Calculating hadronic tensor on the lattice

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χQCD collaboration

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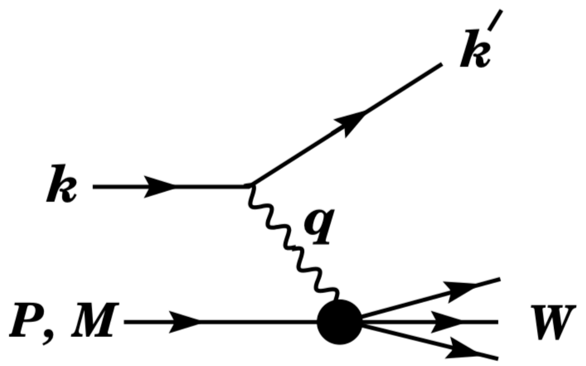
Hadronic tensor



deep ($Q^2 \gg M^2$) inelastic ($W^2 \gg M^2$) scattering (DIS)

to leading order perturbation $$\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4}\sum_j \eta_j L_j^{\mu\nu} W_{\mu\nu}^j$$

the hadronic tensor $$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s \left| \left[J_\mu^\dagger(z) J_\nu(0) \right] \right| p,s \right\rangle$$

Im part of the forward Compton amplitude

for unpolarized cases $$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F_2(x,Q^2)$$

- ✦ for high energy scatterings (DIS), extract PDFs through factorization

$$F_i = \sum_a C_i^a \otimes f_a$$

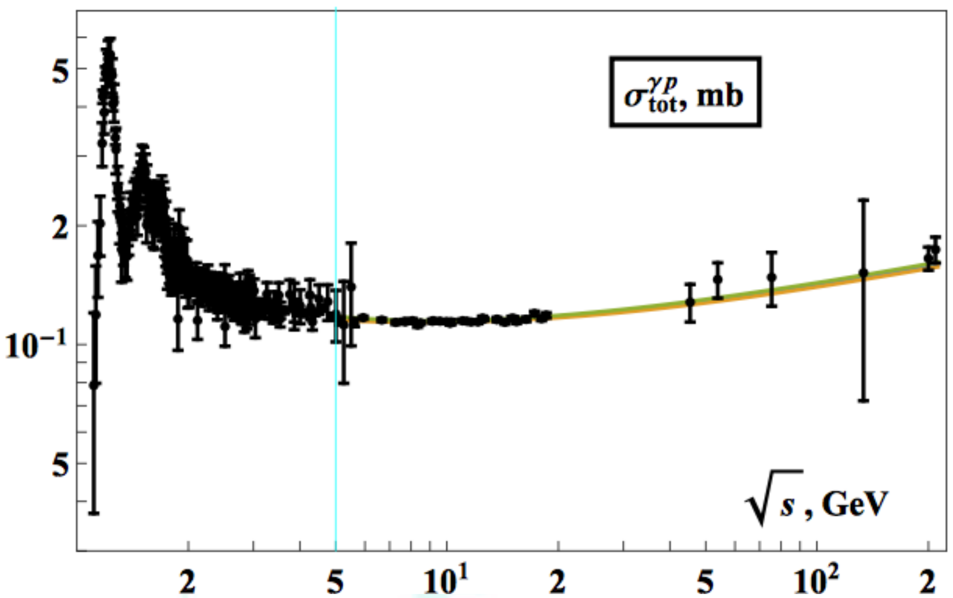
- ✦ for low energy cases (e.g., elastic scatterings), extract form factors

$$F_2^{\mathrm{el}} = \delta(q^2 + 2m_N\nu)\frac{2m_N}{1 - q^2/4{m_N}^2}\left(G_E^2(q^2) - \frac{q^2}{4M_N^2}G_M^2(q^2)\right)$$

Sketch the structure function

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4 z e^{iq\cdot z}\left\langle p,s\left|\left[J^\dagger_\mu(z)J_\nu(0)\right]\right|s,s\right\rangle$$

$$= \frac{1}{4\pi}\sum_n \int \prod_i^n \left[\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\right]\langle p,s|J^\dagger_\mu(z)|n\rangle\langle n|J_\nu(0)|p,s\rangle(2\pi)^3\delta^4(q-p_n+p)$$



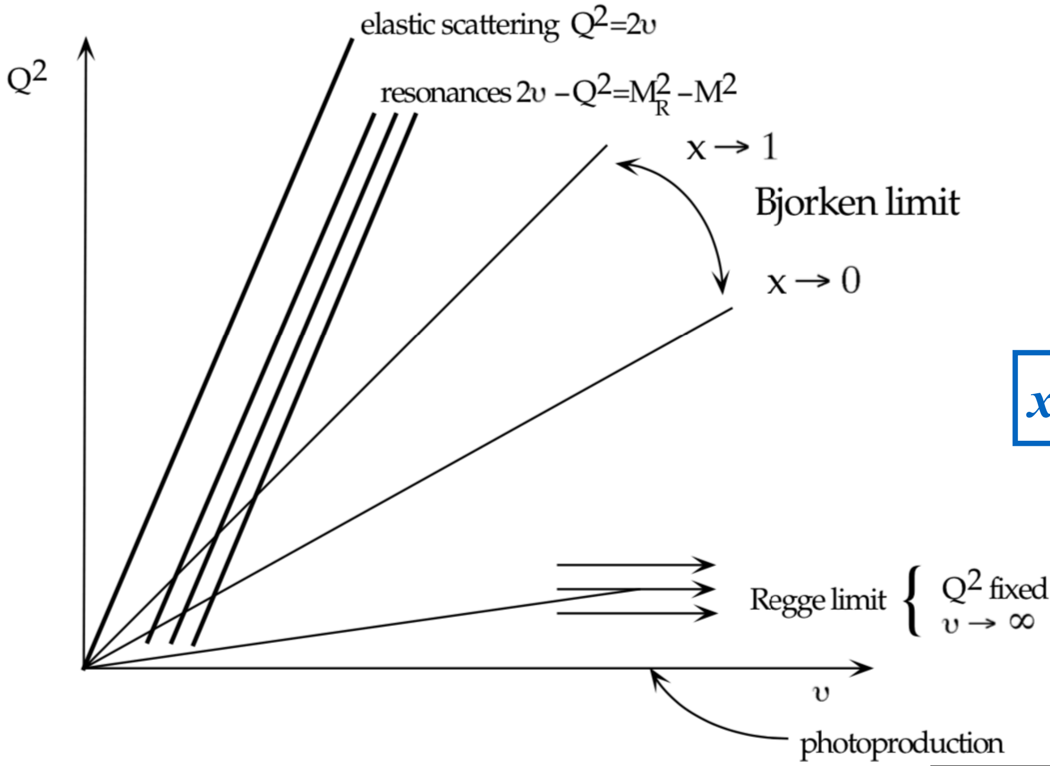
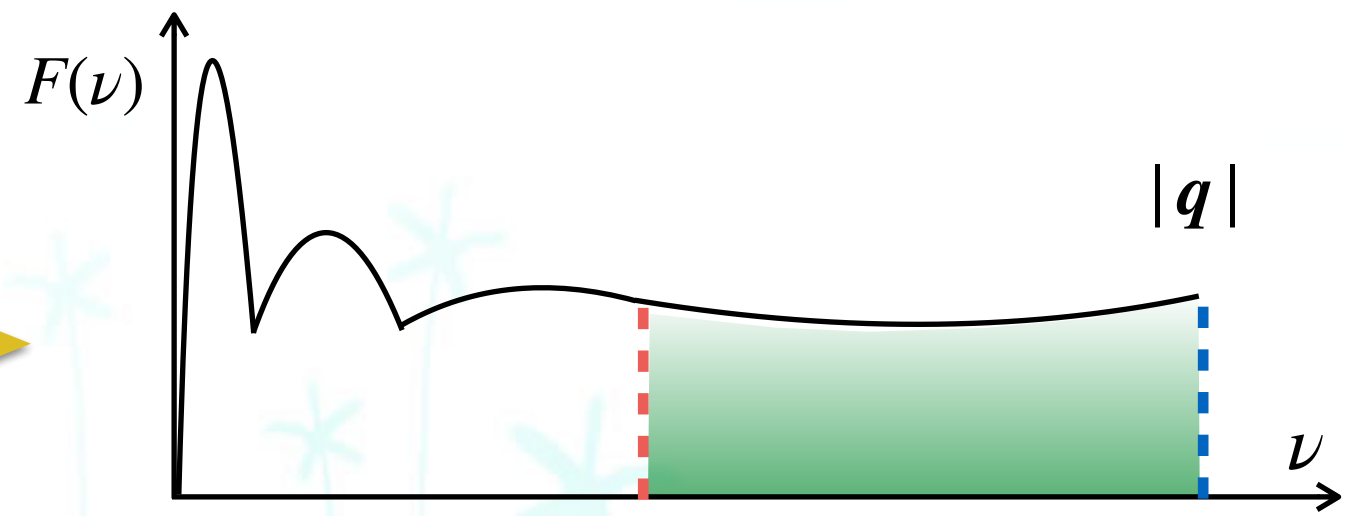


