Stony Brook University

Quasi-pdf

X. Ji, Y. Liu and I. Zahed
arXiv-1807.07528

BNL 19'

Outline

- Scalar QCD-1+1
- QPDF: $1/P^0, 1/P$
- Spinor QCD-1+1
- QPDF: $1/N^0/1/P^0, 1/P, 1/N$
- QPDA in 4E
- Conclusions

QCD-1+1

Scalar QCD1+1

$$\mathcal{L} = \frac{1}{2} tr F_{01}^2 + (D^\mu \phi)^\dagger D_\mu \phi - m^2 \phi^\dagger \phi$$

$$J^a = i\left(\phi^\dagger T^a \pi - \pi^\dagger T^a \phi\right)$$

$$H = H_0 + H_{int} = \int dx (\pi^\dagger \pi + |\partial_1 \phi|^2 + m^2 |\phi|^2) + \frac{g^2}{2} \int dx \left(J^a \frac{-1}{\partial_1^2} J^a \right)$$

$$\phi_\alpha = \int \frac{dk}{\sqrt{4\pi E_k}} e^{-ikx} (a_k + b_{-k}^\dagger)_\alpha$$

$$(\pi^\dagger)_\alpha = i \int \frac{dk}{\sqrt{4\pi E_k}} e^{ikx} E_k (a_k^\dagger - b_{-k})_\alpha$$

Bosonization

$$M(k_1,k_2) = \frac{1}{\sqrt{N_c}}\sum_\alpha a_\alpha(k_1)b_\alpha(k_2)$$

$$N(k_1,k_2) = \sum_\alpha a^\dagger_\alpha(k_1)a_\alpha(k_2) \rightarrow \int dq M^\dagger(k_1,q)M(k_2,q)$$

$$\bar{N}(k_1,k_2) = \sum_\alpha b^\dagger_\alpha(k_1)b_\alpha(k_2) \rightarrow \int dq M^\dagger(q,k_1)M(k_2,q)$$

Holstein-Primakoff 40'
Ikatura 96'

Algebra

$$[M_{12}, M^\dagger_{34}] = \delta_{13}\delta_{24} + \frac{s}{N_c}(\delta_{13}\bar{N}_{42} + \delta_{42}N_{31})$$

$$[M_{12}, N_{34}] = \delta_{13}M_{42}$$

$$[M_{12}, \bar{N}_{34}] = \delta_{23}M_{14}$$

$$[M_{12}, M_{34}] = [N_{12}, \bar{N}_{34}] = 0$$

$$[N_{12}, N_{34}] = \delta_{23}N_{14} - \delta_{14}N_{32}$$

Hamiltonian $1/N_c^{\frac{1}{2}}$

$$H = H_2 + H_4$$

$$H_2 = \int dk(N(k) + \bar{N}(k))\Pi^+(k) + \sqrt{N_c}\int dk(M(k) + M^\dagger(k))\Pi^-(k)$$

$$H_4 = \lambda \int \frac{dk_1 dk_2 dk_3 dk_4}{16\pi} \frac{\delta(k_1+k_2+k_3+k_4)}{(k_1+k_2)^2}$$

$$\times\Bigg(-2f_+(k_1,k_2)f_+(k_3,k_4)M^\dagger(k_1,k_4)M(-k_2,-k_3) + f_-(k_1,k_2)f_-(k_3,k_4)M^\dagger(k_1,k_4)M^\dagger(k_3,k_2)$$

$$+f_-(k_1,k_2)f_-(k_3,k_4)M(k_1,k_4)M(k_3,k_2)) + \mathcal{O}\left(\frac{1}{\sqrt{N_c}}\right)\Bigg)$$

$$f_\pm(k_1,k_2) = \sqrt{\frac{E_2}{E_1}} \pm \sqrt{\frac{E_1}{E_2}}$$

Gap Equation

$$\Pi^{\pm}(k) = \frac{1}{2}\left(\frac{k^2+m^2}{E_k} \pm E_k\right) + \lambda \int \frac{dk_1}{8\pi} \frac{\frac{E_{k_1}}{E_k} \pm \frac{E_k}{E_{k_1}}}{(k+k_1)^2} \xrightarrow{-} 0$$

