Towards TMDPDFs using lattice QCD

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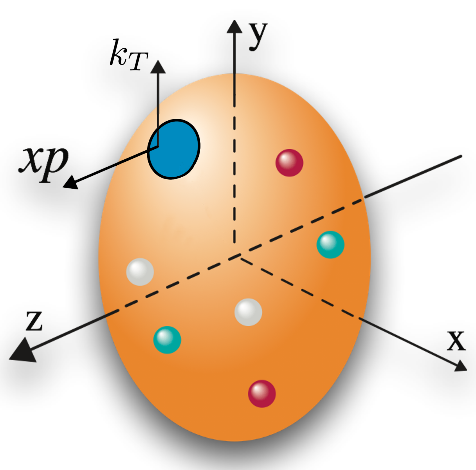
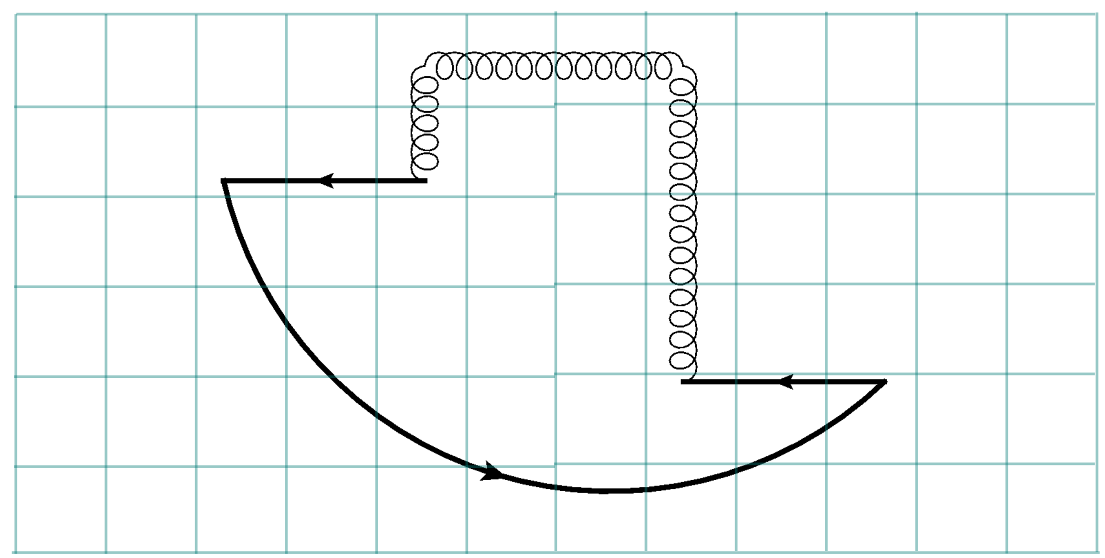
work in progress with

Phiala Shanahan, Yong Zhao





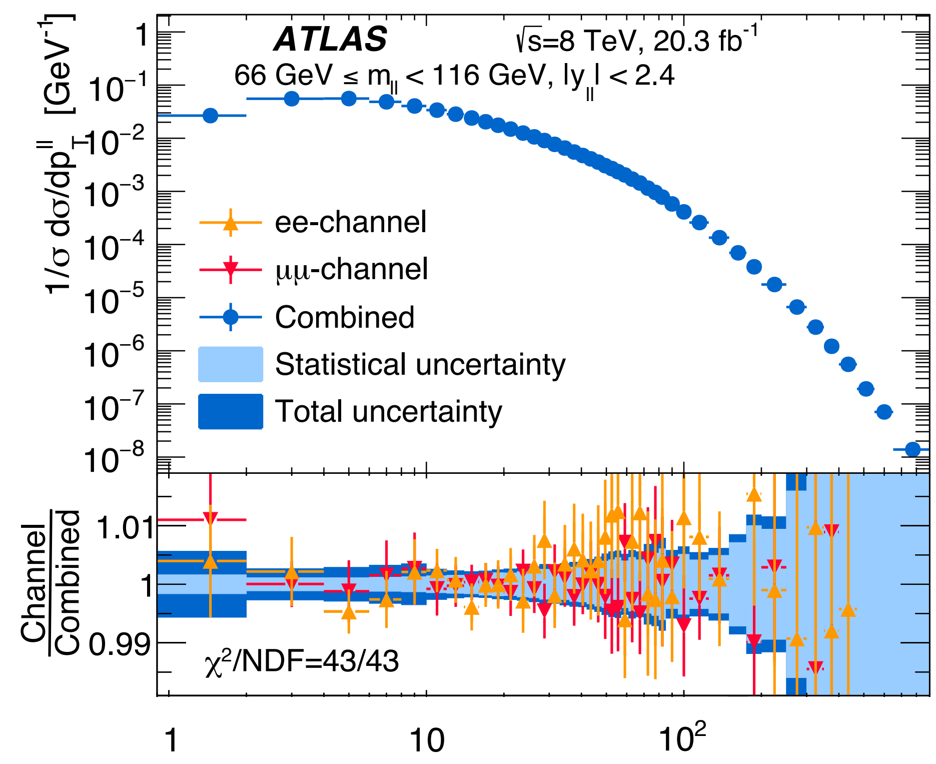
CFNS Workshop on Lattice Parton Distribution Functions

April 18, 2019

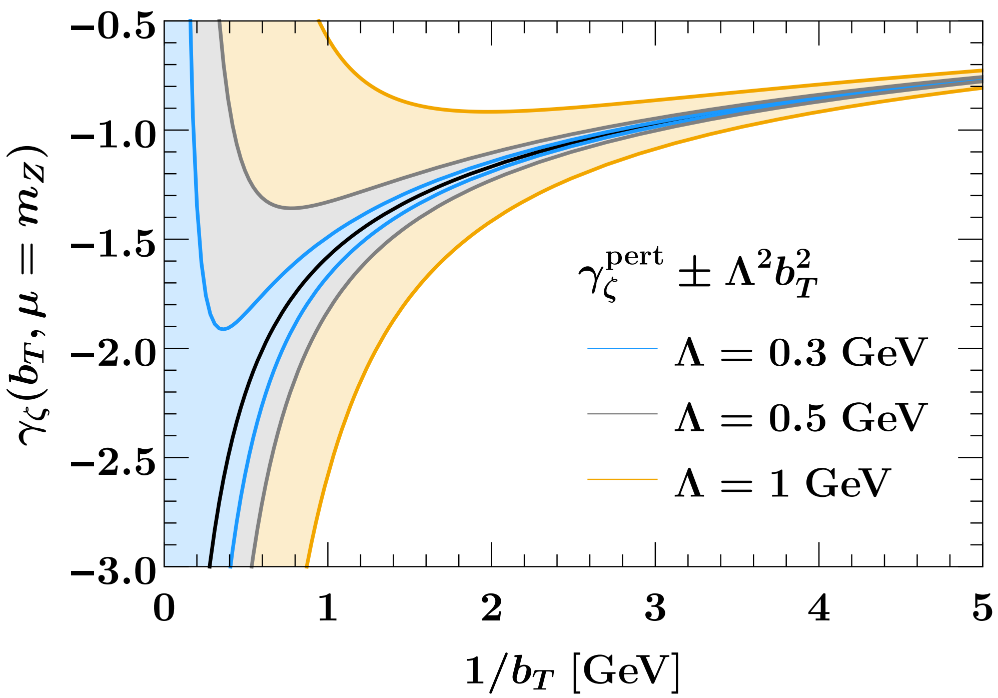
TMD evolution

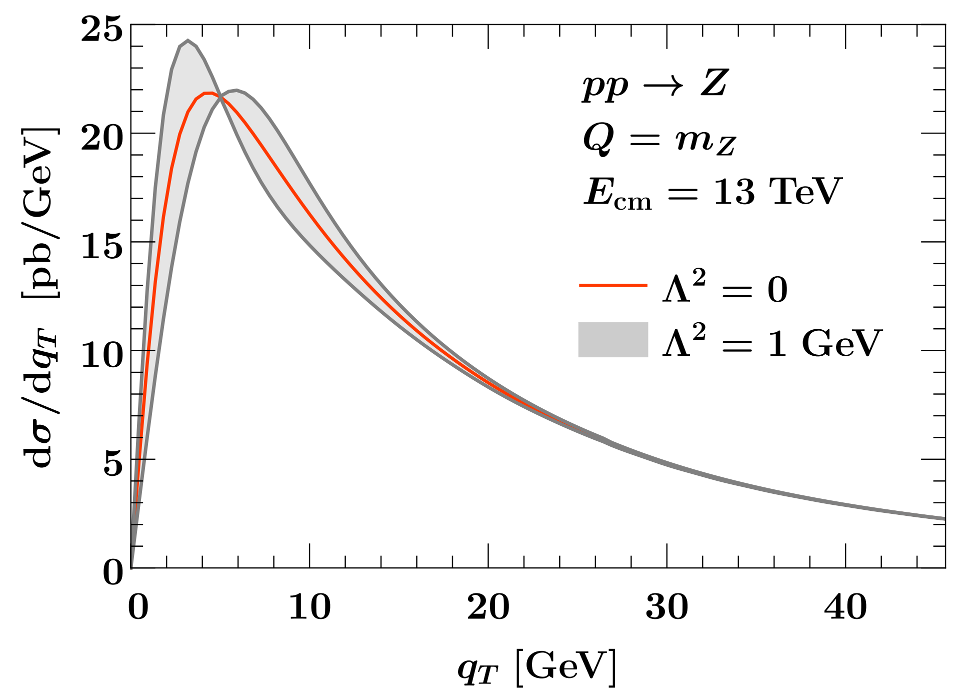
TMDPDFs encode 3D structure of hadrons, q_T dependence of hadronic cross-sections

Drell-Yan q_T dependence measured at LHC, <1% precision



Nonperturbative contributions to rapidity anomalous dimension limit QCD theory predictions to much lower precision



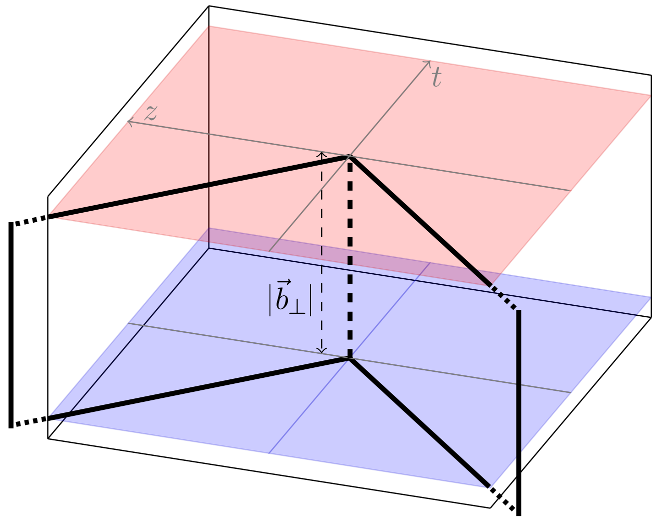


TMD evolution from LQCD

TMDPDFs inaccessible to LQCD with LaMET

— soft factor includes two light-light directions

Ebert, Stewart, Zhao, arXiv:1901.03685



Ratios of TMDPDFs free from soft factors, can be calculated with LQCD

Musch et al, PRD 85 (2012)   Engelhardt et al, PRD 93 (2016)   Yoon et al, PRD 96 (2017)

