BROOKHAVEN NATIONAL LABORATORY *a passion for discovery*

CFNS Workshop on Lattice Parton Distribution Functions

Physics Department, April 17-19, 2019

Good Lattice Cross Sections

Jianwei Qiu

Theory Center, Jefferson Lab

Based on work done with
C. Egerer, T. Ishikawa, J. Karpie, Z.Y. Li, Y.-Q. Ma,
K. Orginos, D.G. Richards, R. Sufian, S. Yoshida, …
and work by many others, …

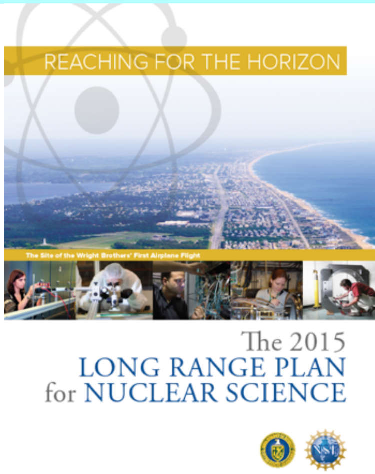
Jefferson Lab EXPLORING THE NATURE OF MATTER

TMD Collaboration

Hadron's Partonic Structure in QCD

Understanding the structure of hadrons in terms of QCD's quarks and gluons is one of the central goals of modern nuclear physics.

– 2015 NSAC Long-Range Plan



BUT, we do not see any quarks and gluons in isolation!

- **Unprecedented intellectual challenge:**
  - ✧ *Any cross section with identified hadron is NOT calculable perturbatively!*
  - ✧ *How to test a theory and quantify the structure of hadrons without being able to see the quarks and gluons?*

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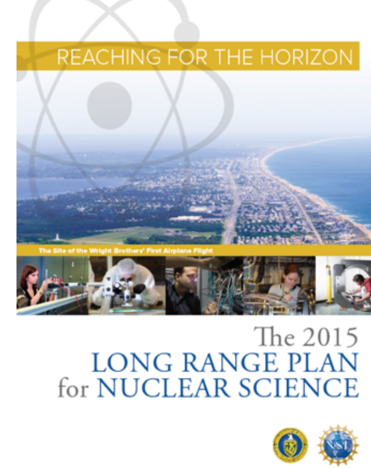

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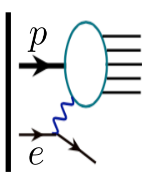
- ❑ **Factorization – approximation:**

➡ $\sigma_{\rm DIS}(x, Q^2) = \left| \text{diagram} \right|^2$



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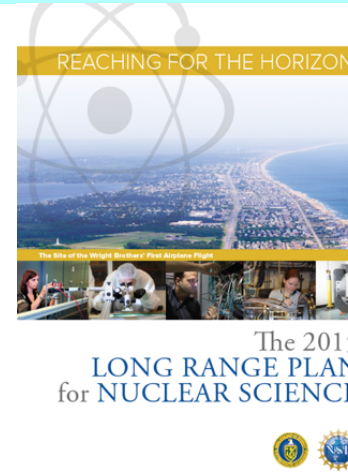

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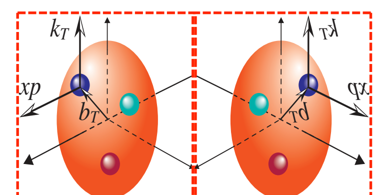
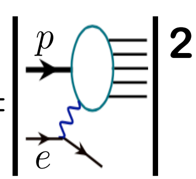
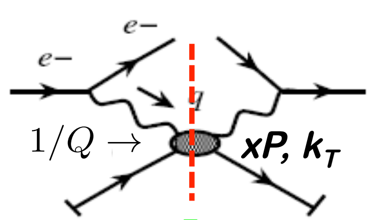


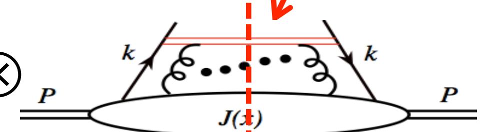
$$\sigma_{\rm DIS}(x, Q^2) =$$



$$\approx$$



$$\otimes$$



$$+ \, O\left(\frac{1}{QR}\right)$$

Factorization

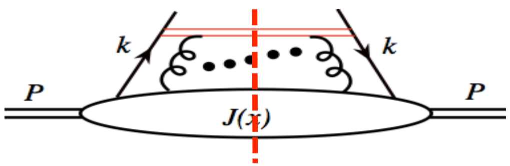
Hard-part
Probe

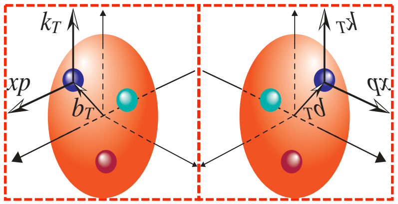
Parton-distribution
Structure

Power corrections
Approximation

Quantify Hadron's Partonic Structure

❑ Parton distribution functions (PDFs):





$f_{q/p}(x,\mu^2)$

$$f_q(x,\mu^2) \equiv \int \frac{dP^+\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle P|\overline{\psi}(\xi^-)\frac{\gamma^+}{2P^+}\exp\left\{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right\}\psi(0)|P\rangle$$

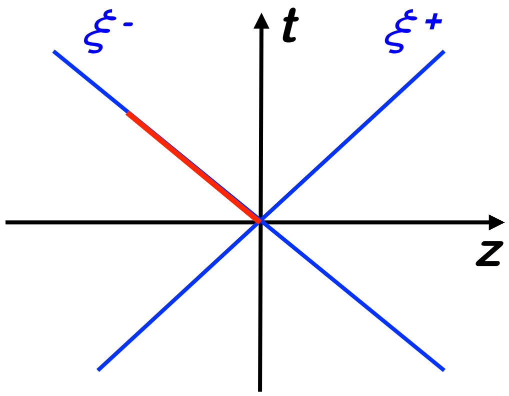
Dominated by the region:

$$\xi^- \lesssim 1/(xP^+) \sim 1/Q$$

Interpreted as:

Probability density
to find a quark with a momentum fraction x

Quantum correlation of quark fields along ξ^- direction!
(Conjugated to the large P^+)



