Valence Pion PDF

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Definitions

quark PDF

$$f_{\psi/h}(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+z^-} \langle h(P)| [\bar{\psi}(\xi^-)\gamma^+ W_L(\xi^-,0)\psi(0)]_\mu |h(P)\rangle$$

quark qPDF [arXiv:1305.1539]

$$q_{\psi/h}(x,P_z,\mu_R^2) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle h(P)| [\bar{\psi}(z)\Gamma W_L(z,0)\psi(0)]_{\mu_R} |h(P)\rangle$$

$$\Gamma \in \{\gamma_t, \gamma_z\}$$

qPDF $\rightarrow$ PDF [arXiv:1709.04933, arXiv:1706.01295]

$$q(x,P^z,\mu_R^2) = \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu_R}{P^z}, \frac{\mu}{|y|P^z}) f(y,\mu) + \mathcal{O}(\frac{m_h^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2})$$

$$\mathcal{O}_\Gamma^\psi(z,0) = \psi(z)\Gamma W_L(z,0)\psi(0)$$

What it is we compute

Isovector quark PDF

$$f_{\pi^+}^{u-d}(x,\mu) = f_{\pi^+}^{u}(x,\mu) - f_{\pi^+}^{d}(x,\mu)$$

Valence quark PDF $x \in [0,1]$

Matching quasi-quark PDF to PDF [arXiv:1709.04933, arXiv:1706.01295]

$$h_{u-d/\pi^+}(x, P^z, \mu_R) = f_{u-d/\pi^+}(x) + \frac{\alpha_s C_F}{2\pi}\int_{-1}^{1} dy \Big[\frac{1}{|y|}\mathcal{F}_1(\frac{x}{y})_+ + |\eta'|\mathcal{F}_2(1+\eta'(x-y))_+ \Big] f_{u-d/\pi^+}(y)$$

Work Flow

(1) Remove Excited States

$$\langle 0 | \, \pi^{+}_{P^z;\Delta t} \mathcal{O}^{u-d}_{\Gamma}(z,0;\tau) \bar{\pi}^{+}_{P^z;0} \, |0\rangle_a \to \left\langle \pi^+(P^z) \right| \mathcal{O}^{u-d}_{\Gamma}(z,0) \left| \pi^+(P^z) \right\rangle_a$$

(2) Renormalize qPDF

$$\left\langle \pi^+(P^z) \right| \mathcal{O}^{u-d}_{\Gamma}(z,0) \left| \pi^+(P^z) \right\rangle_a \to \left\langle \pi^+(P^z) \right| \mathcal{O}^{u-d}_{\Gamma}(z,0) \left| \pi^+(P^z) \right\rangle_{\mu_R}$$

(3) qPDF $\to$ PDF

$$\left\langle \pi^+(P^z) \right| \mathcal{O}^{u-d}_{\Gamma}(z,0) \left| \pi^+(P^z) \right\rangle_{\mu_R} \to f(y,\mu) \text{ via matching formula}$$

Lattice Parameters & Computation Details

- Mixed Action
  - Sea: HotQCD HISQ gauge ensemble [arXiv:1407.6387]
  - Valence: Wilson Clover with 1HYP
- Lattice Spacing: 0.06 fm
- Lattice Size: $48^3 \times 64$
- Pion Mass: 300 MeV
- Configuration Used:
  - N_{CFG}: 100 $|z| > 16$
  - N_{CFG}: 162 $16 \geq |z| > 8$
  - N_{CFG}: 214 $8 \geq |z| > 0$
- Correlators per config computed using All-Mode Averaging (AMA) [arXiv:1402.0244]
  - 1 exact sample
  - 32 sloppy samples

(1) Remove Excited States

2-Point Function

$$C_{2pt}(t) = \sum_n |A_n| e^{-E_n t}$$

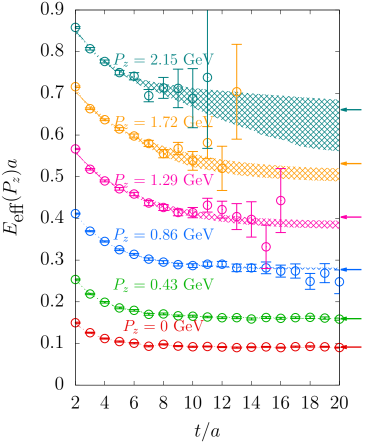
Fit two-point function to sum of exponentials to aquire parameters

