Pion Valence Quark Distribution Using Lattice Cross Sections

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in collaboration with

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Calculations of Parton Distributions on the Lattice

Hadronic tensor (K. F. Liu, S. Dong [PRL 1994, PRD 200])

Position-space correlators (V. M. Braun & D. Müller [EPJ 2008])

Inversion Method (A. Chambers, et al [PRL 2017])

Quasi PDFs (X. Ji [PRL 2013])

Pseudo-PDFs (A. Radyushkin, [PLB 2017])

Extensive efforts and significant achievements in recent years

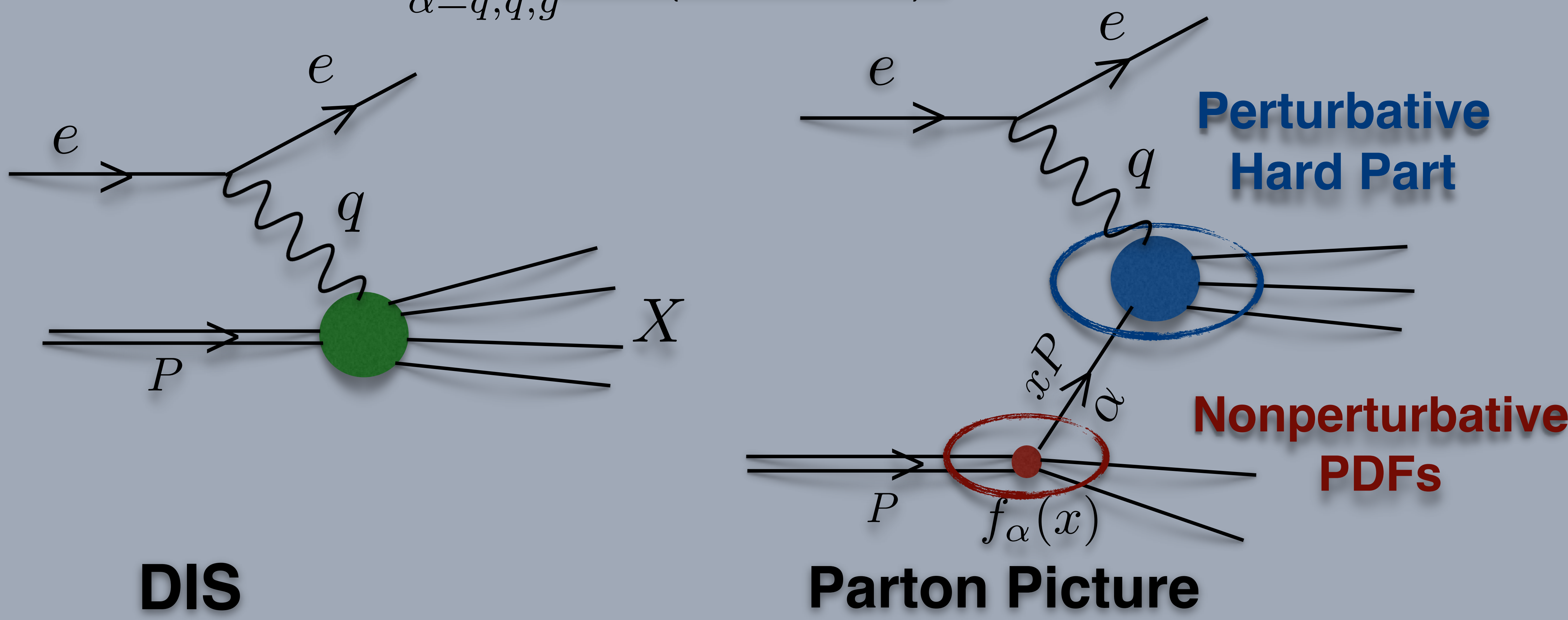
"Good" Lattice Cross Sections (LCSs)
(Y. Q. Ma, J.-W. Qiu, 2014, PRL 2018)

Hadron matrix elements:

- Calculable using lattice QCD with Euclidean time
- Well defined continuum limit ($a \to 0$), UV finite
  i.e. no power law divergence from Wilson line in non-local operator
- Share the same perturbative collinear divergences with PDFs
- Factorizable to PDFs with IR-safe hard coefficients
  with controllable power corrections
- As long as operators have no temporal extent,
  a matrix element calculated in Euclidean space
  is equal its counterpart in Minkowski space.

Parton Distribution Functions (PDFs) & Factorization

$$\sigma^{DIS}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q,\bar{q},g} C_\alpha\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_\alpha(x, \mu^2) + \text{Power Corrections}$$



Factorization scale μ describes which fluctuations should be included in the PDFs and which can be included in the hard scattering part

LCSs: Lattice Calculable + Renormalizable + Factorizable

- **Hadron matrix elements:** $\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{\mathcal{O}_n(\xi)\} | P \rangle$

- **Factorization:**

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{QCD}^2)$$

Nonperturbative PDFs of flavor $a \in \{q, \bar{q}, g\}$

Perturbative hard coefficients

$$f_{\bar{a}}(x, \mu^2) = -f_a(-x, \mu^2)$$

$$\omega = p \cdot \xi$$

Lorentz invariant Ioffe time

Hadron momentum

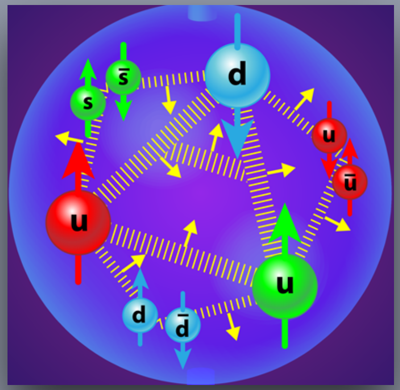
Spatial separation between two currents

Connection to DIS

- P and ξ → $P \to \sqrt{s}$ Collision energy

  $\xi \to \frac{1}{\sqrt{Q^2}}$ Hard **probe (resolving power)**

  Collision kinematics



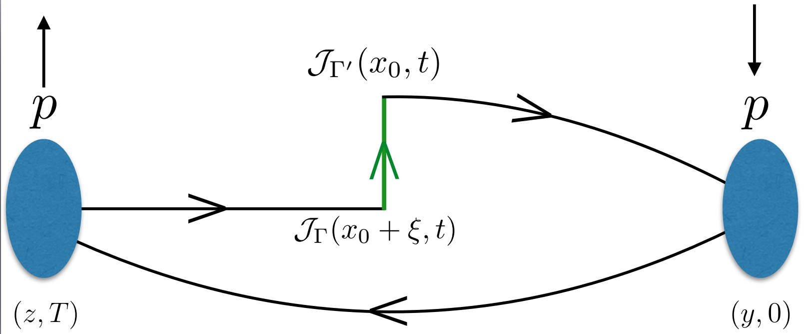
$\xi^2 \Lambda^2_{\rm QCD} \ll 1$

- $\mathcal{O}_n$ → Dynamical Features of LCSs mimics different experiments

- LCSs : Factorization holds for any finite ω and $P^2\xi^2$ if ξ is short distance

Good Lattice Cross Sections (LCSs)

- **Current-current correlators**



$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2} Z_{j_1}^{-1} Z_{j_2}^{-1} j_1(\xi) j_2(0)$$

d_j Dimension of the current

Z_j Renormalization constant of the current

- **Different choices of currents**

$$j_S(\xi) = \xi^2 Z_S^{-1}[\bar{\psi}_q \psi_q](\xi) \qquad j_V(\xi) = \xi Z_V^{-1}[\bar{\psi}_q \gamma \cdot \xi \psi_q](\xi)$$

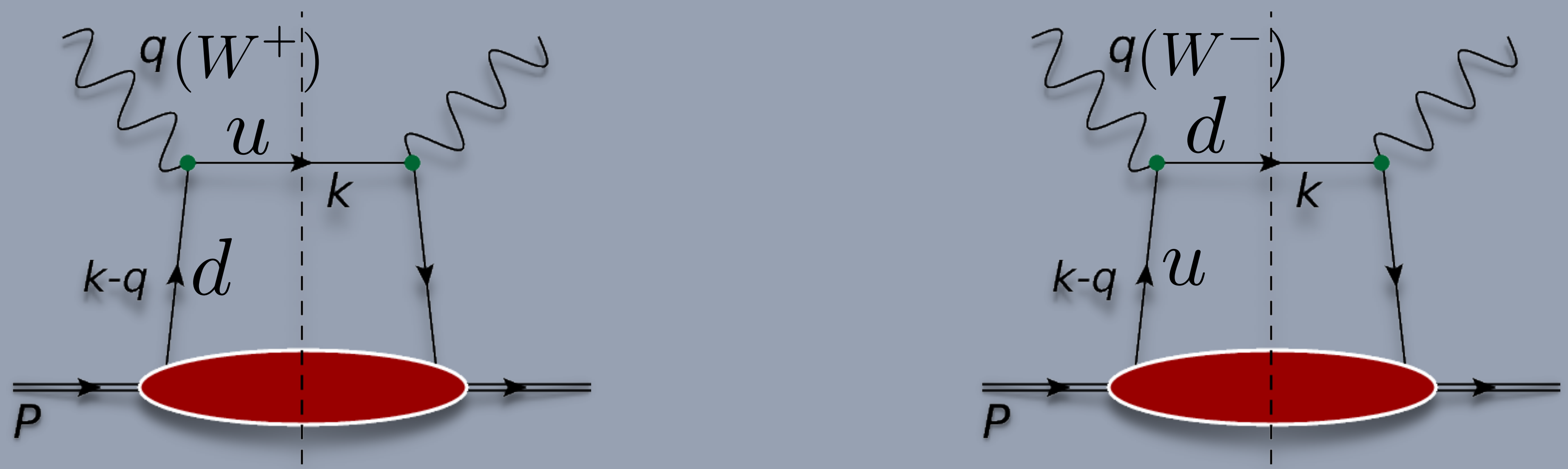
$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\bar{\psi}_q \gamma \cdot \xi \psi_{q'}](\xi) \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4} F_{\mu\nu}^c F_{\mu\nu}^c](\xi)$$

flavor changing current

gluon distribution

Choice of Currents

- Many current combinations are possible
- Which one to pick - neutrino-nucleon scattering can guide



$$W^{\mu\nu} = (-g^{\mu\nu} + q^\mu q^\nu/q^2)F_1(x,Q^2) + \frac{(P^\mu - q^\mu P\cdot q/q^2)(P^\nu - q^\nu P\cdot q/q^2)}{P\cdot q}F_2(x,Q^2) - i\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha P_\beta}{2P\cdot q}F_3(x,Q^2)$$

$$F_3^{W^+} + F_3^{W^-} \sim u - \bar{u} + d - \bar{d}$$

Antisymmetric V-A combination is a good choice to extract valence PDFs

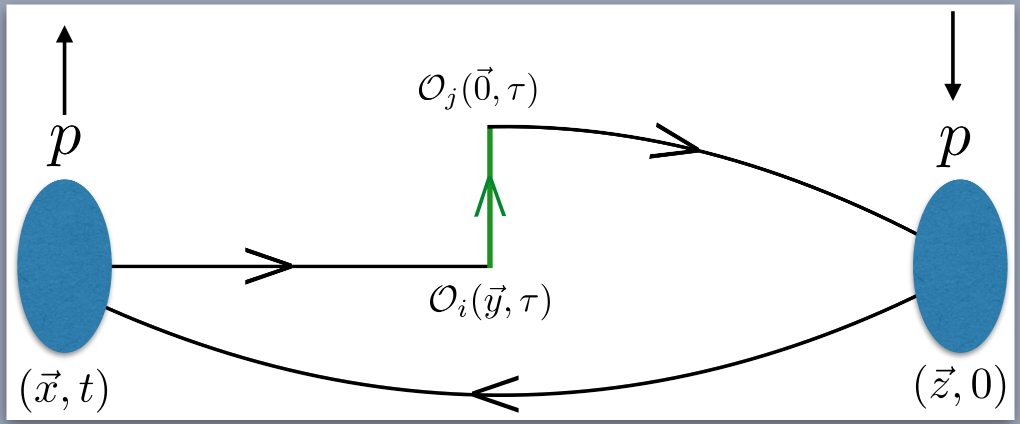
- Two point correlator

$$C_{2\mathrm{pt}}^{\vec{p}}(t) = a^6 \sum_{\vec{x},\vec{z}} e^{-i\vec{p}\cdot(\vec{x}-\vec{z})} \langle 0|\, \Pi(\vec{x},t)\, \Pi^\dagger(\vec{z},0)\, |0\rangle$$

- Four point correlator

$$C_{4\mathrm{pt}}^{ij,\vec{p}}(\vec{y},\tau,t) = a^6 \sum_{\vec{x},\vec{z}} e^{-i\vec{p}\cdot(\vec{x}-\vec{z})} \langle 0|\, \Pi(\vec{x},t)\, \mathcal{O}_i(\vec{y},\tau)\, \mathcal{O}_j(\vec{0},\tau)\, \Pi^\dagger(\vec{z},0)\, |0\rangle$$

- $\Pi(x)$ is a pion interpolator.



possible ξ on/off axis

- $$\langle \pi(p)|\, \mathcal{O}_i(y)\, \mathcal{O}_j(0)\, |\pi(p)\rangle = R_{ij}^{\vec{p}}(\vec{y}) = 2E_{\vec{p}}V \left.\frac{C_{4\mathrm{pt}}^{ij,\vec{p}}(\vec{y},\tau,t)}{C_{2\mathrm{pt}}^{\vec{p}}(t)}\right|_{0\ll\tau\ll t}$$

Why Pion Valence Distribution

- Pion : lightest bound state and associated with dynamical chiral symmetry breaking
- Pion valence distribution at large-x is an unresolved problem
- Large-x region: small configuration constrained by confinement dynamics
- From pQCD and different models : $(1-x)^2$ or $(1-x)^1$?
- C12-15-006 experiment at JLab to explore large-x behavior
- Large -x behavior of pion PDF can serve as a discriminator between different models

Lattice Calculation

- **Present calculation for pion valence quark distribution**

2+1 flavor clover Wilson fermion

$$32^3 \times 96,\ m_\pi \approx 413 \quad \text{MeV}$$

$$a \approx 0.127 \quad \text{fm}$$

- **Analysis being done on**

$$24^3 \times 64,\ m_\pi \approx 413 \ \text{MeV}$$

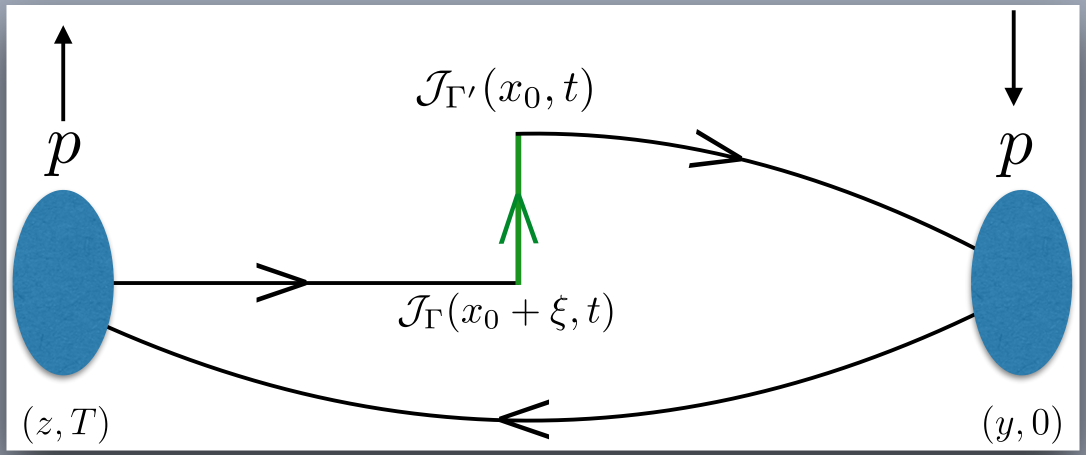
$$a \approx 0.127 \ \text{fm}$$

Lattice spacing, finite volume and pion mass dependence

$$32^3 \times 64, \quad m_\pi \approx 280 \ \text{MeV}$$

$$a \approx 0.09 \quad \text{fm}$$

Lattice Calculation



$$
\begin{aligned}
&C_{4\mathrm{pt}}\,(\xi, p, T, t) \\
&\quad = \langle \Pi_p(\vec{z}, T)\mathcal{J}_{\Gamma}^{\dagger}(x_0 + \xi, t)\mathcal{J}_{\Gamma'}(x_0, t)\overline{\Pi}_p\,(\vec{y}, 0)\rangle \\
&\quad = \sum_{\vec{z},\vec{y}} e^{-i(\vec{z}-\vec{y})\cdot\vec{p}}\langle \bar{\tilde{d}}\gamma^5\tilde{u}\,(\vec{z}, T)\ \ \bar{Q}\Gamma u(x_0 + \xi, t) \\
&\qquad \times \bar{u}\Gamma' Q(x_0, t)\ \bar{\tilde{u}}\gamma^5\tilde{d}\,(\vec{y}, 0)\rangle
\end{aligned}
$$

