Power corrections and renormalons in parton quasi-distributions

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CFNS Workshop on Lattice Parton Distribution Functions

Outline

- Operator Product Expansion

based on: old stuff, '88-'94

- Renormalons

based on: VB, Alexey Vladimirov, Jian-Hui Zhang, PRD **99**, 014013 (2019)

- Normalization from universality

based on: G. S. Bali *et al.*,[RQCD collaboration], PRD **98**, 094507 (2018)

parton quasi-distributions and pseudo-distributions

- Ioffe-time quasi-distributions [Ioffe:1969kf], [Braun:1994jq]

$$\boxed{2(pn)\,\mathcal{I}^{\parallel(\perp)}(z^2,pz) = \langle N(p)|\bar{q}(z)[z,0]\not{n}\,q(0)|N(p)\rangle = C^{\parallel(\perp)}(\mu_F,z)\otimes\int_{-1}^{1}dx\,e^{ixpz}q(x,\mu_F)+\mathcal{O}(z^2)}$$

- Operator Product Expansion (light-ray OPE) [Balitsky:1987bk]

$$\bar{q}(z)[z,0]\not{v}q(0) = C^{\parallel(\perp)}(z)\otimes\Big[\bar{q}(z)[z,0]\not{v}q(0)\Big]_{\mathrm{t2}} + C_4^{\parallel(\perp)}(z)\otimes\Big[\bar{q}(z)[z,0]\not{v}q(0)\Big]_{\mathrm{t4}} + \ldots$$

In what follows often $\boxed{z^\mu \mapsto zv^\mu, \quad z\in\mathbb{R}}$

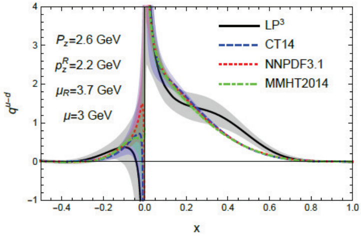
- parton quasi-distributions [Ji:2013dva]

$$\mathcal{Q}^{\parallel(\perp)}(x,p) = (pv)\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{-ixz(pv)}\,\mathcal{I}^{\parallel(\perp)}(z^2v^2,pvz)$$

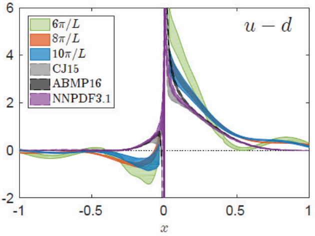
- parton pseudo-distributions [Radyushkin:2017cyf]

$$\mathcal{P}(x,z) = z\int_{-\infty}^{\infty}\frac{d(pv)}{2\pi}e^{-ixz(pv)}\,\mathcal{I}^{\perp}(z^2v^2,pvz)$$

Isovector quark unpolarized PDF



LP3, 1803.04393 & 1807.06566, $m_\pi \approx 135\ MeV, a = 0.09\ fm, L \approx 5.8 fm$



Alexandrou et al, 1803.02685, $m_\pi \approx 130\ MeV,\ \ a = 0.094\ fm, L \approx 4.5 fm$

thanks to J.-H. Zhang

- Power corrections ?

Target (nucleon) mass corrections

- qPDFs

$$\mathcal{Q}^{\parallel}(x,p) = q(x) + \frac{1}{4}\frac{m^2v^2}{(pv)^2}\Big[xq'(x)+q(x)\Big] + \mathcal{O}\left(m^4/p^4\right)$$

$$\mathcal{Q}^{\perp}(x,p) = q(x) + \frac{1}{4}\frac{m^2v^2}{(pv)^2}\Big[xq'(x)+3q(x)\Big] - \frac{1}{2}\frac{m^2v^2}{(pv)^2}\theta(|x|<1)\int_{|x|}^{1}\frac{dy}{y}q(x/y) + \mathcal{O}\left(m^4/p^4\right),$$

