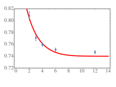
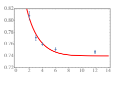


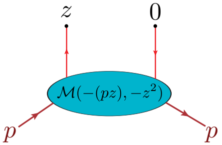
Pseudo-PDF Formalism

A.V. Radyushkin (ODU/Jlab)

CFNS Lattice PDF Workshop
BNL, April 17, 2019



Ioffe-time distributions and Pseudo-PDFs



- Basic matrix element (ignoring spin)

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz),-z^2)$$

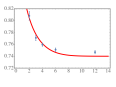
- Lorentz invariance: $\mathcal{M}$ depends on z through $(pz) \equiv -\nu$ and z^2

- Ioffe time ν: $\mathcal{M}(\nu,-z^2) =$ Ioffe-time pseudo-distribution (pseudo-ITD)
- Pseudo $\equiv$ off the light cone
- Using α-representation, it is possible to show that, for any contributing Feynman diagram, for arbitrary z^2 and arbitrary p^2

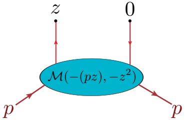
$$\mathcal{M}(\nu,-z^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, \mathcal{P}(x,-z^2)$$

- Limits $-1 \leq x \leq 1$, negative x correspond to anti-particles
- Pseudo-PDF $\mathcal{P}(x,-z^2)$: Fourier transform of pseudo-ITD with respect to ν for fixed z^2
- On the light cone $z_+ = 0$: usual ITD $\mathcal{I}(\nu,\mu^2)$ and usual PDF $f(x,\mu^2)$
- In QCD, $z^2 \to 0$ limit is singular, regularization (like $\overline{\text{MS}}$) is needed, $\mathcal{P}(x,0) \to f(x,\mu^2)$ and we have $\overline{\text{MS}}$ ITD $\mathcal{I}(\nu,\mu^2)$

$$\mathcal{M}(\nu,0)|_{\mu^2} \equiv \mathcal{I}(\nu,\mu^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, f(x,\mu^2)$$



Perturbative corrections and evolution



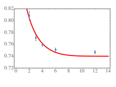
- No lightlike separations on lattice!
- Take $z = (0, 0, 0, z_3)$ (Ji,2013)
- Then $-(pz) \equiv \nu = P z_3$ and $-z^2 = z_3^2$
- "Pseudo-PDF strategy: extract $\mathcal{M}(\nu, z_3^2)$ from lattice and "extrapolate" to $z_3^2 \to 0$ limit
- Implemented by matching between "z_3^2" and $\overline{\mathrm{MS}}$ schemes

- In QCD $\mathcal{M}(\nu, z_3^2)$ has logarithmic singularity in z_3^2
- Generates perturbative evolution. At one loop,

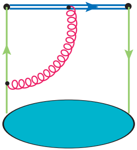
$$\mathcal{M}^{\mathrm{hard}}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi} C_F \ln(z_3^2) \int_0^1 du\, B(u)\, \mathcal{M}^{\mathrm{soft}}(u\nu, 0)$$

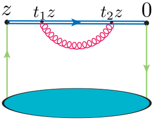
- Altarelli-Parisi (AP) evolution kernel

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$



Gauge link and UV divergences

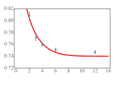




- In QCD, there is one more source of the z^2-dependence: gauge link $\hat{E}(0, z; A)$
- It has specific ultraviolet divergences $\sim z_3/a$, $\ln(1 + z_3^2/a^2)$, with $a \sim$ UV cut-off
- Use Polyakov regularization $1/z^2 \to 1/(z^2 - a^2)$ for gluon propagator in coordinate space
- Effect of the UV cut-off a is similar to that of the lattice spacing
- At one loop, UV singular terms from link self-energy have the structure (Ji et al., 2016)

$$\Gamma_{\rm UV}(z_3, a) \sim -\frac{\alpha_s}{2\pi} C_F \left[2\frac{|z_3|}{a} \tan^{-1}\left(\frac{|z_3|}{a}\right) - 2 \ln\left(1 + \frac{z_3^2}{a^2}\right)\right]$$

