Lattice results using pseudo-PDFs

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Introduction

- PDF cannot be *directly* calculated from Lattice QCD
  - Light cone and Wick Rotations
  - Finite momentum
- Instead Lattice Cross sections
  - Matrix elements with space like separations which can be factorized to PDFs
- The calculation of the matrix element of interest is not the end
  - Can extract moments
  - Can perform Fourier Transforms
- Systematics of Inverse problem methods must be studied

Incomplete list of PDF ideas for the lattice

- “Twist-2” Local Operators
  - Restricted to low moments by reduced rotational symmetry
- Two current Correlations
  - “Light-like” separated Hadronic Tensor

    K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) , Phys. Rev. D62 (2000) 074501
  - Good lattice cross sections

    Y.-Q. Ma J.-W. Qiu (2014) 1404.6860, Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018
- Wilson Line Quark Bilinears
  - Quasi PDF

    X. Ji, Phys.Rev.Lett. 110, (2013), J.-W. Chen et.al. (2018) 1803.04393, C Alexandrou et.al. (2018) 1803.02685
  - Pseudo PDF

    A. Radyushkin Phys.Lett. B767 (2017), K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

Outline

- Feynman-Hellmann summation matrix elements
  - Improved excited state
  - Calculation technique
- Matrix element of interest
  - Renormalization and Reduction
- Ioffe time pseudo-PDF results
  - Matching to MS-bar Ioffe time PDF
- Inverse problems
- Functional forms
- Moments of pseudo-PDF
  - OPE without OPE
- Neural Network Fits
- Bayesian Reconstruction

Matrix Elements from Lattice Field Theory

- Typical Correlation functions

$$C_2(\vec{p}, T) = \langle O_N(-\vec{p}, T)\bar{O}_N(\vec{p}, 0)\rangle$$

$$C_3(\vec{p}, t, T) = \sum_{\vec{x}} \langle O_N(-\vec{p}, T) O_{op}(\vec{x}, t)\bar{O}_N(\vec{p}, 0)\rangle$$

- Feynman-Hellman/Summation extraction C. Bouchard et.al Phys. Rev. D 96, no. 1, 014504 (2017)

$$C_{sum}(\vec{p}, T) = \sum_{\vec{x}, t} \langle O_N(-\vec{p}, T) O_{op}(\vec{x}, t)\bar{O}_N(\vec{p}, 0)\rangle$$

Effective matrix elements and Excited States

- “Traditional Method”

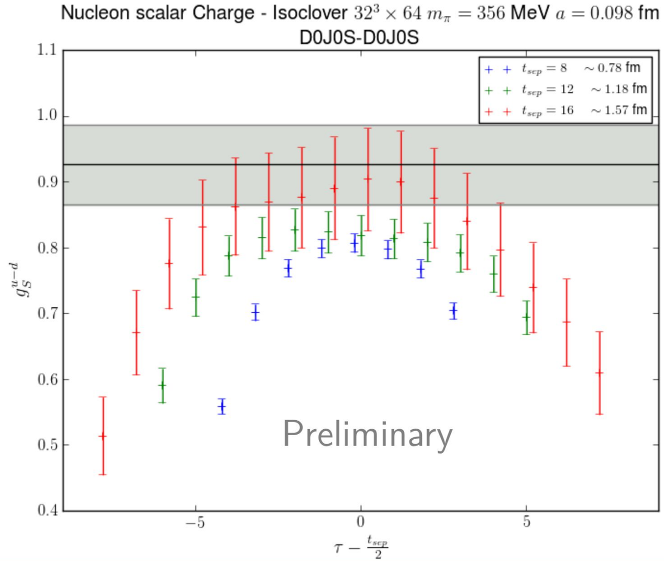
$$M_{eff}(t,T) = \frac{C_3(t,T)}{C_2(T)} = M\left(1 + Ae^{-\Delta(T-t)} + Be^{-\Delta t}\right)$$

- Summation Method

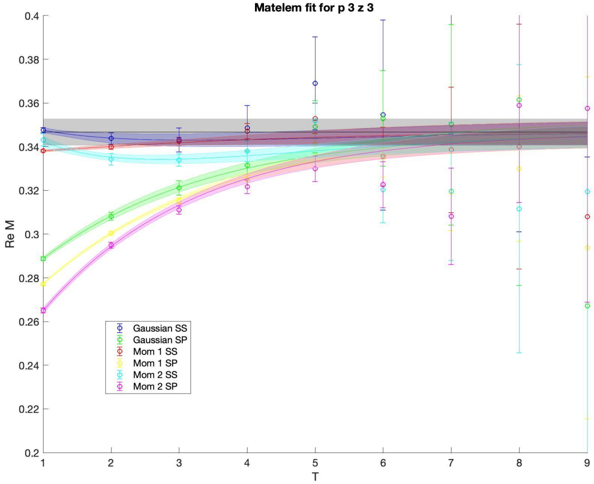
$$M_{eff}(T) = \frac{C_{sum}(T)}{C_2(T)} - \frac{C_{sum}(T+1)}{C_2(T+1)} = M\left(1 + Ae^{-\Delta T} + BTe^{-\Delta T}\right)$$

Feynman Hellmann matrix element extraction

- 2 parameter extraction
- Less clear excited state effects



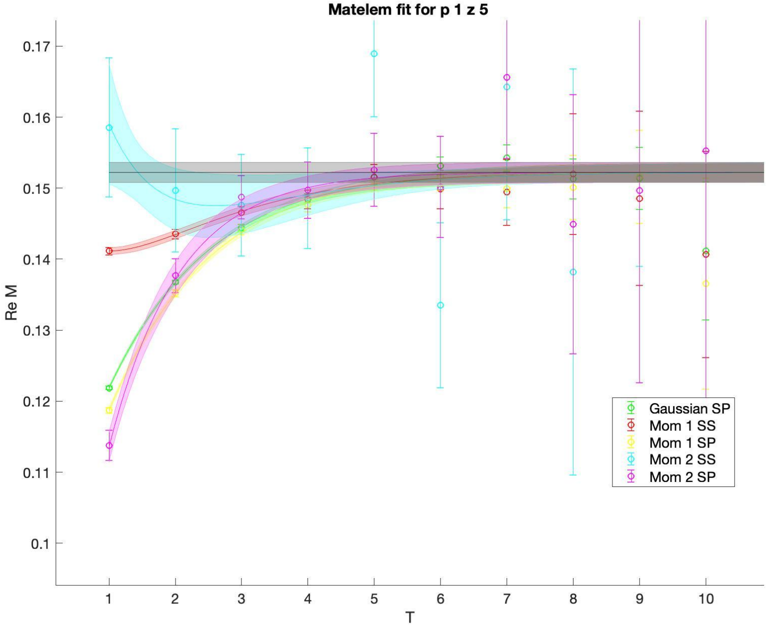
- 1 parameter extraction
- More clear distinction between excited state, plateau, noise regions