- Equal time current insertion : sum over all energy modes

↓

- Use heavy-light flavor changing current to suppress noise from spectator propagator in a systematic way

W. Detmold, D. Lin [PRD 2006]
DIS and the OPE in lattice QCD

- Small improvement is seen by replacing light-quark propagator with strange quark propagator, especially at large current-current separation

- Momentum smearing essential for higher momentum

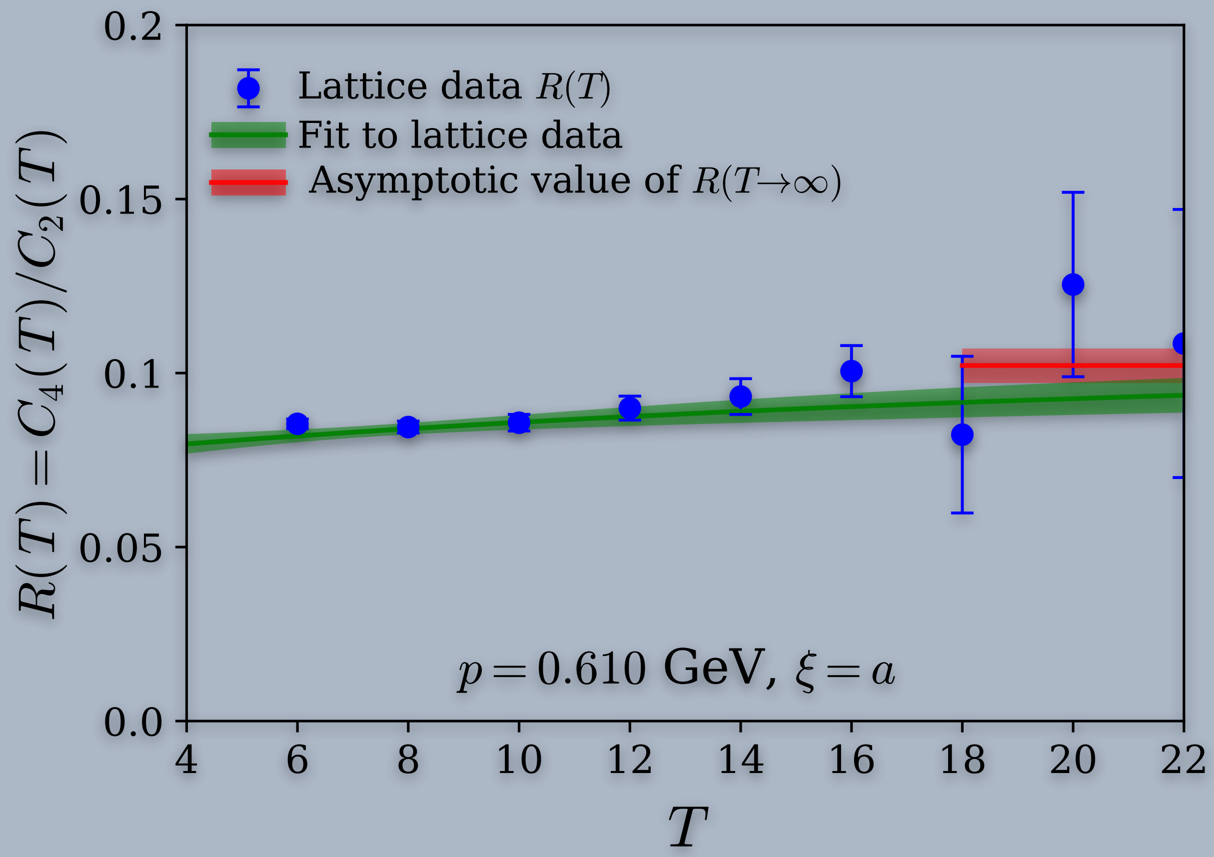
G. S. Bali, et al [PRD 2016]

- Randomly chosen source point (x_0, t)

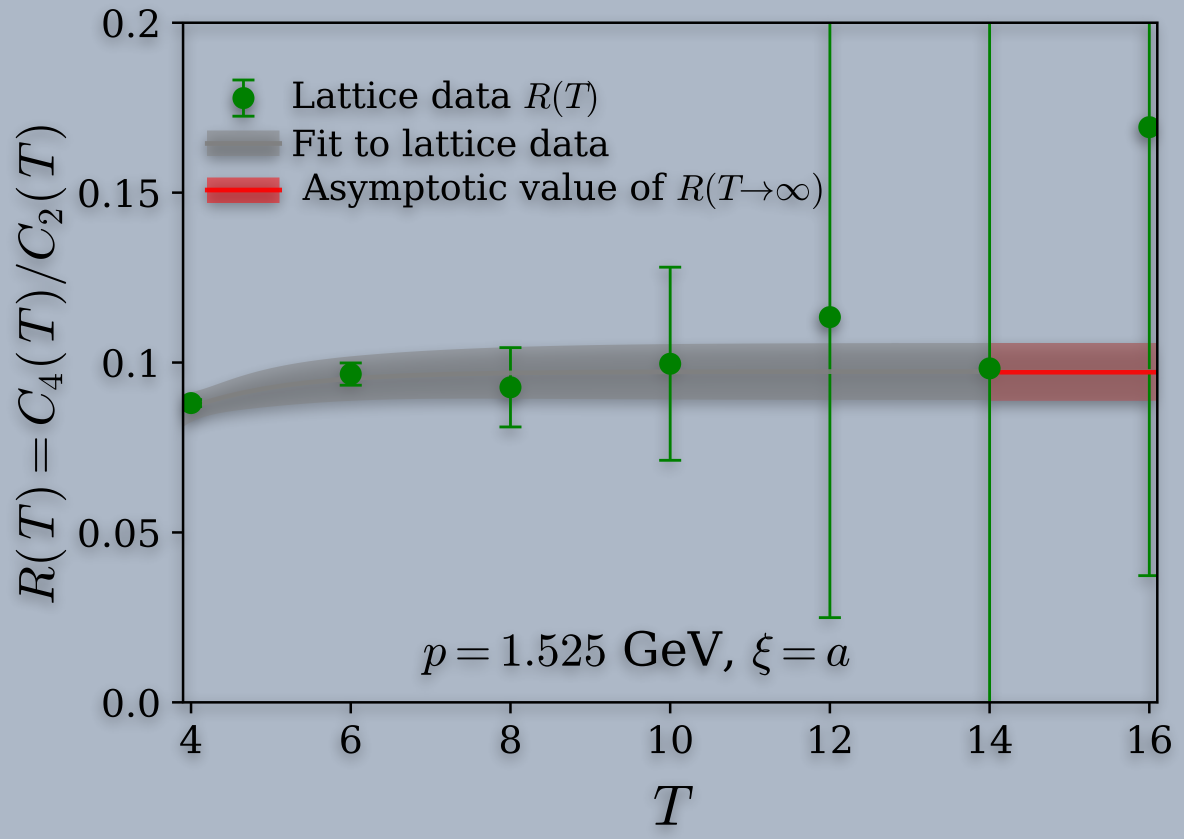
$\vec{p} = [0, 0, p_z]$	ζ	No. of source points (x_0, t)	No. of source-sink separations
$p = 0.610$ GeV	1.75	2	9
$p = 0.915$ GeV	2.50	5	9
$p = 1.220$ GeV	3.75	6	9
$p = 1.525$ GeV	4.50	7	7

- Current insertion at the middle point of source-sink separations

Good Lattice Cross Sections : Matrix Elements



$T_{max} = 2.79\,\mathrm{fm}$

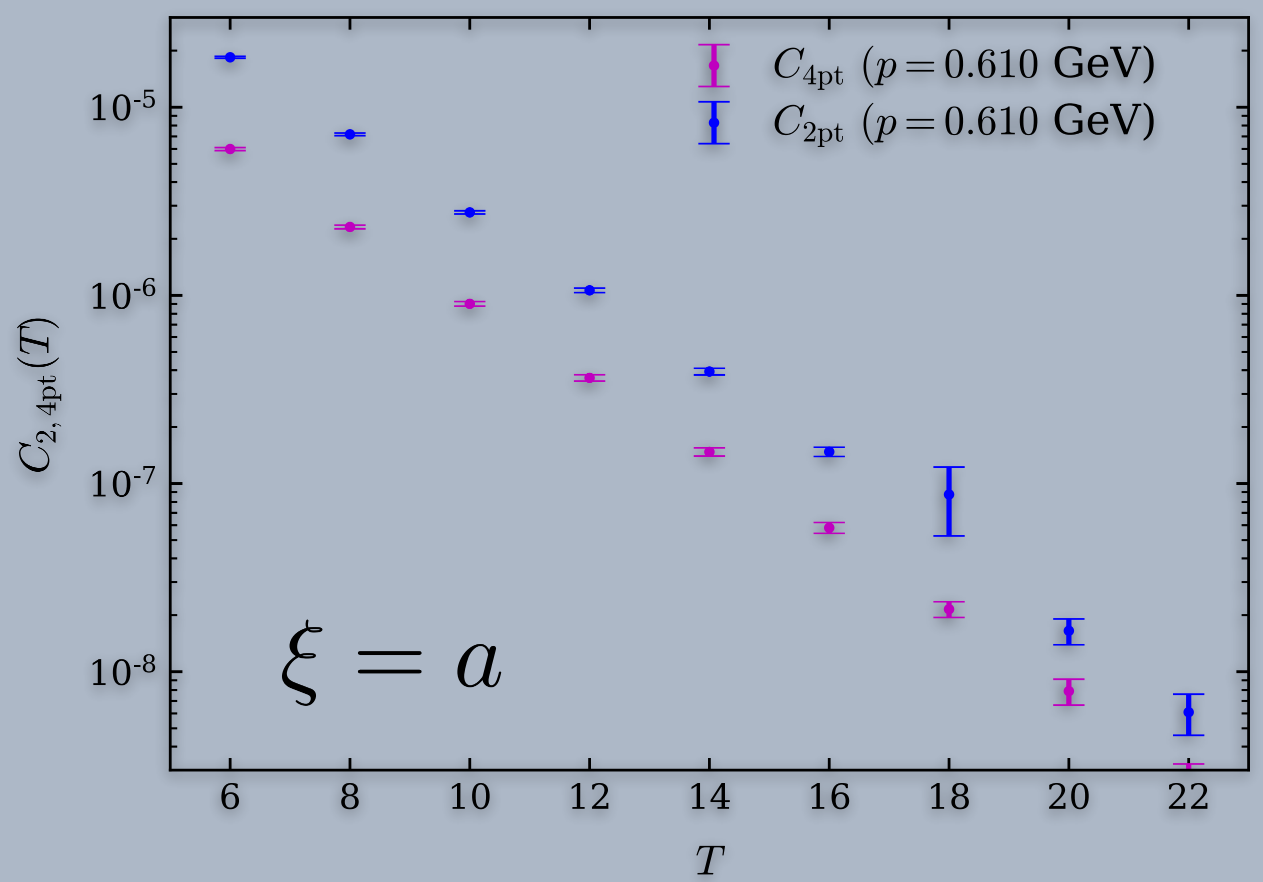


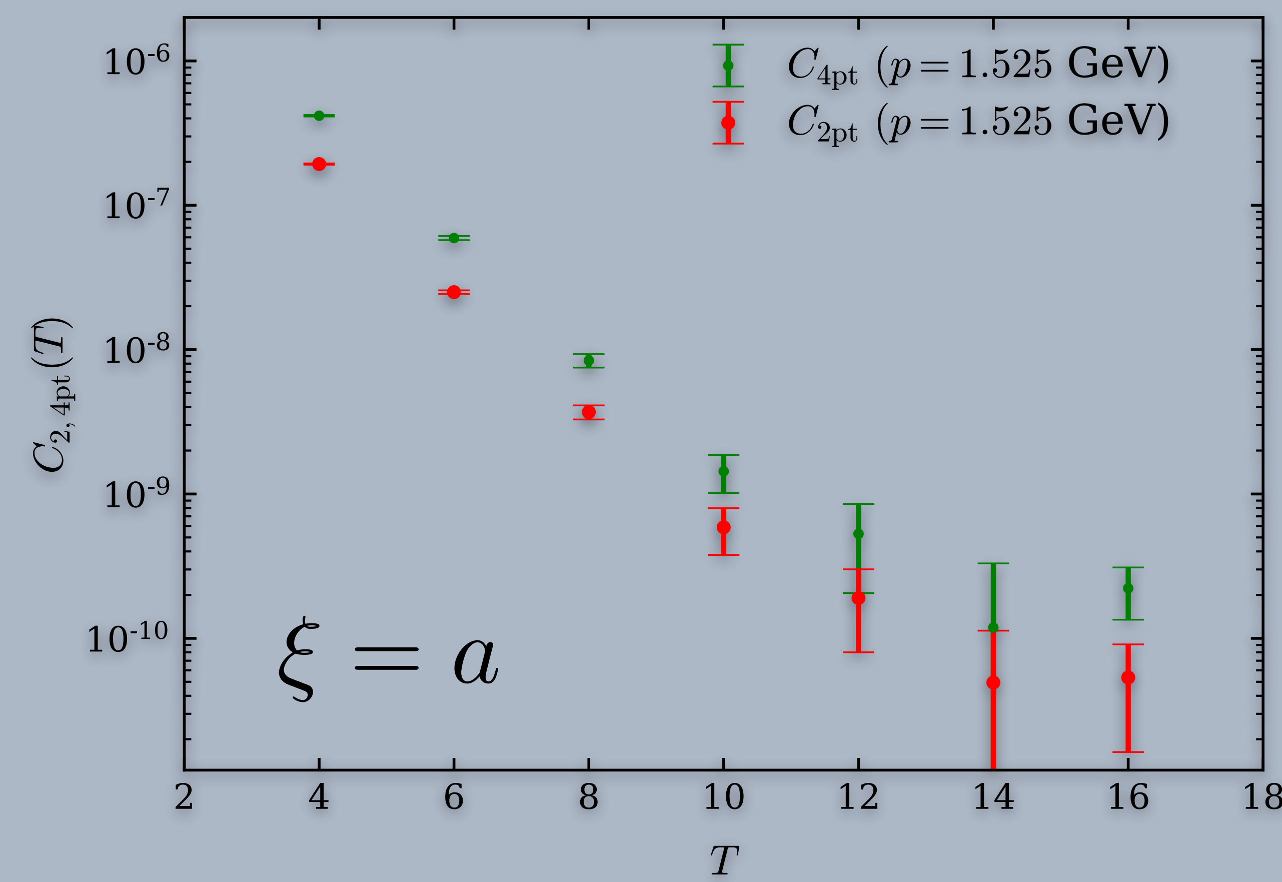
$T_{max} = 2.03\,\mathrm{fm}$

$$R(T) = \frac{C_{4\mathrm{pt}}(T)}{C_{2\mathrm{pt}}(T)} = A + Be^{-\Delta_{\mathrm{eff}}T}$$

p [GeV]	ξ	A	B	Δ_{eff}	χ^2/d.o.f.
0.610 GeV	$1a$	0.102(5)	-0.028(11)	0.054(20)	1.21
1.525 GeV	$1a$	0.097(8)	-0.267(513)	0.809(503)	0.15