PV

$$\frac{k^2+m^2}{E_k} - E_k + \frac{\lambda}{4\pi}\int dk_1 \left(\left(\frac{E_{k_1}}{E_k} - \frac{E_k}{E_{k_1}}\right)\frac{\mathrm{PV}}{|k+k_1|^2} - \frac{E_{k_1}}{E_k}\frac{1}{k_1^2}\right) = 0$$

$$m^2 \to m^2 + \frac{\lambda}{4\pi}\int dk_1 \frac{E_{k_1}}{k_1^2}$$

Canonical

$$H \approx \int dp dq \left(\Pi^+(p) + \Pi^+(q)\right) M^\dagger(p,q) M(p,q) - \frac{\lambda}{16\pi} \int dP \int dk dp \, \frac{A+B}{(p-k)^2}$$

$$A = 2 f_+(p-P, k-P) f_+(p,k) M^\dagger(p-P,p) M(k-P,k)$$
$$B = 2 f_-(p-P, k-P) f_-(p,k) \left(M^\dagger(p,p-P) M^\dagger(k-P,k) + c.c\right)$$

$$M(p-P,p) = \frac{1}{\sqrt{|P|}} \sum_n \left(m_n(P) \phi_n^+(p,P) - m_n^\dagger(-P) \phi_n^-(p-P,-P) \right)$$

$$\left[m_n(P), m_l^\dagger(P' \right] = \delta_{nl} \delta(P-P'),$$
$$|P\rangle \equiv |E_n(P), P\rangle = \sqrt{2E_n(P)}\, m_n^\dagger(P)\, |0\rangle$$

$$i\partial_t M^\dagger(p-P,p)\,|0\rangle = \left[M^\dagger(p-P,p), H \right] |0\rangle$$

$$(\Pi^+(p) + \Pi^+(P-p) \mp P_n^0) \phi_n^\pm(p,P) = \frac{\lambda}{8\pi} \int \frac{dk}{(p-k)^2} \left(S_+(p,k,P) \phi_n^\pm(k,P) - S_-(p,k,P) \phi_n^\mp(k,P) \right)$$

1/P expansion

$$\Pi^+(Px) + \Pi^+((1-x)P) - \sqrt{P^2 + M_n^2} = \frac{1}{2P}\left(\frac{m^2}{x} + \frac{m^2}{1-x} - M_n^2\right) + \mathcal{O}\left(\frac{1}{P^2}\right)$$

$$\left(\frac{m^2}{x} + \frac{m^2}{1-x} - M_n^2\right)\phi_n(x) = \frac{\lambda}{4\pi}\mathrm{PV}\int \frac{dy}{(x-y)^2}\frac{(2-x-y)(x+y)}{\sqrt{x(1-x)y(1-y)}}\phi_n(y)$$

$p = xP, k = yP, \phi^- \to 0$

QPDF

$$\tilde{q}(x,P) = +i\int \frac{dz}{4\pi} e^{iPxz} \langle P|(\partial_1\phi(z))^\dagger W[z,0]\phi(0)|P\rangle - i\int \frac{dz}{4\pi} e^{iPxz} \langle P|(\phi(z))^\dagger W[z,0]\partial_1\phi(0)|P\rangle$$

$$\tilde{q}(x,P) = \frac{E_n(P)}{P}\frac{xP}{E(xP)}\left(|\phi_n^+(xP,P)|^2 + |\phi_n^+(-xP,P)|^2 + |\phi_n^-((xP,P)|^2 + |\phi_n^-(-xP,P)|^2\right)$$

$$\longrightarrow \quad |\phi_n^+(x)|^2$$

$P \to \infty \;: E_n(P) \to P\,, E(xP) \to xP\,, \phi^- \to 0$

1/P-correction

$$\Pi^+ = |P| + \frac{m^2}{2|P|} + \frac{\beta_1}{2|P|^3} + \mathcal{O}\left(\frac{1}{|P|^4}\right)$$

