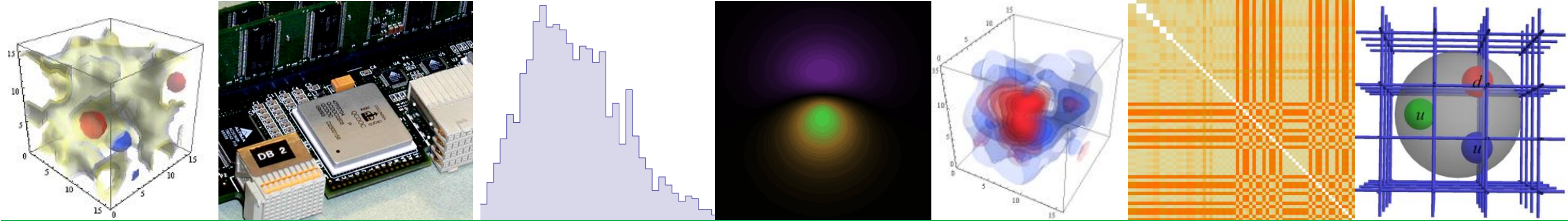


LP³ qPDF Results

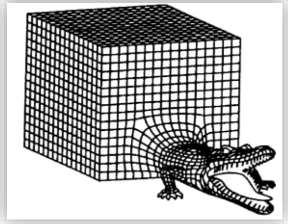
HUEY-WEN LIN

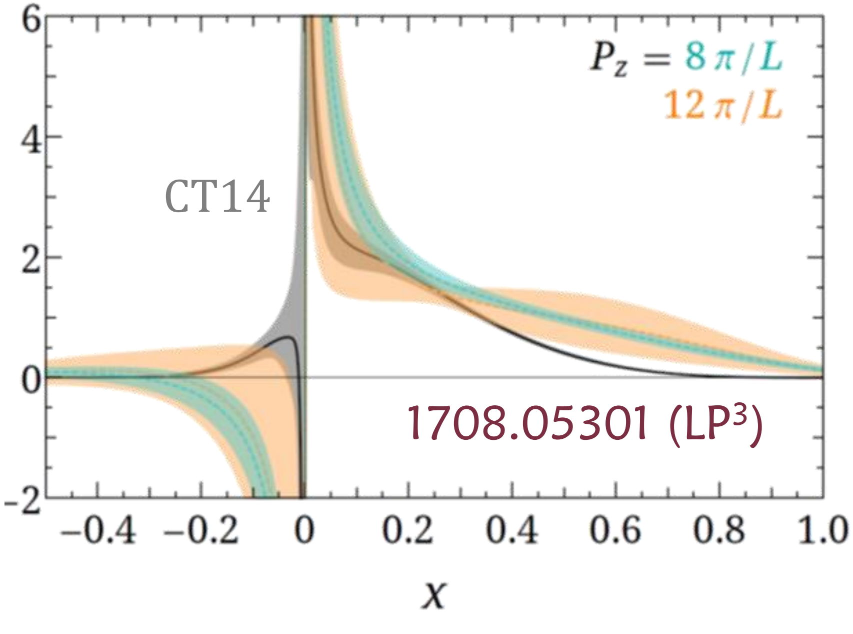
This work is supported by NSF under grant PHY 1653405
"CAREER: Constraining Parton Distribution Functions for New-Physics Searches"

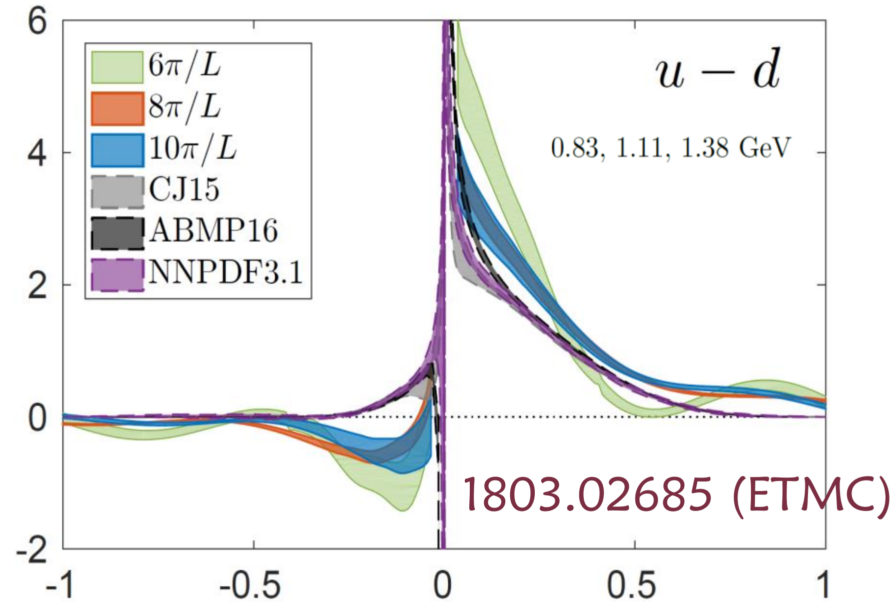
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





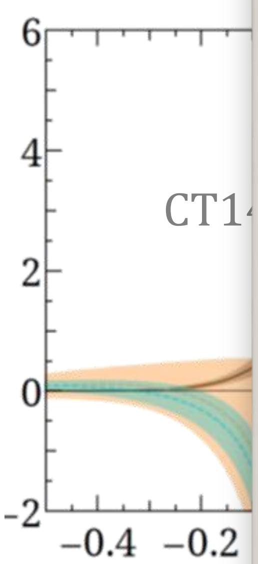


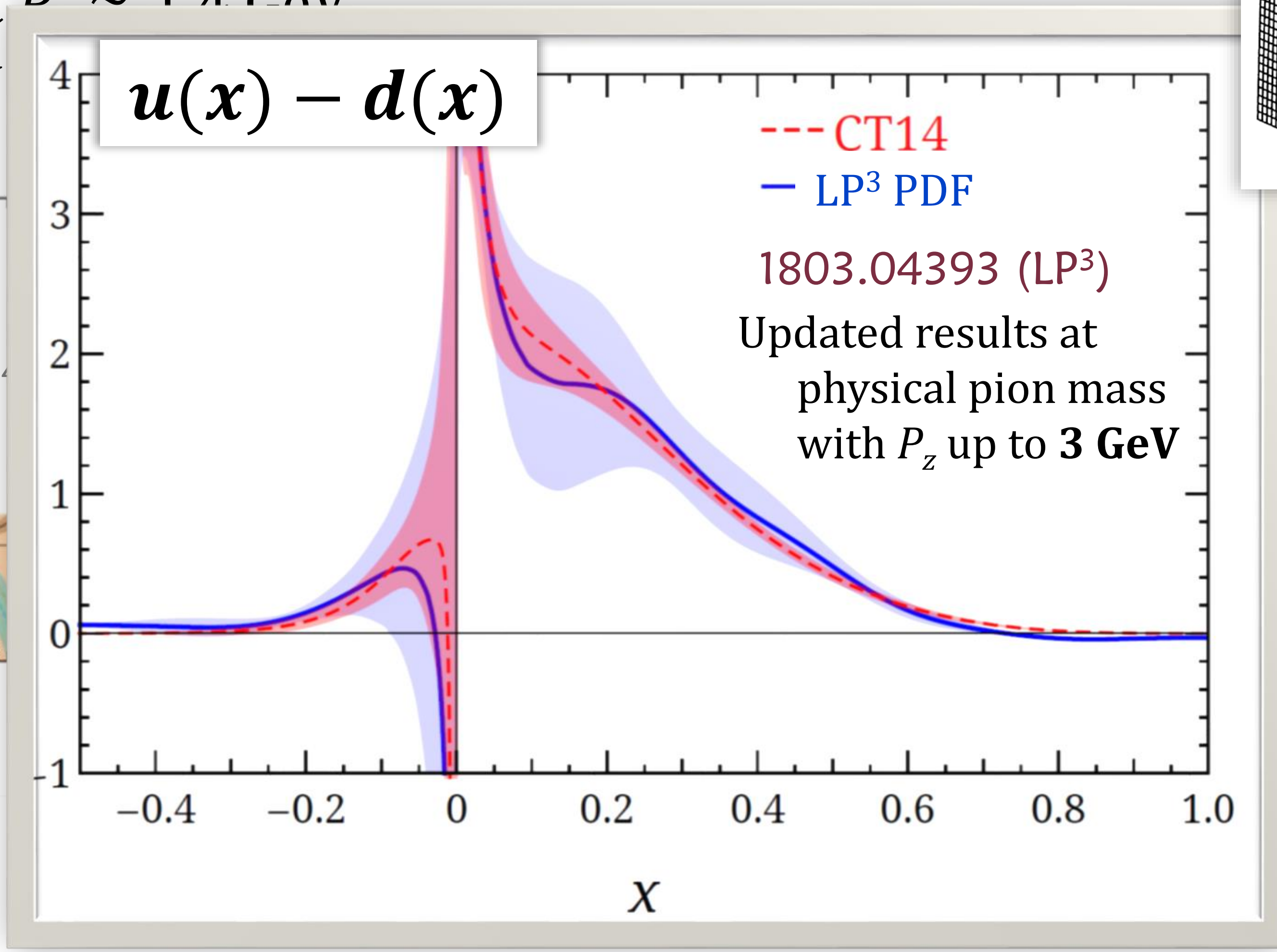
Not using parametrization (e.g. $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$)
Less pretty results; less likely to exactly coincide with global fits.

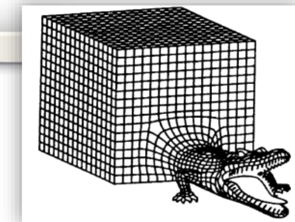
Physical Pion Mass Results

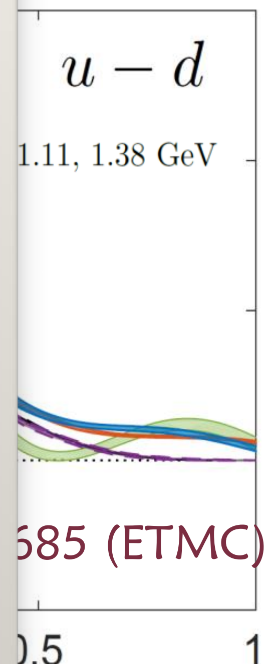
§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Study









Physical Pion Mass Results

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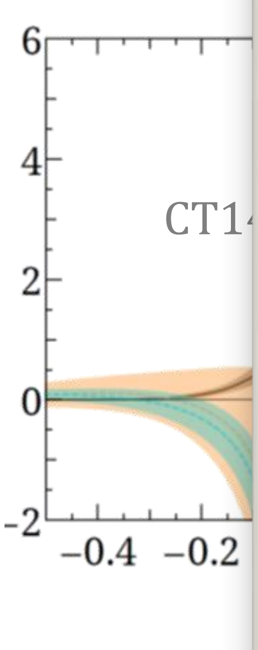
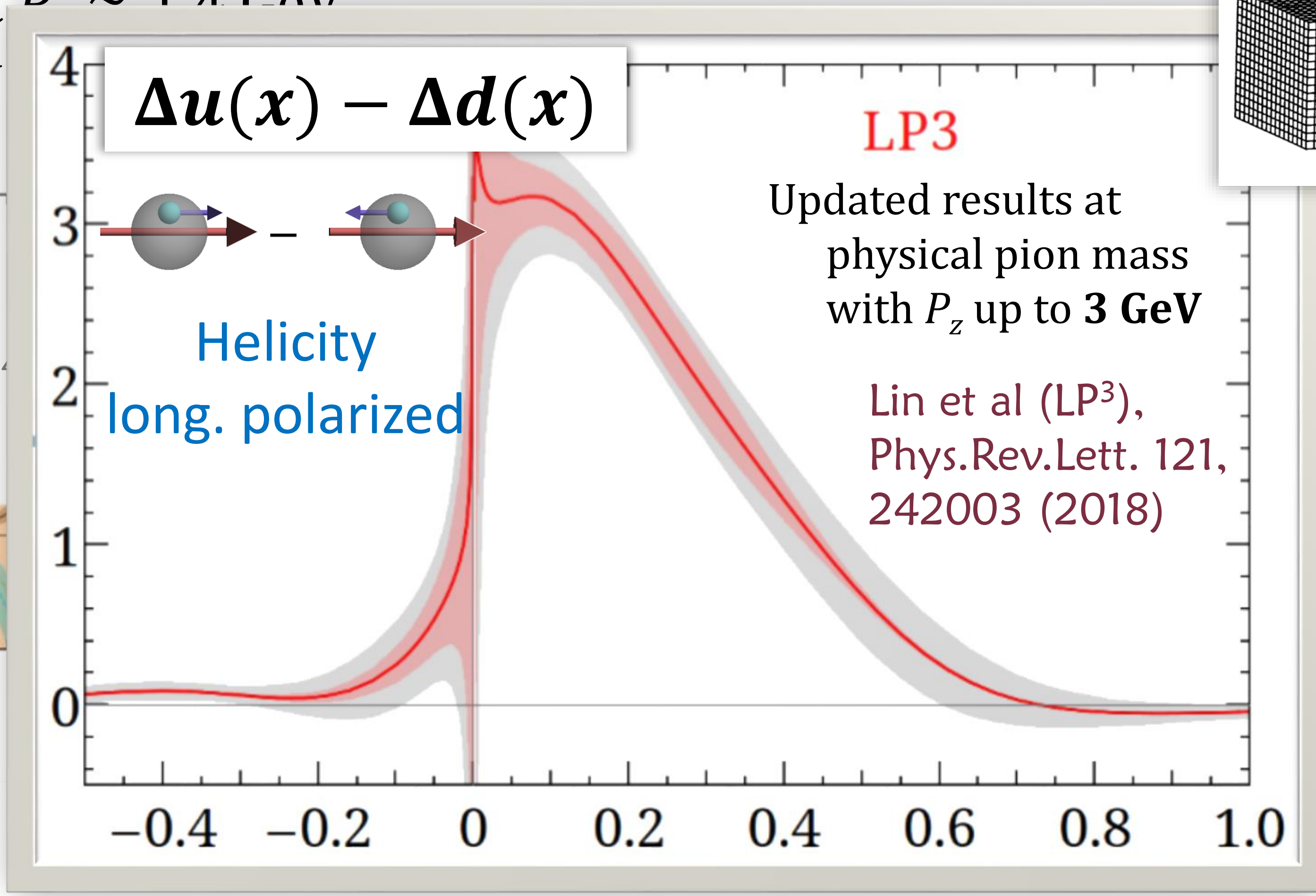
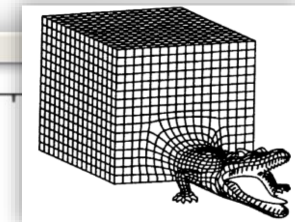
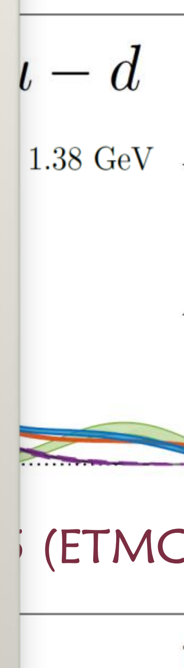
- Boost $P_z \sim 1.4$ GeV
- Study