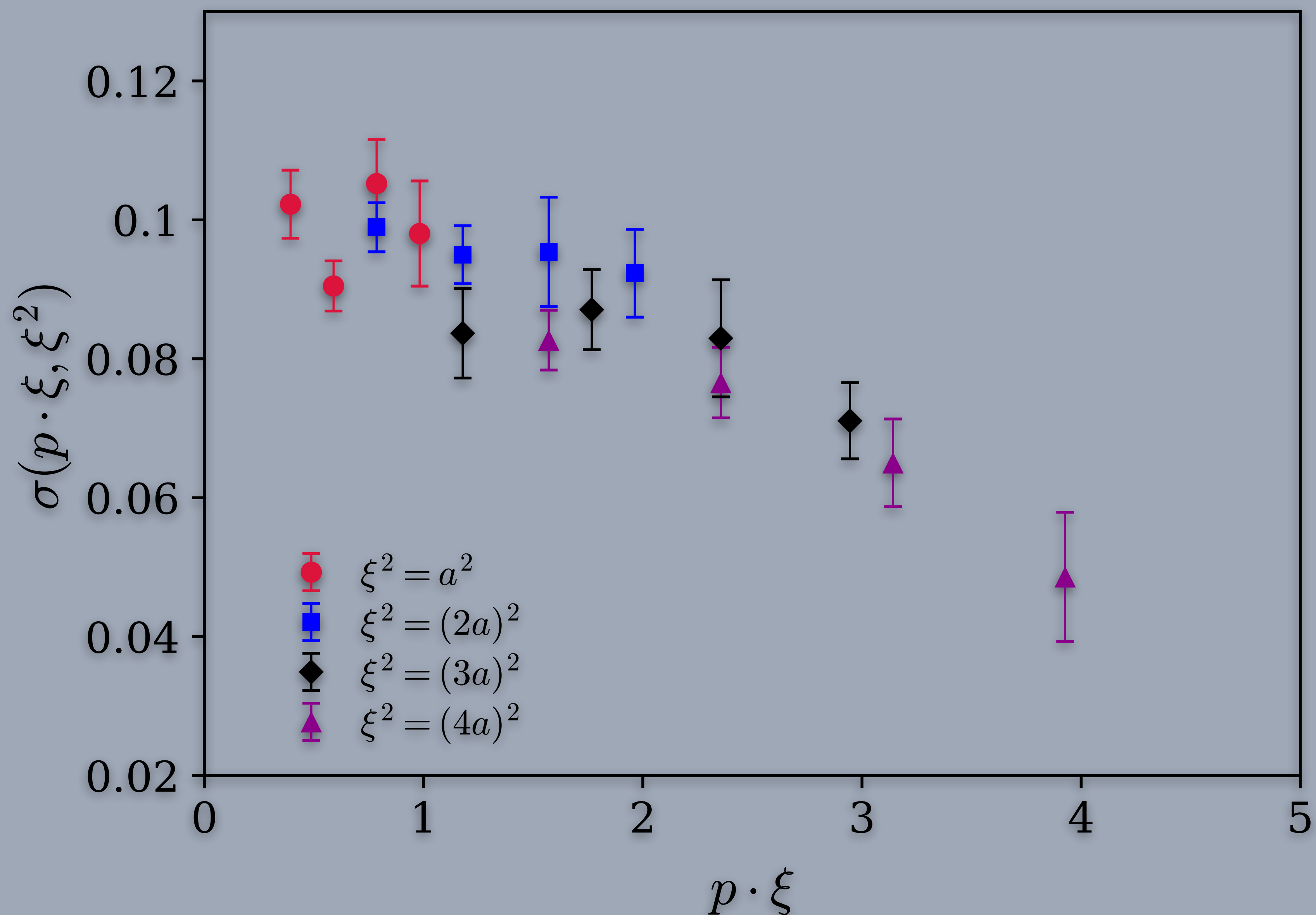
- A simultaneous fit to 2pt and 4pt fit gives to a few % smaller uncertainty in B_0/A_0





$$C_{2\mathrm{pt}}(T) = A_0 e^{-E_0 T}$$
$$C_{4\mathrm{pt}}(T) = e^{-E_0 T}(B_0 + B_1 e^{-\Delta_{\mathrm{eff}} T})$$

- Systematic differences between these two fits are being assessed



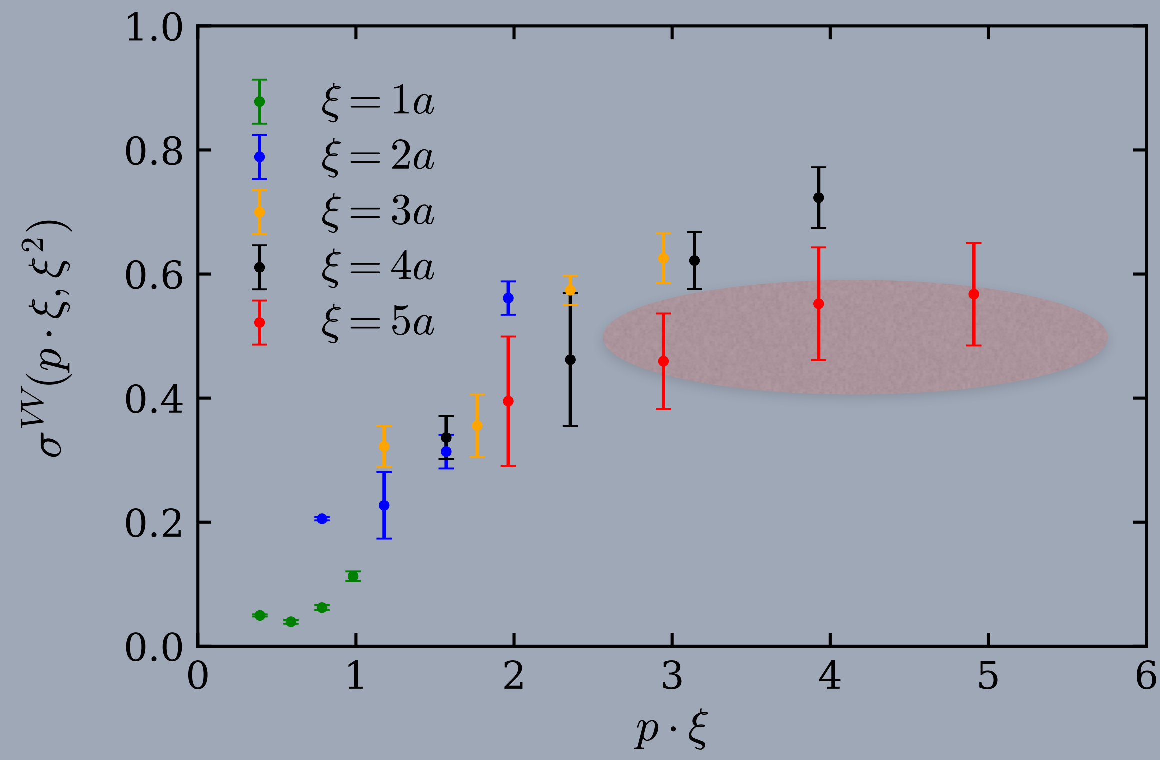
$|\xi| \leq 4a$

$p \in \{0.610 - 1.525\}$ GeV

Antisymmetric combination of V-A matrix elements

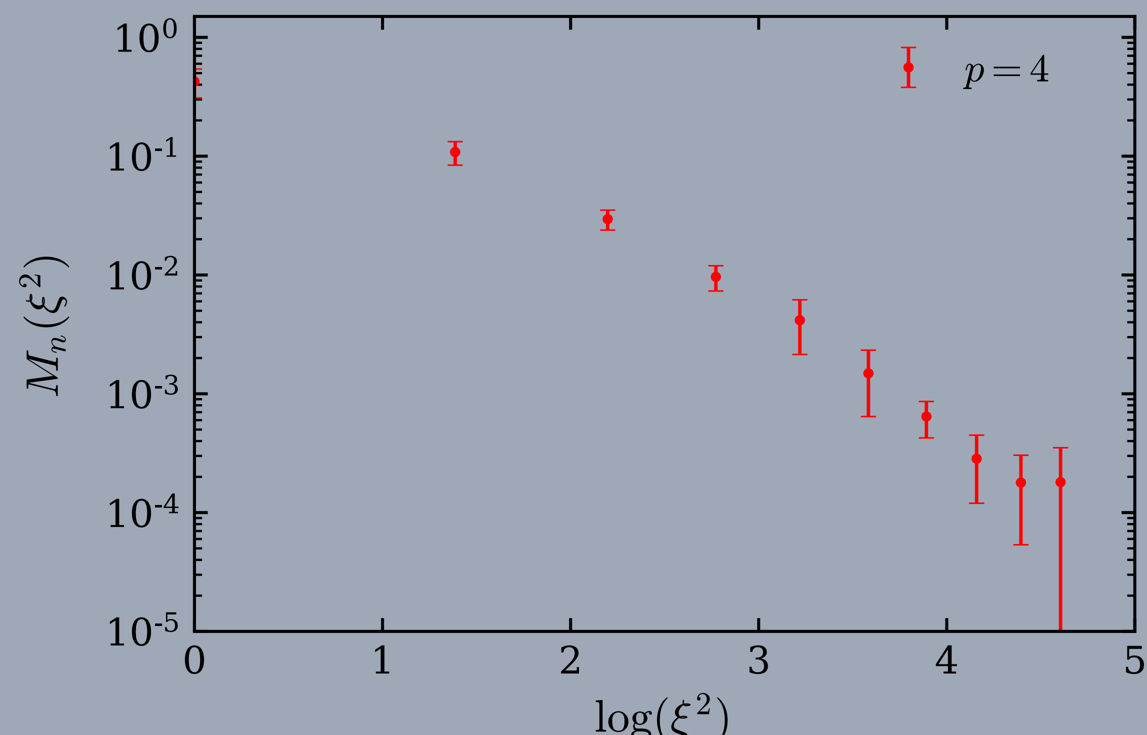
$$\gamma_x - \gamma_y\gamma_5 \quad \text{and} \quad \gamma_y - \gamma_x\gamma_5$$

- $V - V$ combination



Higher twist effect ?

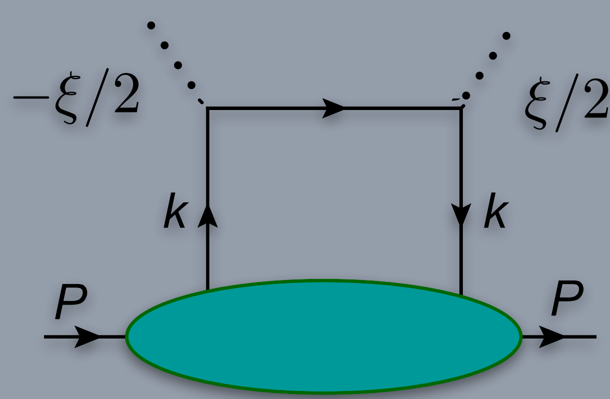
(*Not all multiplicative factors included yet*)

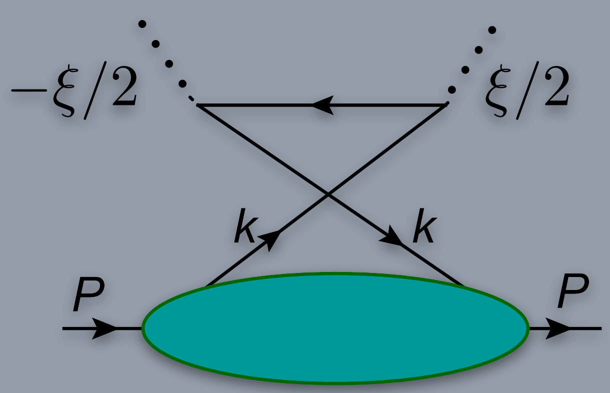


- About 6 different current-current correlations are being analyzed

- A general tree-level calculation first, then we consider V-A case
- Matrix element in pion

$$\sigma_{ij}^{\mu\nu}(\xi,p) = \langle\pi(p)|\,\mathcal{O}_{ij}^{\mu\nu}(\xi)\,|\pi(p)\rangle$$
$$= \xi^4 \langle\pi(p)|\,\mathcal{J}_i^{\mu}(\xi/2)\,\mathcal{J}_j^{\nu}(-\xi/2)\,|\pi(p)\rangle$$





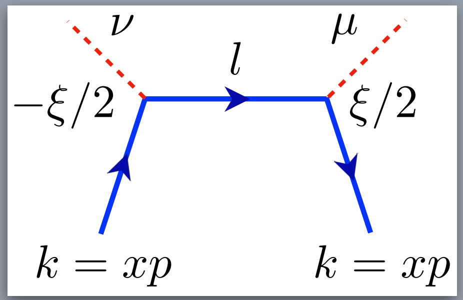
- $h \to q$

$$\sigma_{ij}^{\mu\nu} = \xi^4\Big[\langle q(k,s)|\,\overline{\psi}(\xi/2)\Gamma^{\mu}\psi(\xi/2)\overline{\psi}(-\xi/2)\Gamma^{\nu}\psi(-\xi/2)\,|q(k,s)\rangle$$
$$+\langle q(k,s)|\,\overline{\psi}(-\xi/2)\Gamma^{\nu}\psi(-\xi/2)\overline{\psi}(\xi/2)\Gamma^{\mu}\psi(\xi/2)\,|q(k,s)\rangle\Big]$$

Calculation of Perturbative Matching Kernel

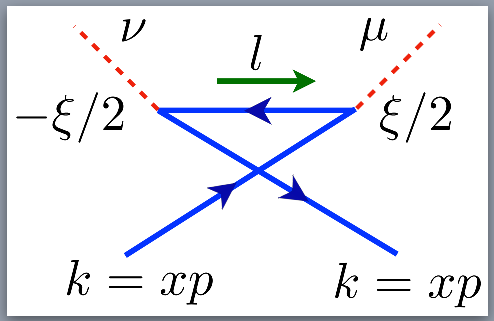
- Write

$$\langle 0|\,\psi(\xi/2)\overline{\psi}(-\xi/2)\,|0\rangle \to \int \frac{d^4 l}{(2\pi)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}$$



- $$= \frac{\xi^4}{2}\sum_s \langle 0|\,\overline{u}^s(k)\,e^{ik\cdot\xi/2}\Gamma_i^\mu\psi(\xi/2)\;\overline{\psi}(-\xi/2)\,\Gamma_j^\nu e^{ik\cdot\xi/2}u^s(k)\,|0\rangle$$

- $$= \frac{\xi^4}{2}e^{ik\cdot\xi}\mathrm{Tr}\big[(\gamma\cdot k)\Gamma^\mu\frac{2i(\gamma\cdot\xi)}{(2\pi)^2\xi^4}\Gamma^\nu\big]$$



$$= -\frac{\xi^4}{2}e^{-ik\cdot\xi}\mathrm{Tr}\big[(\gamma\cdot k)\Gamma^\nu\frac{2i(\gamma\cdot\xi)}{(2\pi)^2\xi^4}\Gamma^\mu\big]$$

Calculation of Perturbative Matching Kernel

- Combine these tree-level diagrams and write $k_\mu = xp_\mu$

$$\sigma_{ij}^{\mu\nu(0)}\left(p\cdot\xi, p; x, \xi\right) = \frac{i}{4\pi^2} x p_\alpha \xi_\beta \left\{ e^{ixp\cdot\xi}\ \mathrm{Tr}\left[\gamma^\alpha \Gamma_i^\mu \gamma^\beta \Gamma_j^\nu\right] - e^{-ixp\cdot\xi}\ \mathrm{Tr}\left[\gamma^\alpha \Gamma_j^\nu \gamma^\beta \Gamma_i^\mu\right] \right\}$$

- For different current-current combinations obtain LO kernels

- Under parity and time reversal transformation

$$\left(\mathcal{PT}\right)\mathcal{J}_A^\mu\left(\xi\right)\left(\mathcal{PT}\right)^{-1} = -\mathcal{J}_A^\mu\left(-\xi\right)$$
$$\left(\mathcal{PT}\right)\mathcal{J}_V^\mu\left(\xi\right)\left(\mathcal{PT}\right)^{-1} = \mathcal{J}_V^\mu\left(-\xi\right)$$

- Consider antisymmetric V-A current combination

$$\frac{1}{2}\left[\sigma_{VA}^{\mu\nu}\left(\xi, p\right) + \sigma_{AV}^{\mu\nu}\left(\xi, p\right)\right]$$

$$= \frac{\xi^4}{2}\langle\pi(p)|\big(\mathcal{J}^\mu{}_{\mathrm{V}}(\xi/2)\mathcal{J}^\nu{}_{\mathrm{A}}(-\xi/2) + \mathcal{J}^\mu{}_{\mathrm{A}}(\xi/2)\mathcal{J}^\nu{}_{\mathrm{V}}(-\xi/2)\big)|\pi(p)\rangle$$

$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1\left(\omega, \xi^2\right) + \left(p^\mu\xi^\nu - \xi^\mu p^\nu\right) T_2\left(\omega, \xi^2\right)$$

$$\frac{1}{2}\left[\sigma_{VA}^{\mu\nu}(\xi,p)+\sigma_{AV}^{\mu\nu}(\xi,p)\right] \equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1\left(\omega,\xi^2\right)+\left(p^\mu\xi^\nu-\xi^\mu p^\nu\right)T_2\left(\omega,\xi^2\right)$$

- T_1, T_2 : Dimensionless functions of the Lorentz invariants
- To isolate structure functions T_1 and T_2

$$p = (p^0, 0, 0, p^3) \qquad \xi = (0, 0, 0, \xi^3)$$

- With $\mu = 1 \text{ and } \nu = 2$

$$T_1\left(\omega,\xi^2\right)=\frac{1}{p^0\xi^3}\frac{1}{2}\left[\sigma_{VA}^{12}(\xi,p)+\sigma_{AV}^{12}(\xi,p)\right]$$

- With $\mu = 0 \text{ and } \nu = 3$

$$T_2\left(\omega,\xi^2\right)=\frac{1}{p^0\xi^3}\frac{1}{2}\left[\sigma_{VA}^{03}(\xi,p)+\sigma_{AV}^{03}(\xi,p)\right]$$

Concentrating on the V-A combinations

- At leading order

$$\sigma_n^{q(0)}\left(\omega,\xi^2\right) = \sum_{a=q,\overline{q},g}\int_0^1 \frac{dx}{x} f_a^{q(0)}\left(x,\mu^2\right)\times K_n^{a(0)}\left(x\omega,\xi^2;\mu^2\right)+\mathcal{O}\left(\xi^2\Lambda_{\rm QCD}^2\right)$$

