Gluon quasi-PDF

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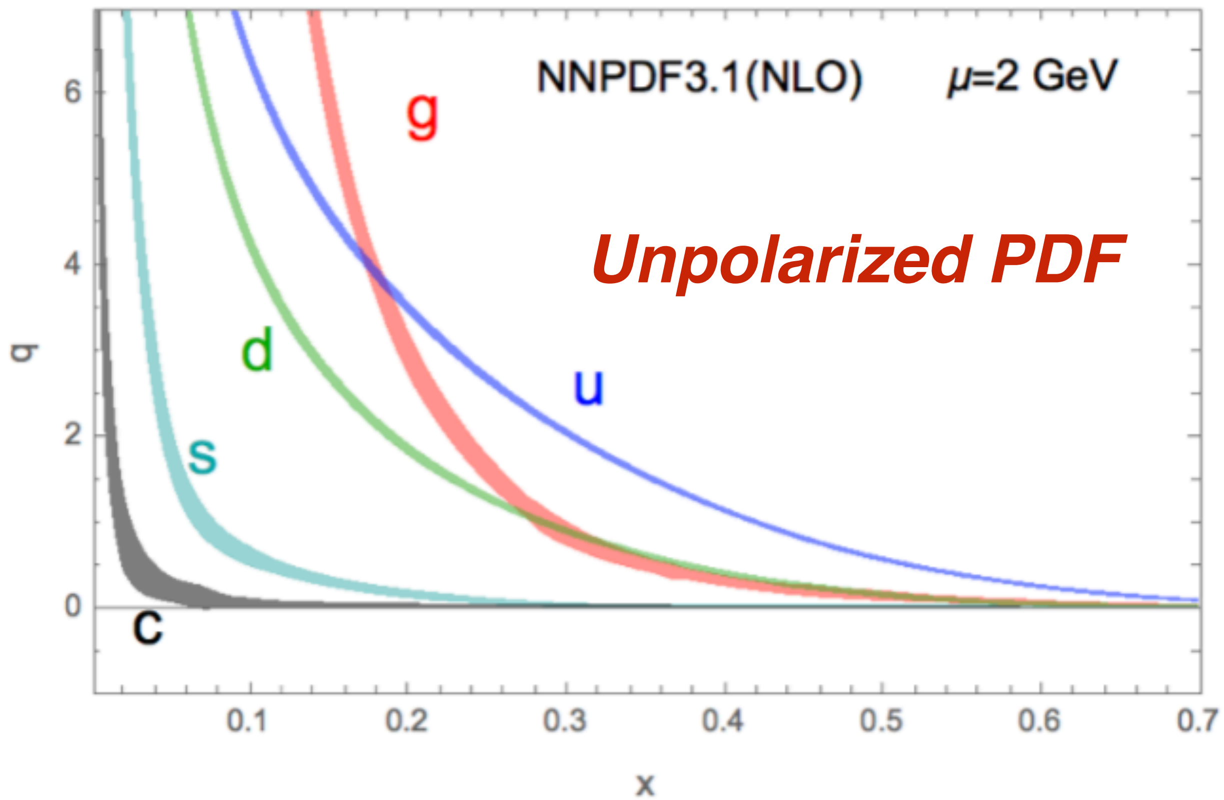
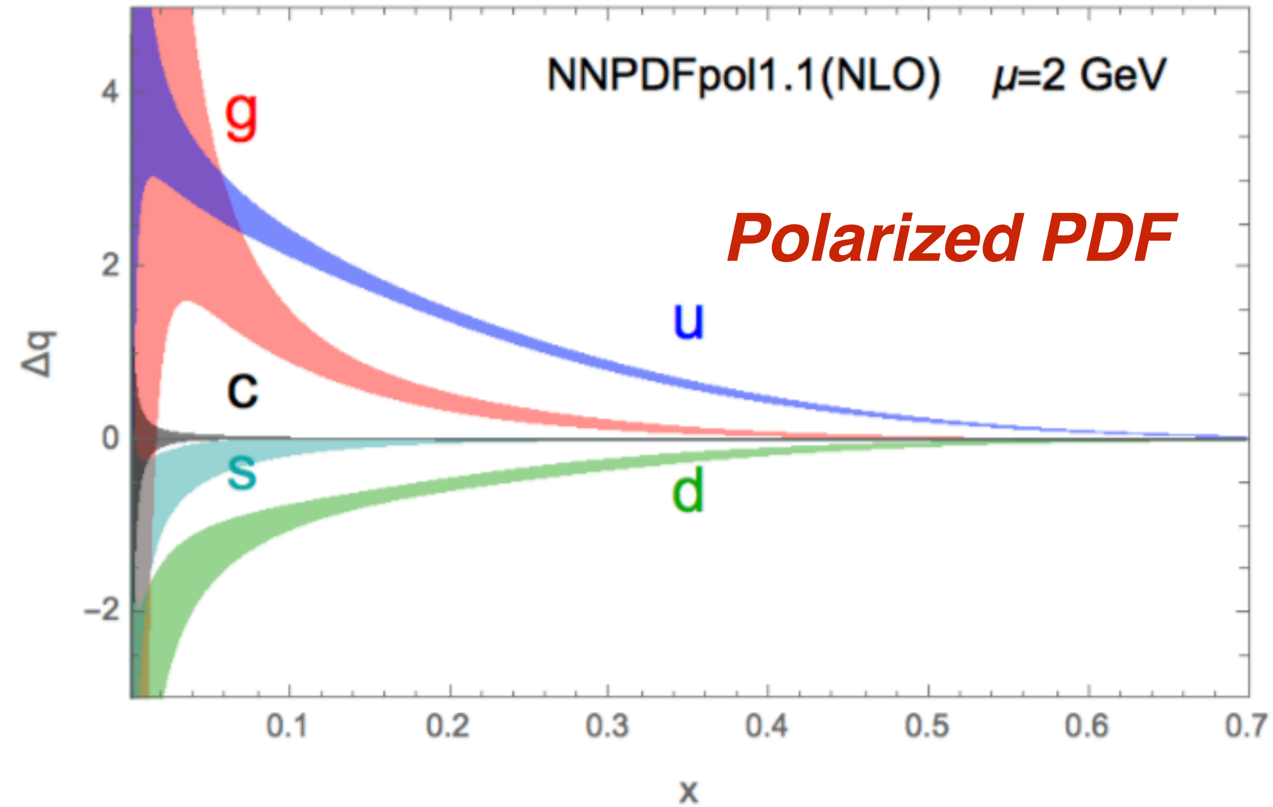
Less known part
of the parton distribution function

R. D. Ball, et al. (NNPDF), EPJC77, 663 (2017), 1706.00428



E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539

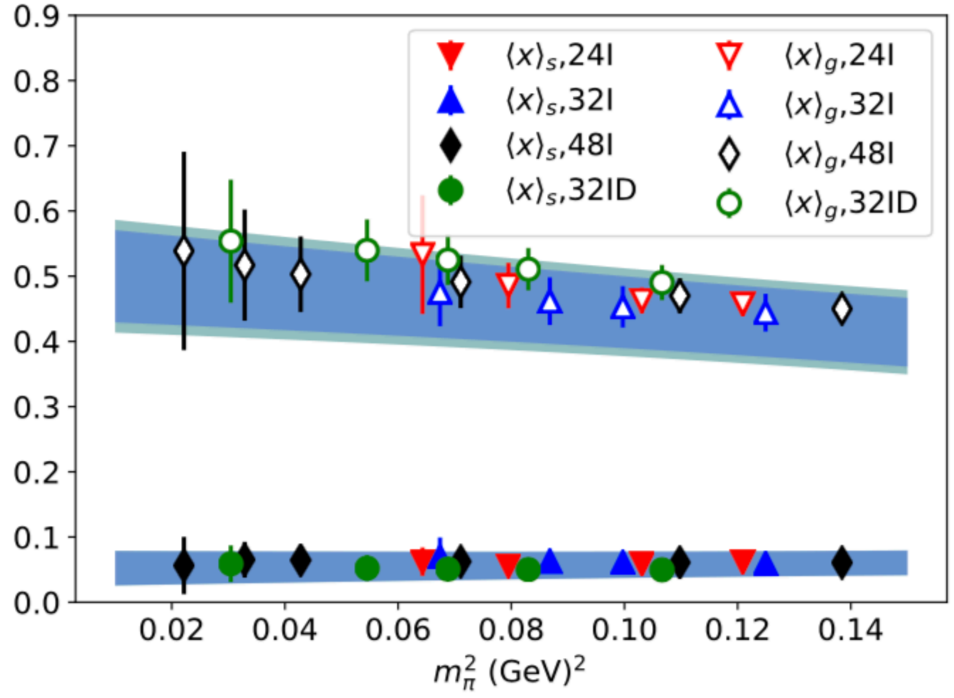


- If we consider the first-moment of the unpolarized PDF and the zeroth-moment of the polarized one, the values of the gluon case are comparable with the quark case.
- But their x-dependence are very different.
- **The gluon PDF is much less unknown from the experiment.**