Quantify Hadron's Partonic Structure

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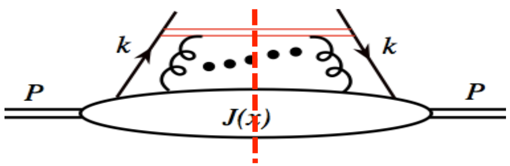

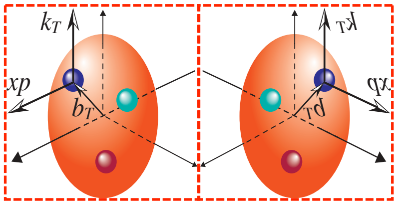

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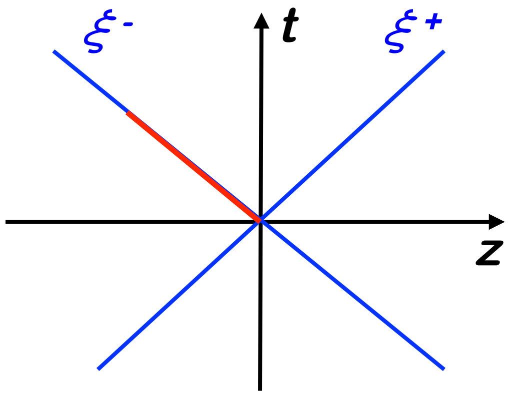


- **Good hadron cross sections:**

  1) **can be measured experimentally with good precision,**
  2) **can be factorized into universal matrix elements of quarks and gluons – parton distribution functions *with controllable approximation***

➡ ***Provide the indirect access to hadron's partonic structure!***

Global QCD Analyses – A Successful Story

- ❑ **World data with "Q" > 2 GeV**

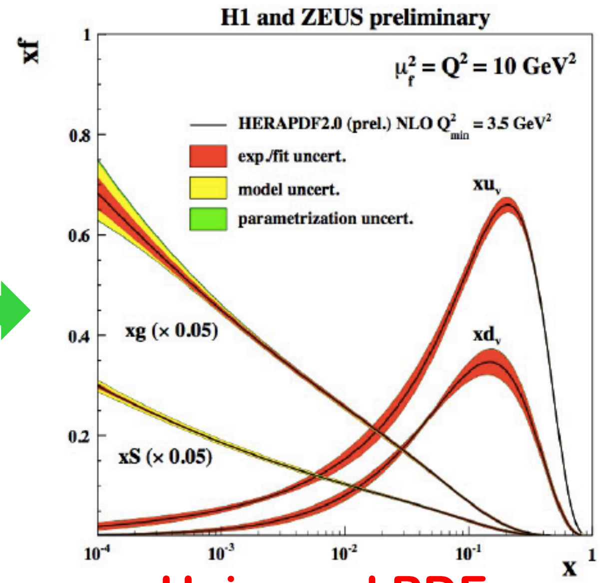
  + QCD Factorization:

e-H: $$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

H-H: $$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

+ DGLAP Evolution:

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$



Universal PDFs

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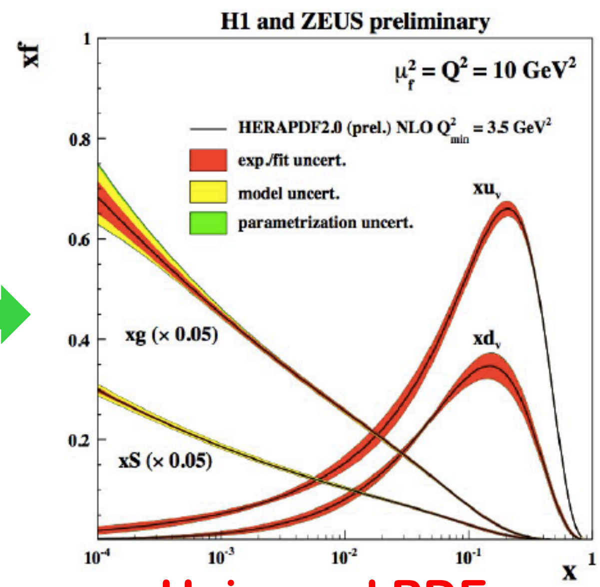

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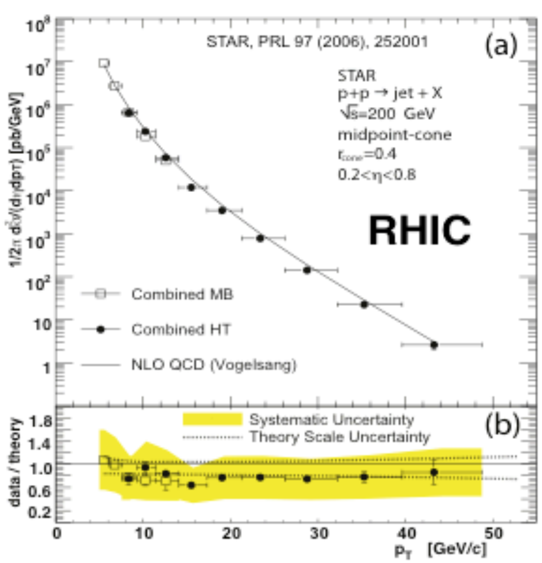