Fig. 4. Kinematic domains in electron-nucleon scattering.



Elastic

***x=1*, form factors**

RES and SIS

N\pi, \Delta, ..., continuous spectrum

DIS region

$$x = \frac{Q^2}{2m\nu}, Q^2 \to \infty, \nu \to \infty$$

$Q^2 = 0$

photoproduction

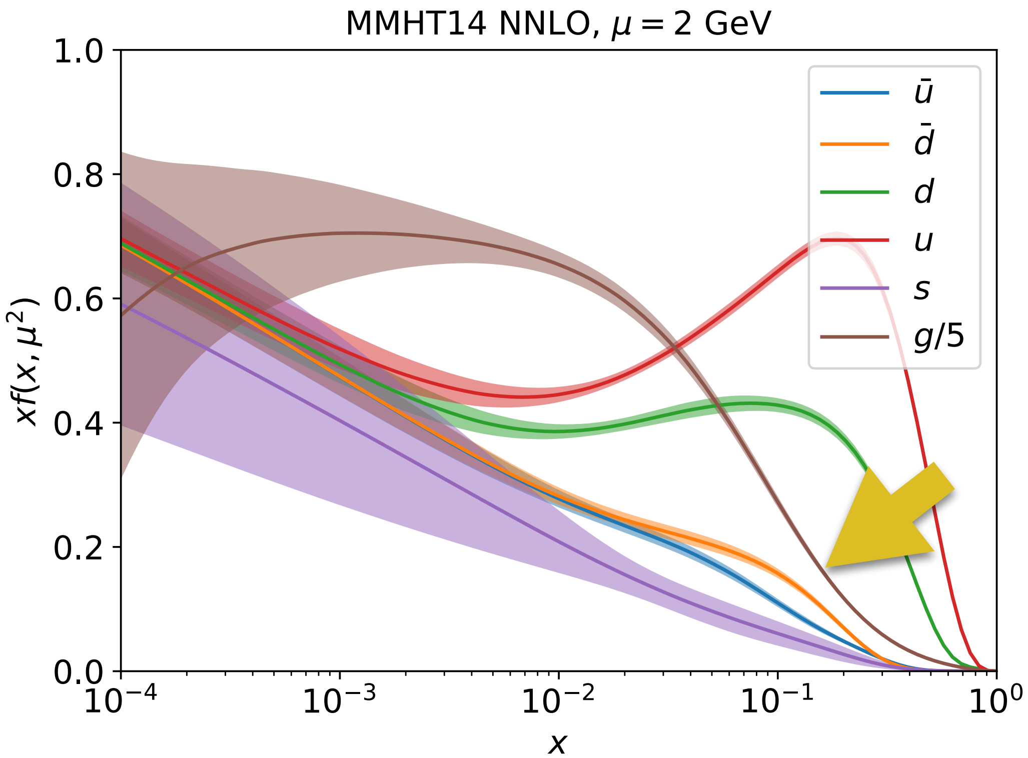
arXiv:hep-ph/9602236

Motivation 1: parton physics

Lattice efforts:

- ✦ Quasi-PDFs and LaMET — *X. Ji, PRL110, 262002 (2013)*; *H.W. Lin et. al., PRL121, 242003 (2018)*
- ✦ Compton amplitude — *A. J. Chambers et. al., PRL118, 242001(2017)*
- ✦ Puedo-PDFs — *A. V. Radyushkin, PRD96, 034025 (2017)*; *K. Orginos et. al., PRD96, 094503 (2017)*
- ✦ Lattice cross sections — *Y.-Q. Ma and J.-W. Qiu, PRL120, 022003 (2018)*; *R. S. Sufian et. al., PRD99, 074507 (2019)*
- ✦ …

✦ **Hadronic tensor is scale independent! No need to do renormalization.**
✦ **Structure functions are frame independent! No need of large external momentum.**



L. A. Harland-Lang et. al., EPJ C75, 204 (2015)

The u-bar and d-bar difference of PDFs (Gottfried sum rule violation) is related to the connected-sea anti-partons.

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

Hadronic tensor provides a direct way to reveal the connected-sea anti-parton contribution.