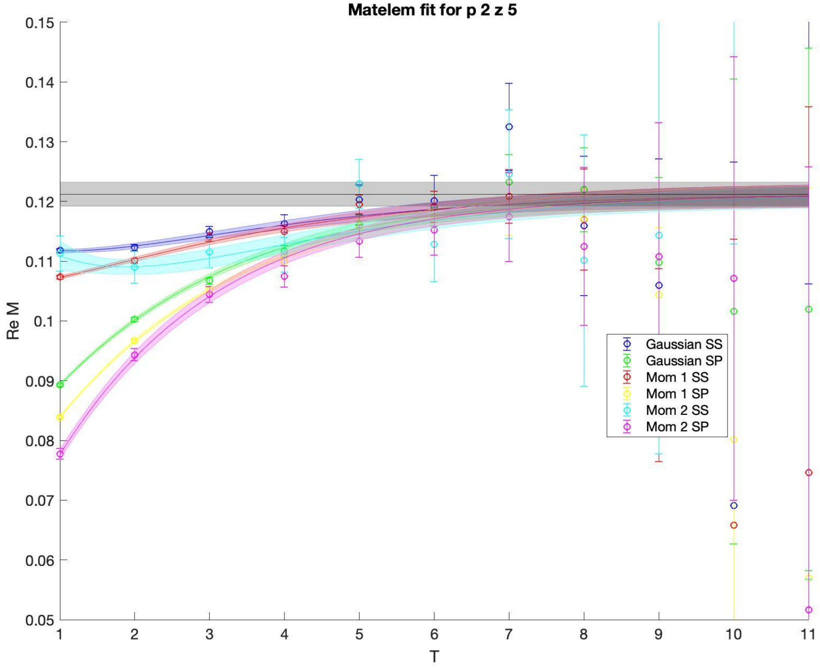


See Axial coupling calculation in Chang et.al. arXiv:1710.06523

Some good extractions

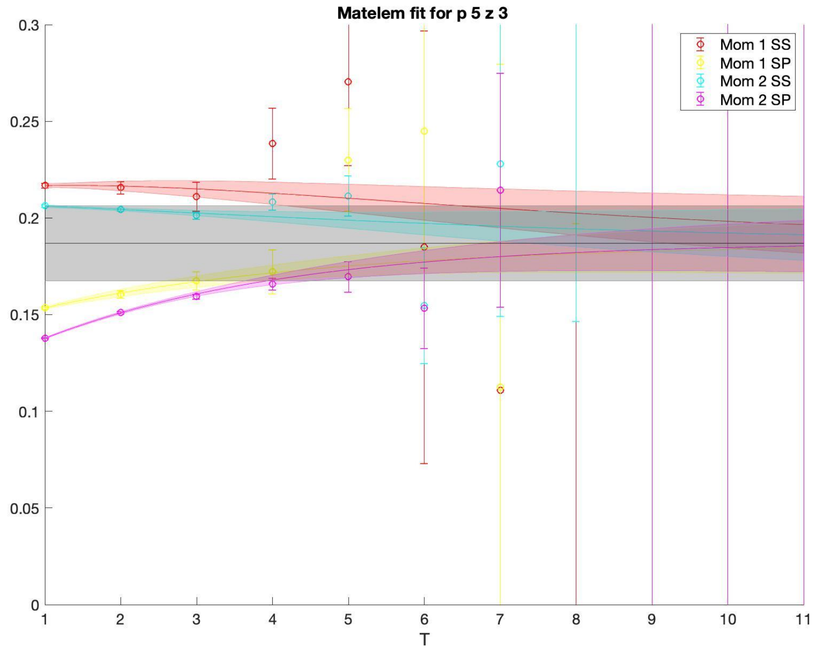
Preliminary

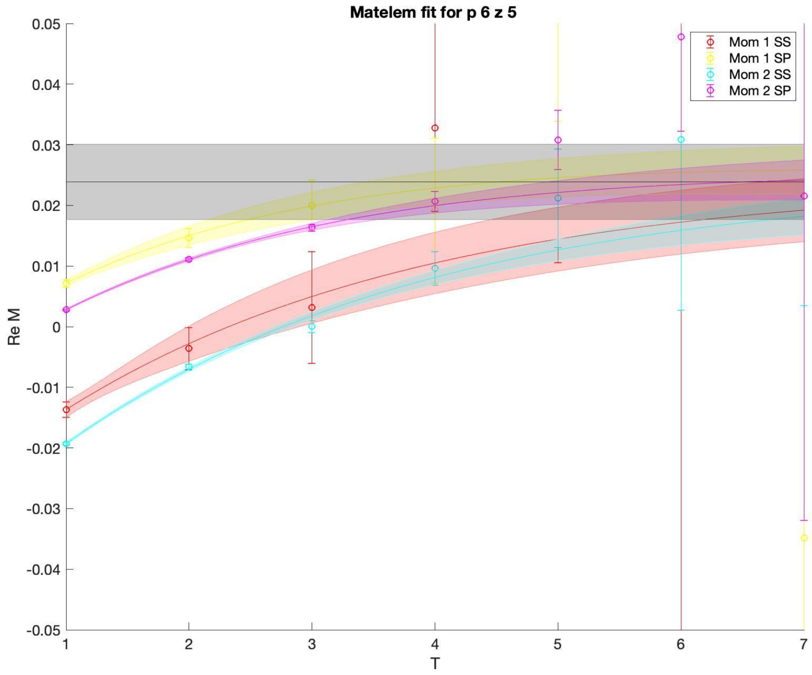




Less good extractions

Preliminary





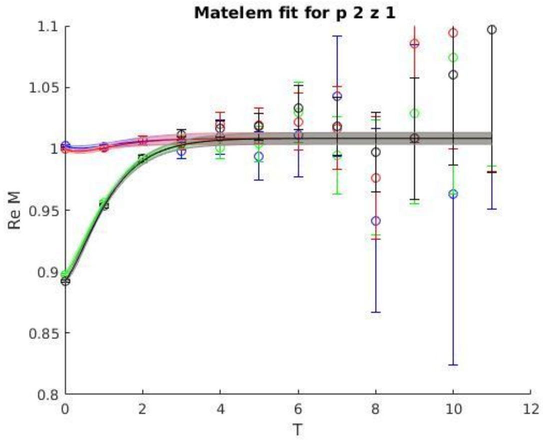
Technical Lattice difficulties for PDFs

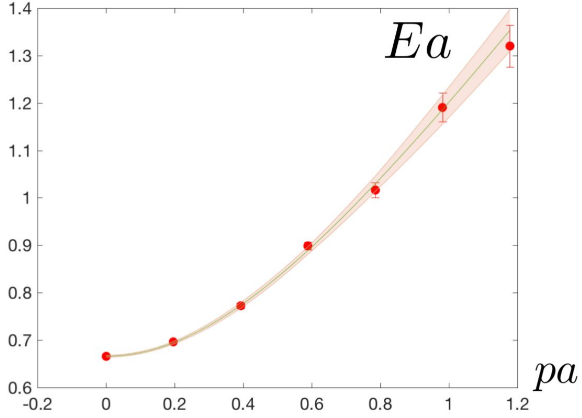
- Excited states contamination
- Signal to noise
    - $C_2(p,T) = \langle O_h(p,T) O_h(p,0)^\dagger \rangle \propto e^{-E_h(p)T}$