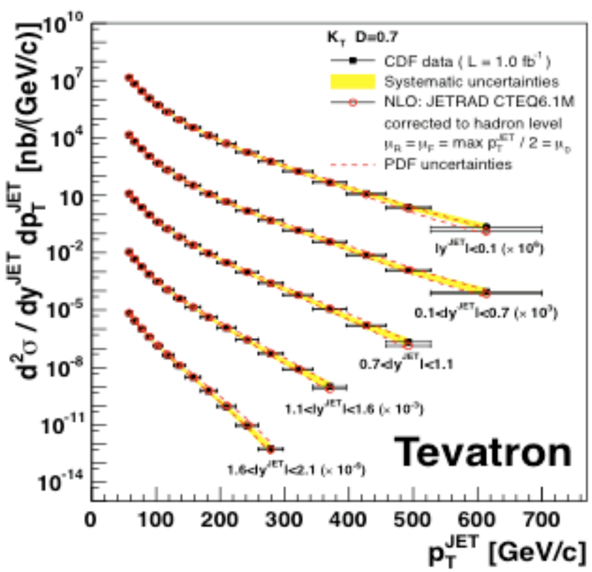

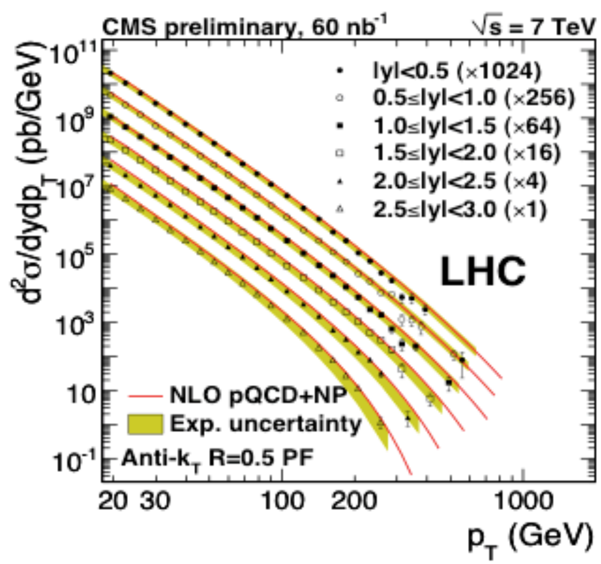

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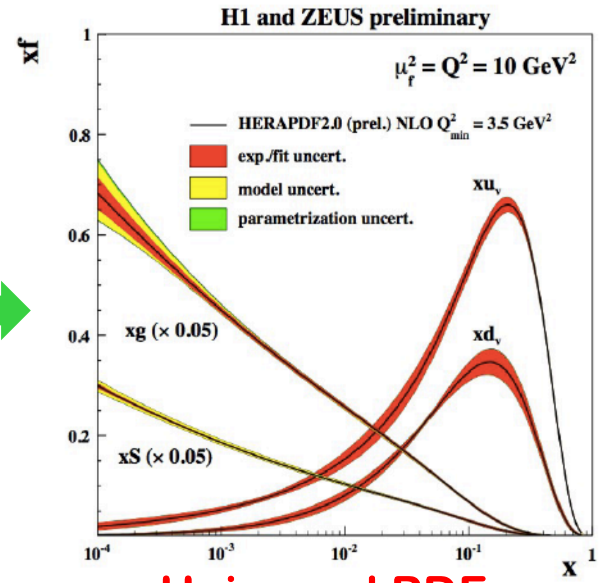

Universal PDFs

- The “BIG” question(s)

Why these PDFs behave as what have been extracted from the fits?

What have been tested is the evolution from μ_1 to μ_2
But, does not explain why they have the shape to start with!

Can QCD calculate and predict the shape of PDFs at the input scale, and other parton correlation functions?

Calculate PDFs in Lattice QCD?

- **Answer:** **Not directly!**

Particle nature of quarks/gluons ⇒ Probes with large Q transfer
Large momentum transfer ⇒ Operators on light-cone
Operators on light-cone ⇒ Can't be calculated in lattice QCD

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- **Quasi-PDFs:**

Ji, arXiv:1305.1539

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

+ gluon contribution beyond LO

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- **Pseudo-PDFs:**

Radyushkin, 2017

$$\mathcal{M}^\alpha(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$
$$\equiv 2p^\alpha\mathcal{M}_p(\nu,\xi^2) + \xi^\alpha(p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha\mathcal{M}_p(\nu,\xi^2)$$

$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi} e^{ix\,\nu} \frac{1}{2p^+}\mathcal{M}^+(\nu,\xi^2)$$ with $\xi^2 < 0$

Off-light-cone extension of PDFs: $f(x) = \mathcal{P}(x,\xi^2=0)$ with $\xi^\mu = (0^+,\xi^-,0_\perp)$

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- **Other approaches, …**

"OPE without OPE" (Chambers et al. 2017), Hadronic tensor (Liu et al. 1994, …), …

Lattice QCD + QCD Factorization

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- Good "Lattice cross sections":

  = Single hadron matrix element:

  p *and* ξ *define collision kinematics*

  $$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle \quad \text{with} \quad \omega \equiv P\cdot\xi,\ \xi^2 \neq 0,\ \text{and}\ \xi_0 = 0;$$

  Plus:

  1) can be calculated in lattice QCD with precision,
     has a well-defined continuum limit (UV+IR safe perturbatively), and
  2) can be factorized into universal matrix elements of quarks and gluons
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Collaboration between lattice QCD and perturbative QCD!

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with the gauge link: $\Phi^{(f,a)}(\xi,0) = \mathcal{P}e^{-ig\int_0^1 \xi\cdot A^{(f,a)}(\lambda\xi)d\lambda}$

  ✧ Quasi-PDFs are given by F.T. of ξ_z with fixed p_z
  ✧ Pseudo-PDFs are given by F.T. of p_z with fixed ξ_z

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Need a UV renormalization scheme:

See Y.Q Ma's talk

✧ Easy to be implemented non-perturbatively, as well as perturbatively
✧ Need to be both IR and CO insensitive!

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Collaboration between lattice QCD and perturbative QCD!

- **Current-current correlators:**

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2} Z_{j_1}^{-1} Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

with

d_j : Dimension of the current

Z_j : Renormalization constant of the current

Sample currents:

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q \gamma \cdot \xi \psi_q](\xi),$$