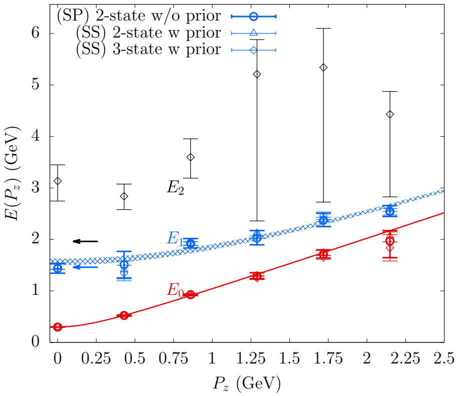
3-Point Function

$$C_{3pt}^{\vec{P},\Gamma}(\Delta t, \tau; z) = \sum_{n,m} e^{-E_n \Delta t} e^{E_n \tau} e^{-E_m \tau} \times A_n^* \langle n| \mathcal{O}_\Gamma^{u-d}(z,0) |m\rangle A_m$$

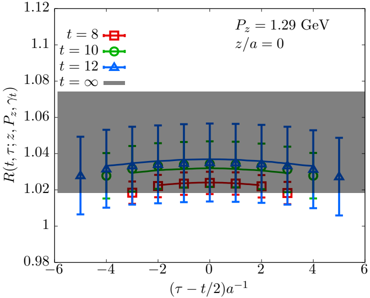
- Define $R(\Delta t, \tau, z) = C_{3pt}(\Delta t, \tau, z)/C_{2pt}(\Delta t)$
  $R_{sum}(\Delta t, z) = \sum_{\tau=t_o}^{\Delta t - t_o} R(\Delta t, \tau, z) = C + (\Delta t - t_o)\mathcal{M}$
- Fit C_{3pt} accross all Δt and τ to get $\mathcal{M}$, fixing A_n and E_n from 2-point fit

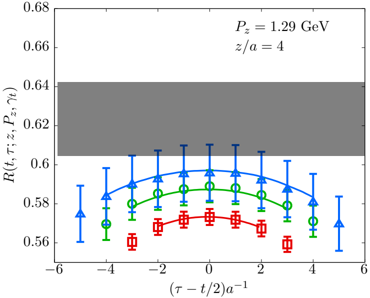
Two-Point Fit Results

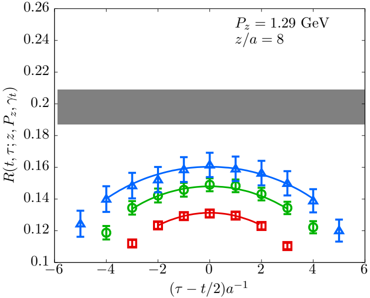




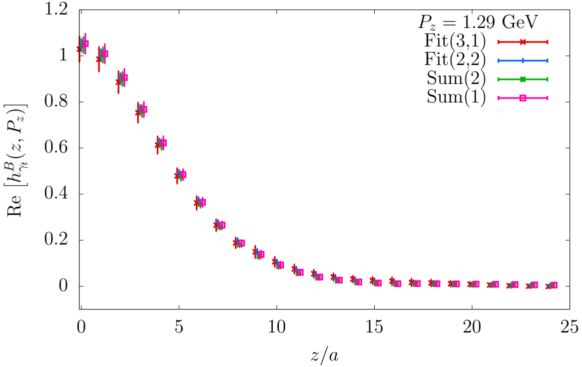
Three-Point Fit Results







Pion qPDF matrix element after extrapolation



(2) qPDF Renormalization

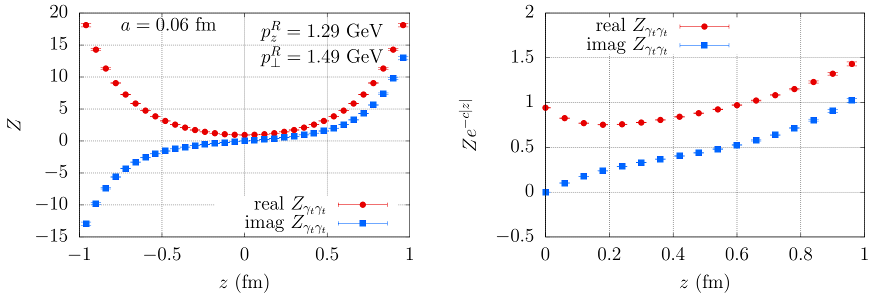
qPDF matrix element can be multiplicatively renormalized

