Collins-Soper Kernel for TMDPDFs from Lattice QCD

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CFNS Workshop on Lattice Parton Distribution Functions
Brookhaven National Lab
April 18, 2019

Based On: M. Ebert, IS, Y. Zhao, Phys. Rev. D99, 034505 (2019) [1811.00026]
M. Ebert, IS, Y. Zhao [1901.03685]

Massachusetts Institute of Technology

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TMD Collaboration

Outline

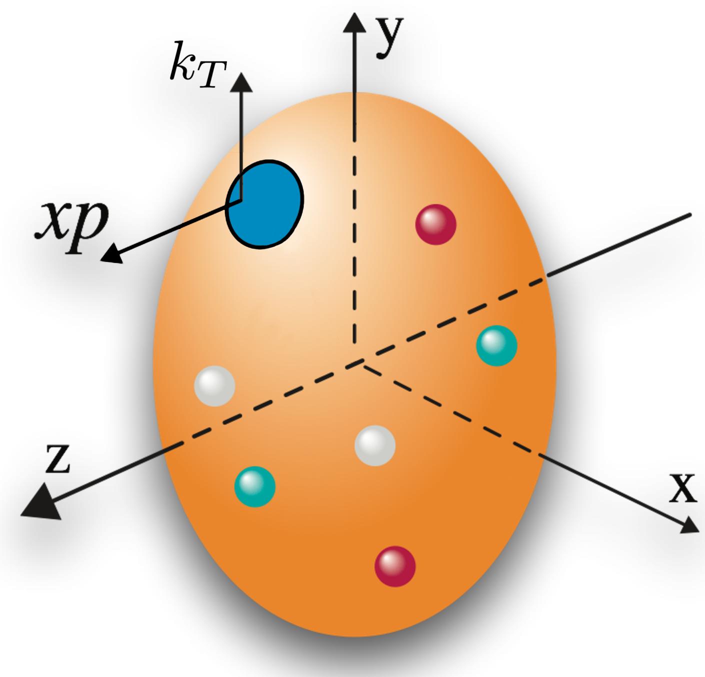
- Introduction
- TMD Factorization Theorems
  - TMDPDFs & their evolution
- TMDPDF definitions
  - hadron and vacuum matrix elements & rapidity divergences
- Quasi-TMDPDFs for Lattice QCD calculations
- Nonperturbative Collins-Soper Evolution Kernel
- Outlook

Parton Distributions

- **provide key information about the structure of hadrons**

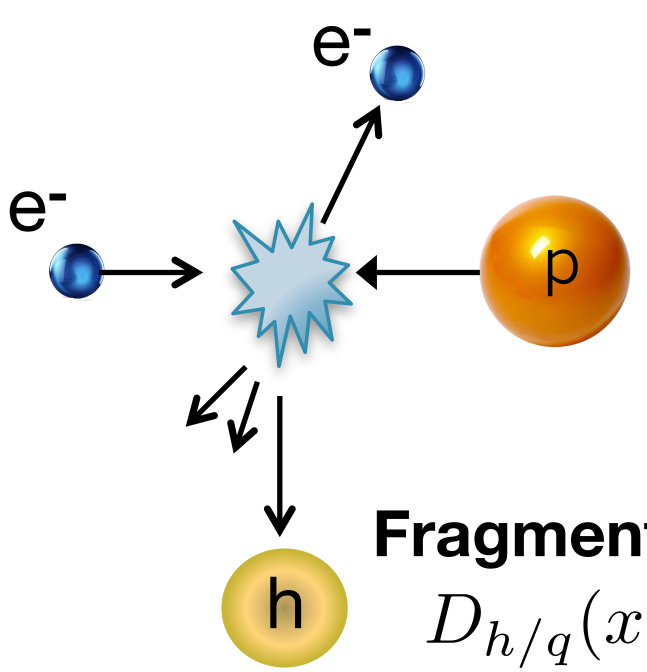
TMD: $f_{q/P}(x, k_T)$

longitudinal & Transverse



Semi-Inclusive DIS

$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$

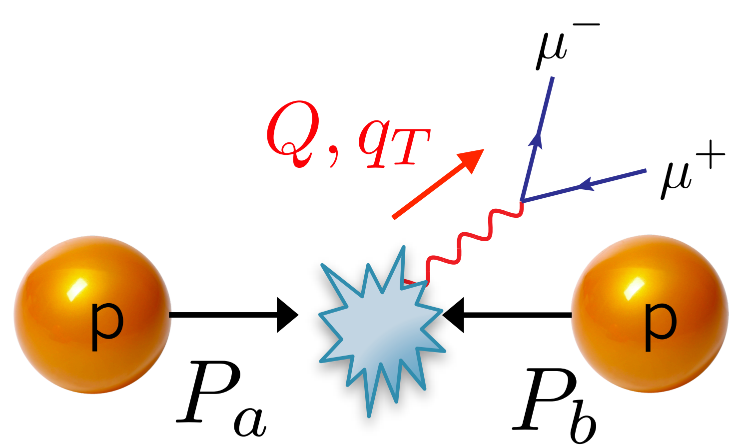


Fragmentation

$D_{h/q}(x, k_T)$

Drell-Yan

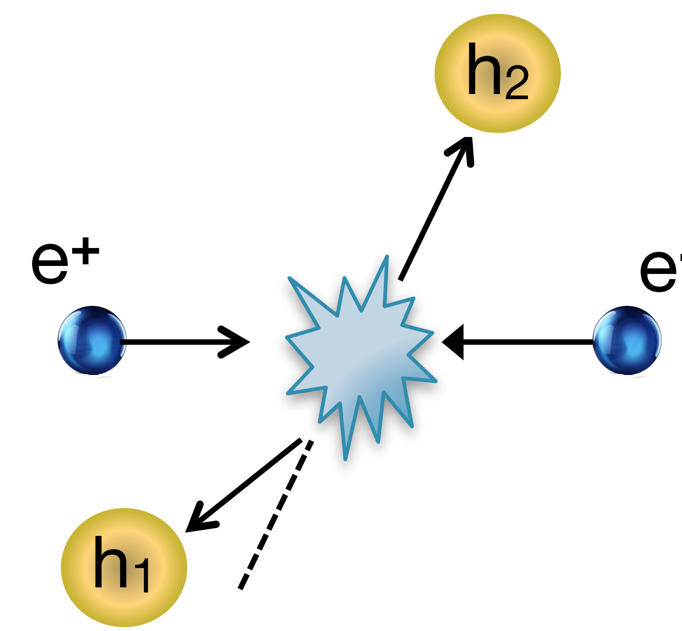
$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$



$q_T \ll Q$

Dihadron in e^+e^-

$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$



TMD Factorization

CSS (Collins, Soper, Sterman)
SCET (Soft Collinear Effective Theory)

- rigorous QFT based derivation of cross sections
- based on analysis of momentum regions

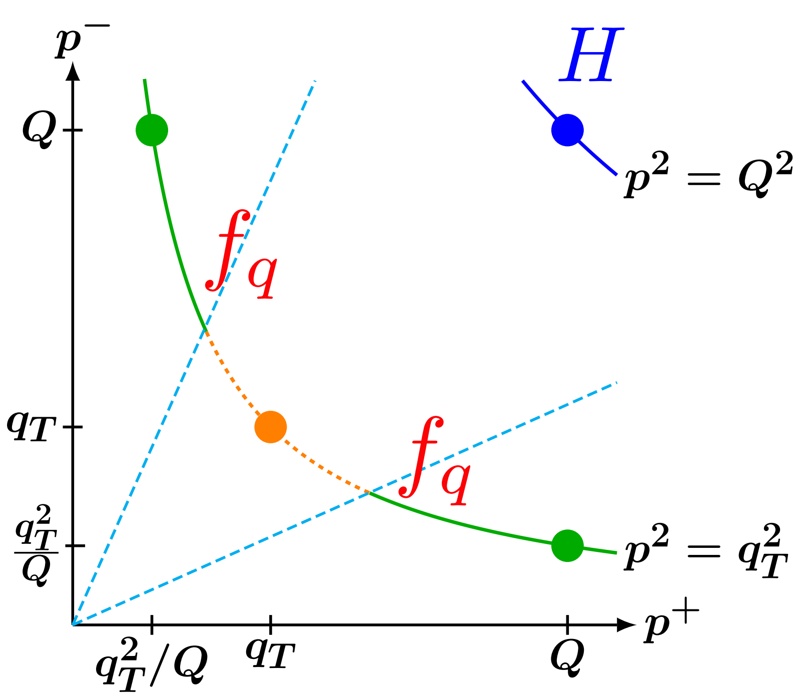
eg. Drell-Yan

$$\sigma(q_T, Q) = H(Q,\mu) \int d^2\vec{b}_T \, e^{i\vec{q}_T\cdot\vec{b}_T} \, f_q(x_a, \vec{b}_T, \mu, \zeta_a) \, f_q(x_b, \vec{b}_T, \mu, \zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Hard virtual corrections