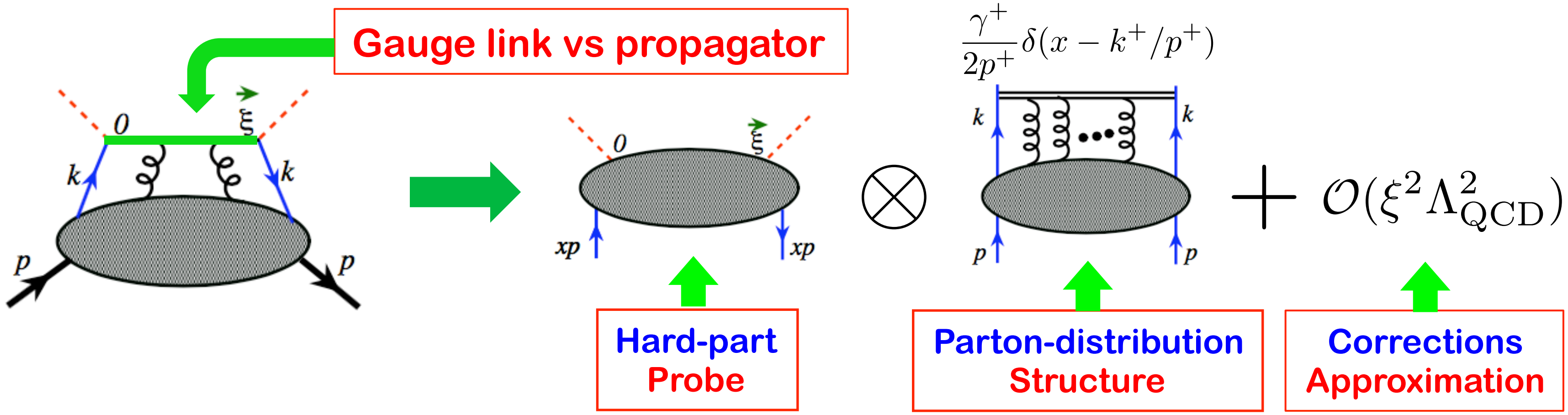
$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q \gamma \cdot \xi \psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4} F_{\mu\nu}^c F_{\mu\nu}^c](\xi), \ldots$$

✧ Easier for UV renormalization, more "lattice cross sections", …
✧ Harder to calculate in lattice QCD, …

Lattice Data + QCD Global Analyses

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- **Factorization:**



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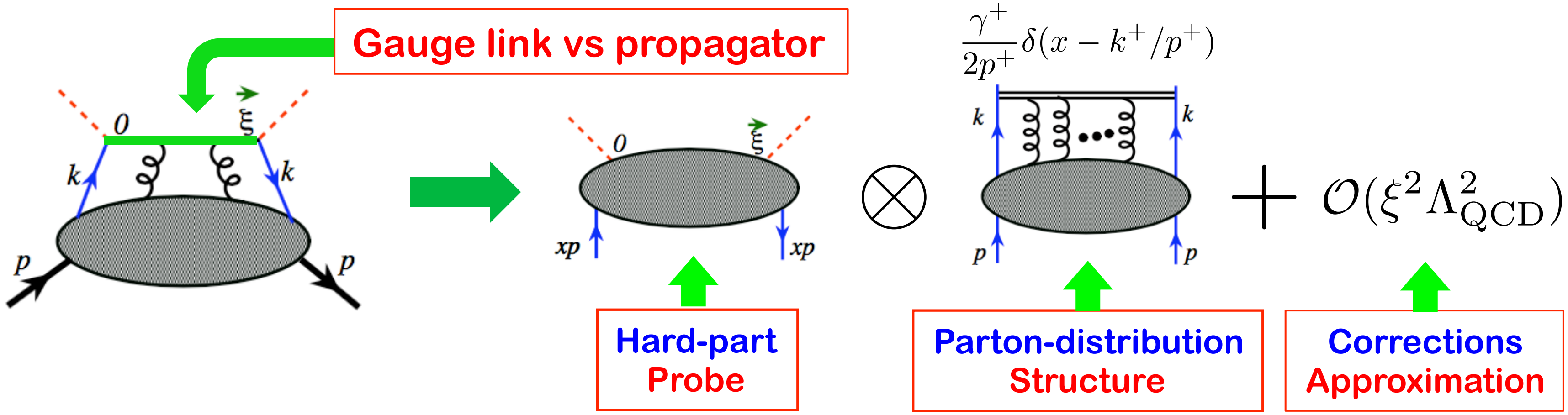

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Need data of "many" good lattice cross sections to be able to extract the x, Q, flavor dependence of the hadron structure, …

Lattice Data + QCD Global Analyses

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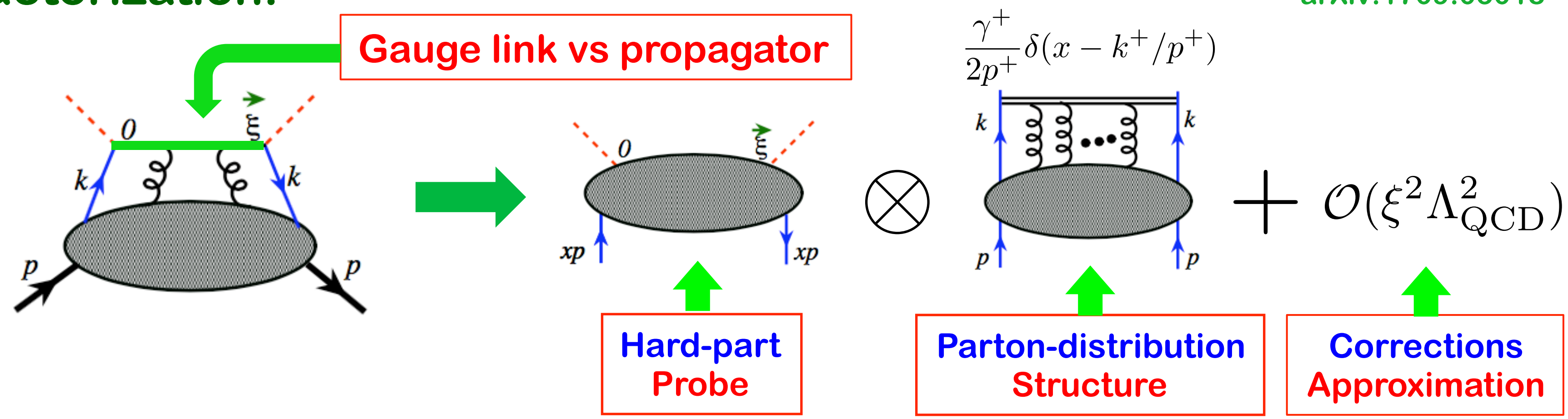

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- ❑ **Complementarity and advantages:**
  - ✧ Complementary to existing approaches for analyzing exp data, large x, …
  - ✧ Complementary between different “lattice cross sections”, …
  - ✧ Have tremendous potentials:

Neutron PDFs, … (no free neutron target!)
Meson PDFs, such as pion, …
More direct access to gluons – gluonic current, quark flavor, …

Factorization

Ma and Qiu, arXiv:1404.6860
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- **Factorized formula for lattice cross section:**

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

with $f_a(x,\mu^2) = -f_a(-x,\mu^2)$

- **Steps needed to prove:**

Let ξ^2 be small but not vanishing, apply OPE to the operator,

$$\sigma_n(\omega,\xi^2,P^2) = \sum_{J=0} \sum_a W_n^{(J,a)}(\xi^2,\mu^2)\, \xi^{\nu_1}\cdots\xi^{\nu_J} \times \langle P|\mathcal{O}^{(J,a)}_{\nu_1\cdots\nu_J}(\mu^2)|P\rangle$$

Local, symmetric and traceless with spin J

with

$$\langle P|\mathcal{O}^{(J,a)}_{\nu_1\cdots\nu_J}(\mu^2)|P\rangle = 2A^{(J,a)}(\mu^2) \times (P_{\nu_1}\cdots P_{\nu_J} - \text{traces})$$

With reduced matrix element: $A^{(J,a)}(\mu^2) = \langle P|\mathcal{O}^{(J,a)}(\mu^2)|P\rangle$

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arXiv:1709.03018

- **Factorized formula for lattice cross section:**

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

with $f_{\bar a}(x,\mu^2) = -f_a(-x,\mu^2)$

- **Steps needed to prove:**

Let ξ^2 be small but not vanishing, apply OPE to the operator,

$$\sigma_n(\omega,\xi^2,P^2) = \sum_{J=0}\sum_a W_n^{(J,a)}(\xi^2,\mu^2)\,\xi^{\nu_1}\cdots\xi^{\nu_J} \times \langle P|\mathcal{O}^{(J,a)}_{\nu_1\cdots\nu_J}(\mu^2)|P\rangle$$

Local, symmetric and traceless with spin *J*

with

$$\langle P|\mathcal{O}^{(J,a)}_{\nu_1\cdots\nu_J}(\mu^2)|P\rangle = 2A^{(J,a)}(\mu^2) \times (P_{\nu_1}\cdots P_{\nu_J} - \text{traces})$$

With reduced matrix element: $A^{(J,a)}(\mu^2) = \langle P|\mathcal{O}^{(J,a)}(\mu^2)|P\rangle$

→

$$\sigma_n(\omega,\xi^2,P^2) = \sum_{J=0}\sum_a W_n^{(J,a)}(\xi^2,\mu^2)\,2A^{(J,a)}(\mu^2) \times \Sigma_J(\omega,P^2\xi^2)$$

with $\Sigma_J(\omega,P^2\xi^2) \equiv \xi^{\nu_1}\cdots\xi^{\nu_J}(P_{\nu_1}\cdots P_{\nu_J} - \text{traces})$

$$= \sum_{i=0}^{[J/2]} C^i_{J-i}(\omega)^{J-2i}\left(-P^2\xi^2/4\right)^i$$

No approximation yet!

Factorization

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- **Approximation – leading power/twist:**

$$A^{(J,a)}(\mu^2) = \frac{1}{S_a}\int_{-1}^{1} dx\, x^{J-1} f_a(x,\mu^2)$$ With symmetry factor: $S_a = 1, 2$ for $a = q, g$

→

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

with $$K_n^a = \sum_{J=1} \frac{2}{S_a} W_n^{(J,a)}(\xi^2,\mu^2)\,\Sigma_J(x\omega, x^2P^2\xi^2)$$

Note: our proof of factorization is valid only when $|\omega| \ll 1$ and $|p^2\xi^2| \ll 1$

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Note: our proof of factorization is valid only when $|\omega| \ll 1$ and $|p^2\xi^2| \ll 1$

- Extrapolate into large ω region:

  - ✧ Validity of OPE guarantees that σ_n is an analytic function of ω, so as its Taylor series of ω around ω=0, defined above
  - ✧ If we fix ξ to be short-distance, while we increase ω by adjusting p, we can't introduce any new perturbative divergence
  - ✧ That is, σ_n remains to be an analytic function of ω unless $\omega = \infty$

Factorization holds for any finite value of ω and $p^2\xi^2$, if ξ is short-distance