TMDPDF rapidity anomalous dimensions (Collins-Soper kernel) calculable from ratios of quasi-TMDPDFs

Ebert, Stewart, Zhao, PRD 99 (2019)

$$\gamma_\zeta^{q,\overline{\mathrm{MS}}}(b_T,\mu) = \zeta \frac{d}{d\zeta} f_q^{\overline{\mathrm{MS}}}(x,b_T,\mu,\zeta)$$

$$= \frac{1}{\ln(p_1^z/p_2^z)} \ln \frac{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xP_2^z) \int db^z e^{ib^z x p_1^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p_1^z)}{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xp_1^z) \int db^z e^{ib^z x p_2^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p_2^z)}$$

LQCD-friendly quasi-beam function

LQCD Setup

$$\gamma_\zeta^{q,\overline{\mathrm{MS}}}(b_T,\mu) = \frac{1}{\ln(p_1^z/p_2^z)} \ln \frac{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xP_2^z) \int db^z e^{ib^z x p_1^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p_1^z)}{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xp_1^z) \int db^z e^{ib^z x p_2^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p_2^z)}$$

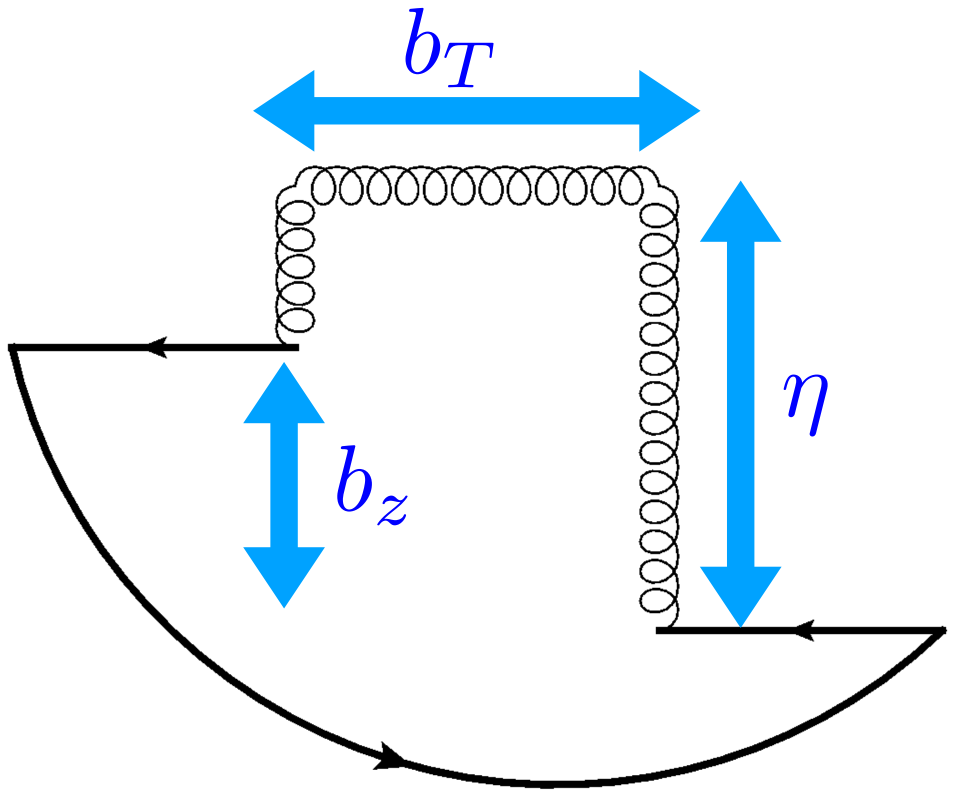
Independent of hadron state, choice of momenta, choice of x

…up to power corrections: $b_T/\eta, \quad 1/(p^z b_T), \; M/p^z$

Exploit independence, calculate for valence pion with $m_\pi \sim 1.2 \text{ GeV}$

Variation of m_π probes power corrections

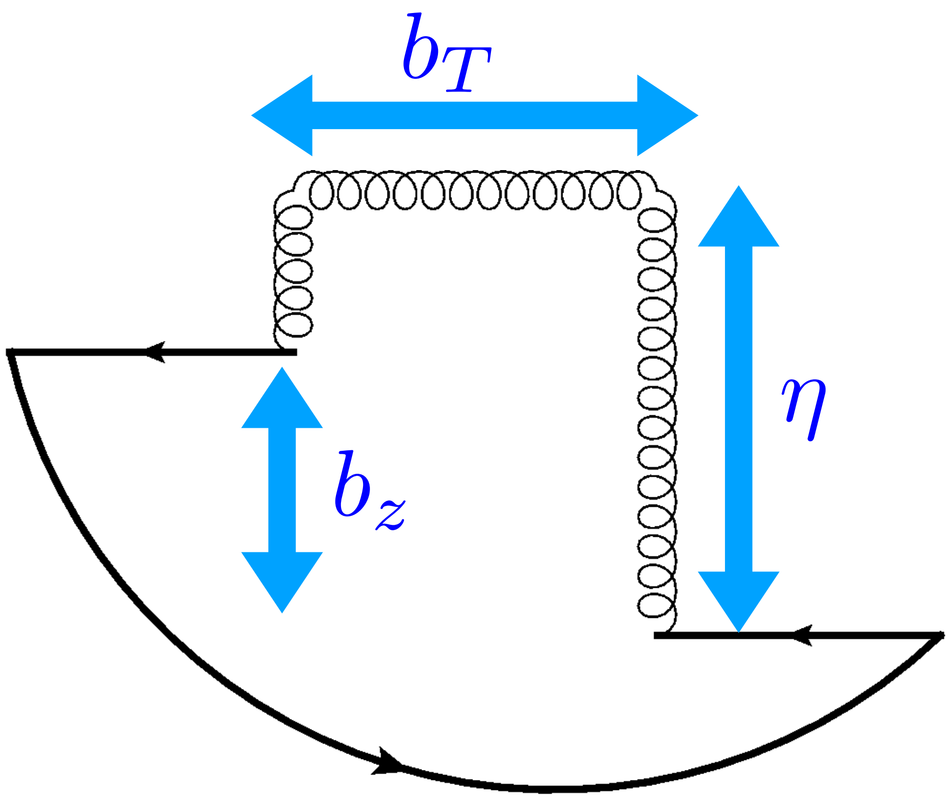
Not independent of sea quark mass, quenched gauge fields used for exploratory calculation



Challenges

$$\widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p^z) = \lim_{t,\tau,|t-\tau|\to\infty} 2E(p)\frac{G^{3pt}(b^z, b_T, \eta, \mu, p^z, t, \tau)}{G^{2pt}(p^z, t)}$$

- Large momentum. Noisy, need fine lattice spacing, …



- Many bare matrix elements useful

$$B(b_z, b_T, \eta, p_z)$$

Redundant η *and* p_z *choices probe systematics*

- And many nonperturbative renormalization factors

$$Z(b_z, b_T, \eta, \mu)$$

$$0 \leq b_z, b_T \leq \eta \leq 10$$

$$p_z = 2,\ 3,\ 4$$

$$p_z^{max} \sim 2.6\ \mathrm{GeV}$$

Analyze 2838 3pt functions and NPR factors

- Fourier transform over b_z

Exploratory ensembles

Quenched Wilson gauge configurations — *generated by Michael Endres*

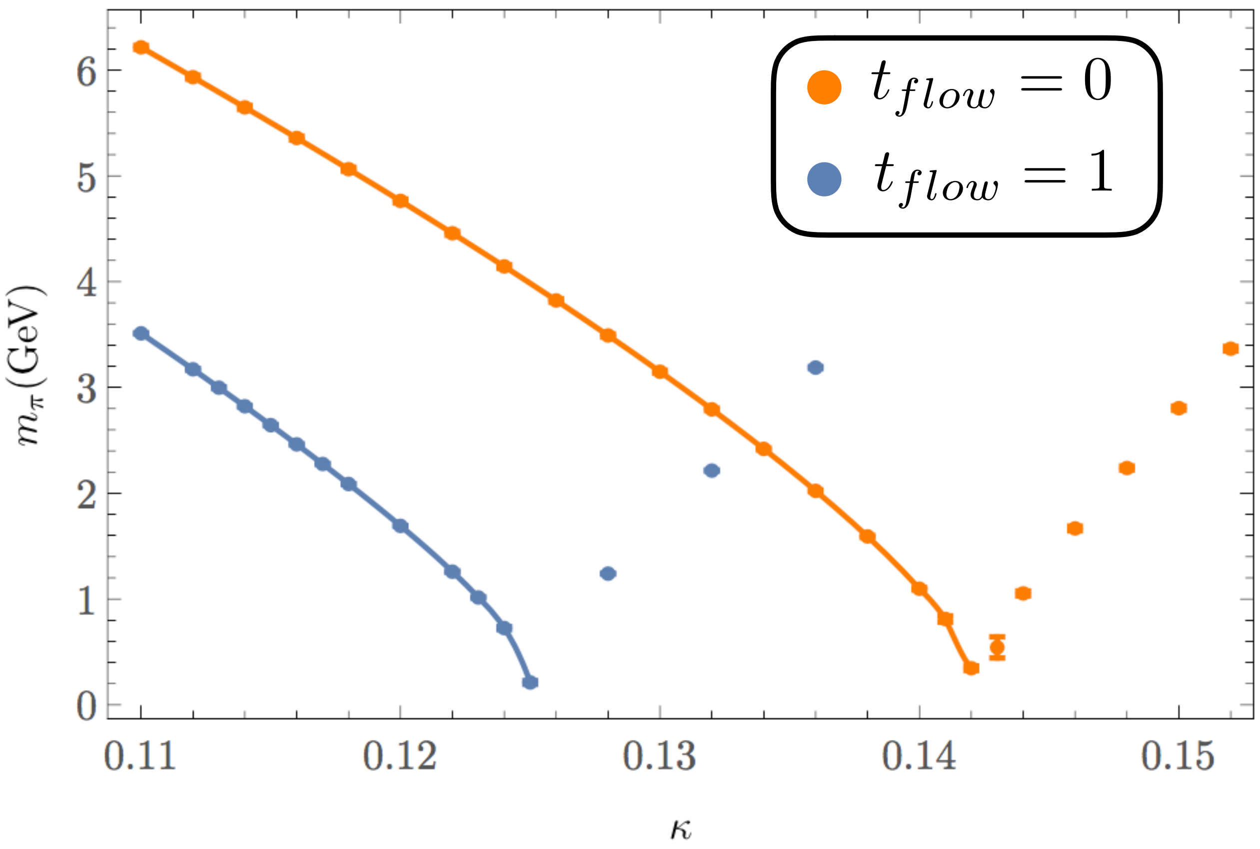
$32^3 \times 64 \qquad \beta = 6.30168$

Lattice spacing determined from gradient flow scale-setting

$a = 0.06(1)\ \mathrm{fm}$

Lüscher, JHEP 1008 (2010)

Borsanyi et al, JHEP 1209 (2012)