- Vertex corrections produce $\ln\left(1 + z_3^2/a^2\right)$ terms only
- For fixed a, these terms vanish when $z_3 \to 0$
- No correction to local current
- Hence, no violation of quark number conservation

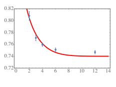


Renormalization

- Link-related UV divergences have the same structure as in HQET
- They are multiplicatively renormalizable (Qiu et al. , Ji et al. , Green et al. 2017)
- UV regulator a appears only in the combination z_3/a
- UV-sensitive terms form a factor $Z(z_3^2/a^2)$
- This factor is an artifact of having a non-lightlike z
- It has nothing to do with the lightcone PDFs
- We should build modified function $Z^{-1}(z_3^2/a^2)\mathcal{M}(\nu, z_3^2; a)$
- To do this, one should know the $Z(z_3^2/a^2)$ factor
- Easier way out: consider reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)} = \lim_{a \to 0} \frac{\mathcal{M}(\nu, z_3^2; a)}{\mathcal{M}(0, z_3^2; a)}$$

- $Z(z_3^2/a^2)$ factors cancel, and $\mathfrak{M}(\nu, z_3^2)$ has finite $a \to 0$ limit
- Reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ is a “physical observable” (like, say, DIS structure functions $F(x, Q^2)$)
- No need to specify renormalization scheme, scale, etc.
- Note: $\nu = 0$ with $z_3 \neq 0$ is obtained by taking $P_3 = 0$
- Still, $\mathcal{M}(0, z_3^2)$ is in perturbative regime as far as z_3^2 is small



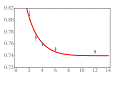
Reduced Ioffe-time pseudo-distribution

- $\mathfrak{M}(\nu, z_3^2)$ is singular in $z_3 \to 0$ limit, $\ln z_3^2$ terms reflect perturbative evolution
- At one loop (with mass-type IR regularization)

$$\mathfrak{M}(\nu, z_3^2) = \mathfrak{M}^{\rm soft}(\nu, 0) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left[\mathfrak{M}^{\rm soft}(w\nu, 0) - \mathfrak{M}^{\rm soft}(\nu, 0)\right] \times \left\{ \frac{1+w^2}{1-w} \left[\ln\left(z_3^2 m^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + 4\, \frac{\ln(1-w)}{1-w} \right\}$$

- For light-cone PDF, one should take $z^2 = 0$ and use some scheme for resulting UV divergence, say, $\overline{\rm MS}$
- Ioffe-time distribution $\mathcal{I}(\nu, \mu^2)$ at one loop (with the same mass-type IR regularization)

$$\mathcal{I}(\nu, \mu^2) = \mathfrak{M}^{\rm soft}(\nu, 0) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \left[\mathfrak{M}^{\rm soft}(w\nu, 0) - \mathfrak{M}^{\rm soft}(\nu, 0)\right] \times \left\{ \frac{1+w^2}{1-w} \ln(m^2/\mu^2) + 2(1-w) \right\}$$

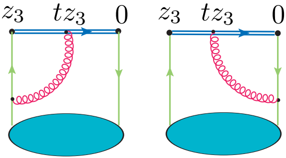


Matching for Ioffe-time distribution

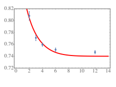
- Writing $\overline{\rm MS}$ ITD in terms of reduced pseudo-ITD gives matching condition (Y. Zhao 2017, A.R. 2017)

$$\mathcal{I}(\nu,\mu^2) = \mathfrak{M}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{M}(w\nu, z_3^2) \times \left\{ B(w) \left[\ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4 \frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}$$

- Altarelli-Parisi kernel $B(w) = \left[(1+w^2)/(1-w) \right]_+$
- Multiplicative scale difference between z_3^2 and $\overline{\rm MS}$ cut-offs $\mu^2 = 4e^{-2\gamma_E}/z_3^2$
- Simple rescaling relation is modified when all terms are taken into account