FT

$$f_q(x_a, \vec{k}_T, \mu, \zeta_a)$$



μ = renormalization scale

ζ = Collins-Soper parameter

$$\zeta_a = (x_a P_a^-)^2 = (2x_a P^z)^2$$

$$\zeta_a \zeta_b = Q^4$$

think: $\zeta \sim Q^2$

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eg. Drell-Yan

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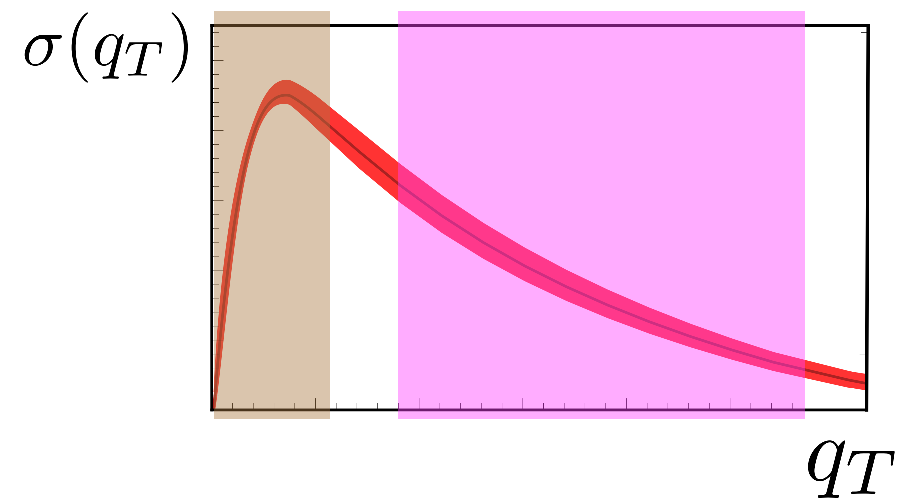

- **nonperturbative**

$$k_T \sim b_T^{-1} \sim \Lambda_{\mathrm{QCD}}$$

$$f_q(x, \vec{k}_T, \mu, \zeta)$$

- **perturbative**

$$k_T \sim b_T^{-1} \gg \Lambda_{\mathrm{QCD}}$$

$$f_q(x, \vec{k}_T, \mu, \zeta) = \sum_i \int \frac{dy}{y} \, C_{qi}\left(\frac{x}{y}, \vec{k}_T, \mu, \zeta\right) f_i(y, \mu)$$

perturbative PDF

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

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$$\mu \frac{d}{d\mu} \gamma_\zeta^q(\mu, b_T) = 2\zeta \frac{d}{d\zeta} \gamma_\mu^q(\mu, \zeta) = -2\Gamma_{\rm cusp}^q[\alpha_s(\mu)]$$

path independent

All Orders form:

$$\gamma_\mu^q(\mu, \zeta) = \Gamma_{\rm cusp}^q[\alpha_s(\mu)] \ln \frac{\mu^2}{\zeta} + \gamma_\mu^q[\alpha_s(\mu)]$$

$$\gamma_\zeta^q(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}^q[\alpha_s(\mu')] + \gamma_\zeta^q[\alpha_s(1/b_T)]$$

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- **Perturbative at short distance** $\mu, b_T^{-1} \gg \Lambda_{\rm QCD}$

$$\gamma_\zeta^q[\alpha_s] = \alpha_s\, \gamma_\zeta^{q(1)} + \alpha_s^2\, \gamma_\zeta^{q(2)} + \alpha_s^3\, \gamma_\zeta^{q(3)} + \ldots$$

3-loop result: Li, Zhu 2016

→ **LL, NLL, NNLL, N3LL, … results**

TMD Evolution:

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3-loop result: Li, Zhu 2016

→ **LL, NLL, NNLL, N3LL, … results**

- **For** $b_T^{-1} \sim \Lambda_{\rm QCD}$ **the CS kernel** $\gamma_\zeta^q(\mu, b_T)$ **becomes nonperturbative**

TMD Evolution:

$$\mu \frac{d}{d\mu} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\mu^q(\mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta) = \gamma_\zeta^q(\mu, b_T)$$

Collins-Soper Equation

Must solve both equations to sum large logarithms:

$$\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$$

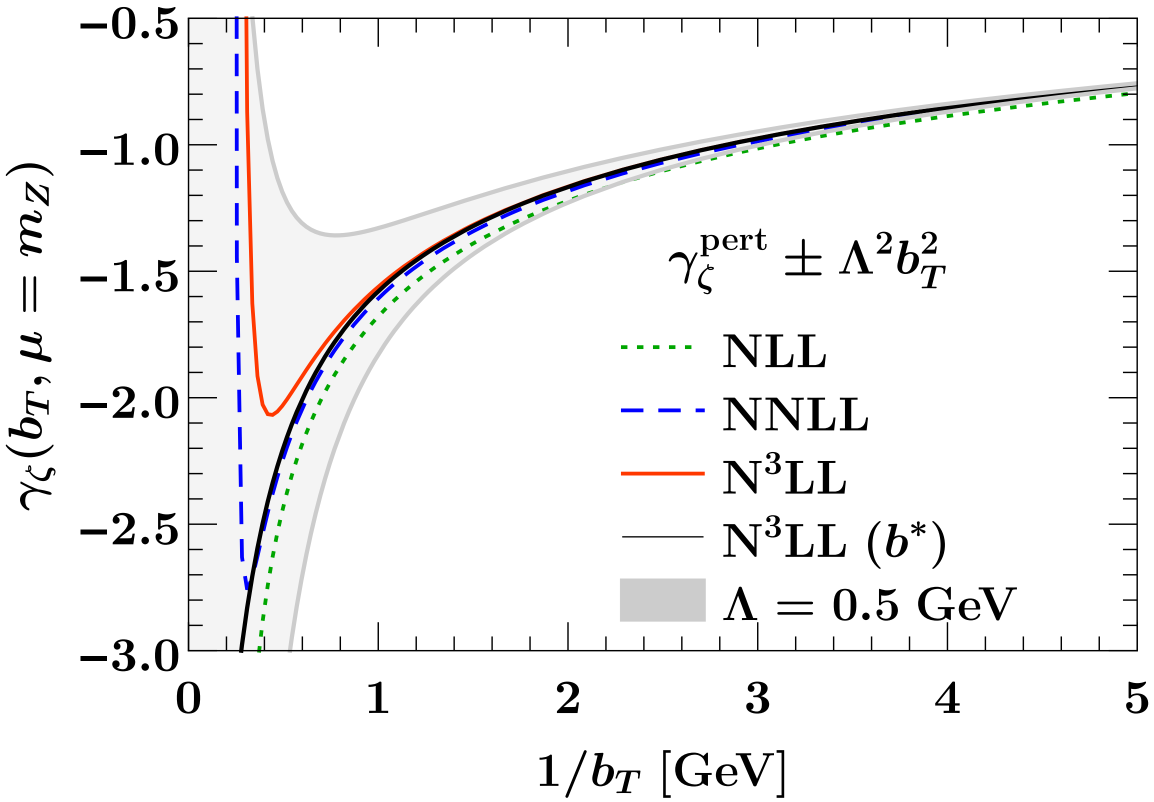
Solution:

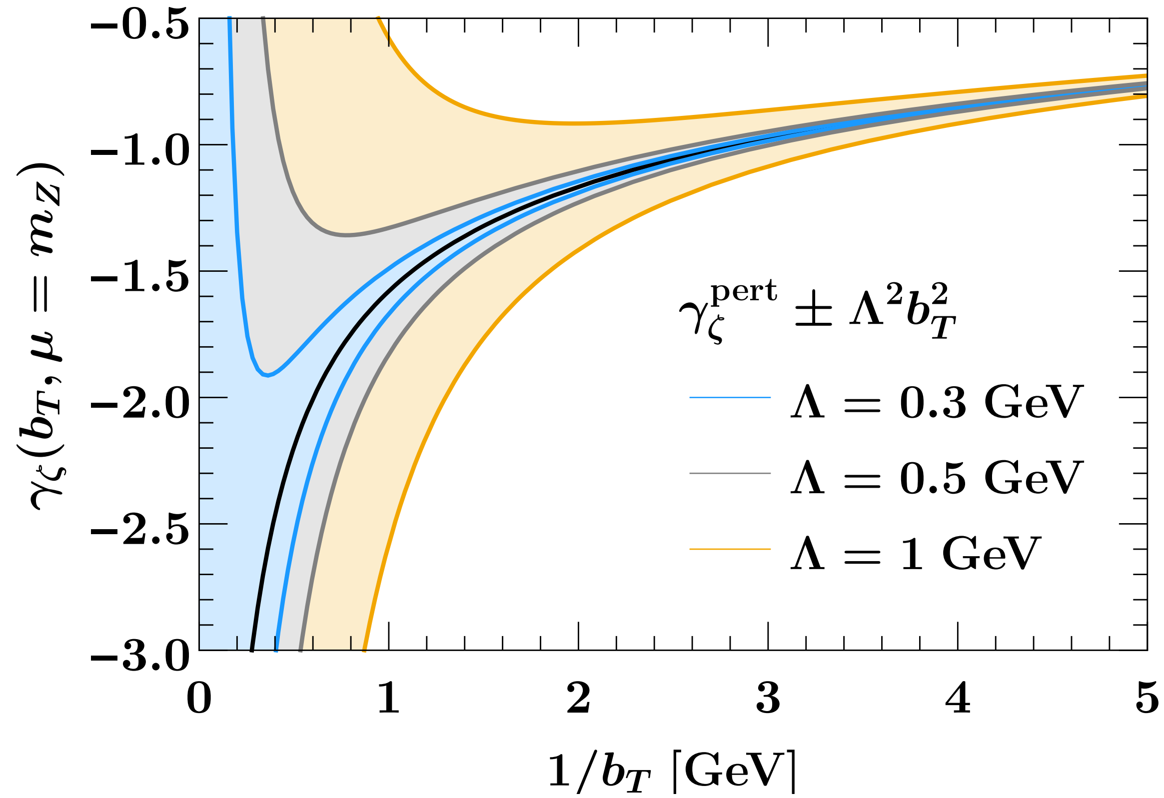
$$f_q(x, \vec{b}_T, \mu, \zeta) = \exp\Big[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^q(\mu', \zeta_0)\Big] \exp\Big[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\Big] \times f_q(x, \vec{b}_T, \mu_0, \zeta_0)$$

Useful: Connect Lattice calculation (or model) with $\mu \sim P^z \sim \text{few GeV}$

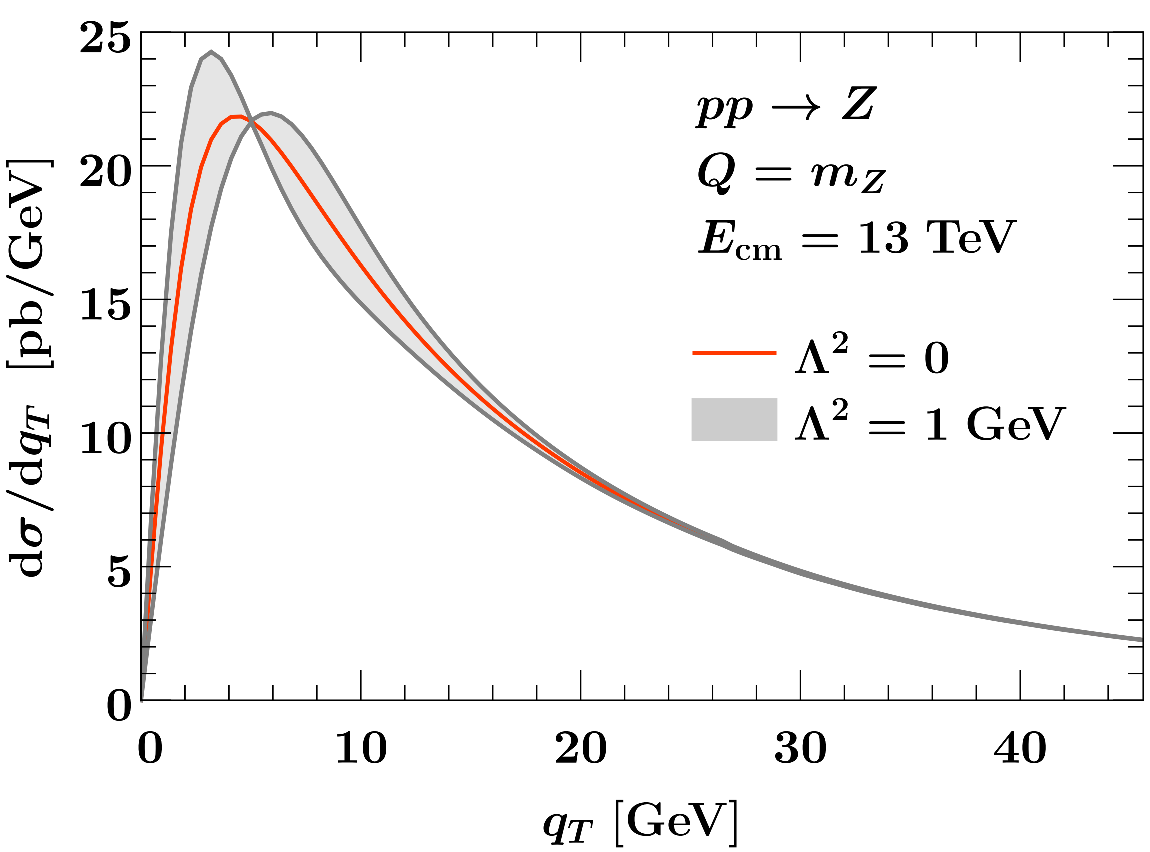
to scales needed in factorization theorem: $\mu \sim Q \ , \ P^z \sim Q/x$

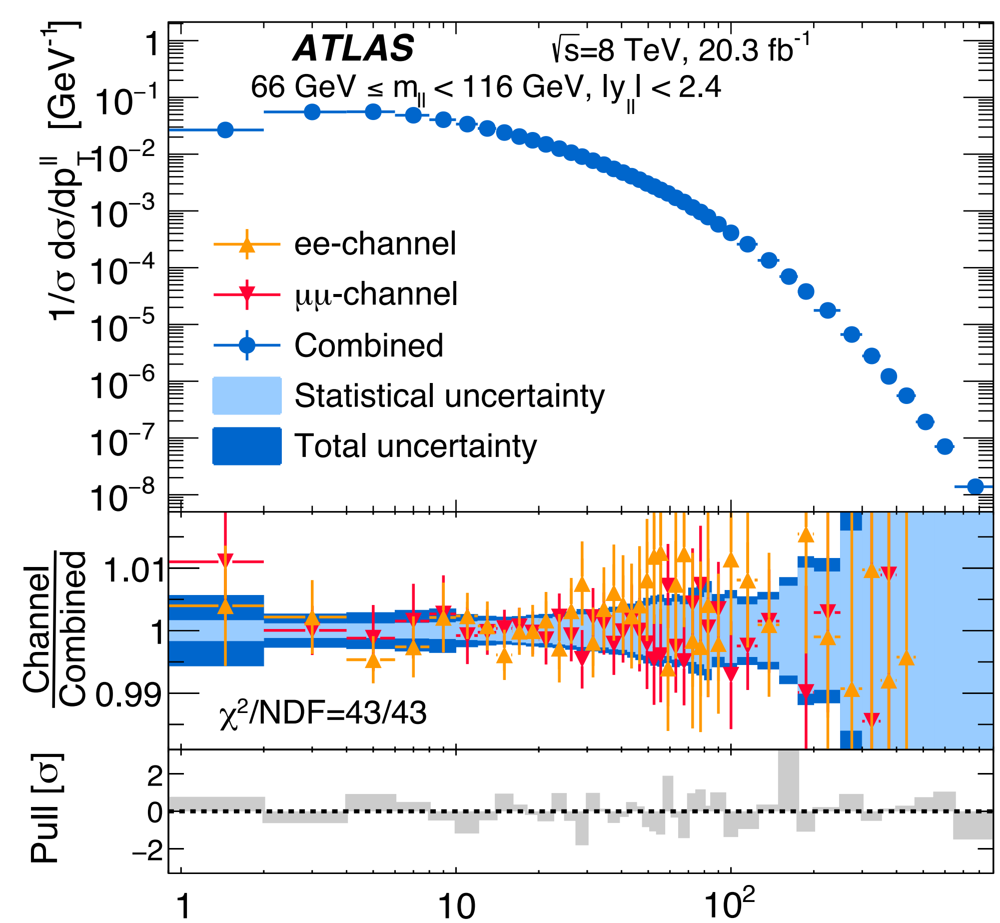
$\gamma_\zeta^q(\mu, b_T)$ Estimates for Size of Nonperturbative Contributions