Physical Pion Mass Results

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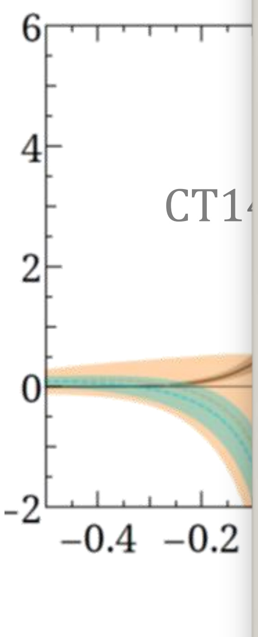
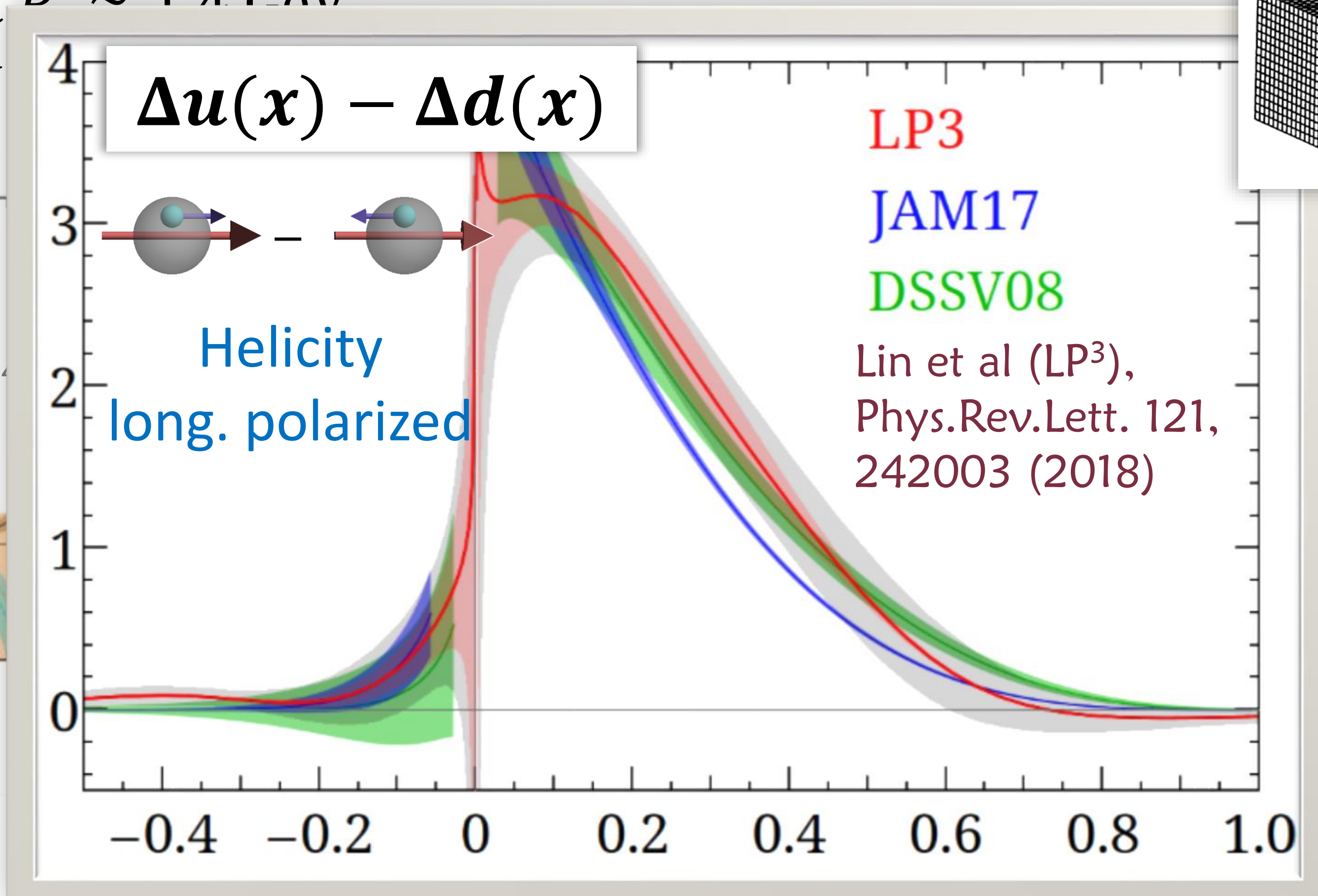
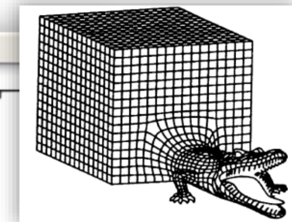
- Boost $P_z \sim 1.4$ GeV
- Study



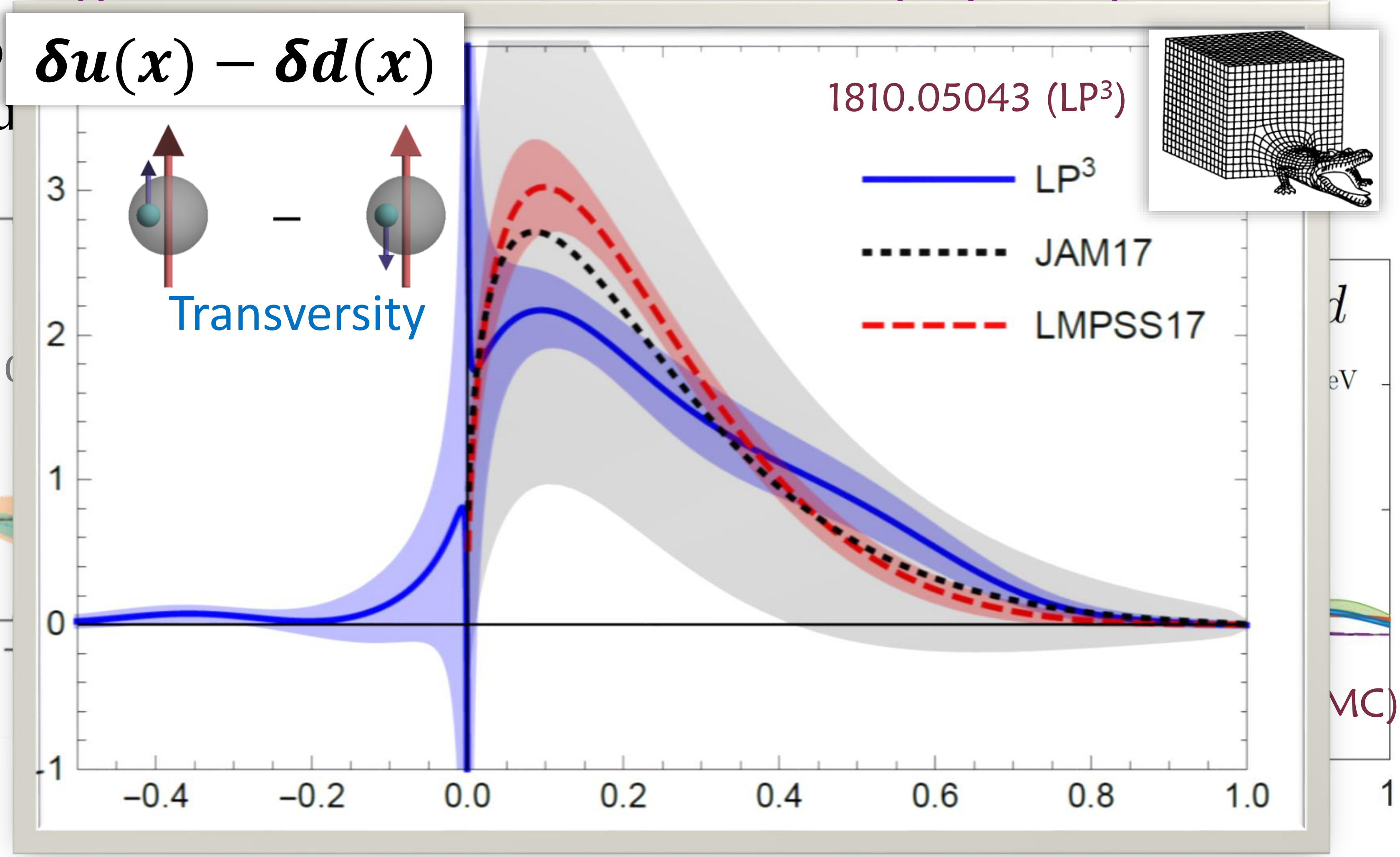




Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Bo
- Stu



Physical Pion Mass Results

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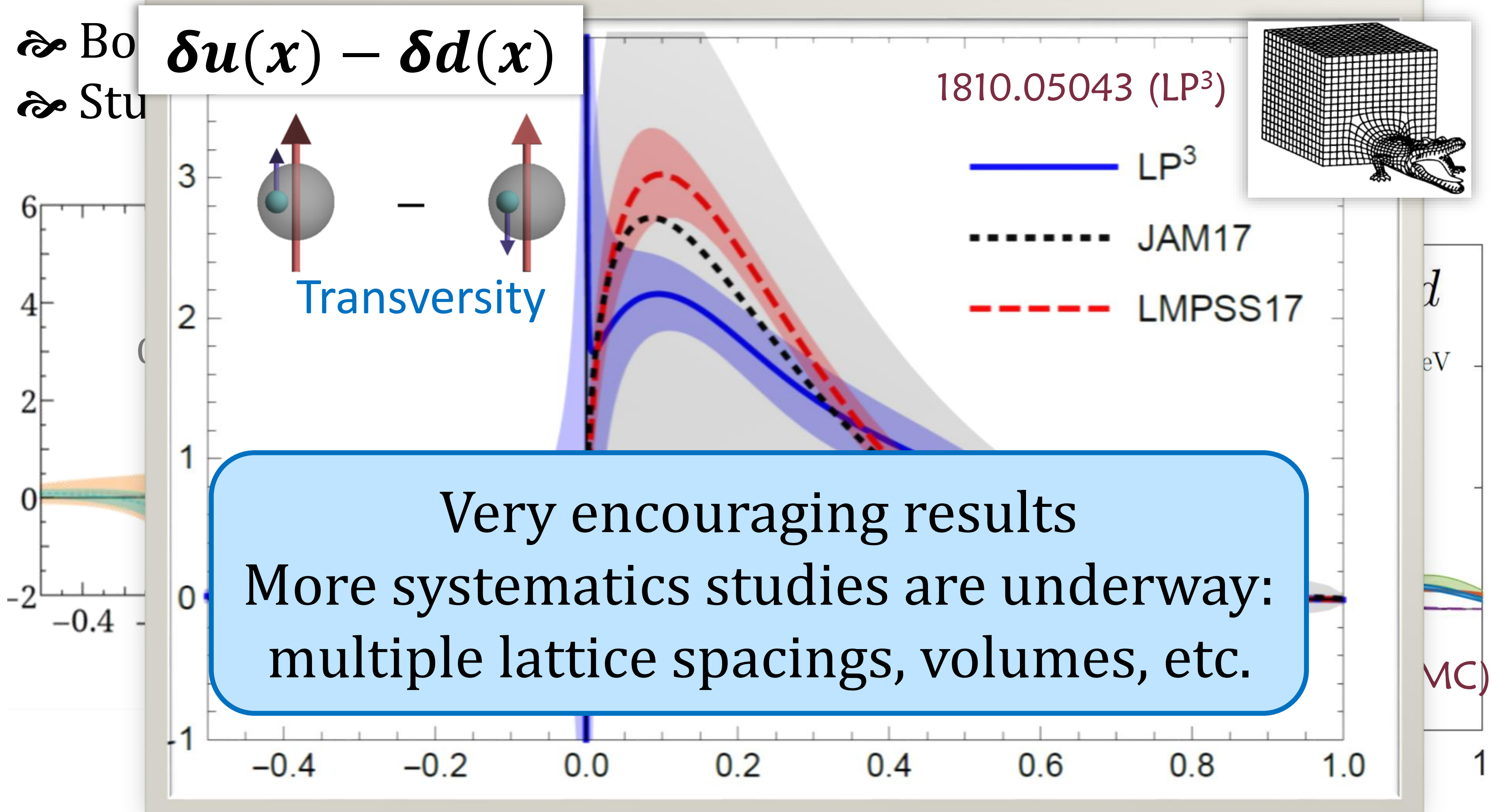
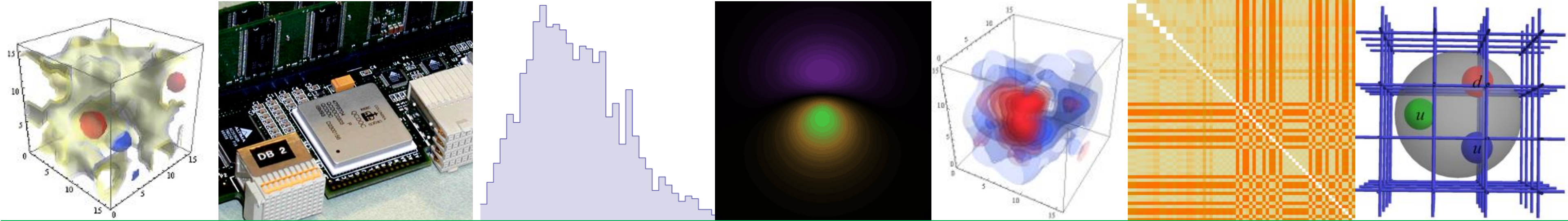


Very encouraging results
More systematics studies are underway:
multiple lattice spacings, volumes, etc.



MythBusting
Criticism of LP³ qPDF Calculations

Huey-Wen Lin

This work is supported by NSF under grant PHY 1653405
“CAREER: Constraining Parton Distribution Functions for New-Physics Searches”

Outline

Apologize in advance for a rather technical talk

Need to clear up many false rumors

- § The need for large boost momentum
  - Nothing to do with lattice
- § Please stop using plateau fit
  - What's the point of getting many wrong answers?
- § I will take more criticism
  - As time allows

The Necessity of Large Boosted Momentum

without modeling

Myth

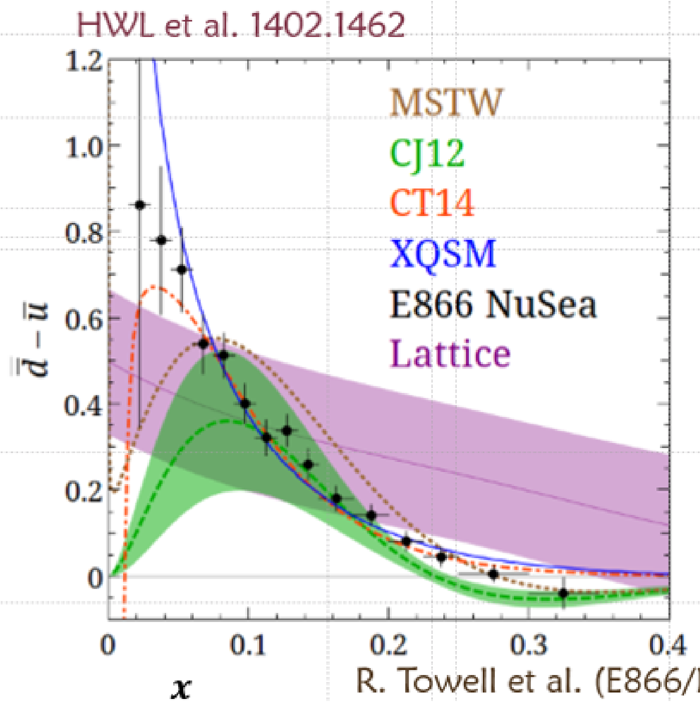
One can recover the full PDF using boost momentum below 2 GeV.