— enhanced as $1/(1-x)$ at $x \to 1$

- pPDF

$$\mathcal{P}(x,z) = q(x) + \frac{1}{4}z^2v^2m^2x^2\theta(|x|<1)\int_{|x|}^{1}\frac{dy}{y}q(x/y) + \mathcal{O}\left(m^4/p^4\right)$$

— suppressed as $O(1-x)$ at $x \to 1$

OPE: Twist-4

To the tree-level accuracy: [Balitsky:1987bk]

$$\Big[\bar q(z)[z,-z]\not{z}\,q(-z)\Big]_{\rm t4} = \frac{1}{4}z^2\int_0^1 du\,\frac{\partial^2}{\partial z_\alpha \partial z^\alpha}\bar q(uz)[uz,-uz]\not{z}\,q(-uz)$$

with [Ball:1998ff]

$$\begin{aligned}\frac{\partial^2}{\partial z_\alpha \partial z^\alpha}\bar q(z)\Gamma q(-z) &= \bar q(z)\Big[\sigma G(z)\Gamma + \Gamma\sigma G(-z)\Big]q(-z) - 2iz^\nu\frac{\partial}{\partial z_\mu}\int_{-1}^1 dv\,v\bar q(z)\Gamma G_{\nu\mu}(vz)q(-z)\\ &+ 2\int_{-1}^1 dv\int_{-1}^v dt\,(1+vt)\bar q(z)\Gamma z^\mu z^\nu G_{\mu\rho}(vz)G^\rho{}_\nu(tz)q(-z)\\ &+ iz^\nu\int_{-1}^1 dv\,(1+v^2)\bar q(z)\Gamma[D_\mu, G^\mu{}_\nu](vz)q(-z) + \text{total derivatives}\end{aligned}$$

- $\langle N(p)|\ldots|N(p)\rangle \Rightarrow$ multiparton $\bar q G q$, $\bar q G G q$, $\bar q q \bar q q$ PDFs [Jaffe:1983hp]

  — impractical, many unknown functions

- $\langle 0|\ldots|M(p)\rangle \Rightarrow$ conformal PWE, higher-twist LCDAs [Braun:1989iv]

  — established technique

Concept

M. Beneke, Phys.Rept. 317 (1999)
M. Beneke, V. Braun, hep-ph/0010208

- Leading twist calculation “knows” about the necessity to add a power correction

Example:

$$F_2(x,Q^2) = 2x\int_x^1 \frac{dy}{y} C(y,Q^2/\mu^2) q(\tfrac{x}{y},\mu^2) + \frac{1}{Q^2} D_2(x)$$

$$C(y) = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1}, \qquad \alpha_s = \alpha_s(\mu)$$

One-loop result:

$$D_2(x) = \varkappa \Lambda^2_{\rm QCD} 2x \int_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y})\,,$$

$$d_2(x) = -\frac{4}{[1-x]_+} + 4 + 2x + 12x^2 - 9\delta(1-x) - \delta'(1-x) \qquad \varkappa = \mathcal{O}(1)$$

with ONE parameter $\varkappa = \mathcal{O}(1)$

Cut-off scheme

Imagine the separation between CFs and MEs is done using exlicit cutoff at $|k| = \mu$.
CFs will be modified compared to usual calculation by terms $\sim \mu^2/Q^2$

$$C(y)|^{\text{cut}} = \delta(1-y) + \sum_{n=0}^{\infty} c_n \alpha_s^{n+1} - \frac{\mu^2}{Q^2} d(x) + \mathcal{O}\left(\frac{\mu^4}{Q^4}\right)$$

The dependence on μ must cancel:

- Logarithmic terms $\ln Q^2/\mu^2$ in CFs against μ-dependence in PDFs
- Power-terms μ^2/Q^2 against the higher-twist contributions

This means that $D_2(x)$ *in the cutoff scheme* must have the form

$$D_2(x) = \mu^2 2x \int_x^1 \frac{dy}{y} d_2(x) q(\tfrac{x}{y}) + \delta D_2(x)$$

— related to quadratic UV divergences in matrix elements of twist-4 operators (in this scheme!)