      $var\left[C_2(p,T)\right] = \langle O_h(p,T) O_h(p,T)^\dagger O_h(p,0) O_h(p,0)^\dagger \rangle \propto e^{-n_q m_\pi T}$

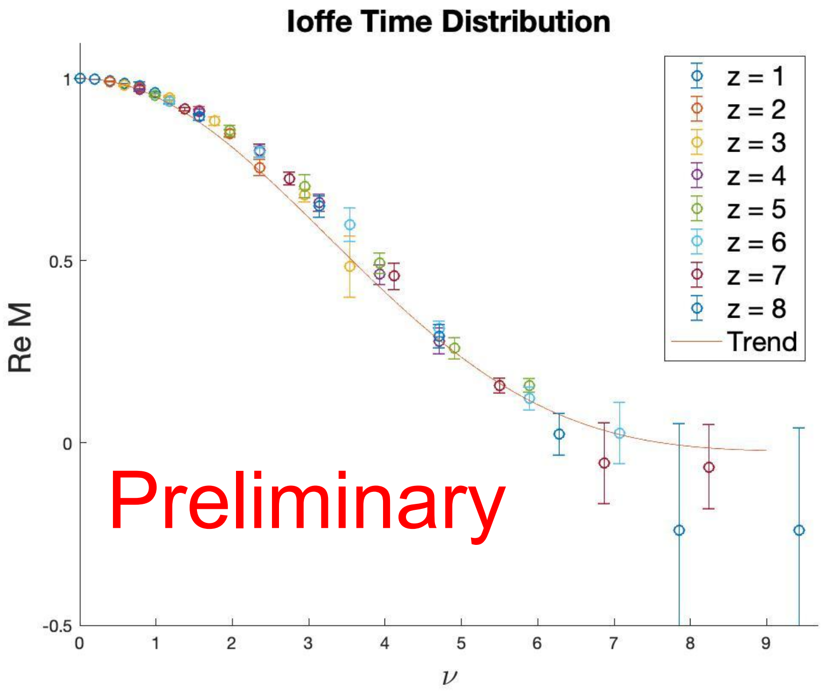
      $\frac{var\left[C_2(p,T)\right]^{\frac{1}{2}}}{C_2(p,T)} \propto e^{(E_h(p) - n_q m_\pi/2)T}$
        - Momentum smearing Bali et.al. Phys. Rev. D 93, 094515 (2016)
- Use of heavy pions
- Connected and disconnected
- Lattice spacing errors

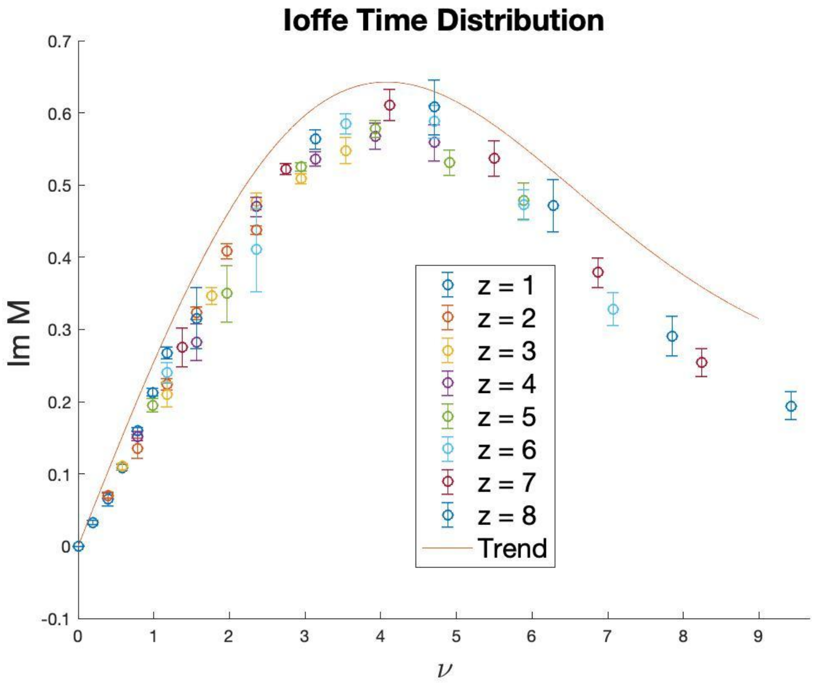
$$apmax = \frac{2\pi}{L}\left(\frac{L}{4}\right) = \frac{\pi}{2} \sim O(1)$$