Gradient flow also used as link smearing to reduce noise

— Bare quark mass tuned after smearing

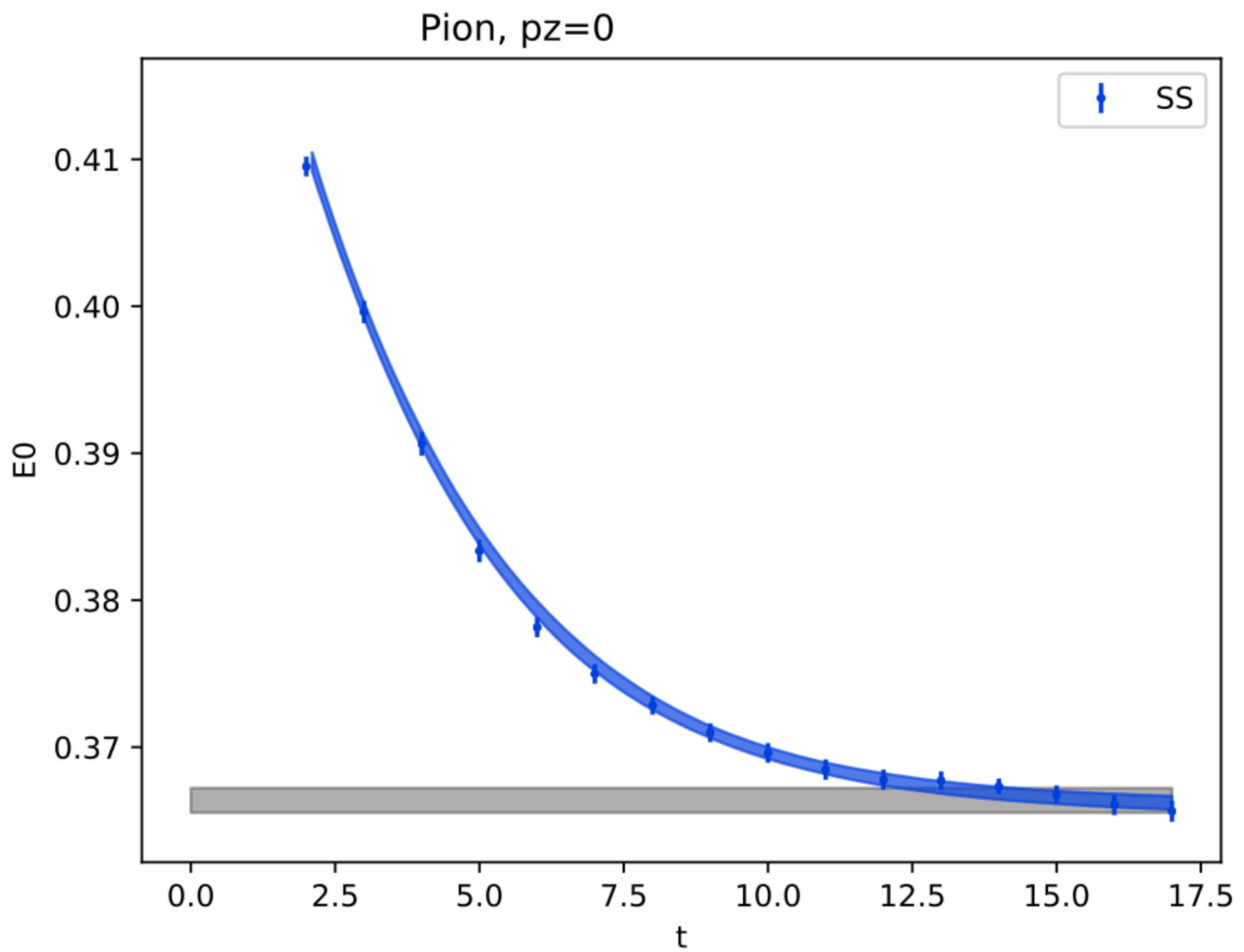
Low statistics:

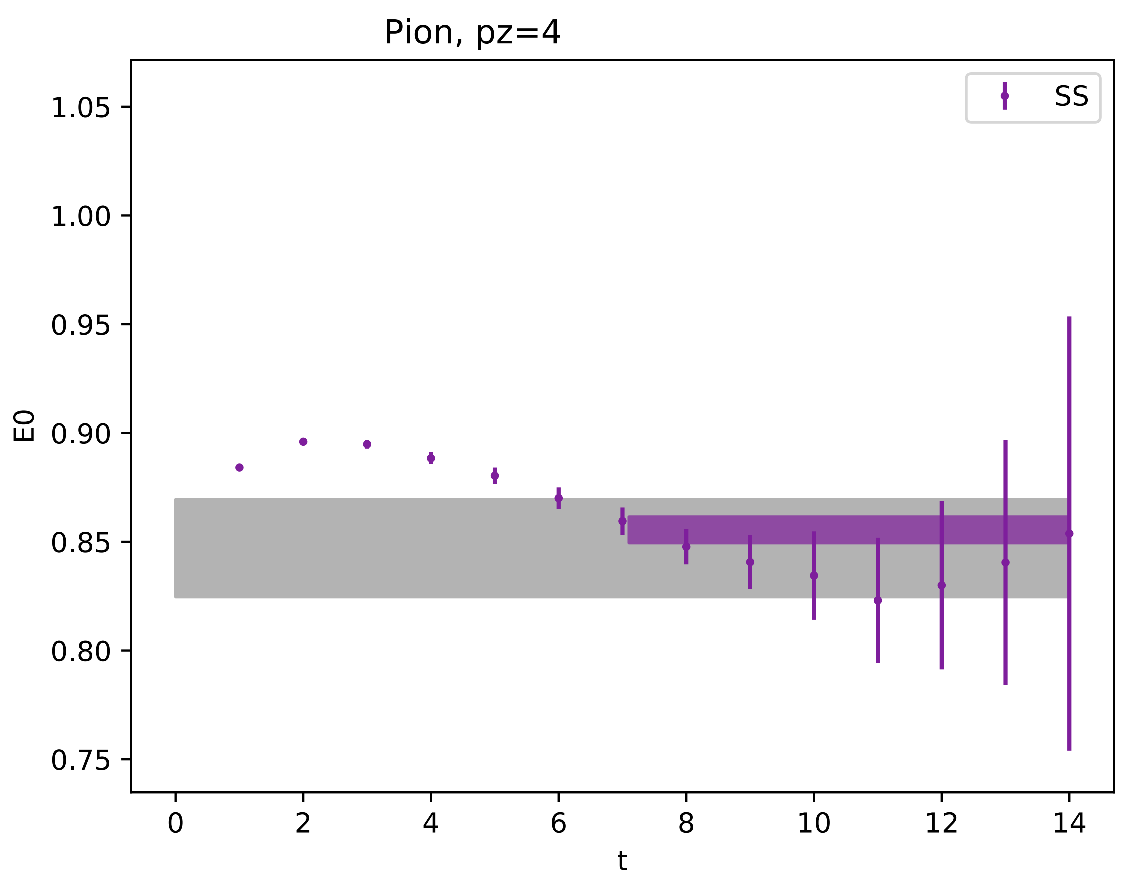
N = 197 cfgs x 2 sources/cfg

Pion correlation functions

Pion energy fit from 2pt function

$$G_\pi(p,t) = \sum_x e^{ip\cdot x} \left\langle \pi(x,t)\pi(0)^\dagger \right\rangle$$
$$= \sum_n Z_n(p) e^{-E_\pi(p)t}$$





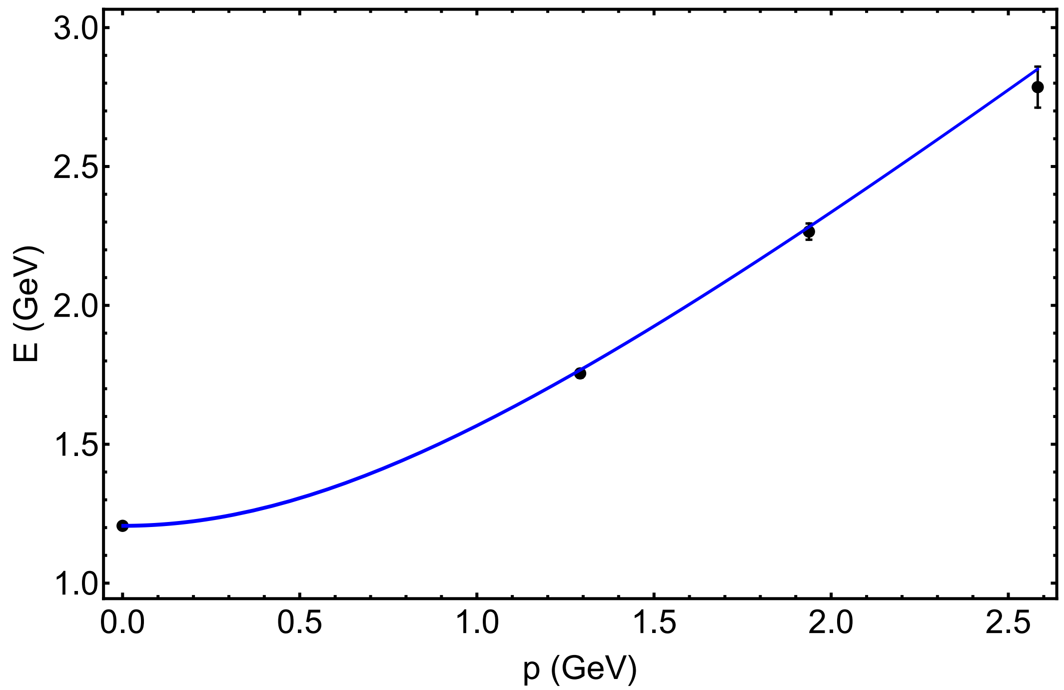
Boosted pion has exponential signal-to-noise problem

$$\mathrm{StN}(t) \propto \sqrt{N} e^{-(E_\pi(p)-m_\pi)t}$$

Momentum smearing increases overlap and reduces noise

Bali, Lang, Musch, Schäfer, PRD 93 (2016)