- At tree level

$$T_1^{q(0)}(x\omega,\xi^2) = \frac{x}{\pi^2}\left(e^{ix\omega}+e^{-ix\omega}\right)$$

- No contribution from

$$T_2^{q(0)}(x\omega,\xi^2) = 0$$

- Perform a Fourier transform in ω

$$\tilde{T}_1(\tilde{x},\xi^2) \equiv \int \frac{d\omega}{2\pi} e^{-i\tilde{x}\omega}\, T_1\left(\omega,\xi^2\right)$$

$$= \int \frac{d\omega}{2\pi} e^{-i\tilde{x}\omega} \int_0^1 \frac{dx}{x} q\left(x\right) \frac{x}{\pi^2}\left(e^{ix\omega} + e^{-ix\omega}\right)$$

$$= \frac{1}{\pi^2}\left\{q\left(\tilde{x}\right) + q\left(-\tilde{x}\right)\right\}$$

$$= \frac{1}{\pi^2}\left\{q\left(\tilde{x}\right) - \bar{q}\left(\tilde{x}\right)\right\} = \frac{1}{\pi^2}\, q_{\mathrm{v}}(\tilde{x})$$

- $$T_1\left(\omega,\xi^2\right) \equiv \sigma(p\cdot\xi,\xi^2) = \int_0^1 dx \frac{1}{\pi^2} \cos(x\omega) q_{\mathrm{v}}^{\pi}(x)$$

Lattice data

Matching kernel

PDF

- Fit lattice data similar global fits of PDFs using

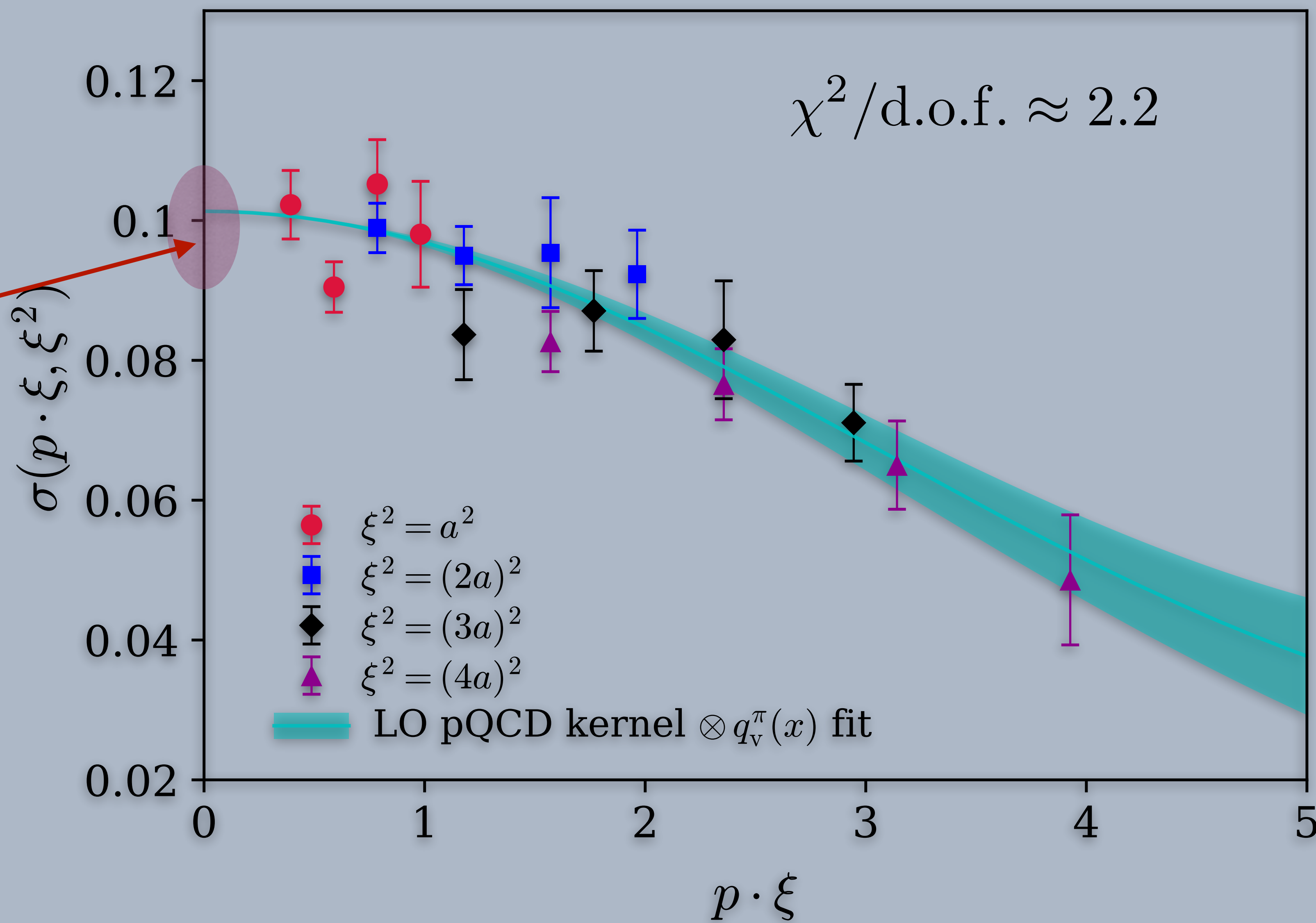
$$q_v^\pi(x) = N x^\alpha (1-x)^\beta (1 + \rho\sqrt{x} + \gamma x)$$

- $$\sigma(p\cdot\xi, \xi^2) = \int_0^1 dx \frac{1}{\pi^2} \cos(x\omega) q_v^\pi(x)$$

Set constraint

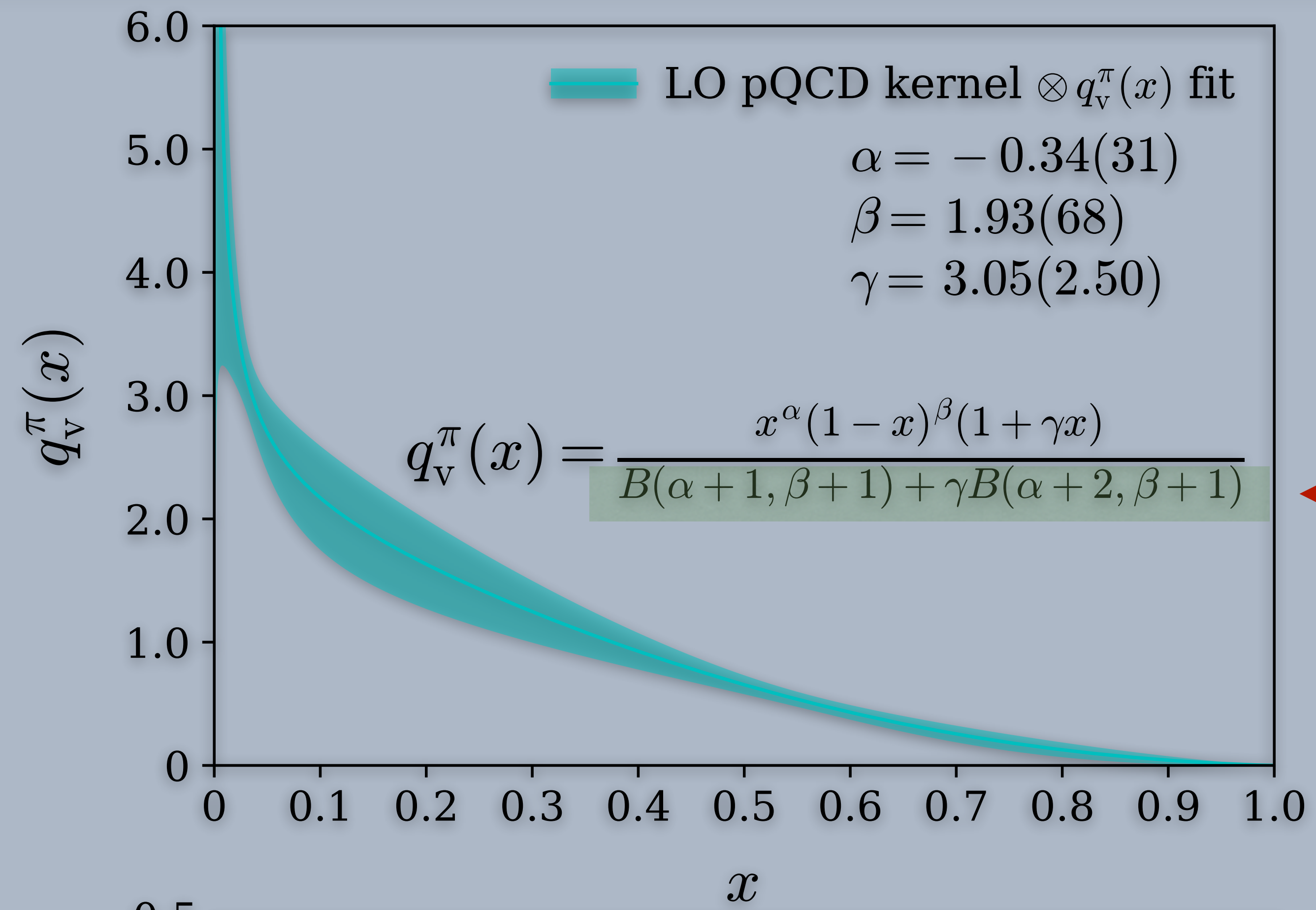
$\alpha < 0, \quad 0 < \beta < 4$

Fit constrained by **theoretical** value at $\omega = 0$

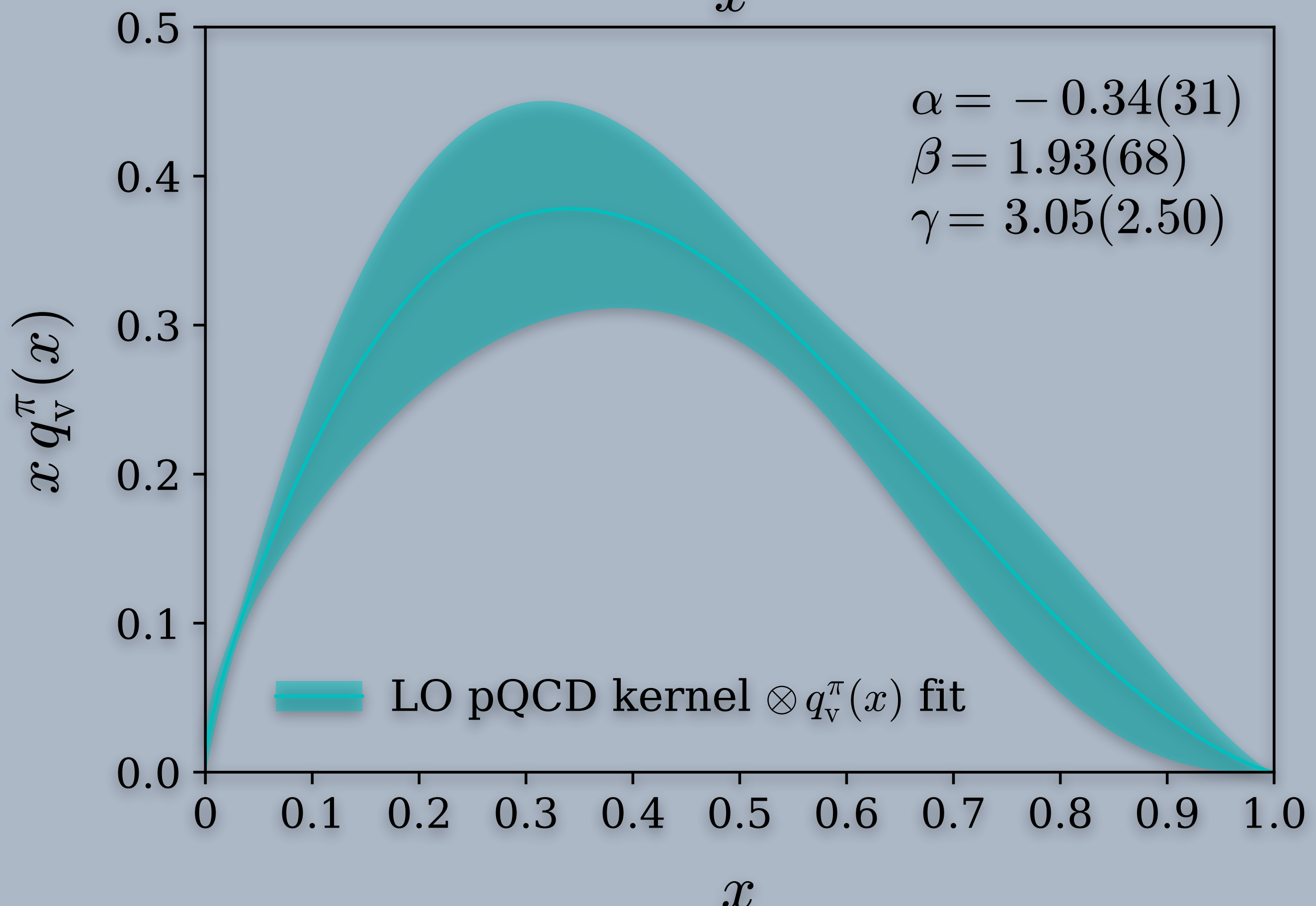


Significant discretization error

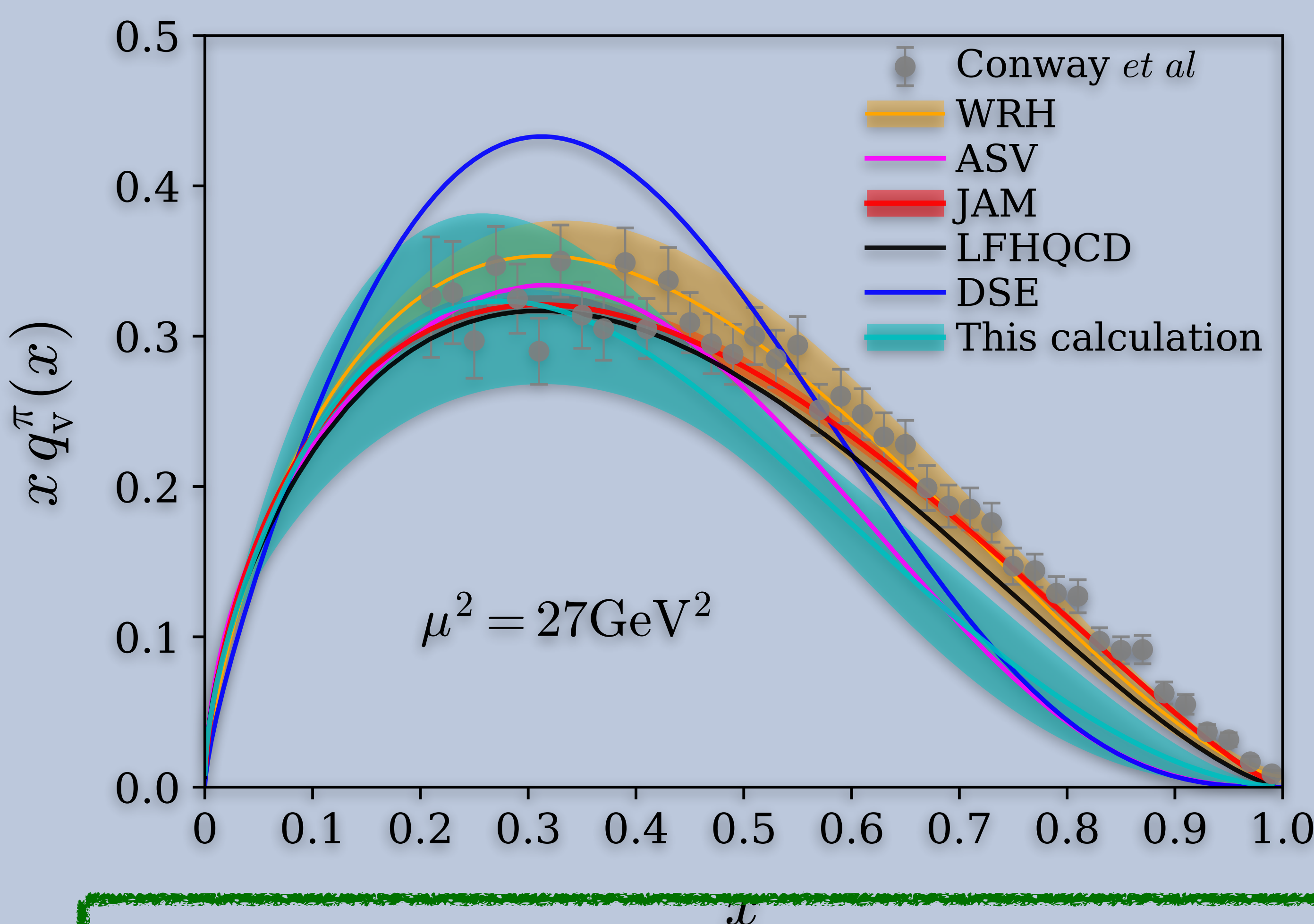
PDF extraction from lattice data



$$\int_0^1 dx\, q_v^\pi(x) = 1$$



Comparison with Global Fits and Phenomenological Calculations



$\mu_0 \approx 1\,\text{GeV}$

Good Lattice Cross Sections

***RSS*, Karpie, Egerer, Orginos, Qiu, Richards**
Phys. Rev. D 99, 074507 (2019)

From pQCD and different models : $(1-x)^2$ or $(1-x)^1$?