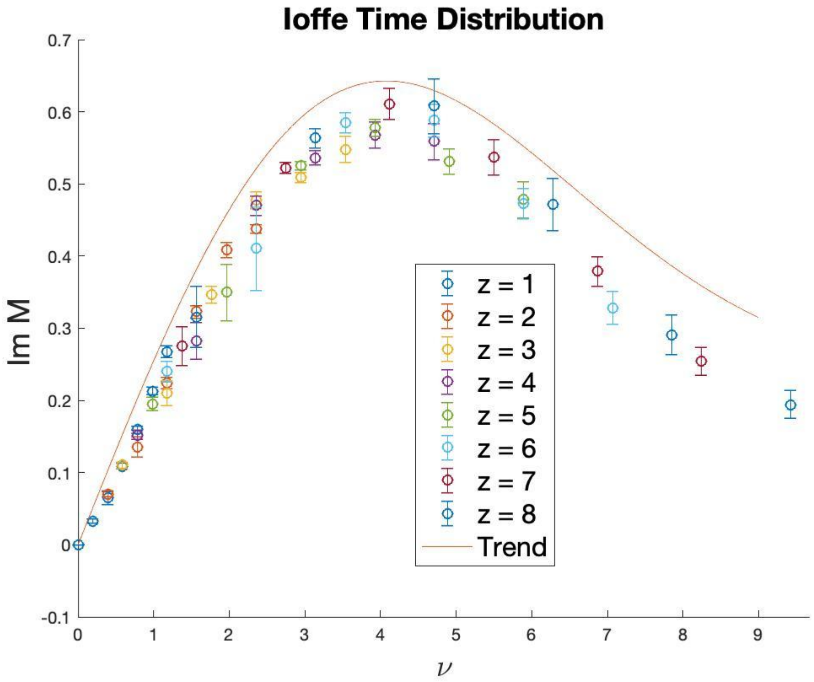


Pseudo-ITDF Results a127m440L



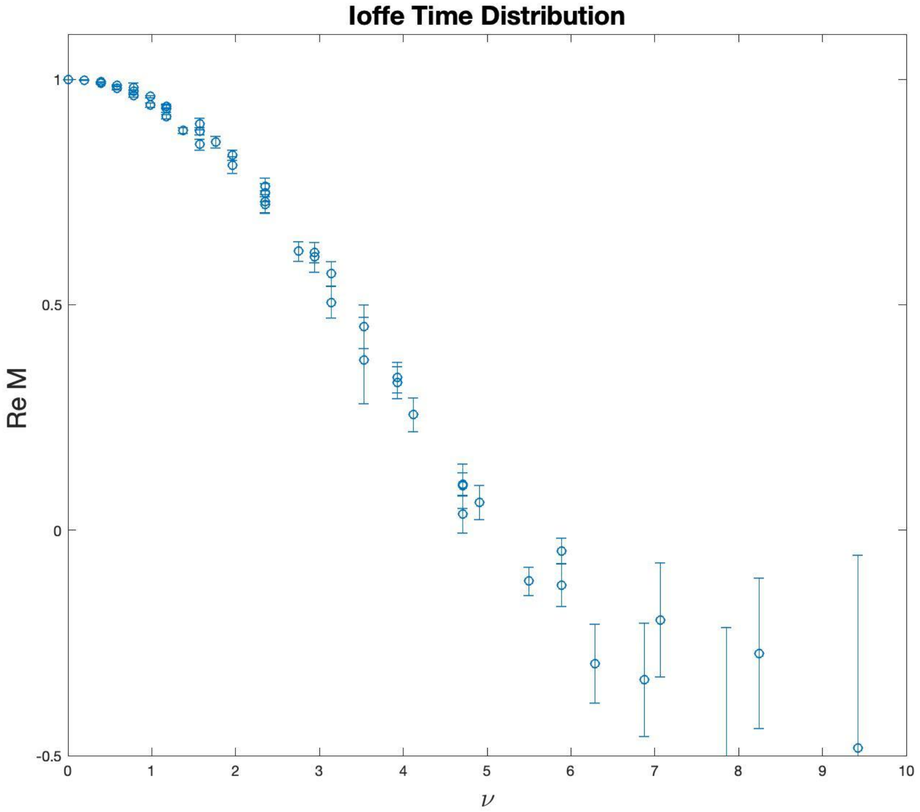


Imaginary Component and Anti-Quarks



- Imaginary component mixes valence, sea, and anti-quark distributions
- Use real component to find valence contribution, the rest is the sea and anti-quark distribution
- Identify anti-quark distribution without need of needing to perform inaccurate Fourier transforms and requiring the unreliable low negative x region
- Qualitatively it gives proper sign for both quenched and dynamical iso-vector quarks

ITDF MS bar matched Results

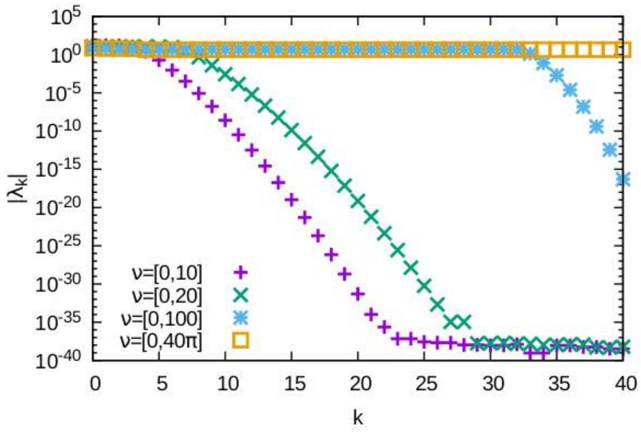


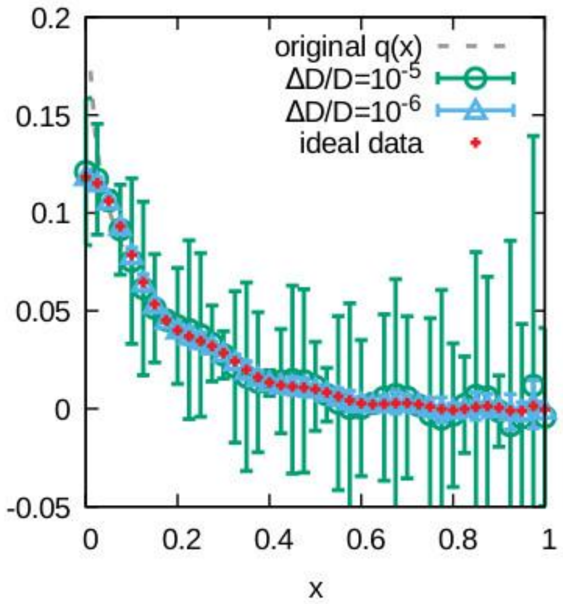
Preliminary

We have data, but now what?

Full PDF Reconstruction is hard

- Truncated discretized Fourier (cosine) transform is unreliable for realistic lattice data
  - Ill posed inverse problem
  - Consider problem as matrix equation $\mathcal{M}_i = \sum_j C_{ij} q_j$
- Mock test to reconstruct PDF from 40 evenly spacing Ioffe time PDF points given Gaussian noise.
  - Noisy data requires either unreasonably large ranges of Ioffe Time unreasonably precise data to reproduce model PDFs.





Real component and the Valence Quark distribution

- In first attempt to avoid ill posed inverse Fourier transform
- A general model PDF used by JAM collaboration for fitting

$$f_{abcd}(x) = N_{abcd}\ x^a(1-x)^b(1+c\sqrt{x}+d\,x)$$

$$N_{abcd} = B(a+1,b+1) + cB(a+\frac{3}{2},b+1) + dB(a+2,b+1)$$

- Expected lowest order behaviors
  - Regge $a = -\frac{1}{2}$
  - Quark counting $b = 3$
  - Small Corrections $c \sim 0 \sim d$

