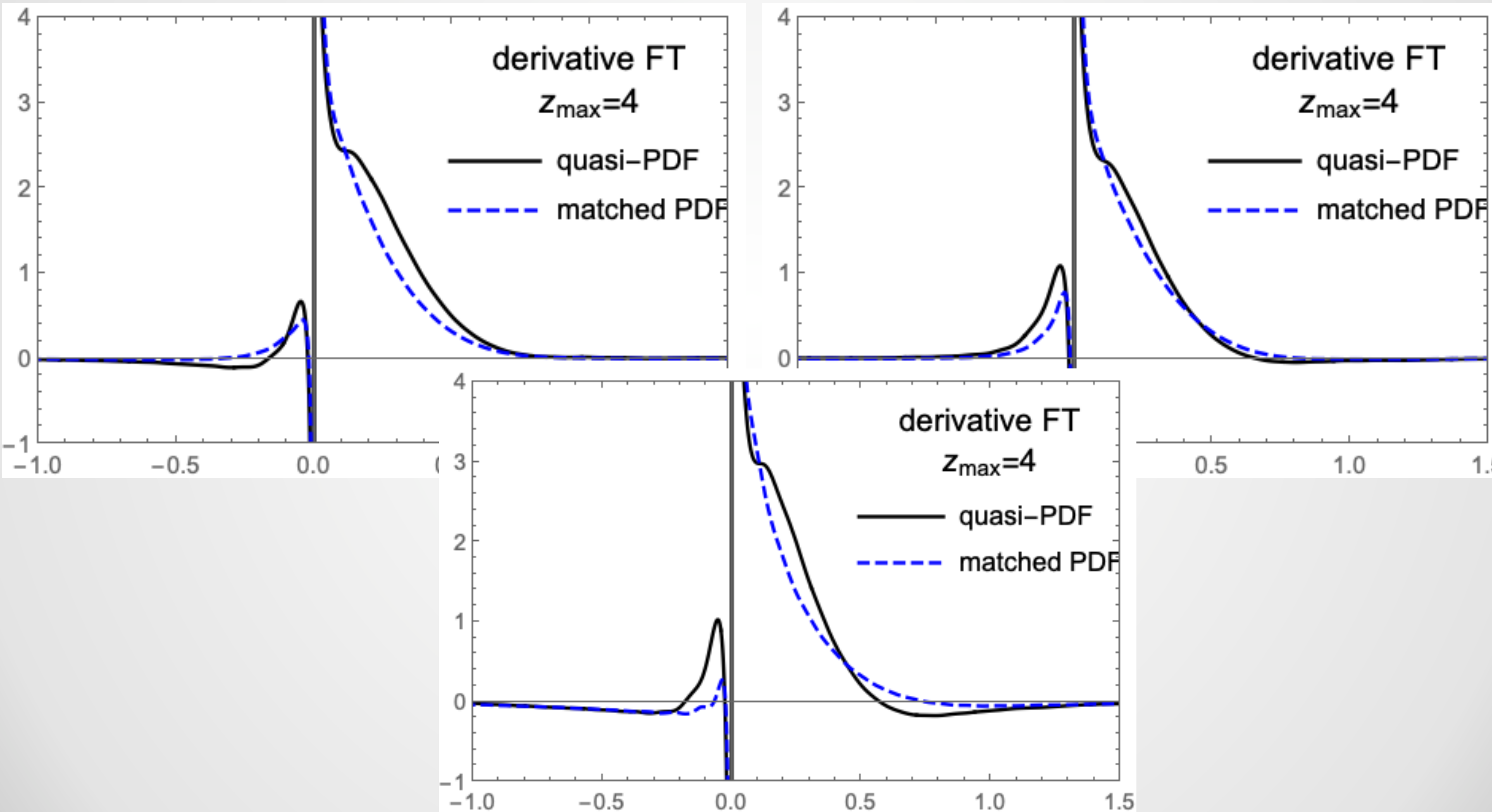
Ratio Scheme



Effect of Matching

One-step matching

Ratio Scheme



MMS Scheme

Conclusion

- The systematic procedure of lattice renormalization and perturbative matching for the quasi-PDF is already setup;
- Two-step matching is more effective in studying the renormalization scale dependence in coordinate space, while one-step matching is free from truncation errors in the Fourier transform of the conversion factor;
- The ratio scheme has a smaller conversion factor in coordinate space, which is favorable for perturbation theory and suffers less from the truncation effects in Fourier transform.

Outlook:

- Two-step matching is easier for higher-loop calculations. The scheme conversion factor can be calculated numerically, while the matching can be calculated with on-shell partons.