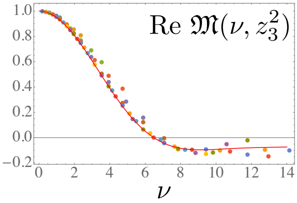
- Term with $[\ln(1-w)]/(1-w)$ produces large negative contribution
- In Feynman gauge, it comes from vertex diagrams
- Gluon is attached to running tz_3 position on the link
- z_3-dependence is generated then by effective scale smaller than z_3



Evolution in lattice data

- Exploratory lattice study of reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ for the valence $u_v - d_v$ parton distribution in the nucleon [Orginos et al. 2017]
- When plotted as function of ν, data both for real and imaginary parts of $\mathfrak{M}(\nu, z_3^2)$ lie close to respective universal curves
- Data show no polynomial z_3-dependence for large z_3 though z_3^2/a^2 changes from 1 to ~ 200
- Apparently no higher-twist terms in the reduced pseudo-ITD
- Real part corresponds to the cosine Fourier transform of $q_v(x) = u_v(x) - d_v(x)$

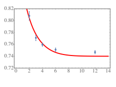
$$\mathfrak{R}(\nu) \equiv \mathrm{Re}\, \mathfrak{M}(\nu) = \int_0^1 dx\, \cos(\nu x)\, q_v(x)$$



- Overall curve corresponds to the function

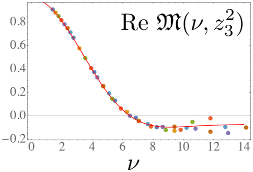
$$f(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$

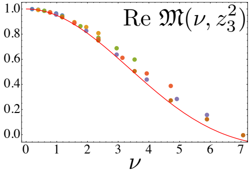
- Obtained by forming cosine Fourier transforms of $x^a(1-x)^b$-type functions and fitting a, b
- Shape is dominated by points with smaller values of Re $\mathfrak{M}(\nu, z_3^2)$

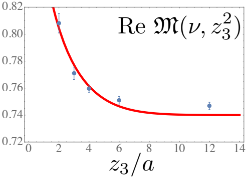


Evolution in lattice data, cont.

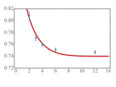
Pseudo-&Quasi-PDFs
Parton Densities
Pseudo-distributions
Pseudo-PDF
UV divergences
Renormalization
Reduced pseudo-ITD
Evolution in lattice data
Data
Building $\overline{\rm MS}$ ITD
Results
Summary







- Points corresponding to $7a \leq z_3 \leq 13a$
- Some scatter for points with $\nu \gtrsim 10$
- Otherwise, practically all the points lie on the universal curve based on $f(x)$
- No z_3-evolution visible in large-z_3 data
- Points in $a \leq z_3 \leq 6a$ region
- All points lie higher than universal curve
- Perturbative evolution increases real part of the pseudo-ITD when z_3 decreases
- Conjecture that the observed higher values of $\mathfrak{R}$ for smaller-z_3 points may be a consequence of evolution
- z_3-dependence of the lattice points for "magic" Ioffe-time value $\nu = 3\pi/4 = 12\frac{\pi}{16}$
- Shape of eye-ball fit line is $\text{const} + \Gamma(0, z_3^2/30a^2)$
- $\Gamma(0, z_3^2/30a^2)$ has "perturbative" $\ln(1/z_3^2)$ behavior for small z_3, rapidly vanishes for $z_3 > 6a$



Building $\overline{\text{MS}}$ ITD

- Data show a logarithmic evolution behavior in small z_3 region
- Starts to visibly deviate from a pure logarithmic $\ln z_3^2$ pattern for $z_3 \gtrsim 5a$
- This sets the boundary $z_3 \le 4a$ on the "logarithmic region"
- "Evolution" part of 1-loop correction

$$\mathcal{I}_R^{\text{ev}}(\nu,\mu^2) = \mathfrak{R}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{R}(w\nu, z_3^2) B(w)\, \ln\left(z_3^2\mu^2 \frac{e^{2\gamma_E}}{4}\right)$$

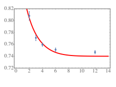
- For $z_3 = 2e^{-\gamma_E}/\mu$, the logarithm vanishes, and we have

$$\mathcal{I}_R^{\text{ev}}(\nu,\mu^2) = \mathfrak{R}(\nu, (2e^{-\gamma_E}/\mu)^2) = \mathfrak{R}(\nu, (1.12/\mu)^2)$$