OPE without OPE

Chambers et al. [hep/lat] 1703.01153
Martinelli [hep/lat] 9810013

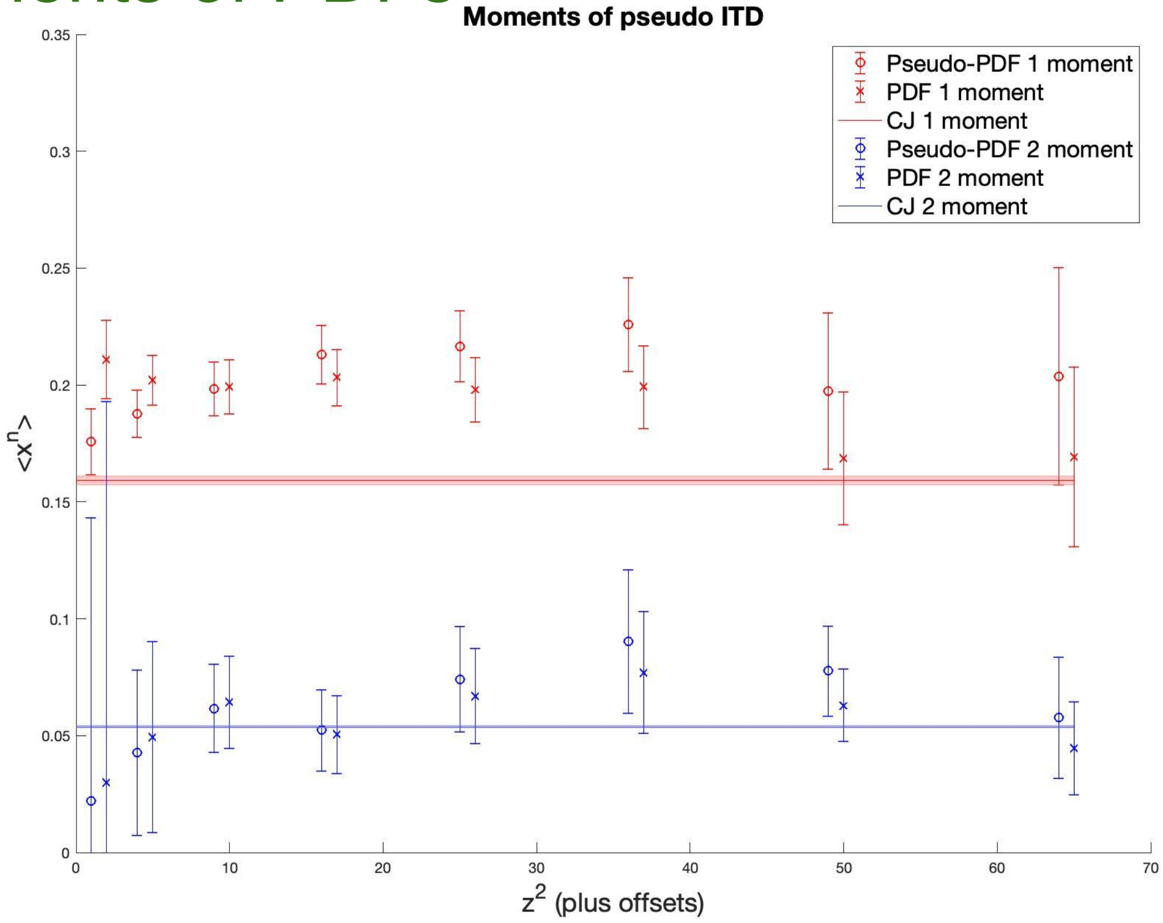
- Use of non-local matrix elements to extract local matrix elements for the moments.
- Using matching relationships, properties of the non-local matrix element can be related to MS-Bar PDF moments
- PDF reconstruction requires solution to the Inverse problem related to Mellin transforms.

Moments of PDFs

- Taylor Expansion coefficients of Ioffe Time Distributions give the moments of PDFs
  - Even moments from Real component of ITDF
  - Odd moments from Imaginary component of ITDF
- With sufficient data there is no limit on what moments can be calculated
  - The reduced matrix element is renormalization group invariant and could not have any divergent lattice spacing dependence
  - Power divergences in the local matrix elements of the OPE are canceled by power divergences in the Wilson Coefficients
  - Practically, only lowest moments are currently accessible

JK, K. Orginos, S. Zafeiropoulos 1807.10933

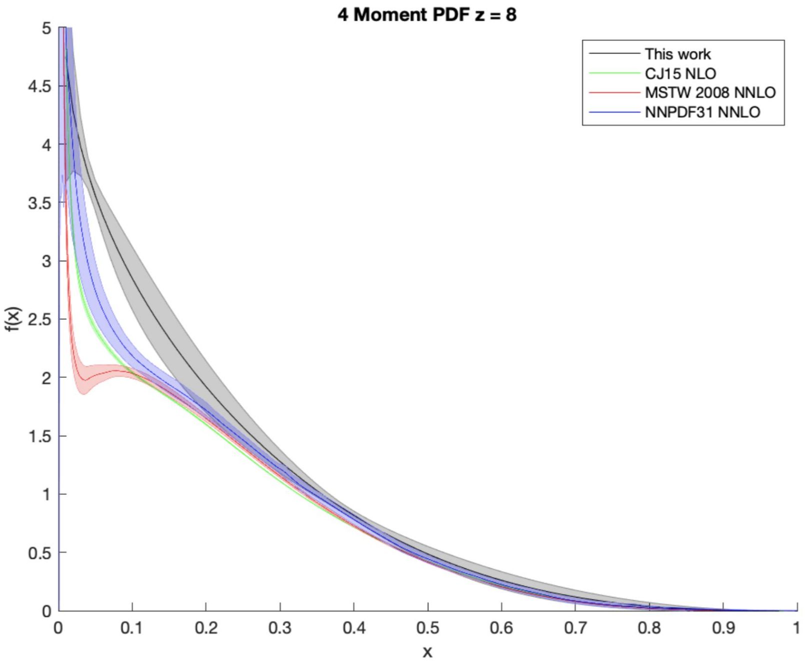
Moments of PDFs

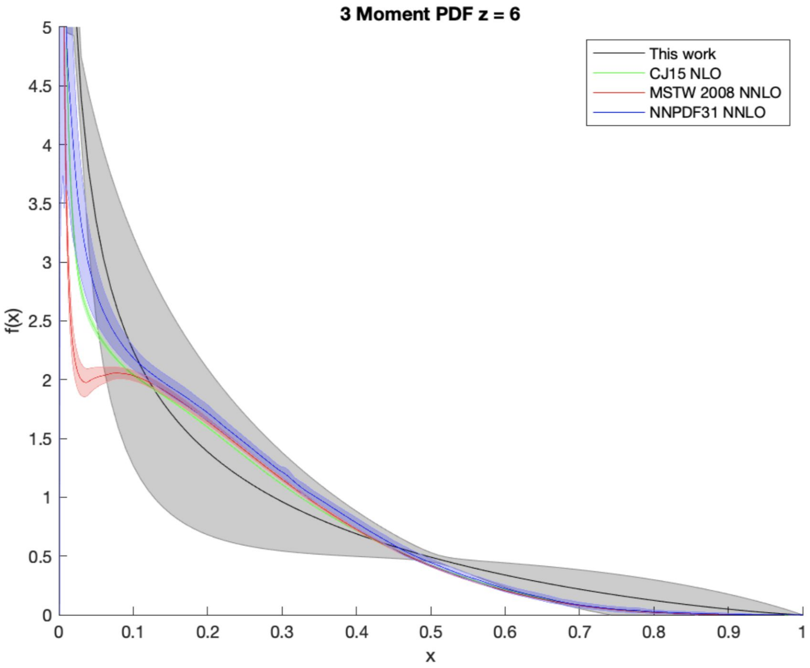


Without many moments the inverse in hard

The inverse Mellin transform of these moments can be used to extract PDF.