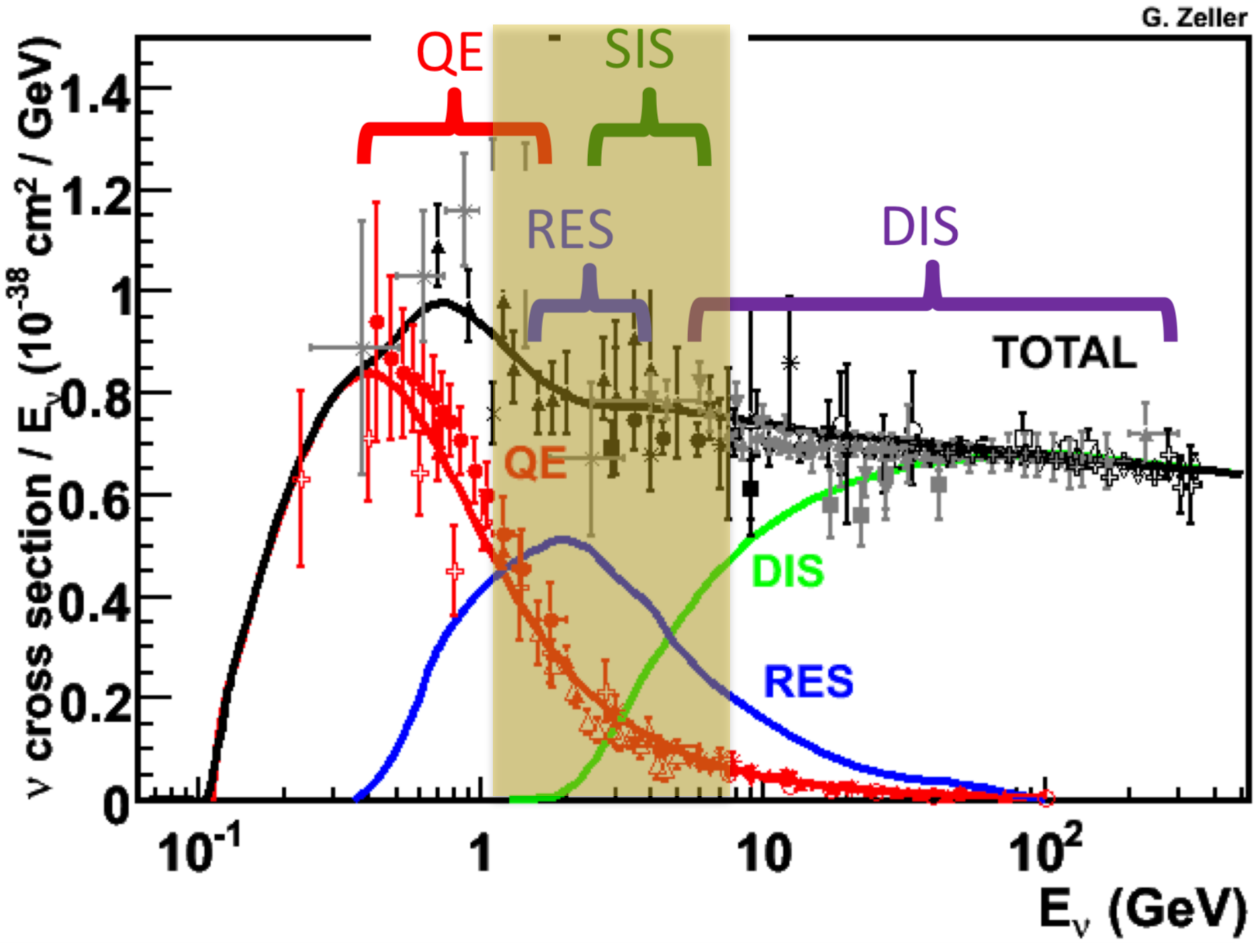
K.-F. Liu, PRD62, 074501 (2000)
K.-F. Liu, PoS LATTICE2015, 115 (2016)
J. Liang et. al., EPJ Web Conf. 175, 14014 (2018)

Similar proposals:

U. Aglietti et. al., PLB432, 411 (1998)
W. Detmold and C. J. D. Lin, PRD73, 014501 (2006)
M. T. Hansen et. al., PRD96, 094513 (2017)

Motivation 2: neutrino-nucleus scattering

- one of the most important tasks in High Energy Physics is to understand the properties of neutrinos.
- DUNE@LBNF FERMILAB with neutrino energy ~1-~7 GeV



J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012); Teppei Katori's talk

- $\nu A \rightarrow \nu N$, theoretical input about nucleon structure is needed to help map out the original neutrino beam energy and flux.
- For elastic contribution, nucleon FFs can be calculated by lattice or models.
- But soon enough, one will not be able to tell one state from another and will need INCLUSIVE hadron tensor (the resonance and shallow inelastic scattering (SIS) region).
- The only way that lattice QCD can help as far as we know.

Lattice QCD

Euclidean field theory using the path-integral formalism,

$$t \to -i\tau \qquad \langle O \rangle = \frac{1}{Z} \int \mathscr{D}A_\mu \mathscr{D}\psi \mathscr{D}\bar{\psi} O e^{-S}$$

Euclidean time correlation functions:

$$C_2(t) = \mathrm{Tr}\left[\Gamma \langle O(t)\bar{O}(0)\rangle\right] = \sum_n \left| \langle 0 | O | n \rangle \right|^2 e^{-E_n t} \qquad O(t) = e^{\hat{H}t} O(0) e^{-\hat{H}t}$$

$$C_3(t,\tau) = \mathrm{Tr}\left[\Gamma \langle O(t) C(\tau) \bar{O}(0)\rangle\right] = \sum_{mn} \langle 0 | O | n \rangle \langle n | C | m \rangle \langle m | O | 0 \rangle e^{-E_n(t-\tau)} e^{-E_m \tau}$$

time dependent matrix element can be problematic (e.g., light-cone PDFs)

Minkowski $$W_{\mu\nu} = \frac{1}{4\pi} \int d^4 z e^{iq\cdot z} \left\langle p, s \left| \left[J_\mu^\dagger(z) J_\nu(0) \right] \right| s, s \right\rangle$$

$$= \frac{1}{2} \sum_n \int \prod_i^n \left[\frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \right] \langle p, s | J_\mu^\dagger(0) | n \rangle \langle n | J_\nu(0) | p, s \rangle (2\pi)^3 \delta^4(q - p_n + p)$$

Euclidean $$W'_{\mu\nu} = \frac{1}{4\pi} \sum_n \int dt e^{\left(\nu - (E_n - E_p)\right)t} \int d^3 \mathbf{z} e^{i\mathbf{q}\cdot\mathbf{z}} \langle p, s | J_\mu^\dagger(\mathbf{z}) | n \rangle \langle n | J_\nu(0) | p, s \rangle$$

$$= \frac{1}{4\pi} \sum_n \frac{e^{\left(\nu - (E_n - E_p)\right)T} - 1}{\nu - (E_n - E_p)} \int d^3 \mathbf{z} e^{i\mathbf{q}\cdot\mathbf{z}} \langle p, s | J_\mu^\dagger(\mathbf{z}) | n \rangle \langle n | J_\nu(0) | p, s \rangle$$

Hadronic tensor on the lattice

four-point function with two currents

$$C_4 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \sum_{\boldsymbol{x}_2 \boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \left\langle \chi_N(\boldsymbol{x}_f, t_f) J_\mu^\dagger(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

nucleon two-point function

$$C_2 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \left\langle \chi_N(\boldsymbol{x}_f, t_f) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

Euclidean hadronic tensor

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \frac{E_p}{m_N} \frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \to \sum_{\boldsymbol{x}_2 \boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \langle p, s \,|\, J_\mu(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \,|\, p, s \rangle$$

$$= \sum_n A_n e^{-(E_n - E_p)\tau}, \ \tau \equiv t_2 - t_1$$

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD62, 074501 (2000)