Backstory

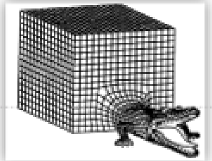
- Many of you are old enough to remember this:

Sea Flavor Asymmetry

- First time in LQCD history to study antiquark distribution!
  - $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**



$\bar{q}(x) = -q(-x)$



Lost resolution in small-x region

Future improvement: larger lattice volume

$$\int dx\, \left(\bar{u}(x) - \bar{d}(x)\right) \approx -0.16(7)$$

Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

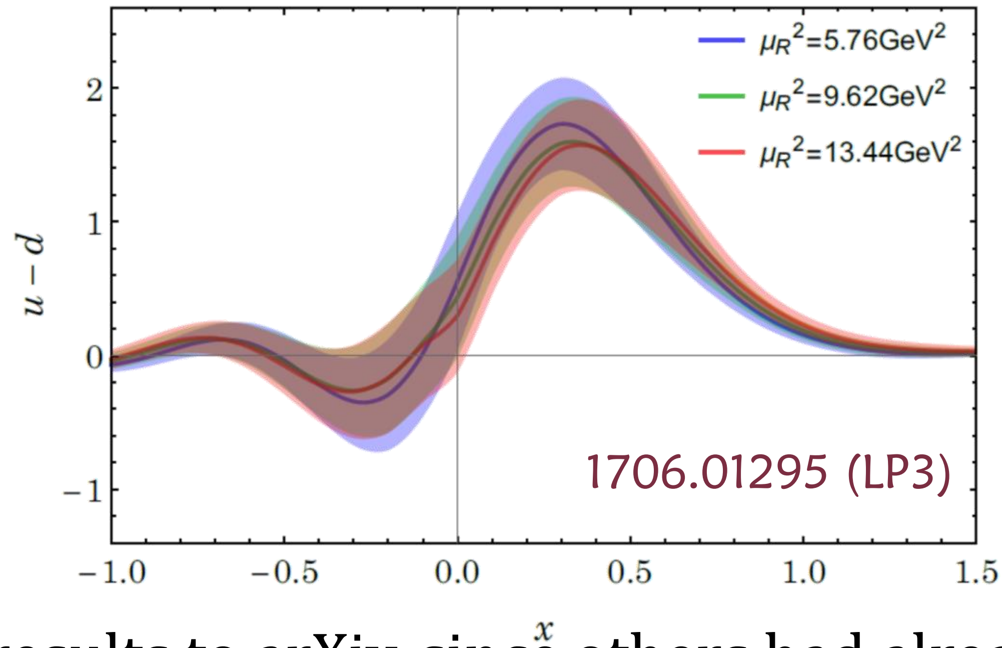
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Caveat: These matrix elements are not properly renormalized

Backstory

§ Efforts by multiple collaborations have been devoted into working on lattice renormalization

- We finally obtained the renormalized ME, and the renormalized PDF results puzzled us for months!

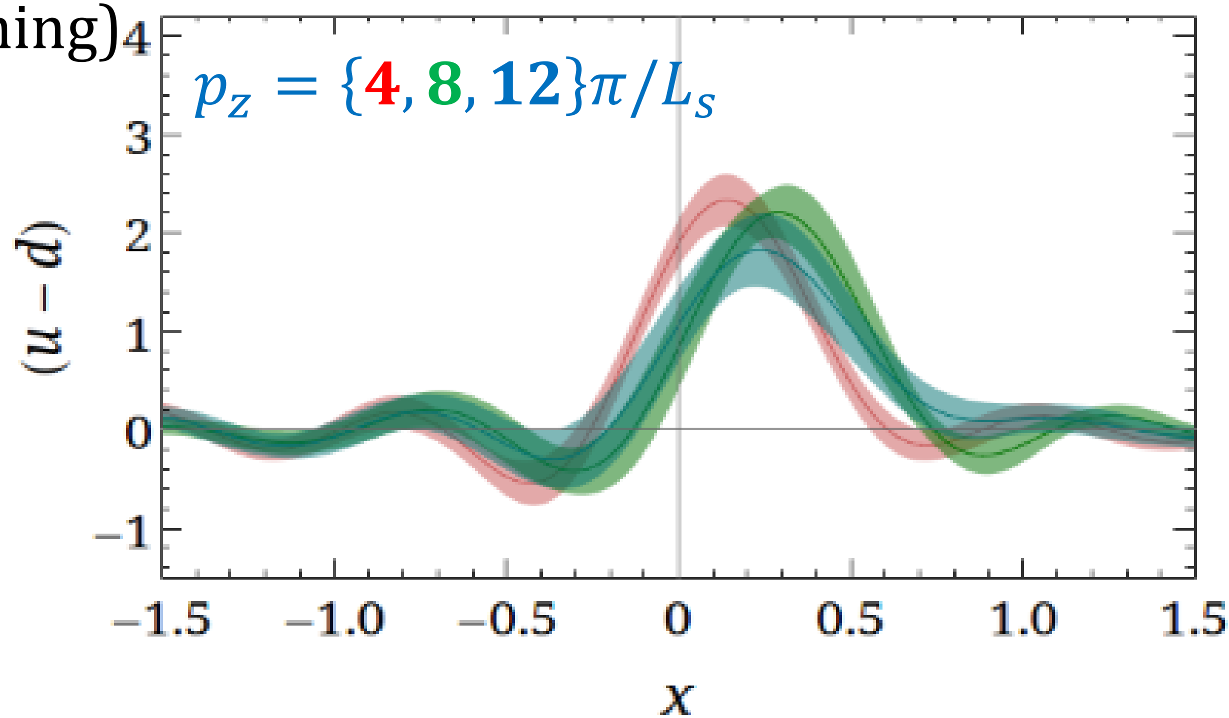


- We finally posted the results to arXiv, since others had already posted their renormalized result

Backstory

§ Immediately, we checked a different lattice and observed the same thing and worse, since we had more “z” data!

- Results from 2017 Summer at physical pion mass (before matching)

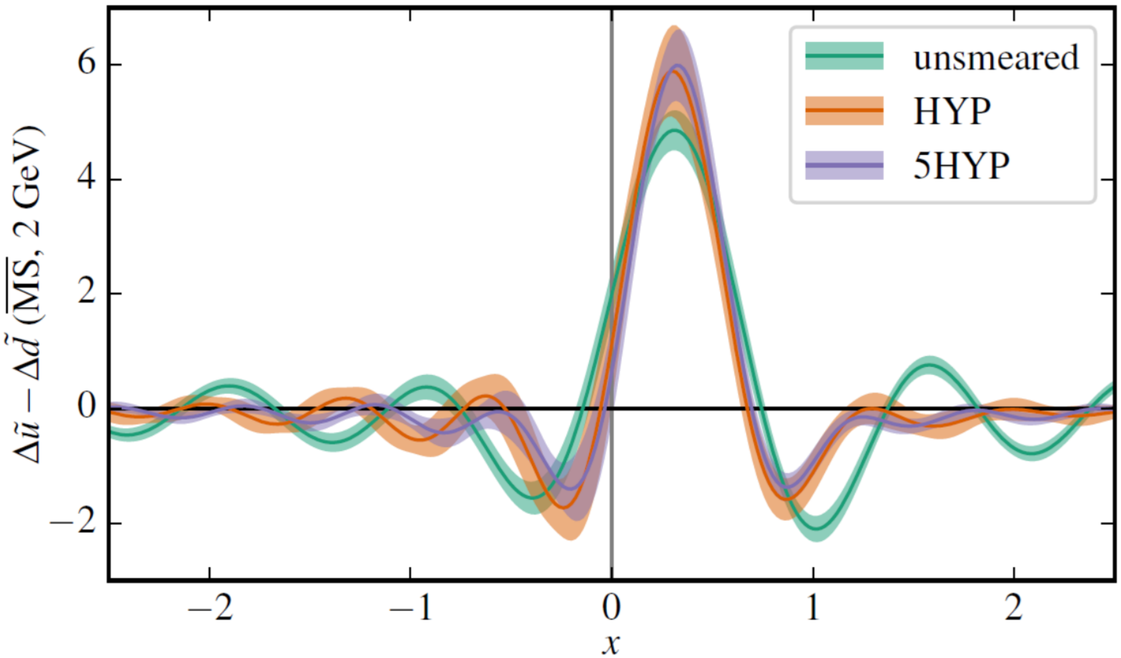


Backstory

§ Something is obviously wrong!

- This effect was missed by earlier ETMC work due to the small *z* and momentum combination

J. Green et al 1707.07152

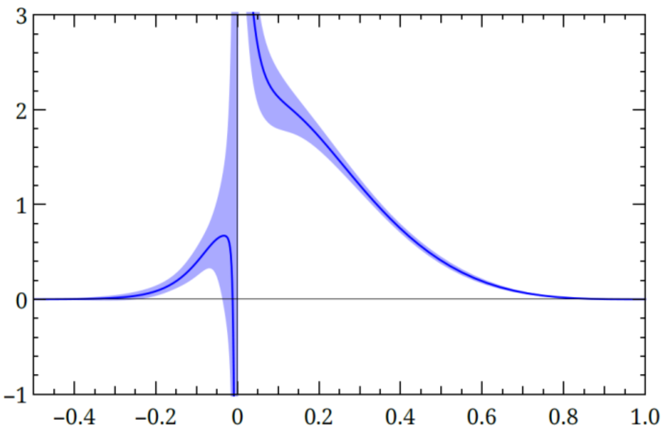


- We called it “oscillation” and F.T. truncation artifacts
- For a while, people had no idea what we are talking about

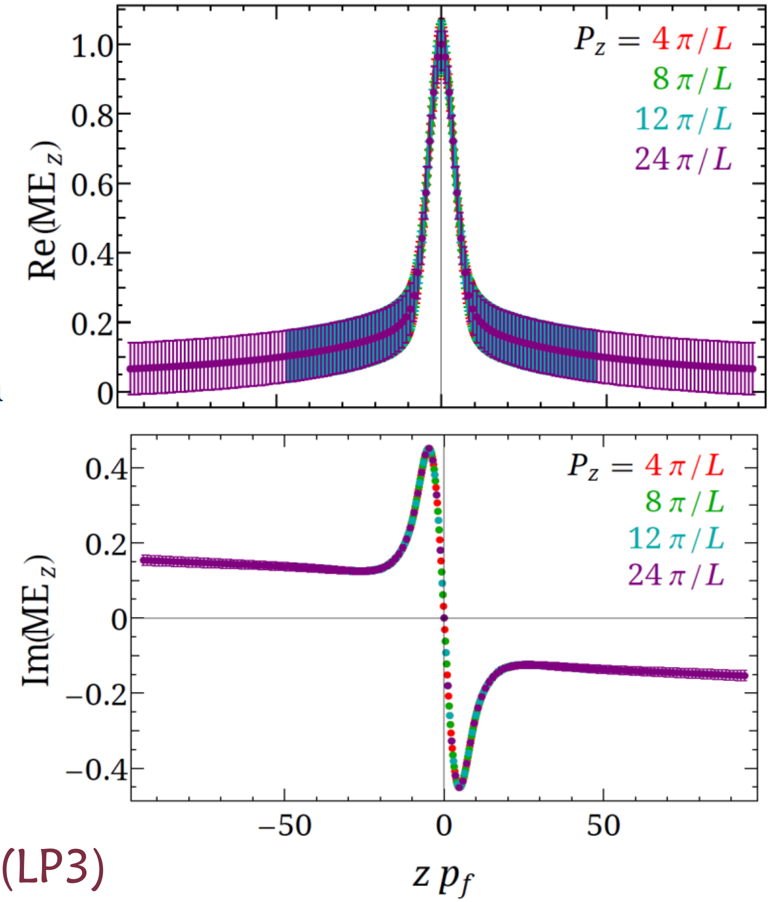
Focus on Continuum

- Not a lattice problem but a Fourier-transform issue
- Simple exercise with CT14 PDF 1506.07443

Pick your favorite PDF (CT14 here)



Fourier to coordinate space at some momentum

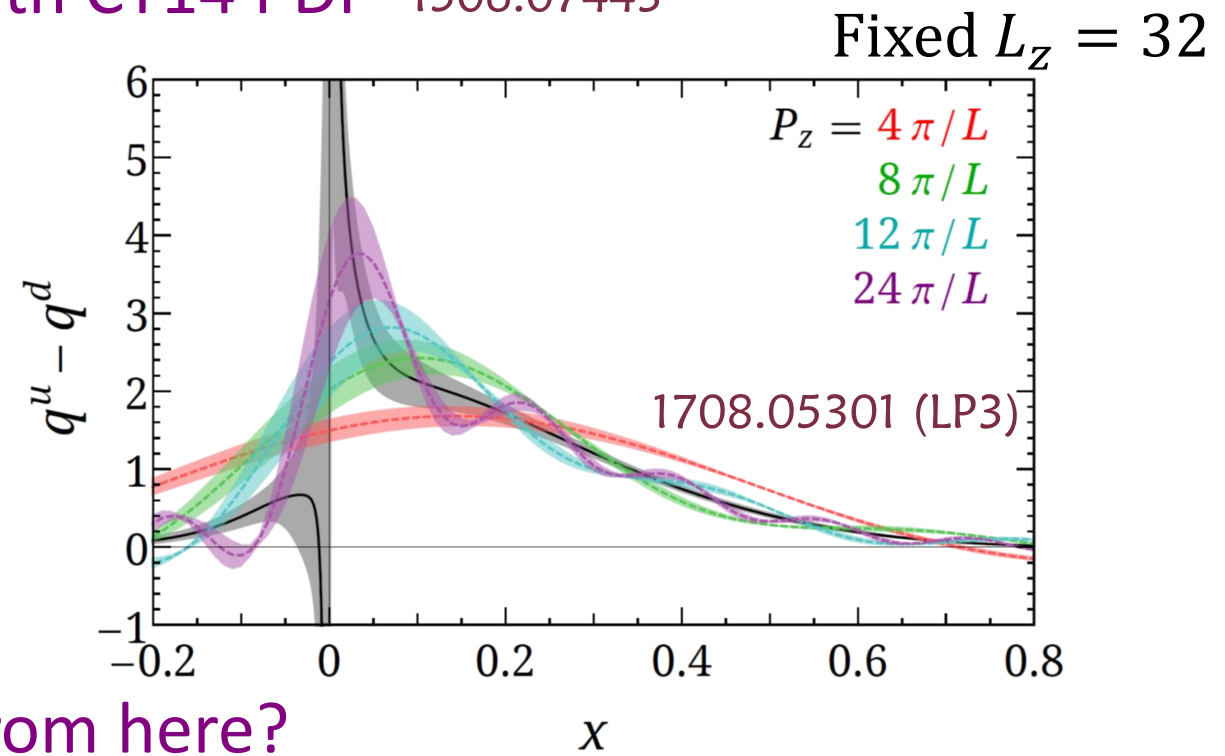


1708.05301 (LP3)

Focus on Continuum

- § Not a lattice problem but a Fourier-transform issue
- § Simple exercise with CT14 PDF 1506.07443

Inverse Fourier transform back to momentum space

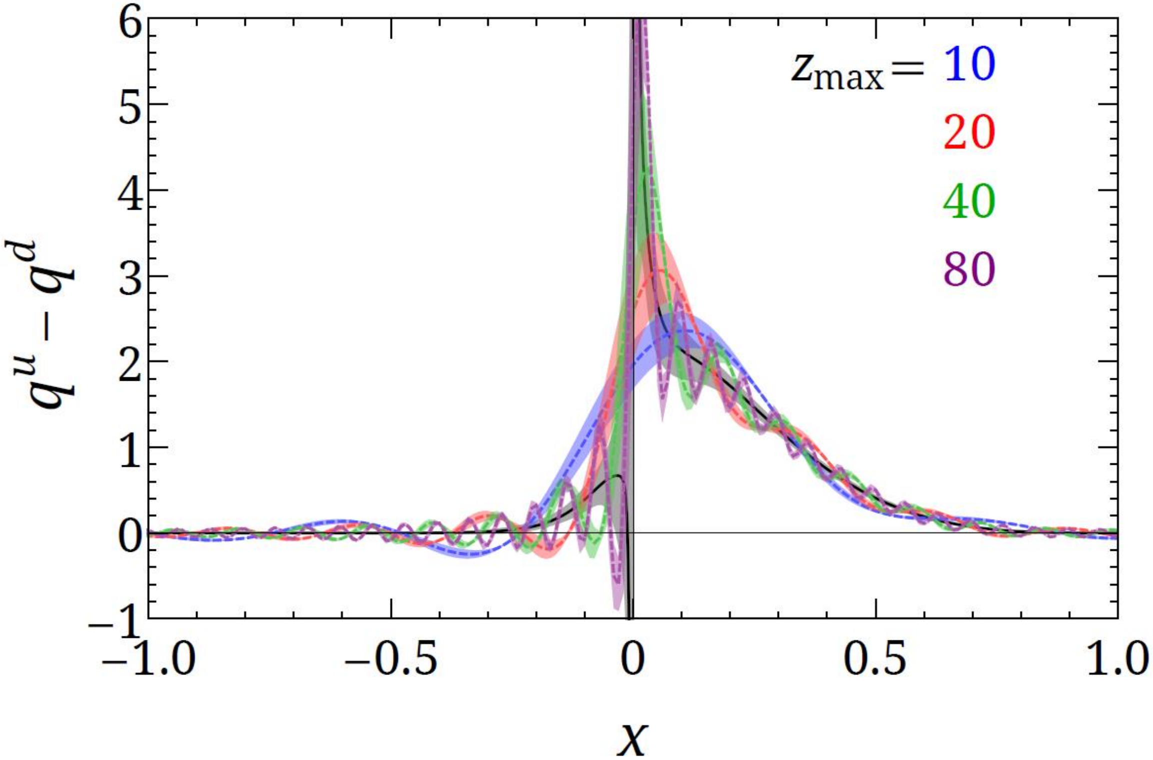


- § What do you do from here?
  - Throw up your hands and add 100% errorbars across all x
  - Find some way to salvage as much information as possible

Continuum

§ Fix P_z and use large z_{max}?