Conway et al. , PRD 1989 (LO extraction of Drell-Yan data)

Wijesooriya, Reimer, Holt, PRC 2005 (NLO fit)

Aicher, Schafer, Vogelsang, PRL 2010 (NLO fit + **soft gluon re-summation**)

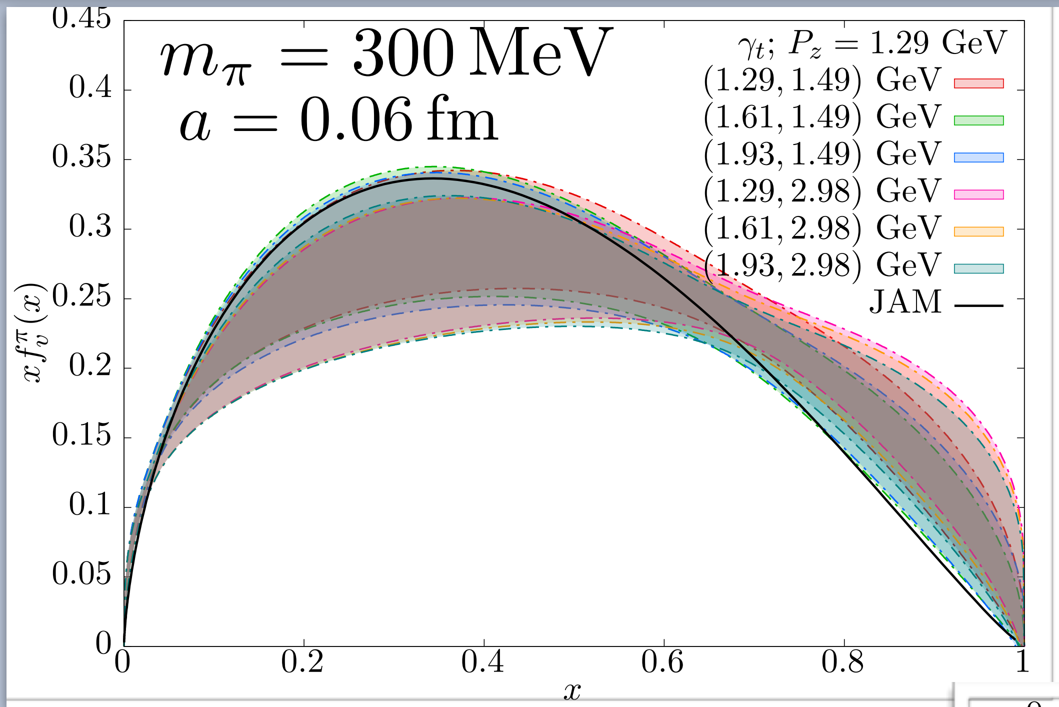
$\sigma \sim C \otimes q(x)$

Bary, Sato, Melnitchouk, PRL 2018 (NLO fit)

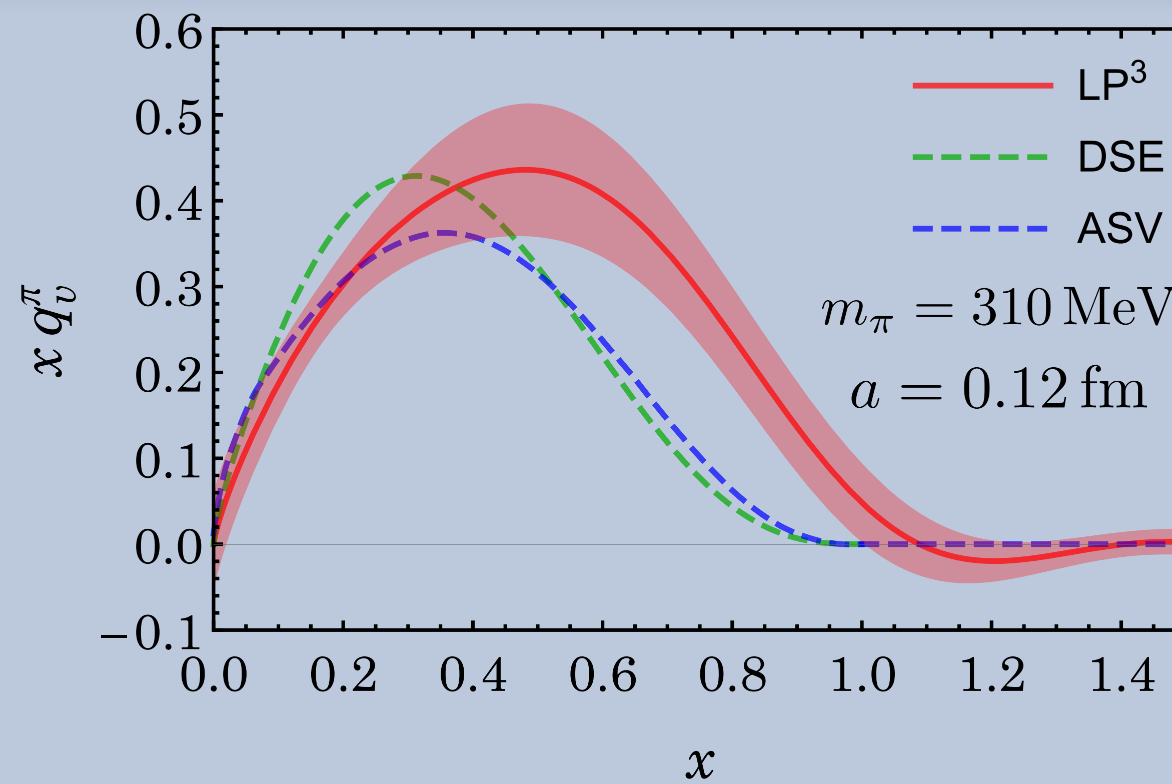
de Teramond, Liu, ***RSS***, Dosch, Brodsky, Deur, PRL 2018

Chen, Chang, Roberts, Wan, Zong, PRD 2016

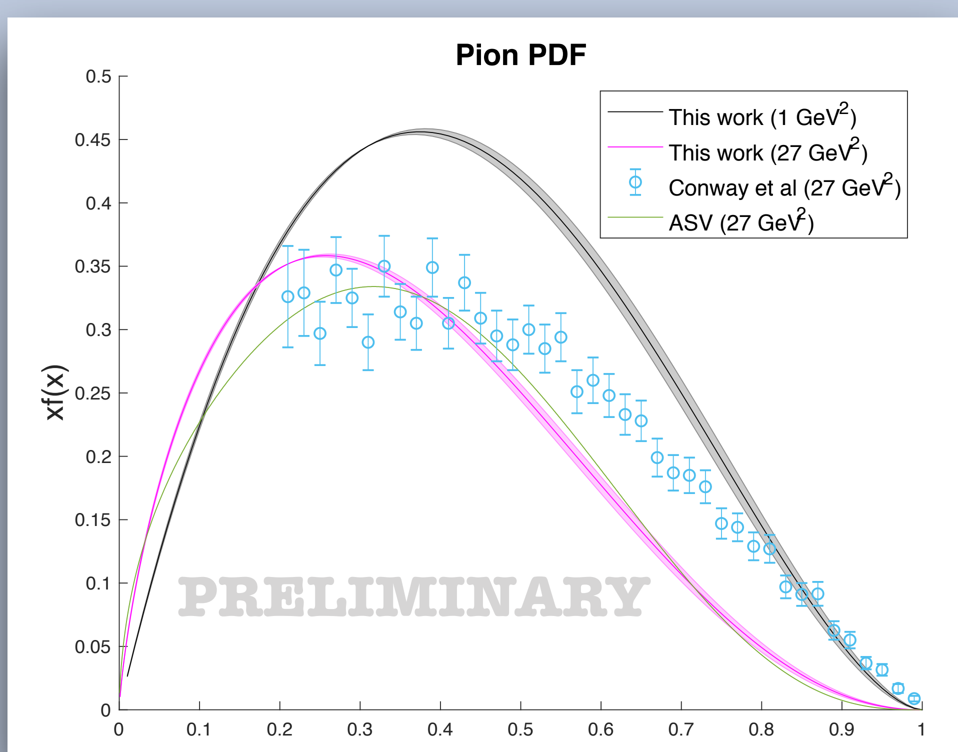
Comparison with other lattice calculations



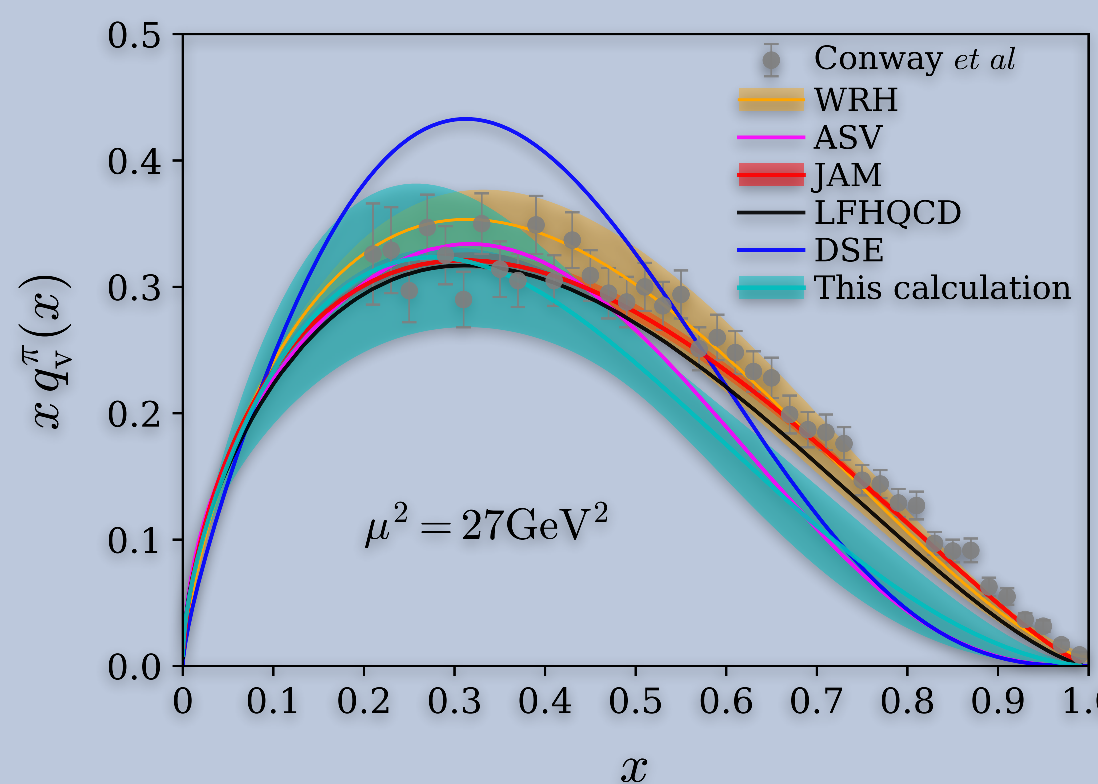
N. Karthik [APS Meeting 2019]



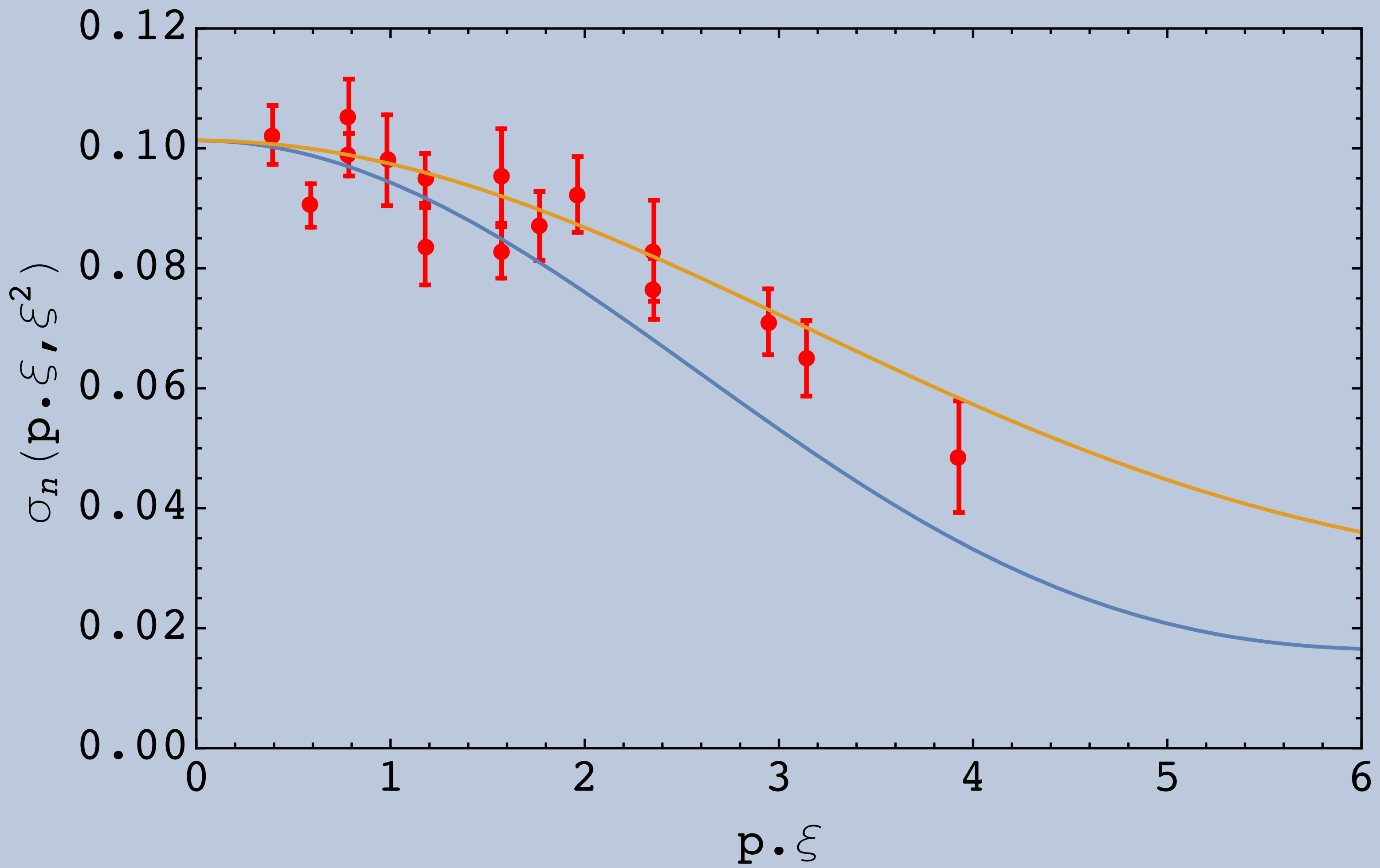
Chen et al, LP3 [arXiv:1803.04393]



Pseudo PDF



$(1-x)$ or $(1-x)^2$? – Not there yet



$$q_v^\pi(x) = N x^\alpha (1-x)^\beta (1+\gamma x)$$

Orange: $\alpha = -0.5,\ \beta = 2.0,\ \gamma = 3.0$

Blue: $\alpha = -0.5,\ \beta = 1.0,\ \gamma = 3.0$

- Precise lattice data at large Ioffe time required

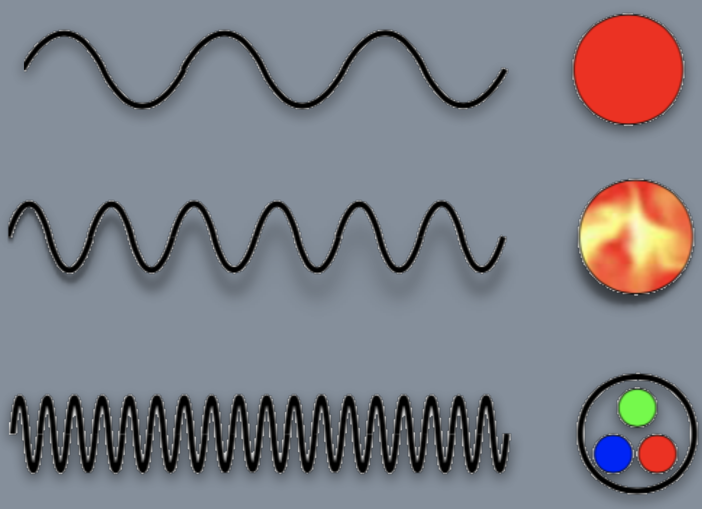
Outlook

- K_n^a at LO and NLO for different currents being calculated
- Different current combinations being analyzed to obtain different sets of PDFs
- Collaboration between lattice QCD and perturbative QCD
- Understanding and control of various systematics required
- Extensions such as kaon, nucleon PDFs on their way
- Goal is to be complementary to global fits of PDFs