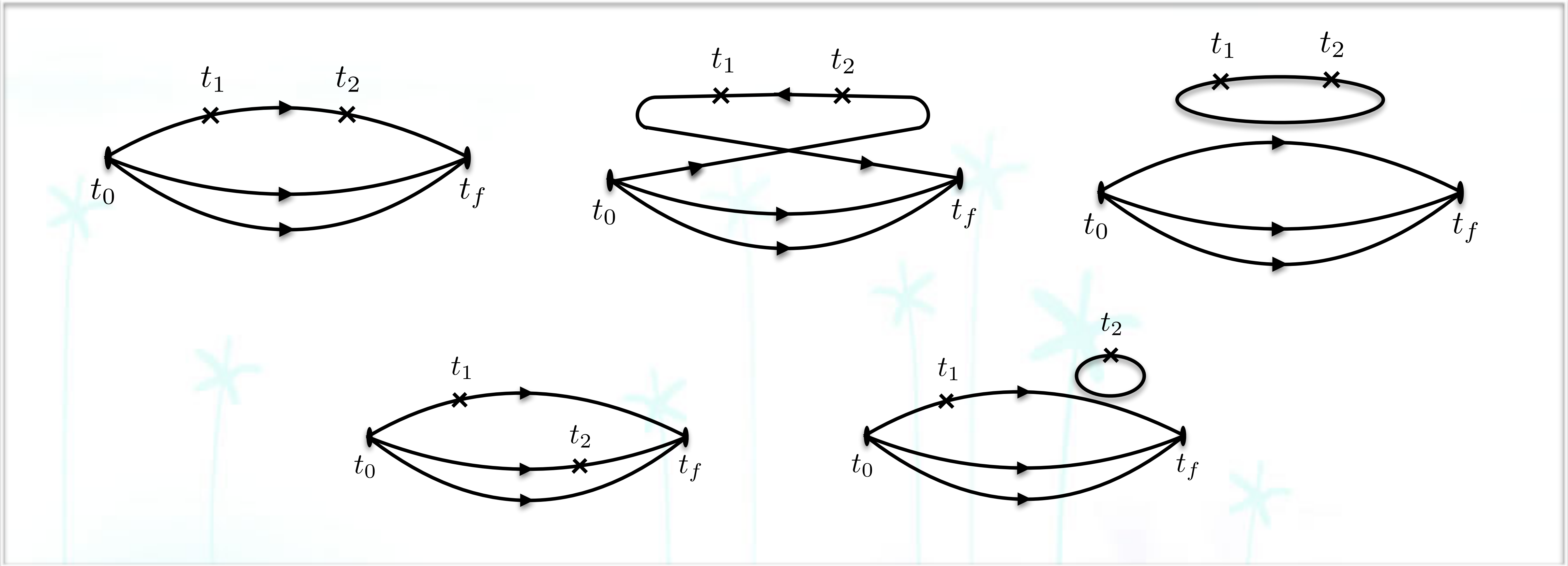
J. Liang et. al., EPJ Web Conf. 175, 14014 (2018)

exponential behavior w.r.t. the time difference between the two currents

Contractions

$$C_4 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \sum_{\boldsymbol{x}_2 \boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \left\langle \chi_N(\boldsymbol{x}_f, t_f) J_\mu(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

$$\chi_N = \left[u_1^T C\gamma_5 d\right] u_2$$



More contractions if we consider different types of the two currents: vector, axial vector, neutral or charged, various quark flavors …

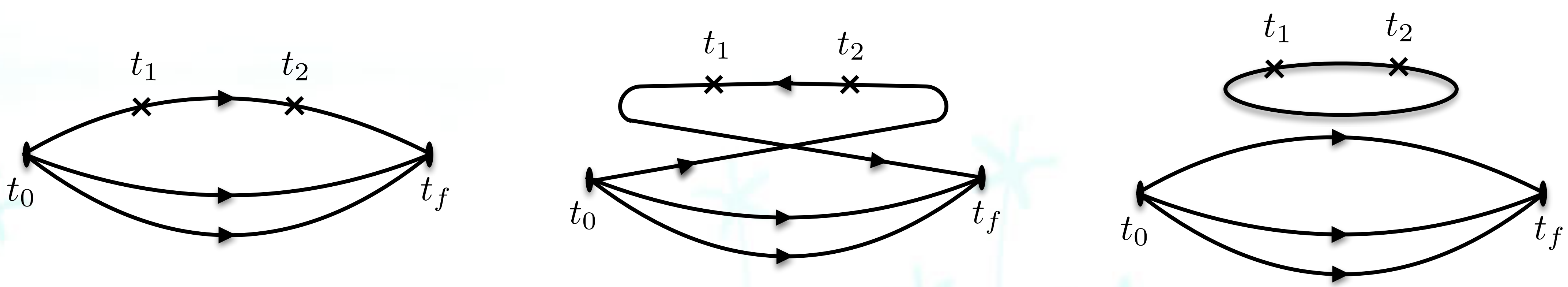
No disconnected insertions are considered in the current plan.

The latter two are suppressed when the momentum and energy transfers are large.

Contractions

$$C_4 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \sum_{\boldsymbol{x}_2\boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \left\langle \chi_N(\boldsymbol{x}_f, t_f) J_\mu(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

$$\chi_N = \left[u_1^T C\gamma_5 d\right] u_2$$



valence and connected-sea parton

connected-sea anti-parton (Gottfried sum rule violation)

disconnected-sea parton and anti-parton

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

For the moment, we will focus on the first one.

Back to the Minkowski space

Euclidean hadronic tensor

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) \sim \sum_n A_n e^{-\nu_n \tau}, \nu_n \equiv E_n - E_p$$

Formally, an inverse Laplace transform will do

$$W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau e^{\nu\tau}\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau)$$

Practically, need to solve the inverse problem of the Laplace transform

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu)e^{-\nu\tau}$$

several (O(10)) discrete data points — **continuous function w.r.t. \nu**

lack of information, an ill-posed problem

Tests on two-point functions

$$C_2(\tau) = e^{-m_1\tau} + e^{-m_2\tau} + e^{-m_3\tau}$$

$$C_2(\tau) = \int d\omega \rho(\omega) e^{-\omega\tau}$$

- **Backus-Gilbert (BG)**

  G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)

- **Maximum Entropy (ME)**

  E Rietsch et. al., JOURNAL OF GEOPHYSICS, 42:489 (1977)