Dimensional regularization

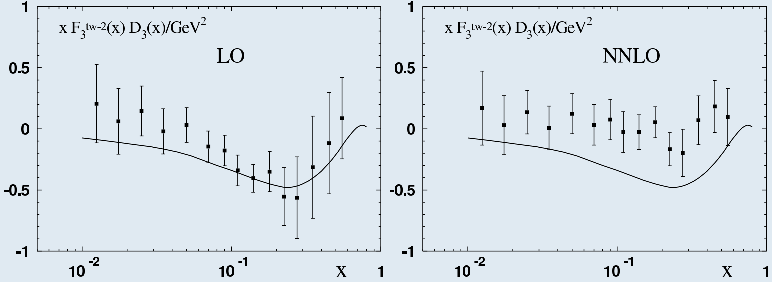
- In dim.reg. power-like terms in the CFs do not appear. Instead, the coefficients c_k (e.g., in $\overline{\text{MS}}$) diverge factorially with the order k

— The factorial divergence implies that the sum of the pert. series is only defined to a power accuracy and this ambiguity (renormalon ambiguity) must be compensated by adding a non-perturbative higher–twist correction

— Detailed analysis [Beneke:2000kc]: the asymptotic large-order behavior of the coefficients (the renormalons) is in one-to-one correspondence with the sensitivity to extreme (small or large) loop momenta

— Infrared renormalons in the l.t. CF are compensated by ultraviolet renormalons in the MEs of twist-four operators. At the end the same picture re-appears: only the details depend on the factorization method

Example: Power correction in CCFR data on $F_3(x,Q^2)$ vs. renormalon model:



Borel transform and renormalons

- light-ray OPE

$$\bar q(zv)\not{v}[zv,0]q(0) = \int_0^1 d\alpha\, H^{\parallel}(z,\alpha,\mu,\mu_F)\left[\bar q(\alpha z v)\not{z}q(0)\right]^{\mu_F}_{t2} + \dots$$

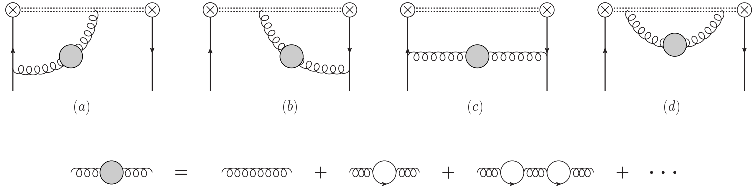
$$H = \delta(1-\alpha) + \sum_{k=0}^{\infty} h_k a_s^{k+1}\,, \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}\,, \qquad h_k \propto k!$$

- A convenient way to handle such a series is to consider the Borel transform

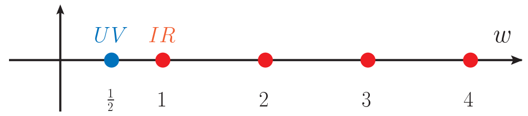
$$B[H](w) = \sum_{k=0}^{\infty}\frac{h_k}{k!}\left(\frac{w}{\beta_0}\right)^k \qquad H = \delta(1-\alpha) + \frac{1}{\beta_0}\int_0^{\infty} dw\, e^{-w/(\beta_0 a_s)}B[H](w)$$

However, integration is obscured by singularites on the integration path — renormalons

- Bubble-chain approximation 't Hooft, '77



Singularities of the Borel transform



- Ultraviolet renormalon at $w = 1/2$ M. Beneke, VB, '94

$$B[H^{\parallel(\perp)}] \overset{w\to 1/2}{=} \frac{4C_F}{w-1/2}\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)^{1/2}$$

— removed by normalization or by explicit subtraction (next slide)