Gluon unpolarized PDF from Lattice QCD

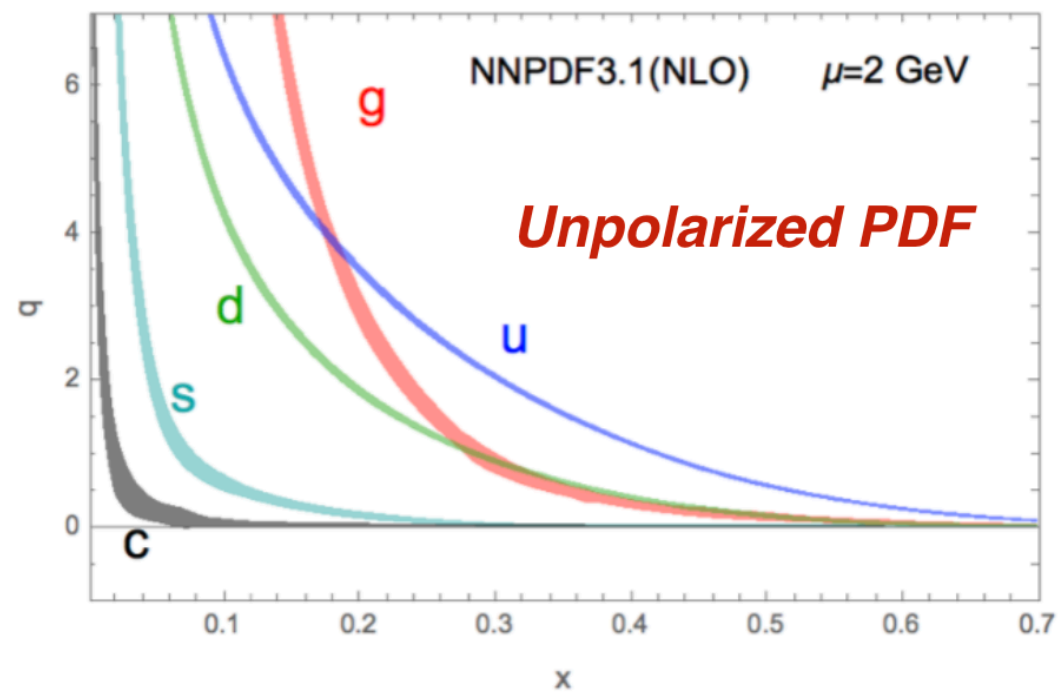
Beyond the first moment

YBY, J. Liang, et.al (χQCD), PRL121, 212001 (2018), 1808.08677



- From the first moment calculation, the central value is insensitive to the pion mass dependence but the uncertainty is NOT;
- Start from much heavier quark mass can be **a good starting point**.

- The value of g(x) at small x is much larger than any quark PDF;
- Give the first moment is roughly the same, the higher moments of g(x) will be smaller than those of q(x) and then hard to be calculated precisely.

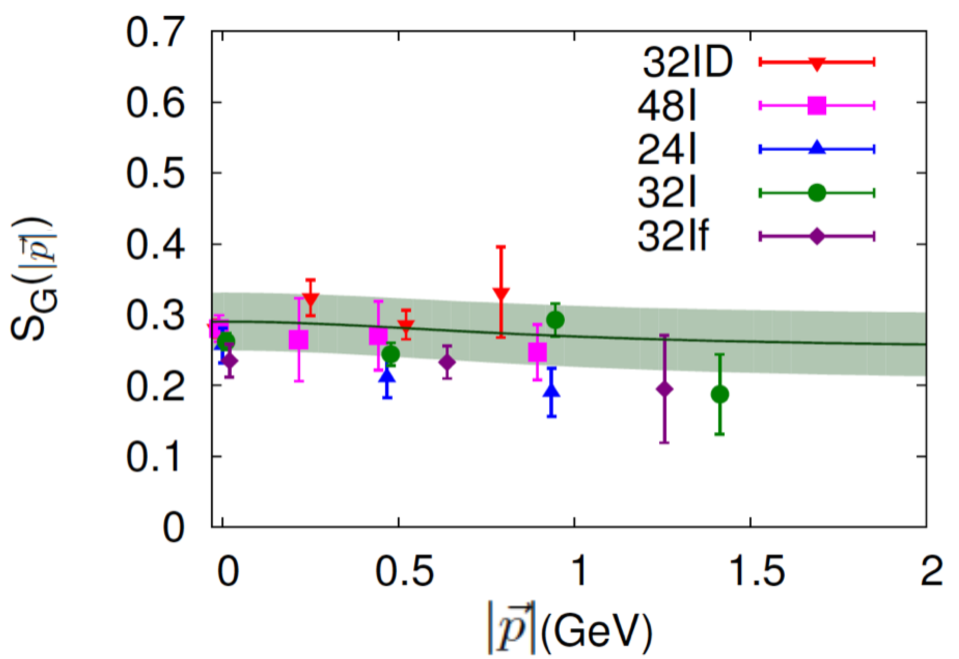


R. D. Ball, et al. (NNPDF), EPJC77, 663 (2017), 1706.00428

Gluon unpolarized PDF from Lattice QCD

Revisit the zeroth moment

YBY, R. Sufian, et al., (χQCD), PRL118, 042001(2017), 1609.05937

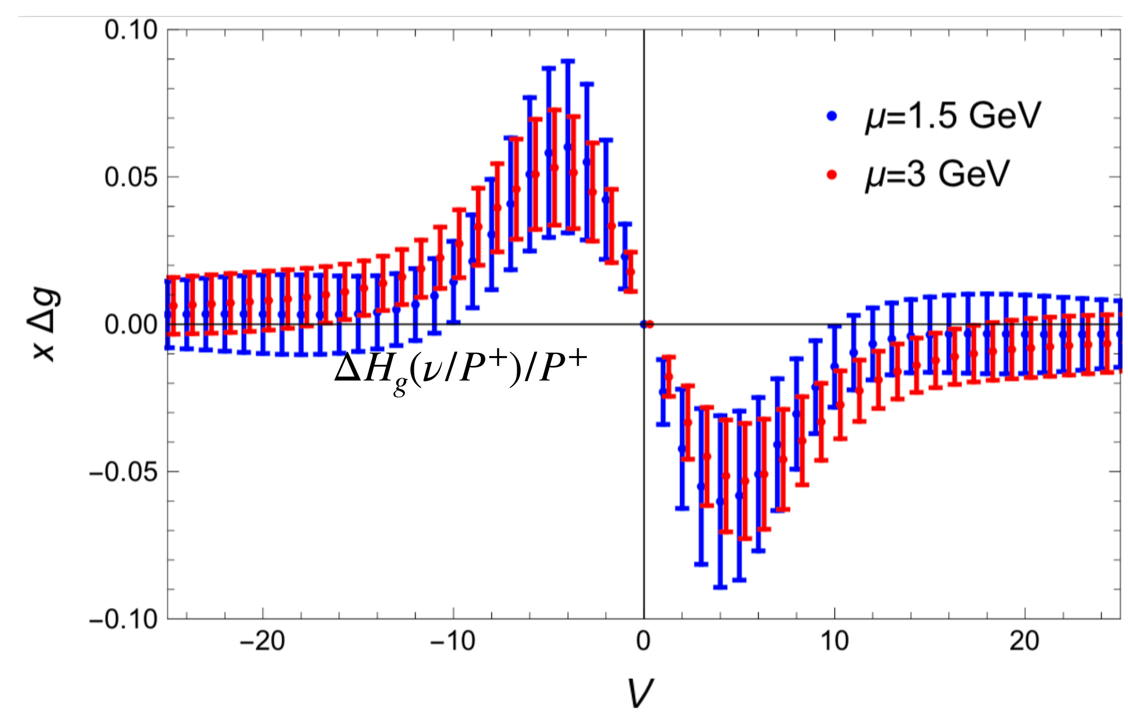


Gluon spin under the Coulomb gauge

- *Sizable contribution to the proton spin;*
- *Convergence of the LaMET matching is poor at 1-loop level;*
- *Gauge dependence should be checked with the calculation under the other gauge conditions.*