Drell-Yan Cross Section:



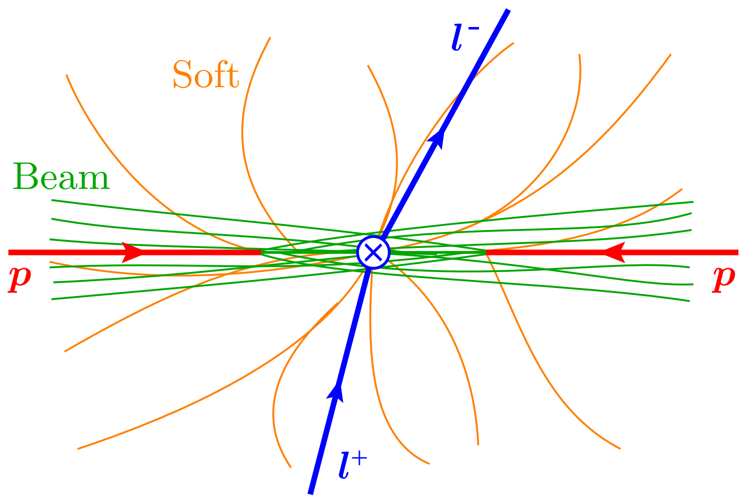


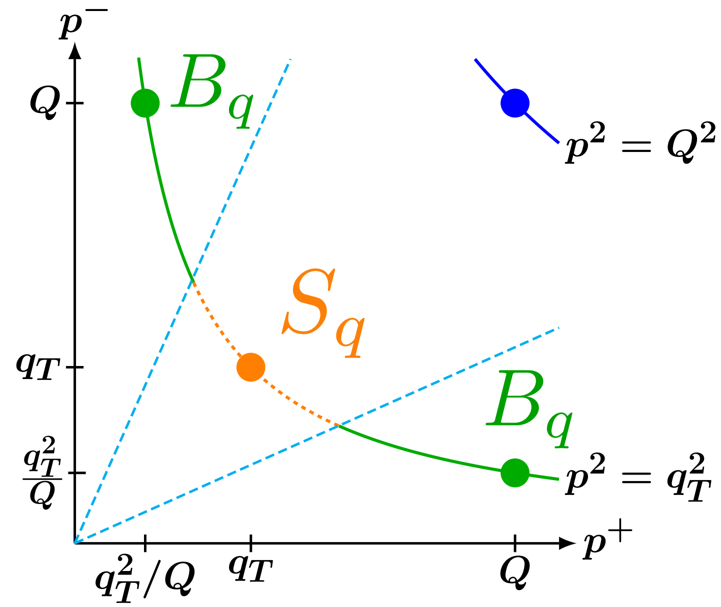
[ATLAS '15]

TMD Definitions

collinear region **soft region**

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\rm uv}(\epsilon, \mu, \zeta) B_q(x, \vec{b}_T, \epsilon, \tau, \zeta) \sqrt{S_q(b_T, \epsilon, \tau)}$$



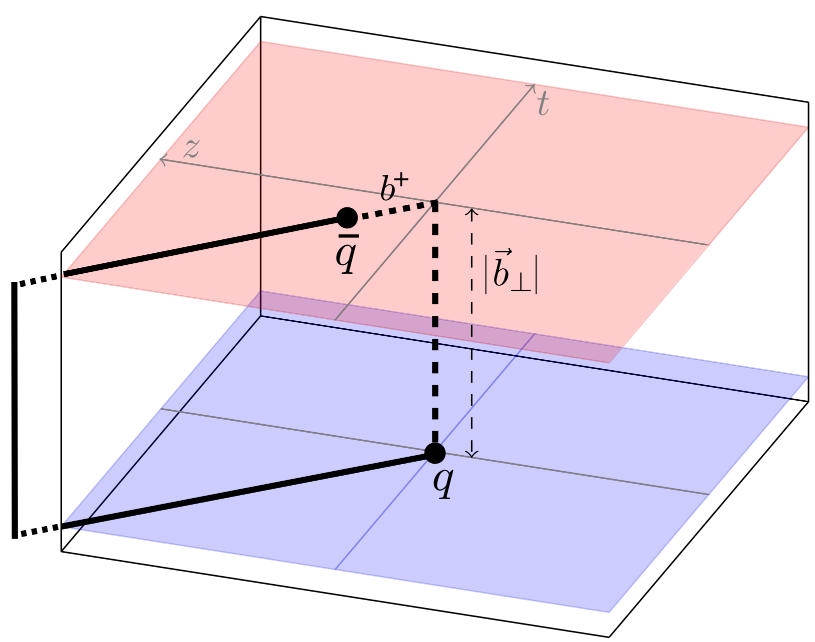


Wilson Lines:

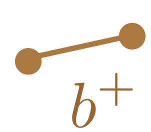
$B_q = \langle p|O_B|p\rangle$

O_B :

staple shaped

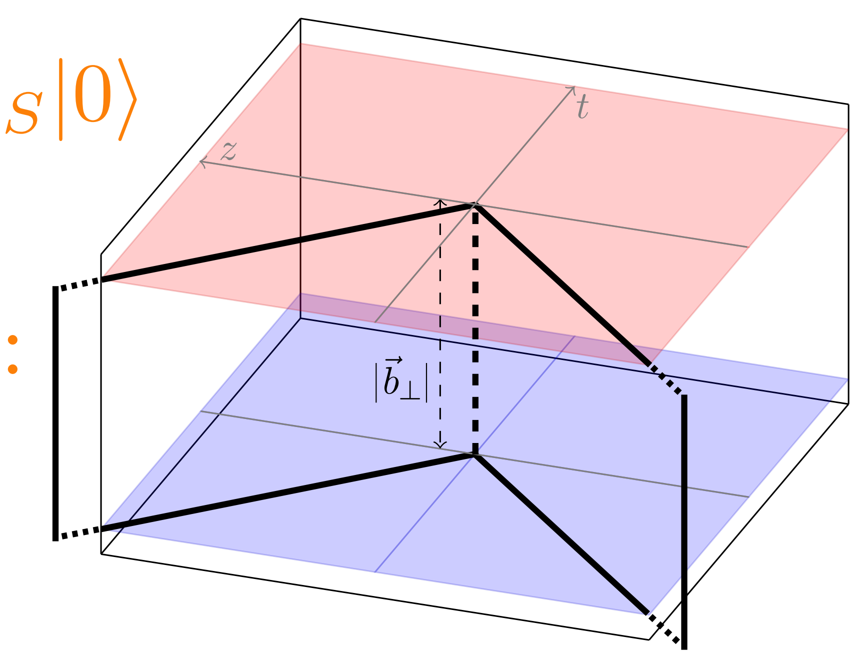


cf. PDF $(b_T = 0)$:



$S_q = \langle 0|O_S|0\rangle$

O_S :



two light-cone directions

depends on color rep. (q or g)

TMD Definitions

"Beam Function" "Soft Factor"

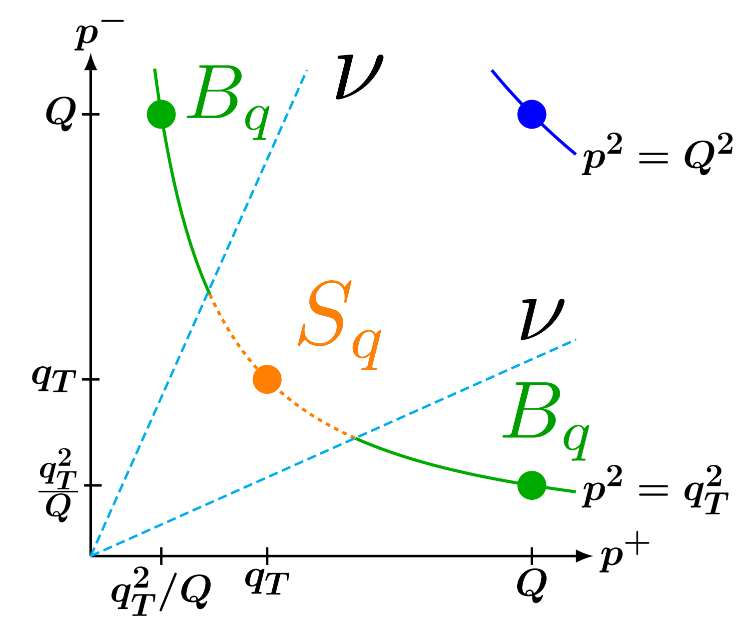
$$f_q(x,\vec{b}_T,\mu,\zeta) = \lim_{\epsilon\to 0,\tau\to 0} Z_{\rm uv}(\epsilon,\mu,\zeta) B_q(x,\vec{b}_T,\epsilon,\tau,\zeta)\ \Delta_q(b_T,\epsilon,\tau)$$