$$E(P) = |P| + \frac{\beta_2}{|P|} + \mathcal{O}\left(\frac{1}{|P|^2}\right)$$

$$\Rightarrow$$

$$x = 0, 1 \ !!$$

$$\phi_n^-(x) = \frac{1}{P^2}\left(\frac{\lambda}{24\pi\sqrt{x(1-x)}}\int_0^1 dy\,\frac{\phi_n(y)}{\sqrt{y(1-y)}}\right)$$

$$\phi_n^+(x) = \phi_n(x) + \frac{1}{P^2}\phi_{n1}^+(x)$$

Spinor QCD1+1

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \psi^\dagger(iD - m)\psi$$

$$\psi(x) = \int_0^\infty \frac{dp^+}{2\pi}(a(p^+)e^{-ip^+x^-} + b^\dagger(p^+)e^{-ip^+x^-})$$

$$\psi(x) = \int \frac{dp}{2\pi}e^{ipx}(a(p)u(p) + b^\dagger(-p)v(-p))$$

$$u(p) = e^{-\frac{1}{2}\theta(p)\gamma^1}(1,0)^T$$

$$v(-p) = e^{-\frac{1}{2}\theta(p)\gamma^1}(0,1)^T$$

$$p\cos(\theta(p)) - m\sin(\theta(p)) = \frac{\lambda}{2}\,\mathrm{PV}\int dk\,\frac{\sin(\theta(p)-\theta(k))}{(p-k)^2}$$

Bars-Green 78’

Bosonization

$$M(xP,(1-x)P)=\frac{1}{\sqrt{P}}\sum_n m_n(P)\phi_n(x)$$

$$M(k_1,P-k_1)=\frac{1}{\sqrt{|P|}}\sum_n \left(\phi_n^+(k_1,P)m_n(P)-\phi_n^-(-k_2,-P)m_n^\dagger(-P)\right)$$

$$\text{Leading } \frac{1}{P},\frac{1}{N}\ :\ \text{qpdf}=\text{pdf}$$

1/N-Algebra

$$M = M^0 + \frac{1}{N_c} M^1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$$N = N^0 + \frac{1}{N_c} N^1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

$$\begin{aligned} N^0_{12} &= \int d3\, M^{0\dagger}_{13} M^0_{23} \\ \bar{N}^0_{12} &= \int d3\, M^{0\dagger}_{31} M^0_{32} \\ M^1_{12} &= \mp\frac{1}{2}\int d3d4\, M^{0\dagger}_{34} M^0_{14} M^0_{32} \\ N^1 &= 0 \end{aligned}$$

$A^- = 0$ gauge

$$\frac{\lambda}{4\pi\sqrt{N_c}} \int \frac{dPdP_1}{P^{\frac{3}{2}}} \left(m_i^\dagger(P_1) m_j^\dagger(P-P_1) m_k(P) f_{ijk}(\frac{P_1}{P}) + \text{c.c}\right) + \frac{1}{N_c} m^\dagger m^\dagger m m$$

$$\left(\frac{1}{N_c} m^\dagger m^\dagger m m\right) m^\dagger |0\rangle = 0$$

1/N-PDF

$$\delta q_i(x) = \sum_{kl} \int \frac{dkdq}{2\pi} \phi_k \left(\frac{xP}{xP+q} \right) \phi_l \left(\frac{k}{k+q} \right) {}^1\langle P_i| \left(\frac{m_k^\dagger(xP+q) m_l(k+q)}{\sqrt{(xP+q)(k+q)}} \right) |P_i\rangle^1$$

$$|P\rangle^1 = \frac{\lambda}{2\sqrt{2\pi N_c}} \int dP_1 \sum_{kl} \left(\frac{f_{kli}(\frac{P_1}{P}) m_k^\dagger(P_1) m_l^\dagger(P-P_1)}{\frac{m_k^2}{x} + \frac{m_l^2}{1-x} - m_i^2} \right) |0\rangle$$

$$\delta q_i(x) = \frac{\lambda^2}{\pi^2 N_c} \int_0^{1-x} dy \sum_{kk'l} \frac{F_{kli}(x,y) F_{k'li}(x,y)}{x+y} \ ,$$