  M. Asakawa et. al., Prog. Part. Nucl. Phys. 46, 459 (2001)

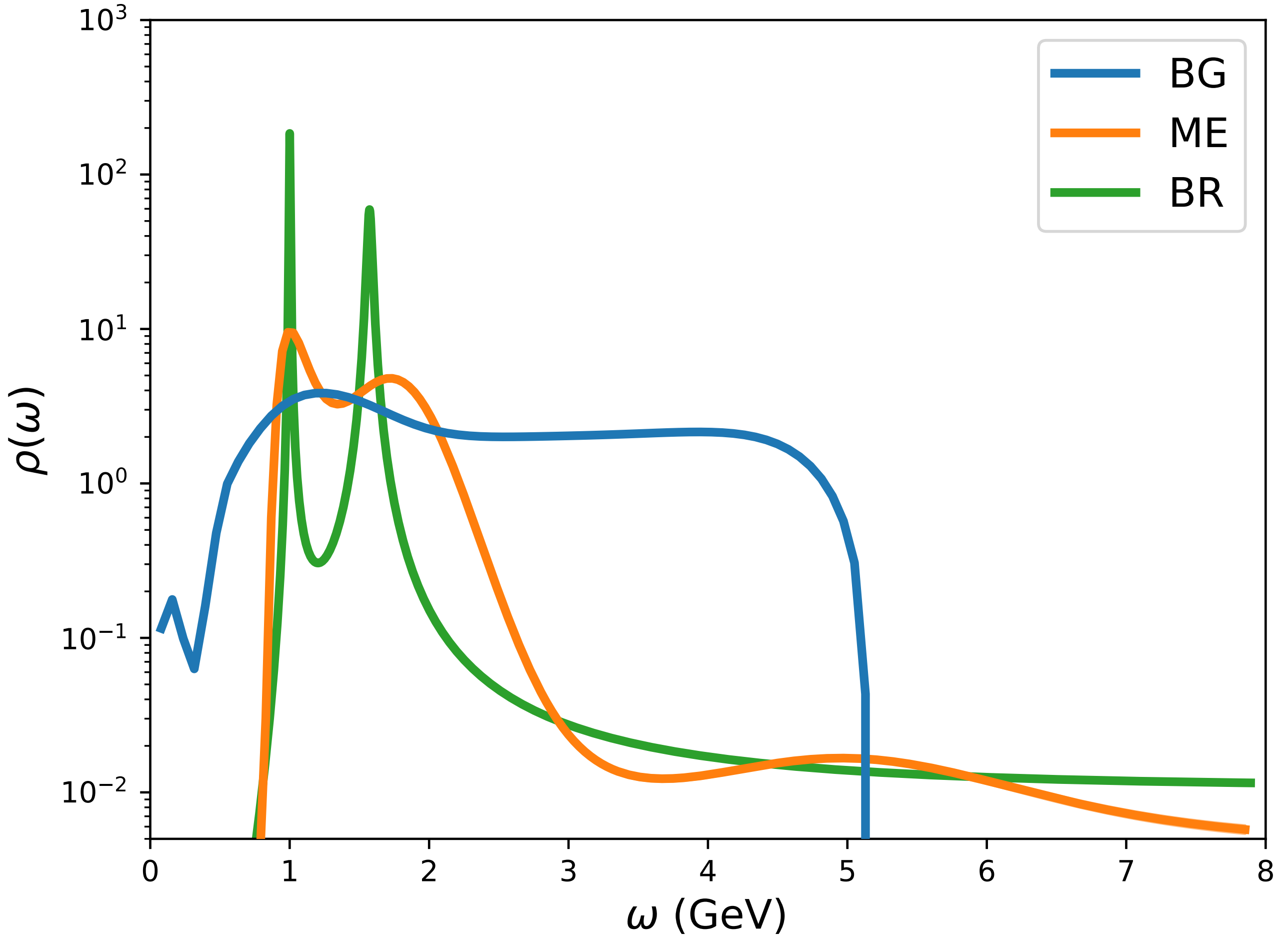
- **Bayesian Reconstruction (BR)**

  Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

mock two-point function data: 3 single exponentials with mass 1.0, 1.5 and 1.8 GeV respectively, a~0.1 fm, Nt=20, S/N=100

bad resolution of BG

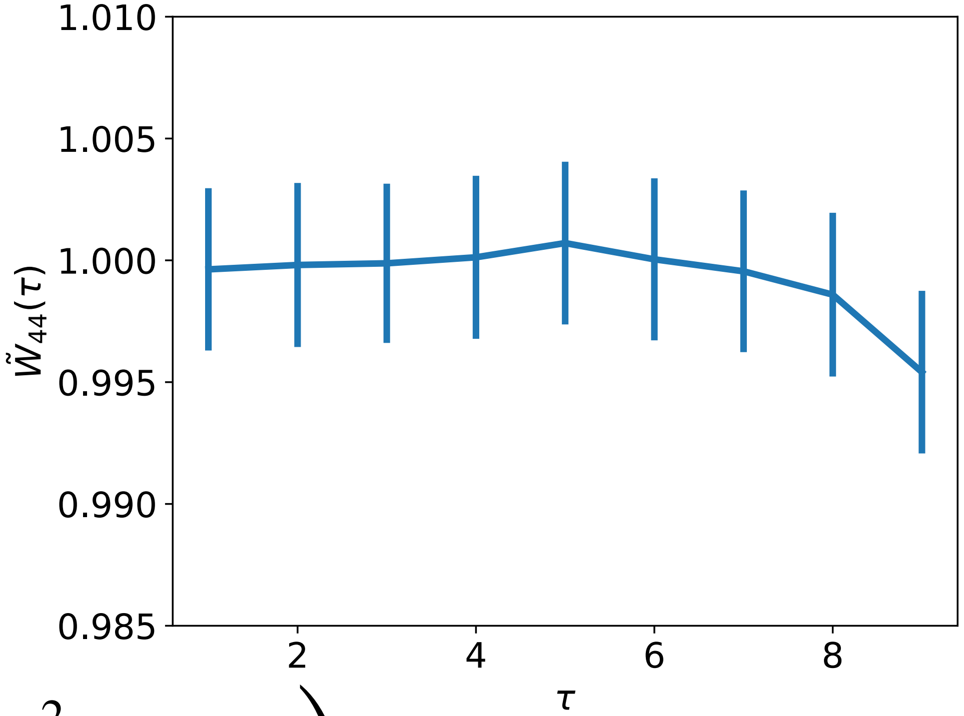
BR is shaper and more stable than ME



The elastic case

normalized vector current $J_4 = \bar{\psi}\gamma_4\psi$

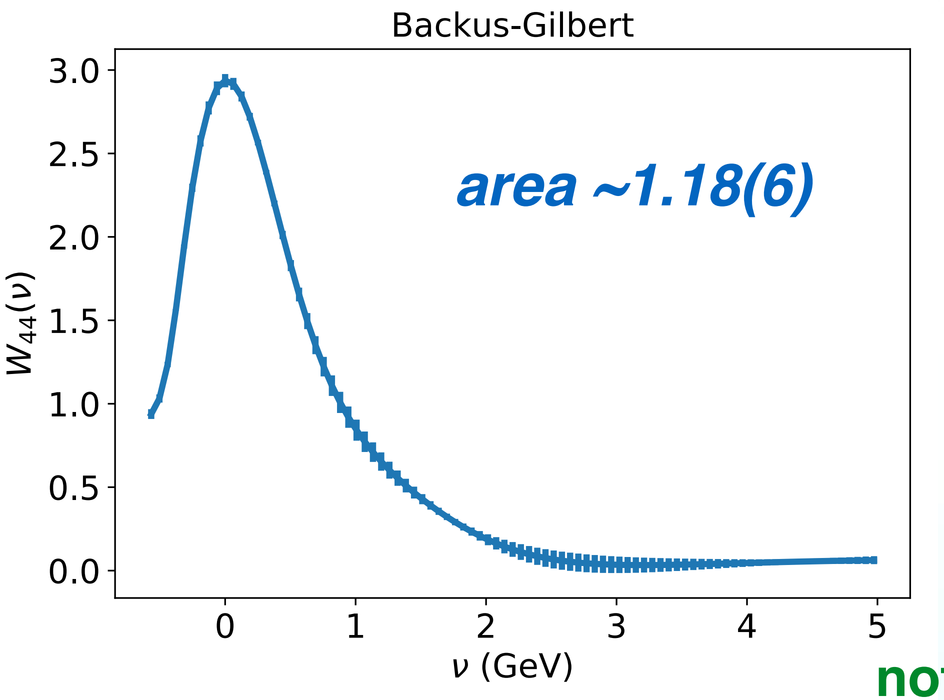
$$\tilde{W}_{44}(\boldsymbol{p}=0,\,\boldsymbol{q}=0,\,\tau) \overset{\tau\to\infty}{=} |\langle N|J_4|N\rangle|^2 e^{-(M_p-M_p)\tau}$$
$$= F_1^2(q^2=0) = g_V^2 = 1$$