Fixed $P_z = 24\pi/L$



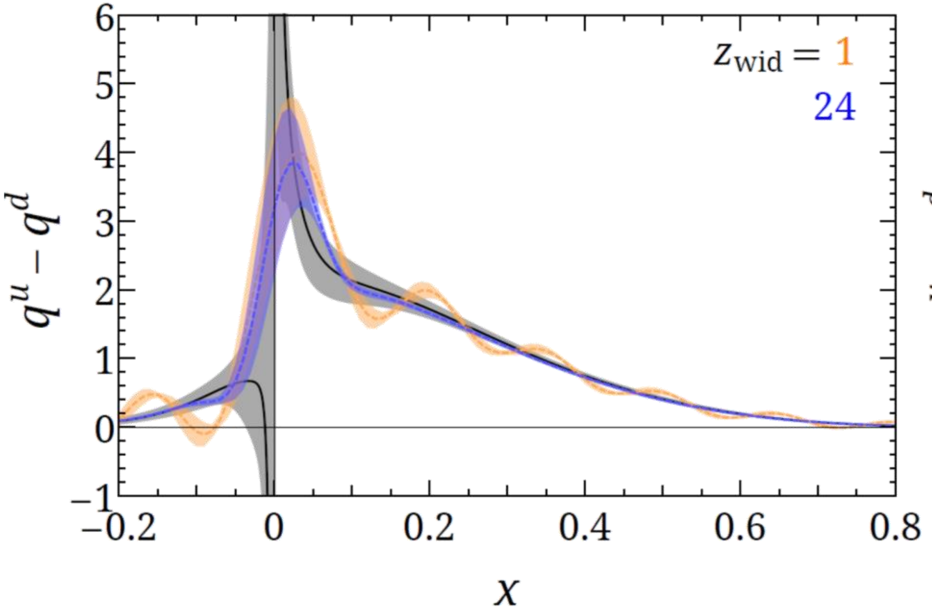
1708.05301 (LP[3])

Continuum

- Luckily, we know the answer
- Two possible solutions proposed (likely more) 1708.05301 (LP[3])

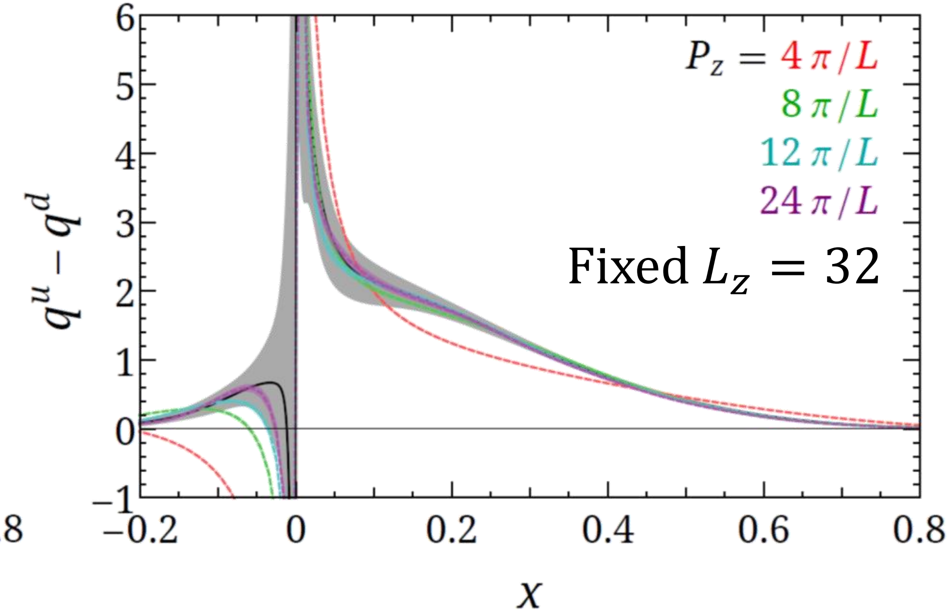
Filter approach

$$F(z_{\text{lim}}, z_{\text{wid}}) = \frac{1+\text{erf}\left(\frac{z+z_{\text{lim}}}{z_{\text{wid}}}\right)}{2}\,\frac{1-\text{erf}\left(\frac{z-z_{\text{lim}}}{z_{\text{wid}}}\right)}{2}$$

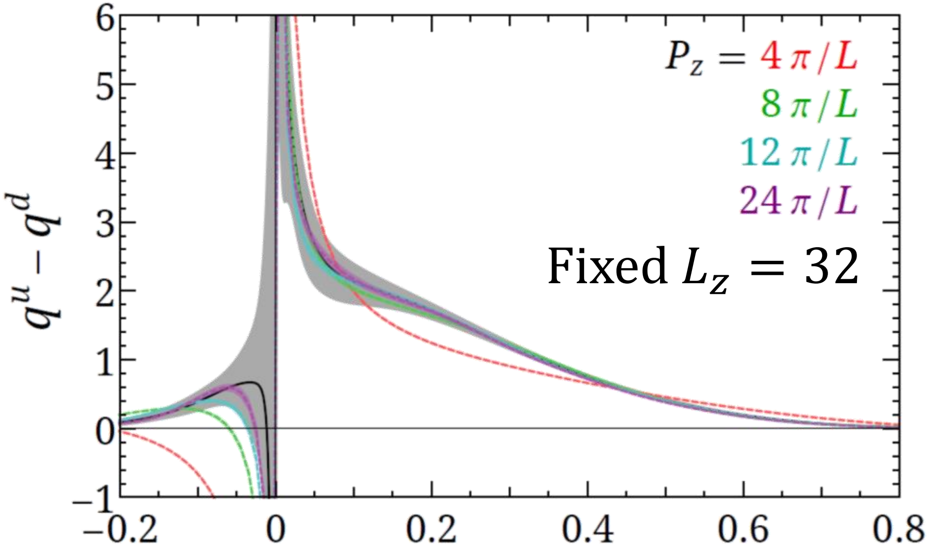


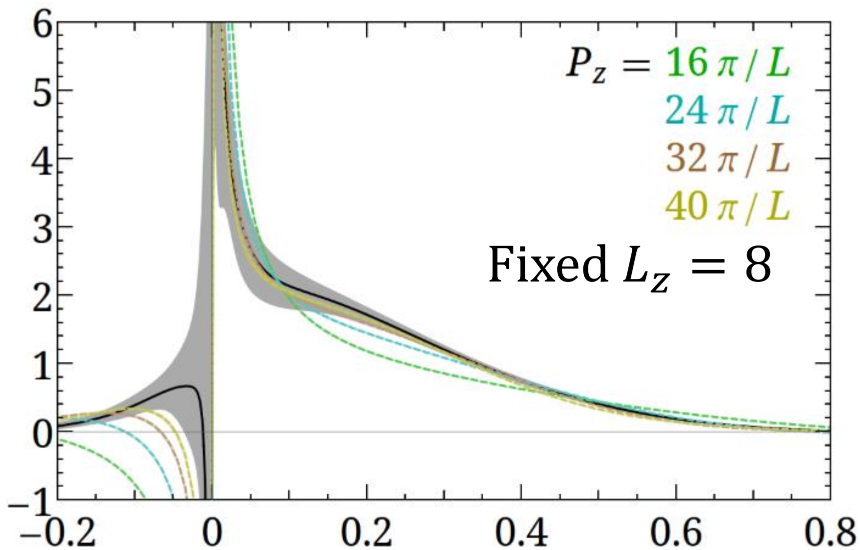
Derivative approach

$$q(x) = \int_{-z_{\max}}^{+z_{\max}} dz\, \frac{-P_z}{2\pi} \frac{e^{ixP_z z}}{iP_z x} h'(z)$$



Continuum

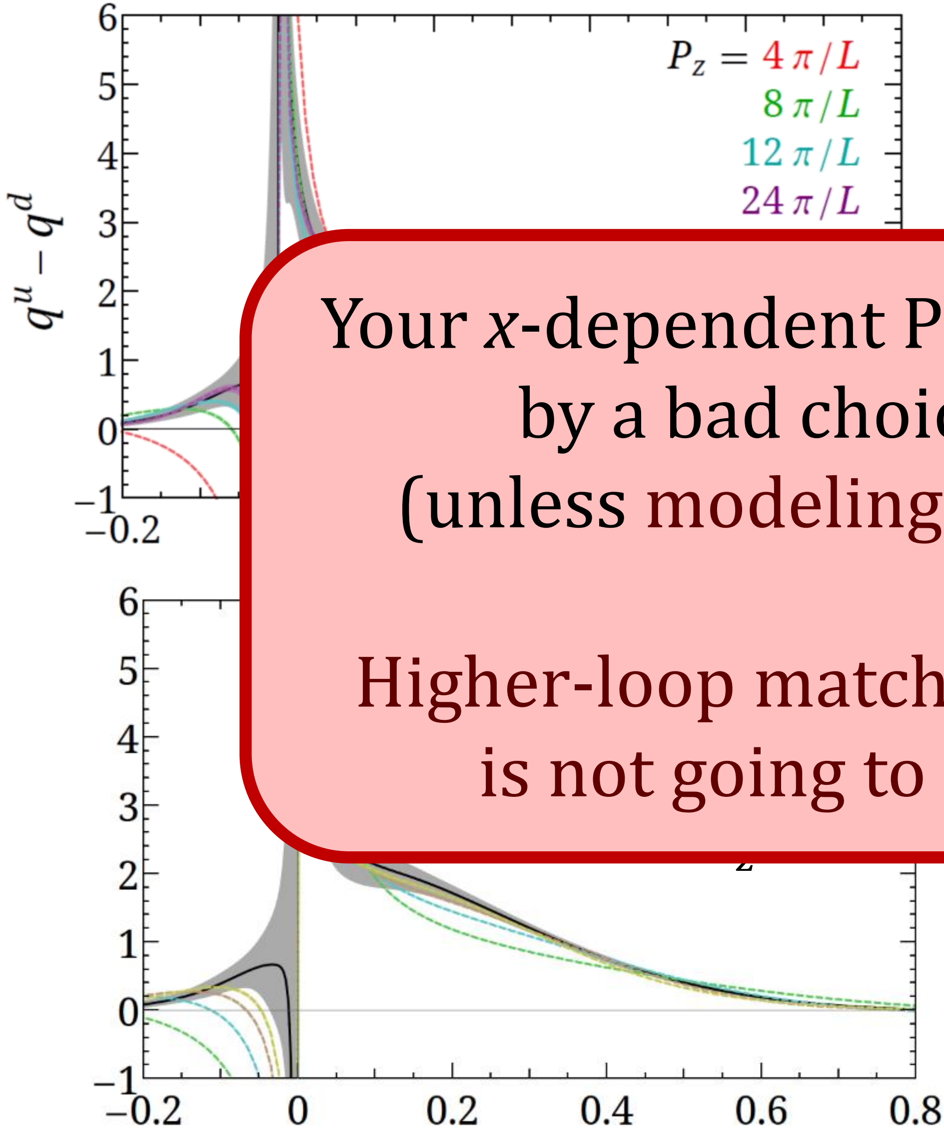




§ What lessons learned here?

- Given $L_z \approx 15$, you need large momentum to just get the sign of the antiquark correct!
- With small zP_z, you will miss over the majority of x
- Not just a quasi-PDF problem
- Going for large P_z is an unavoidable direction for any method that requires steps similar to Fourier transformation

Continuum



§ What lessons learned here?

- Given $L_z \approx 15$, you need large momentum to just get … uark … vill miss … problem … n … on for any method that requires steps similar to Fourier transformation

Your x-dependent PDFs will be doomed by a bad choice of max zP_z!
(unless modeling zP_z dependence)

Higher-loop matching in LaMET later is not going to do much for it!

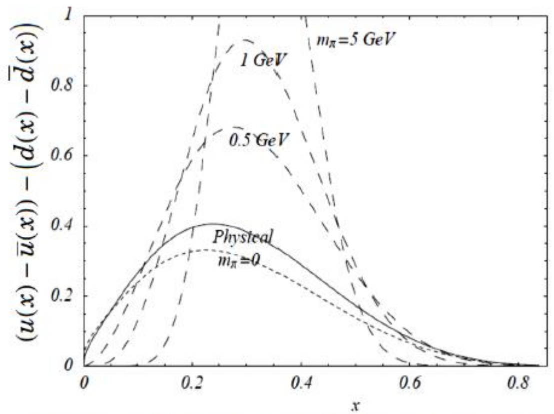
Alternative Solution

- One can model the lattice data?
  - Replaces cutoff systematic by model dependence
  - No obvious advantage for direct-*x* approach from moments (*already did this in 2001!*)
  - Nor guide the global PDF with correct *x*-forms or improve that fit-form dependence

§ What can we learn about the *x*-distribution?

- Make an ansätz of some smooth form for the distribution and fix the parameters by matching to the lattice moments

$$x q(x) = a x^b (1-x)^c \left(1 + \epsilon \sqrt{x} + \gamma x\right)$$



Cannot separate valence-quark contribution from sea

New idea needed to access the sea!

W. Detmold et al, Eur.Phys.J.direct C3 (2001) 1-15

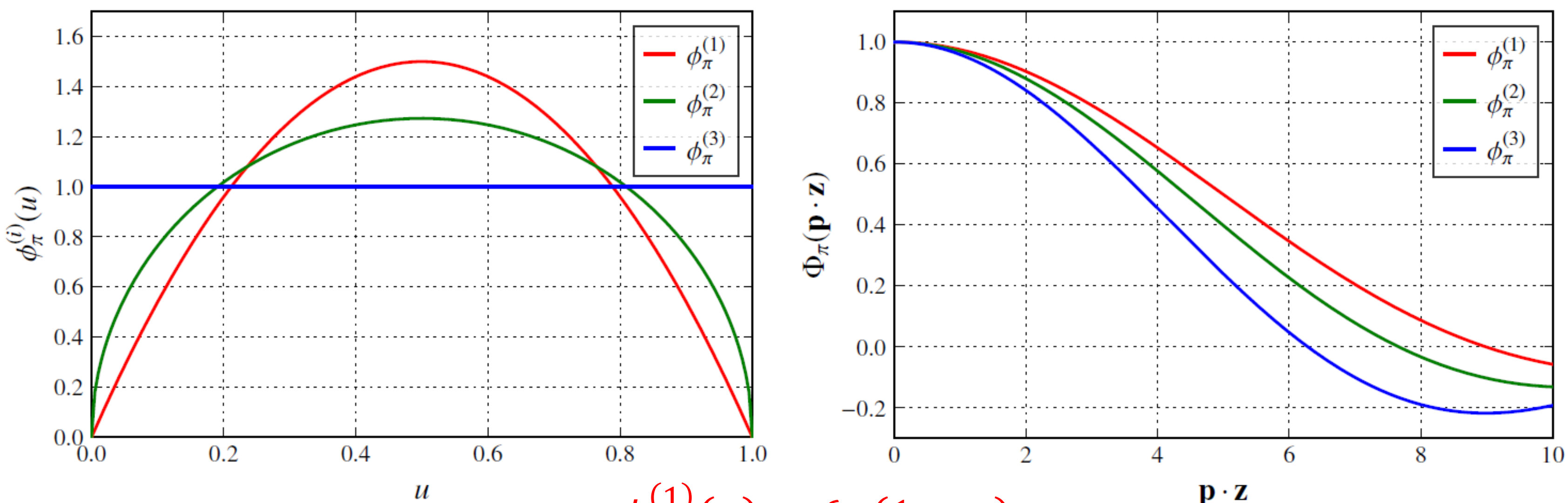
UNIVERSITY OF WASHINGTON   Huey-Wen Lin — Lattice 2013 @ Mainz   3

Alternative Solution

§ Coordinate-space PDFs?