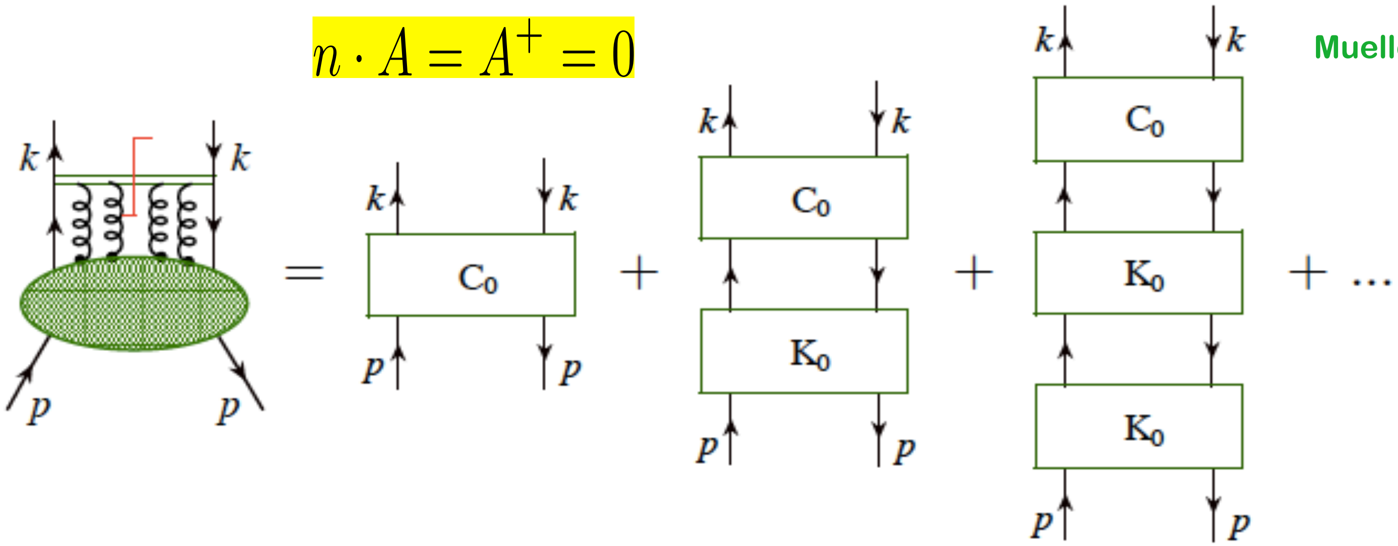
Factorization of CO Divergence

Ma and Qiu, arXiv:1404.6860

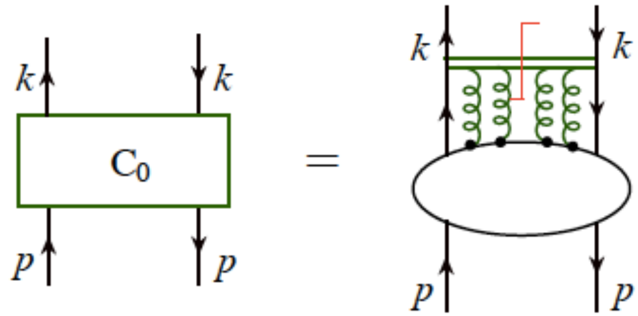
- **Generalized ladder decomposition in a physical gauge**

$n \cdot A = A^+ = 0$

Mueller, PRD 1974



- $C_0, \; K_0$ **:2PI kernels**
  - ✧ **Only process dependence:**



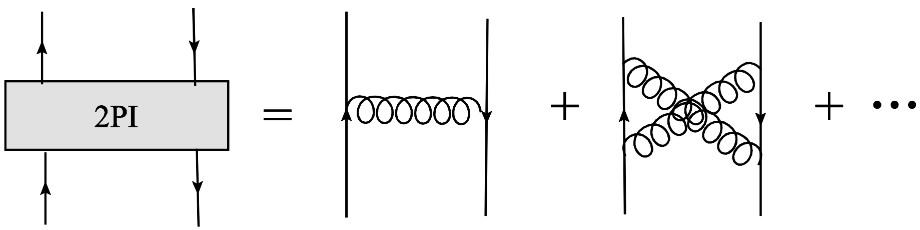
  - ✧ **2PI are finite in a physical gauge for fixed *k* and *p*:**

Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

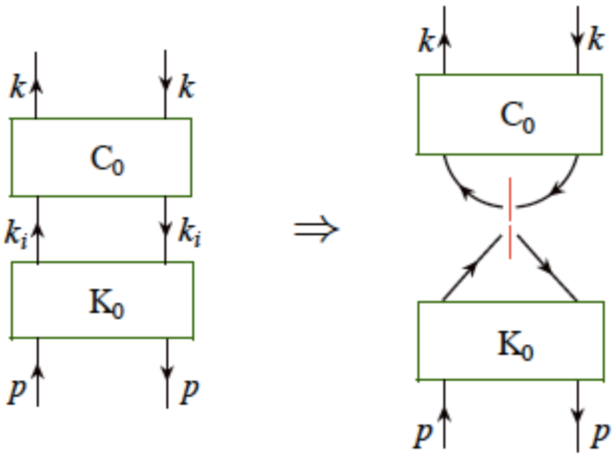
Arguments valid on if UV divergences are multiplicatively renormalizable

Factorization of CO Divergence

❑ 2PI kernels – Diagrams:



❑ Ordering in virtuality: $P^2 \ll k^2 \lesssim \tilde{\mu}^2$ – Leading power in $\frac{1}{\tilde{\mu}}$



$\leftarrow \frac{1}{2}\gamma\cdot p$

$\leftarrow \frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x_i - \frac{k_i\cdot n}{p\cdot n}\right)$

+ power suppressed

Cut-vertex for normal quark distribution
Logarithmic UV and CO divergence

❑ Renormalized kernel - parton PDF:

$$K \equiv \int d^4k_i\,\delta\left(x_i - \frac{k^+}{p^+}\right)\mathrm{Tr}\left[\frac{\gamma\cdot n}{2p\cdot n}\,K_0\,\frac{\gamma\cdot p}{2}\right] + \mathrm{UVCT}_{\mathrm{Logarithmic}}$$

Factorization of CO Divergence

- **Projection operator for CO divergence:**

$$\widehat{\mathcal{P}}\, K$$ Pick up the logarithmic CO divergence of *K*

- **Factorization of CO divergence:**

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K^i + \mathrm{UVCTs}$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\widehat{\mathcal{P}})K\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\widehat{\mathcal{P}})K\right]^i\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$\Rightarrow \quad \tilde{f}_{q/P} = \left[C_0 \frac{1}{1-(1-\widehat{\mathcal{P}})K}\right]_{\mathrm{ren}} \left[\frac{1}{1-\widehat{\mathcal{P}}K}\right]$$

CO divergence free

Normal Quark distribution

All CO divergence of quasi-quark distribution

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All CO divergence of quasi-quark distribution

$$\Rightarrow \quad \tilde{f}_{i/h}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_j \int_0^1 \frac{dx}{x}\, \mathcal{C}_{ij}\left(\frac{\tilde{x}}{x}, \tilde{\mu}^2, P_z\right) f_{j/h}(x, \mu^2)$$

UV finite?