1/N-QPDF

$$H_1 = \frac{1}{\sqrt{N_c}} \sum_{123} f_{123} m_1^\dagger m_2^\dagger m_3$$

$$H_2 = \frac{1}{N_c} \sum_{1234} f_{1234} m_1^\dagger m_2^\dagger m_3^\dagger m_4$$

$$|i\rangle^1 = \frac{1}{\sqrt{N_c}} \sum_{12} |12\rangle\, \alpha_{12i}$$

$$|i\rangle^2 = \frac{1}{N_c} \sum_{123} |123\rangle\, \alpha_{123i}$$

$$\begin{aligned}\delta\tilde{q}_i(x) = \quad & 2\sin\frac{\theta(xP)}{2} \sum_{kk'l} \frac{\alpha_{kli}\alpha_{k'li}}{|k|} \phi_k^+(xP, p_k)\phi_{k'}^+(xP, p_k) \\ & +2\sin\frac{\theta(xP)}{2} \sum_{kk'l} \frac{\alpha_{kli}\alpha_{k'li}}{|k|} \\ & \times \phi_k^+(xP + p_k, p_k)\phi_{k'}^+(xP + p_k, p_k)\end{aligned}$$

Summary

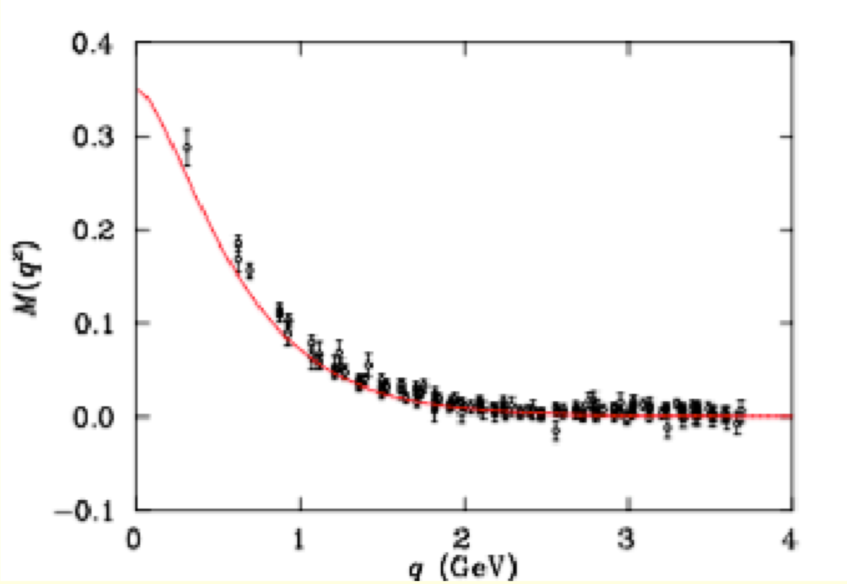
Ji's proposal OK

$$x \to 0, 1 \;\; \text{after} \;\; P \to \infty$$

QCD-4E

Instanton vacuum: Pion

$$\kappa = \frac{N\rho^4}{2V_4 N_c} \equiv \alpha^2\rho^4 \approx 2\,10^{-3}$$



$$M(p) = \frac{\alpha}{\sqrt{2}} \frac{|p\varphi'(p)|^2}{||q\varphi'^2||^2} + \mathcal{O}(\alpha^2)$$

$$g_\pi f_\pi = \langle i\psi^\dagger\psi\rangle = \frac{4\alpha N_c}{\sqrt{2}} \frac{||\varphi'||^2}{||p\varphi'^2||^2}$$

$$\psi_{0I,\bar{I}}(p) = \sqrt{2}\varphi'(p)\,\chi^\pm \equiv \sqrt{2}(4\pi\rho\, e^{-p\rho}/p)\chi^\pm$$