- Infrared renormalon at $w = 1$

$$B[H^{\parallel}](w) \overset{w\to 1}{=} \frac{-4C_F}{1-w}\left[\alpha+\bar\alpha\ln\bar\alpha\right]\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)$$

$$B[H^{\perp}](w) \overset{w\to 1}{=} \frac{-4C_F}{1-w}\left[\alpha+\bar\alpha\ln\bar\alpha+\alpha\bar\alpha\right]\left(-\frac{v^2z^2\mu^2e^{5/3}}{4e^{-2\gamma_E}}\right)$$

Normalized (scale-invariant) quasi-distributions

Quasidistributions are scale-dependent (Z_q in axial gauge) and in renormalization schemes not based on dim. reg. also suffer from a linear UV divergence of the Wilson line. Problem can be avoided by considering a scale-independent ratio:

- diving out the value at zero proton momentum [Orginos:2017kos]

$$\mathbf{I}(z,pv) = \mathcal{I}(z,pv,\mu)/\mathcal{I}(z,0,\mu)$$

- or, alternatively, normalizing the qITD to the vacuum correlator

$$\widehat{\mathbf{I}}(z,pv) = \mathcal{I}(z,pv,\mu)/\mathcal{N}(z,\mu)\,, \qquad \mathcal{N}(z,\mu) = \left(\frac{2iN_c}{\pi^2 z^2}\right)^{-1} \langle 0|\bar{q}(z)\not{z}[z,0]q(0)|0\rangle$$

In this way one can define the scale-independent qPDF/pPDF

$$\mathbf{Q}(x,p) = (pv)\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{-ixz(pv)}\,\mathbf{I}(z,pv)$$

$$\mathbf{P}(x,z) = z\int_{-\infty}^{\infty}\frac{d(pv)}{2\pi}e^{-ixz(pv)}\,\mathbf{I}(z,pv)$$

and similarly for $\widehat{\mathbf{Q}}(x,p)$ and $\widehat{\mathbf{P}}(x,z)$

- normalization to the vacuum correlator does not affect leading $\mathcal{O}(z^2)$ power corrections, whereas the normalization to zero momentum, as we will see, has a nontrivial effect.

Leading power corrections to Ioffe-time quasidistributions

- Ioffe time: $\tau = (p \cdot v)z$

$$\mathcal{I} = I(\tau)\Big\{1 + \kappa(v^2 z^2 \Lambda^2)\mathcal{R}_{\mathcal{I}}(\tau)\Big\},$$

$$\mathbf{I} = I(\tau)\Big\{1 + \kappa(v^2 z^2 \Lambda^2)\mathbf{R}_{\mathcal{I}}(\tau)\Big\},$$

- Non-normalized qITDs

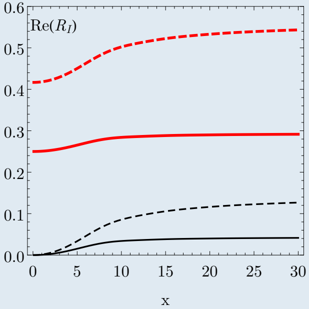
$$\mathcal{I}^{\parallel}(\tau) = I(\tau) + (v^2 z^2 \Lambda^2)\int_0^1 d\alpha\,(\alpha + \bar\alpha\ln\bar\alpha)I(\alpha\tau)$$

$$\mathcal{I}^{\perp}(\tau) = I(\tau) + (v^2 z^2 \Lambda^2)\int_0^1 d\alpha\,(\alpha + \bar\alpha\ln\bar\alpha + \alpha\bar\alpha)I(\alpha\tau)$$

- qITDs normalized to zero momentum

$$\mathbf{I}^{\parallel}(\tau) = I(\tau) + (v^2 z^2 \Lambda^2)\int_0^1 d\alpha\,[\alpha + \bar\alpha\ln\bar\alpha]_+ I(\alpha\tau)$$

$$\mathbf{I}^{\perp}(\tau) = I(\tau) + (v^2 z^2 \Lambda^2)\int_0^1 d\alpha\,[\alpha + \bar\alpha\ln\bar\alpha + \alpha\bar\alpha]_+ I(\alpha\tau)$$



