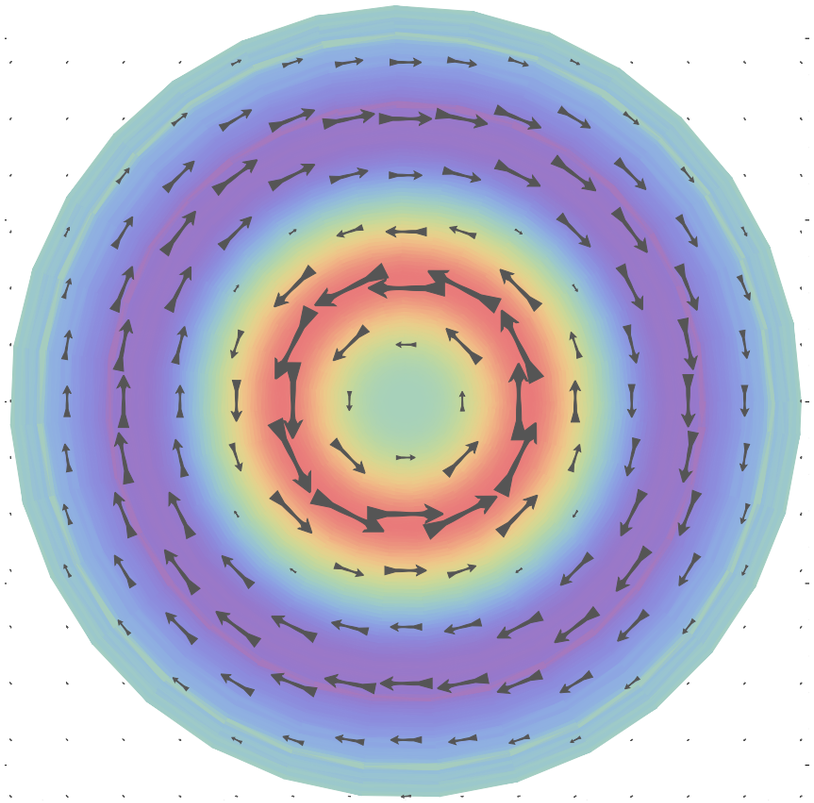
Gluon structure of nucleons and nuclei



Phiala Shanahan

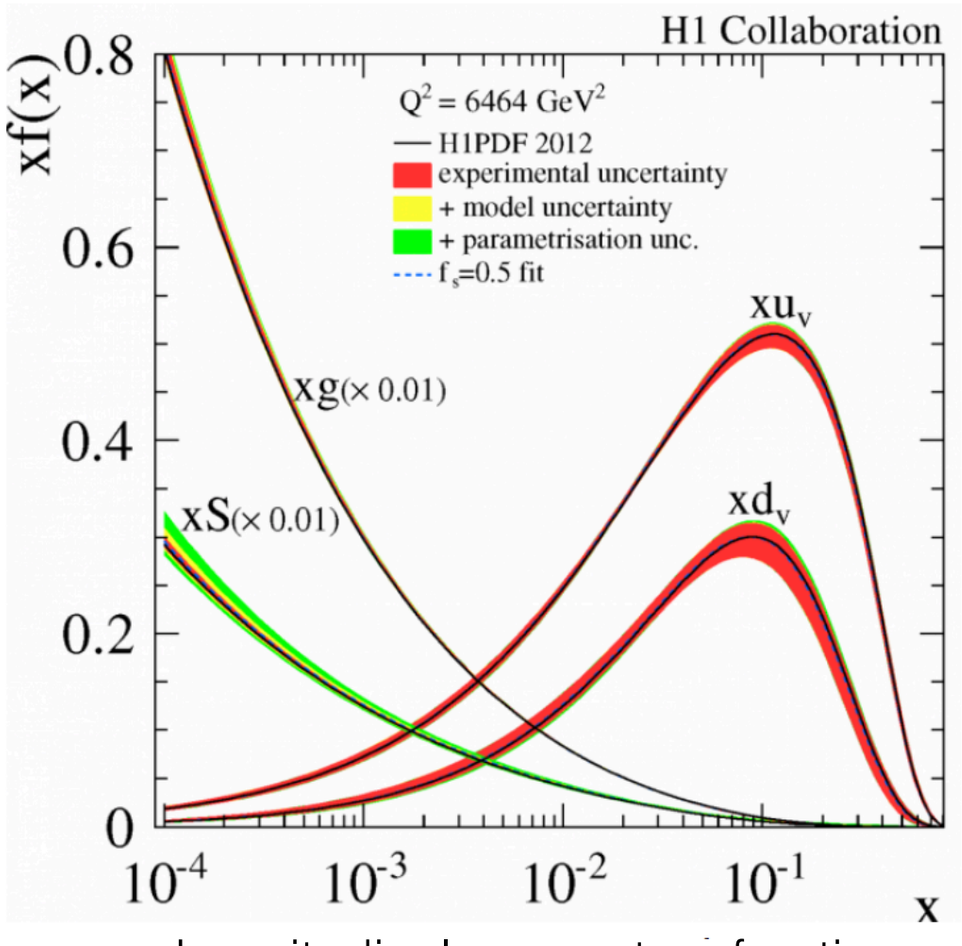
Massachusetts Institute of Technology

Gluon structure

Gluons offer a new window on nuclear structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon PDF dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

Parton distributions in the proton



Longitudinal momentum fraction carried by parton

Gluon structure

First-principles QCD calculations

→ QCD benchmarks and predictions ahead of experiment

Cover image from EIC whitepaper arXiv::1212.1701

Gluon structure

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

Gluon structure

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- Momentum
- Spin
- Mass
- D-term

1

What is the gluon distribution in a proton

- PDFs, GPDs, TMDs
- Pressure, Shear
- 'Gluon radius'

Gluon structure

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

1

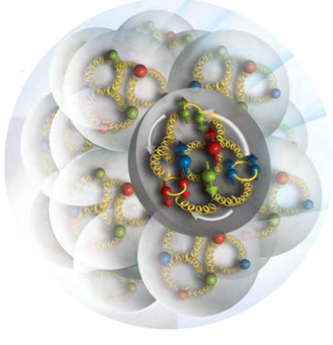
What is the gluon distribution in a proton

- PDFs, GPDs, TMDs
- Pressure, Shear
- 'Gluon radius'

2

How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue



Energy-momentum tensor

Many gluon structure properties derived from **Energy-Momentum Tensor** (conserved Noether current associated with Lorentz translations)

Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^a_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s \rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t)\, \frac{i\, P_{\{\mu} \sigma_{\nu\}\rho} \Delta^\rho}{2M_N} + D_g(t)\, \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Three generalised gluon form factors $A_g(t),\ B_g(t),\ D_g(t)$
- Sum rules with quark pieces in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental!

D-term

D-term GFF encodes the **pressure** and **shear** distributions in the **nucleon** (Breit frame)

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r),$$

$$\widetilde{D}(r) = \int \frac{d^3\vec{p}}{2E(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{r}}\, D(-\vec{p}^{\,2})$$

- Quark and gluon shear forces individually well-defined (i.e., scale-dependent partial contributions $s_{q,g}(r)$
- Pressure defined from D only for the total system (pieces depend also on GFFs related to the trace terms of the EMT that cancel in the sum)

Generalised parton distributions

GFFs correspond to lowest moments of GPDs:

$$\int_0^1 \mathrm{d}x\; H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)\,, \qquad \int_0^1 \mathrm{d}x\; E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

$$\int_{-1}^1 \mathrm{d}x\; x\, H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t)\,, \qquad \int_{-1}^1 \mathrm{d}x\; x\, E_q(x,\xi,t) = B_q(t) - \xi^2 D_q(t)$$

- **Quark GPDs:** constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV
- **Gluon GPDs:** almost unknown from experiment, future constraints are a central goal of EIC

Leading twist nucleon gluon GPDs:

Gluon field-strength tensor

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[\mathcal{U}^{(A)}_{[-\frac{\lambda}{2}n, \frac{\lambda}{2}n]}\right]_{ab} G_{b\alpha}^{\nu\}}(\frac{\lambda}{2}n) | p, s\rangle$$

$$= \frac{1}{2}\left(H_g(x,\xi,t)\bar{U}(p',s')P^{\{\mu}\gamma^{\nu\}}U(p,s) + E_g(x,\xi,t)\bar{U}(p',s')\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M}U(p,s)\right) + \dots\,,$$

GPDs(Bjorken x, skewness, mom transfer)

$$\Delta_\mu = p'_\mu - p_\mu$$
$$P_\mu = (p_\mu + p'_\mu)/2$$
$$t = \Delta^2$$
$$n^2 = 0$$

D-term from JLab DVCS

Recent experimental determination of DVCS D-term and extraction of proton pressure distribution

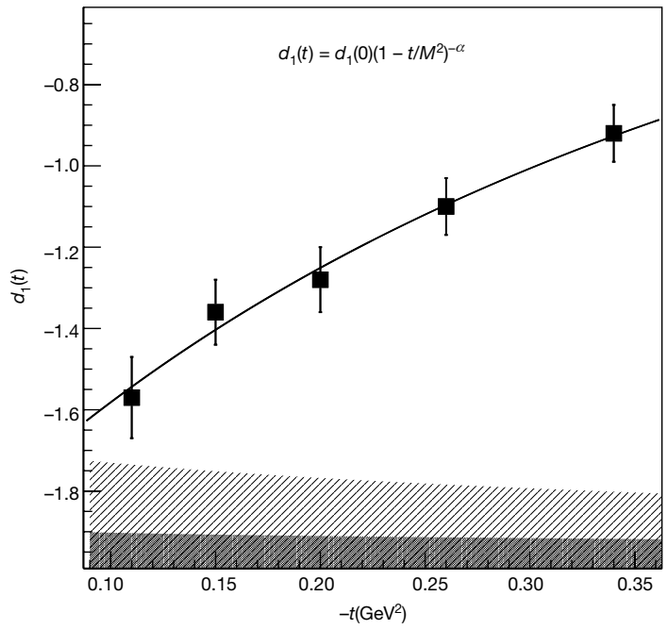
V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396 (2018)

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

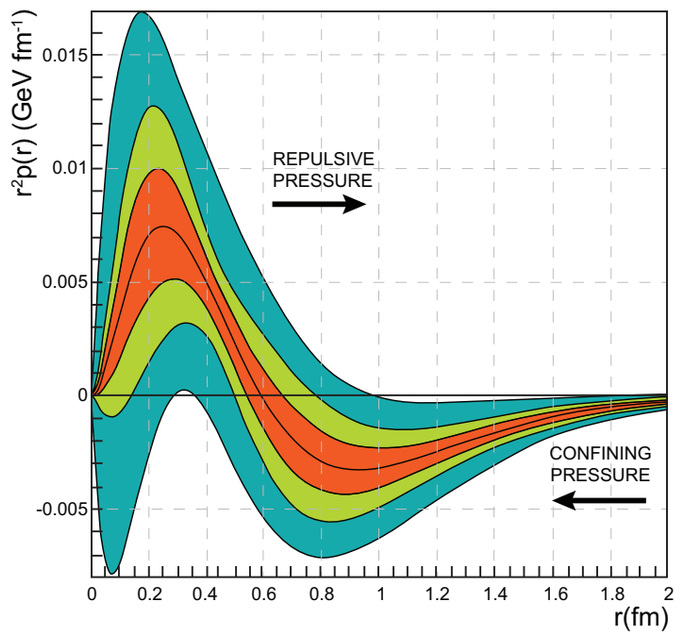
- Strong repulsive pressure near the centre of the proton
- Binding pressure at greater distances.
- Peak pressure near the centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: **gluon D-term same as quark term, tripole form factor model,** $D_u(t,\mu) = D_d(t,\mu)$

Use lattice QCD to test assumptions in pressure extraction

DVCS (quark) D-term



Radial pressure distribution



Gluon GFFs from LQCD

Construct system of equations for **generalised gluon form factors**

Ratios of 3pt and 2pt correlation functions:

$$R_{s;\mathfrak{R},i}(\vec{p},\vec{p}',t_f,\tau) \;=\; \frac{C^{\rm 3pt}_{s;\mathfrak{R},i}(\vec{p},\vec{p}',t_f,\tau)}{C^{\rm 2pt}_s(\vec{p}',t_f)}\sqrt{\frac{C^{\rm 2pt}_s(\vec{p},t_f-\tau)C^{\rm 2pt}_s(\vec{p}',t_f)C^{\rm 2pt}_s(\vec{p}',\tau)}{C^{\rm 2pt}_s(\vec{p}',t_f-\tau)C^{\rm 2pt}_s(\vec{p},t_f)C^{\rm 2pt}_s(\vec{p},\tau)}} \overset{t_f\gg\tau\gg 0}{\longrightarrow} \frac{{\rm Tr}\left[\Gamma_s(\not{p}'+M_N)\mathcal{F}_i[A_g,B_g,D_g](\not{p}+M_N)\right]}{8\sqrt{E^{(N)}_{\vec{p}}E^{(N)}_{\vec{p}'}(E^{(N)}_{\vec{p}}+M_N)(E^{(N)}_{\vec{p}'}+M_N)}}$$

$$\mathcal{F}_{\mu\nu}[A_g,B_g,D_g] = A_g(t)\,\gamma_{\{\mu}P_{\nu\}} + B_g(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D_g(t)\,\frac{\Delta_{\{\mu}\Delta_{\nu\}}}{4M_N}$$

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = -\Delta^2$

- Nucleon spin up/down: $\Gamma_{s=\pm 1}$
- Sink and operator momenta:

$$|\vec{p}\,'|^2 \;\leq\; 5(2\pi/L)^2$$

$$|\vec{\Delta}|^2 \leq 18(2\pi/L)^2$$

- Operator index choices: two different irreducible representations of H(4)

$$\mathcal{O}^{\tau_3^{(6)}}_{i=\{1,\ldots,6\}} = \left\{\frac{(-i)^{\delta_{\nu 0}}}{\sqrt{2}}\left(\mathcal{O}_{\mu\nu}+\mathcal{O}_{\nu\mu}\right),\;\; 0\leq\mu<\nu\leq 3\right\}$$

$$\mathcal{O}^{\tau_1^{(3)}}_{1} = \frac{1}{2}\left(\mathcal{O}_{11}+\mathcal{O}_{22}-\mathcal{O}_{33}+\mathcal{O}_{00}\right),\;\; \cdots,$$

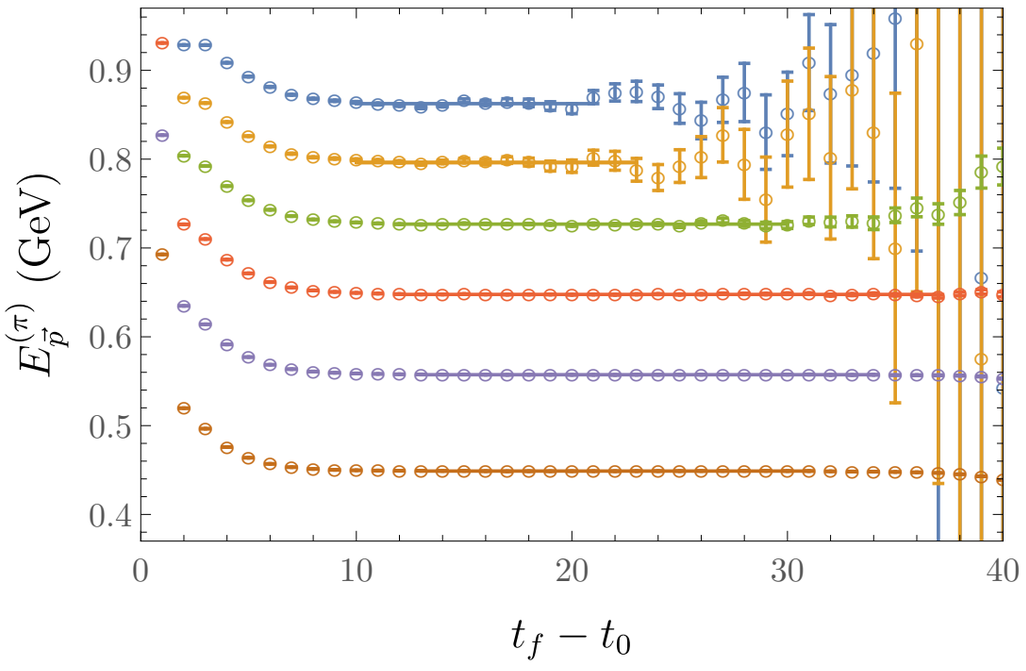
Gluon GFFs from LQCD

One ensemble, $\mathbf{m_\pi}$ **~450 MeV**

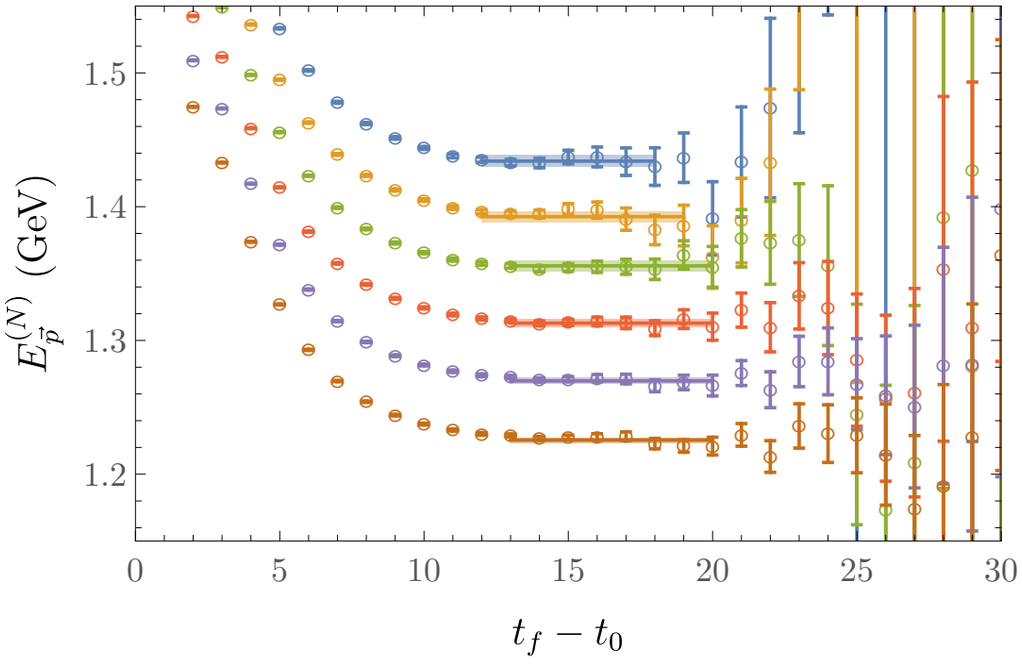
L/a	T/a	β	am_l	am_s	a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)	$m_\pi L$	$m_\pi T$	$N_{\rm cfg}$	$N_{\rm meas}$
32	96	6.1	-0.2800	-0.2450	0.1167(16)	3.7	11.2	450(5)	596(6)	8.5	25.6	2821	203