- Lose sensitivity in constraining PDF forms

1709.04325(RQCD)



$$\phi_\pi^{(1)}(u) = 6u(1-u)$$

$$\phi_\pi^{(2)}(u) = \frac{8}{\pi}\sqrt{u(1-u)}$$

$$\phi_\pi^{(3)} = 1$$

Please Stop Using Plateau Fit

Myth

We should all use the plateau fit!

We can control excited states using large source-sink separation.

What is Plateau Fit?

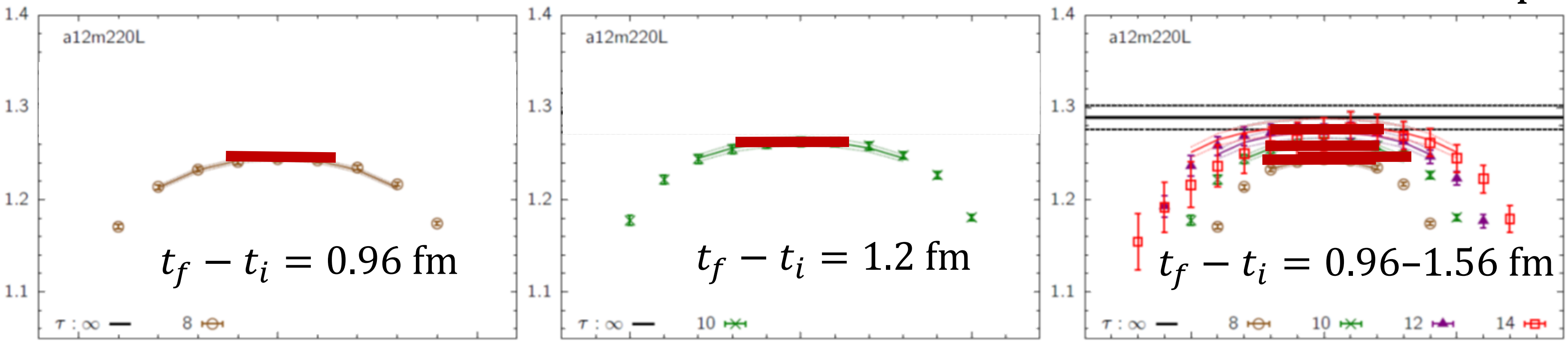
§ A “ruler” fit from a ground-state dominated world

- Advantage: simple to eyeball a straight line

$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$

$$C^{2\mathrm{pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)}$$

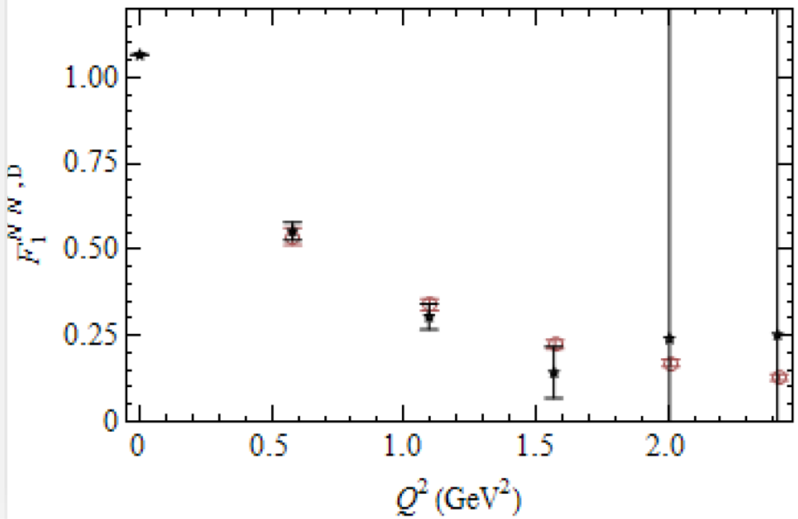
- You get **wrong** answers many times until you get to large t_{sep}



No More Plateau Fit!

- § Since 2008, I have advocated abandoning plateau fits at large source-sink separation
- § Why waste precious computing resources in the regions where noise dominates?

- Results



- Consistent with conventional ratio approach
- Smaller errorbar at larger Q^2
- Normally smearings are tuned to eliminate higher-energy contribution

HL@Lattice2008

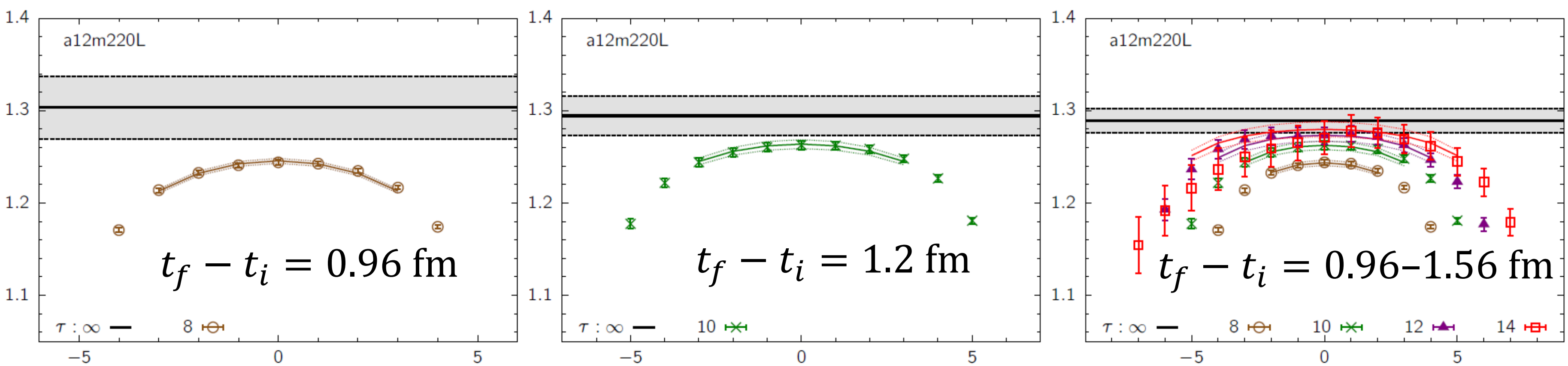
❧ The only people who listens mainly to claim credits:

We have been doing this for meson cases for ages! This is nothing new!

What is Plateau Fit?

§ What isn't plateau fit?

- Multistate fit to multiple separations
- Converts excited-state systematic into statistical uncertainty

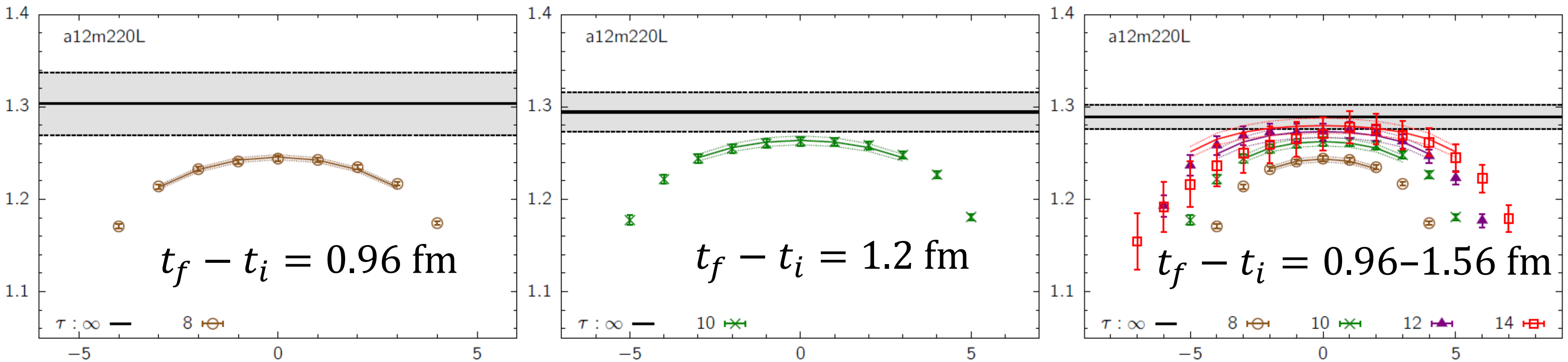


$$C^{3\text{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$
$$C^{2\text{pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)}+\ldots$$

No More Plateau Fit!

§ If one can get to the right answer sooner, why not?

- Errors are large at small t_{sep}; errors are bigger than plateau fit
- They are consistent
- More t_{sep} data helps to further reduce the errors



$$C^{3pt}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^* \langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1 \langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)} e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2 \langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

$$C^{2pt}(t_f, t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)} + \ldots$$

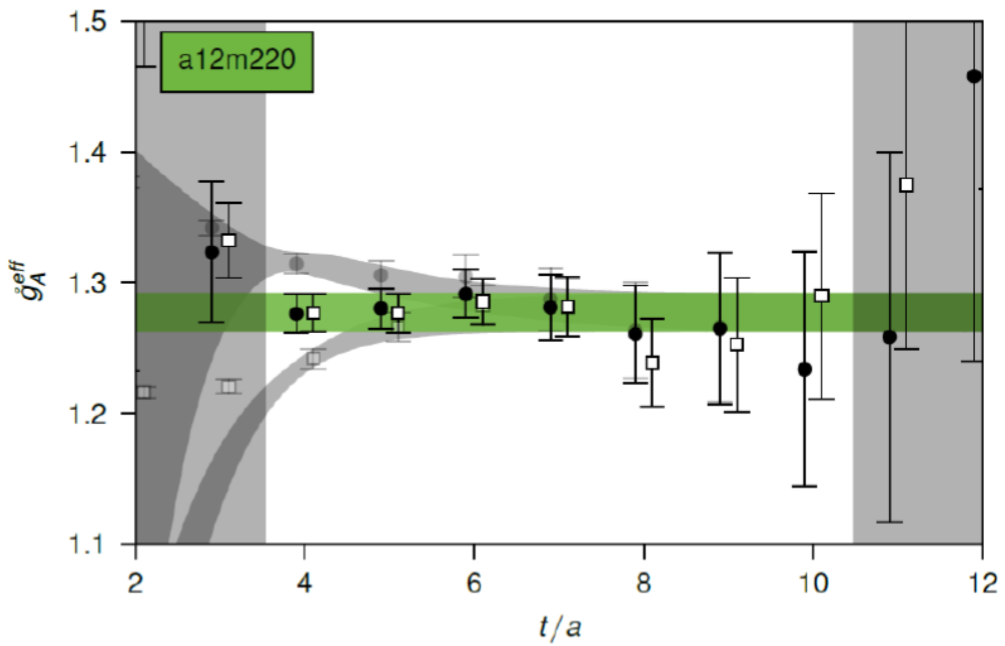
Myth

The two-state ground-state extraction will be dominated by the smallest source-sink separation

No More Plateau Fit!

§ When the smallest source-sink separation dominates the ground-state errors

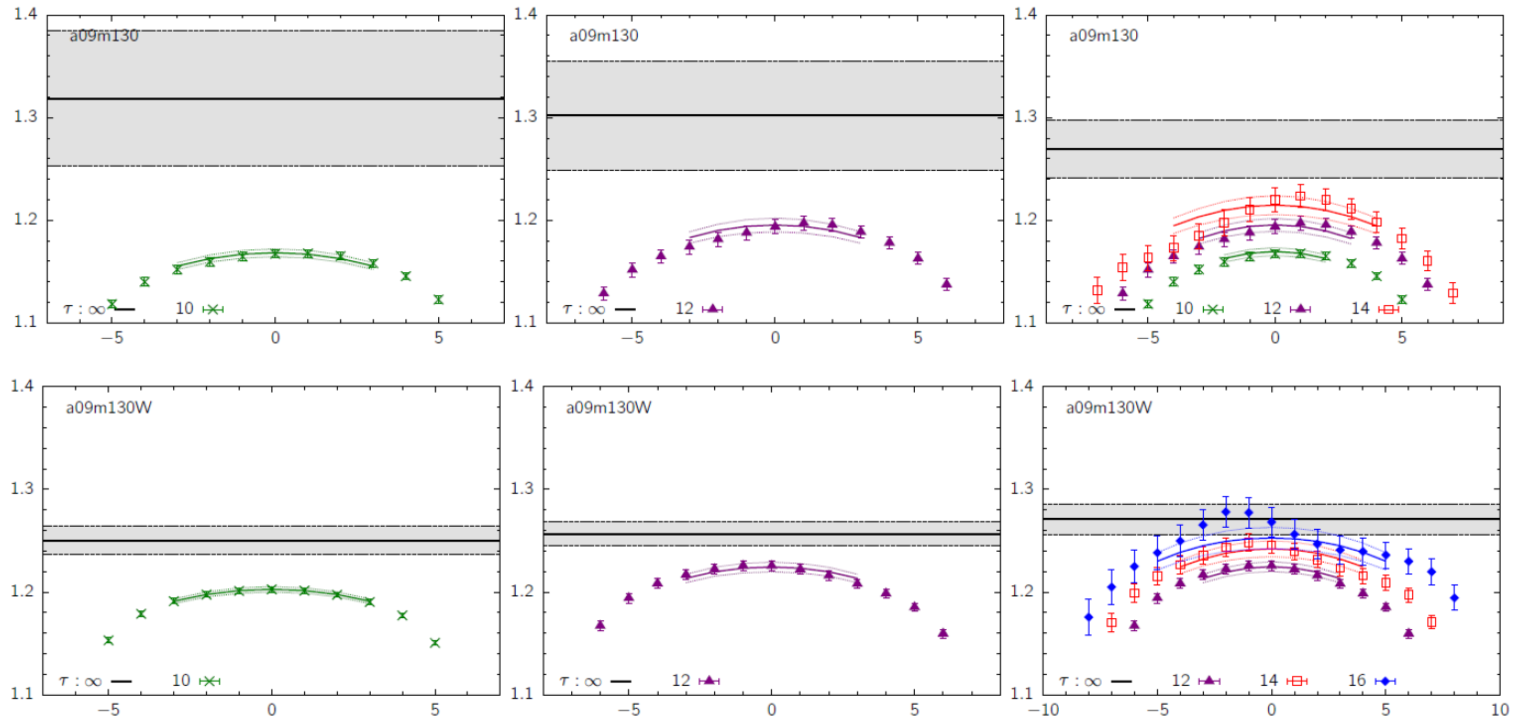
CalLat, *Nature* 558 (2018) no.7708, 91-94



No More Plateau Fit!