Boosted pion energies



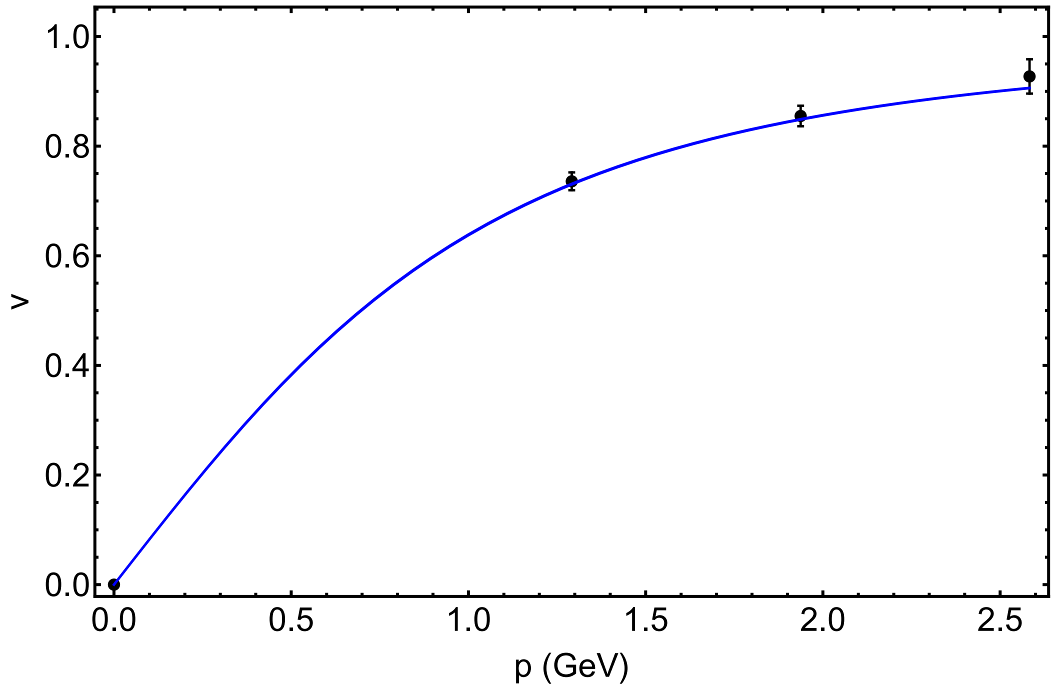
Results described well by
continuum dispersion relation

$$p_z^{max} \sim 2.6 \text{ GeV}$$

$$a^{-1} \sim 3.3 \text{ GeV}$$

Boosted pions highly relativistic

$$v^{max} \sim 0.93$$



Three-point functions

Ground-state matrix elements extracted from ratios of 3pt/2pt functions

$$\frac{G^{3pt}(b^z, b_T, \eta, \mu, p^z, t, \tau)}{G^{2pt}(p^z, t)} = \frac{\sum_{n,m} \mathcal{M}_{nm}\sqrt{Z_n Z_m} e^{-E_n(p^z)\tau} e^{-E_n(p^z)(t-\tau)}}{\sum_n Z_n e^{-E_n(p^z)\tau}}$$

Rearranging, matrix elements accessible from **linear** fit

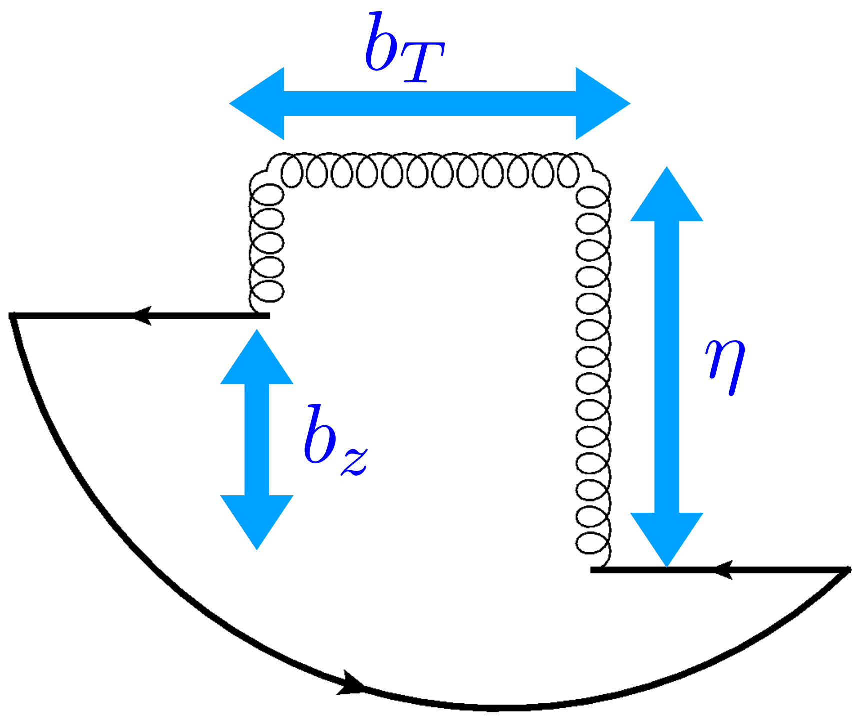
$$\left(1 + \sum_{n>0} \frac{Z_n}{Z_0} e^{-[E_n(p^z)-E_0(p^z)]t}\right) \frac{G^{3pt}(b^z, b_T, \eta, \mu, p^z, t, \tau)}{G^{2pt}(p^z, t)} =$$

$$\mathcal{M}_{00} + \sum_{n,m>0} \widetilde{\mathcal{M}}_{nm} e^{-[E_n(p^z)-E_0(p^z)]\tau} e^{-[E_n(p^z)-E_0(p^z)](t-\tau)}$$

Three source/sink separations:

$$t = 0.48,\ 0.6,\ 0.72\ \text{fm}$$

Simultaneous fit to all $t,\ \tau$



Fitting sums of exponentials

What fit range(s)?

tmax: signal-to-noise > 2 (results mostly insensitive)

tmin: use all choices where operators are separated by width of transfer matrix

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Start with 1.

Try 2. Keep if preferred by information criterion, e.g. AIC

Try 3….

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How to estimate the covariance matrix?

Bootstrap

Optimal shrinkage (interpolates between correlated and uncorrelated fit)

Ledoit, Wolf, Journal of Multivariate Analysis 88 (2004)

Rinaldi, Syritsyn, MW et al, arXiv:1901.07519

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How should acceptable fits be combined?

Weighted average, e.g. weight = p-value/variance

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We’re not using plateau fits!

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Start with 1.

Try 2. Keep if preferred by information criterion, e.g. AIC

Try 3….

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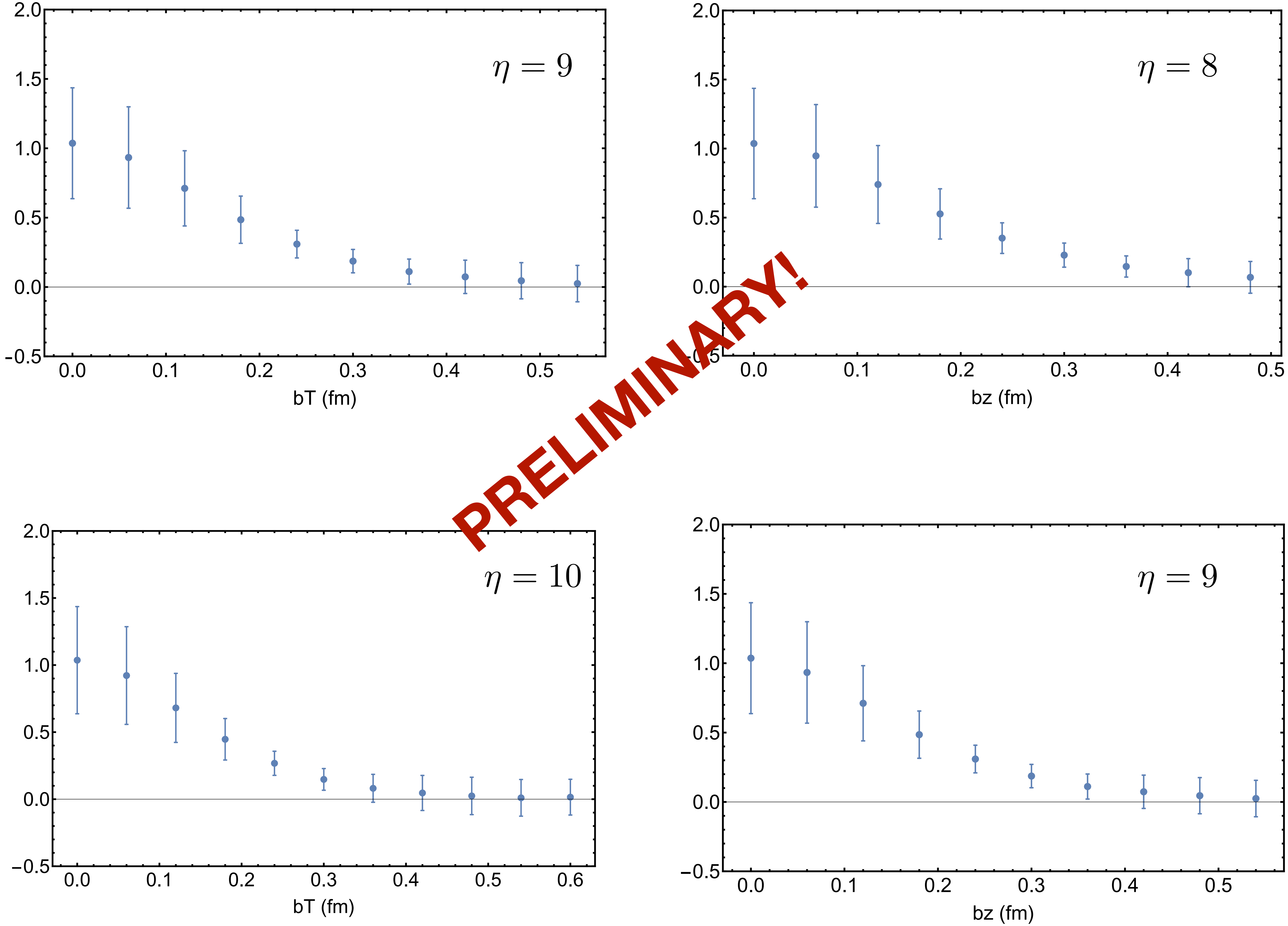
Bare beam functions



Quark field renormalization

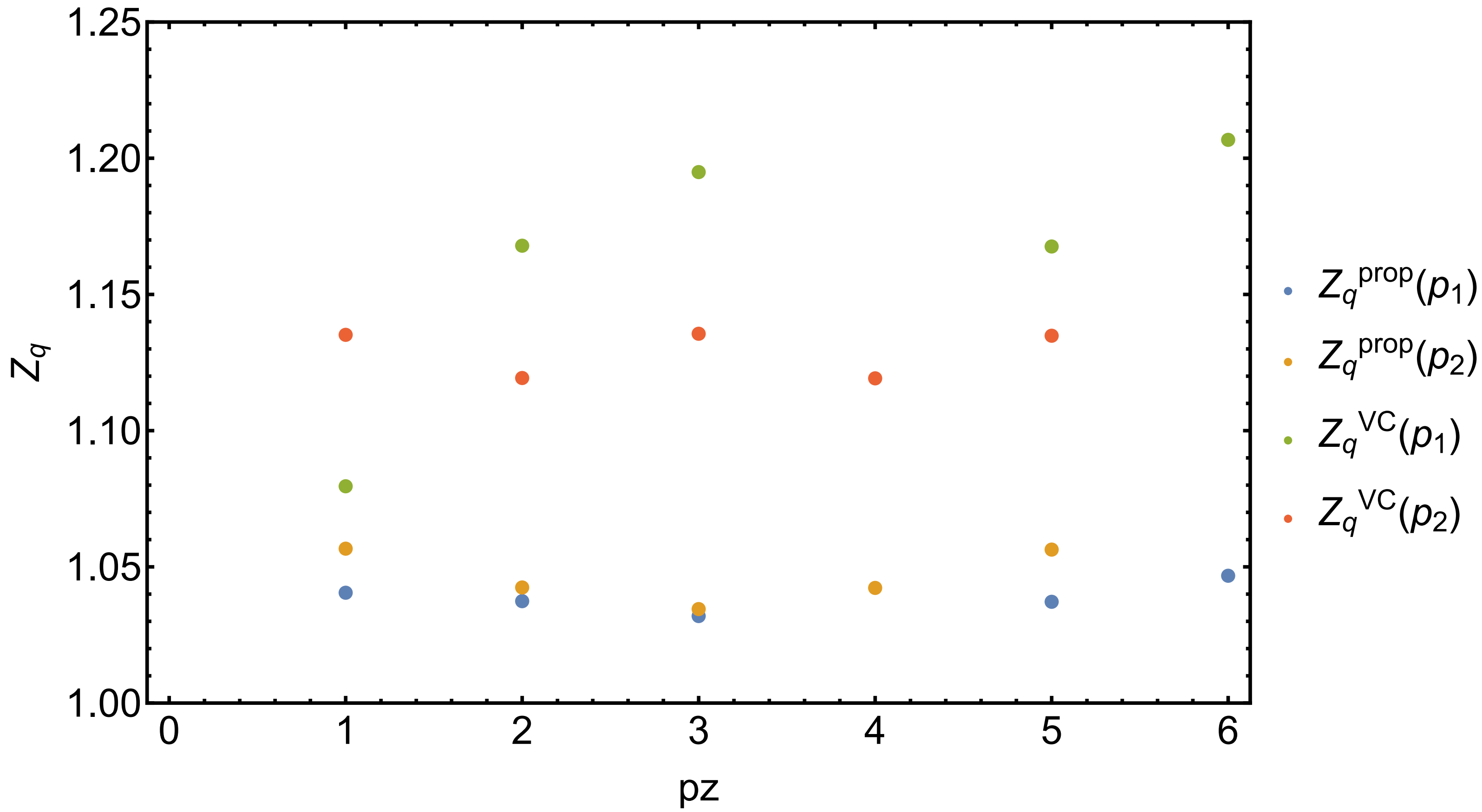
(Landau) gauge-fixed quark propagators accessible to LQCD and perturbation theory