Figure from Yu-Sheng Liu. Based on

E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539



Gluon spin can also be obtained through the following gauge invariant definition:

$$\Delta\tilde{g} = \int_0^\infty dz \Delta\tilde{H}_g(z)|_{P_z\to\infty} = \int_0^\infty dz \Delta H_g(z) + \mathcal{O}(\alpha_s) = \Delta g + \mathcal{O}(\alpha_s)\,.$$

$$\Delta\tilde{H}_g(z) = \sum_{i=x,y} \langle PS|F_{iz,a}(z)(e^{\int_0^z igA_z(z')dz'})_{ab}\tilde{F}_{iz,b}(0)|PS\rangle$$

$$\Delta H_g(z) = \sum_{i=x,y} \langle PS|F_{+\mu,a}(\xi^-)(e^{\int_0^{\xi^-} igA^+(\eta^-)d\eta^-})_{ab}\tilde{F}^{+,b}_{\mu}(0)|PS\rangle = \int dx P^+ x e^{ix\xi^- P^+} g(x)$$

Outline

- ***Theoretical preparation***
- *Simulation results*
- *Outlook*

Gluon PDF and its moments

The gluon PDF is defined by:

$$g(x,\mu) = \int \frac{\mathrm{d}\xi^-}{\pi x} e^{-ix\xi^- P^+} \langle P | F_\mu^{+}(\xi^-) U(\xi^-,0) F^{\mu +}(0) | P \rangle\,,$$

And its odd moments can be defined through local operators, likes the first moment:

$$\begin{aligned}\langle x \rangle_g &\equiv \int_0^1 x\ g(x)\mathrm{d}x = \frac{1}{P^+}\langle P | F_\mu^{+}(0) F^{\mu +}(0) | P \rangle \\ &= \frac{1}{P_z}\langle P | \overline{T}^{tz}(0) | P \rangle \\ &= \frac{P_0 \langle P | \overline{T}^{zz}(0) | P \rangle}{\frac{1}{4}P_0^2 + \frac{3}{4}P_z^2} = \frac{P_0 \langle P | \overline{T}^{tt}(0) | P \rangle}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2}\,,\end{aligned}$$

All the operators belong to the traceless part of the gauge EMT.

Gluon quasi-PDF

So the gluon quasi-PDF can be defined through the quasi-PDF matrix elements $\tilde{H}_0$ **:**

$$\tilde{g}(x, P_z^2, \mu) = \int \frac{\mathrm{d}z}{\pi x} e^{-ixzP_z} \tilde{H}_0^R(z, P_z, \mu),$$

where $\tilde{H}_0(z, P_z) = \langle P|\mathcal{O}_0(z)|P\rangle$ *and* $\mathcal{O}_0$ *is defined by*

$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t{}_\mu, F^{\mu t}; z) - \frac{1}{4} g^{tt} \mathcal{O}(F^\mu{}_\nu, F^\nu{}_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2},$$

or the other alternative choice:

$$\mathcal{O}_1(z) \equiv \frac{1}{P_z} \mathcal{O}(F_{t\mu}, F_{z\mu}; z),$$

$$\mathcal{O}_2(z) \equiv \frac{P_0\left(\mathcal{O}(F_{z\mu}, F_{\mu z}; z) - \frac{1}{4} g^{zz} \mathcal{O}(F_{\mu\nu}, F_{\nu\mu}; z)\right)}{\frac{1}{4}P_0^2 + \frac{3}{4}P_z^2},$$

Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), 1808.02077

all of them provide ***the same first moment*** *of the gluon PDF, while only* $\mathcal{O}_1$ *can be multiplicative renormalizable.*

Renormalization property of the gluon quasi-PDF operators

Jian-Hui Zhang, et.al, PRL122, 142001 (2019), 1808.10824

Zheng-Yang Li, et.al, PRL122, 062002 (2019), 1809.01836

- Suppose the Wilson link is along the **z** direction:
- The renormalization factor of the operator $F_{\mu\nu}(z)U(z,0)F_{\rho\sigma}(0)$ takes the form:

$$Z_{\mu\nu}Z_{\rho\sigma}e^{\alpha_s C_A \delta m \frac{z}{a}},$$

where $Z_{z\alpha} = Z_1, Z_{\alpha\beta} = Z_2, (\alpha, \beta \neq z)$.

- Since Z_1!=Z_2, not all the operators defined in previous slide can be renormalized multiplicatively:

$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t{}_\mu, F^{\mu t}; z) - \frac{1}{4}g^{tt}\mathcal{O}(F^\mu{}_\nu, F^\nu{}_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2}, \quad \times$$

$$\mathcal{O}_1(z) \equiv \frac{1}{P_z}\mathcal{O}(F_{t\mu}, F_{z\mu}; z), \quad \checkmark$$

$$\mathcal{O}_2(z) \equiv \frac{P_0\left(\mathcal{O}(F_{z\mu}, F_{\mu z}; z) - \frac{1}{4}g^{zz}\mathcal{O}(F_{\mu\nu}, F_{\nu\mu}; z)\right)}{\frac{1}{4}P_0^2 + \frac{3}{4}P_z^2}, \quad \times$$

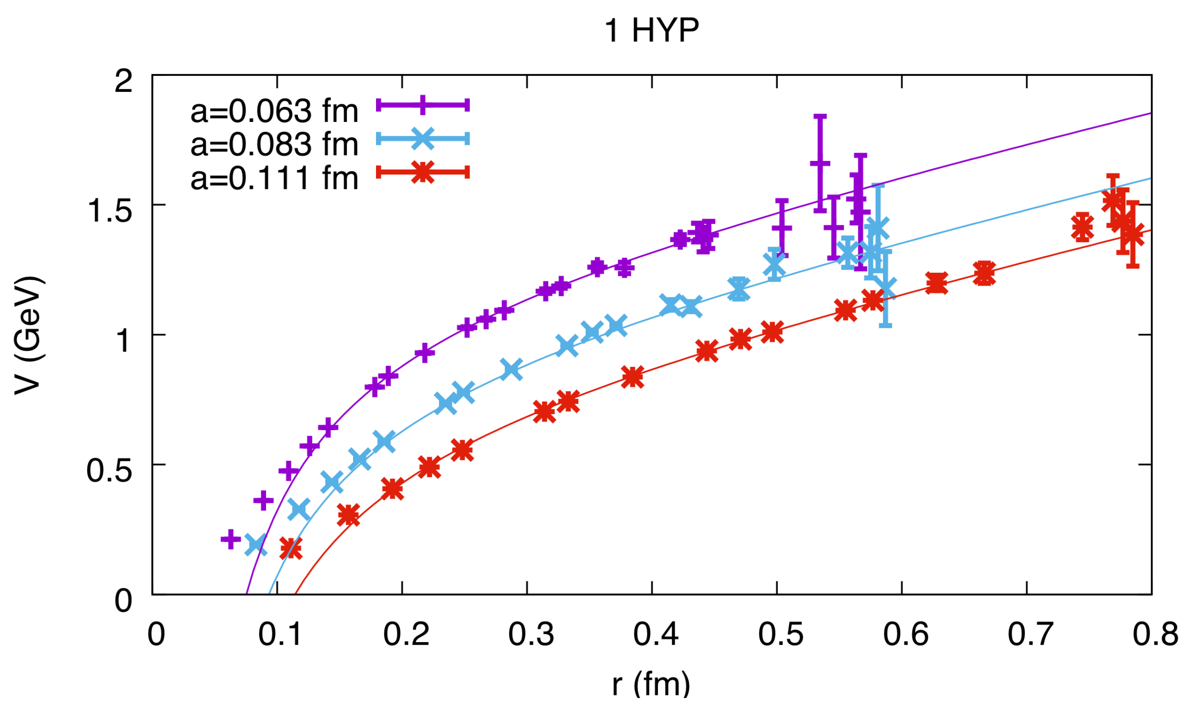
while the linear divergence of them are still the same.