- When the smallest source-sink separation DO NOT dominate the errors

PNDME, Phys. Rev. D98 (2018) 034



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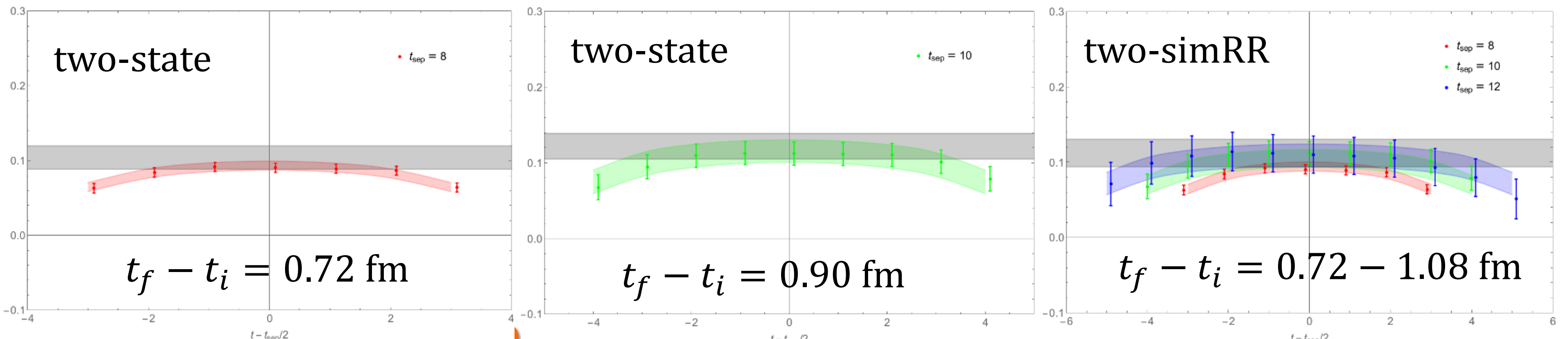
$P_z = 2.6$ GeV, $z = 4$, real (plot by Zhouyou Fan)



$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$

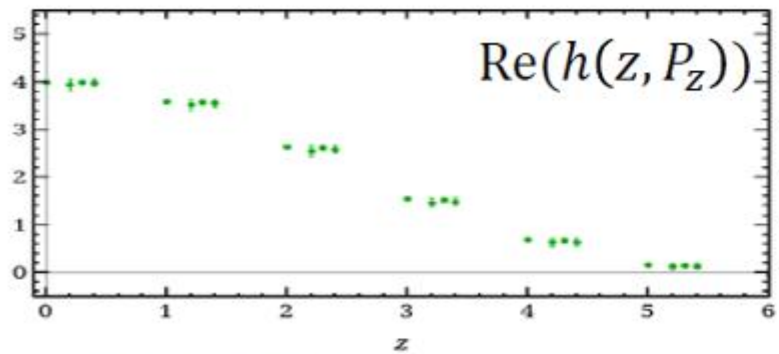
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

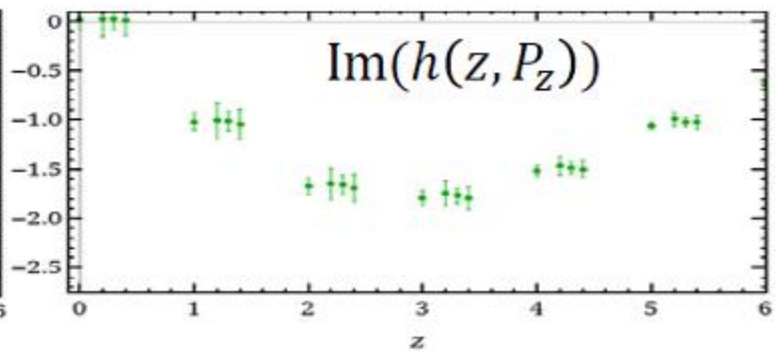
$$C^{\mathrm{2pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)} + \ldots$$

We Do Not Use Plateau Fit

1) Calculate nucleon matrix elements on the lattice

$$h(z,Pz) = \left\langle P \left| \bar{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$





$P_z =$ 2.6 GeV

$M_\pi \approx$ **135 MeV**, $a \approx 0.09$ fm

LP³ 1804.01483

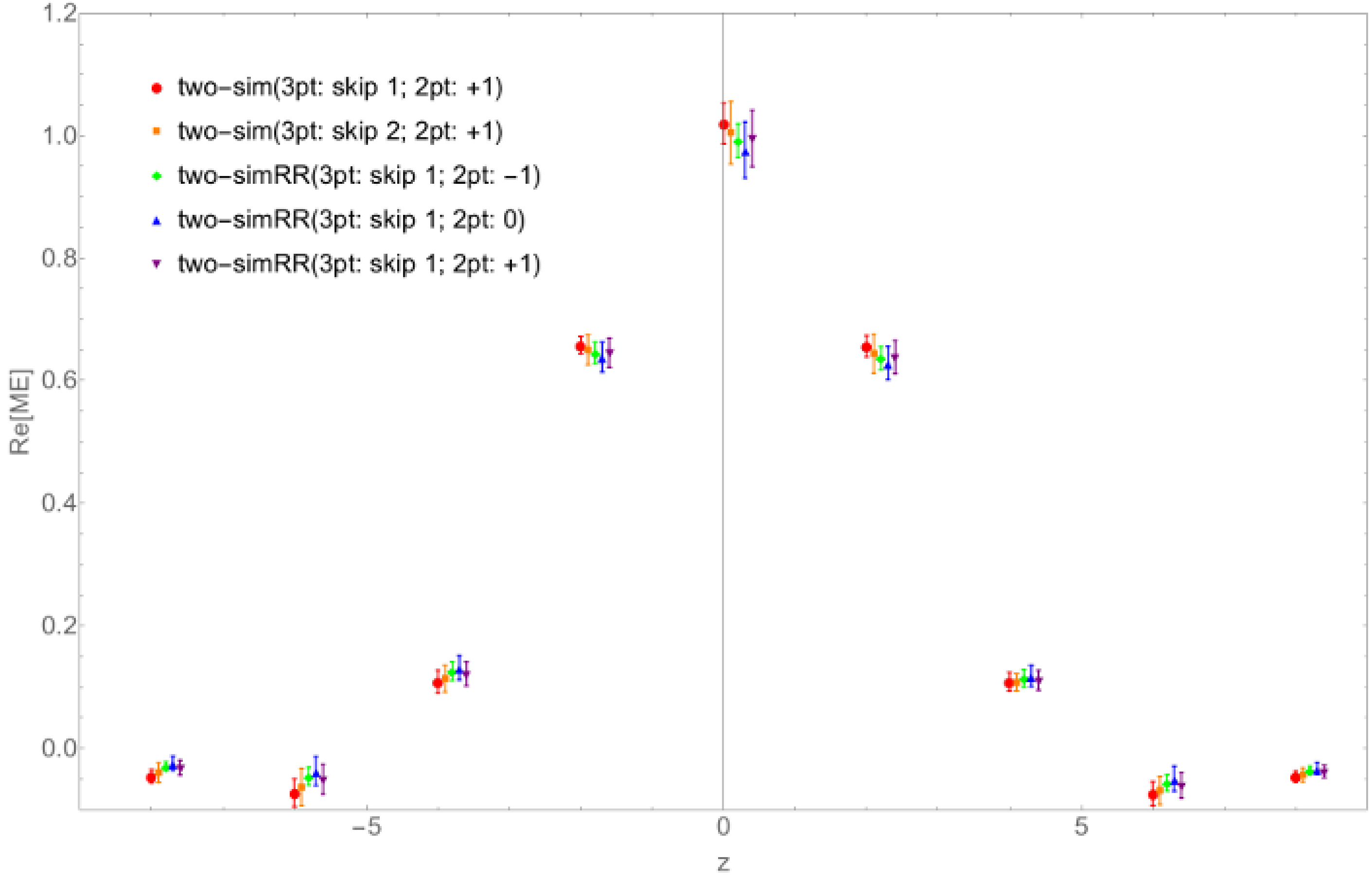
Ruizi Li

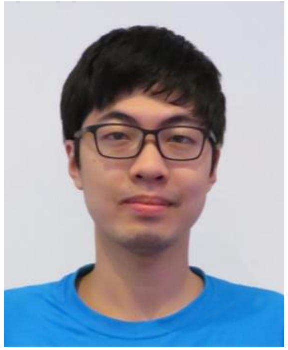
Blinded 3-state fits produced consistent results

HL@INT2018, DNP2018,…

We Do Not Use Plateau Fit

$P_z = 2.6$ GeV, transversity (plot by Zhouyou Fan)



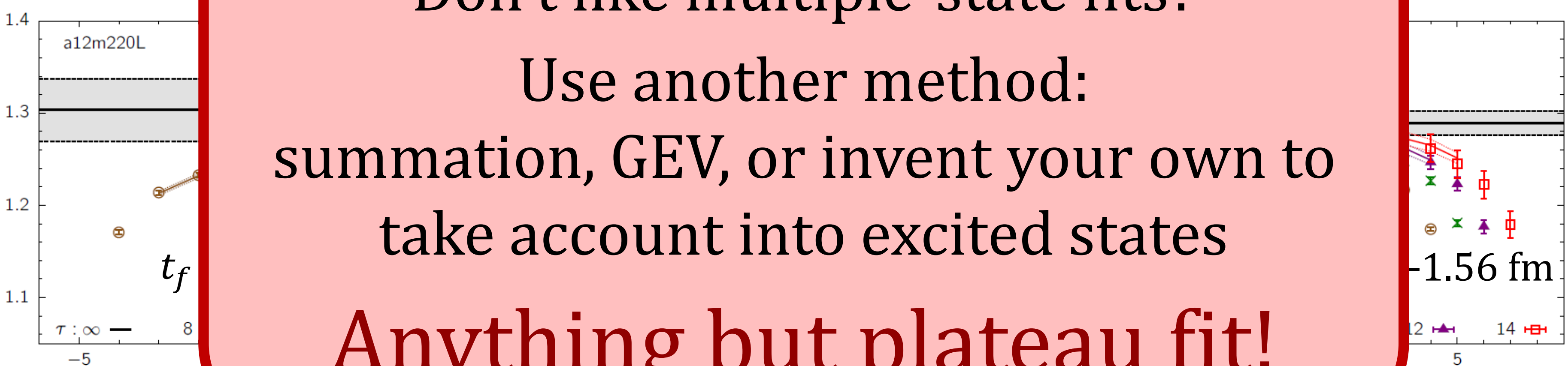


Zhouyou Fan

No More Plateau Fit!

§ If one can get to the right answer sooner, why not?

- Errors are large at small t_{sep}; errors are bigger than plateau fit
- They are consistent
- More



Don't like multiple-state fits?

Use another method:

summation, GEV, or invent your own to take account into excited states

Anything but plateau fit!

$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

$$C^{2\text{pt}}(t_f, t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)} + \ldots$$

No More Plateau Fit!

§ Avoid getting signal from noisy areas!

- Many people are doing this!

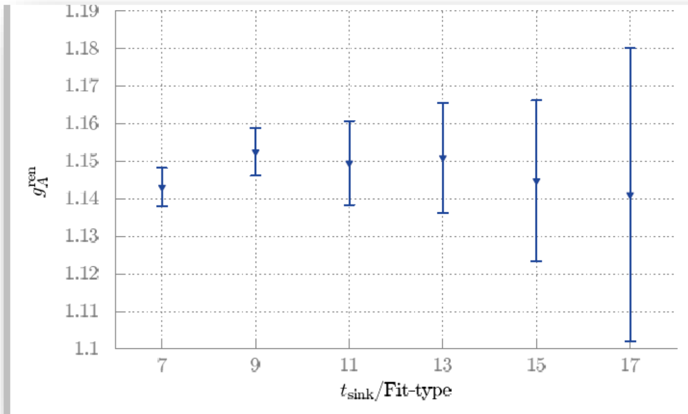
- FIND EVAN'S SLIDE AND SOME NPLQCD

Myth

One needs 1 fm source-sink separation to control the excited state for nucleon matrix elements

Smearing Dependent

§ Even with the plateau method



❶ Plateau Method: single-state

⇐ **RQCD (2014):**

[G.Bali et al. (RQCD), 2014]

- m_π=285MeV
- g_A not sensitive on $T_{\rm sink}$: 0.49-1.19 fm

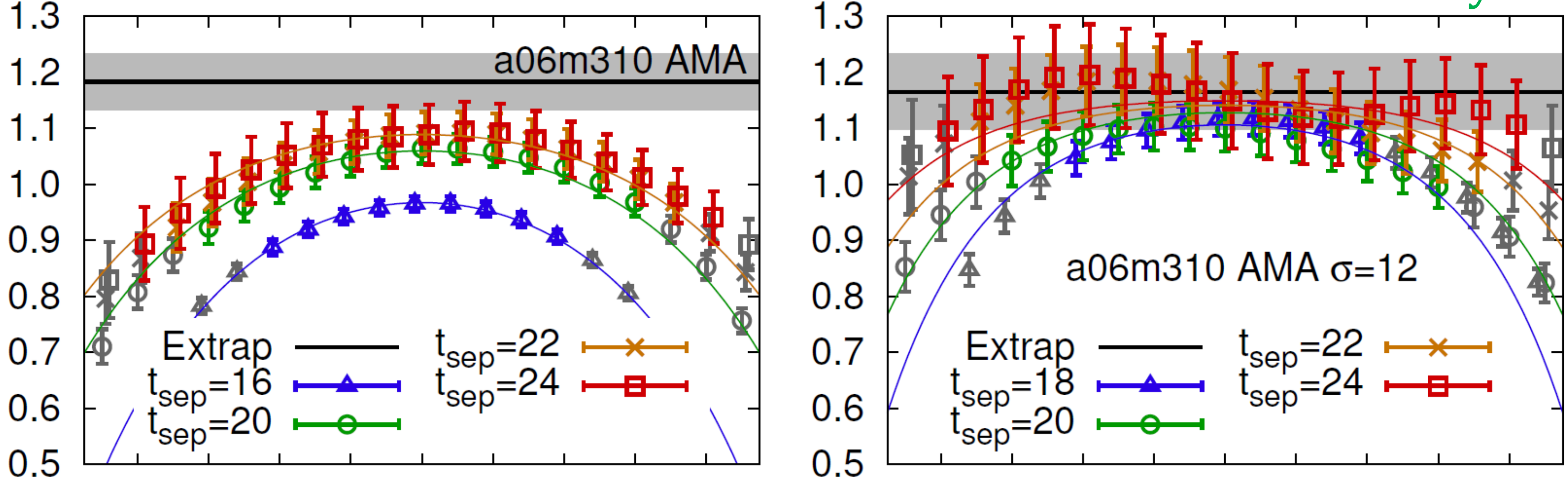
M. Constantinou @ Lattice 2014

Smearing Dependent

- An example study on excited-state from PNDME work (42K)
  - Robustness of the 2-state fit $a = 0.06$ fm, 220-MeV pion

1606.07049

Small smearing param. σ_G=6.5 g_A^{bare} Bigger smearing param. σ_G=10 needed for one-state analysis