— $\mathcal{R}_{\mathcal{I}}(\tau)$ and $\mathbf{R}_{\mathcal{I}}(\tau)$
for $q(x) = x^{-1/2}(1-x)^3$
— Solid for Re $R^{\parallel}$, dashed for Re $R^{\perp}$

Leading power corrections to quasi-PDFs

$$\boxed{\begin{aligned}\mathcal{Q}^{\parallel}(x,p) &= q(x) - \frac{v^2\Lambda^2}{x^2(pv)^2}\int_{|x|}^{1}\frac{dy}{y}\left[\frac{y^2}{[1-y]_+} + 2\delta(\bar{y}) + \delta'(\bar{y})\right]q(\tfrac{x}{y})\\ \mathcal{Q}^{\perp}(x,p) &= q(x) - \frac{v^2\Lambda^2}{x^2(pv)^2}\int_{|x|}^{1}\frac{dy}{y}\left[\frac{y^2}{[1-y]_+} + 3\delta(\bar{y}) + \delta'(\bar{y}) - 2y^2\right]q(\tfrac{x}{y})\end{aligned}}$$

where $\Lambda = \mathcal{O}(\Lambda_{\mathrm{QCD}})$

For a numerical study, present the result in the form

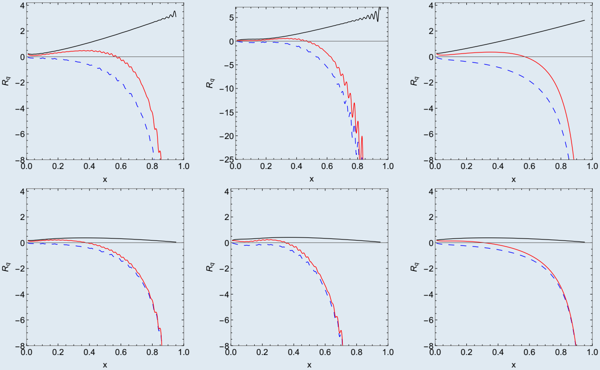
$$\mathcal{Q}^{\parallel(\perp)}(x,p) = q(x)\left\{1 - \frac{v^2\Lambda^2}{x^2(1-x)(pv)^2}\mathcal{R}_{\mathcal{Q}}^{\parallel(\perp)}(x)\right\}$$

Leading power corrections to quasi-PDFs

u-quark d-quark $q(x) = x^{-1/2}(1-x)^3$

$\mathcal{R}^{\parallel}_{\mathcal{Q}}$

$\mathcal{R}^{\perp}_{\mathcal{Q}} - \mathcal{R}^{\parallel}_{\mathcal{Q}}$



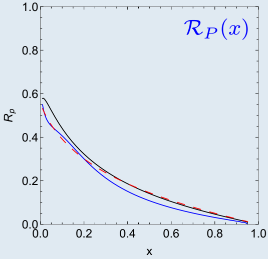
black: non-normalized qPDFs (original Ji's definition)
blue: subtraction terms from normalization at zero momentum
red: normalized qPDFs (after subtraction at zero momentum)

used: MSFW NLO valence quark PDFs at 2 GeV

Leading power corrections to pseudo-PDFs

$$\mathcal{P}(x,z,\mu) = q(x) + (v^2z^2\Lambda^2)\int_{|x|}^{1} \frac{dy}{y}\,(y+\bar y\ln\bar y + y\bar y)\,q(\tfrac{x}{y}) \;\equiv\; q(x)\Big\{1+(v^2z^2\Lambda^2)\mathcal{R}_P(x)\Big\}$$

$$\mathbf{P}(x,z,\mu) = q(x) + (v^2z^2\Lambda^2)\int_{|x|}^{1} \frac{dy}{y}\,[y+\bar y\ln\bar y + y\bar y]_+\,q(\tfrac{x}{y}) \;\equiv\; q(x)\Big\{1+(v^2z^2\Lambda^2)\mathbf{R}_P(x)\Big\}$$