$$h^R_{\gamma_t} = Z_{\gamma_t\gamma_t}(z; p^R) h^o_{\gamma_t}(z; P^z, a)$$

Renormalization coefficient renormalized under RI-MOM scheme

$$Z_{\gamma_t\gamma_t}(z, p^R) q_{\gamma_t}(z, p)|_{p=p^R} = e^{ip^R_z z}$$

where q is the quark-in-quark PDF



LHS: qPDF RI-MOM Z-factor

Divergent contribution $\sim e^{c|z|}$

RHS: qPDF Z-factor after factoring out Wilson Line UV-divergent term

c = number evaluated from heavy-quark effective potential [arXiv:1804.10600]

Comparison to 1-Loop Perturbation Theory

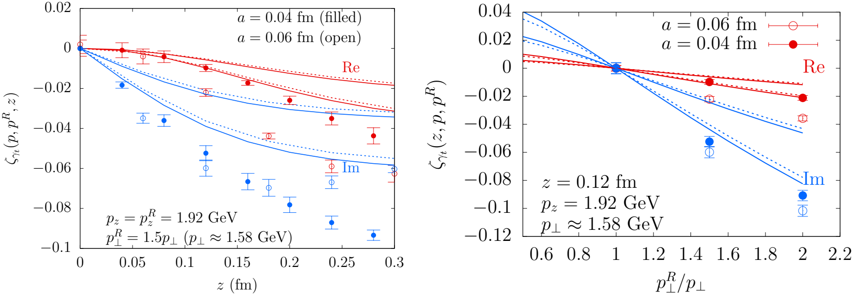
How well does $q(z, P^z, \mu_R^2)_{\text{lat}}$ match with $q^{\text{1-Loop}}(z, P^z, \mu_R^2)$?
Study the behavior of

$$\zeta_\Gamma(z, p, p^R) = \frac{q^R(z, p, p^R)}{q^R(z, p, p)} - 1$$

In both 1-Loop Perturbation Theory and on the the Lattice.

No running of α_s for ζ in 1-Loop Perurbation theory.
Compute $\zeta^{\text{1-Loop}}$ using α_s from scales $\mu = \frac{1}{2}p^R$ to $2p^R$

ζ z and p^R dependence



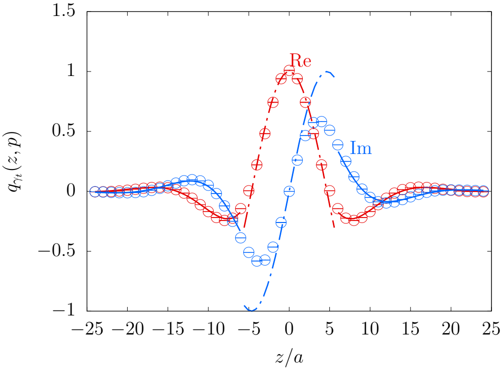
Identifying Perturbative/Nonperturbative Regime in Matrix Element

For what quark-antiquark separation do we expect to see nonperturbative physics to be prominent in real-space qPDF?
Model bare quark qPDF using the following ansatz:

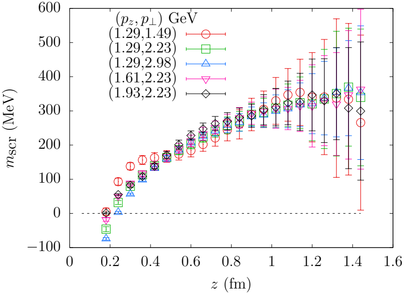
$$q(z,P) = Ae^{i\omega z}e^{-m_{\mathrm{scr}}|z|}e^{-c|z|}$$

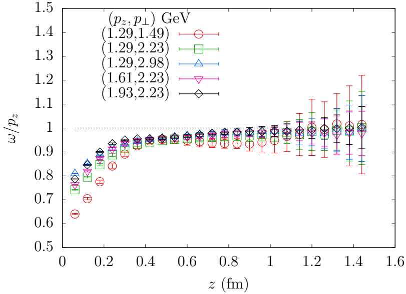
- $e^{i\omega z}$ represents tree-level bare quark-qPDF
- $e^{-c|z|}$ removes UV divergent piece of qPDF; c determined from heavy-quark potential calculation [arXiv:1804.10600]
- $e^{-m_{\mathrm{scr}}|z|}$ models the damping of plane-wave quark via inverse screening length $m_{\mathrm{scr}} \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$

How well does it fit the data?