$$S_{\alpha\beta}(p) = \sum_{x,y} e^{ip\cdot(x-y)} \langle q_\alpha(x)\overline{q}_\beta(y)\rangle$$

RI/MOM condition: full propagator = tree-level propagator (at one scale)

$$Z_q S(p)\big|_{p^2=\mu_R^2} = S^{\rm tree}(p)$$

$$\Longrightarrow\ Z_q = \frac{1}{12}{\rm Tr}\left[S^{-1}(p)S^{\rm tree}(p)\right]_{p^2=\mu_R^2}$$



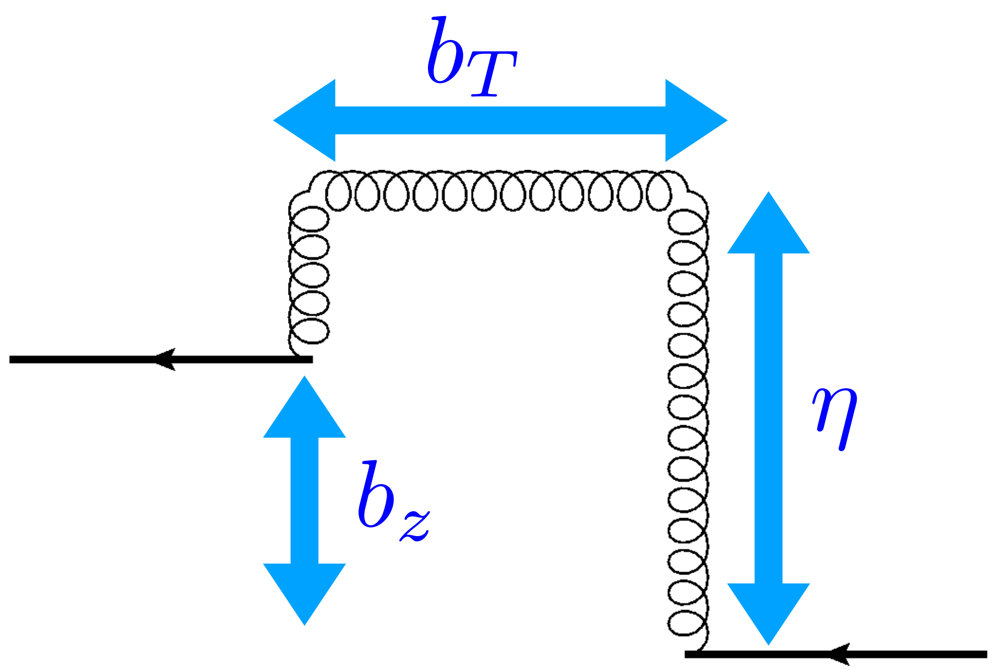
Momentum dependence O(10%) 1-loop effect

Alternative definition from conserved current

Staple renormalization

Correlation function for nonlocal operator in gauge-fixed quark state

$$G_{\alpha\beta}(p) = \sum_{x,y,z} \mathrm{e}^{\mathrm{i}p\cdot(x-y)} \langle q_\alpha(x) \mathcal{O}(z+b,z) \bar{q}_\beta(y) \rangle$$
$$= \sum_{z} \langle \gamma_5 S^\dagger(p, b+z) \gamma_5 \tilde{W}(\eta; b+z, z) \frac{\Gamma}{2} S(p,z) \rangle_{\alpha\beta}$$



Vertex function accessible to LQCD and perturbation theory

$$\Lambda(p) = \left(\gamma_5 \left[S^{-1}(p) \right]^\dagger \gamma_5 \right) G(p) S^{-1}(p)$$

RI/MOM condition

$$Z_q^{-1} Z_{\mathcal{O}} \mathrm{Tr} \left[P_\Gamma \Lambda(p) \right] = \mathrm{Tr} \left[P_\Gamma \Lambda^{\mathrm{tree}}(p) \right] = 6 e^{ip\cdot b}$$

Constantinou, Panagopoulos and Spanoudes, arXiv:1901.03862

One-loop matching

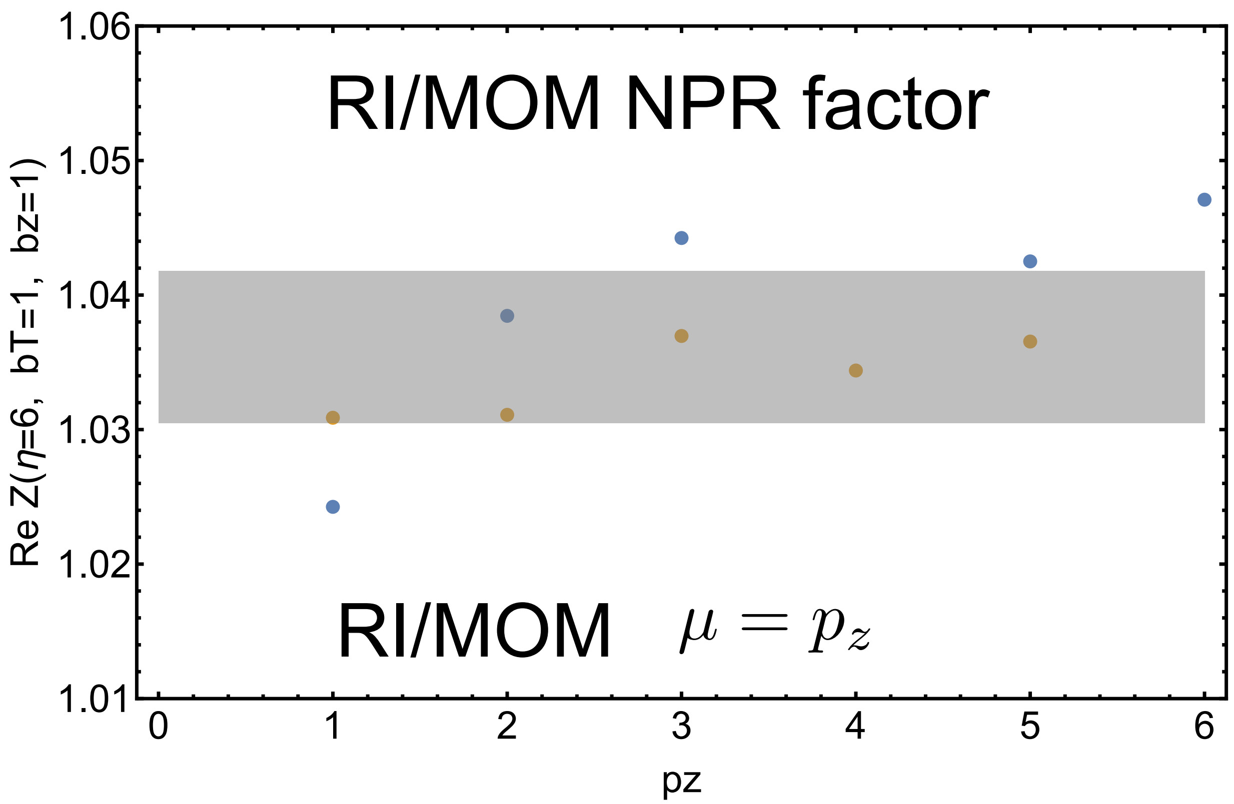
Dependence on p_z arises from perturbative RG and lattice artifacts

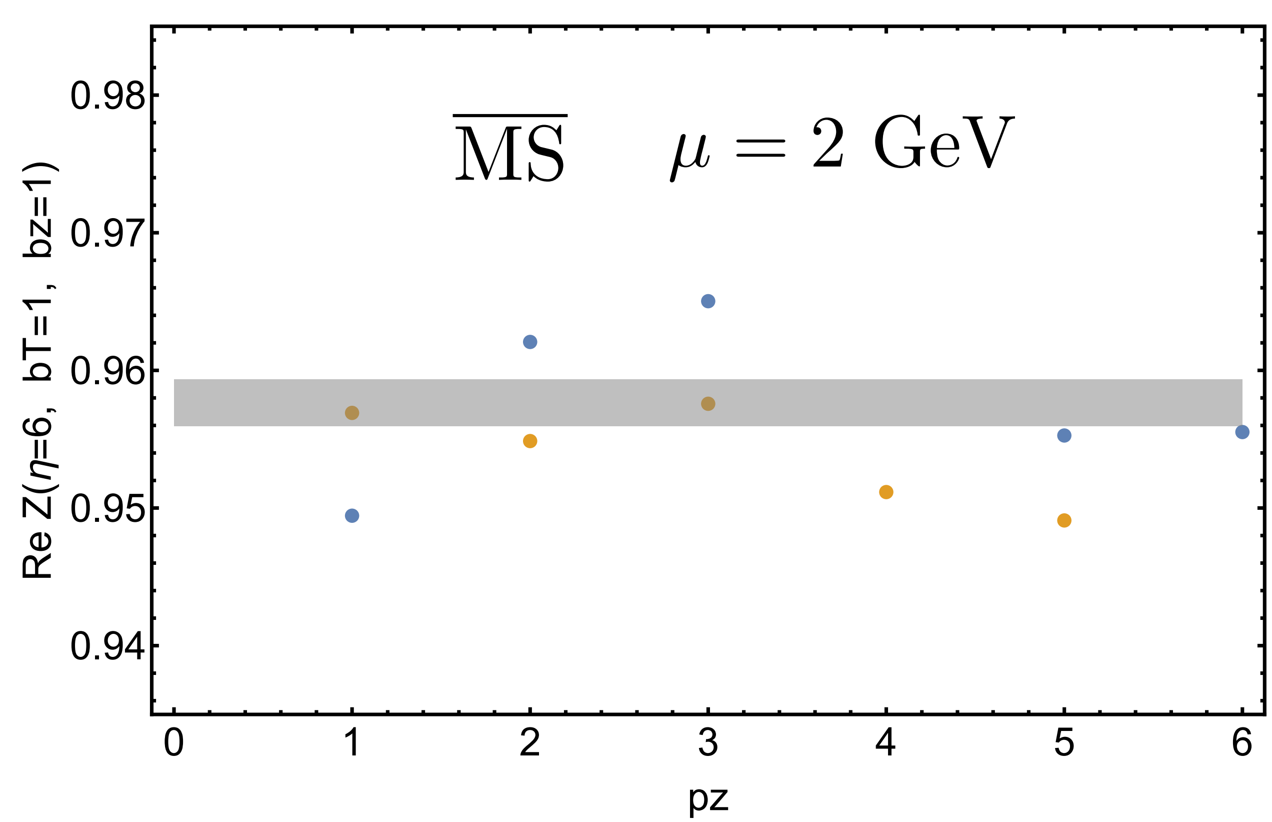
One-loop matching from RI/MOM $\mu = p_z$ to $\overline{\text{MS}}$ $\mu = 2$ GeV reduces p_z dependence