TMDPDF

ϵ : regulates UV divergences

τ : regulates rapidity divergences

$$\int_{q_T}^{Q} \frac{dk^+}{k^+} = \int_0^{Q} \frac{dk^+}{k^+} R(k^+,\tau,\nu) + \int_{q_T}^{\infty} \frac{dk^+}{k^+} R(k^+,\tau,\nu)$$
$$= \left(-\frac{1}{\tau} + \ln\frac{Q}{\nu}\right) + \left(\frac{1}{\tau} + \ln\frac{\nu}{q_T}\right)$$



<u>**Schemes:**</u> **Same f_q, ie. universal (across most schemes)**
Different B_q & Δ_q

- **Wilson lines off the light cone** (Modern Collins '11) $\Delta_q = 1/\sqrt{S_q}$
- **Delta regulator** $(k^\pm + i\delta^\pm)$ (Echevarria,Idilbi,Scimemi '11) $\Delta_q = 1/\sqrt{S_q}$
- **η regulator** $|k^z/\nu|^{-\eta}$ (Chiu,Jain,Neill,Rothstein '12) $\Delta_q = \sqrt{S_q}$
- **Exponential regulator** $e^{-k^0\tau}$ (Li,Neill,Zhu '16) $\Delta_q = 1/\sqrt{S_q}$

TMD Definitions

"Beam Function" "Soft Factor"

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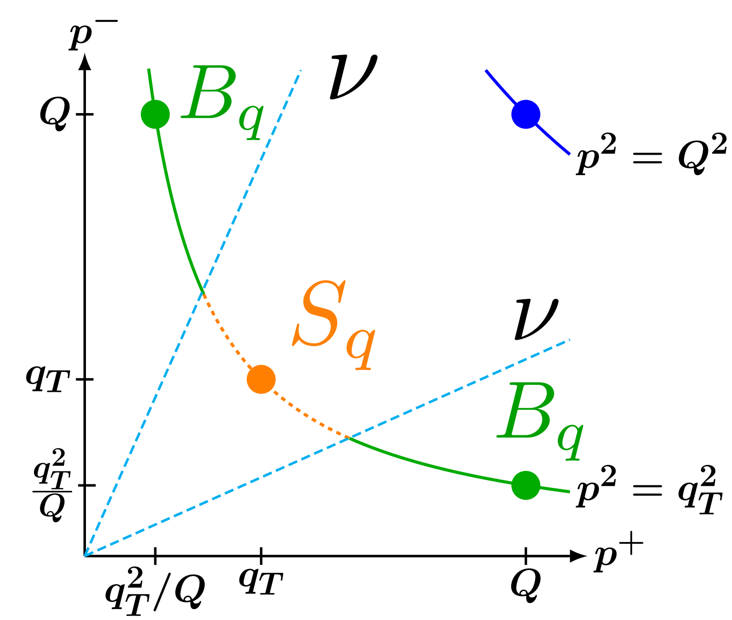


Schemes: Same f_q, ie. universal (across most schemes)
Different B_q & Δ_q

$$f_q(x,\vec{b}_T,\mu,\zeta) = B_q^{\rm ren}(x,\vec{b}_T,\mu,\nu^2/\zeta)\ \Delta_q^{\rm ren}(b_T,\mu,\nu)$$

$$\gamma_\zeta(b_T,\mu) = 2\zeta\frac{d}{d\zeta}\ln f_q = -\nu\frac{d}{d\nu}\ln B_q^{\rm ren} = \frac{1}{2}\nu\frac{d}{d\nu}\ln\Delta_q^{\rm ren}$$

CS kernel
= rapidity anom.dim.

Vacuum matrix element, so clearly independent of hadronic state.

Quasi-PDFs: (Ji 2013) Many talks at this workshop.

Now on a rigorous footing.

Quasi-TMDs

Calculate Nonperturbative $\gamma_\zeta^q(\mu, b_T)$ M. Ebert, IS, Y. Zhao, 1811.00026

Calculate Nonperturbative $f_q(x, b_T, \mu, \zeta)$ M. Ebert, IS, Y. Zhao, 1901.03685

(harder!)

[compare also to Ji, Jin, Yuan, Zhang, Zhao 1801.05930]

Quasi-Beam Functions

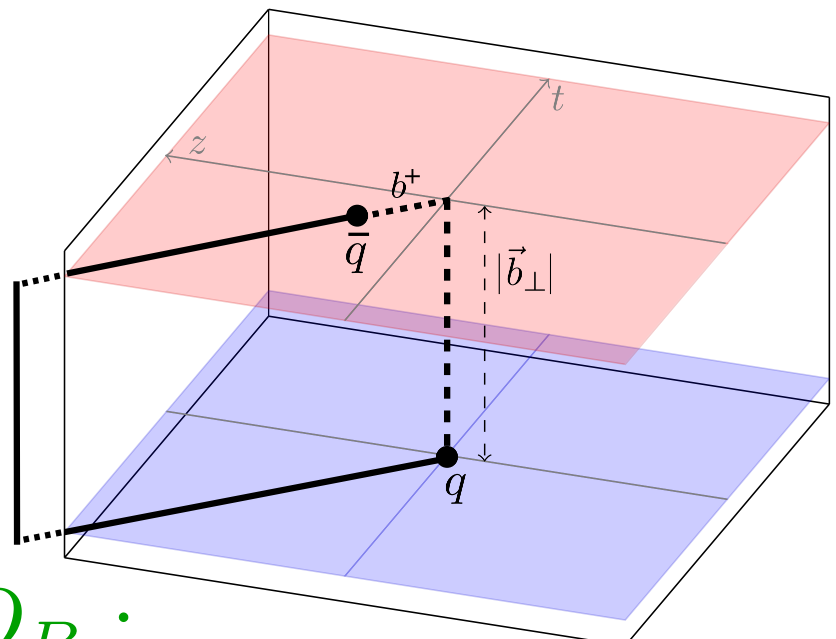
Beam Function

$B_q = \langle p | O_B | p \rangle$

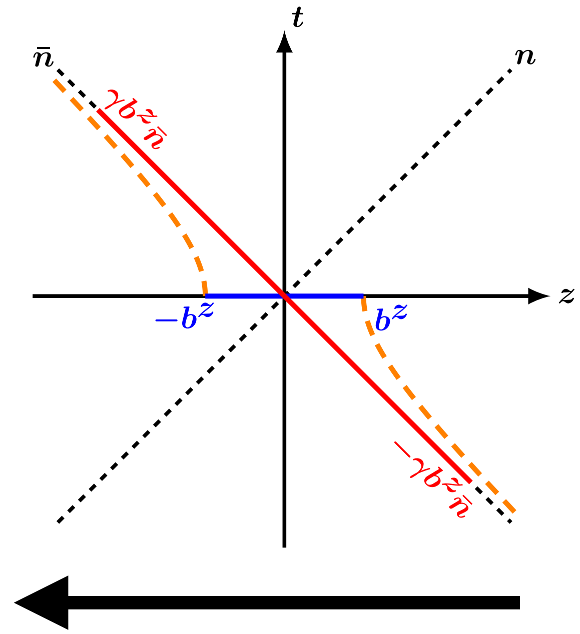
Natural Quasi-Beam Function

(from boost picture)