$m_{\rm scr}$ and ω



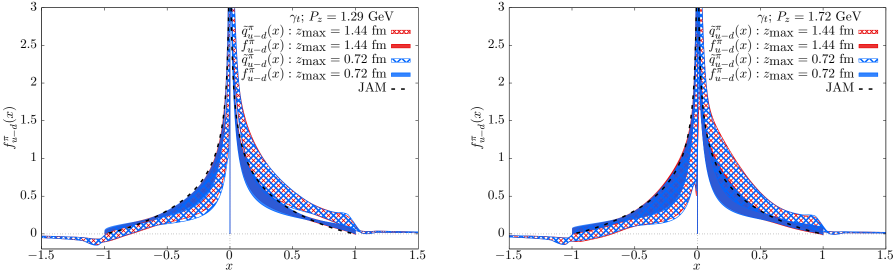


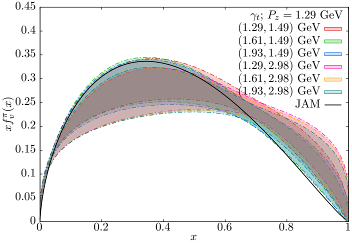
(3) qPDF $\rightarrow$ PDF Matching

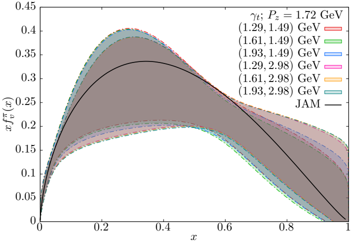
Strategy

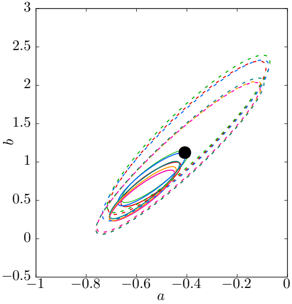
- A possible strategy. One could parameterize the real-space data $h^R(z, P^z, p^R)$ over available z then model the behavior for larger z. Then perform Fourier Transform
  - Advantages: No model dependence when dealing with available data
  - Disadvantages: Must model real-space qPDF beyond available z-data. Performing Fourier Transform involves performing an integral in non-perturbative region of qPDF
- What we do. Use PDF ansatz $f(x; a, b) = Ax^a(1-x)^b$, transform the data into real-space quasiPDF, then fit that to data, minimizing.
  - Disadvantages: Introduce model dependence
  - Advantages: Model already used by phenomenologists fitting PDF's from data
    Can control which z-data we can fit to

Pion qPDF and PDF




γ_t; $P_z = 1.29$ GeV
$(1.29, 1.49)$ GeV
$(1.61, 1.49)$ GeV
$(1.93, 1.49)$ GeV
$(1.29, 2.98)$ GeV
$(1.61, 2.98)$ GeV
$(1.93, 2.98)$ GeV
JAM
$xf_v^\pi(x)$
x


γ_t; $P_z = 1.72$ GeV
$(1.29, 1.49)$ GeV
$(1.61, 1.49)$ GeV
$(1.93, 1.49)$ GeV
$(1.29, 2.98)$ GeV
$(1.61, 2.98)$ GeV
$(1.93, 2.98)$ GeV
JAM
$xf_v^\pi(x)$
x


b
a

Conclusions

- We presented a calculation of the Pion valence-quark PDF using the quasi-PDF approach

$$m_\pi = 300 \text{ MeV}, \text{ a} = 0.06 \text{ fm}, P^z = 1.29 \text{ GeV}, 1.72 \text{ GeV}$$

- We studied the running of the renormalization coefficient
  - Qualitative agreement between lattice renormalization and 1-loop
  - Smaller Lattice Spacing improves quantitative agreement
  - 2-Loops may be necessary
- More statistics are necessary to overcome noise in large-momentum data