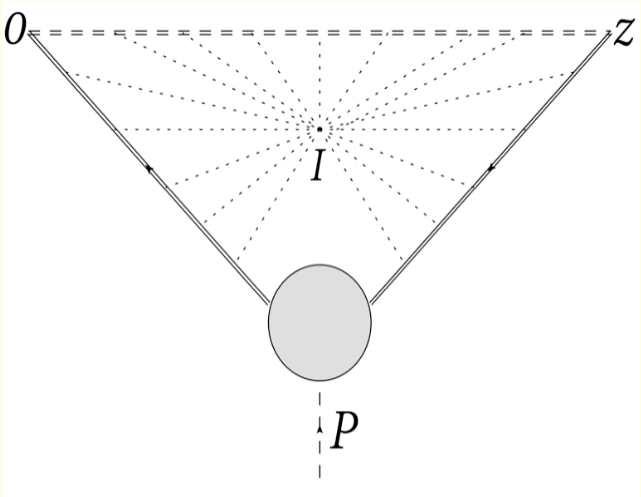
regular gauge

QPDA in 4E

$$\tilde{\phi}_\pi(x, P_z) = \frac{i}{f_\pi} \int \frac{dz}{2\pi}\, e^{-i(x-\bar{x})zP_z}\ \langle \pi(p)|\, \psi^\dagger(z_-)\gamma^z\gamma^5\, [z_-, z_+]\, \psi(z_+)\, |0\rangle$$

$$\int dx\, \tilde{\phi}_\pi(x, P_z) = \frac{i}{f_\pi P_z} \langle \pi(p)|\psi^\dagger(0)\gamma^z\gamma^5\psi(0)|0\rangle = 1$$

1/N-QPDA



$$\langle J_5 \mathbb{J}_{5z} \rangle$$

$$\tilde{\phi}_\pi(x, P_z) = \lim_{P^2 \to 0} \frac{-i}{f_\pi P_z} \frac{P^2}{g_\pi}$$
$$\times \int \frac{d^4k}{(2\pi)^4} \delta\left(x - \frac{1}{2} - \frac{k_z}{P_z}\right) \mathrm{Tr}\left(\gamma^z \gamma^5 S_1 O_5(p_1, p_2) S_2\right)$$

$$p_1^2 = \left(k_4 \pm \frac{i}{2} E_\pi\right)^2 + k_\perp^2 + x^2 P_z^2$$
$$p_2^2 = \left(k_4 \mp \frac{i}{2} E_\pi\right)^2 + k_\perp^2 + \bar{x}^2 P_z^2$$

$$g_\pi f_\pi = \langle i\psi^\dagger \psi \rangle = \frac{4\alpha N_c}{\sqrt{2}} \frac{||\varphi'||^2}{||p\varphi'^2||^2}$$