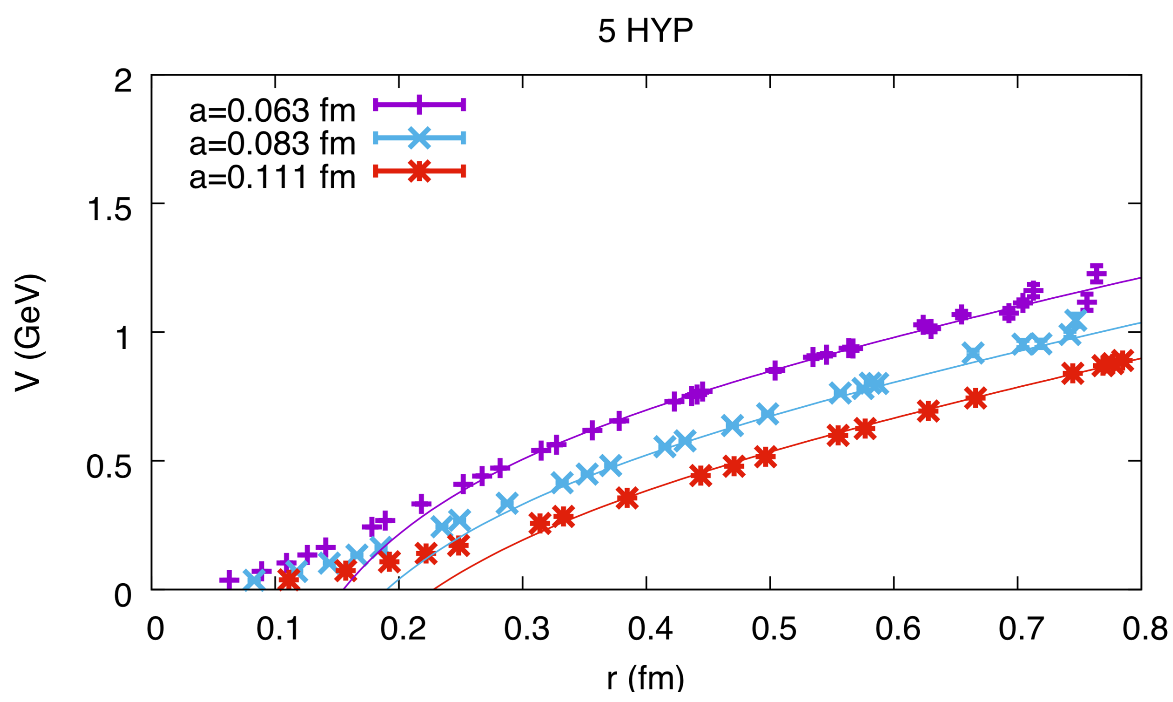
δm Renormalization from the statical potential

- In principle, the δm in the renormalization factor $Z_{\mu\nu}Z_{\rho\sigma}e^{\alpha_s C_A \delta m \frac{z}{a}}$ can be obtained through the calculation of the statical potential of the heavy quark:

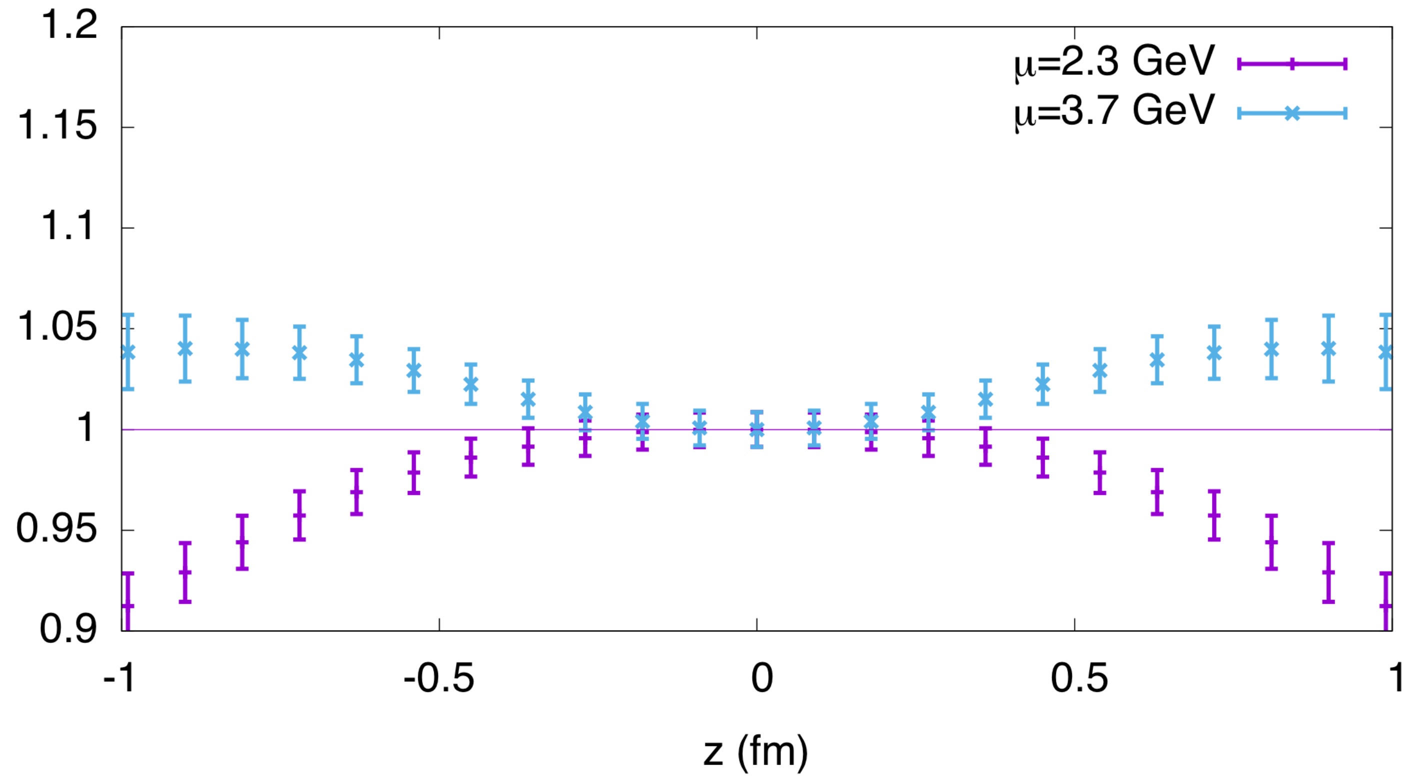
$$V(r,a) = \frac{e}{r} + V_0 + \alpha_s C_F \frac{r}{a} + \sigma r$$

- But in practical, the above formula can not describe the entire curve well, after the HYP smearing applied on the gauge field.
- Even though, the linear divergence effect can be caught through the fit with relatively large z.





The “ratio renormalization”



$$\frac{\langle N|\bar{q}(z)\gamma_t U(z,0)q(0)|N\rangle}{\langle q|\bar{q}(z)\gamma_t U(z,0)q(0)|q\rangle}\Big|_{P_z=p_z=0}$$

Example of the RI/MOM renormalized quark ME with $P_Z=p_Z=0$:

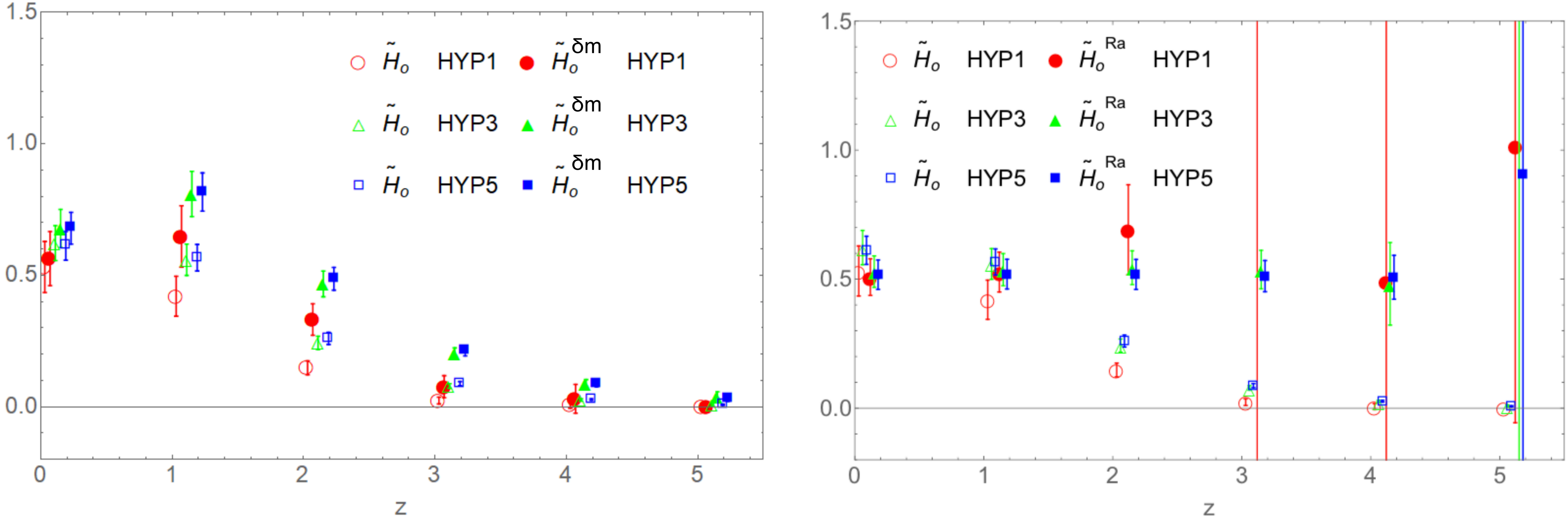
- *Almost a constant in the region $|z|<0.5$ fm up to 5%-10% deviation;*
- *The deviation can be a scale dependence as it varies with different μ.*

Thus if we define the “ratio renormalized” matrix element likes the following,

$$\tilde{H}_0^{Ra}(z,P_z,\mu)=\frac{\tilde{H}_0^{\overline{\rm MS}}(0,0,\mu)}{\tilde{H}_0(z,0)}\tilde{H}_0(z,P_z),$$

then the linear divergence of the gluon quasi-PDF operator can be removed.