Renormalization

- Current-current correlators – take care by construction:

  Construct operators by using renormalizable, or conserved currents

- Renormalization of quasi-parton operators:

  Bad: UV power divergence, mixing, …
  Good: UV divergences are multiplicative!
  "Ugly": Several proposals, impact to high orders, not converging yet, …

See talks by
Y. Zhao, Y.Q. Ma
J.H. Zhang, …

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- Basic requirements/considerations:

  Z. Li, Y.Q. Ma, J. Qiu
  See talk by Y.Q. Ma

  ✧ Fact: UV divergence is only sensitive to the operator, not to the state defining the matrix elements

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  e.g., $\overline{MS}$ scheme vs DIS scheme (no gluonic contribution to F_2)

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  ✧ Our choice:

  - Easy to be implemented non-perturbatively, as well as perturbatively
  - Not to be sensitive to the hadron state, especially, its CO behavior

  Need to be both IR and CO insensitive – it is a choice

  e.g., Pseudo-PDFs suppress certain power correction

  V. Braun's talk:

Renormalization

- **For multiplicative renormalizable quasi-parton operators:**

Focus on the continuum formalism for now

$$\mathcal{O}_q^{\nu}(\xi)_B = \overline{\psi}_q(\xi)\gamma^{\nu}\Phi^{(f)}(\xi,0)\psi_q(0)$$

$$\mathcal{O}_g^{\mu\nu\alpha\beta}(\xi)_B = F^{\mu\nu}(\xi)\Phi^{(a)}(\xi,0)F^{\alpha\beta}(0)$$

No sum over any indices! Effectively, a "scalar" operator for any given indice

- **Lattice cross sections - observables:**

$$\sigma_n(\omega,\xi^2,p^2) \equiv Z_n^{-1}(\xi^2)\langle P|T\{\mathcal{O}_n(\xi)_B\}|P\rangle$$

$$Z_n(\xi^2) = \frac{\langle\Omega|T\{\mathcal{O}_n(\xi)_B\}|\Omega\rangle}{\langle\Omega|T\{\mathcal{O}_n(\xi)_B\}|\Omega\rangle^{(0)}}$$

- Hard scale: $1/\xi^2$
- IR and CO safe!
- Calculable with a UV regulator

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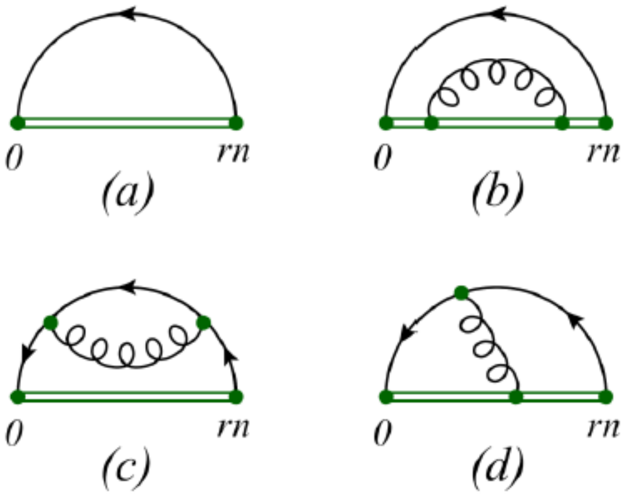

$$Z_q(\xi^2) = 1 + \frac{\alpha_s C_F}{\pi}\left[2 - \frac{3}{4}\gamma_E - \frac{1}{4}\ln\pi + \frac{1}{3}\pi^2 + \frac{3}{4\epsilon}(-\xi^2\mu^2)^{\epsilon} - \frac{|\xi|\mu}{1-2\epsilon}\right]$$

Completely IR and CO safe, independent of hadron state

See also Y.Q. Ma's talk

Renormalization

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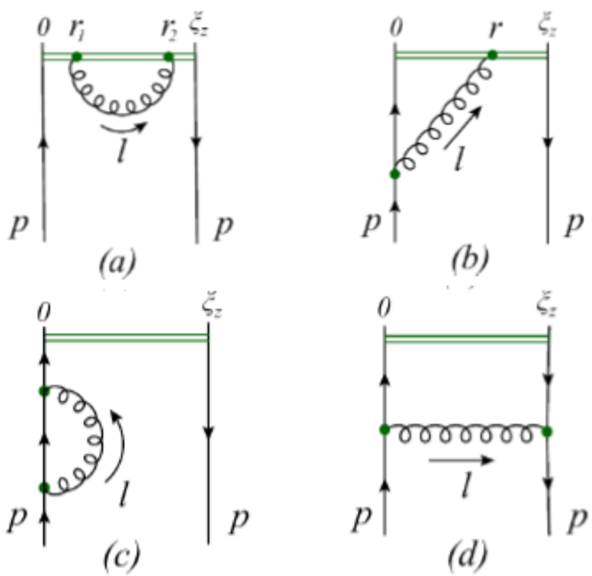

$$\sigma_{q/q}(\omega,\xi^2) = \int_{-1}^{1}\frac{dx}{x} f_{q/q}^{\overline{\rm MS}}(x,\mu_f^2)\, e^{ix\omega} H_q^{\nu}(x\omega,\xi^2,\mu_f^2)$$

$$f_{q/q}^{\overline{\rm MS}}(x,\mu_f^2) = \delta(1-x) - \frac{\alpha_s C_F}{\pi}\frac{1}{2\epsilon}\left[\frac{1+x^2}{1-x}\right]_+ \theta(x)(4\pi\mu^2/\mu_f^2)^{\epsilon}$$

$$H_q^{\nu}(x\omega,\xi^2,\mu_f^2) = \frac{i\xi^{\nu}}{-\xi^2}H_1(x\omega,\mu_f^2) + xp^{\nu}H_2(x\omega,\mu_f^2)$$

H_1 and H_2 are given in Y.Q. Ma's talk

Current-Current Correlators

Ma, Qiu, PRL (2018)

Sufian et al. JLab PRD99 (2019) 074507

- **Pion/Keon PDFs:**
  - using a vector-axial-vector correlation as an example

✧ Parity-Time-reversal invariance:

$$\frac{1}{2}\left[T^{\mu\nu}_{v5}(\xi,p) + T^{\mu\nu}_{5v}(\xi,p)\right] = \frac{\xi^4}{2}\langle h(p)|\left(\mathcal{J}^\mu_v(\xi/2)\,\mathcal{J}^\nu_5(-\xi/2) + \mathcal{J}^\mu_5(\xi/2)\,\mathcal{J}^\nu_v(-\xi/2)\right)|h(p)\rangle$$
$$\equiv \epsilon^{\mu\nu\alpha\beta}\,p_\alpha\xi_\beta\,\widetilde{T}_1(\omega,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)\,\widetilde{T}_2(\omega,\xi^2)$$

✧ Collinear factorization:

$$\widetilde{T}_i(\omega,\xi^2) = \sum_{f=q,\bar{q},g}\int_0^1 \frac{dx}{x}\, f(x,\mu^2)\, C_i^f(\omega,\xi^2;x,\mu^2) + \mathcal{O}\left[|\xi|/\mathrm{fm}\right]$$