Clean plateaus in effective masses for $|\vec{p'}|^2 \leq 5(2\pi/L)^2$

Pion

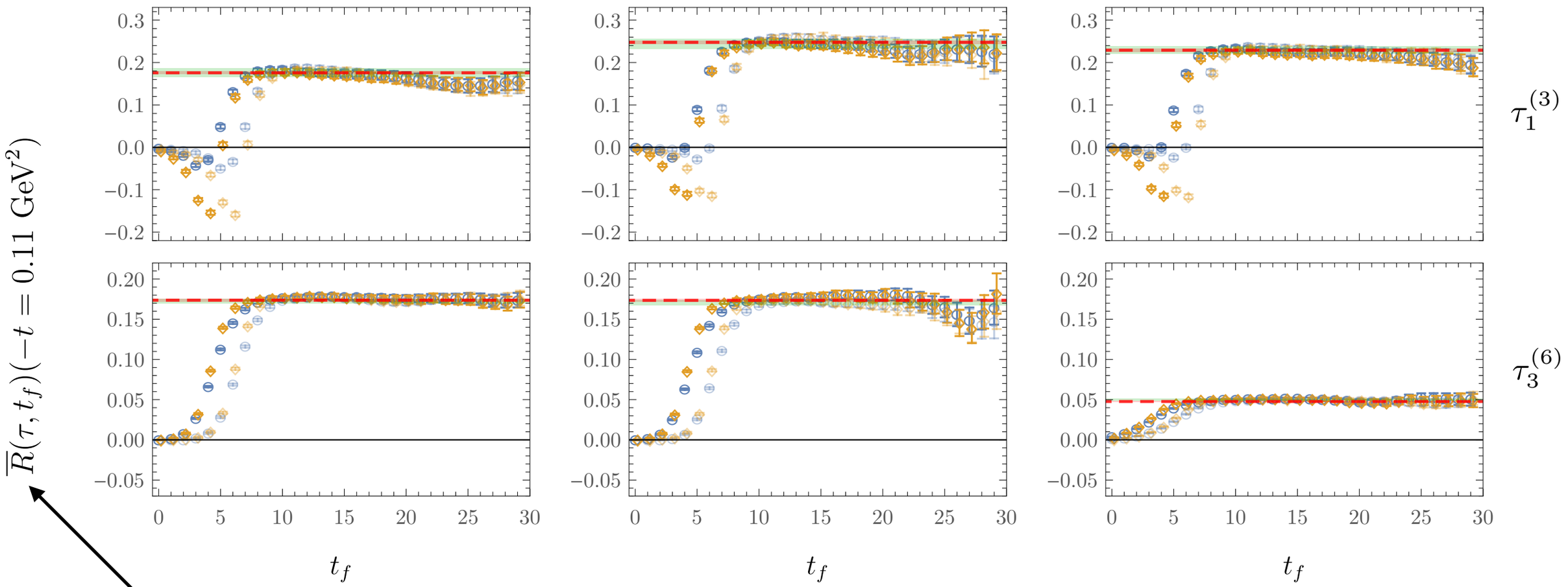


Nucleon



Gluon GFFs from LQCD

PION: Clean signals in 3pt/2pt ratios (examples)



Ratios that give same linear combination of GFFs are averaged

Plateau fits

Solution of system of equations projected back

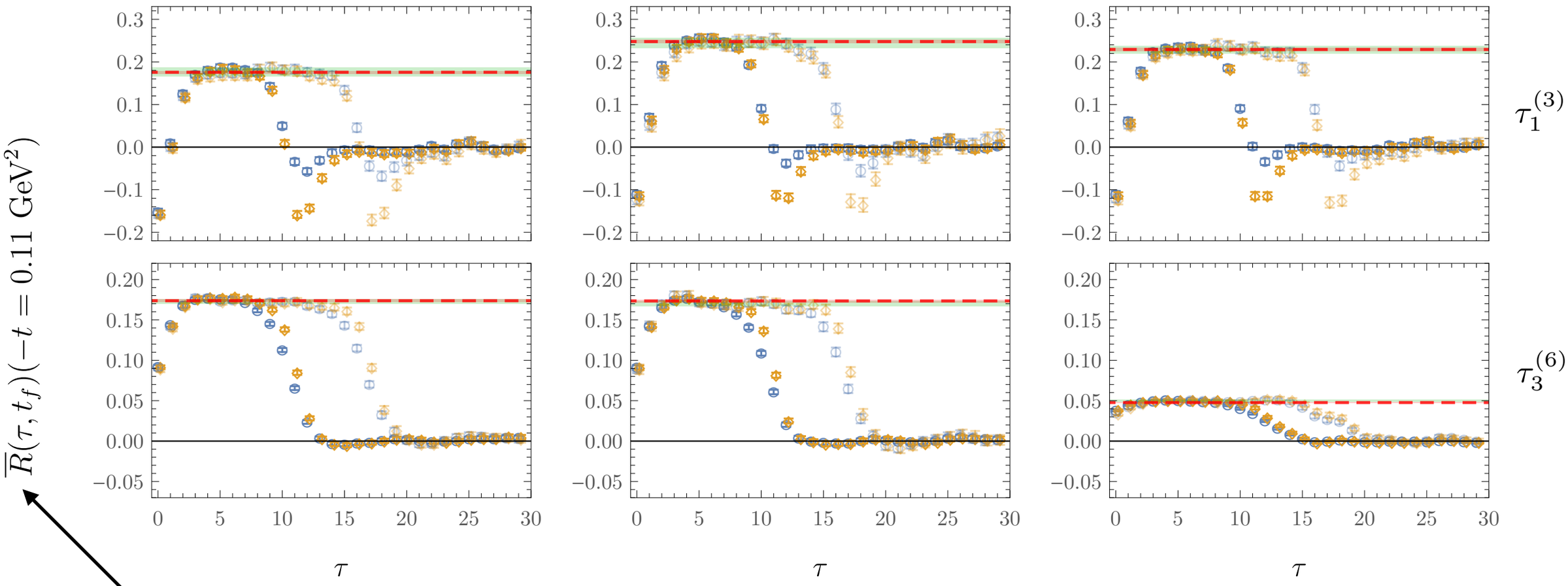
$\tau = 5$ (dark points), τ=7 (pale points)

smeared-point

smeared-smeared

Gluon GFFs from LQCD

PION: Clean signals in 3pt/2pt ratios (examples)



Ratios that give same linear combination of GFFs are averaged

Plateau fits

Solution of system of equations projected back

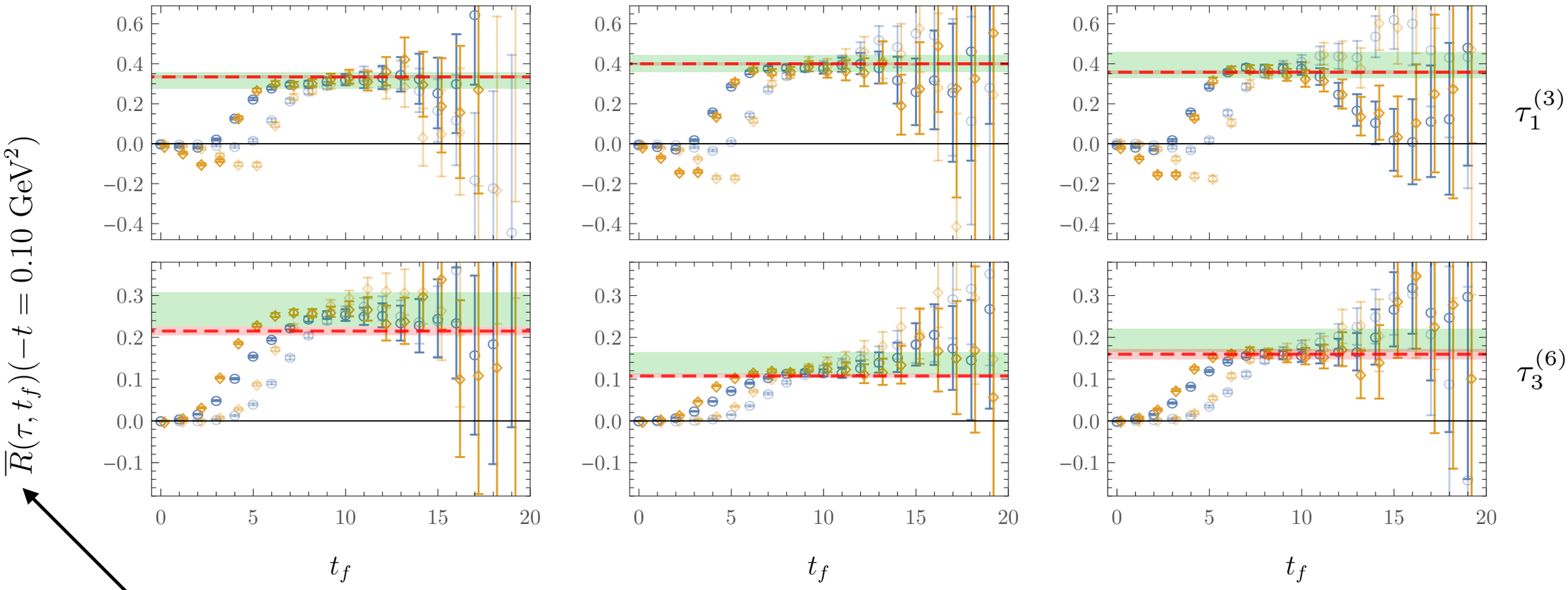
$t_f = 13$ (dark points), t_f=18 (pale points)

smeared-point

smeared-smeared

Gluon GFFs from LQCD

NUCLEON: Clean signals in 3pt/2pt ratios (examples)



Ratios that give same linear combination of GFFs are averaged

Plateau fits

Solution of system of equations projected back

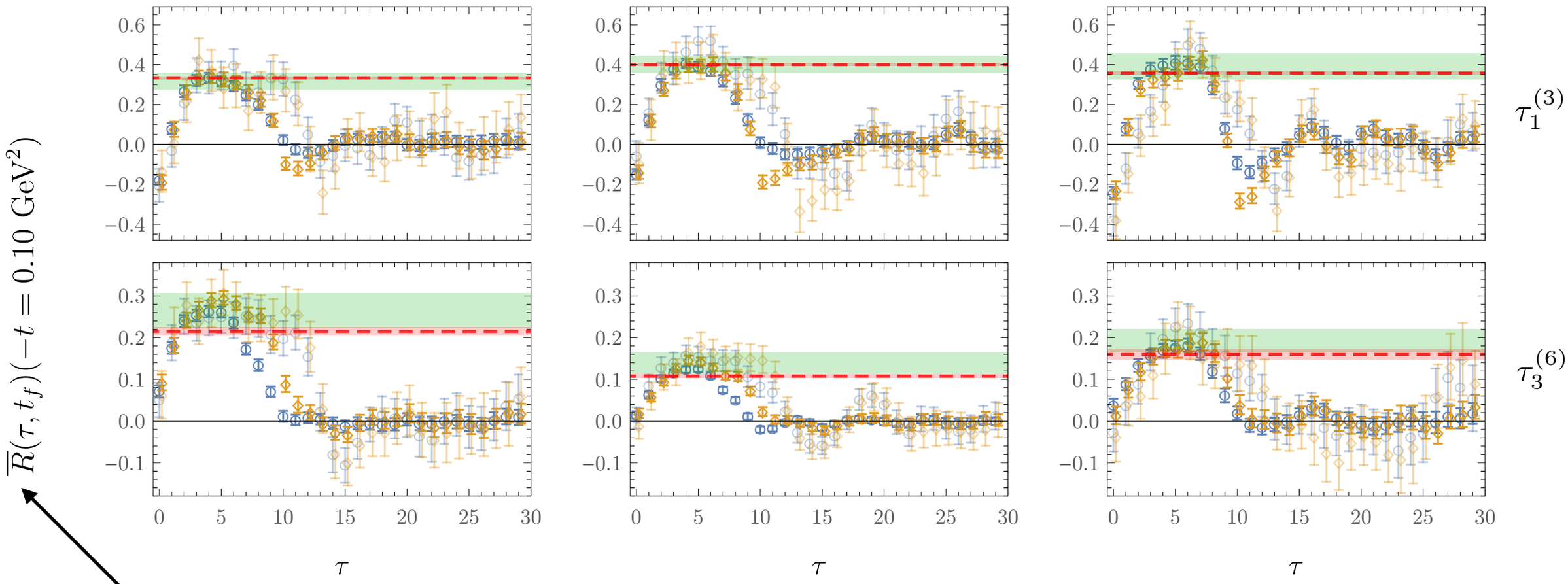
$\tau = 4$ (dark points), τ=6 (pale points)

smeared-point

smeared-smeared

Gluon GFFs from LQCD

NUCLEON: Clean signals in 3pt/2pt ratios (examples)



Ratios that give same linear combination of GFFs are averaged

Plateau fits

Solution of system of equations projected back

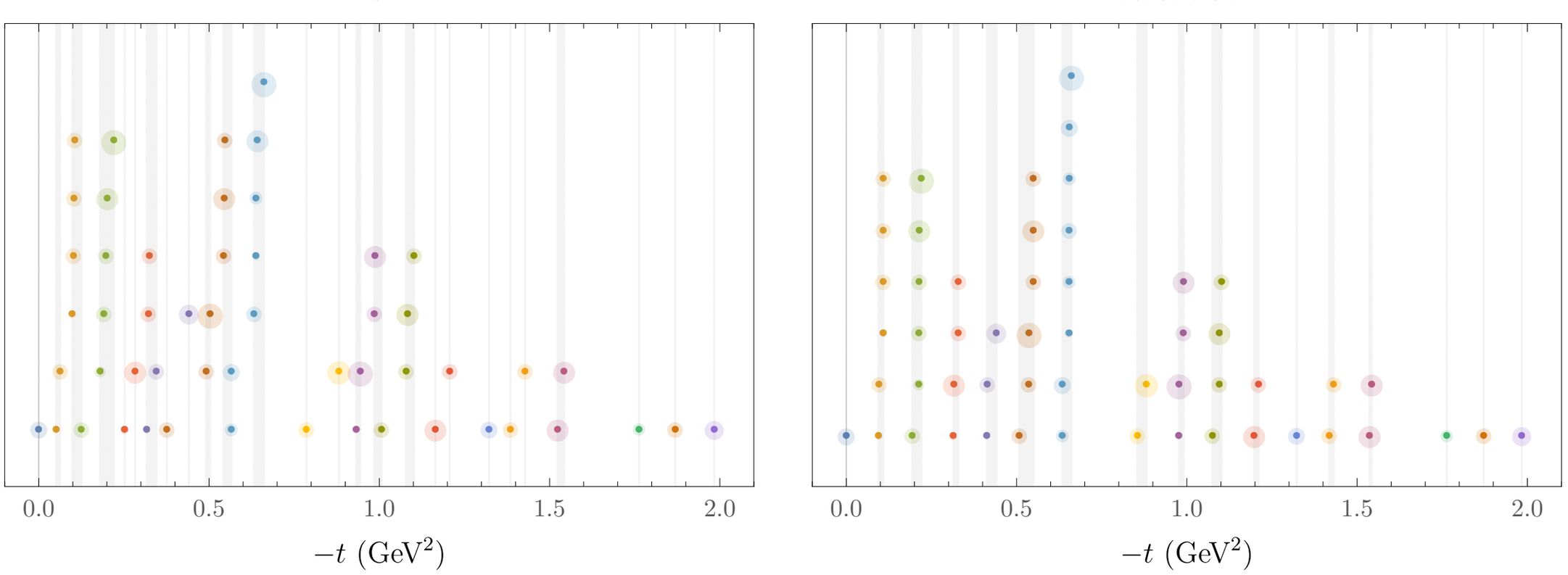
$t_f = 12$ (dark points), t_f=14 (pale points)

smeared-point

smeared-smeared

Gluon GFFs from LQCD

Solve system of equations for GFFs in bins in $t = (p' - p)^2$



- Colour coding: three momentum transfer $\vec{\Delta}^2 = (\vec{p}' - \vec{p})^2$
- Point size $\propto$ number of three-momenta at that $\vec{\Delta}^2$
- Grey bands: bins in t