1-loop matching for asymmetric staple: Ebert, Stewart, Zhao *in preparation*

Residual p_z dependence from 2-loop RG and lattice artifacts

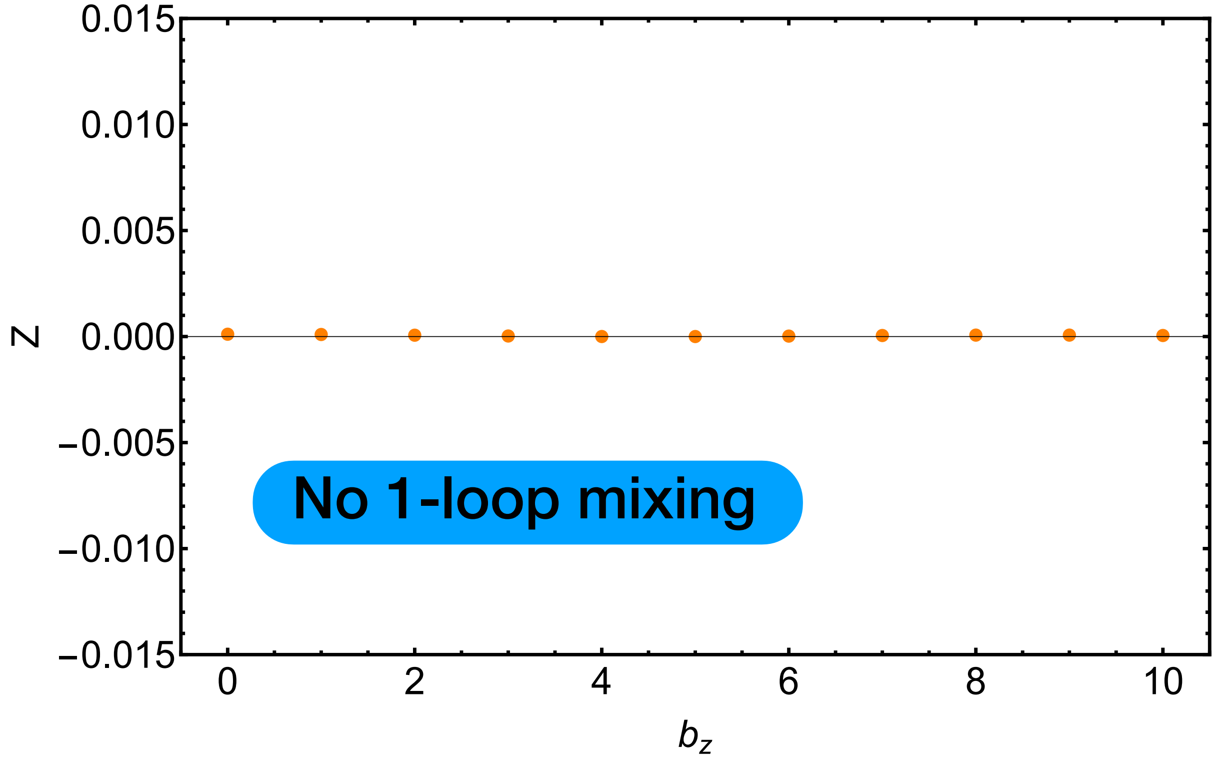
See e.g. Blossier et al (ETM), PRD 91 (2015)



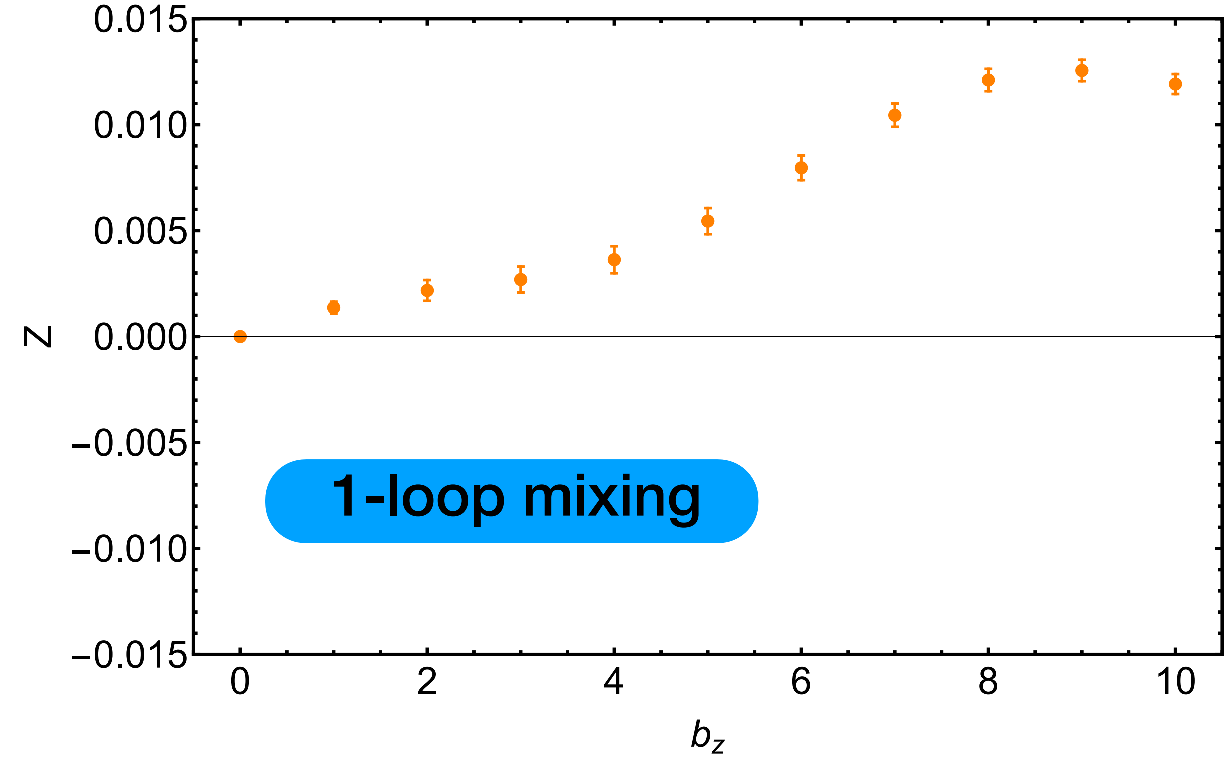


Operator mixing results

Straight line: $\Lambda = \gamma_t$, $P = \sigma_{zt}$

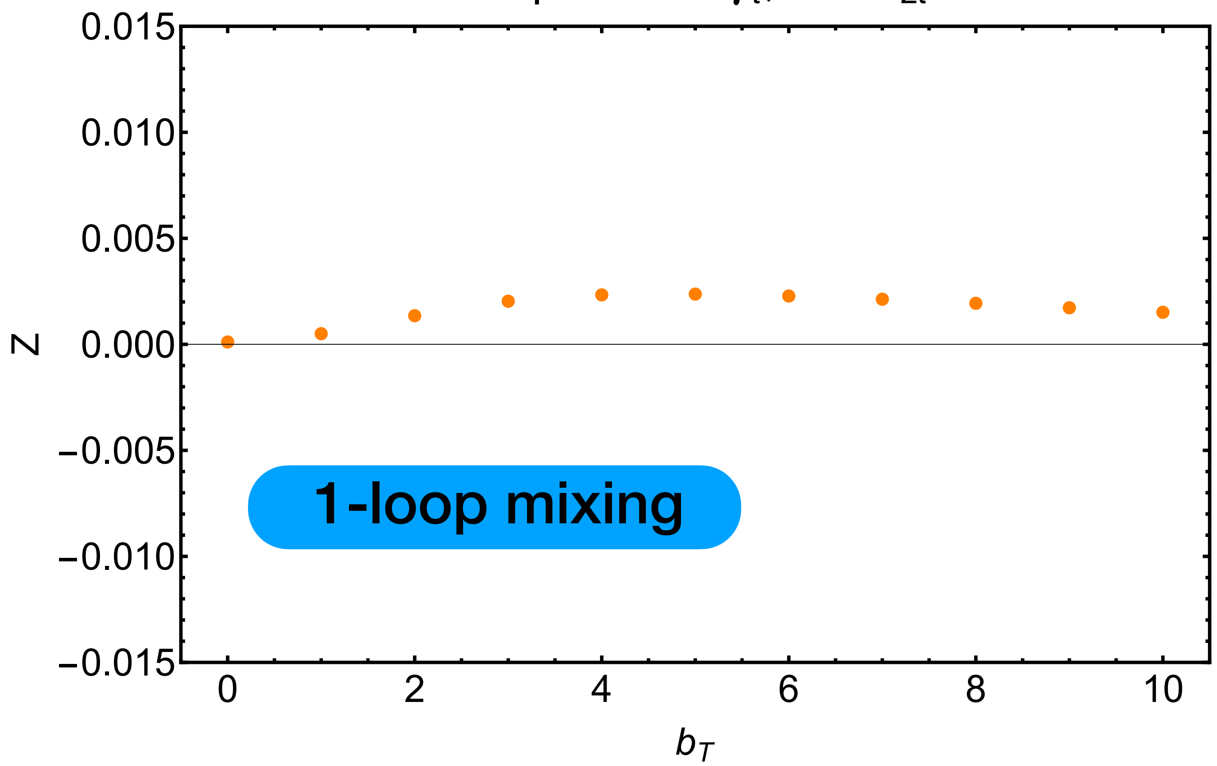


Straight line: $\Lambda = \gamma_z$, $P = 1$



Lattice perturbation theory: Xiong, Ji, Zhang, Zhao, PRD 90 (2014)
Constantinou, Panagopoulos, PRD 96 (2017)
Chen et al, arXiv:1710.01089
Green, Jansen, Steffense, PRL 121 (2018)