$\tilde{B}_q = \langle p | \tilde{O}_B | p \rangle$

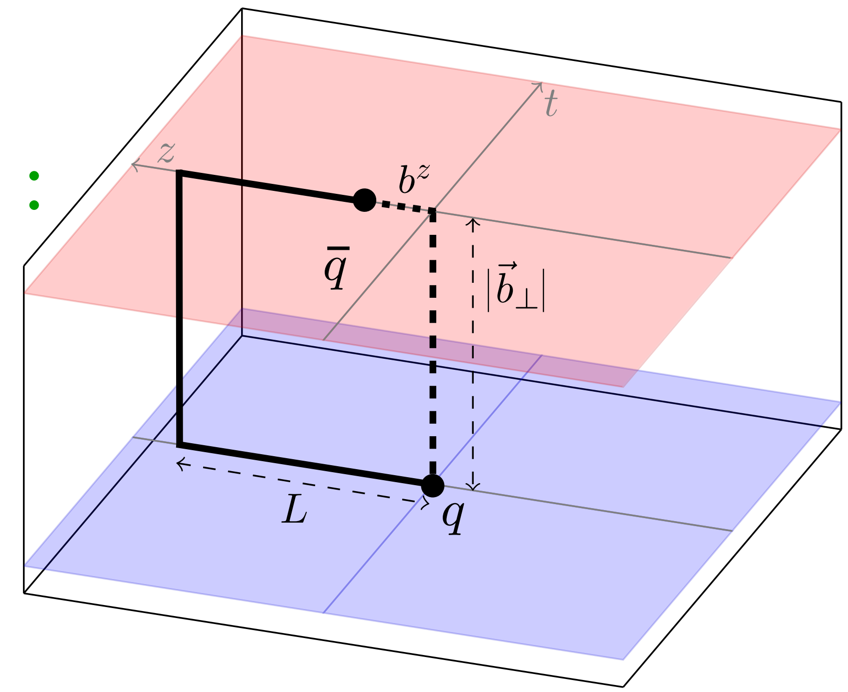


O_B :



Boost?

$\tilde{O}_B$:



- **Boost valid for unregulated functions. Impact of regularization?**
- **Finite length L for Wilson lines, no rapidity divergences** $\dfrac{1 - e^{-\mathrm{i}k^z L}}{k^z}$

$$\frac{1}{P^z} \ll b_T \ll L$$

- **Spatial lines, so have power law UV divergence** $\propto \text{length} = 2L + b_T - b^z$

Quasi-Soft Function

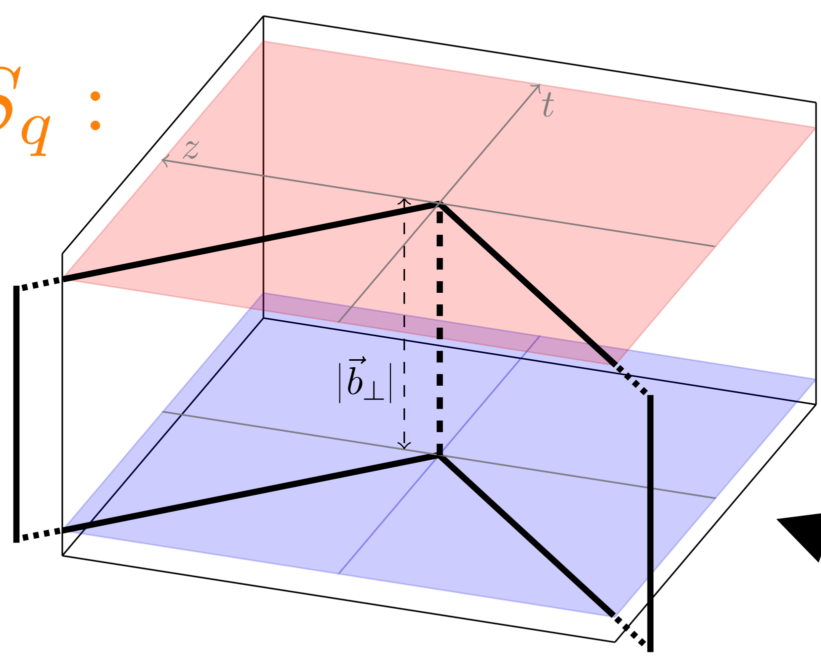
$\tilde{f}_q \sim \tilde{B}_q \tilde{\Delta}_S^q$, $\tilde{\Delta}_S^q = 1/\sqrt{\tilde{S}_q}$

- **Cancel power law dependence on L, length =** $2(2L + b_T)$
- **Also needed to reproduce infrared structure.**
- **Free to invent a** $\tilde{S}_q$ **to achieve this.**

Quasi-Soft Function = ?

Soft Function

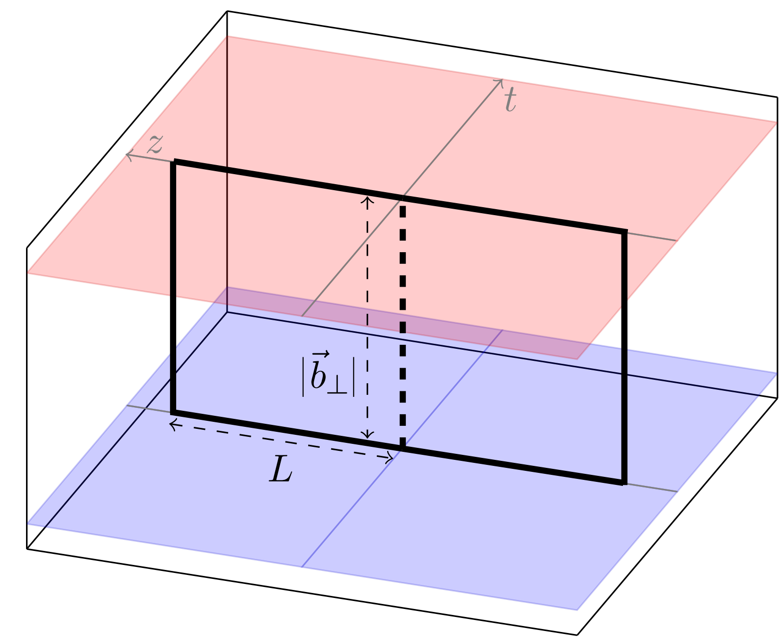
S_q :



?

(1) “naive” quasi-soft function

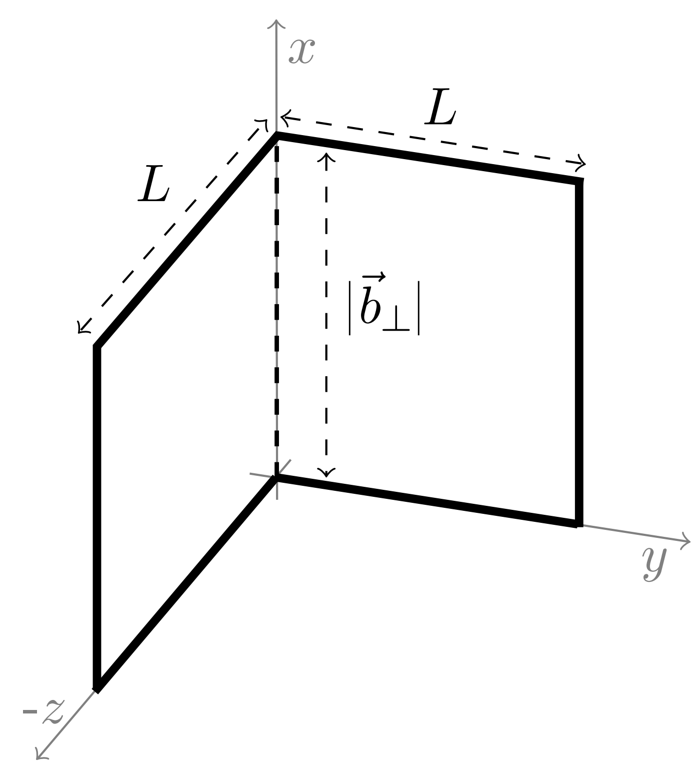
$\tilde{S}_q$:



?

(2) “bent” quasi-soft function

$\tilde{S}_q$:



No connection via a boost.

Quasi-TMDPDF

a = lattice spacing (UV regulator)

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \lim_{\substack{a\to 0 \\ L\to\infty}} \tilde{Z}'_q(b^z, \mu, \tilde{\mu}) \tilde{Z}^q_{\rm uv}(b^z, \tilde{\mu}, a)$$

$$\times \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z) \tilde{\Delta}^q_S(b_T, a, L)$$

- linear divergences in L cancel
- $\tilde{Z}^q_{\rm uv}$ multiplicative, and removes linear b^z/a divergence
- $\tilde{Z}'_q$ converts lattice friendly scheme ($\tilde{\mu}$) to $\overline{\rm MS}$ (μ)

In order to match onto TMDPDF the IR physics of quasi-TMPDF $\tilde{f}_q$ and TMDPDF f_q must agree (collinear singularity & $b_T \sim \Lambda_{\rm QCD}^{-1}$ dependence)

- We use this to test choices for $\tilde{\Delta}^q_S$ perturbatively.