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$$\equiv \epsilon^{\mu\nu\alpha\beta}\,p_\alpha\xi_\beta\,\widetilde{T}_1(\omega,\xi^2)+(p^\mu\xi^\nu-\xi^\mu p^\nu)\,\widetilde{T}_2(\omega,\xi^2)$$

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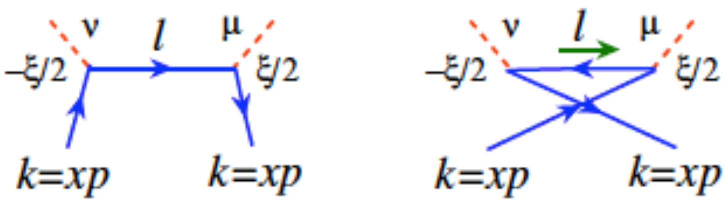
✧ **Lowest order coefficient functions:**

$$C^{q(0)}_1(\omega,\xi^2;x)=\frac{1}{\pi^2}\,x\left(e^{ix\omega}+e^{-ix\omega}\right)$$
$$C^{q(0)}_2(\omega,\xi^2)=0$$
$$T_1(\tilde{x},\xi^2)\equiv\int\frac{d\omega}{2\pi}e^{-i\tilde{x}\omega}\widetilde{T}_1(\omega,\xi^2)$$
$$=\frac{1}{\pi^2}\left(q(\tilde{x},\mu^2)-\bar{q}(\tilde{x},\mu^2)\right)\equiv\frac{1}{\pi^2}q_v(\tilde{x},\mu^2)$$



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✧ **Lattice QCD calculation results with 1-loop matching coefficient**

Current-Current Correlators

- First lattice QCD calculation of pion PDFs:

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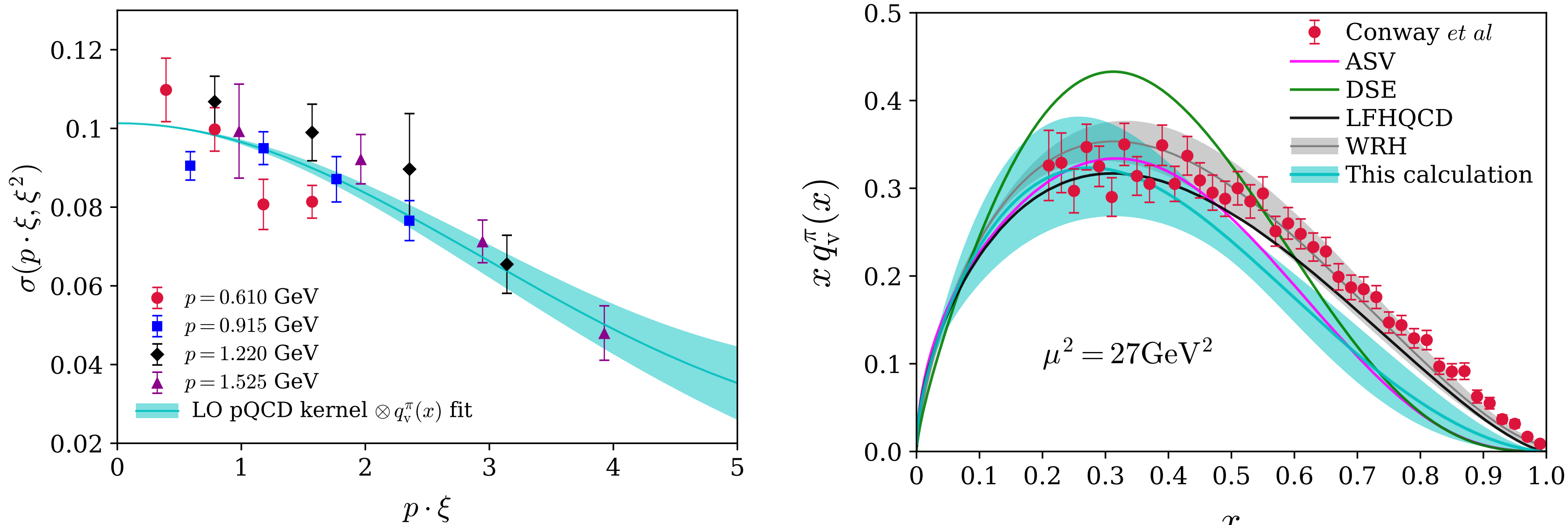
See Sufian's talk



- Global analysis of LQCD data – complementary to experimental data:

QCD factorization of "lattice cross section"

$$\sigma_n^h\left(\omega,\xi^2,p^2\right)=\sum_{a=q,\bar{q},g}\int_0^1\frac{dx}{x}f_a\left(x,\mu^2\right)K_n^a\left(x\omega,\xi^2,x^2p^2,\mu^2\right)+\mathcal{O}\left(\xi^2\Lambda_{\rm QCD}^2\right)$$

LQCD calculable | Global analyses | PQCD calculable

Complementary to experimental data!

Summary and outlook

- Good "lattice cross sections" = single hadron matrix elements
  calculable in Lattice QCD, renormalizable + factorizable in QCD

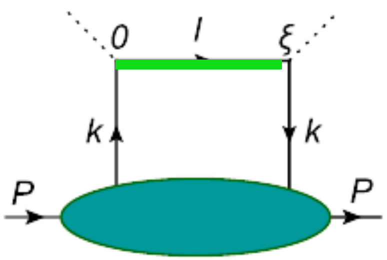
Going beyond the quasi-parton operators

- Extract PDFs by global analysis of data of "Lattice x-sections"

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

with $f_{\bar{a}}(x,\mu^2) = -f_a(-x,\mu^2)$

- Conservation of difficulties – an example:



Use heavy-light flavor changing current to suppress noise from the middle propagator:

$$\Rightarrow f_q(x,\mu^2) + f_Q(x,\mu^2) \approx f_q(x,\mu^2) \quad \text{if} \quad m_Q \sim \mu$$

No free-lunch! But, we are trying to find a better way to get our lunch!

- Lattice QCD can be used to study hadron structure, but, more works are needed!

Thank you!