MSFW u-quarks (black)
MSFW d-quarks (blue)
$q(x) = x^{-1/2}(1-x)^3$ (red)

$$\mathcal{R}_P(x) \overset{x\to 1}{=} \mathcal{O}(1-x)\,, \qquad ☺$$

$$\mathbf{R}_P(x) = \mathcal{R}_P(x) - \frac{5}{12}\,, \qquad ☹$$

Summary from renormalons:

- Power corrections for qPDFs have a generic behavior

$$\mathcal{Q}(x,p) = q(x)\left\{1 + \mathcal{O}\left(\frac{\Lambda^2}{p^2}\cdot\frac{1}{x^2(1-x)}\right)\right\}$$

Normalization to zero momentum considerably reduces the correction at $0.1 < x < 0.6$ at the cost of a strong enhancement at $x > 0.6$

- Power corrections for pPDFs have a generic behavior

$$\mathcal{P}(x,z) = q(x)\Big\{1 + \mathcal{O}\big(z^2\Lambda^2(1-x)\big)\Big\}$$

but the suppression at $x \to 1$ is lifted by the normalization to zero momentum

- Position space PDFs (qITDs) have flat power corrections at large Ioffe times

How to fix an overall normalization? Universality!

Power corrections from universality: RQCD exploratory study: 1709.04325 and 1807.06671

- Consider several correlation functions simultaneously; in this study

$$\langle 0|\bar{q}(z/2)\Gamma_A q(z/2)\bar{q}(-z/2)\Gamma_B q(-z/2)\}\pi^0(p)\rangle \sim \Phi_{AB}(p\cdot z, z^2)$$

with $\Gamma_A, \Gamma_B \to \gamma_\mu, \gamma_\mu\gamma_5, \gamma_5, \mathbb{I}$

- Normalization/prefactors adjusted such that at tree level $\forall A, B$

$$\Phi_{AB}(p\cdot z, z^2) = \int_0^1 du\, e^{i(u-1/2)p\cdot z}\phi_\pi(u)$$

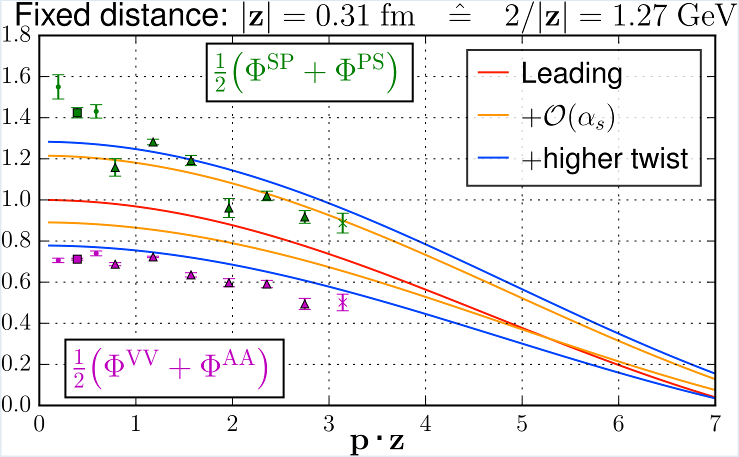
— position-space pion LCDA

- Beyond tree level Φ_{AB} differ because of perturbative and higher-twist corrections
  — "quasi-LCDAs"

— take into account the α_s correction (one loop)

— higher-twist correction $\mathcal{O}(z^2)$ involves one parameter to be fitted from data