Relation between Quasi-TMDPDF & TMDPDF

(isovector operators from here on)

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

nonperturbative quasi-TMDPDF	perturbative kernel	nonperturbative factor that changes based on choice for $\tilde{\Delta}_S^q$	nonperturbative CS kernel	nonperturbative TMDPDF

(Only confirmed so far at one-loop)

Determination of $\gamma_\zeta^q(\mu, b_T)$

- **independent of** $g_q^S(b_T, \mu)$

Determination of $f_q(x, \vec{b}_T, \mu, \zeta)$

- **requires full matching formula. <u>Only</u> exists if** $g_q^S(b_T, \mu) = 1$
- **take** $\zeta = (2xP^z)^2$ **to obtain matching:**

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, f_q(x, \vec{b}_T, \mu, \zeta)$$

(Note: no convolution in x)

Collins-Soper Kernel from Lattice

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\mathrm{TMD}}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMDPDF

Hold ζ fixed, take ratio with two different P^zs:

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C^{\mathrm{TMD}}(\mu, xP_2^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_1^z)}{C^{\mathrm{TMD}}(\mu, xP_1^z)\, \tilde{f}_q(x, \vec{b}_T, \mu, P_2^z)}$$

$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C^{\mathrm{TMD}}(\mu, xP_2^z) \int db^z e^{ib^z xP_1^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\mathrm{uv}}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_1^z)}{C^{\mathrm{TMD}}(\mu, xP_1^z) \int db^z e^{ib^z xP_2^z}\, \tilde{Z}'_q\, \tilde{Z}^q_{\mathrm{uv}}\, \tilde{B}_q(b^z, \vec{b}_T, a, L, P_2^z)}$$

quasi-Beam fns.

- **only needs $\tilde{B}_q$, does not require $\tilde{\Delta}_S^q$**
- **LHS independent of $P_1^z, P_2^z, x,$ hadron state, spin structure**

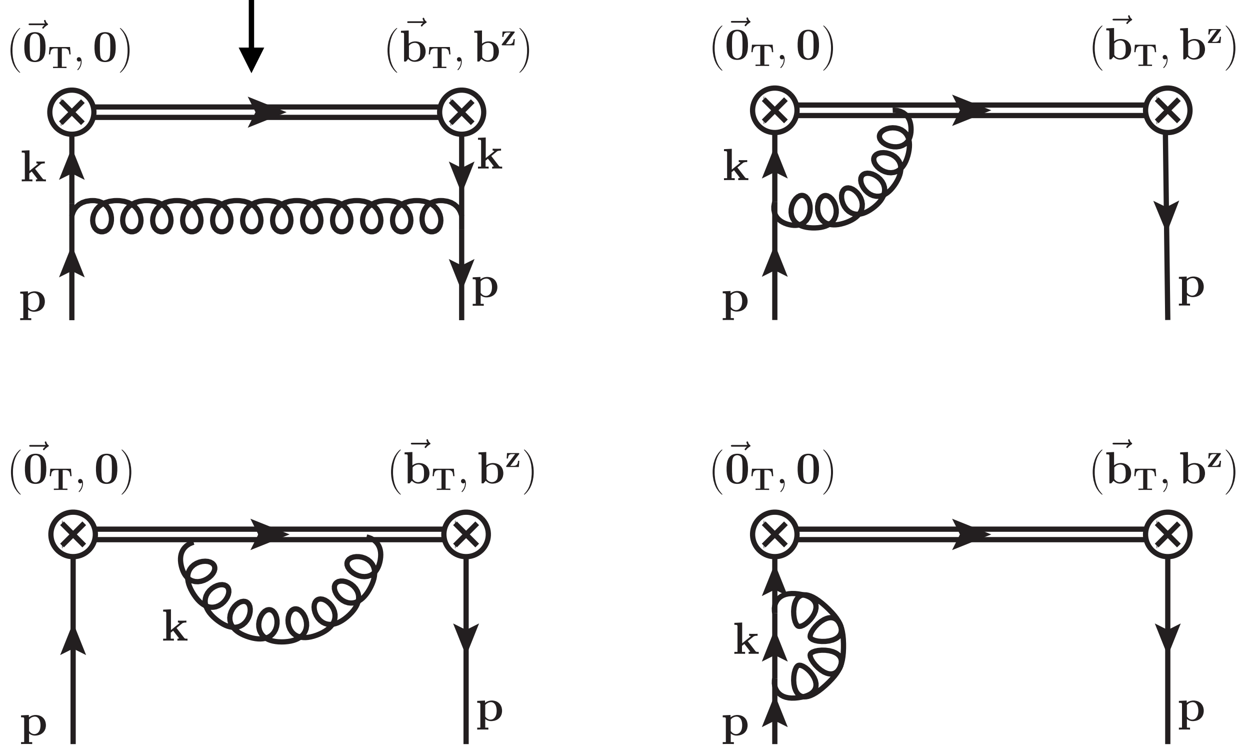
Important universal QCD function obtainable from Lattice QCD

Ratios of proton $\tilde{B}_q$s also studied by [Musch et al '10'12; Engelhardt et al '15; Yoon et al '17] (same P^z)