Renormalisation

Non-perturbative RI-MOM renormalisation of gluon operator

- Mixing with quark operator neglected
  Found to be small in lattice PT e.g., Alexandrou et al., 1611.06901
- One-loop perturbative matching to $\overline{\mathrm{MS}}$ scheme: Yang et al., 1612.02855

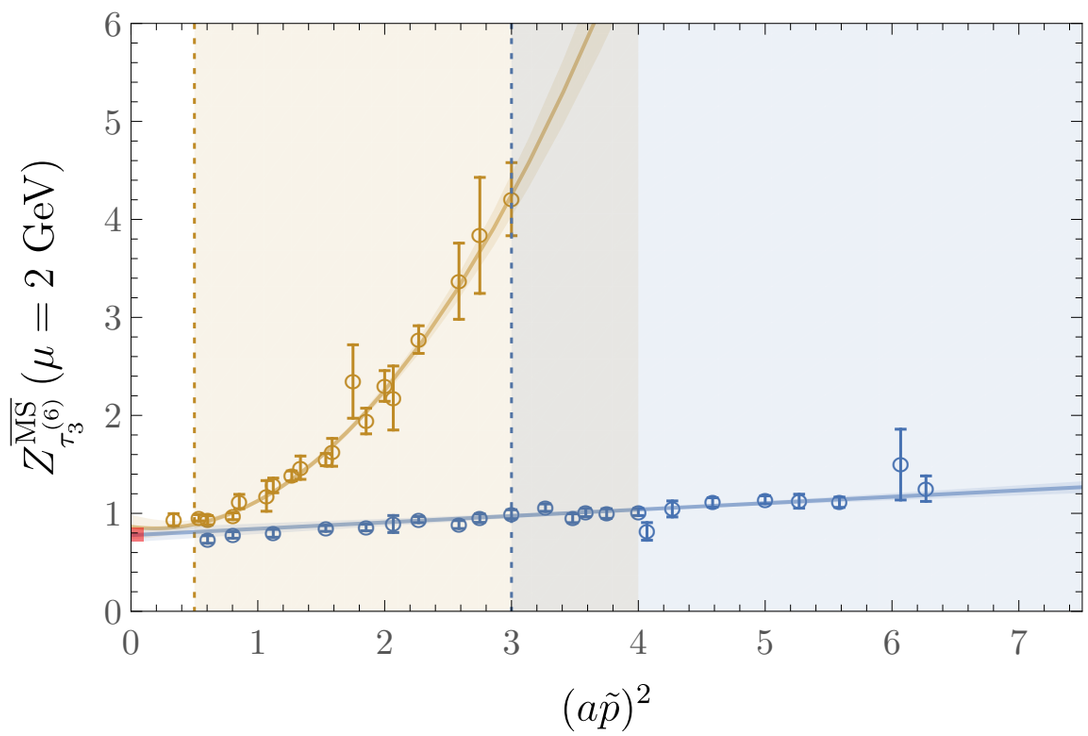
$$\mathcal{O}^{\mathrm{MS}}(\mu^2) = Z_{\mathcal{O}}^{\mathrm{MS}}(\mu^2)\mathcal{O}^{\mathrm{latt}} = \mathcal{R}^{\mathrm{MS}}(\mu^2, \mu_R^2) Z_{\mathcal{O}}^{\text{RI-MOM}}(\mu_R^2)\mathcal{O}^{\mathrm{latt}}$$

Calculate RI-MOM coefficient using Landau-gauge fixed gluon 2pt function

$$\left(Z_{\hat{\mathcal{O}}}^{\text{RI-MOM}}(\mu_R^2)\right)^{-1} = \frac{4p^2\langle\hat{\mathcal{O}}_{\alpha\beta}\mathrm{Tr}[A_\tau(p)A_\tau(-p)]\rangle}{\Lambda_{\hat{\mathcal{O}}}^{\mathrm{tree}}(p)\langle\mathrm{Tr}[A_\tau(p)A_\tau(-p)]\rangle}\Bigg|_{\substack{p^2=\mu_R^2 \\ \tau\neq\alpha\neq\beta \\ p_\tau=0}}$$

$$\Lambda_{\hat{\mathcal{O}}}^{\mathrm{tree}}(p) = \langle\hat{\mathcal{O}}_{\alpha\beta}^{\mathfrak{R}}\mathrm{Tr}[A_\tau(p)A_\tau(-p)]\rangle_{\mathrm{amp.}}^{\mathrm{tree}}$$

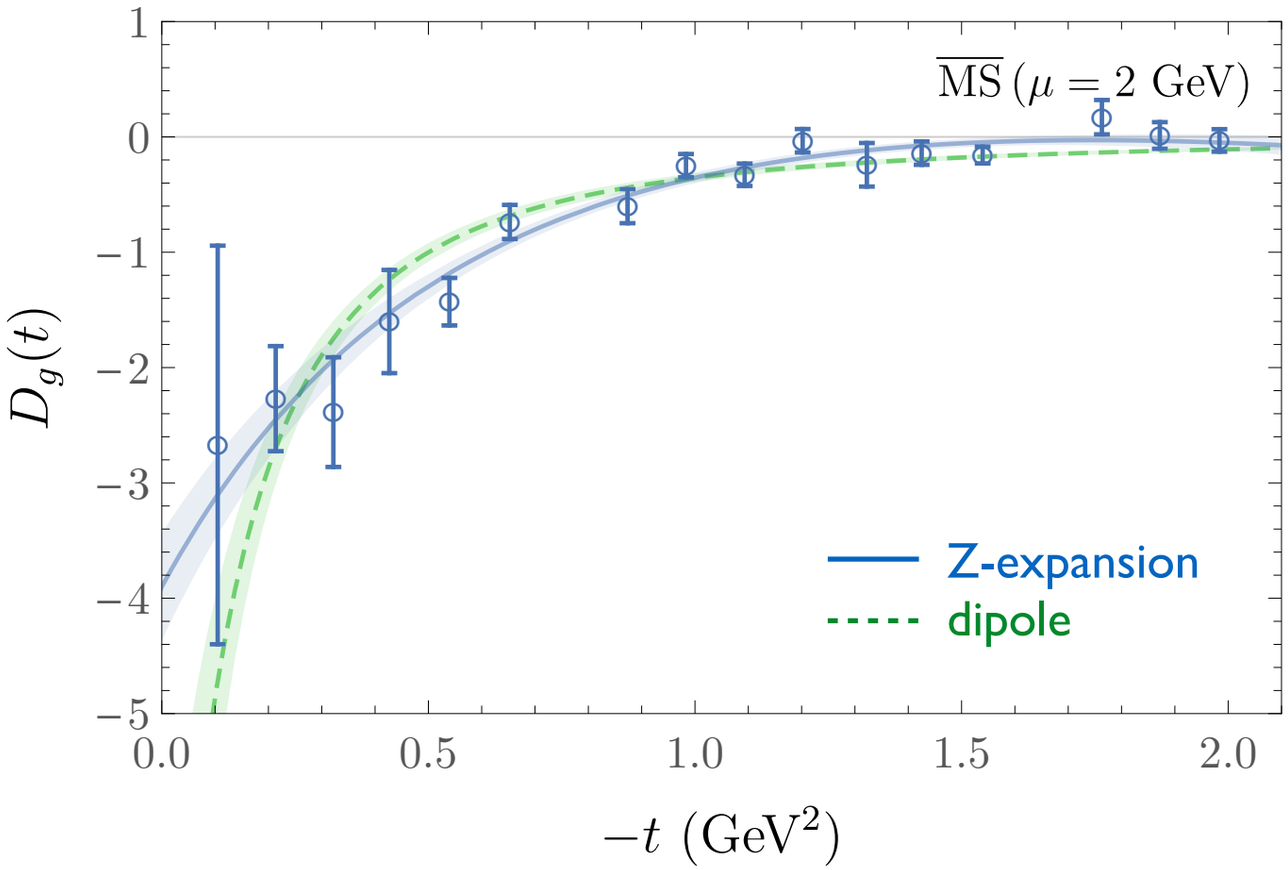
- Wilson-flowed gluon 2pts
- No flow in 2pts

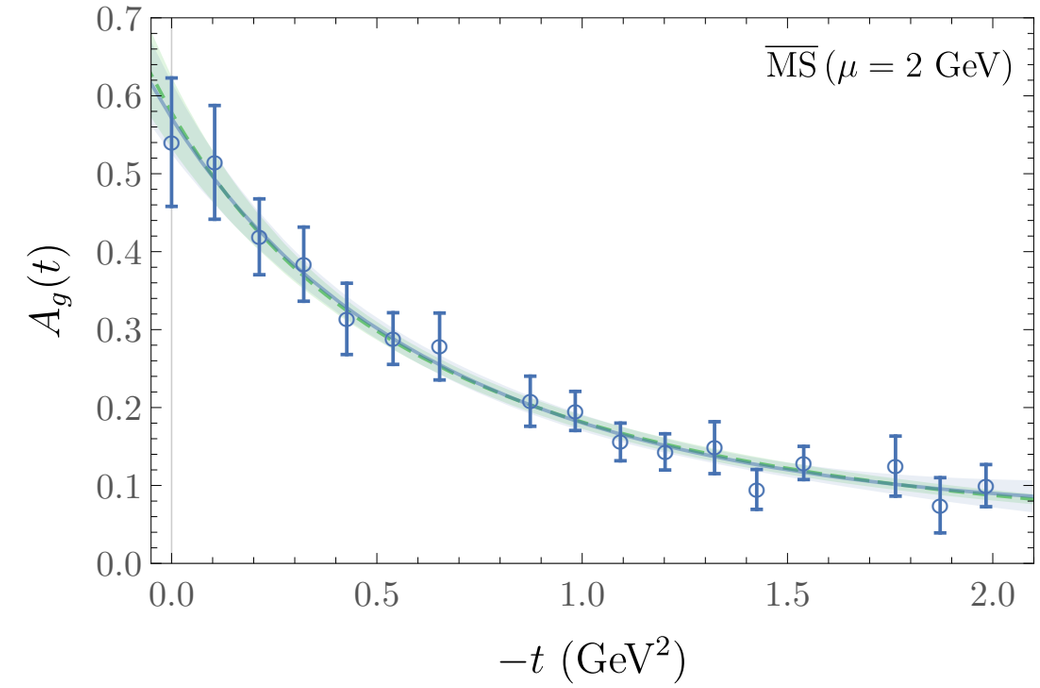


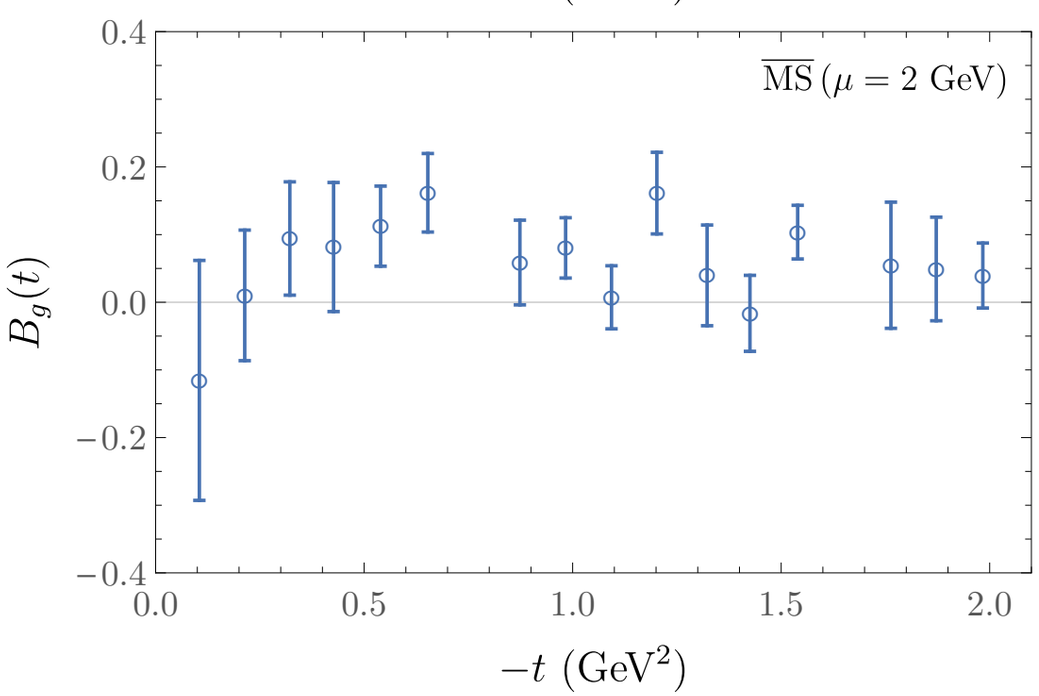
LQCD Nucleon GFFs

LQCD results for nucleon gluon GFFs
m_π ~450 MeV

Dipole-like fall-off with momentum transfer







Shanahan, Detmold, PRD 99, 014511, PRL 122 072003 (2019)