Staple: $\Lambda = \gamma_t$, $P = \sigma_{zt}$



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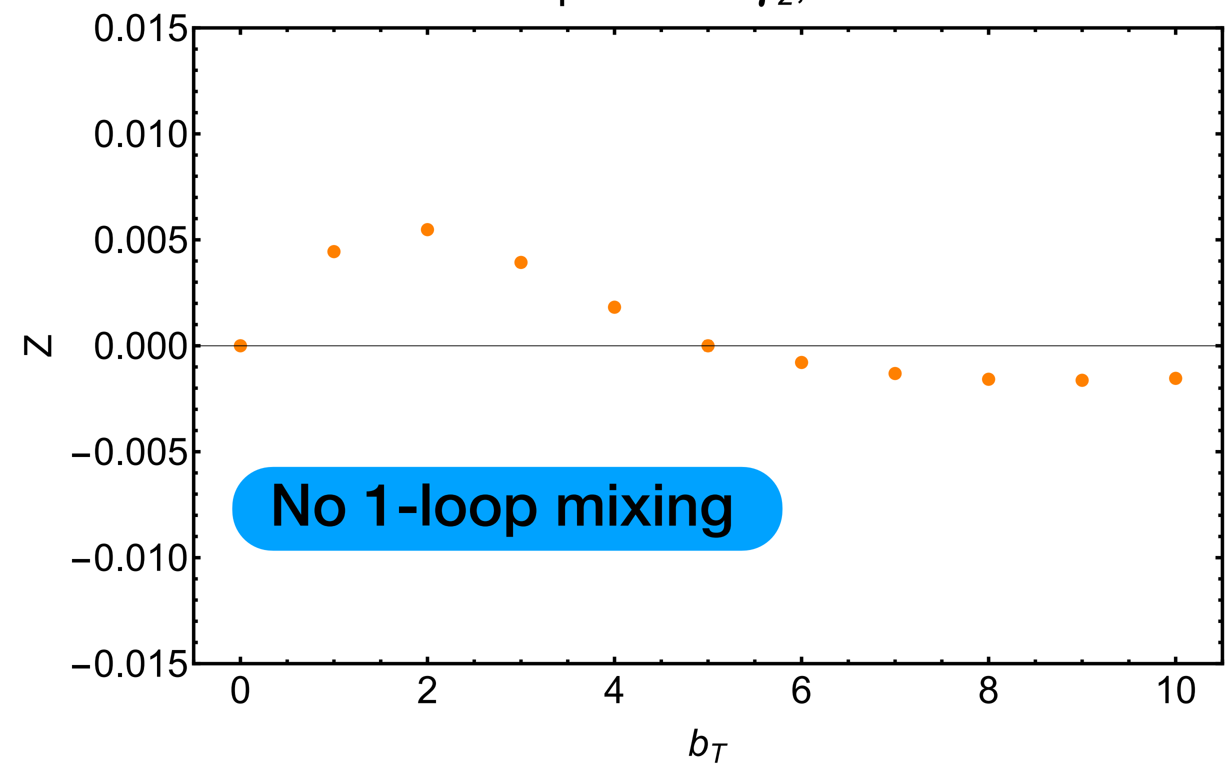
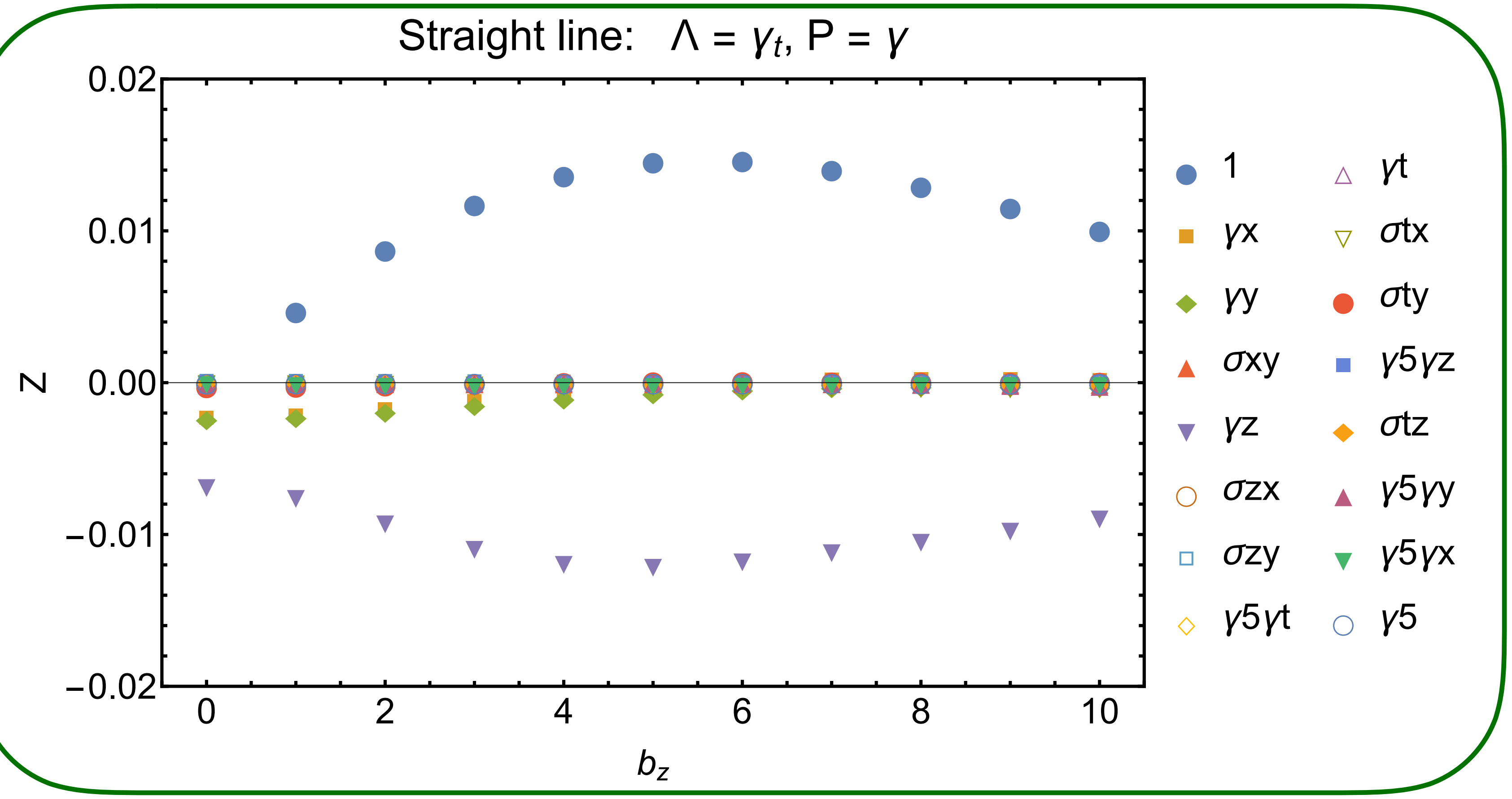
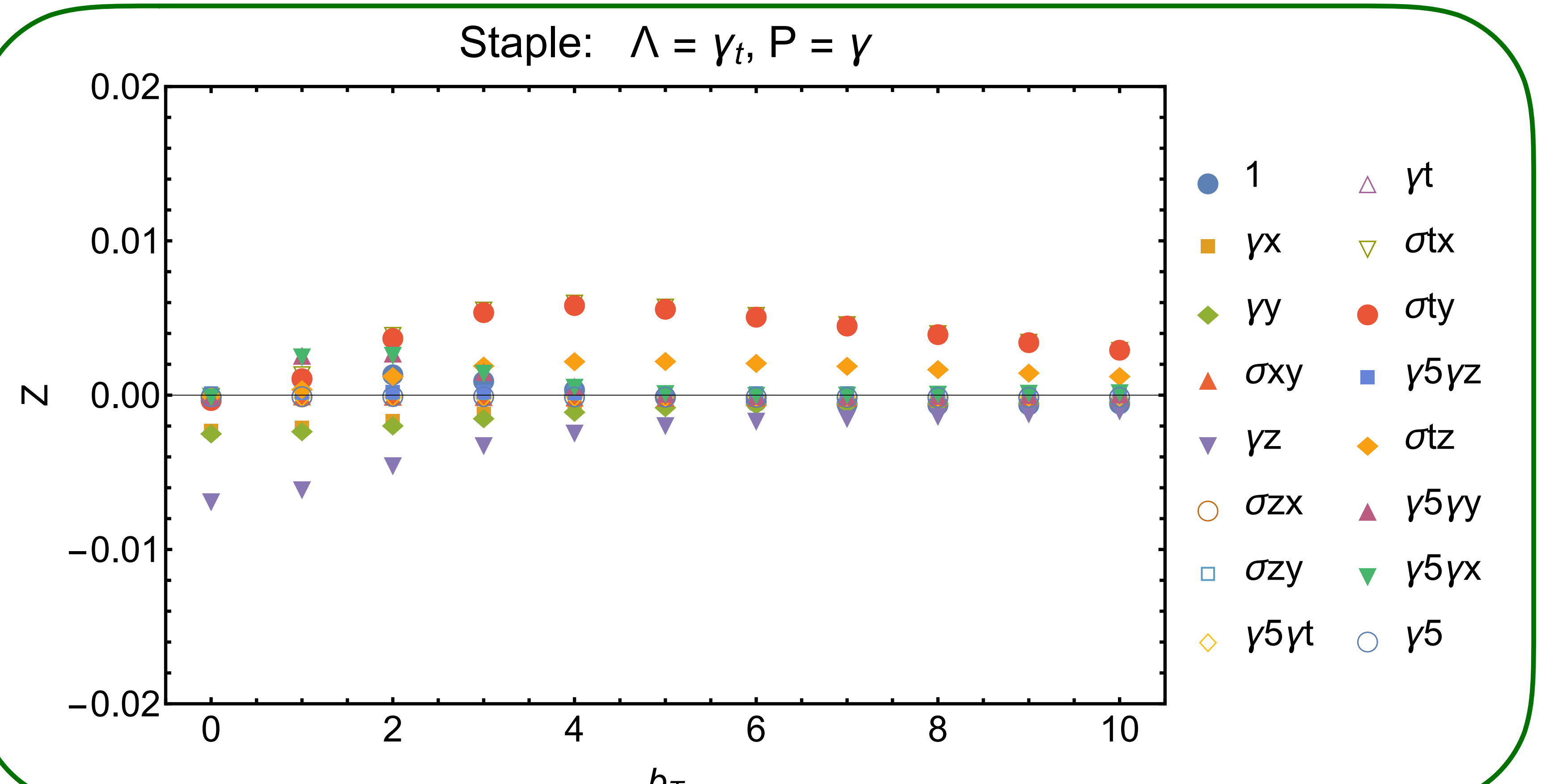


Lattice perturbation theory: Constantinou, Panagopoulos and Spanoudes, arXiv:1901.03862