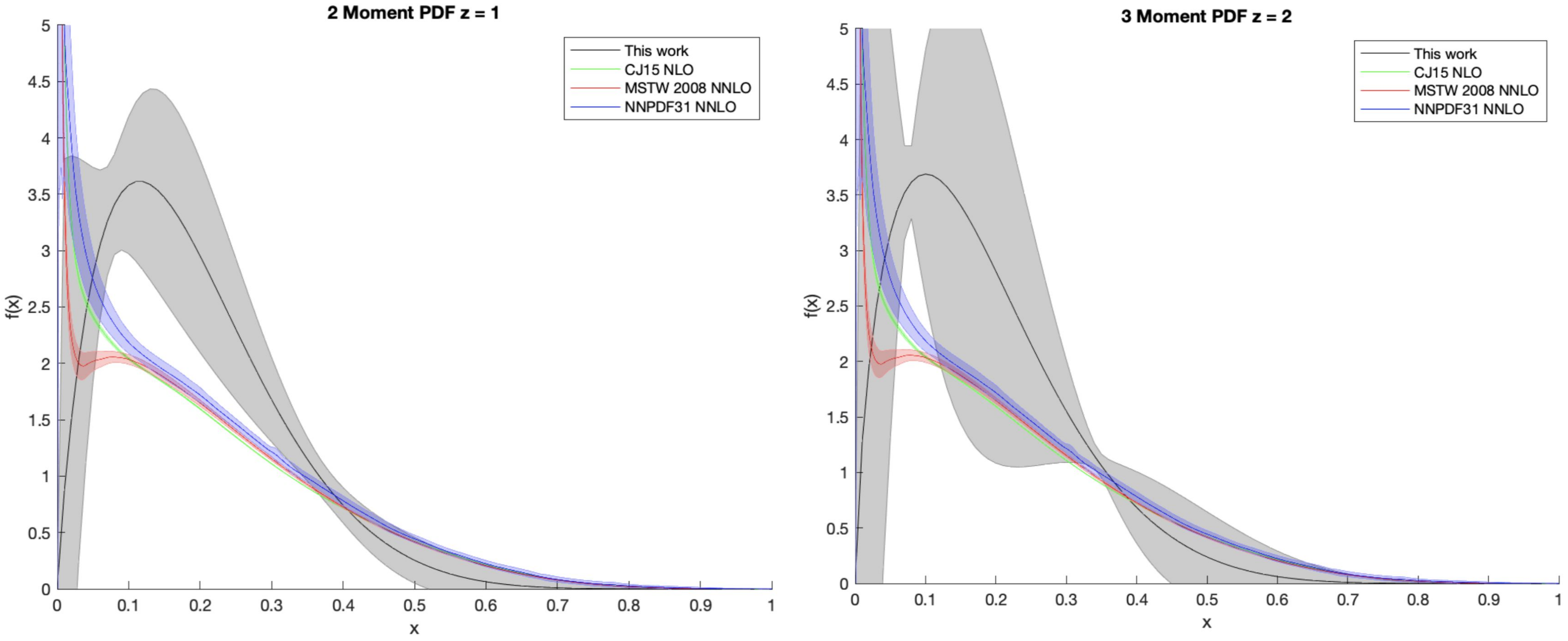
Some cases work appear to well





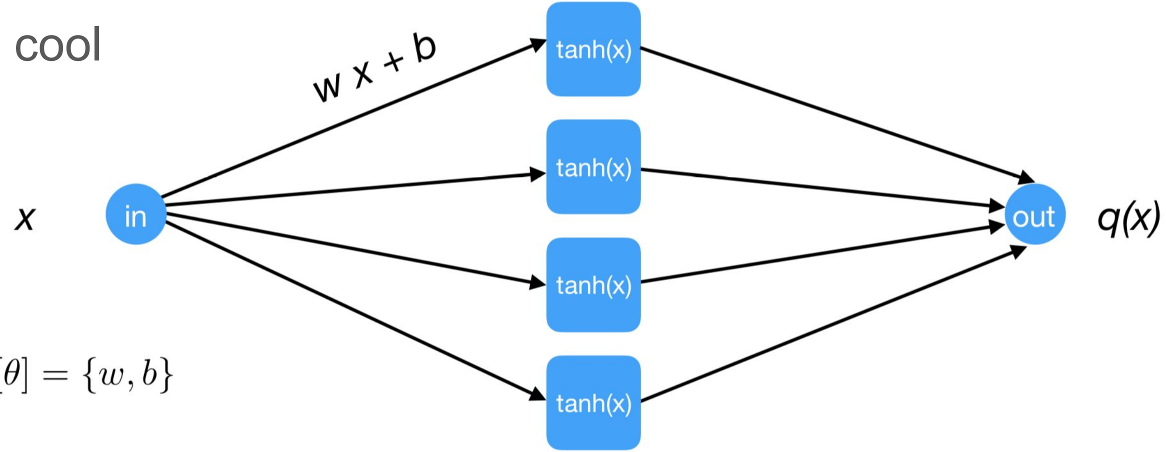
Without many moments the inverse in hard

And some cases don't



Neural Network Reconstruction

- In the style of NNPDF, a series of neural networks can be constructed to represent the ill posed inverse transformation.
    - Many choices of Network geometry and activation functions need to be explored
- Even a small Neural net can be used to reconstruct PDF to accuracy of other methods.
- Machine Learning is cool



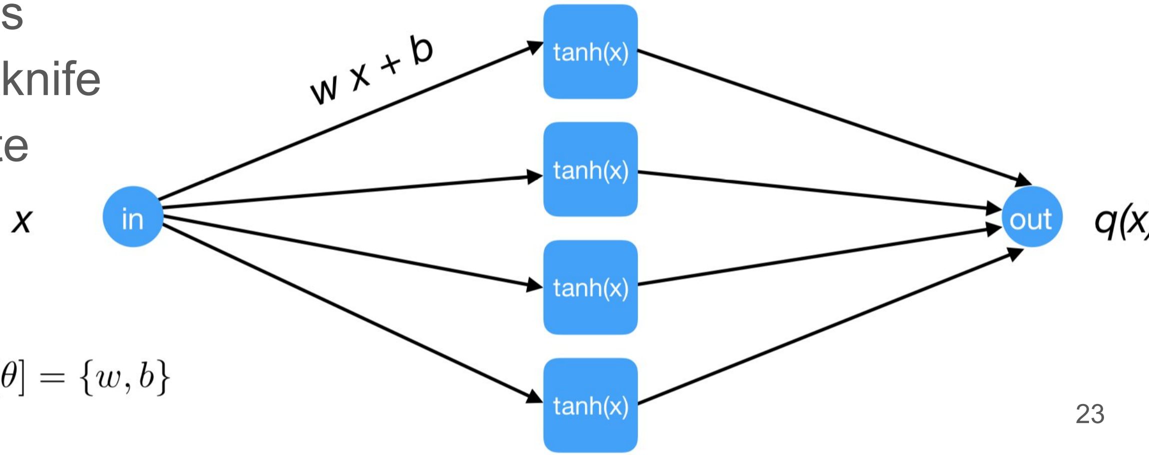
$$[\theta] = \{w, b\}$$

Neural Network Procedure

- Choose network geometry and activation function
- Using the full dataset, minimize network with respect to