Order α^0

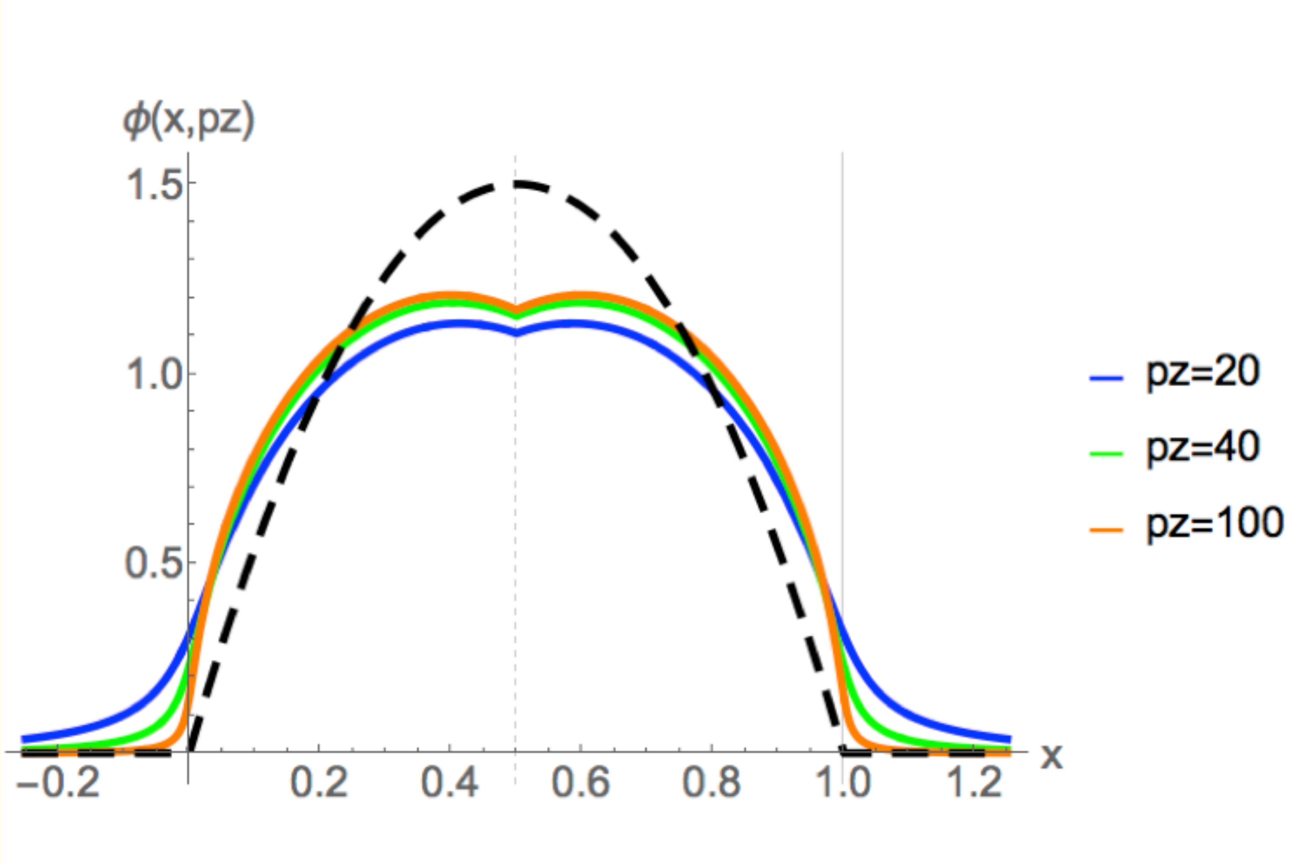
$$\phi_\pi(x,P_z) = \lim_{P^2\to 0} \frac{P^2}{\langle -2i\psi^\dagger\psi|_\alpha \sigma_{00}\rangle} \int_{I+\bar{I}} \frac{d^4k d^3p}{(2\pi)^7}$$
$$\times\left(\mathrm{Tr}\left(\gamma^z\gamma^5\delta G^I(p,k)\gamma^5\Gamma(k,P)\psi_{0I}(k^-)\psi^\dagger_{0I}(p^-)\right)\right.$$
$$\left.+\mathrm{Tr}\left(\gamma^z\gamma^5\psi_{0I}(p)\psi^\dagger_{0I}(k)\gamma^5\Gamma(k,P)\delta G^I(k^-,p^-)\right)\right)$$

$$\int_0^1 dx\phi_\pi(x,P_z) = 1$$

$$p_z = xP_z$$
$$k^- k - P\,,p^- = p - P$$

$$\delta G^I(p,k) = G^I(p,k) + (2\pi)^4\delta(k-p)\frac{1}{\not{k}}$$
$$\Gamma(p,P) = \lambda(P)\frac{|p||p-P|\varphi'(p)\varphi'(p-P)}{||p\varphi'^2||^2}$$
$$\lambda(P)\left(1 - \frac{\int \frac{d^4k}{(2\pi)^4}\varphi'^2(k)\varphi'^2(k-P)(k^2 - k\cdot P)}{||p\varphi'^2||^2}\right) = 1$$

QPDA: Preliminary



Koch-Liu-Zahed 19’

Summary

Instantons : QPDA, QPDF, QTMD