“δm renormalized” and “ratio renormalized” matrix elements



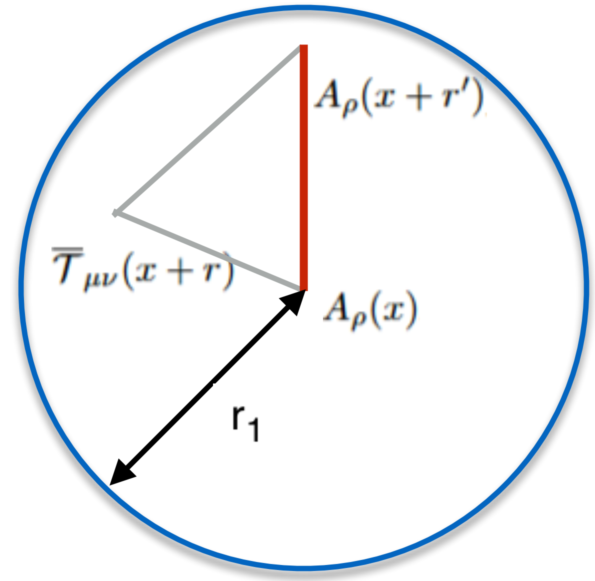
- *The Pz=0.46 GeV case with 1/3/5 steps of HYP-smearing. a=0.11 fm.*
- *HYP-smearing itself can not remove all the linear divergence.*
- *δm is not so helpful either but the ratio can do a better job?*
- *The RI/MOM renormalization?*

Gluon RI/MOM with CDER

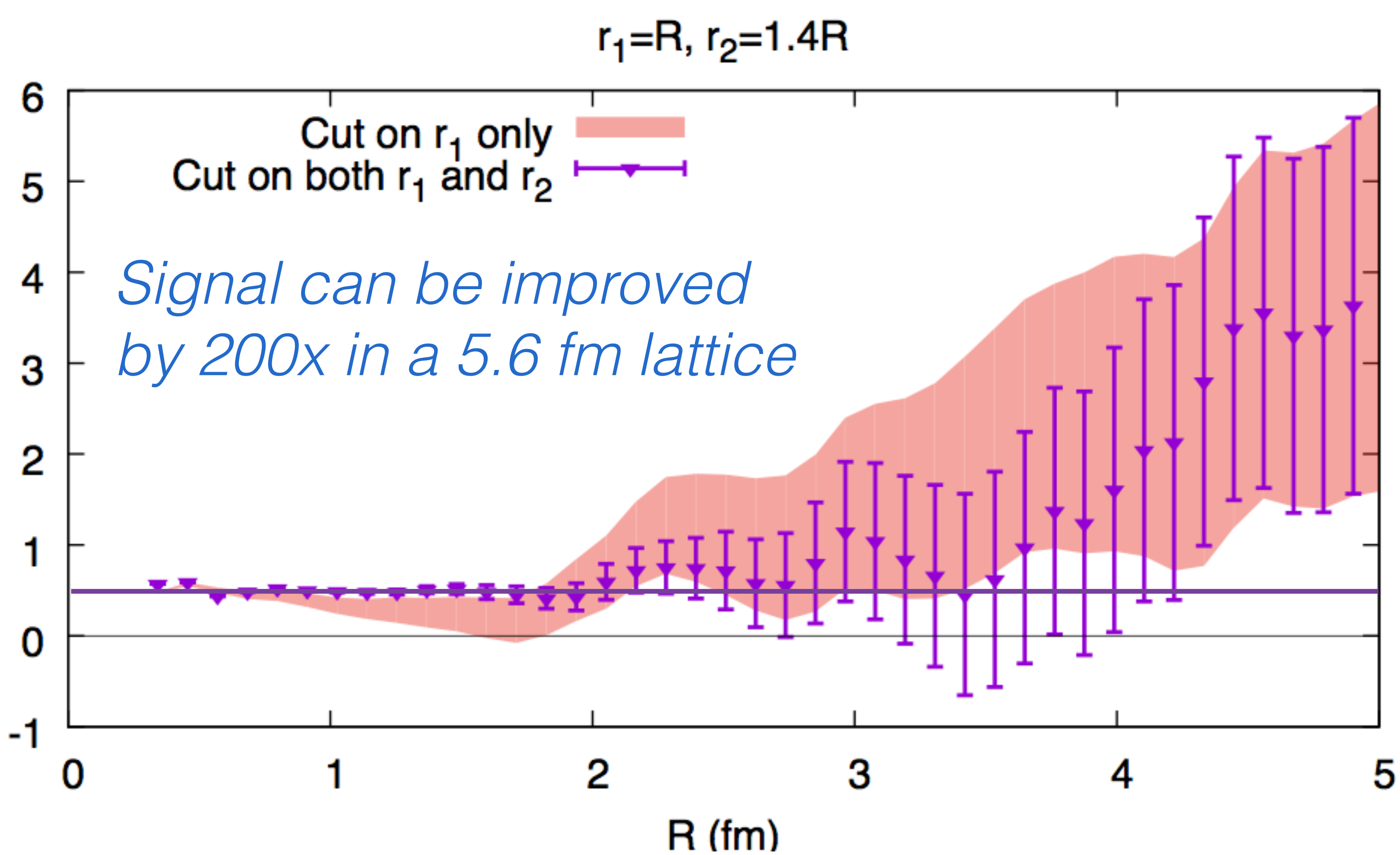
K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

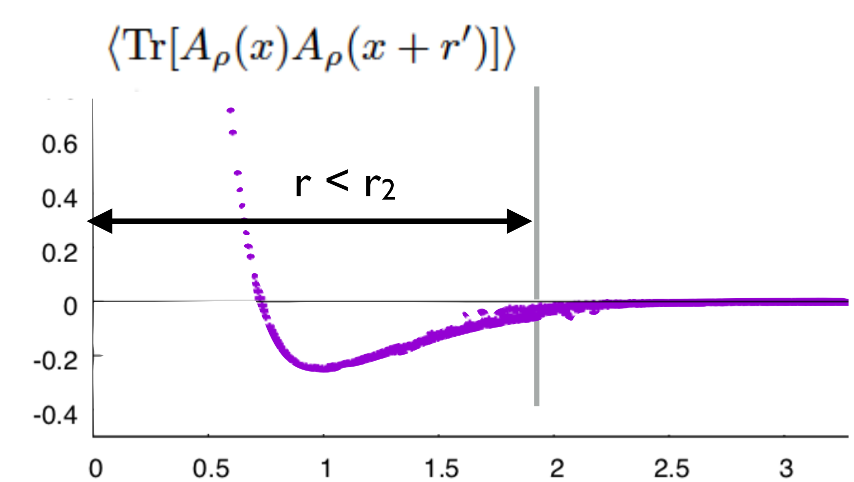
$$Z_T^{-1}(\mu_R^2) = \left.\frac{p^2\langle(\overline{T}_{\mu\mu} - \overline{T}_{\nu\nu})\,\mathrm{Tr}[A_\rho(p)A_\rho(-p)]\rangle}{2p_\mu^2\langle \mathrm{Tr}[A_\rho(p)A_\rho(-p)]\rangle}\right|_{\substack{p^2=\mu_R^2,\\ \rho\neq\mu\neq\nu,\\ p_\rho=0,\\ p_\nu=0}}$$

$$C_3(p) = \left\langle \int d^4x\, d^4y\, d^4z\, e^{ip\cdot(x-y)}\overline{T}_{\mu\nu}(z)\,\mathrm{Tr}[A_\rho(x)A_\rho(y)] \right\rangle$$
$$\simeq \left\langle \int_{|r|<r_1} d^4r \int_{|r'|<r_2} d^4r' \int d^4x\, e^{ip\cdot r'}\overline{T}_{\mu\nu}(x+r)\,\mathrm{Tr}[A_\rho(x)A_\rho(x+r')] \right\rangle$$