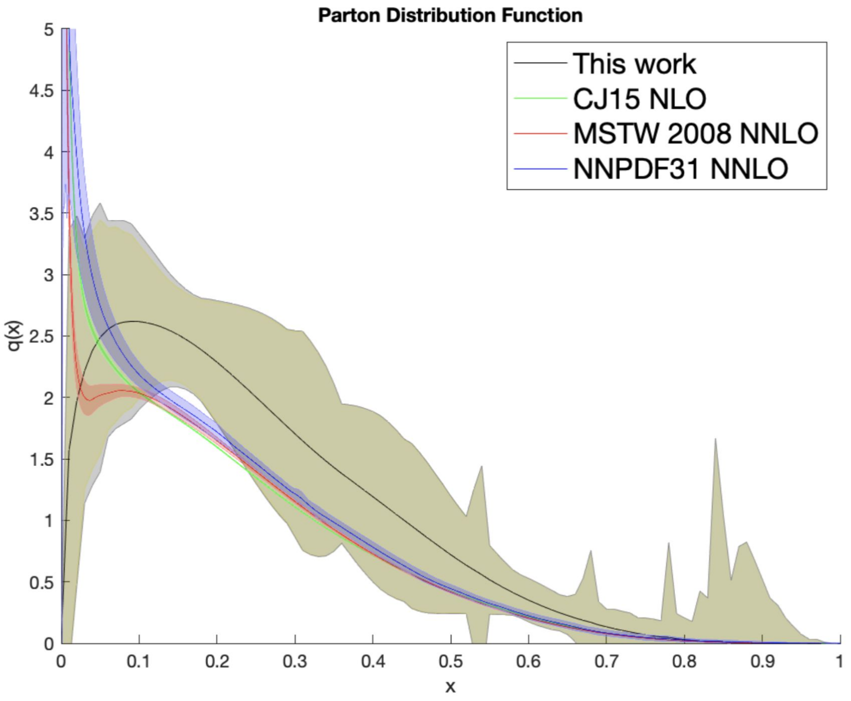
$$\chi^2 = (M_k - \int_0^1 dx cos(\nu_k x) q_{[\theta]}(x)) C^{-1} (M_k - \int_0^1 dx cos(\nu_k x) q_{[\theta]}(x))^T$$

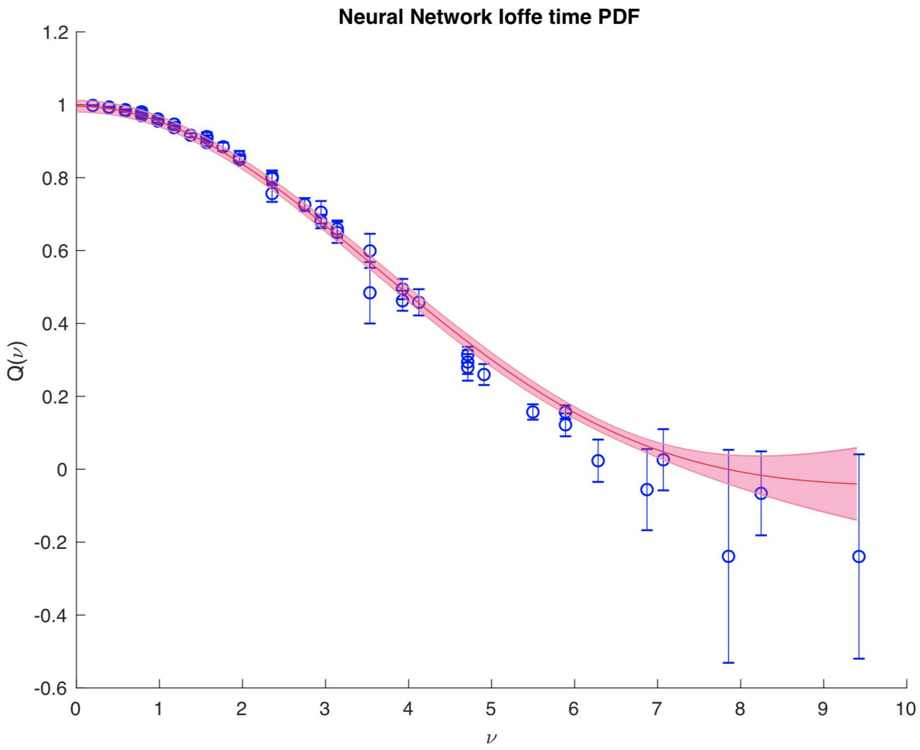
  several times, removing networks with largest value
- Repeat for a few generations
- Retrain each replica on jackknife samples to get error estimate