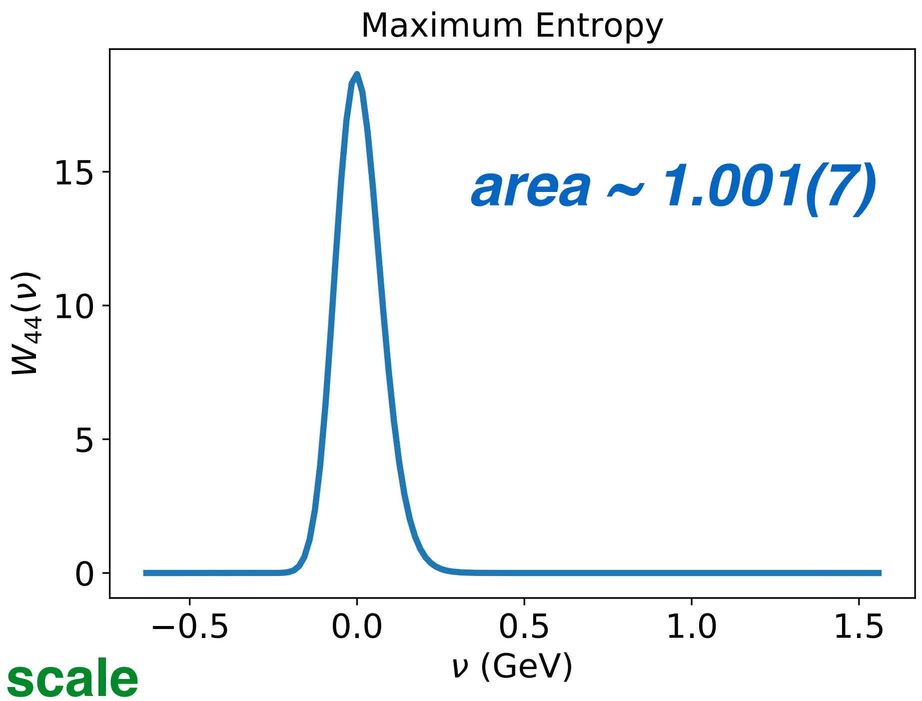


inverse $\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu)e^{-\nu\tau}$

$$W_{44}(q^2,\,\nu) = \delta(q^2+2m_N\nu)\frac{2m_N}{1-q^2/4{m_N}^2}\left(G_E^2(q^2) - \frac{q^2}{4M_N^2}G_M^2(q^2)\right)$$
$$\overset{q^2=0}{=} \delta\nu G_E^2(q^2=0) = \delta\nu g_V^2 = \delta\nu$$

delta function at zero





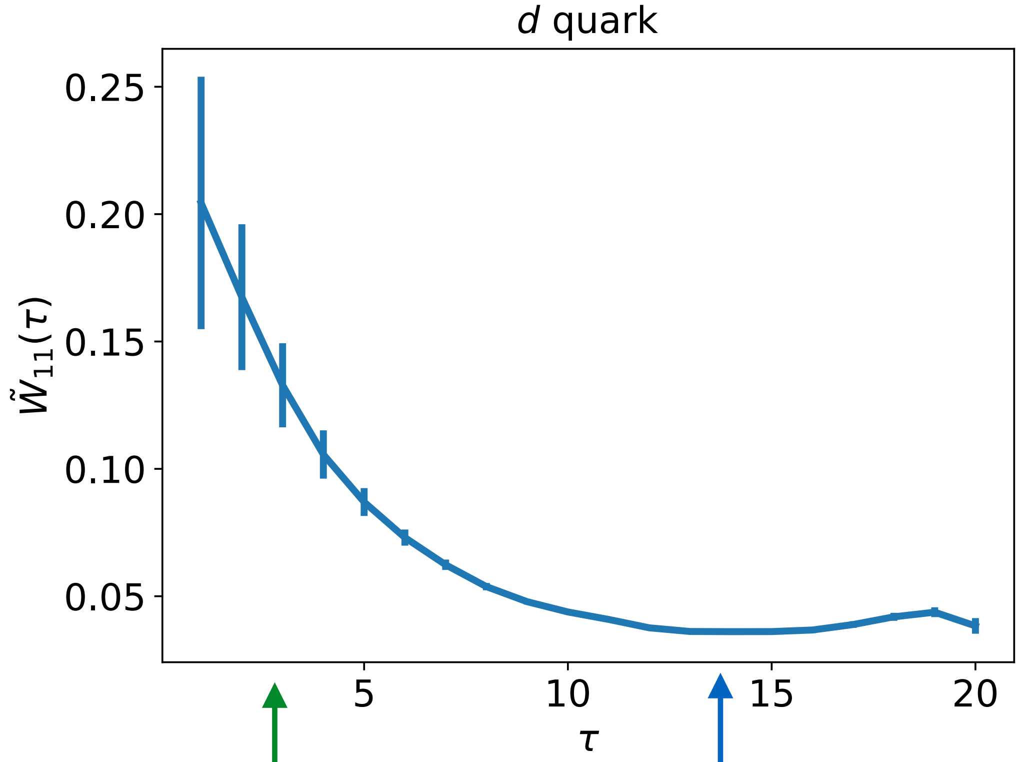
note, different x scale

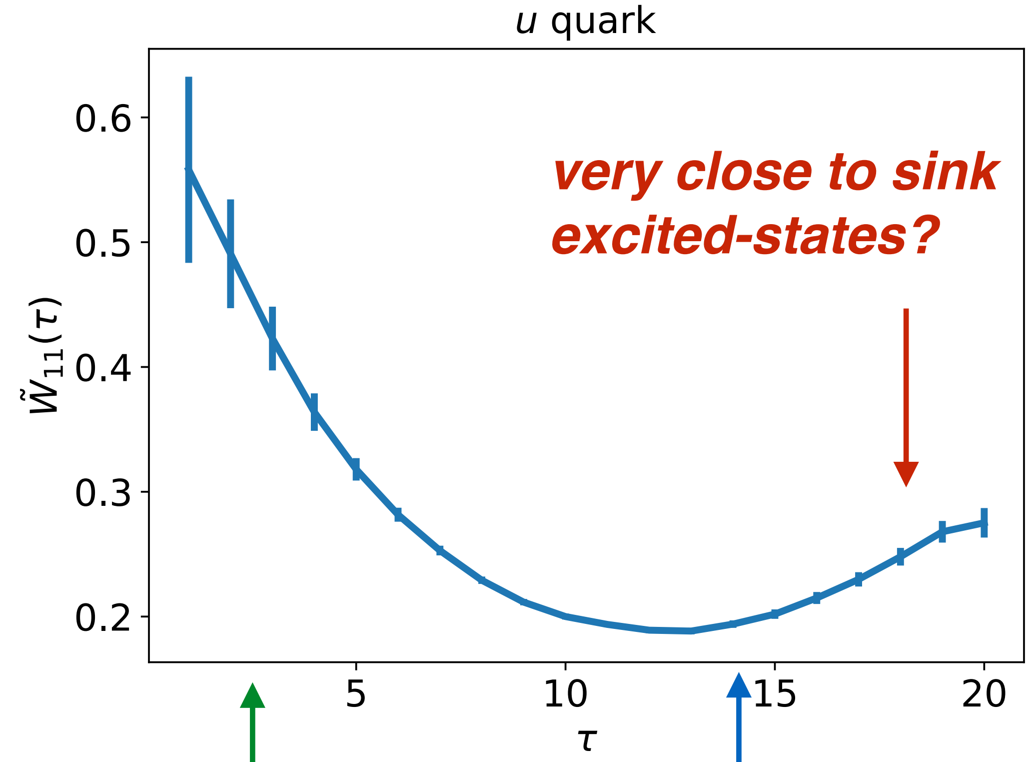
Large momentum transfer

$\boldsymbol{p}$	$\boldsymbol{q}$	E_p	$E_{n=0}$	$\|\boldsymbol{q}\|$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]