r_1: Cut on the $T_{\mu\nu}$ - A_ρ correlation



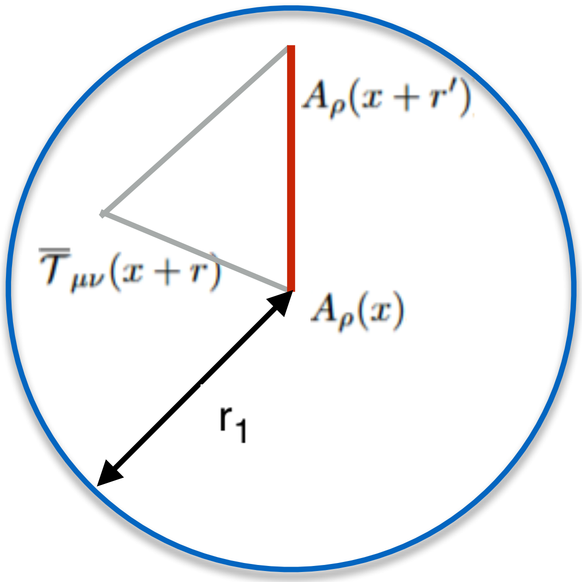


r_2: Cut on the A_ρ - A_ρ correlation

YBY, et. al., (χQCD), PRD 98, 074506 (208), 1805.00531

Ri/MOM renormalization for the gluon quasi-PDF operator

- *Similar strategy can be applied to the quasi-PDF operator.*
- *The present statical uncertainty of the RI/MOM renormalization constant for the z=0 case is ~3%*
- *Based on the z-dependence of the relative uncertainty in the hadron matrix element, the uncertainty of the RI/MOM renormalization constant would be* ***over 100%*** *with z~1 fm.*



$$F_{t\mu}(0)F_{z\mu}(0) \to F_{t\mu}(\frac{-z}{2})U(\frac{-z}{2},\frac{z}{2})F_{z\mu}(\frac{z}{2})$$

Thus the renormalization constant would be safely approximated by **proper** hadronic ratio with **corresponding perturbative matching**, up to $O(z^2)$ corrections.

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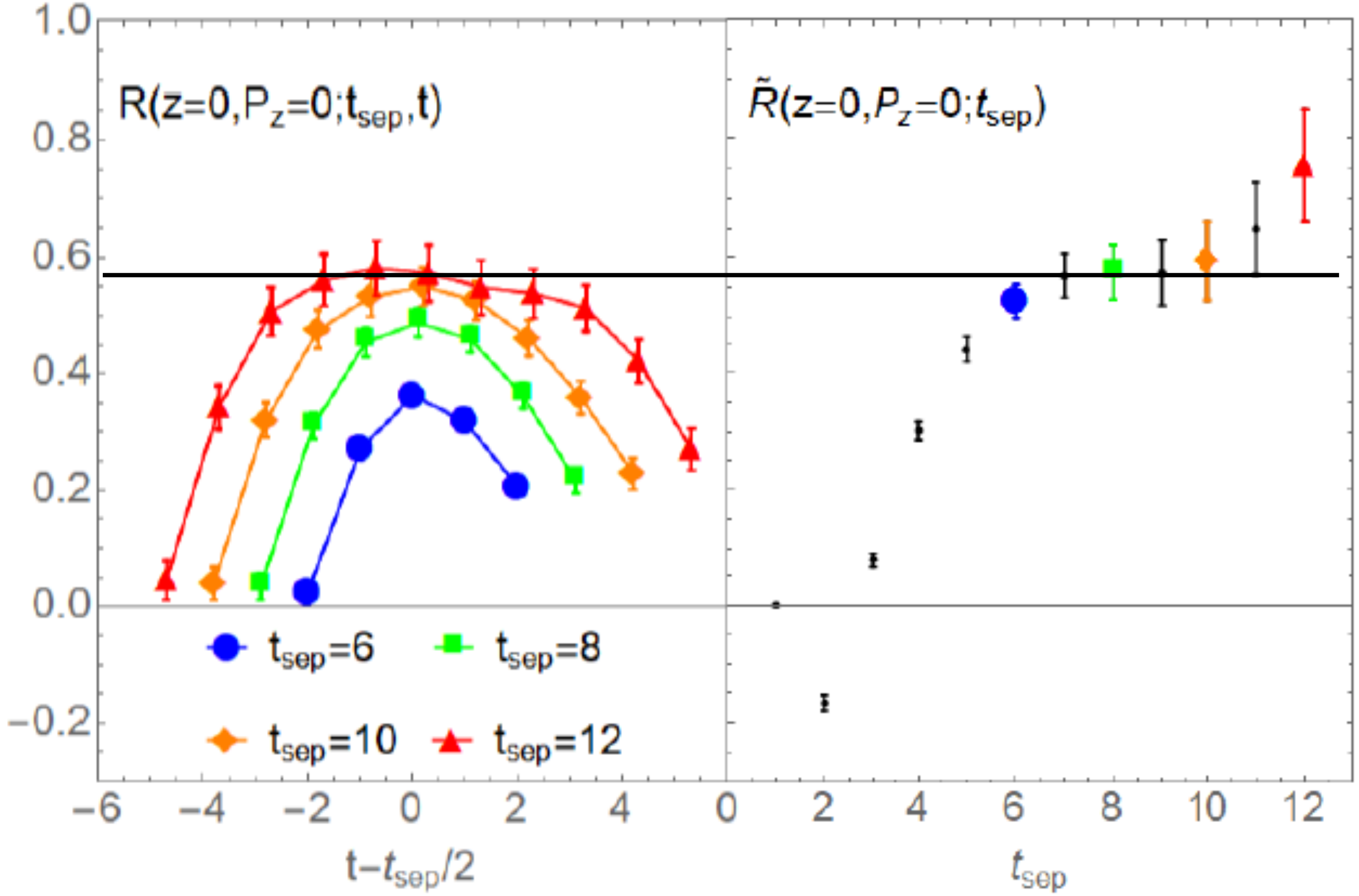
Bare glue momentum fraction

Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), 1808.02077



- *The z=0 case:*
- *Corresponds to the bare glue momentum fraction;*
- *Z=0.9(1) based on NPR;*
- *Two treatments on the excited state contamination provide consistent result.*

$$\tilde{R}(z,P_z;t_{\rm sep}) = \sum_{0<t<t_{\rm sep}} R(z,P_z;t_{\rm sep},t) - \sum_{0<t<t_{\rm sep}-1} R(z,P_z;t_{\rm sep}-1,t) = \tilde{H}_0(z,P_z) + \mathcal{O}(e^{\Delta m t_{\rm sep}}),$$

where

$$R(z,P_z;t_{\rm sep},t) \equiv \frac{E\langle 0|\Gamma^e \int {\rm d}^3y e^{-iy\cdot P}\chi(\vec{y},t_{\rm sep})\mathcal{O}_0(z;t)\chi(\vec{0},0)|0\rangle}{(\frac{3}{4}E^2+\frac{1}{4}P_z^2)\langle 0|\Gamma^e \int {\rm d}^3y e^{-iy_3P_3}\chi(\vec{y},t_{\rm sep})\chi(\vec{0},0)|0\rangle}$$