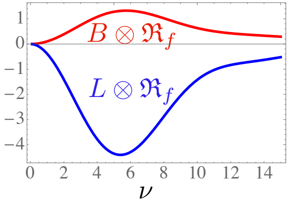
- This happens only if, for some α_s, the $\ln z_3^2$-dependence of the 1-loop term cancels actual z_3^2-dependence of the data, visible as scatter in the data
- Fitted value: $\alpha_s/\pi \approx 0.1$
- Remaining part of $\mathcal{I}(\nu,\mu^2)$ is due to corrections beyond the leading log approximation

$$\mathcal{I}_R^{\text{NL}}(\nu) = \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{R}_f(w\nu) \left\{ B(w) + \left[4\frac{\ln(1-w)}{1-w} - 2(1-w)\right]_+ \right\}$$

$$\equiv \frac{\alpha_s}{2\pi} C_F \left[B \otimes \mathfrak{R}_f + L \otimes \mathfrak{R}_f\right]$$

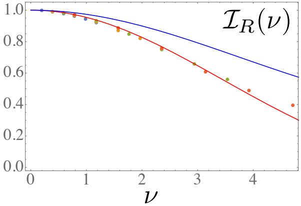


Numerical results



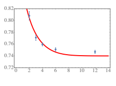
- $L \otimes \mathfrak{R}_f$ is negative and rather large
- In $\nu < 5$ region, $L \otimes \mathfrak{R}_f \approx -3.5 B \otimes \mathfrak{R}_v$
- Combined effect is close to LLA evolution with modified rescaling factor

$$\mathcal{I}_R(\nu, \mu^2) \approx \mathfrak{R}(\nu, (4/\mu)^2)$$

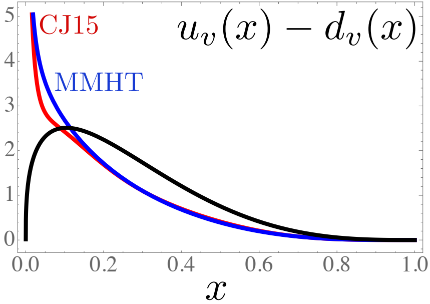


- Actual calculations should be done using "exact" formula
- We choose $\mu = 1/a$ which, at lattice spacing of 0.093 fm is $\approx$ 2.15 GeV
- Using $\alpha_s/\pi = 0.1$ and $z_3 \leq 4a$ data, we generate the points for $\mathcal{I}_R(\nu, (1/a)^2)$
- Upper curve corresponds to the ITD of the CJ15 global fit PDF for μ =2.15 GeV

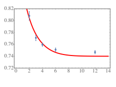
- Evolved points are close to some universal curve with a rather small scatter
- The curve itself corresponds to the cosine transform of a normalized $\sim x^a(1-x)^b$ distribution with $a = 0.35$ and $b = 3$



Numerical results, cont.



- $\sim x^{0.35}(1-x)^3$ PDF compared to CJ15 and MMHT global fits for $\mu = 2.15$ GeV
- Unable to reproduce $\sim x^{-0.5}$ Regge behavior
- Possible reasons: quenched approximation, large pion mass
- Maybe, a crude way used for doing Fourier transformation



Summary

- Formulated algorithm for extraction of light-cone PDF from lattice data
- First step: measure reduced ITD $\mathfrak{M}(\nu, z_3^2) = \mathcal{M}(\nu, z_3^2)/\mathcal{M}(0, z_3^2)$
- No UV renormalization needed
- UV-generated z_3^2-dependence also cancels
- Second step: check that small-z_3^2 behavior has $\ln z_3^2$ structure
- Check the DGLAP evolution equation
- Build $\overline{\rm MS}$ ITD $\mathcal{I}(\nu, \mu^2)$
- Last step: invert Fourier to get $\overline{\rm MS}$ PDF
- Due to the limited range of ν, this is a rather dirty exersize
- Instead, one can skip the last step and just compare the lattice ITD with that obtained from phenomenological fits

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