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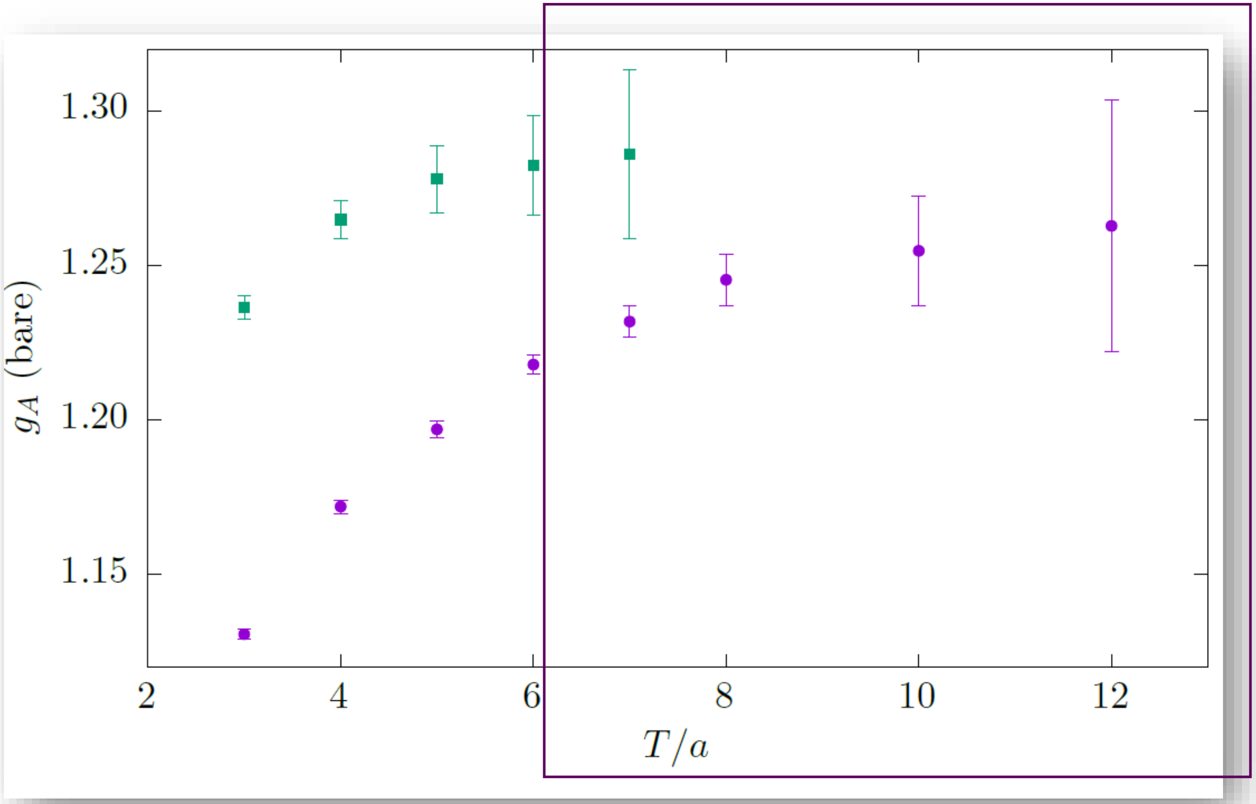
- If done right, the stat. error is NOT dominated by smaller tsep data
- With a~0.09fm, σ~4 with 1.2fm source-sink with primitive one-state fit suffers large excited-state contamination

Analysis Method Dependent

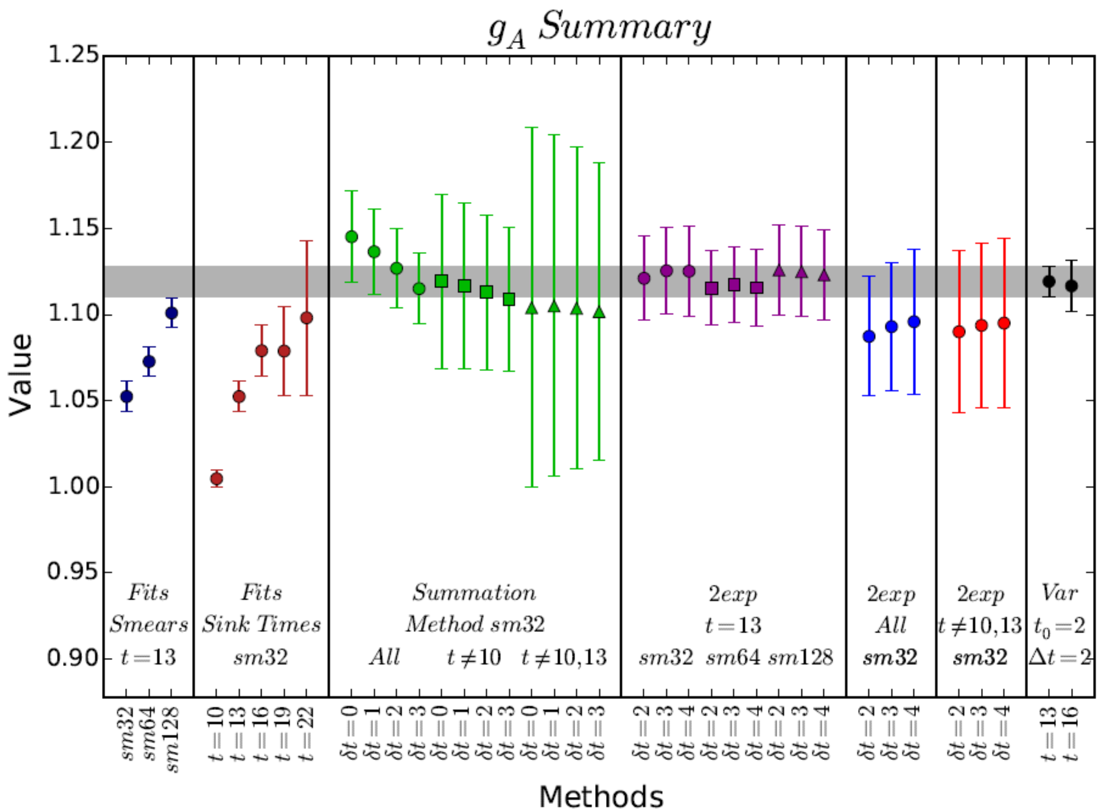
§ Summation method

- 0.116 fm at physical pion mass

1812.10574 J. Green



Analysis Method Dependent



J. Green@Lattice 2018

$m_\pi = 460$ MeV, $a = 0.074$ fm

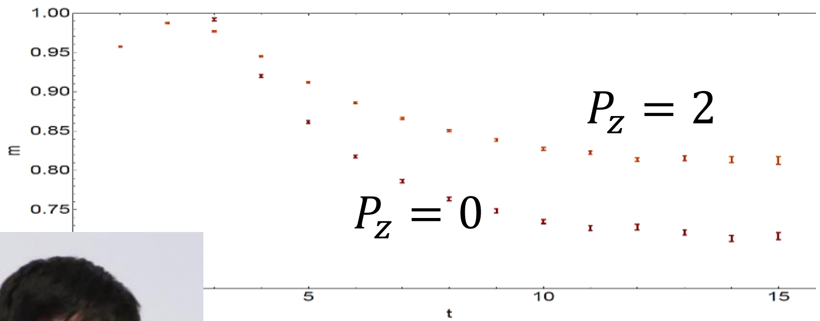
J. Dragos *et al.*, PRD **94**, 074505 (2016) [1606.03195]

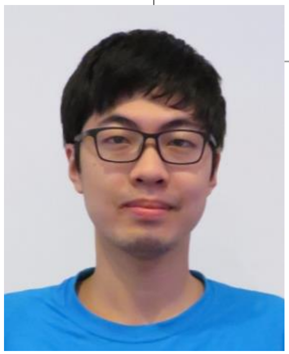
Myth

You need way too much computer to reach high boost momentum

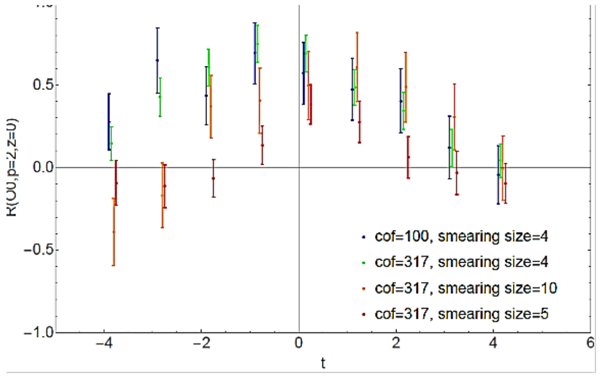
The Case of 3-GeV P_z

- § The signal-to-noise ratio depends on many parameters
- § Be careful what one extrapolates
  - Here is an example from my student
  - If one simply extrapolates from this data, the conclusion:
    - One needs more statistics to get zero-momentum proton mass and moments; obviously NOT true

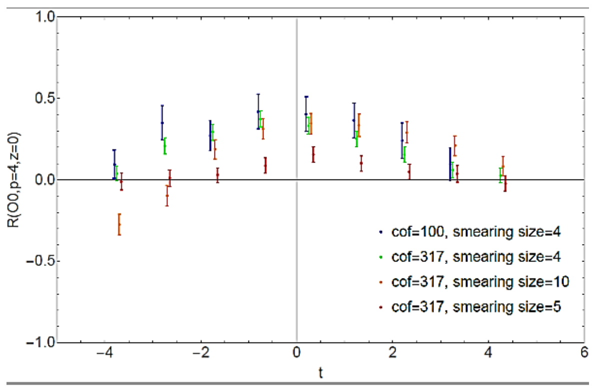




Zhouyou Fan



$P_z = 2$



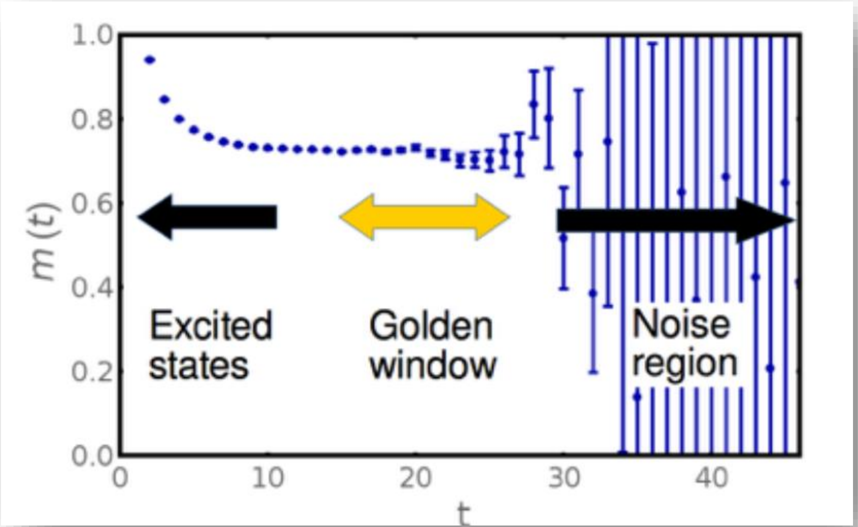
$P_z = 4$

The Case of 3-GeV P_z

§ Our original target was not 3 GeV

- We were hoping to get *some* signal at 2.6 GeV with momentum smearing fine-tuned to this point
- We thought that we would have 2.2 GeV to fall back, since it's not far from 2.6 GeV
- 3.0 GeV was a surprise to me too

No one thought NPLQCD could calculate *NN* interactions when they first started, but now they are the main force in lattice nuclear physics



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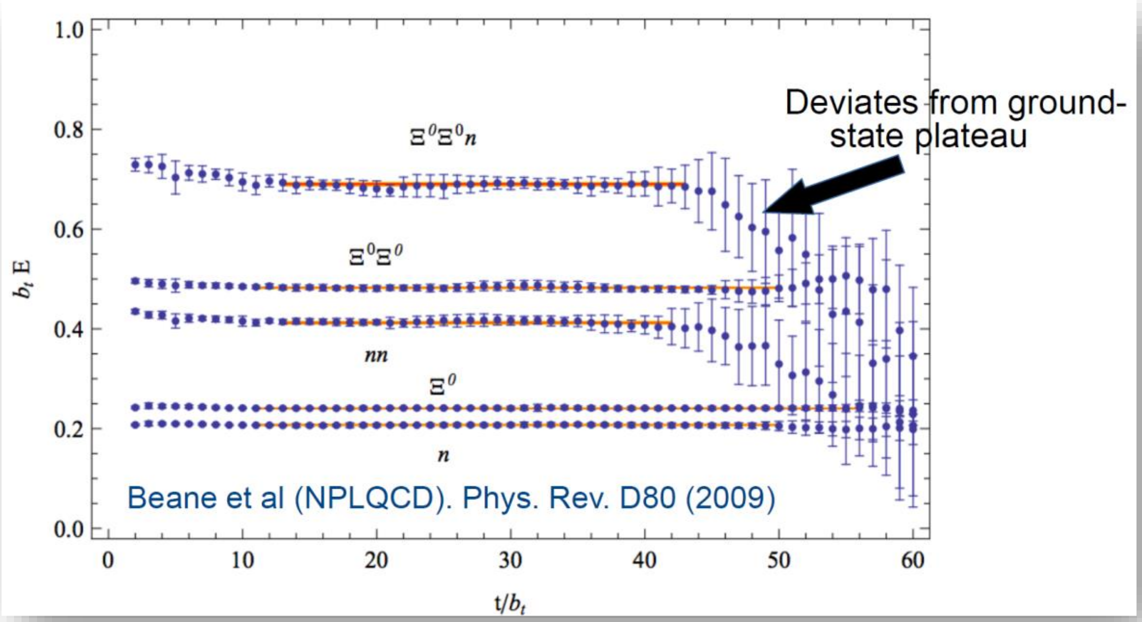

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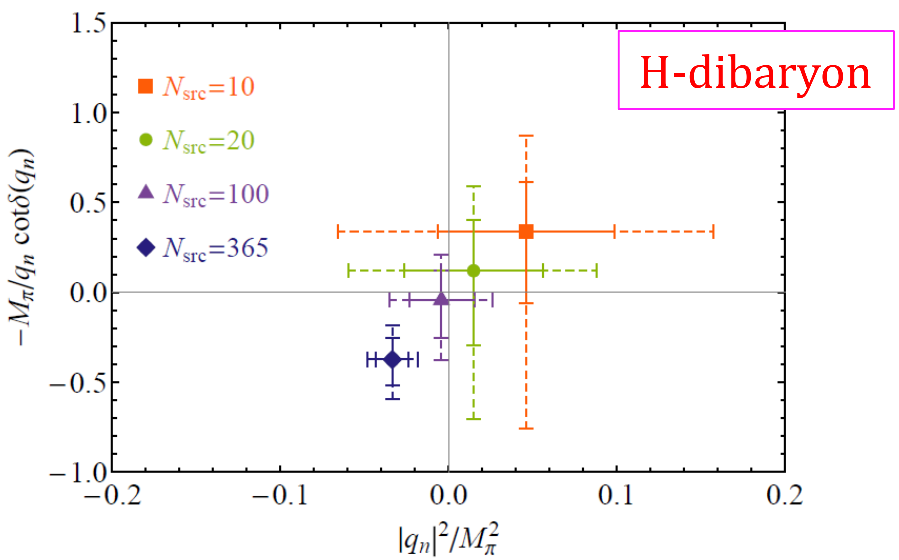
Words of advice:
Similar to hadron-interaction calculations, one can easily reach wrong conclusions with insufficient statistics



HWL@INCP 2010

Excited States at 3-GeV P_z

- How about excited states?
- We do what we can on our ensembles
  - If only we were not bounded by finite computer resources
- Future direction in progress
  - Smaller lattice spacing
  - Variational method to catch more excited sates
  - Happy to hear more suggestions
    - (...unless they involve plateau fits)

Conclusions

Apologize in advance for a rather technical talk

Need to clear up many false rumors

§ We still need large boost momentum

- The most non-model dependent way to access small-x physics for EIC
- Let's work together to figure out the best way to do it

§ Please stop using plateau fit

- Just stop

Further Criticism?