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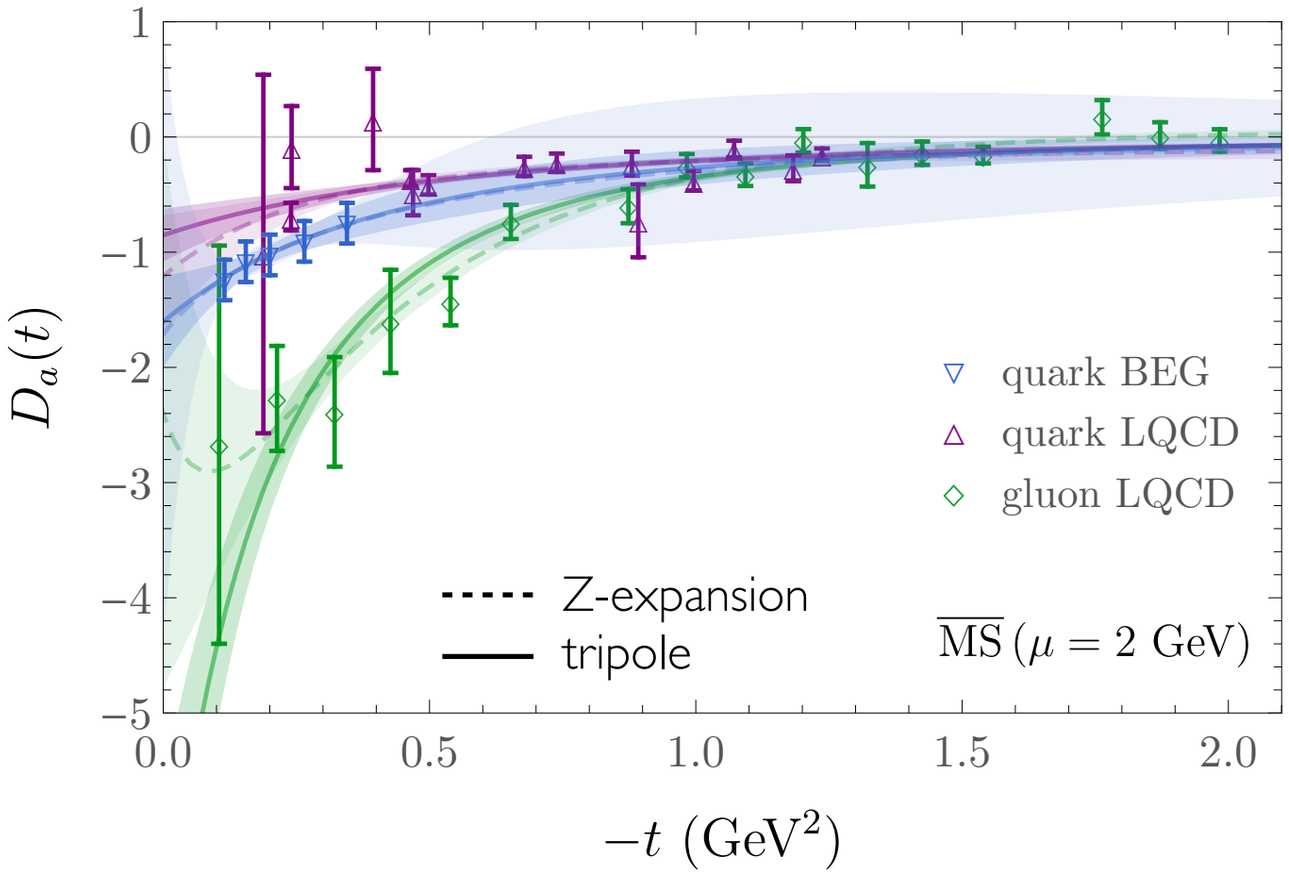
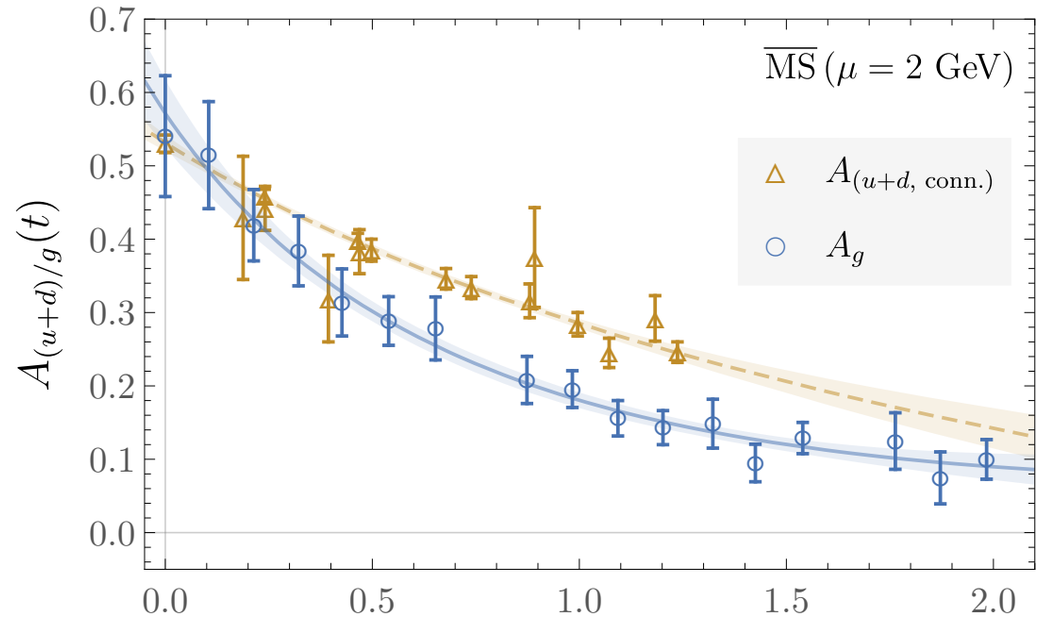
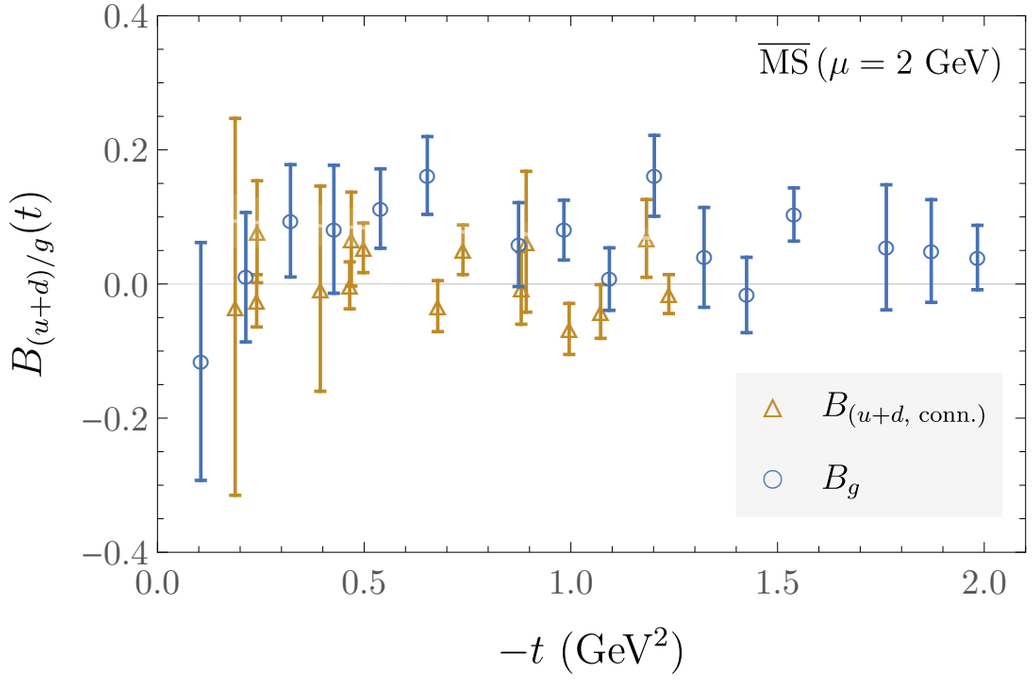






Gluon GFFs: Shanahan, Detmold, PRD 99, 014511, PRL 122 072003 (2019)

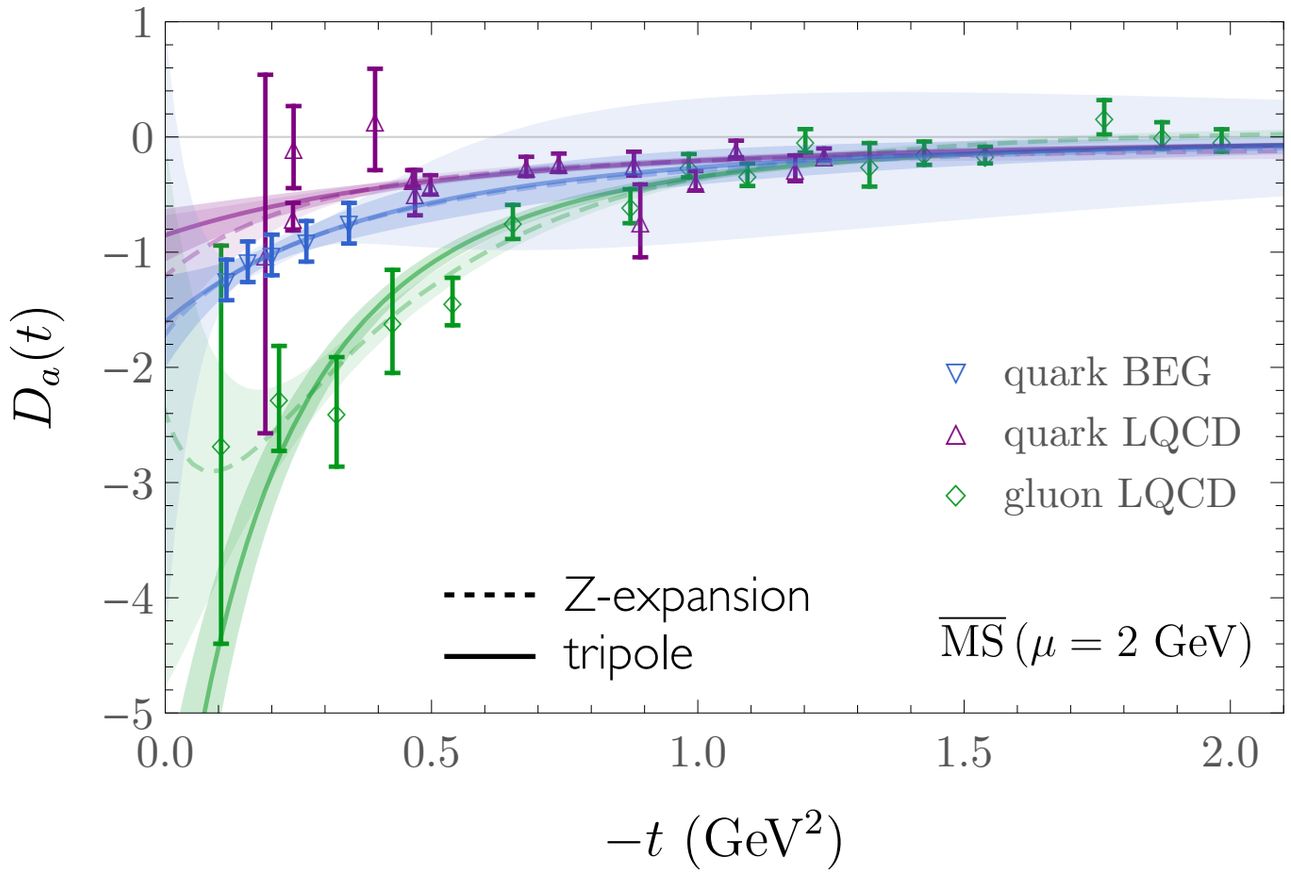
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Nucleon D-term GFFs

LQCD results for nucleon gluon GFFs
m_π ~450 MeV

Tripole-like fall-off with momentum transfer



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Key assumptions in pressure extraction from DVCS

- **Gluon D-term same as quark term in magnitude and shape**

  Factor of ~2 difference in magnitude, somewhat different t-dependence

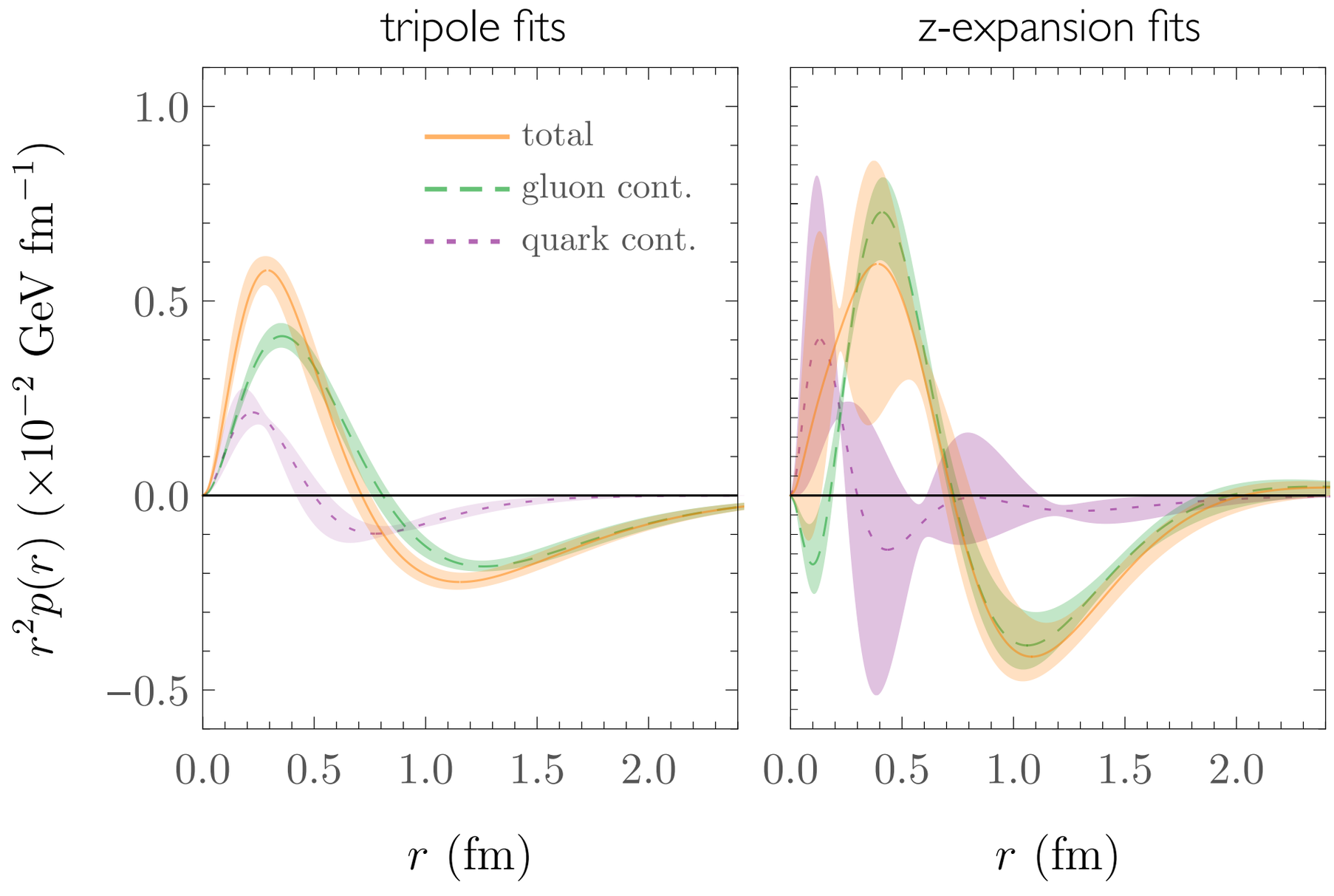
- **Tripole form factor model**

  LQCD results consistent with ansatz, but more general form is less well constrained

- **Isovector quark D-term vanishes**
  $D_{u-d}(t)$ ~0 from other LQCD studies

LQCD proton pressure

Nucleon pressure using LQCD results for quark and gluon GFFs, m_π ~450 MeV



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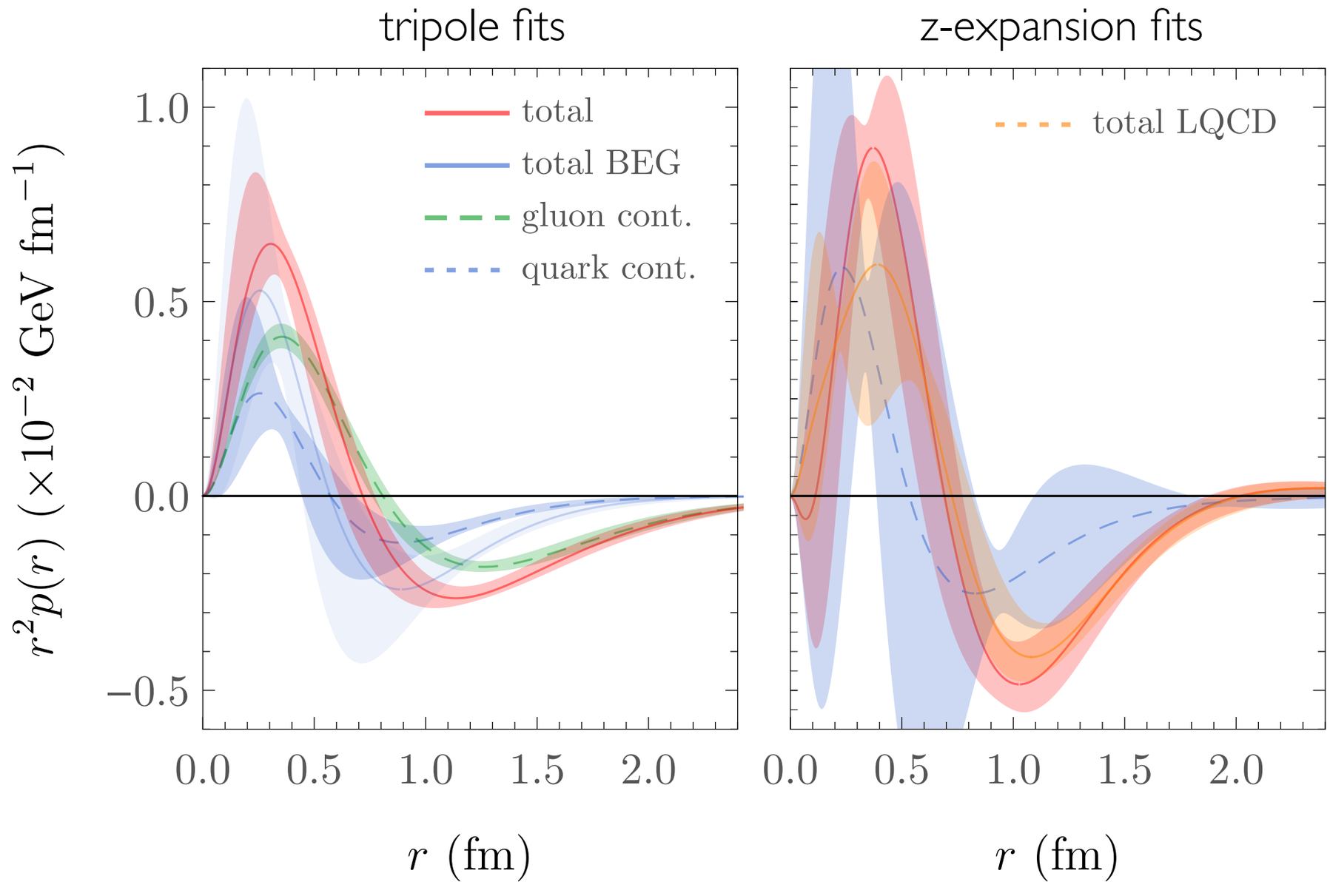
LQCD + EXP proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF



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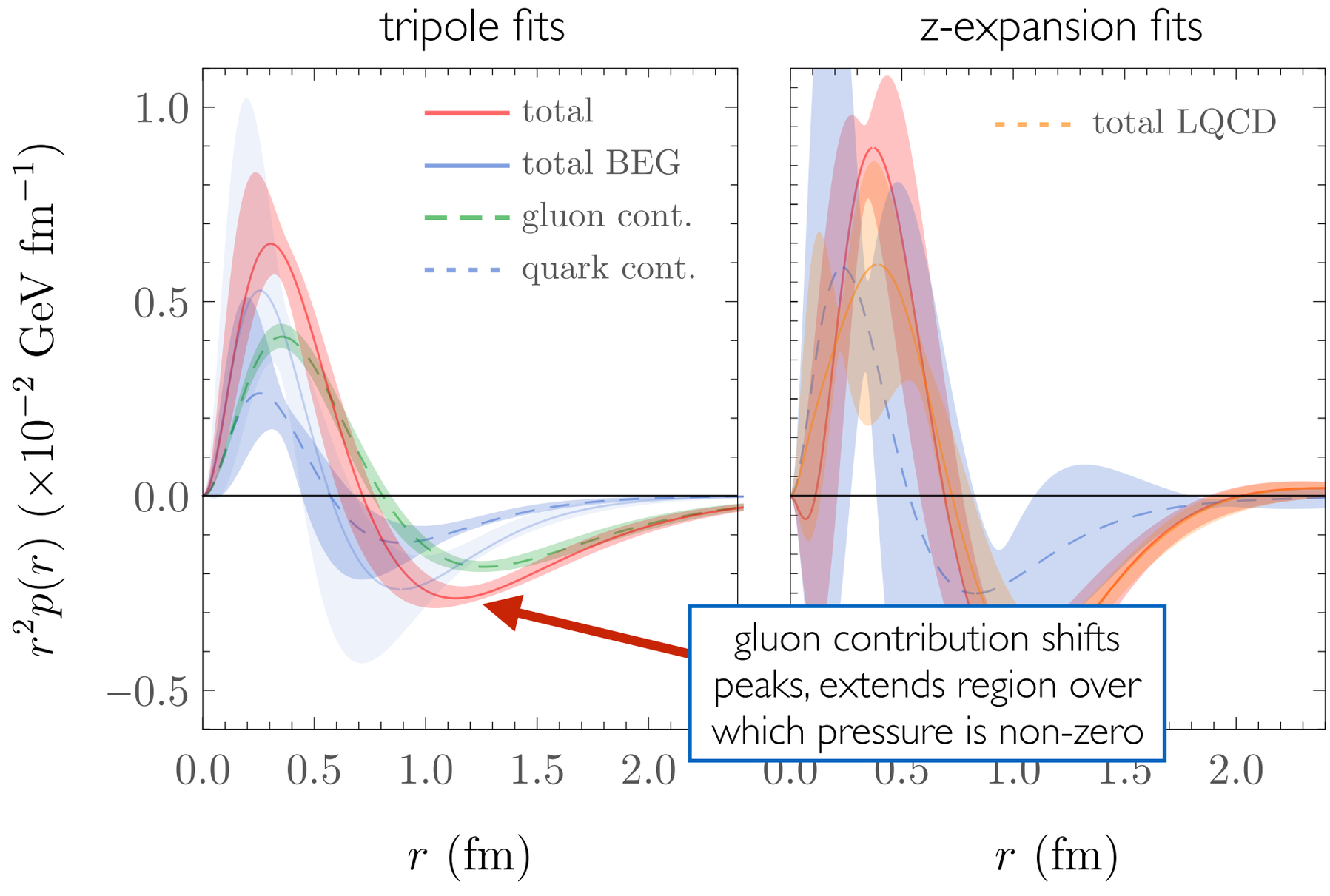
LQCD + EXP proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF



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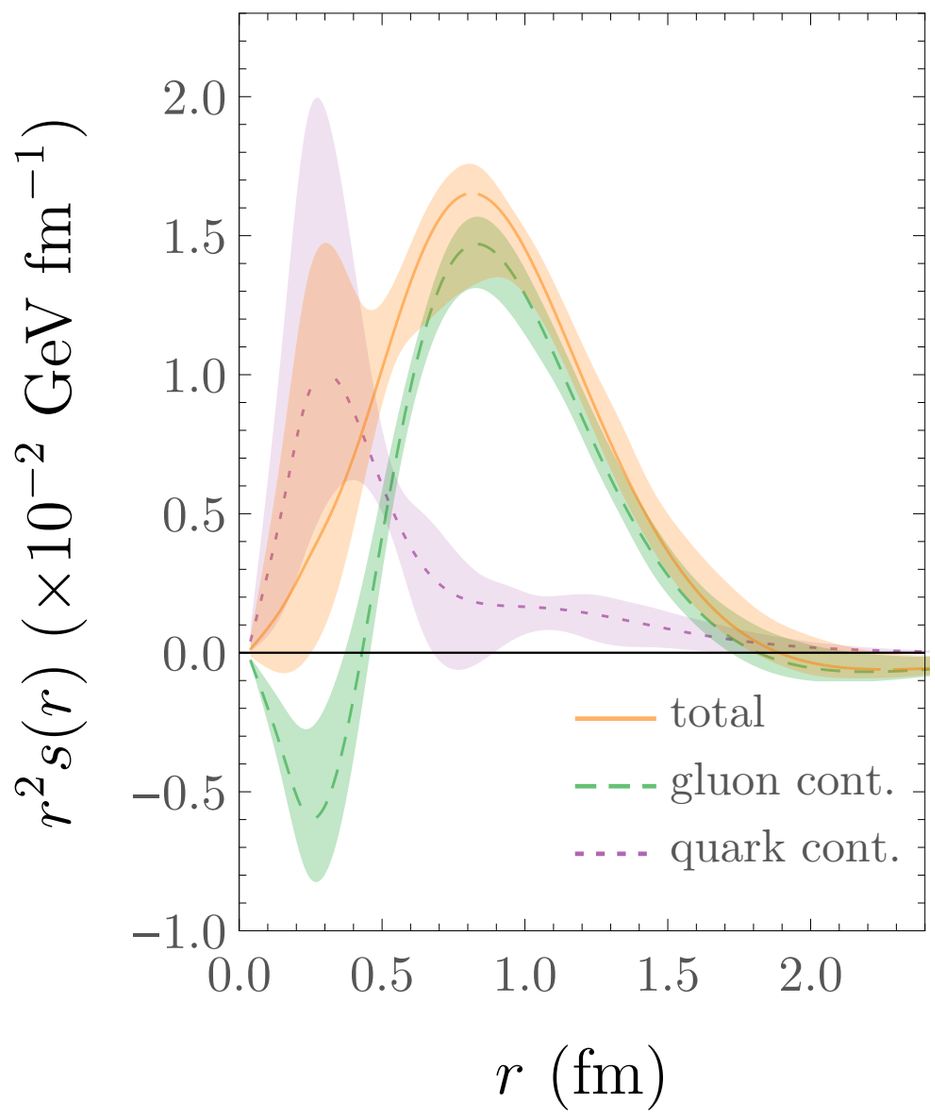
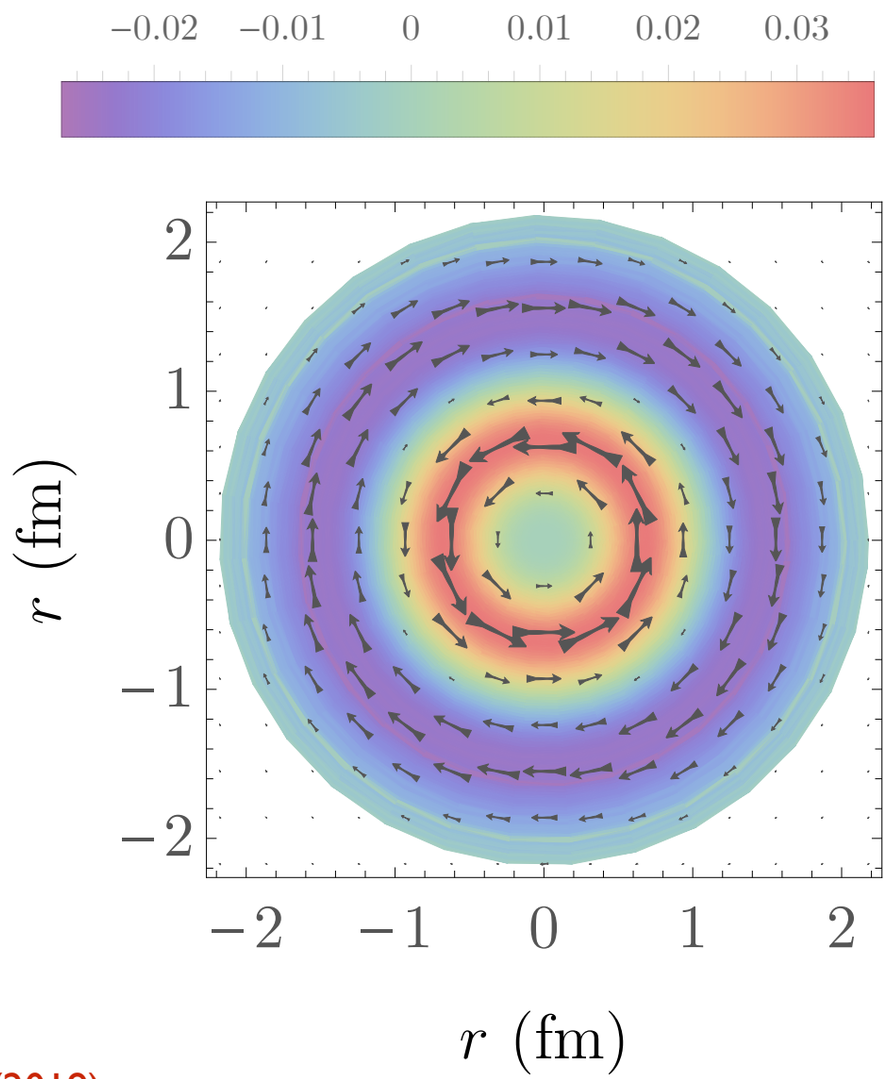
LQCD proton shear

z-expansion fits



Tangential shear vector field $4\pi r^2 T_{ij} e^{\phi}_{j}$



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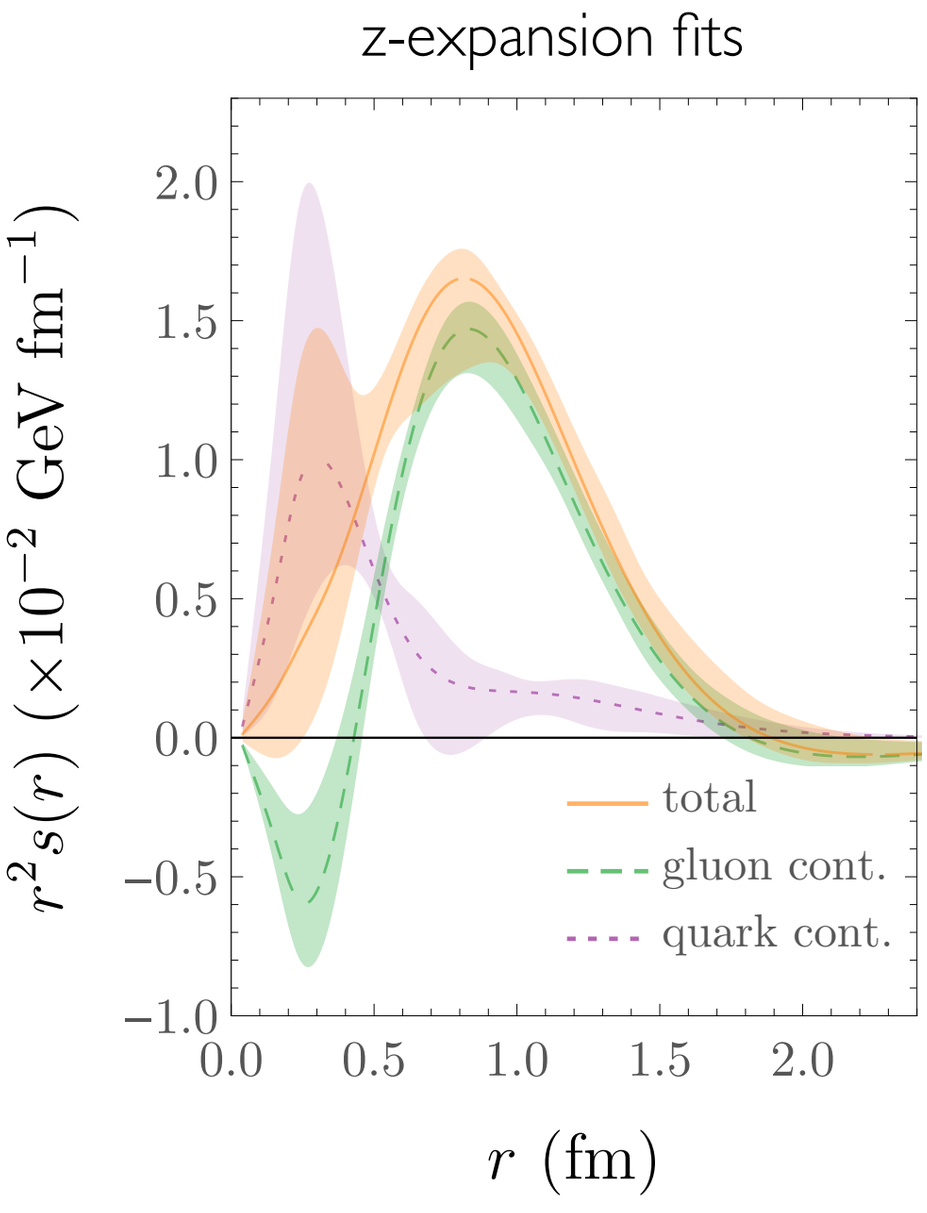
LQCD proton shear