Thank You

EXTRA

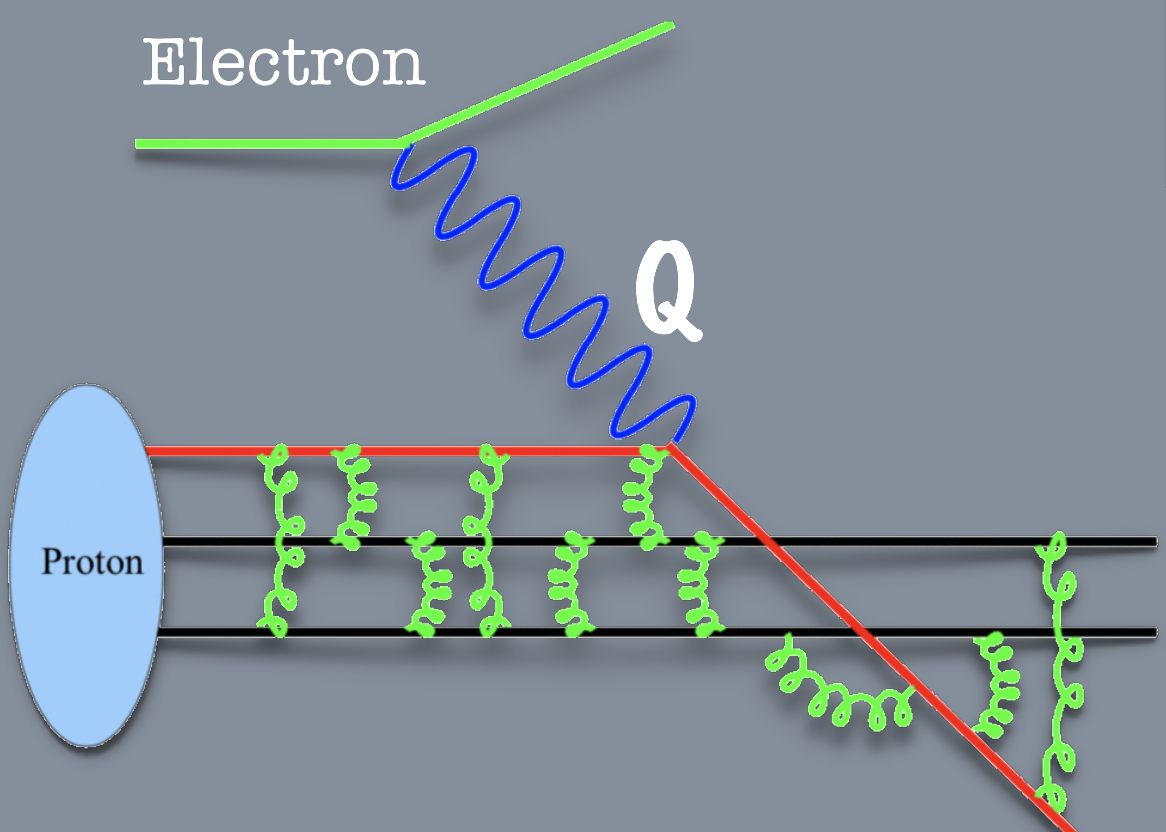
Parton Distribution Functions



Soft interaction

↓

Hard interaction



Parton picture is valid when

$t_h << t_s$ & $Q >> \Lambda_{QCD}$

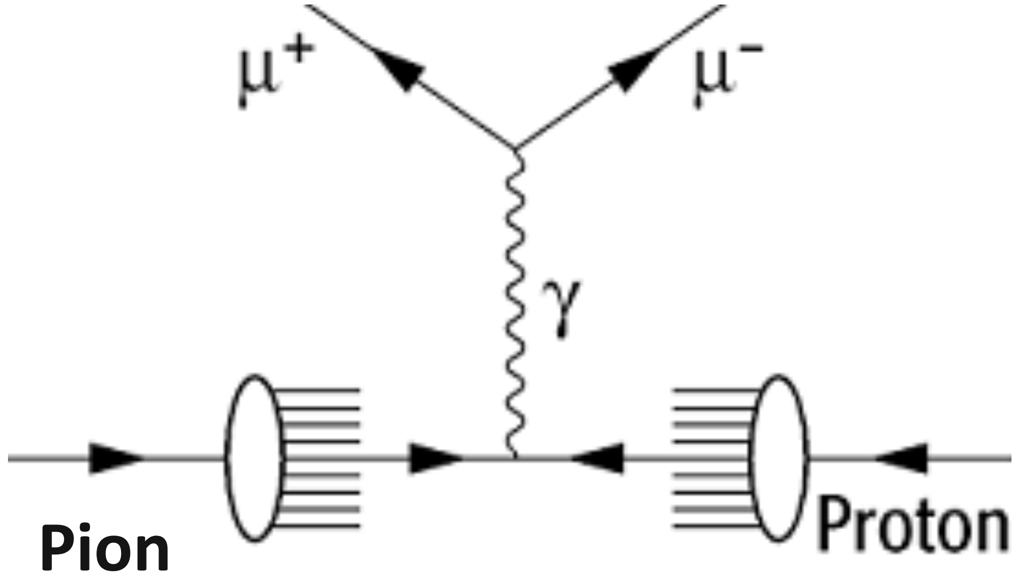
PDF gives the probability of finding a parton of flavor $i\ (q, \bar{q}, g)$ carrying a fraction x of the proton momentum probed at interaction scale Q

$$\sigma_n^{q(0)}\left(\omega,\xi^2\right) = \sum_{a=q,\bar{q},g} \int_0^1 \frac{dx}{x} f_a^{q(0)}\left(x,\mu^2\right) \times K_n^{a(0)}\left(x\omega,\xi^2;\mu^2\right) + \mathcal{O}\left(\xi^2\Lambda_{\mathrm{QCD}}^2\right)$$

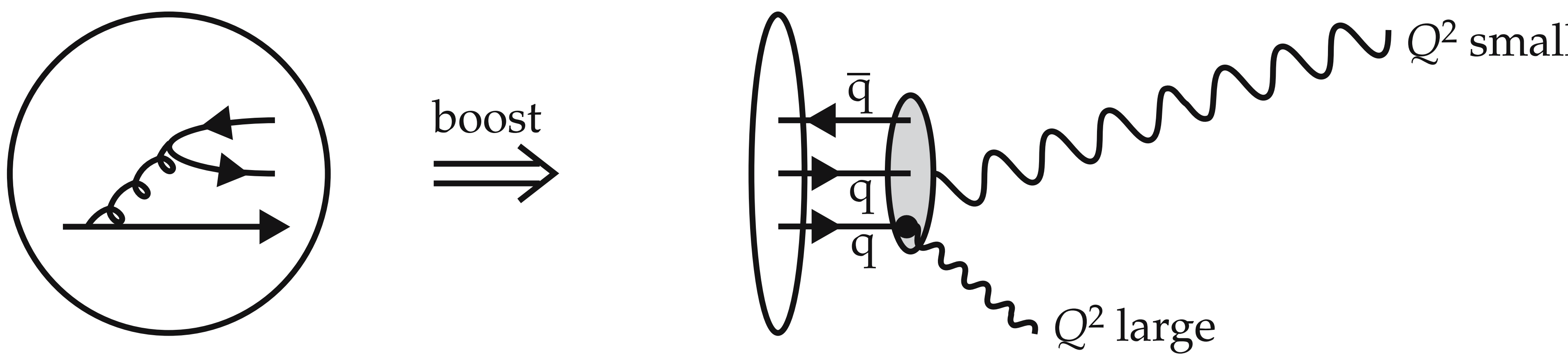
$$f_a^{q(0)}\left(x,\mu^2\right) = \delta\left(1-x\right)\delta^{qa}$$

Quark distribution of an asymptotic quark at zeroth order in α_s

$$\sigma_n^{q(0)}\left(\omega,\xi^2\right) = K_n^{q(0)}\left(\omega,\xi^2\right)$$



$$\pi p \to u^+ \mu^- X$$



Role of Q^2 as transverse resolving power [Greiner, Schramm, Stein]

Calculation of Perturbative Kernel

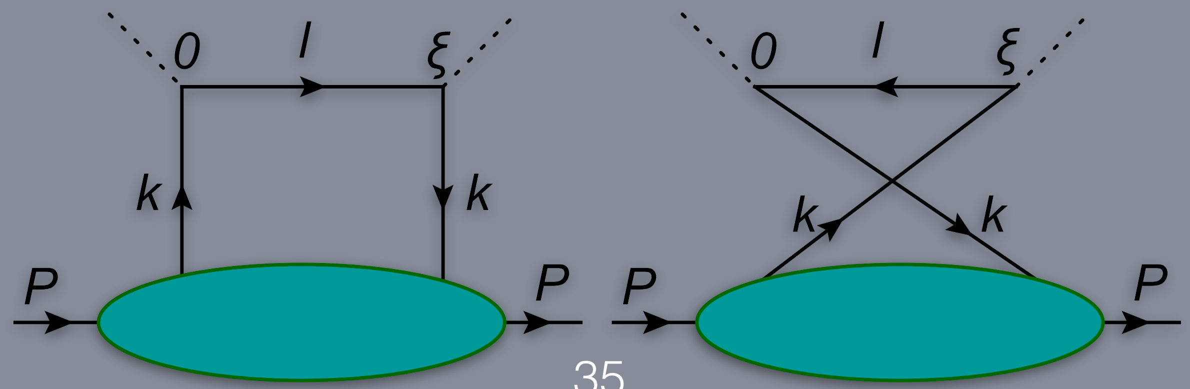
★ For small and nonzero ξ^2 , applying OPE to the nonlocal operator $\mathcal{O}_n(\xi)$

$$\sigma_n(\omega,\xi^2,P^2)=\sum_{J=0}\sum_a W_n^{(J,a)}(\xi^2,\mu^2)\,\xi^{\nu_1}\cdots\xi^{\nu_J}\times\langle P|\mathcal{O}^{(J,a)}_{\nu_1\cdots\nu_J}(\mu^2)|P\rangle$$

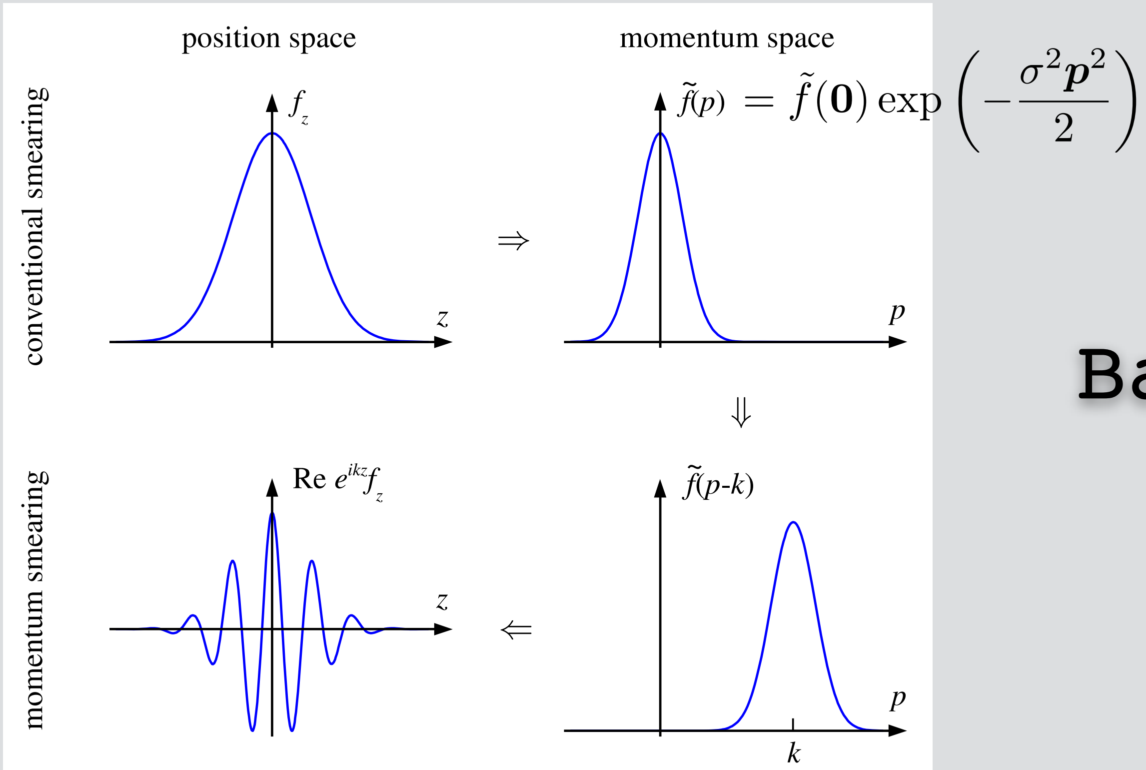
local, symmetric, traceless operator of spin J

★ $$\sigma_n^h(p\cdot\xi,\xi^2,P^2)=\sum_a\int_{-1}^{1}\frac{dx}{x}f_a^h(x,\mu^2)K_n^a(x\omega,\xi^2,x^2P^2,\mu^2)$$

Project “h” to “q”