Nucleon momentum dependence of the ME with kinds of the operators

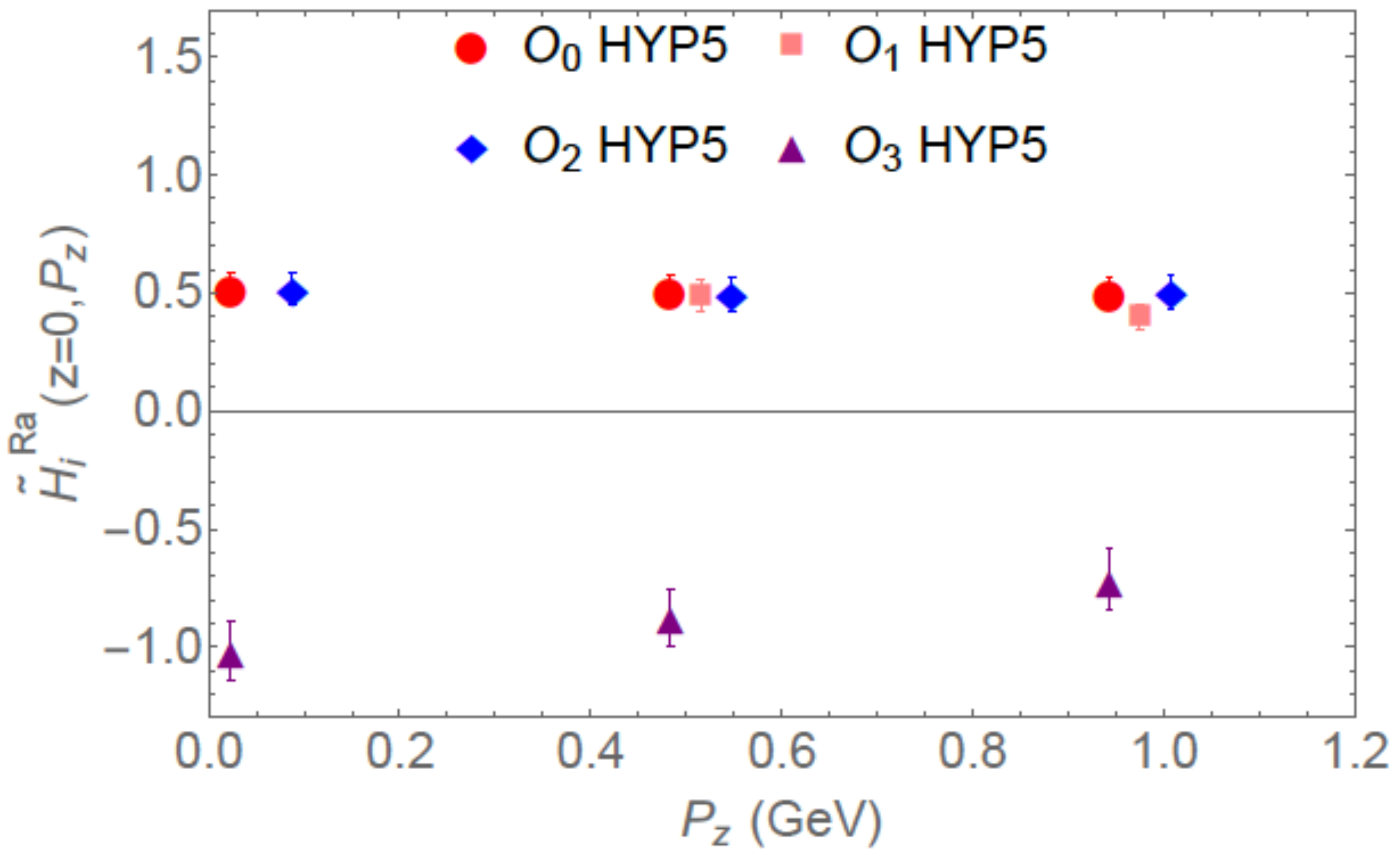
Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), 1808.02077

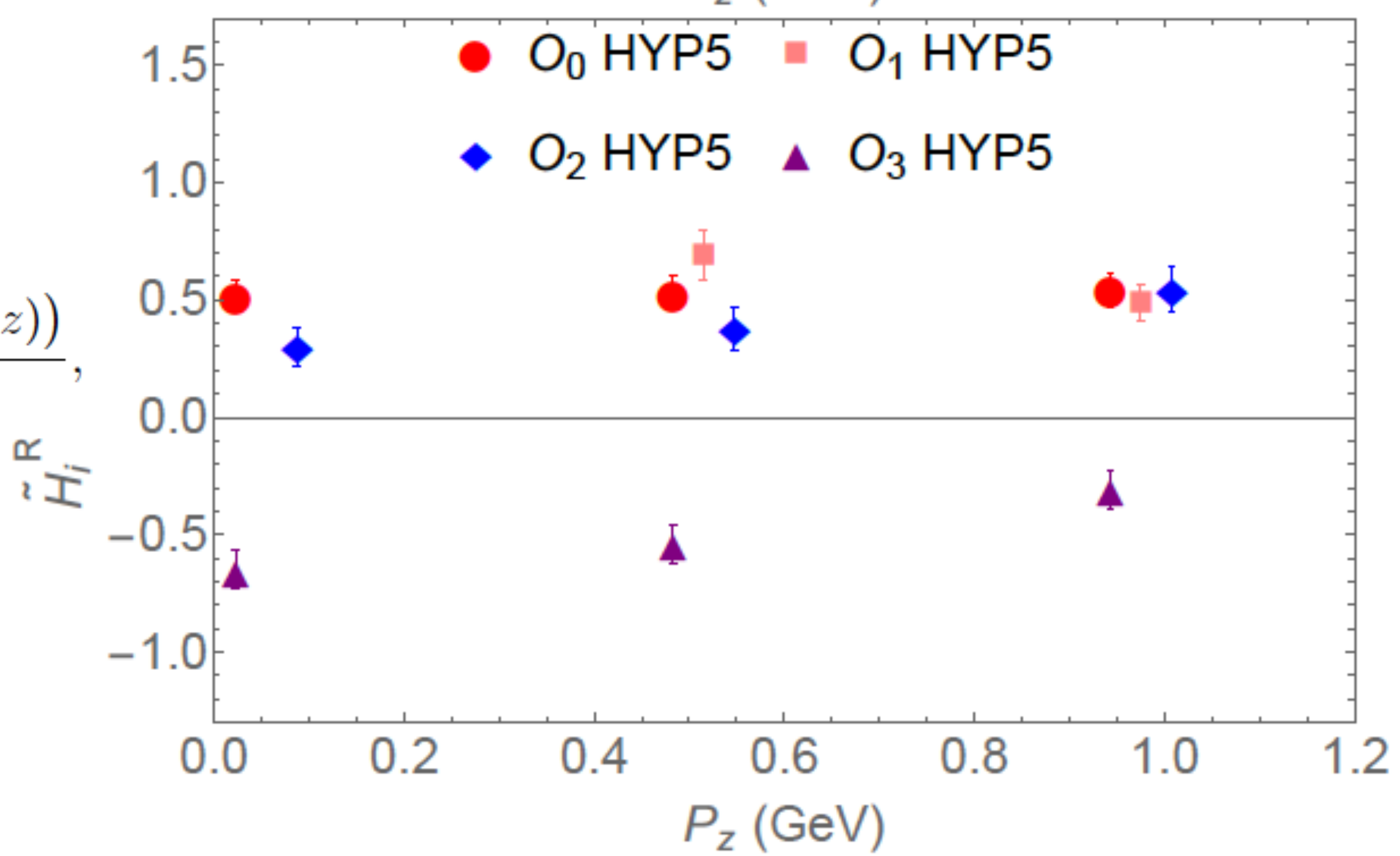
$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t_\mu, F^{\mu t}; z) - \frac{1}{4} g^{tt} \mathcal{O}(F^\mu_\nu, F^\nu_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2},$$

$$\mathcal{O}_1(z) \equiv \frac{1}{P_z}\mathcal{O}(F^t_\mu, F^{z\mu}; z),$$

$$\mathcal{O}_2(z) \equiv \frac{P_0\left(\mathcal{O}(F^z_\mu, F^{\mu z}; z) - \frac{1}{4} g^{zz} \mathcal{O}(F^\mu_\nu, F^\nu_\mu; z)\right)}{\frac{1}{4}P_0^2 + \frac{3}{4}P_z^2},$$