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LQCD Pion GFFs

Pion gluon GFFs m_π ~450 MeV

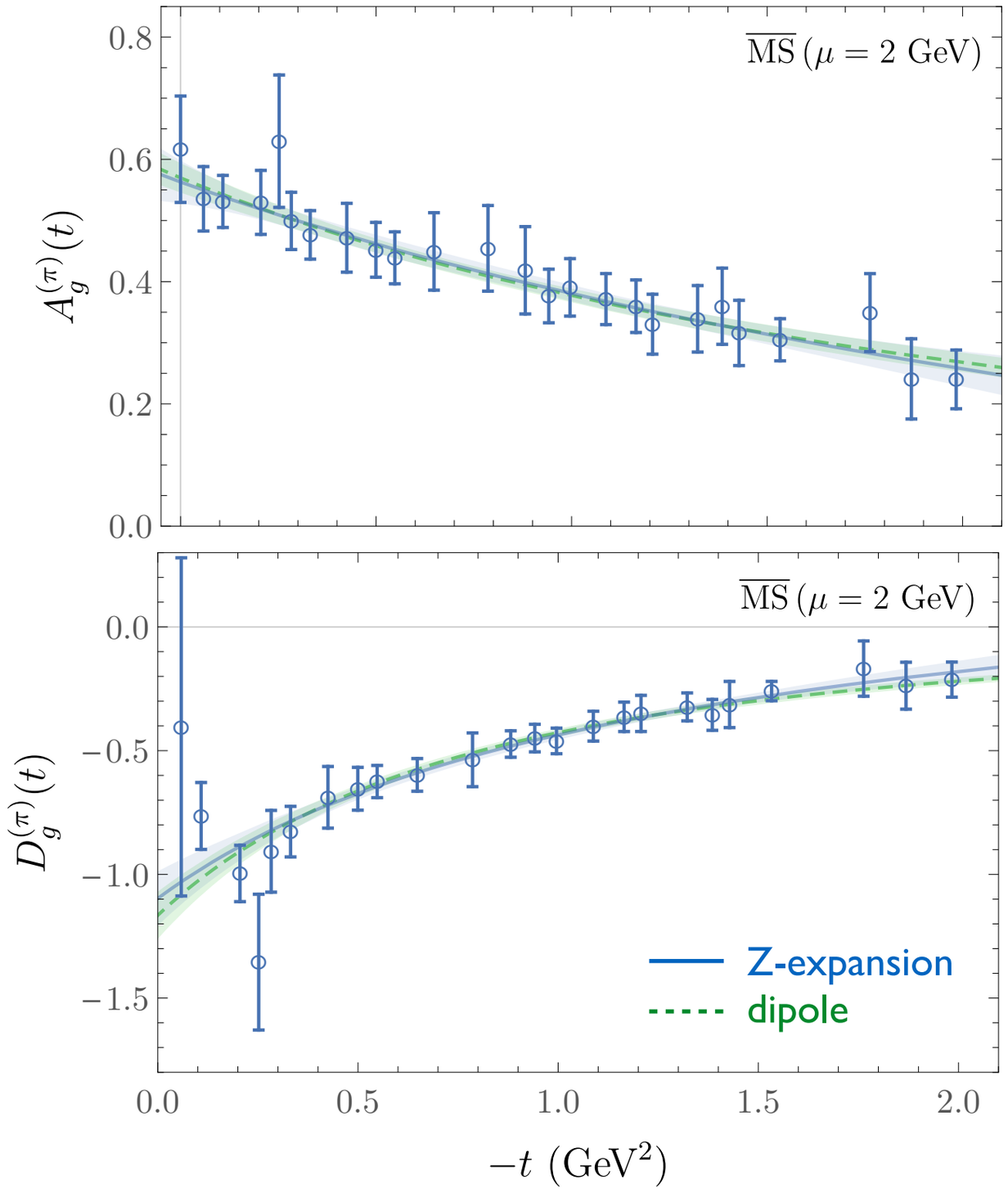
Solve system of equations simultaneously for both hypercubic irreps for each binned four-momentum transfer

Dipole-like fall-off with momentum transfer

- Momentum fraction $A_a(0) = \langle x \rangle_a$

  → $\sum_{a=q,g} A_a(0) = 1$

- D-terms $D_a(0)$ related to pressure and shear distributions



Shanahan, Detmold,
PRD 99, 014511, PRL 122 072003 (2019)

LQCD Pion GFFs

Pion gluon GFFs m_π ~450 MeV

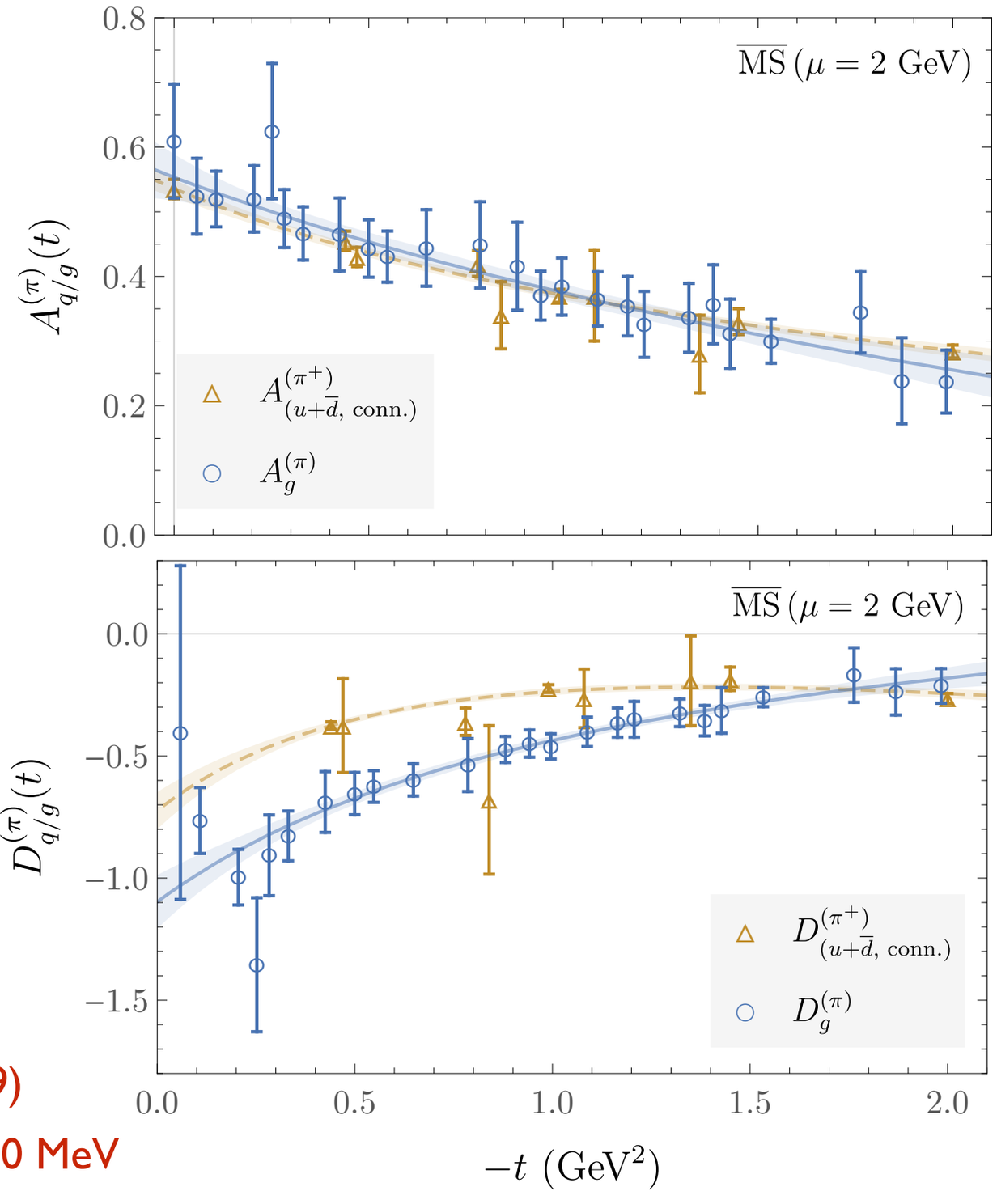
Solve system of equations simultaneously for both hypercubic irreps for each binned four-momentum transfer

Dipole-like fall-off with momentum transfer

- Momentum fraction $A_a(0) = \langle x \rangle_a$

  $\Rightarrow \sum_{a=q,g} A_a(0) = 1$

- D-terms $D_a(0)$ related to pressure and shear distributions

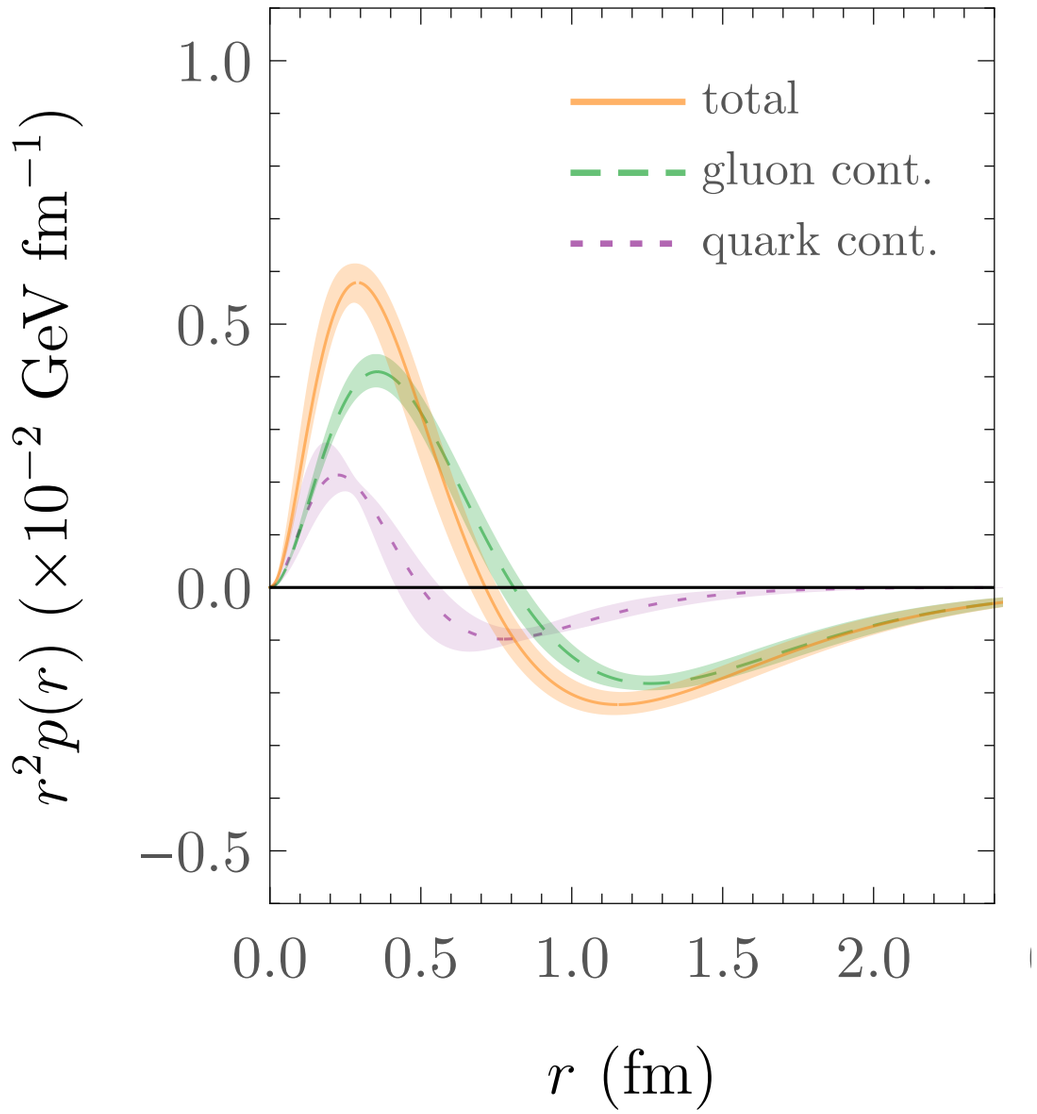


gluon: Shanahan, Detmold, PRD 99, 014511 (2019)

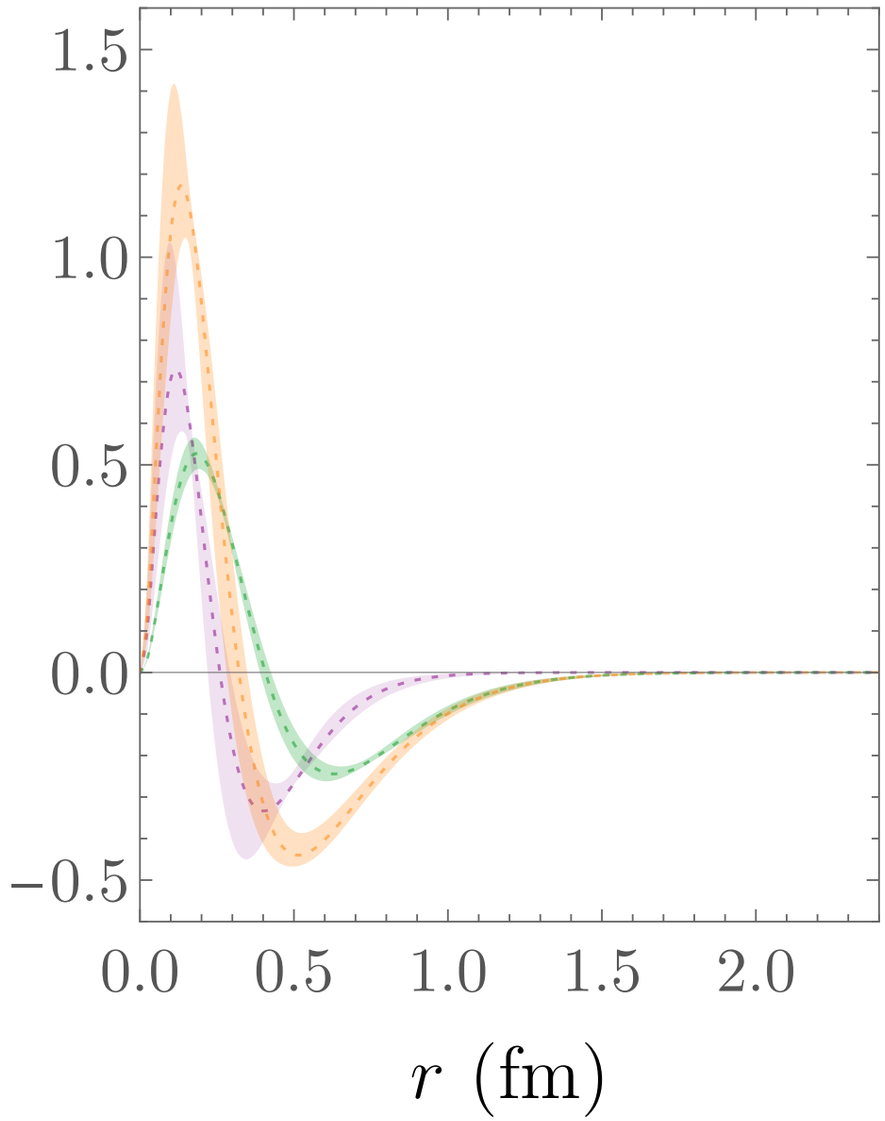
quark: Dirk Brömmel Ph.D. thesis (2007) m_π ~840 MeV

LQCD pion pressure

Nucleon pressure distribution



Pion pressure distribution

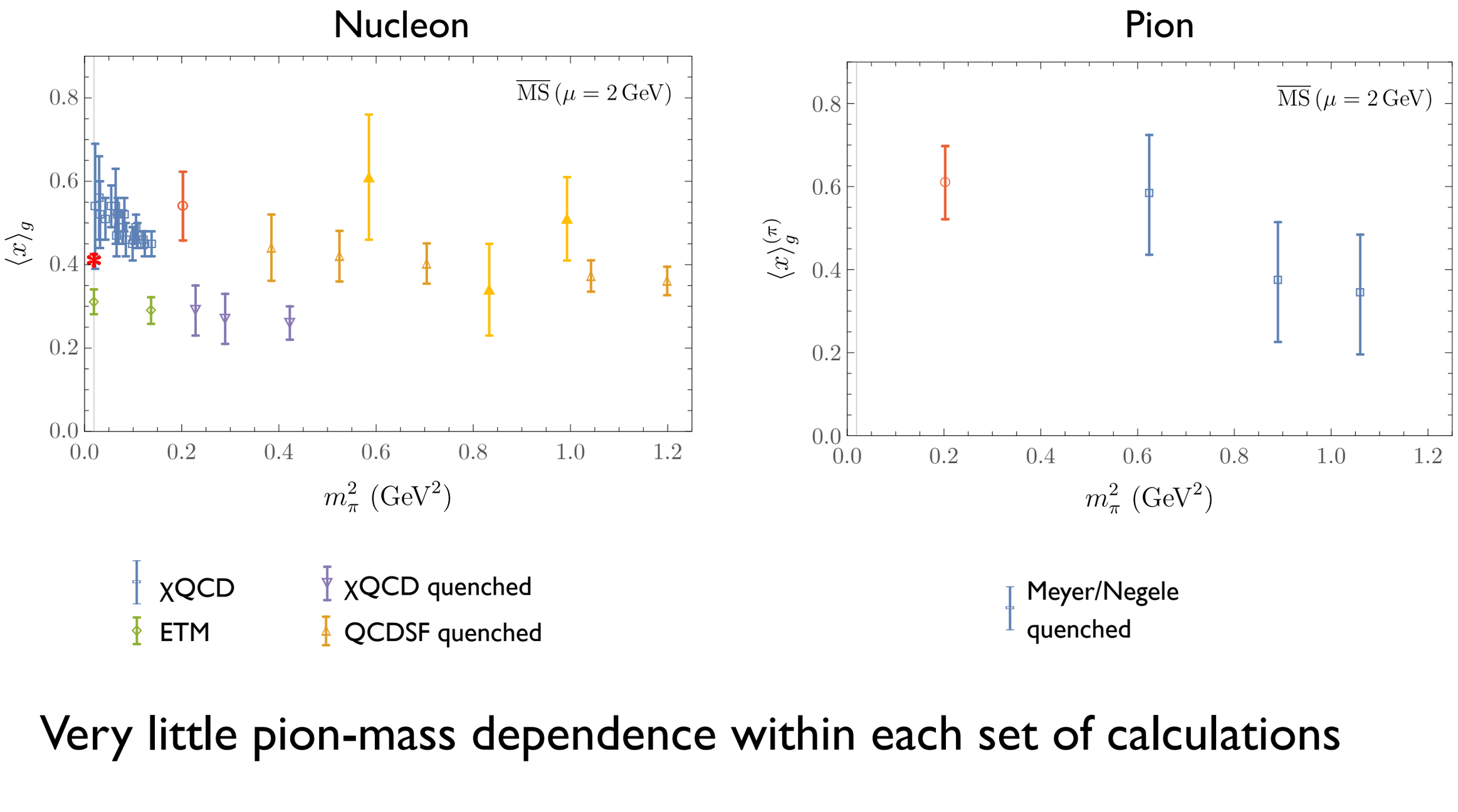


m_π ~450 MeV, tripole fits

gluon: Shanahan, Detmold, PRD 99, 014511 (2019)
quark (nucleon): P. Hägler et al. (LHPC), PRD77, 094502 (2008)
quark (pion): Brommel Ph.D. thesis (2007) m_π ~840 MeV