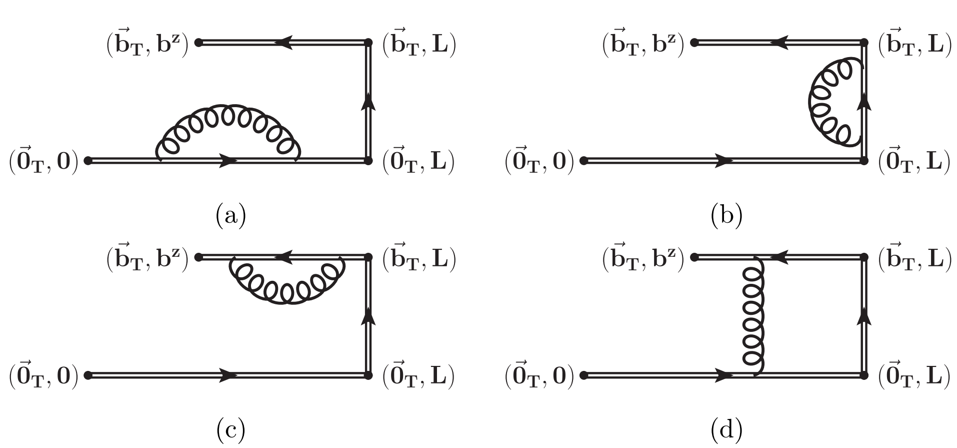
One-Loop Diagrams

Quasi-Beam Function

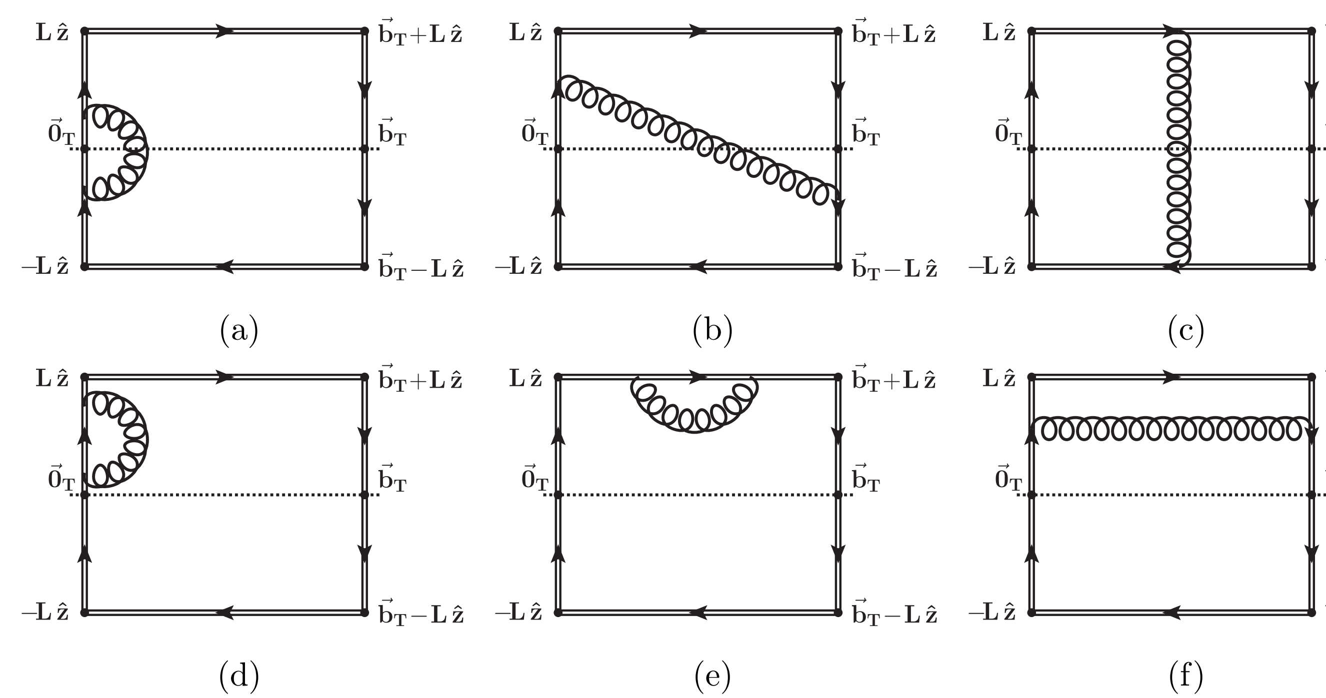
staple!



eg. this is equal to



Quasi-Soft Function (eg. of naive case)



One-Loop Analysis

Legend: match the colors!

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

$$L_b = \ln\frac{b_T^2 \mu^2}{4e^{-2\gamma_E}}$$

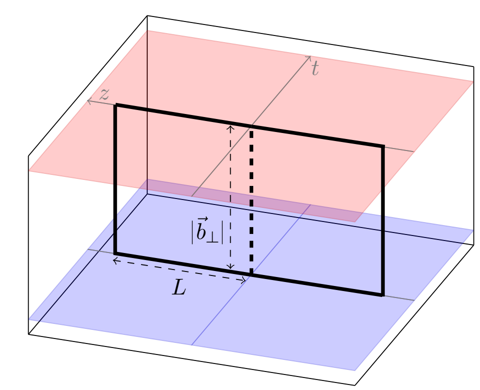
$$L_{P_z} = \ln\frac{\mu^2}{(2xP^z)^2}$$

TMDPDF:

$$f_q^{(1)}(x, \vec{b}_T, \mu, \zeta) = \frac{C_F \alpha_s}{2\pi}\left[-\left(\frac{1}{\epsilon_{\rm IR}} + L_b\right)P_{qq}(x) + (1-x)_+ + \delta(1-x)\left(-\frac{1}{2}L_b^2 + \frac{3}{2}L_b + L_b \ln\frac{\mu^2}{\zeta} + \frac{1}{2} - \frac{\pi^2}{12}\right)\right]$$

quasi-TMDPDF:

(1) "naive" $\tilde{S}_q$:



$$\tilde{f}_q^{(1)}(x, \vec{b}_T, \mu, P^z) = \frac{C_F \alpha_s}{2\pi}\left[-\left(\frac{1}{\epsilon_{\rm IR}} + L_b\right)P_{qq}(x) + (1-x)_+ + \delta(1-x)\left(-\frac{1}{2}L_b^2 + \frac{3}{2}L_b + L_b + L_b \ln\frac{\mu^2}{(2xP^z)^2} - \frac{1}{2}L_{P_z}^2 - L_{P_z} - \frac{3}{2}\right)\right]$$

IR log (matching fails, fine for γ_ζ^q)

agree: [Ji, Jin, Yuan, Zhang, Zhao '18] [Ebert, IS, Zhao '18]

One-Loop Analysis

Legend: match the colors!

$$\tilde{f}_q(x, \vec{b}_T, \mu, P^z) = C^{\rm TMD}(\mu, xP^z)\, g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right] f_q(x, \vec{b}_T, \mu, \zeta)$$

$$L_b = \ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}}$$

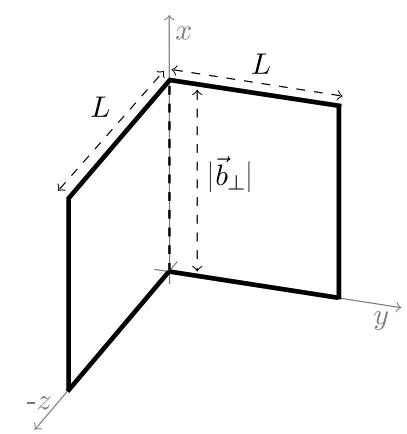
$$L_{P_z} = \ln\frac{\mu^2}{(2xP^z)^2}$$

TMDPDF:

$$f_q^{(1)}(x, \vec{b}_T, \mu, \zeta) = \frac{C_F\alpha_s}{2\pi}\Big[-\Big(\frac{1}{\epsilon_{\rm IR}} + L_b\Big)P_{qq}(x) + (1-x)_+ + \delta(1-x)\Big(-\frac{1}{2}L_b^2 + \frac{3}{2}L_b + L_b\ln\frac{\mu^2}{\zeta} + \frac{1}{2} - \frac{\pi^2}{12}\Big)\Big]$$

quasi-TMDPDF:

(2) "bent" $\tilde{S}_q$:



$$\tilde{f}_q^{(1)}(x, \vec{b}_T, \mu, P^z) = \frac{C_F\alpha_s}{2\pi}\Big[-\Big(\frac{1}{\epsilon_{\rm IR}} + L_b\Big)P_{qq}(x) + (1-x)_+ + \delta(1-x)\Big(-\frac{1}{2}L_b^2 + \frac{3}{2}L_b + L_b\ln\frac{\mu^2}{(2xP^z)^2} - \frac{1}{2}L_{P_z}^2 - L_{P_z} - \frac{3}{2}\Big)\Big]$$

(matching works, and fine for γ_ζ^q)

$C^{\rm TMD}(\mu, xP^z)$ available in $\overline{\rm MS}$ at 1-loop

[Ebert, IS, Zhao '19]

Details on IR Logs

$$L_b = \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}}$$

Regulator	Beam function B_q	Soft factor Δ_S^q	TMDPDF $f_q^{\mathrm{TMD}} = B_q \Delta_S^q$
Collins	$-\frac{1}{2}L_b^2\,,\ \frac{5}{2}L_b$	$-L_b$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
δ regulator	$\frac{3}{2}L_b$	$-\frac{1}{2}L_b^2$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
η regulator	$\frac{3}{2}L_b$	$-\frac{1}{2}L_b^2$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
Exp. regulator	$-L_b^2\,,\ \frac{3}{2}L_b$	$\frac{1}{2}L_b^2$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$
	quasi $\tilde{B}_q$	quasi $\tilde{\Delta}_S^q$	quasi $\tilde{f}_q^{\mathrm{TMD}} = \tilde{B}_q \tilde{\Delta}_S^q$
Finite L, naive $\tilde{\Delta}_S^q$	$-\frac{1}{2}L_b^2\,,\ \frac{9}{2}L_b$	$-2L_b$	$-\frac{1}{2}L_b^2\,,\ \frac{5}{2}L_b$
Finite L, bent $\tilde{\Delta}_S^q$	$-\frac{1}{2}L_b^2\,,\ \frac{9}{2}L_b$	$-3L_b$	$-\frac{1}{2}L_b^2\,,\ \frac{3}{2}L_b$

- **Matching for $\tilde{B}_q \leftrightarrow B_q$ fails in all schemes. So boost argument fails for all known regulated beam functions.**
- **Matching for $\tilde{\Delta}_S^q \leftrightarrow \Delta_S^q$ also fails.**
- **Matching works for $\tilde{f}_q \leftrightarrow f_q$ at 1-loop with bent quasi-soft factor.**

Summary

- In general TMDs are much less constrained by experiment
- TMDs complicated by Wilson line paths, rapidity divergences, hadronic matrix element & vacuum soft matrix element
- Proposed a method to determine CS kernel with Lattice QCD.
- Proposed matching formula for obtaining full TMDPDF using a bent quasi-TMDPDF construction (only checked at 1-loop)

Future

- Proof for matching relation needed (start with 2-loop analysis)
- Lattice Simulations for γ_ζ^q (Mike Wagman's talk next!)