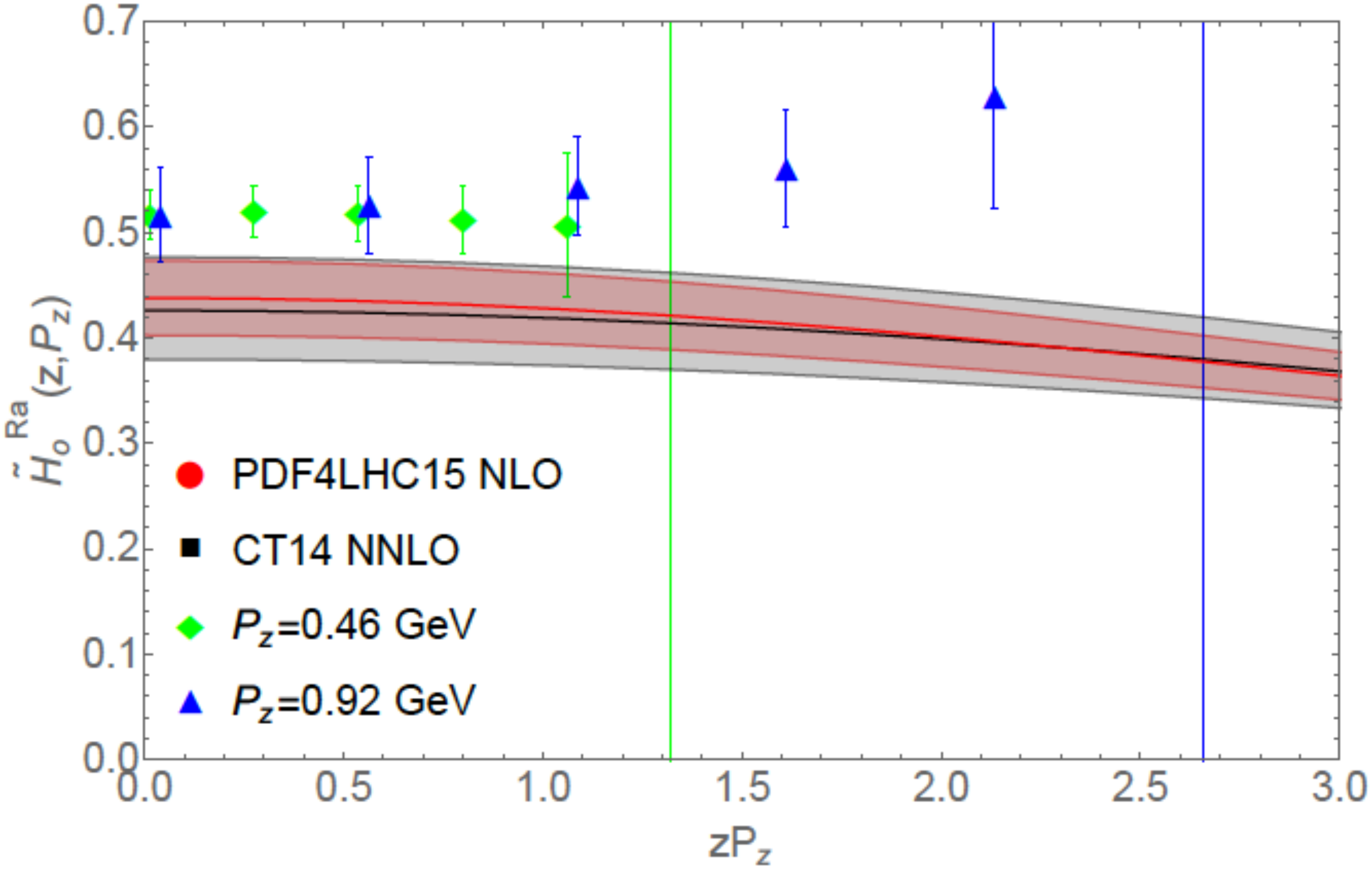
$$\mathcal{O}_3(z) \equiv \frac{1}{P_0}\mathcal{O}(F^z_\mu, F^{z\mu}; z)$$





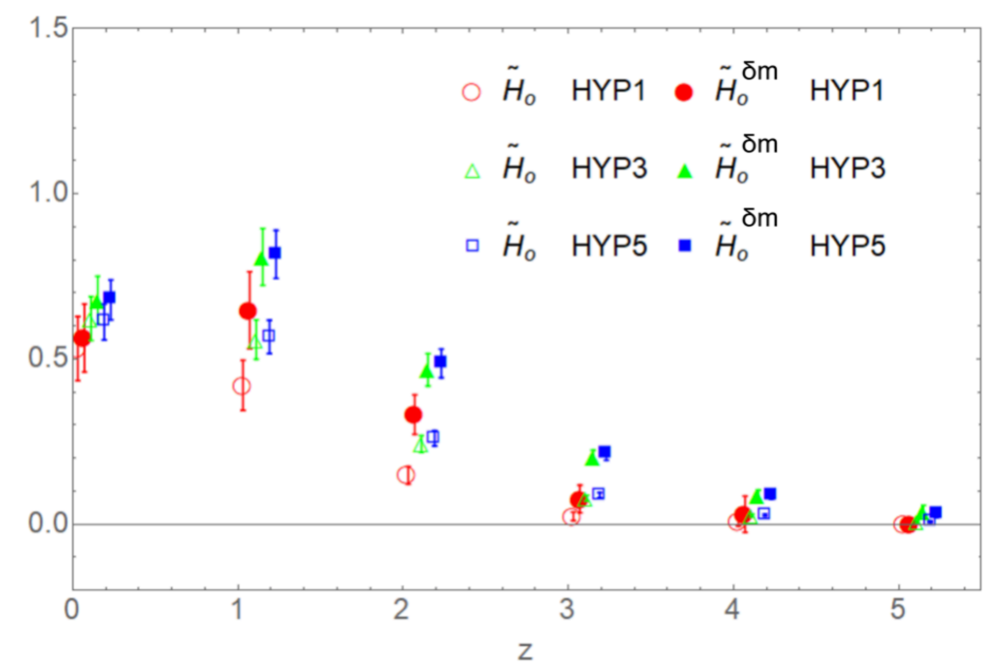
FT of the PDF vs. that of quasi-PDF

Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), 1808.02077



$$H(\tau,\mu) = \frac{1}{2}\int_{-1}^{1} e^{i\tau x} x g(x,\mu) \mathrm{d}x$$

- **Nucleon** with 678 MeV pion mass, 5HYP glue operators, "ratio renormalization"
- **Unpolarized case**, WITHOUT matching and mixing
- Much larger **momentum** and **statistics** are required.



- ***δm is not so helpful either but the ratio can do a better job***

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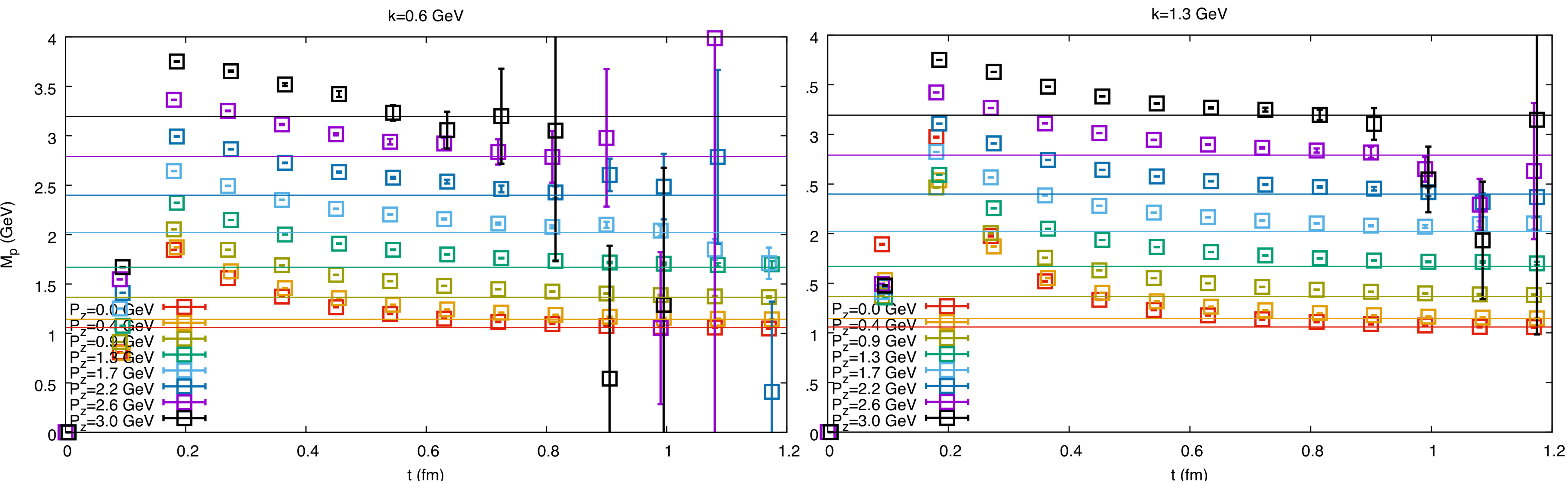
Momentum smearing

G. S. Bali, et.al, PRD93, 094515 (2016), 1602.05525



a=0.09 fm, m_π=310 MeV, 737 configurations, 96 2-2-2 grid sources

- *0.5 M measurements in total;*
- *A larger momentum smearing parameter is (almost) harmless for the smaller nucleon momentum;*
- *Very tiny uncertainty at t~0.5 fm **(~0.1% with P=3.0 GeV)**;*
- *Uncertainty increases significantly at larger t due to the contaminations from the other grid points.*

Summary

- Access the gluon PDF through LaMET approach from Lattice QCD is possible.

- The result can be improved much with the momentum smearing+CDP

- RI/MOM renormalization, or the improved ratio renormalization, can be applied with proper matching.

- Will calculate the unpolarized/polarized PDF, and also the gluon spin under kinds of the gauge conditions.