Gluon momentum fraction

Gluon momentum fraction $A_a(0) = \langle x \rangle_a$



Very little pion-mass dependence within each set of calculations

Gluon structure

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

1

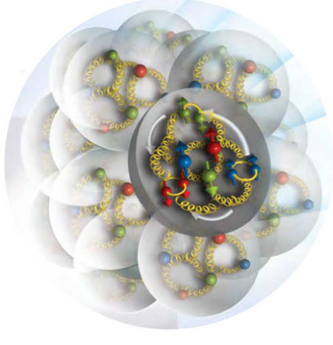
What is the gluon distribution in a proton

- PDFs, GPDs, TMDs
- Pressure, Shear
- 'Gluon radius'

2

How is the gluon structure of a proton modified in a nucleus

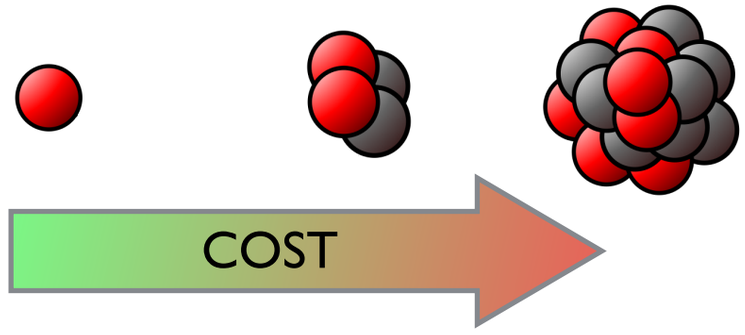
- Gluon 'EMC' effect
- Exotic glue



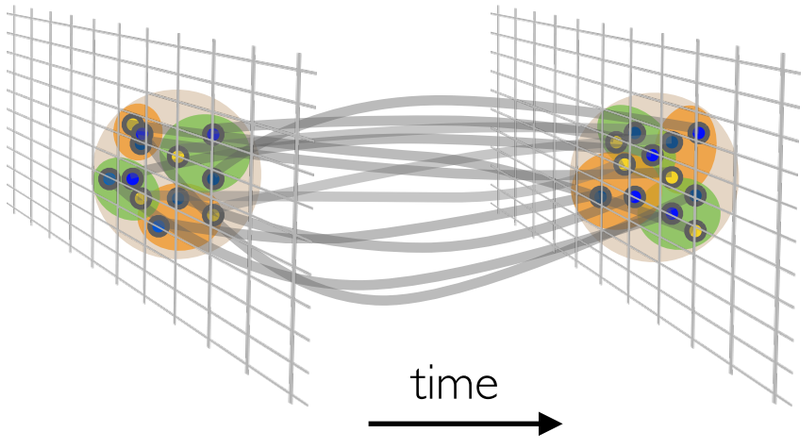
Nuclear physics from LQCD

Nuclei on the lattice: HARD

- **Noise:**
  Statistical uncertainty grows exponentially with number of nucleons



- **Complexity:**
  Number of contractions grows factorially



Calculations possible for A<5 (unphysically heavy quark masses)

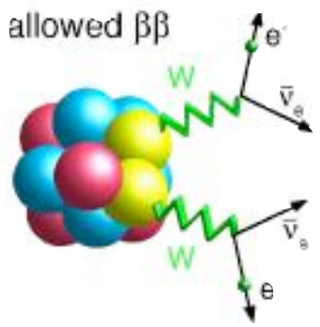
Unphysical nuclei

- Nuclei with A<5
- QCD with unphysical quark masses

  m_π~800 MeV, m_N~1,600 MeV

  m_π~450 MeV, m_N~1,200 MeV

- Proton-proton fusion and tritium β-decay
  [PRL **119**, 062002 (2017)]
- Double β-decay
  [PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)]

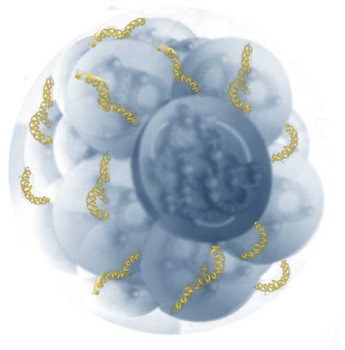


- Nuclear structure: magnetic moments, polarisabilities

  [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]

- First nuclear reaction: np→dγ
  [PRL **115**, 132001 (2015)]

- Gluon structure of light nuclei
  [PRD **96** 094512 (2017)]
- Scalar, axial and tensor MEs
  [PRL **120,** 152002 (2018)]



NPLQCD

Gluon structure of nuclei

2 **How does the gluon structure of a nucleon change in a nucleus?**

Ratio of structure function F_2 per nucleon for iron and deuterium

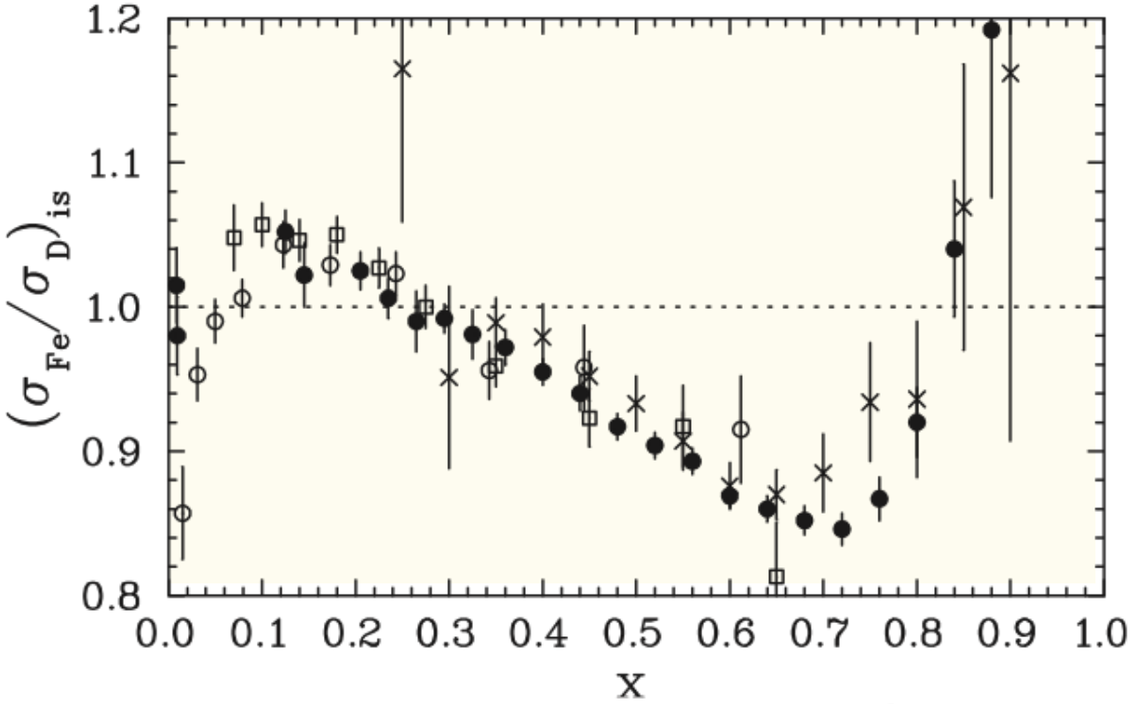
$$F_2(x,Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2\, [q(x,Q^2) + \overline{q}(x,Q^2)]$$

Number density of partons of flavour q

European Muon Collaboration (1983): "EMC effect"

Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?



Longitudinal momentum fraction

Nuclear glue, m_π ~450 MeV

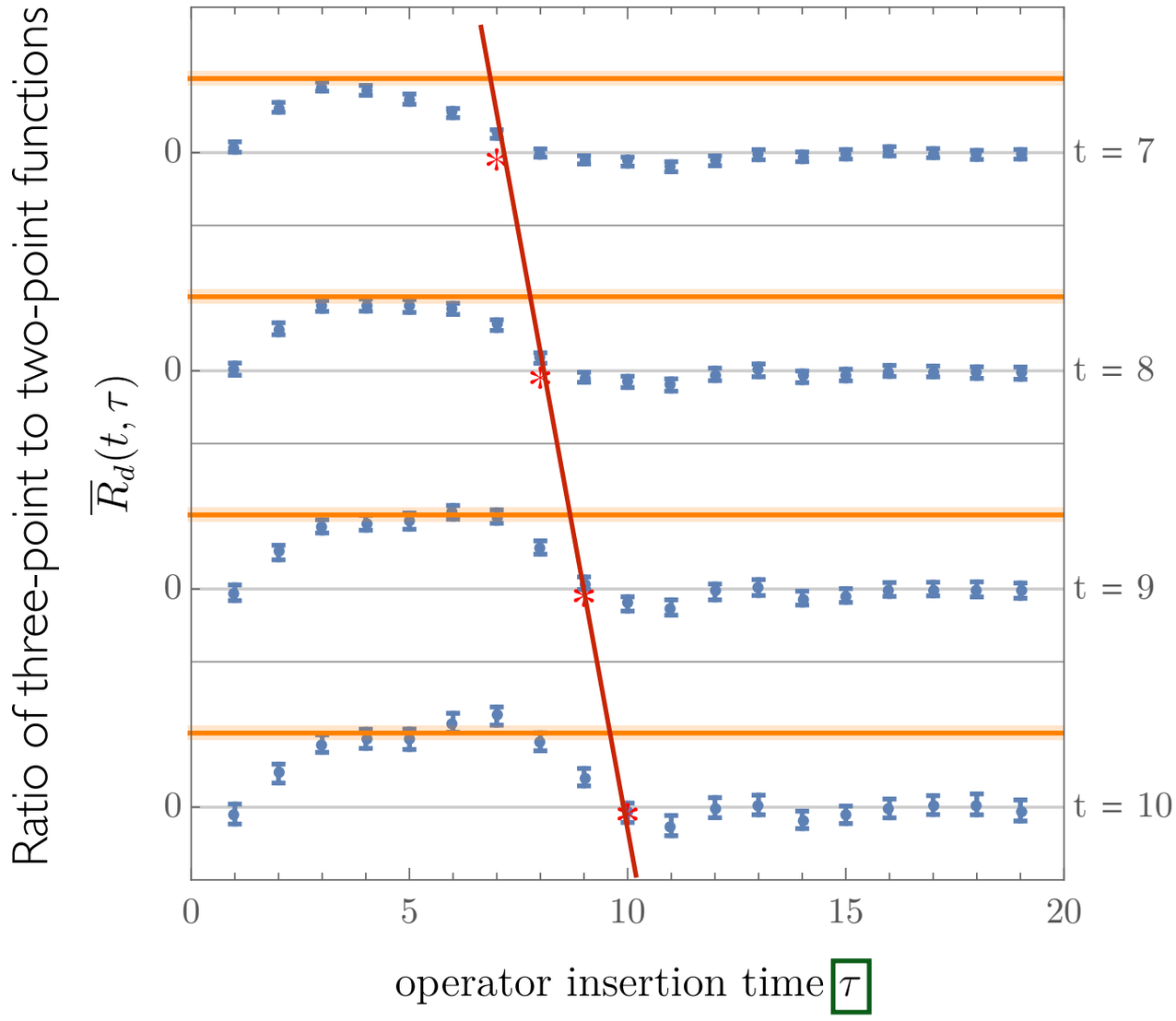
Look for **nuclear (EMC-type) effects** in the first moments of the spin-independent gluon structure function

Doubly challenging

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)

Deuteron gluon momentum fraction

Ratio $\propto$ matrix element for $0 \ll \boxed{\tau} \ll \boxed{t}$



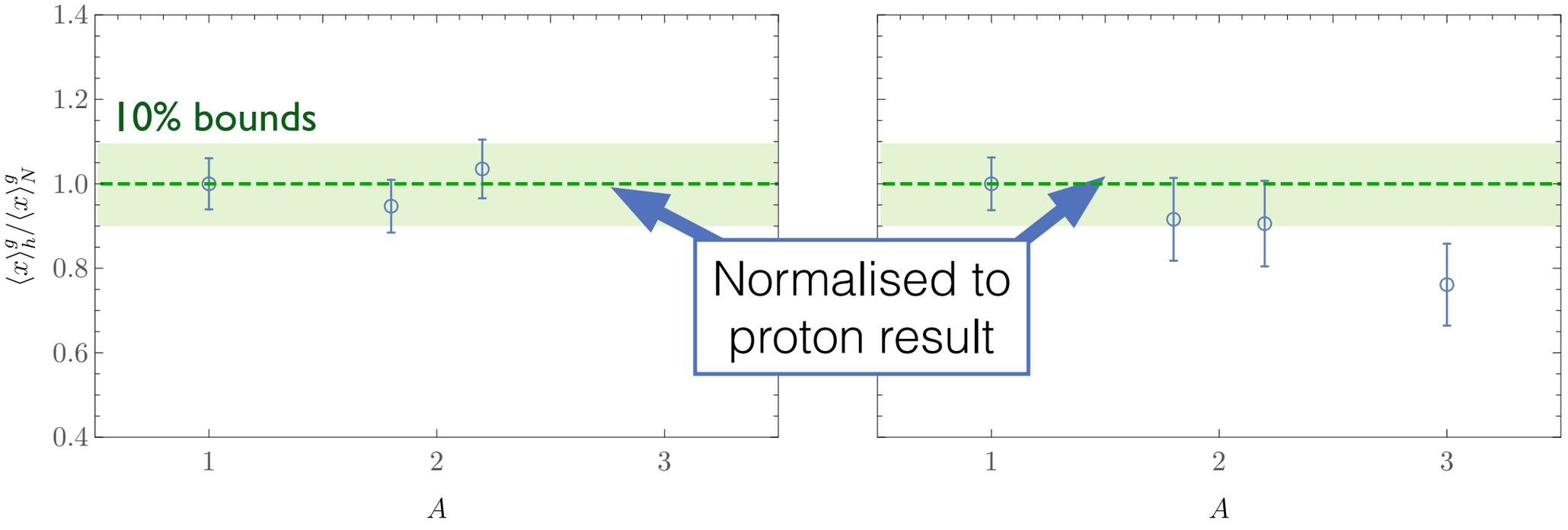
NPLQCD Collaboration PRD96 094512 (2017)

Gluon momentum fraction

NPLQCD Collaboration PRD96 094512 (2017)

- Matrix elements of the **spin-independent gluon operator** in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators

Ratio of gluon momentum fraction in nucleus to nucleon



m_π ~450 MeV

m_π ~800 MeV

Gluon structure of nuclei

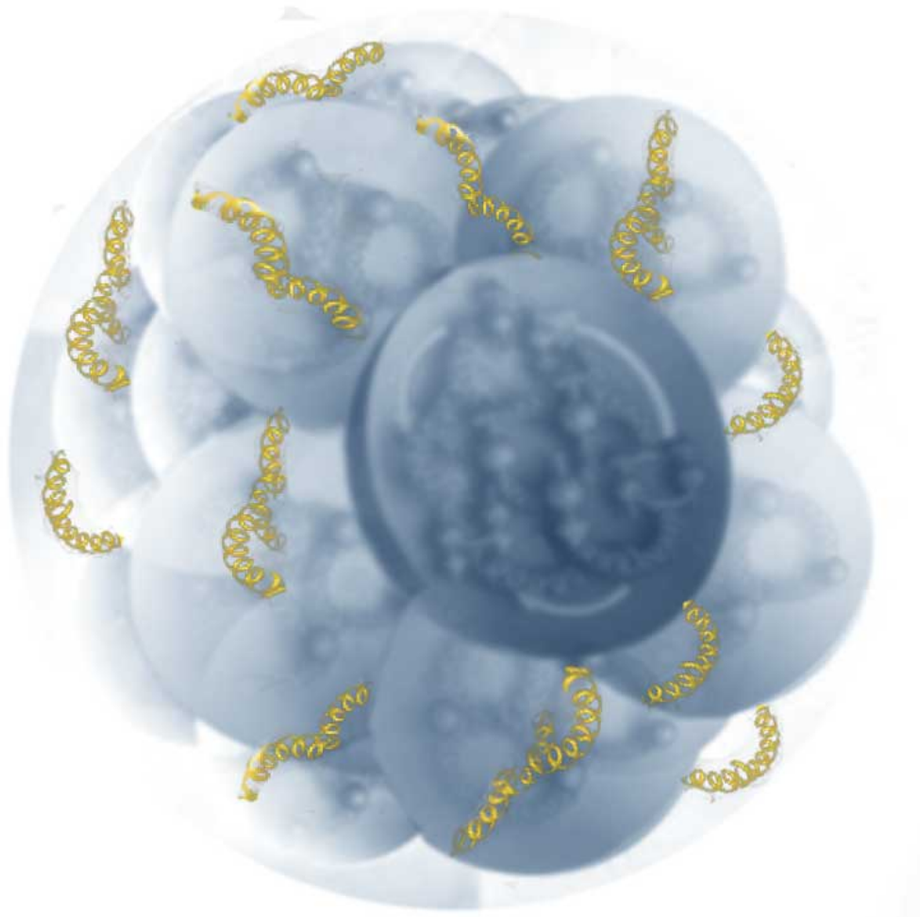
Exotic Glue

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$

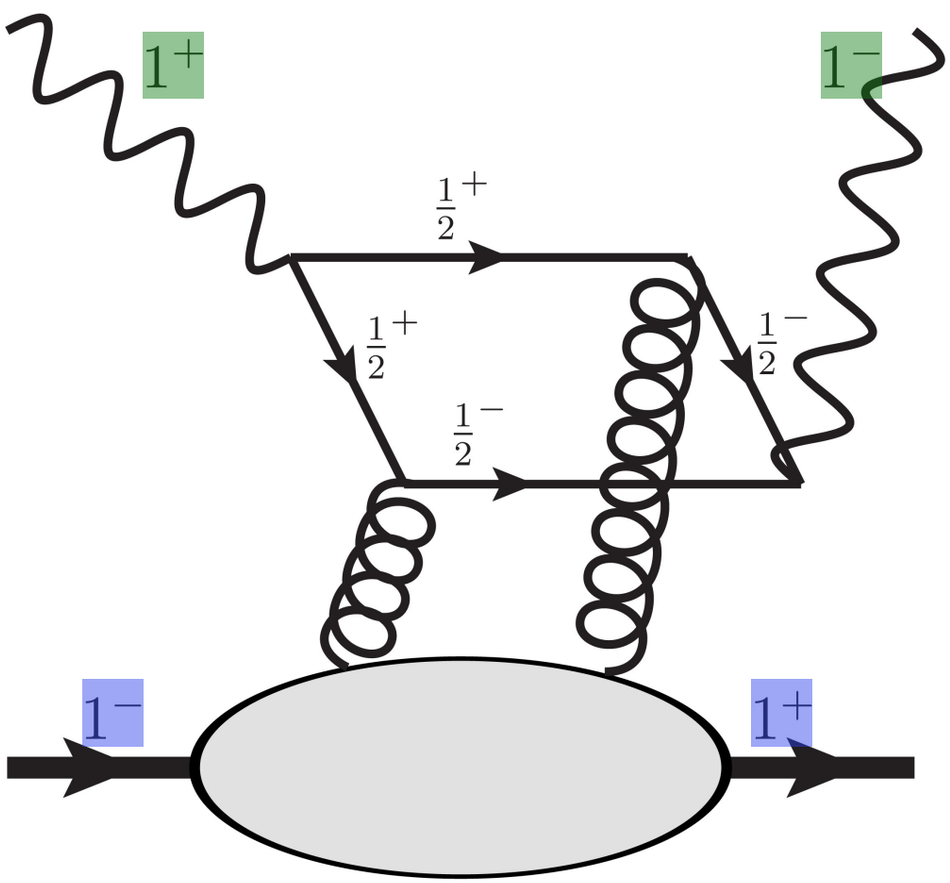


Jaffe and Manohar, "Nuclear Gluonometry"
Phys. Lett. B223 (1989) 218

Gluonic Transversity

Double helicity flip structure function $\Delta(x,Q^2)$

Changes both photon and target helicity by 2 units



- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin≥ 1
- **Experimentally measurable** in unpolarised electron DIS
  on polarised target
  - Nitrogen target: JLab LoI 2015
  - Polarised nuclei at EIC
- **Moments calculable in LQCD**

Non-nucleonic glue in deuteron

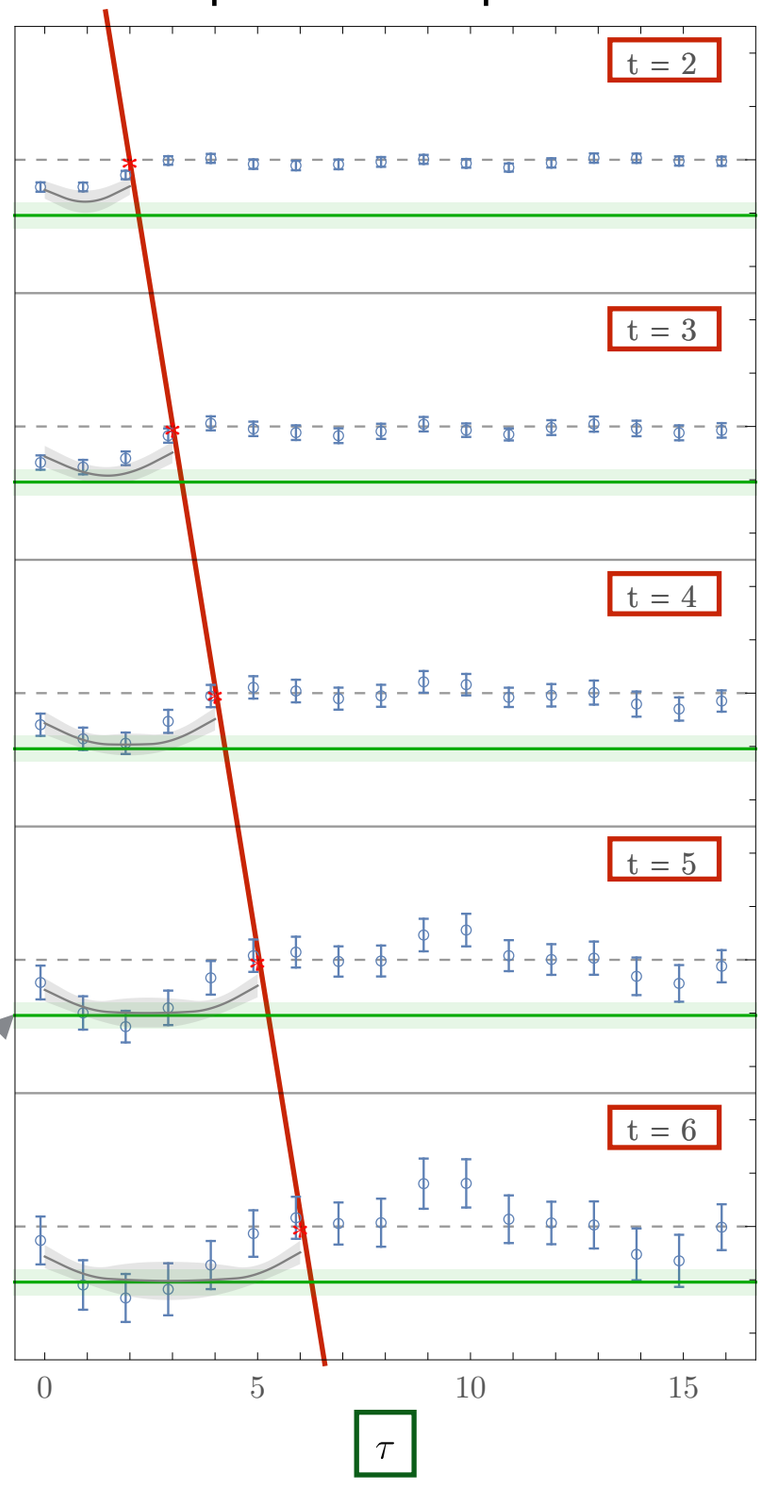
NPLQCD Collaboration PRD96 094512 (2017)

First moment of gluon transversity distribution in the deuteron, m_π ~800 MeV

- **First evidence for non-nucleonic gluon contributions to nuclear structure**
- **Hypothesis of no signal ruled out to better than one part in 10^7**
- Magnitude relative to momentum fraction as expected from large-N_c

Ratio ∝ matrix element for $0 \ll \tau \ll t$

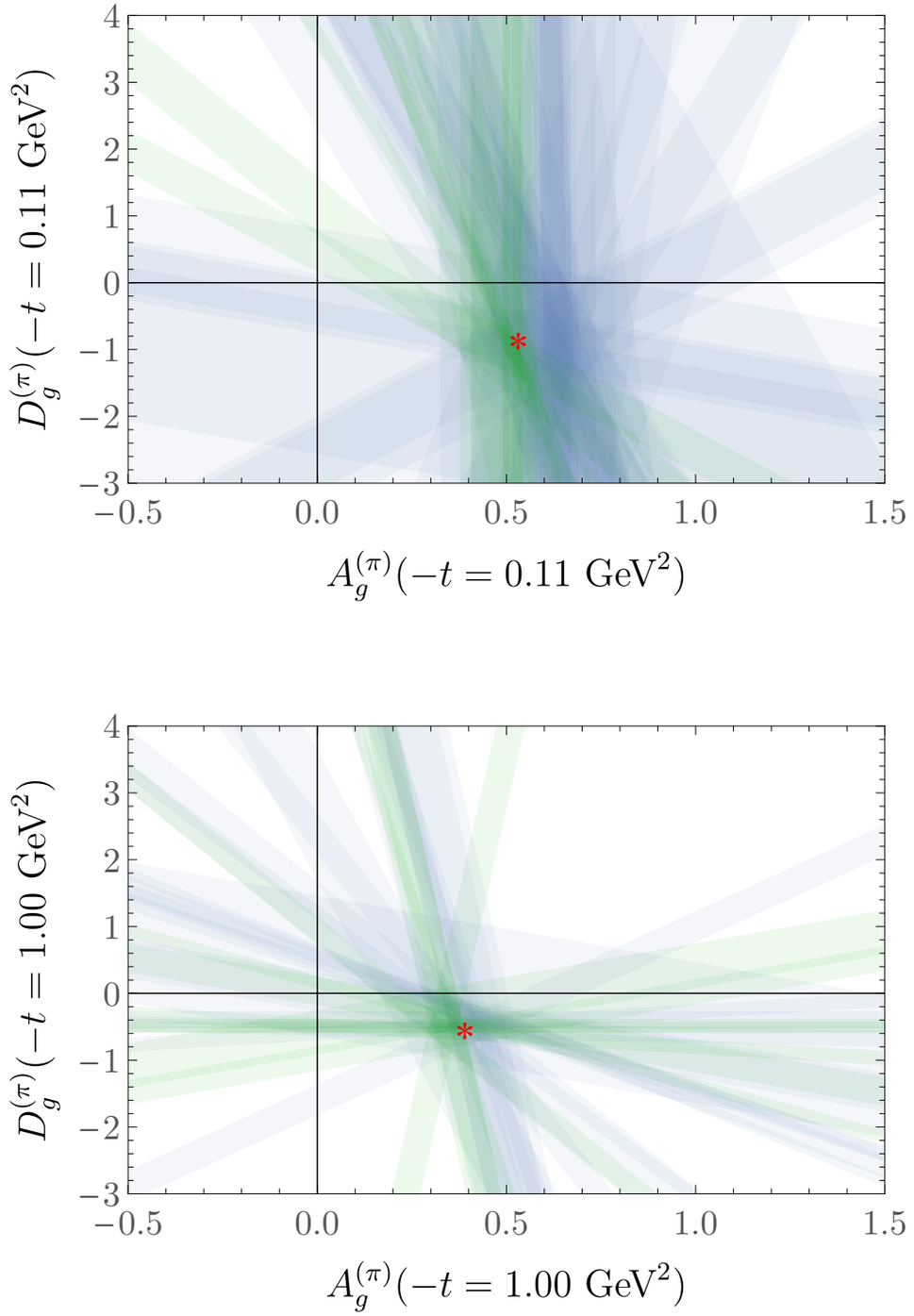
Ratio of 3pt and 2pt functions



Gluon structure from LQCD

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
  - **First determination of gluon contributions to shear and pressure distributions in the proton**
    - Supports analysis assumptions in recent experimental determination
    - Suggests target kinematics for future model-independent extractions at JLab12 and EIC
  - Compare quark and gluon distributions in hadrons and nuclei
- **Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature**

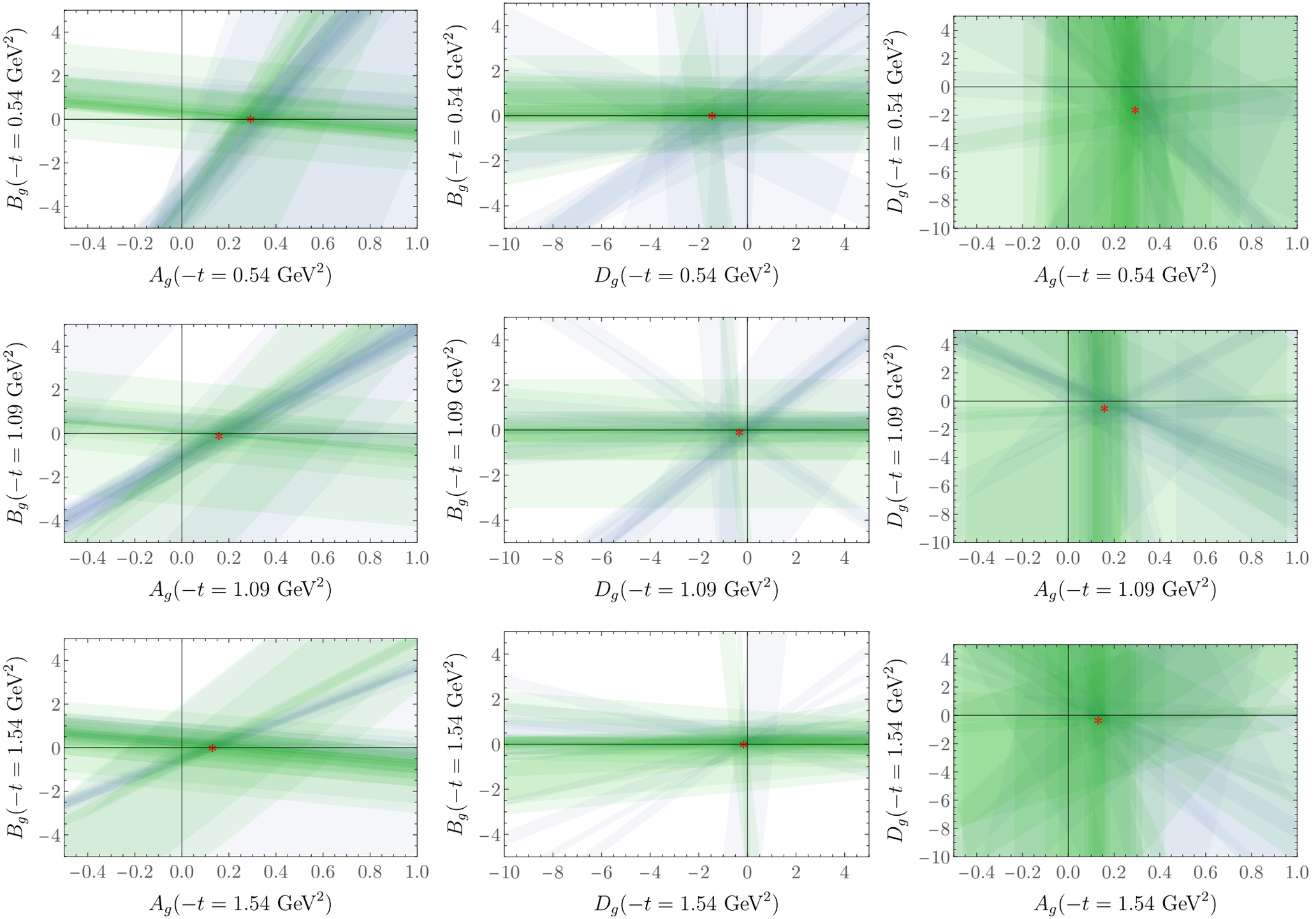
Gluon GFFs from LQCD



Uncertainties from renormalisation not shown

$\tau_1^{(3)}$ $\tau_3^{(6)}$

Gluon GFFs from LQCD



Cross-sections: GFF not shown in each projection taken to its central value

$\tau_1^{(3)}$

$\tau_3^{(6)}$