Exploring universality and higher-twist effects, 1807.06671



$N_f = 2$, $32^3 \times 64$, $a \simeq 0.071$ fm, $m_\pi \simeq 295$ MeV

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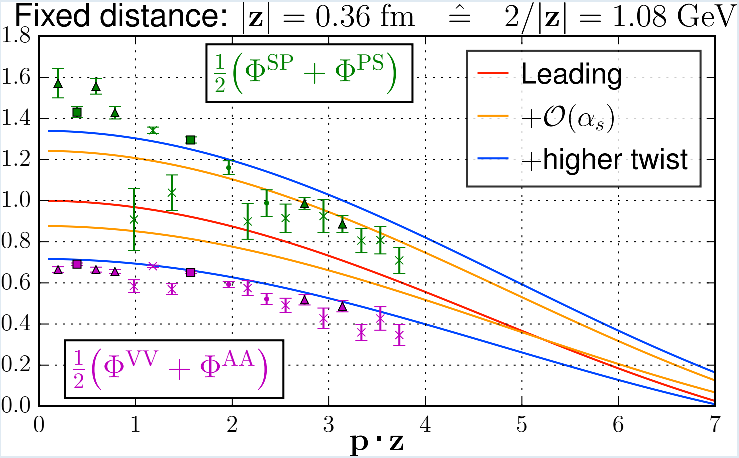

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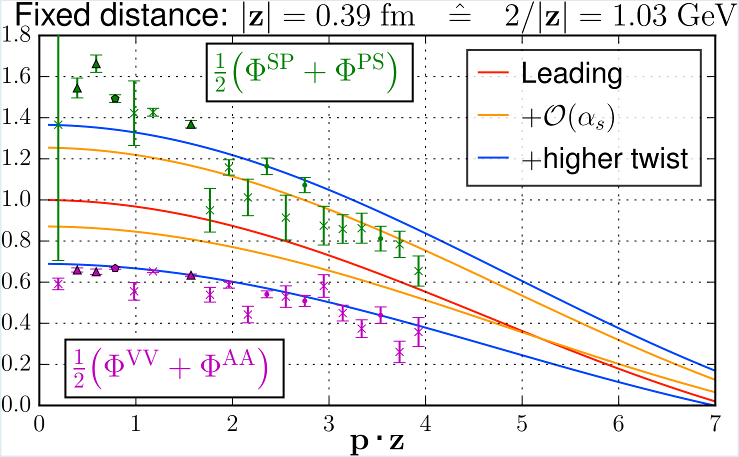

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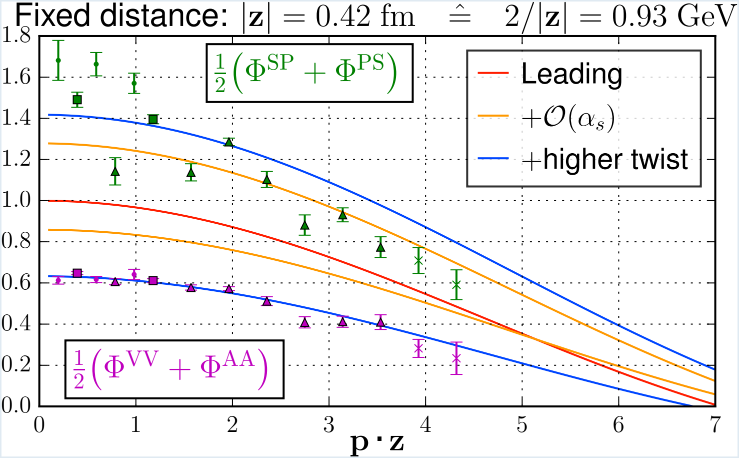

$N_f = 2$, $32^3 \times 64$, $a \simeq 0.071$ fm, $m_\pi \simeq 295$ MeV

Higher-twist correction

- The fit returns “reasonable” results for the pion LCDA ($a_2(2\text{ GeV}) \sim 0.2 - 0.3$) **uncorrelated** with the higher-twist parameter δ_π^2

$$\langle 0|\bar{u}(0)\gamma^\rho ig\widetilde{G}_{\rho\mu}u(0)|\pi^0(p)\rangle = p_\mu F_\pi \delta_2^\pi; \quad 0.20\text{ GeV}^2 \leq \delta_2^\pi \leq 0.25\text{ GeV}^2$$

- First determination in lattice QCD