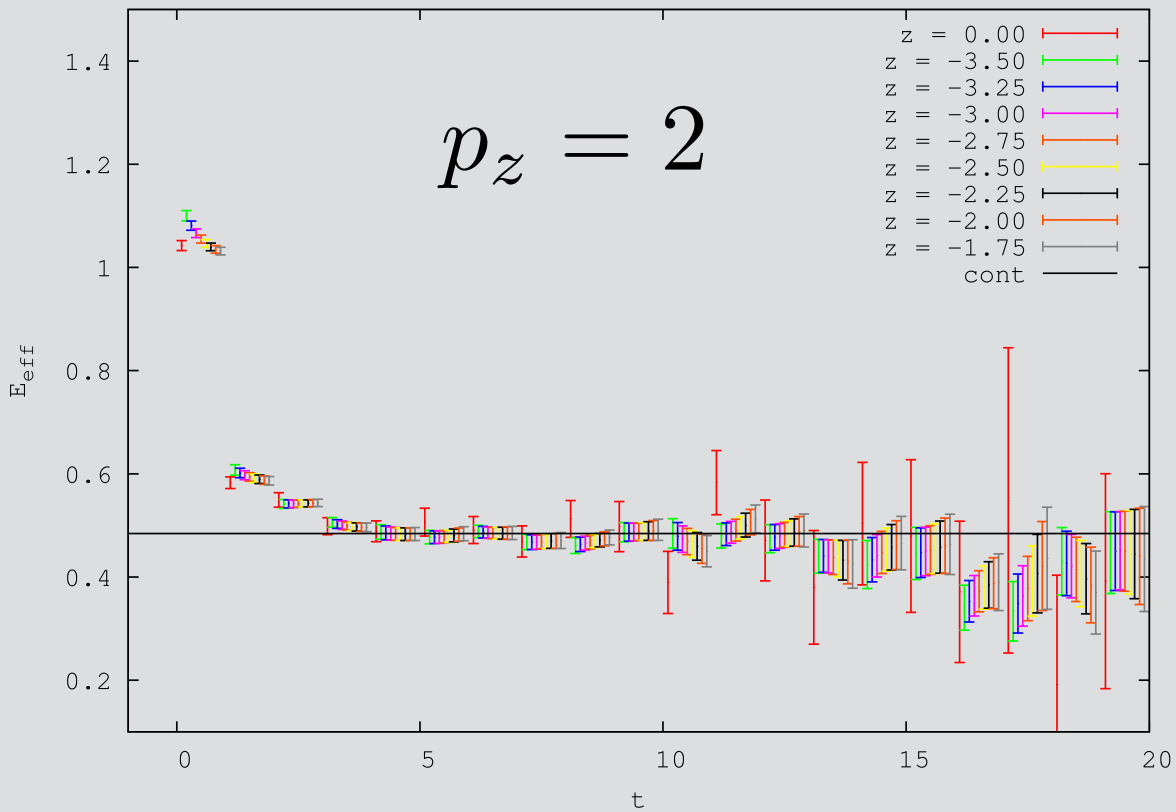


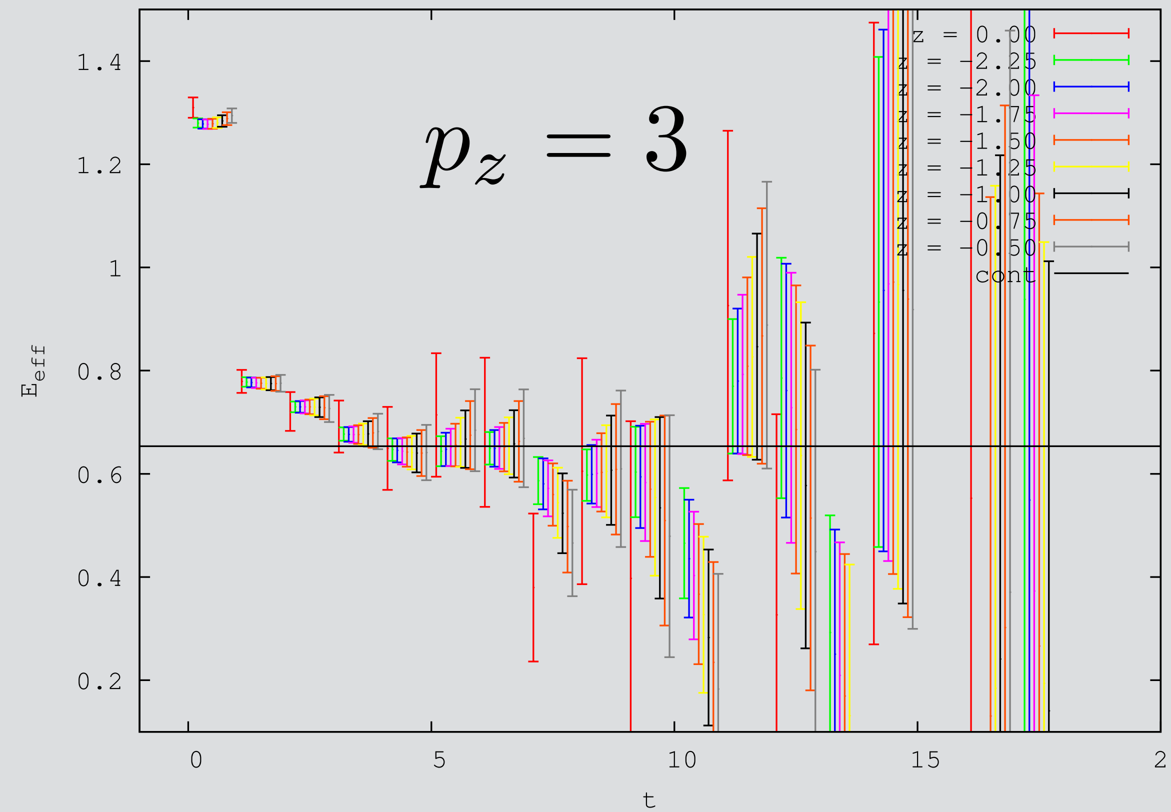
★ Momentum smearing used higher momentum



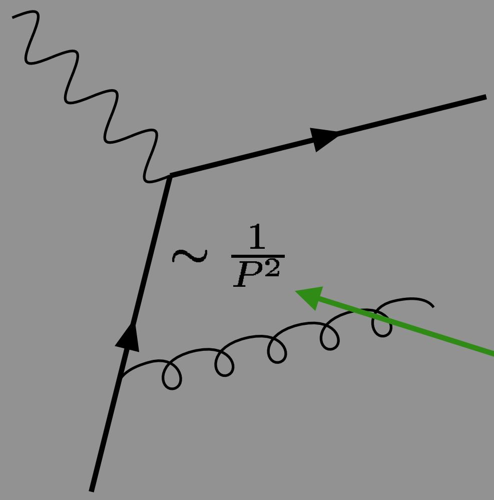
Bali, et al(PRD 2016)

★ Plots from Colin Egerer (only 50 configs, one source)





Near threshold, large logarithms coming from soft radiation become the leading corrections to scattering cross section



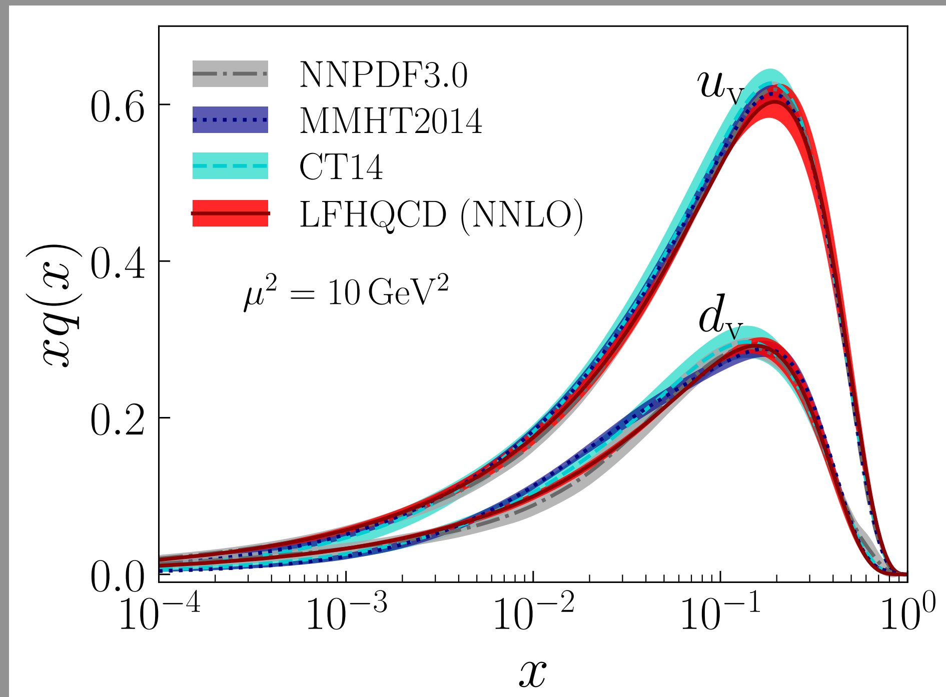
Interaction of a quark with a photon including a gluon correction

$$p^2 = 0$$

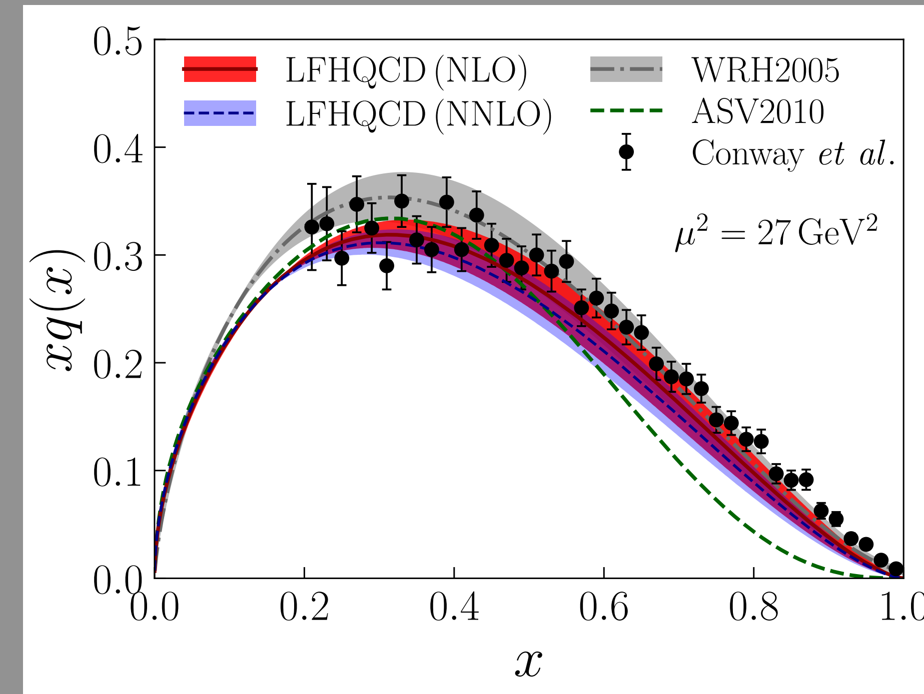
1. If momentum of gluon small, internal quark propagator almost on–shell.

2. Can lead to large logarithmic corrections after the infrared divergences are canceled.

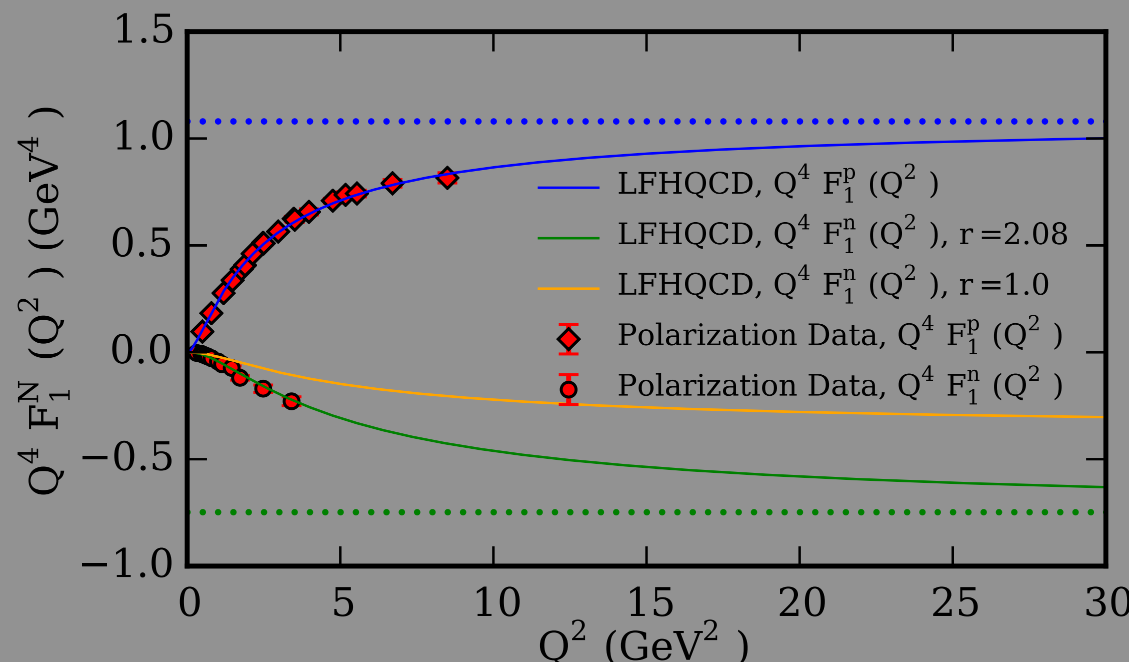
3. This gluon radiation consists of an infinite number of soft gluons, which would make perturbation theory an unusable method for computing physical cross sections.



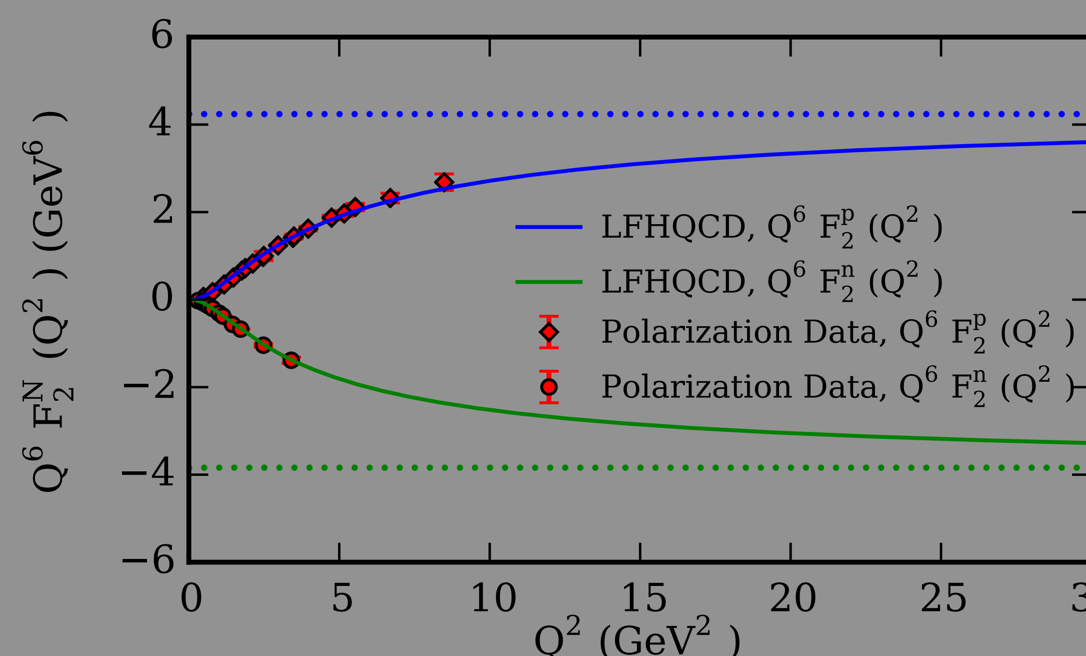
NNPDF3.0
MMHT2014
CT14
LFHQCD (NNLO)
u_v
d_v
$\mu^2 = 10\,\mathrm{GeV}^2$
$xq(x)$
x



LFHQCD (NLO)
LFHQCD (NNLO)
WRH2005
ASV2010
Conway et al.
$\mu^2 = 27\,\mathrm{GeV}^2$
$xq(x)$
x



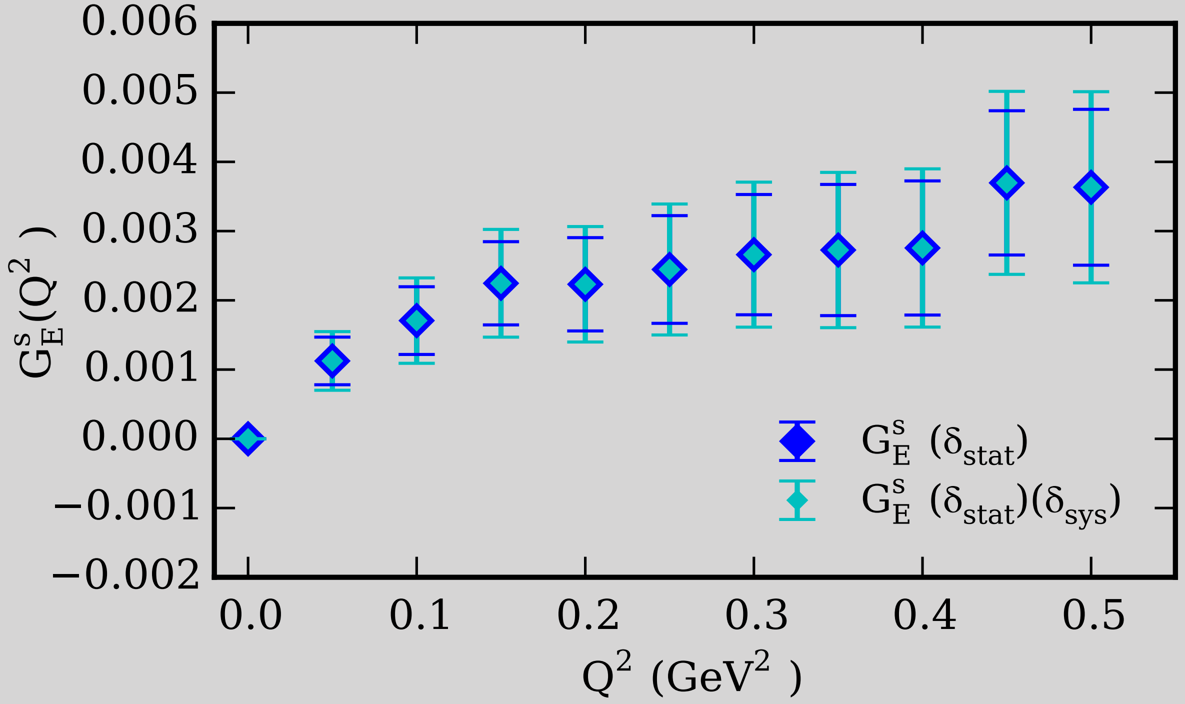
LFHQCD, $Q^4\, F_1^p\, (Q^2)$
LFHQCD, $Q^4\, F_1^n\, (Q^2)$, r =2.08
LFHQCD, $Q^4\, F_1^n\, (Q^2)$, r =1.0
Polarization Data, $Q^4\, F_1^p\, (Q^2)$
Polarization Data, $Q^4\, F_1^n\, (Q^2)$
$Q^4\, F_1^N\, (Q^2)$ (GeV^4)
Q^2 (GeV^2)