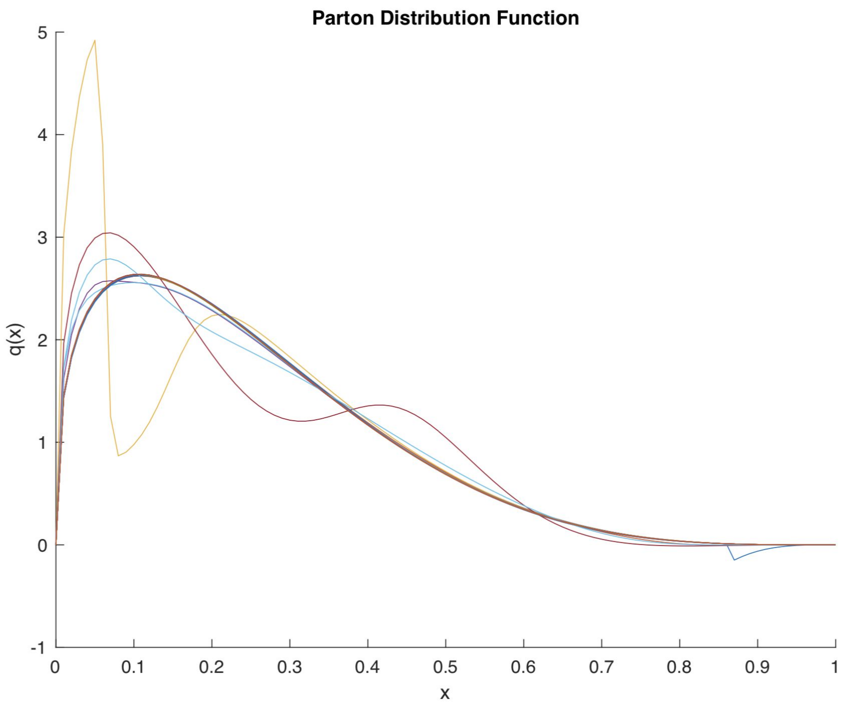
$$[\theta] = \{w, b\}$$

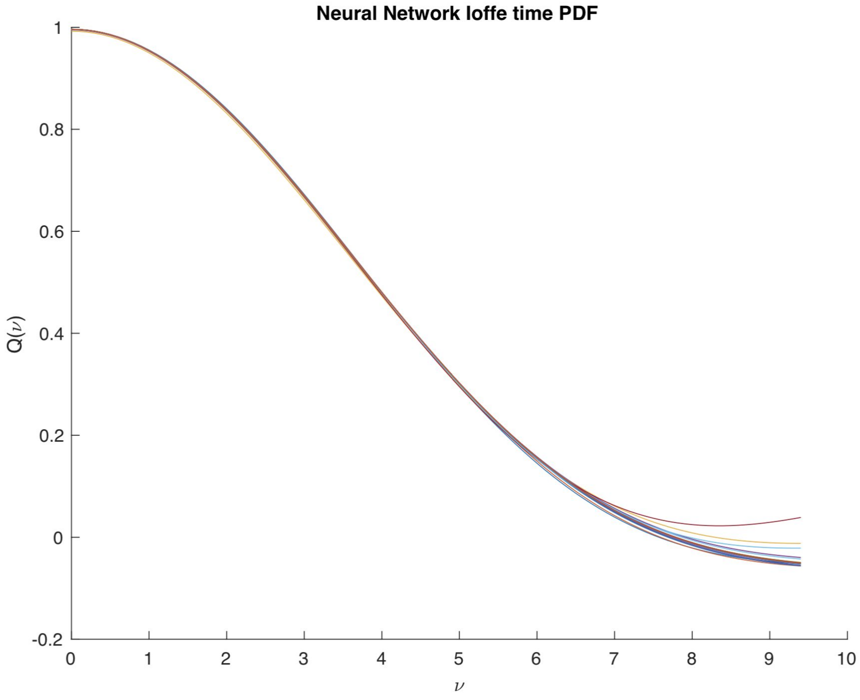
Tests of Neural Network Reconstructions





Neural networks





Bayesian Reconstruction

Y. Burnier and A. Rothkopf (2013) 1307.6106

- Technique based upon Bayes Theorem

$$P[q|M,I] = \frac{P[M|q,I]P[q|I]}{P[M|I]}$$

- Acknowledging the ill posed nature of the problem and that a unique solution require addition of further information
- Parameterize the probabilities and extremize the posterior probability
- Developed for extraction of quark spectral function which is a much harder application

$$P[M|q,I] = exp[-L]$$

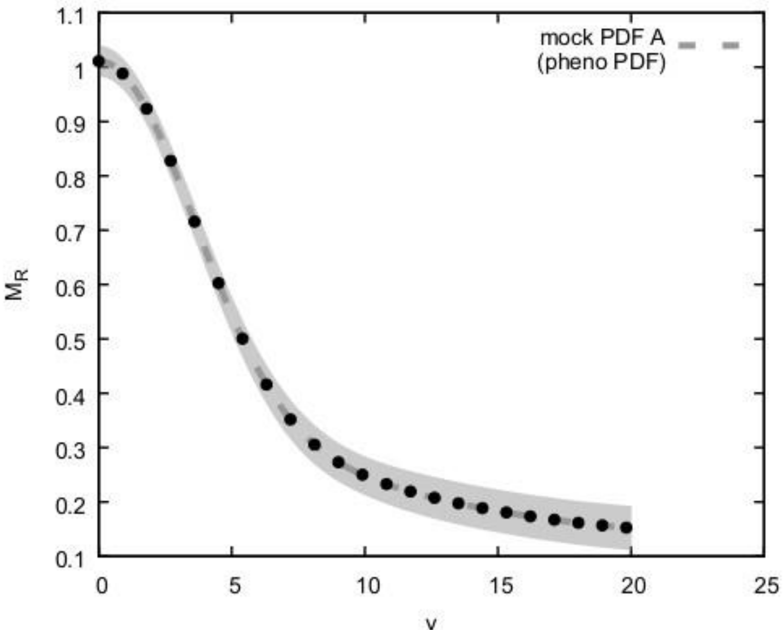
$$P[q|I] = exp[-S] \qquad S = \sum_n \Delta x_n (1 - \frac{q_n}{m_n} + \log \frac{q_n}{m_n})$$

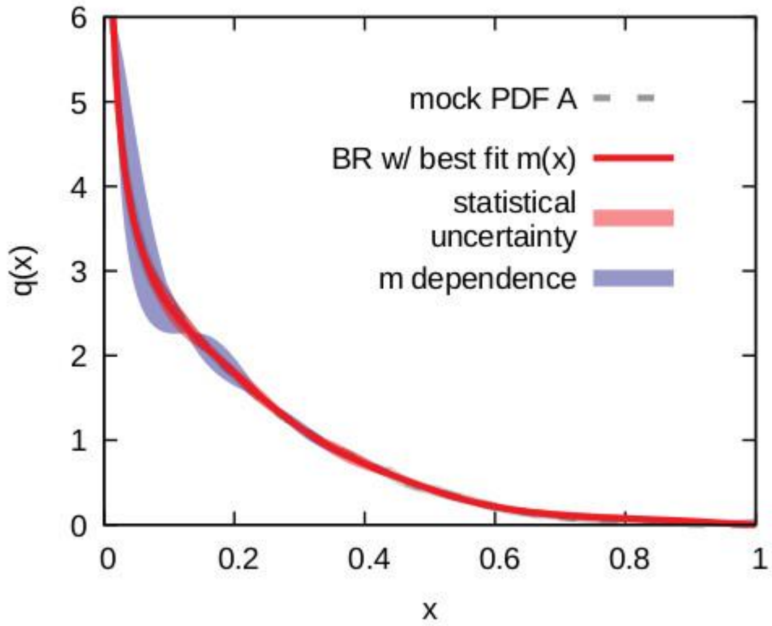
$$P[M|I] = N$$

Mock Tests of Bayesian Reconstruction

- Tests use NNPDF31_nnlo_as_0118 data set with artificial errors.
- Reconstructions are more stable and reliable than direct inversion or fits.

  Bayesian Reconstruction





Summary

- Simple extraction PDF moments are possible from lattice data
- Systematic issues with Fourier transform are controllable with inverse problem techniques
- Most extractions have qualitative agreement with PDFs despite few systematics under control
  - Statistics in some methods are lacking
  - Systematics of methods need to be studied
- Once these techniques are understood and controlled then any light cone distribution is within reach of the lattice

Thank you for your attention!