$\mu = \nu = 1$ and $p_1 = q_1 = 0$ $\quad W_{11}(\nu) = F_1(x, Q^2)$

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \qquad \boldsymbol{p} + \boldsymbol{q} = -\boldsymbol{p} \qquad E_0 = (m_N^2 + |\boldsymbol{p} + \boldsymbol{q}|^2) = E_p$$





very close to sink excited-states?

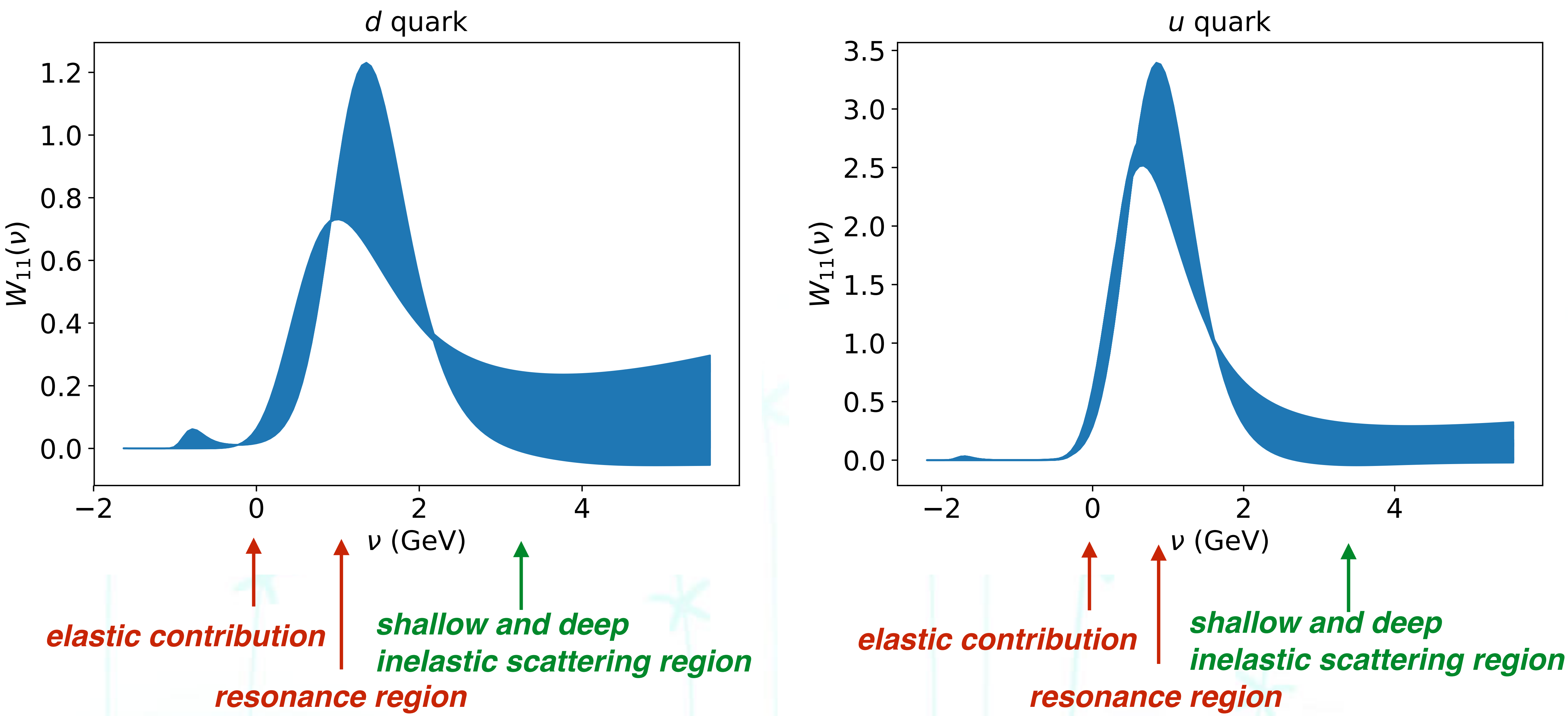
elastic contribution (flat)

higher intermediate-states contribution (exponentially decay)

elastic contribution (flat)

higher intermediate-states contribution (exponentially decay)

Minkowski hadronic tensor (after ME)