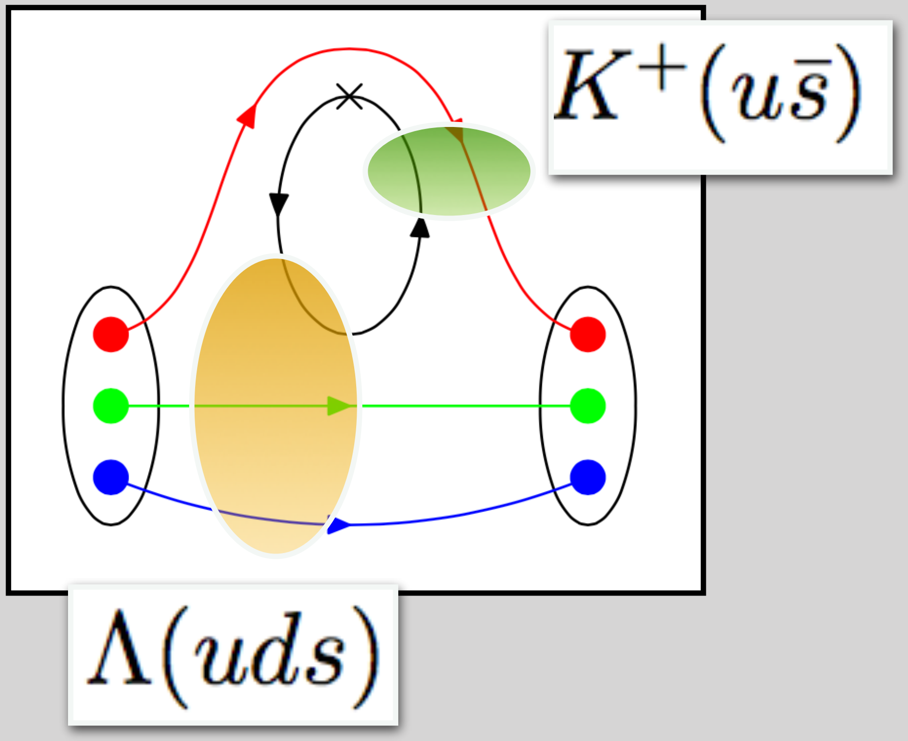
LFHQCD, $Q^6\, F_2^p\, (Q^2)$
LFHQCD, $Q^6\, F_2^n\, (Q^2)$
Polarization Data, $Q^6\, F_2^p\, (Q^2)$
Polarization Data, $Q^6\, F_2^n\, (Q^2)$
$Q^6\, F_2^N\, (Q^2)$ (GeV^6)
Q^2 (GeV^2)

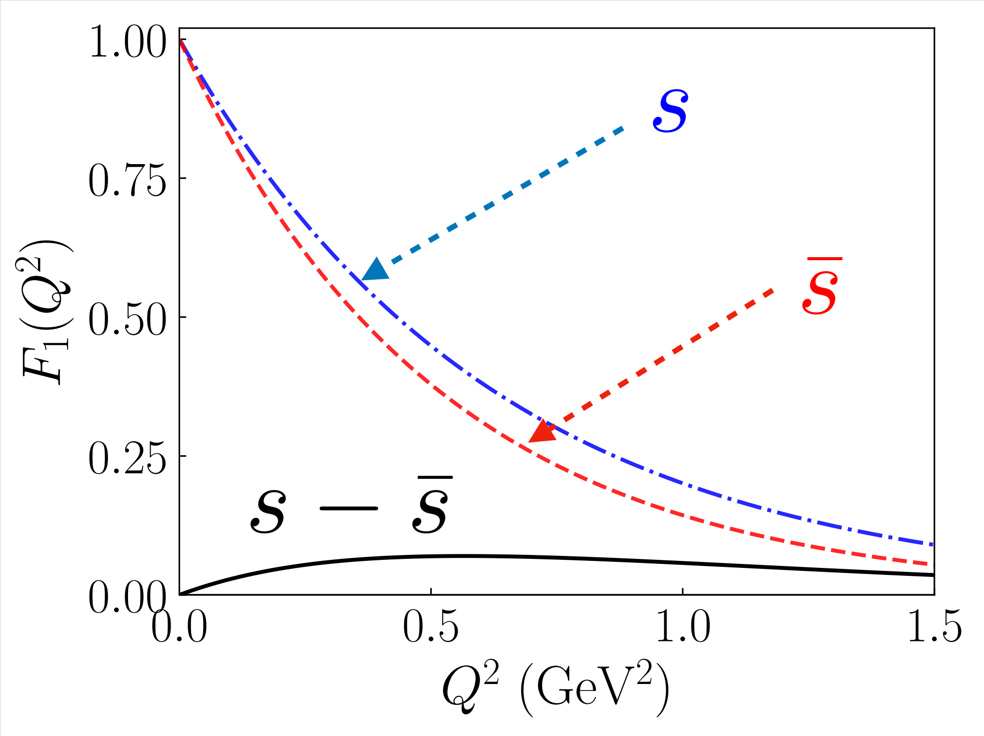
More Applications: Strange Electric FF

★ Nonzero strange electric form factor



Signal, Thomas
PLB 191, 205 (1987)

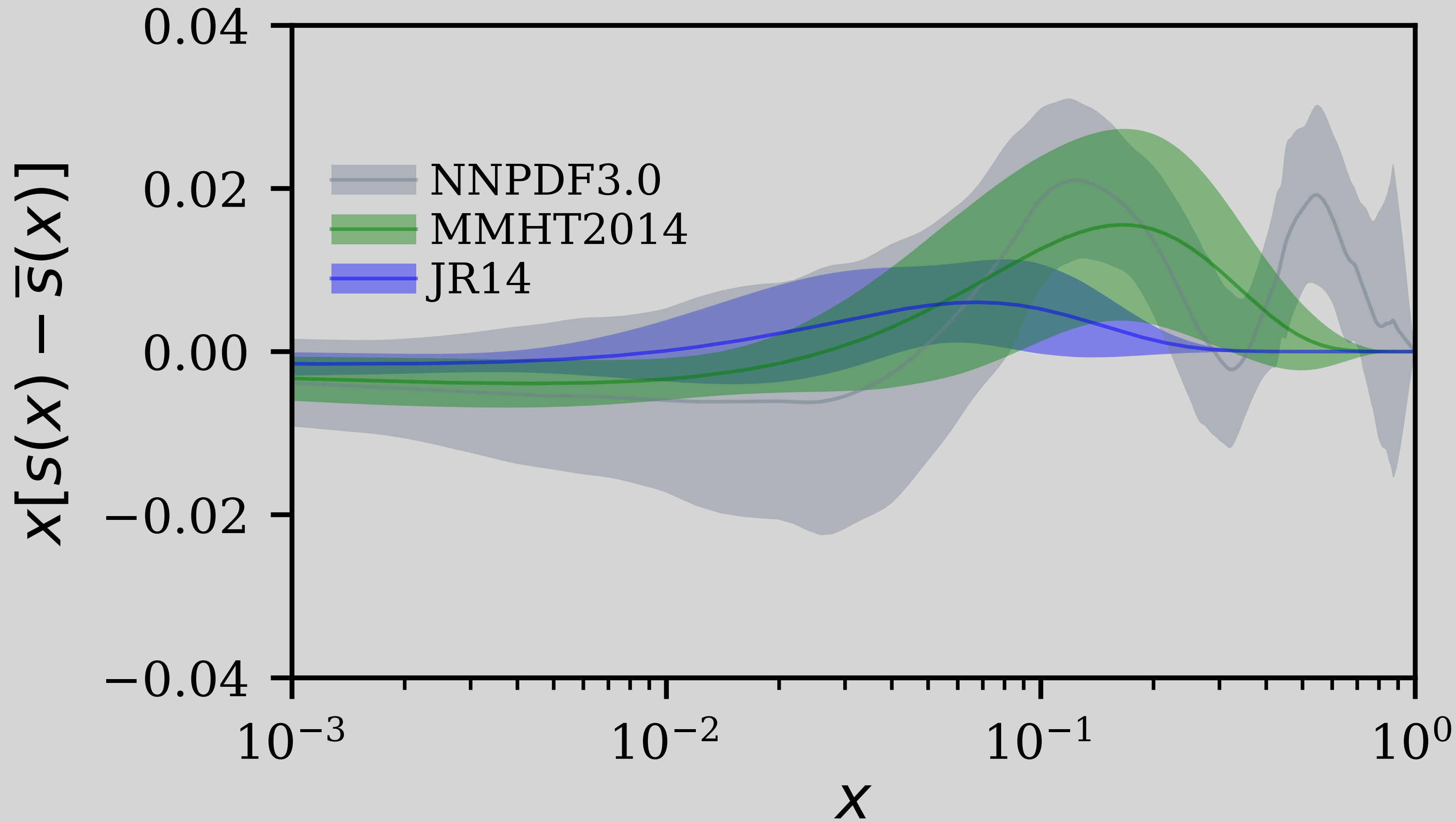




Fourier transform:
a narrower distribution in coordinate space corresponds to a wider distribution in momentum space

Constraints on strange-antistrange asymmetry

★ No definitive results from global fits



★ Nonzero strange electric form factor can be connected to strange-antistrange asymmetry in the nucleon's wave function

★ One Goal: Constrain unknown normalization of model calculations using Lattice QCD

Constraints on strange-antistrange asymmetry

★ Light-front holographic QCD : A semiclassical approach to relativistic bound state equations followed from the holographic embedding of light-front dynamics in a higher dimensional gravity theory

★ EM form factors for a bound state hadron of twist τ

$$F_\tau(t) = \frac{1}{N_\tau} B\big(\tau - 1, 1 - \alpha(t)\big)$$

$$N_\tau = \Gamma(\tau - 1)\Gamma(1 - \alpha(0))/\Gamma(\tau - \alpha(0))$$

$$\alpha(t) = \frac{1}{2} + \frac{t}{4\lambda} - \frac{\Delta M^2}{4\lambda}$$

de Téramond, Liu, RSS,
Dosch, Brodsky, Deur
PRL 2018

★ Write beta function in a reparametrization invariant form

$$B(u, v) = \int_0^1 dx\, w'(x)\, w(x)^{u-1} \left(1 - w(x)\right)^{v-1}$$

$$w(0) = 0, \quad w(1) = 1, \quad w'(x) \geq 0$$

$$w(x) = x^{1-x} e^{-a(1-x)^2}$$

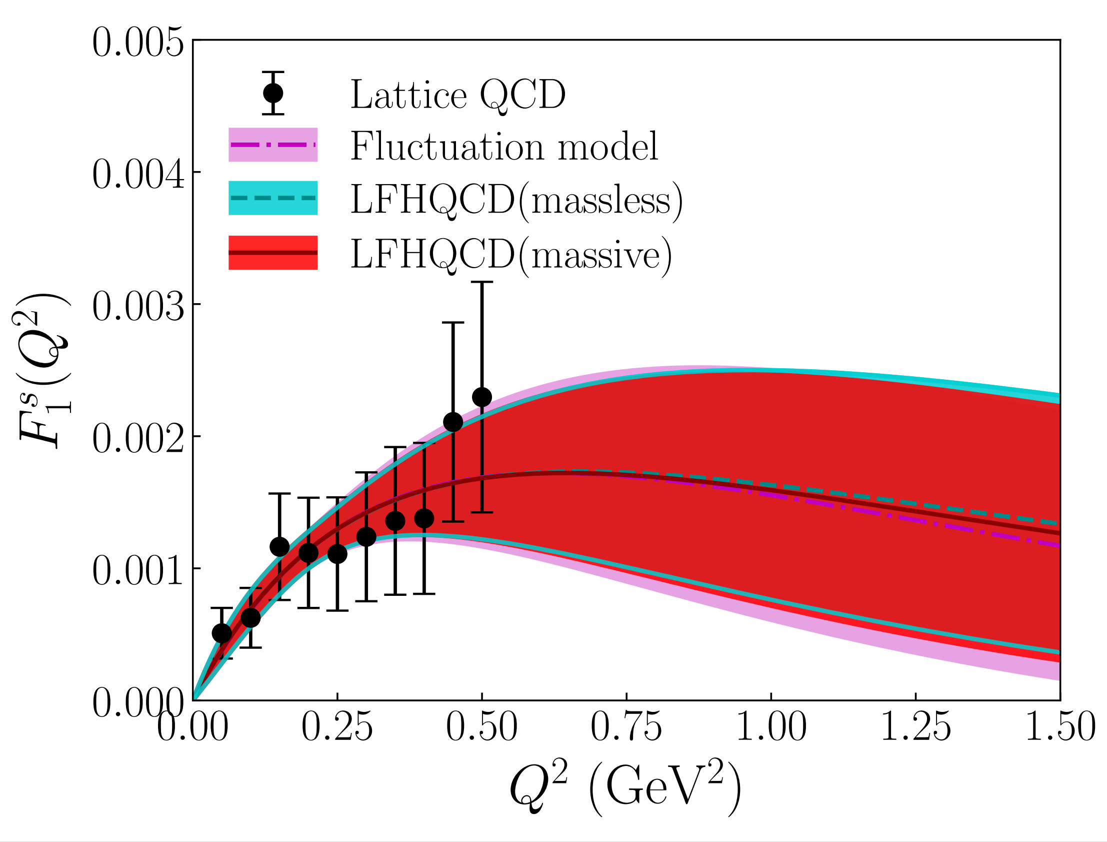
Constraints on strange-antistrange asymmetry

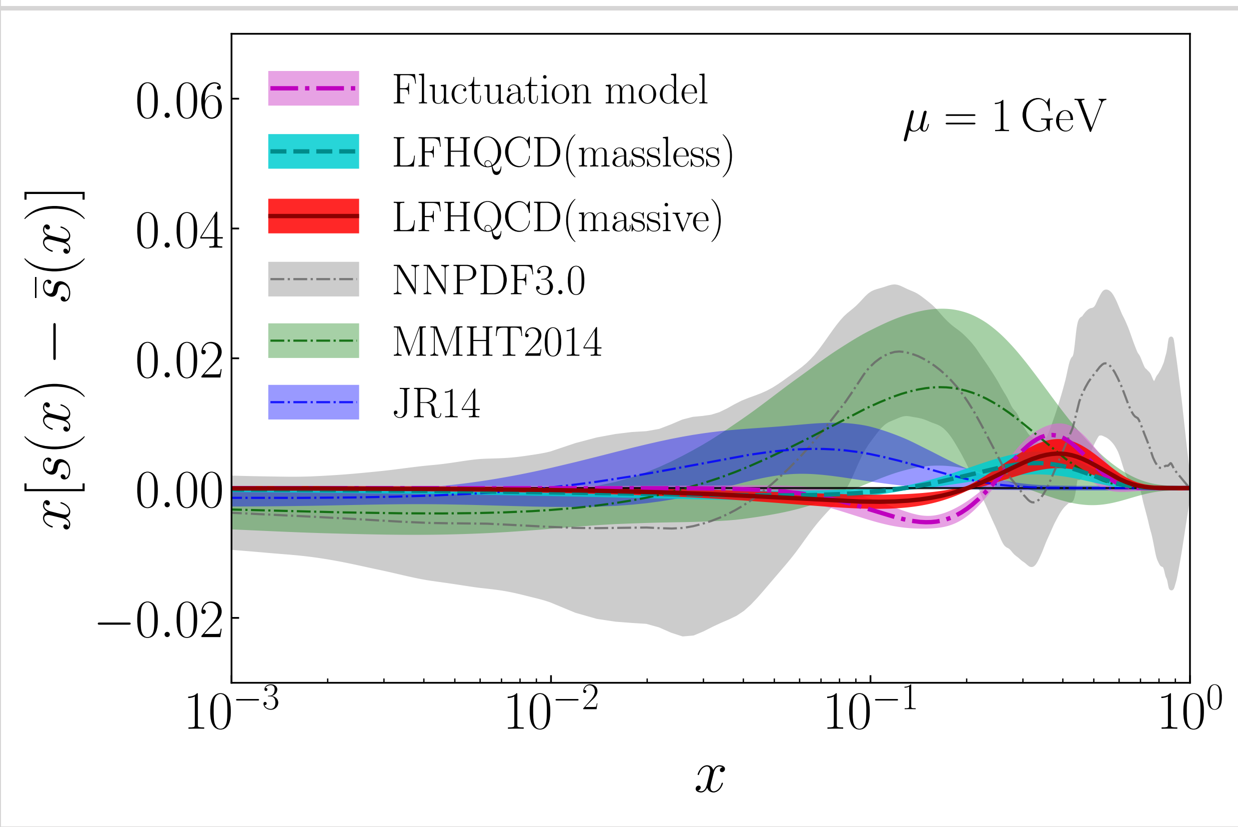
★ **Re-write form factor**

$$F_\tau(t) = \frac{1}{N_\tau}\int_0^1 dx\, w'(x) w(x)^{-\frac{t}{4\lambda}-\frac{1}{2}}\left[1-w(x)\right]^{\tau-2} e^{-\frac{\Delta M^2}{4\lambda}\log\left(\frac{1}{w(x)}\right)}$$

★ **Quark distribution**

$$q_\tau(x) = \frac{1}{N_\tau}[1-w(x)]^{\tau-2} w(x)^{-\frac{1}{2}} w'(x) e^{-\frac{\Delta M^2}{4\lambda}\log\left(\frac{1}{w(x)}\right)}$$





RSS, Liu, de Téramond, Dosch, Brodsky, et. al.
PRD 2018

$$\langle S_- \rangle = 0.0011(4)$$