Backup Slides

Control Excited States

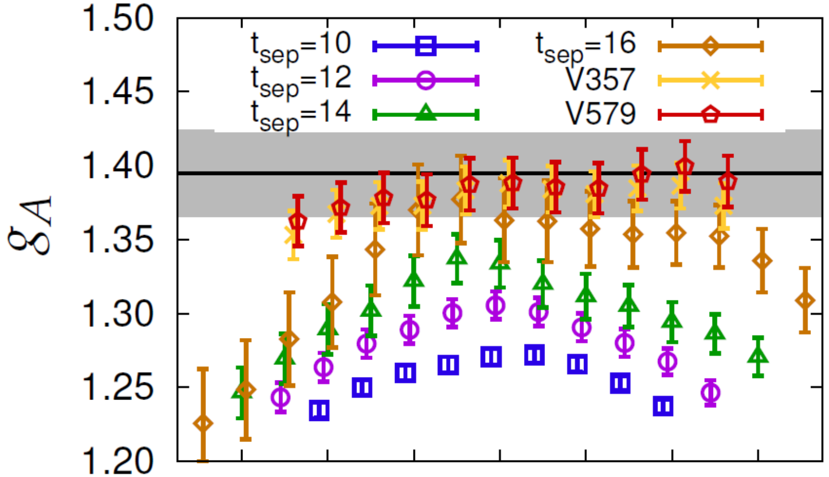
- Same conclusion from another study on excited-state from NME (Jlab,PNDME,LHPC) work

$a = 0.08$ fm, 312-MeV pion

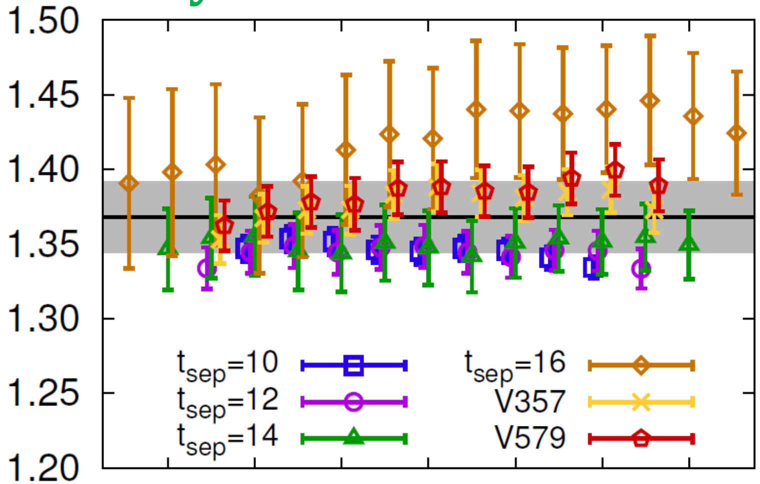
1602.07737

- Robustness of the 2-state fit; consistent with vibrational method

Small smearing param. σ_G=5



Bigger smearing param. σ_G=10 needed for one-state analysis



- V357 and V579 are results from different 3x3 correlator matrix analysis
- Ground-state stat. error is NOT dominated by smallest tsep

Many Such Examples

§ An example study on excited-state from PNDME work

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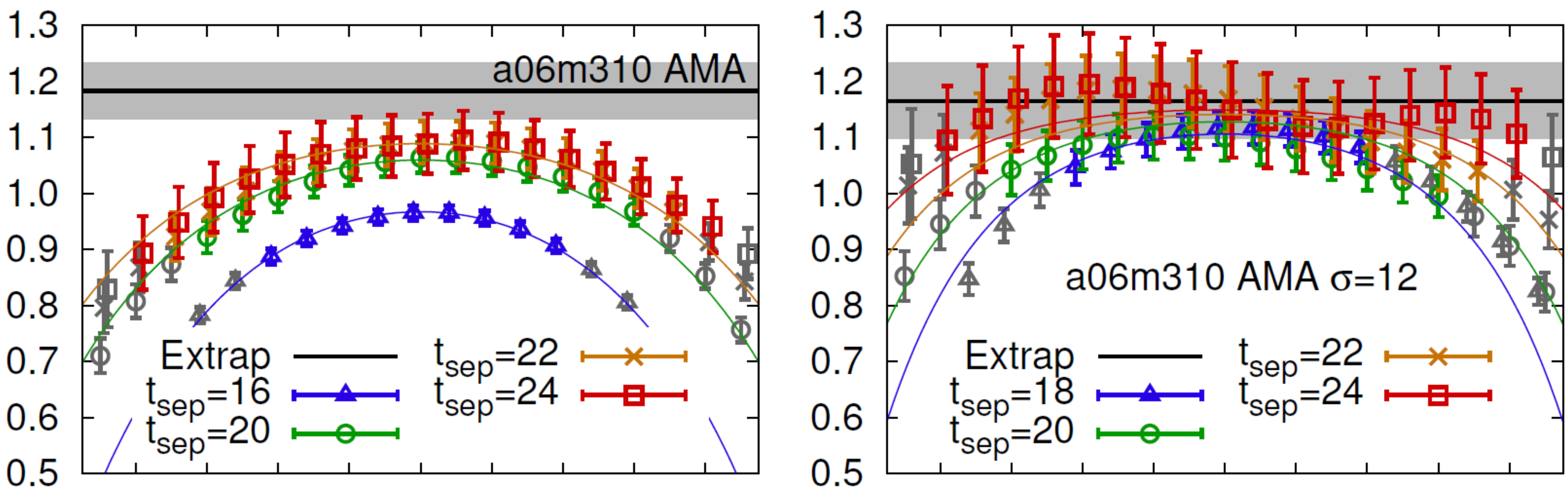
$\sigma_G=6.5$ g_S^{bare} $\sigma_G=11$



1606.07049, Plots by Boram Yoon

§ Consistent ground-state matrix element by allowing a small excited state

§ Very small gain by focusing on analysis with only ground state

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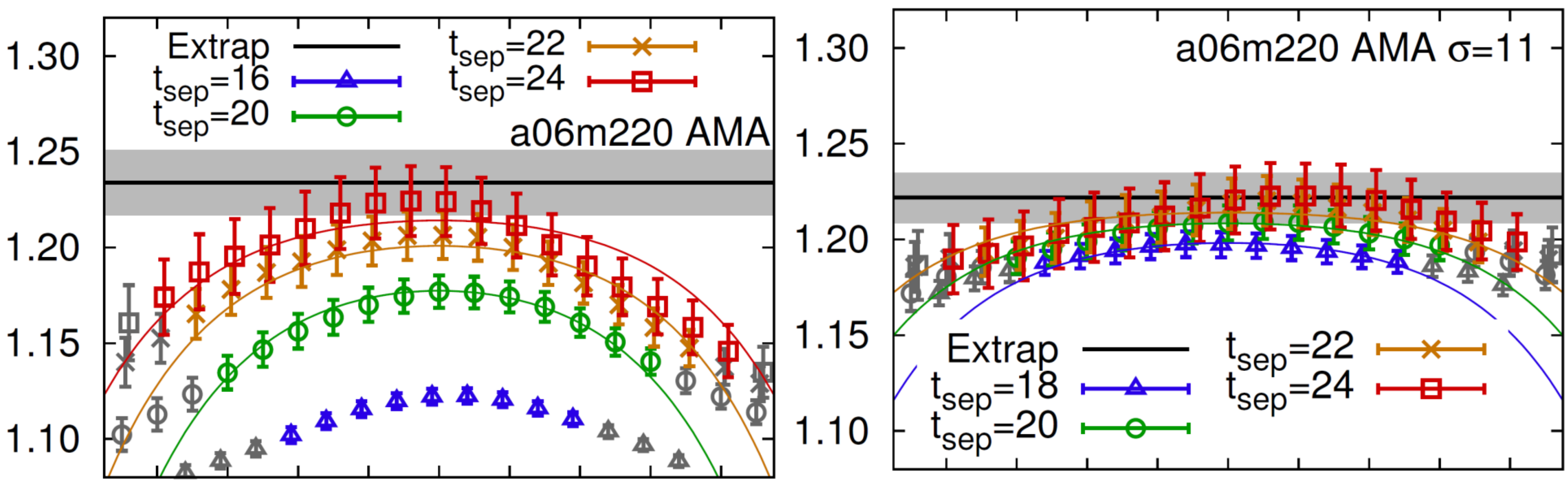

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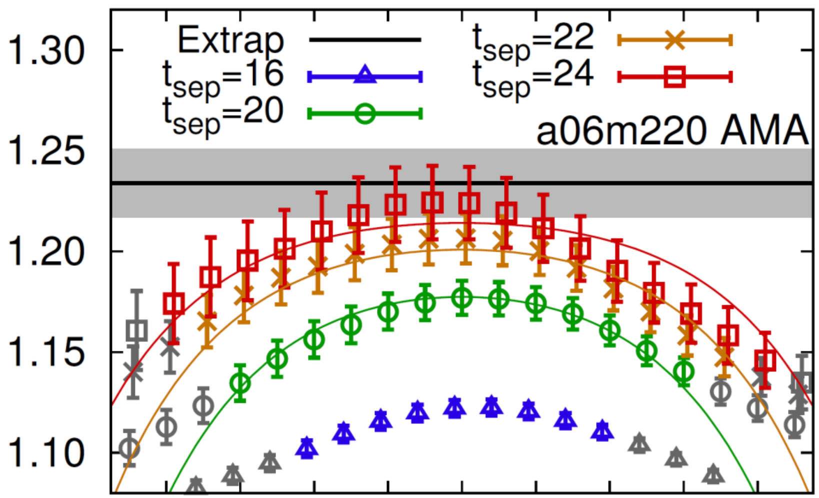

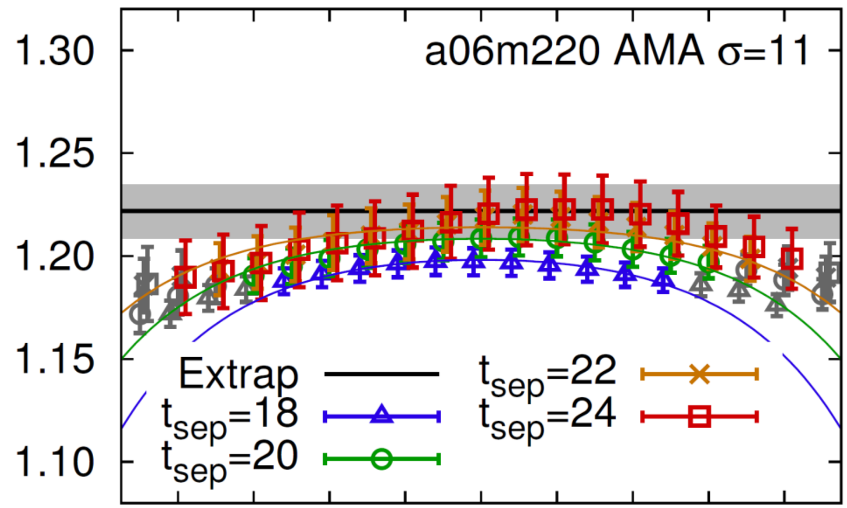

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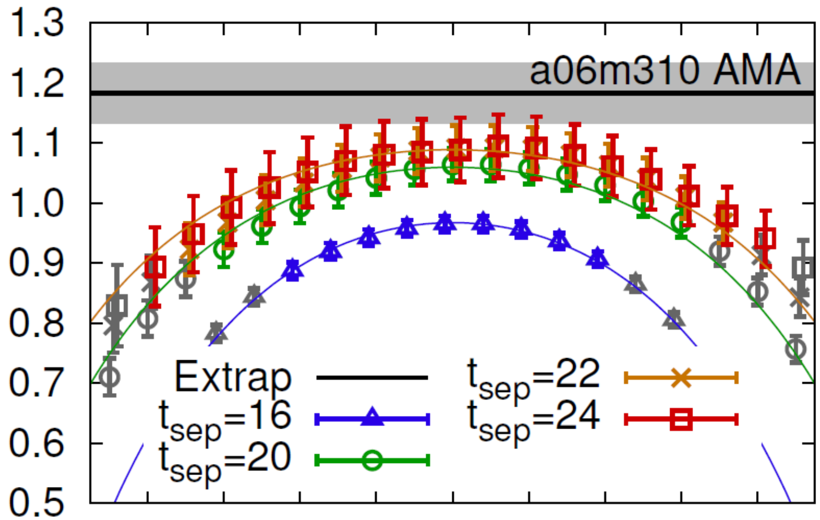

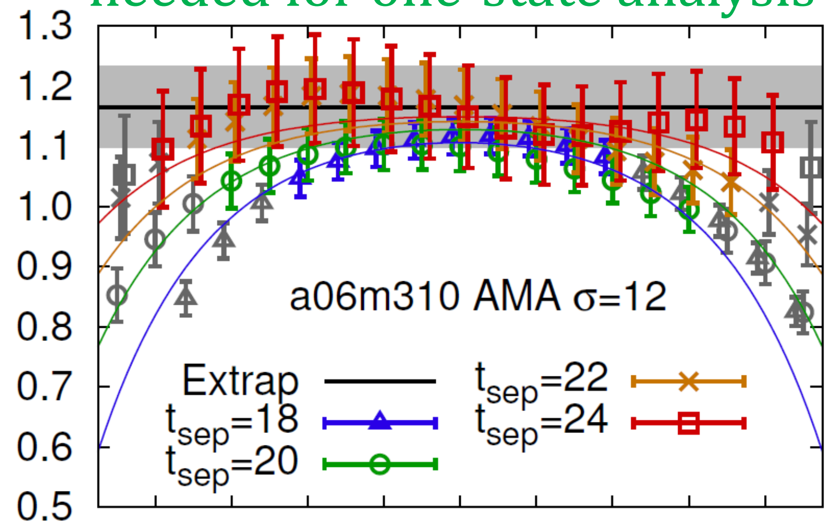

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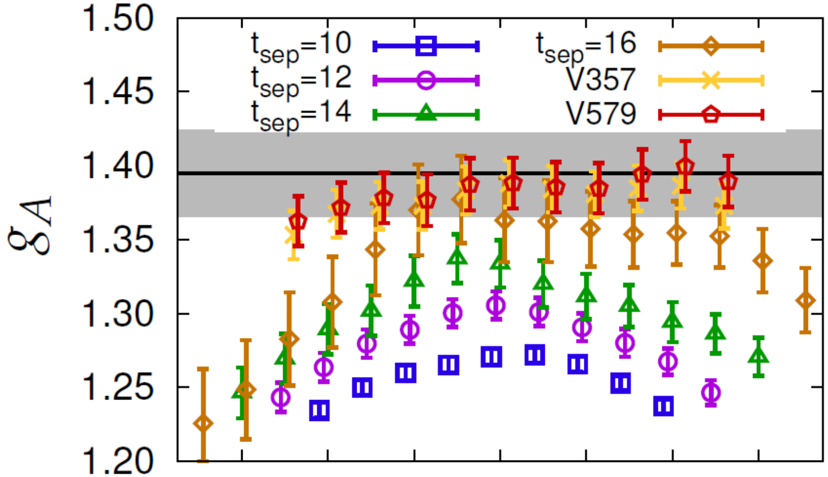

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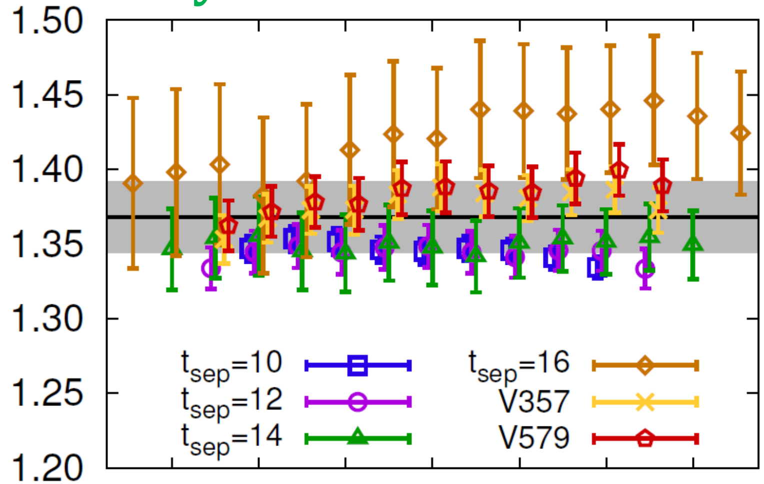

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