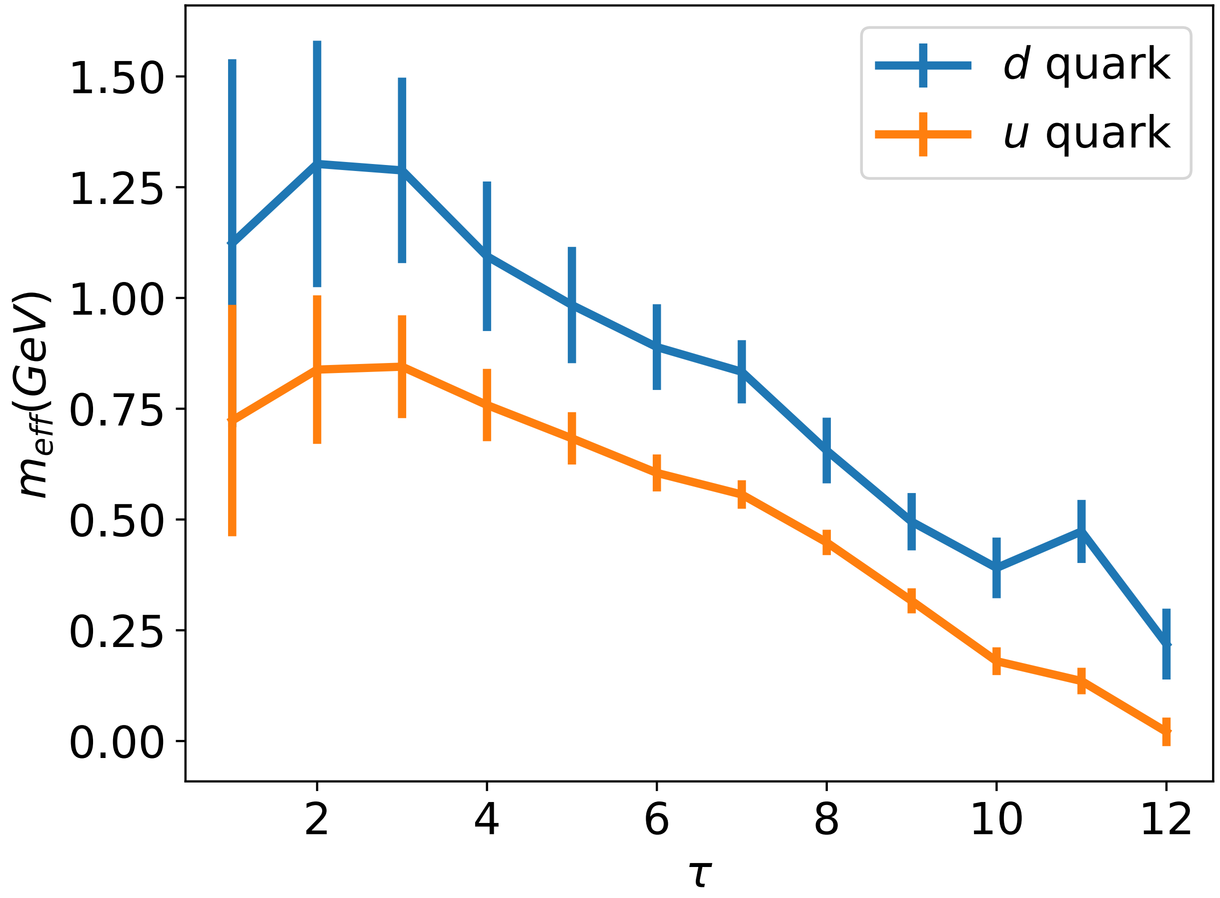
- Elastic contribution is suppressed by the large momentum transfer. $G^2(0) \propto \dfrac{1}{\left(1+\frac{Q_{\rm el}^2}{\Lambda^2}\right)^4}$
  $Q^2 \sim 13\ \mathrm{GeV}^2,\ G^2(0) \sim 10^{-5}$
- RES contribution is large and relatively stable.
- Large error in the SIS and DIS region, no enough constraint from the data

Check the effective mass

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \qquad E_p \sim 2.15 \text{ GeV}$$

one can check the effective mass of the Euclidean hadronic tensor

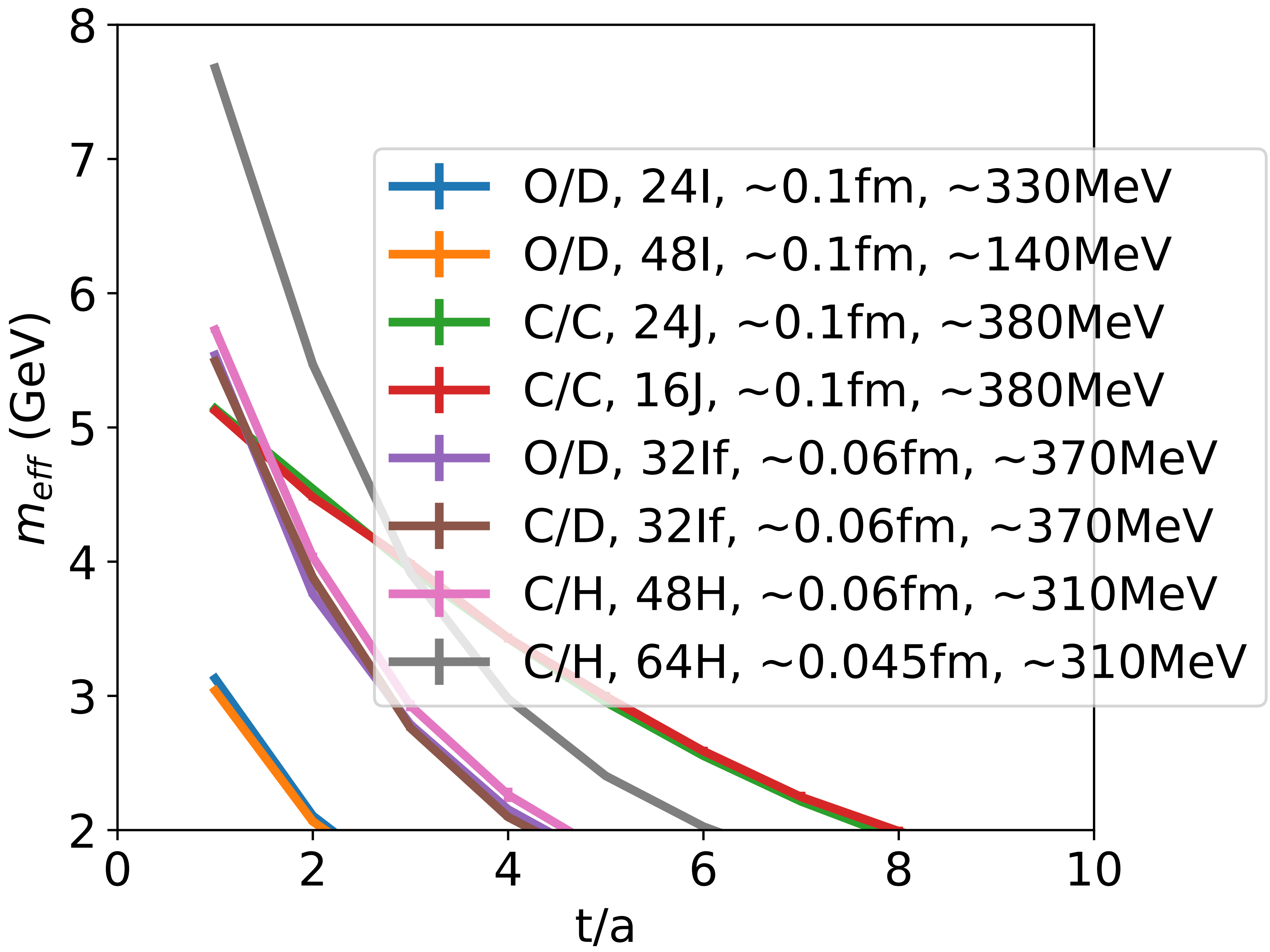
$$m_{\rm eff} = \log[\tilde{W}(\tau)/\tilde{W}(\tau+1)]$$



$\nu \sim E_n - E_p \sim 1$ GeV $\;E_n \sim 3.2$ GeV NOT large enough energy transfer!

lattice artifacts: finite volume (resulting in discrete momenta and discrete spectrum)? finite lattice spacing (an UV cutoff)? and/or unphysical pion mass (unphysical multi-particle states)?

Learn more from two-point functions



- ✦ **It seems how high we can reach is mainly connected to the lattice spacings.**
- ✦ **Other factors are not significant.**
- ✦ **The a~0.045 fm lattice can be a much better choice.**

Summary and outlook

- Calculating the hadronic tensor on the lattice would be helpful to the neutrino experiments and to understand more about the nucleon structure.
- This might be the only lattice approach that can have inclusive results in the ELASTIC, RES and SIS region.
- We can have reasonable results for the elastic contributions.
- We find that the lattice spacing plays an important role to reach higher excited states (large energy transfers).
- We are working on lattices with smaller lattice spacings to have better results.
- Everything is still in its early stage but the approach has great potential. More applications.

Thank you for your attention!