Operator mixing is generic

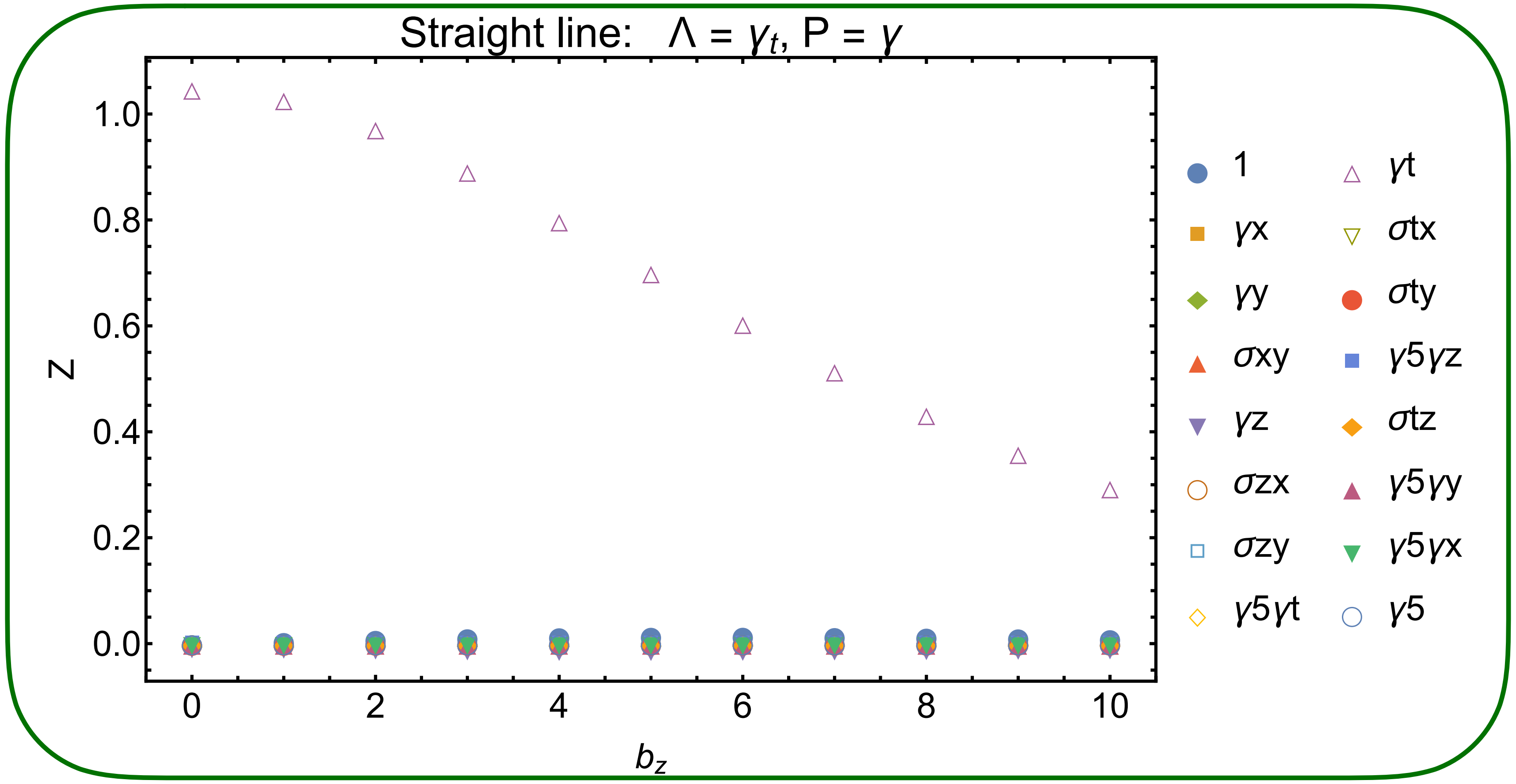


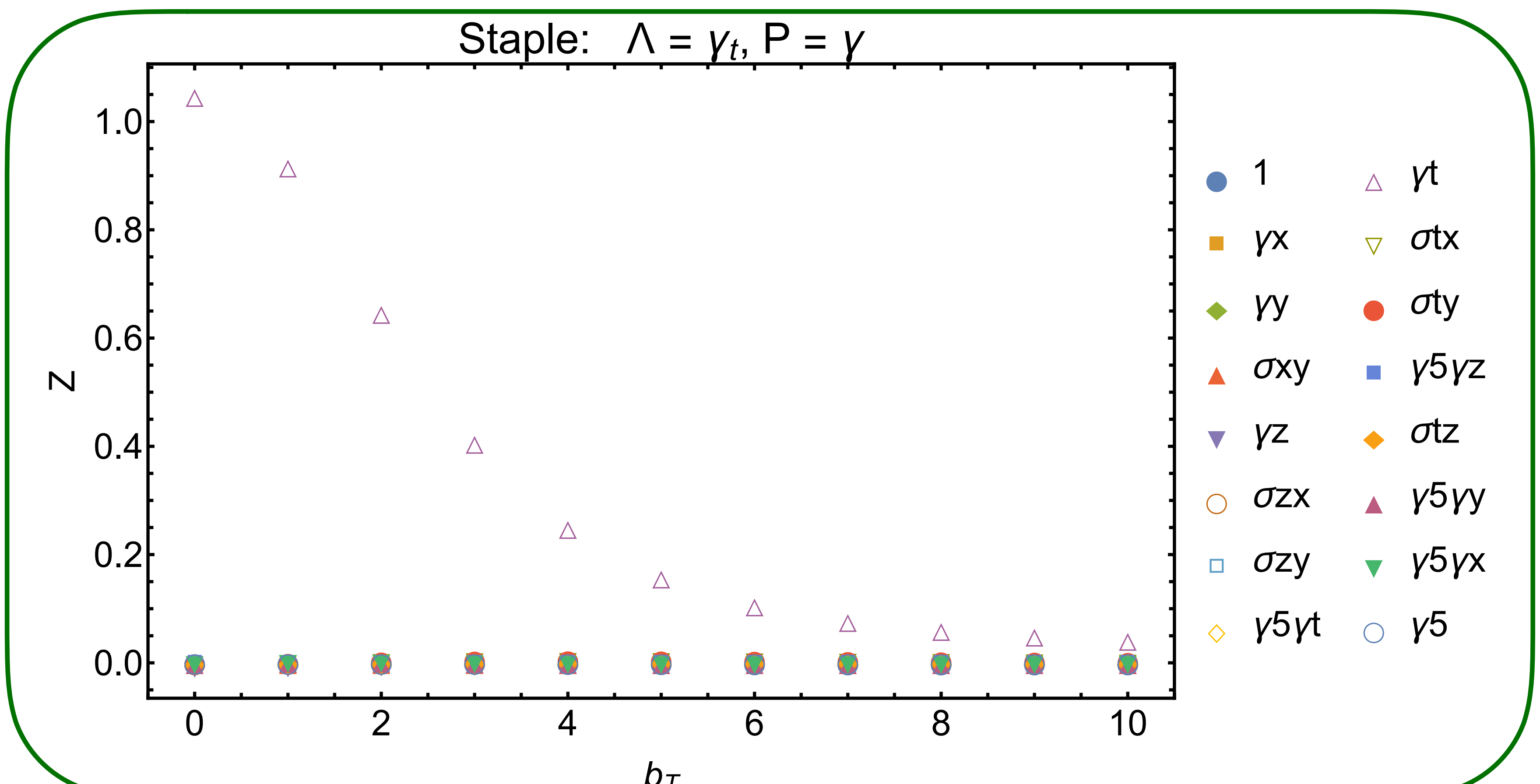
Nonperturbative mixing between different currents generic for nonlocal operators



Hierarchies between mixings visible but differ from 1-loop mixing pattern

Operator mixing is small



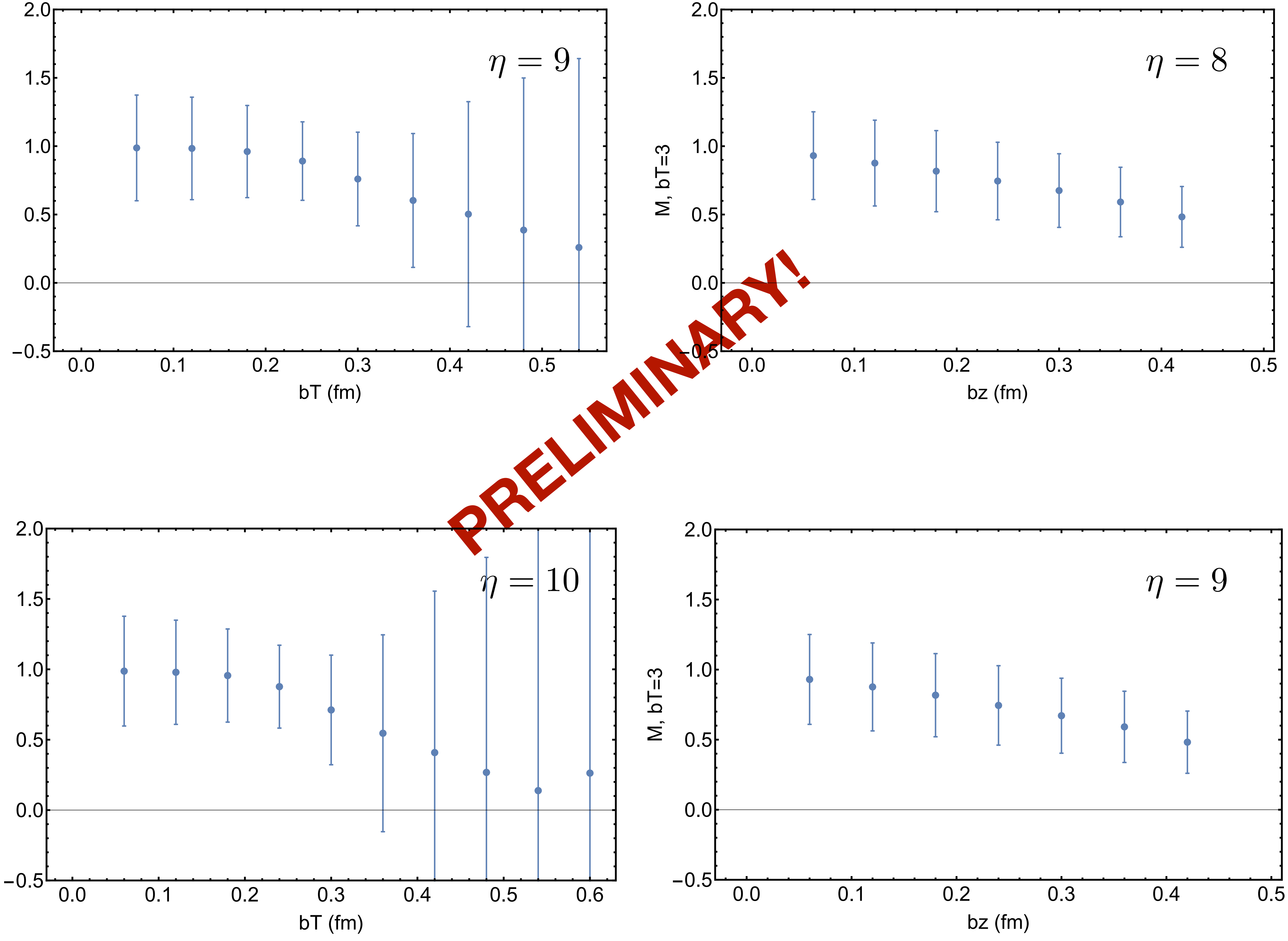


Small nonperturbative mixing between different currents generic for nonlocal operators

O(1%) effects negligible for exploratory calculations

Present in precision PDF calculations

Beam functions



Towards TMD evolution from LQCD

Next: Fourier transform beam function and form ratios

$$\gamma_\zeta^{q,\overline{\mathrm{MS}}}(b_T,\mu) = \frac{1}{\ln(p_1^z/p_2^z)} \ln \frac{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xP_2^z) \int db^z e^{ib^z x p_1^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p_1^z)}{C_{\mathrm{TMD}}^{\overline{\mathrm{MS}}}(\mu, xp_1^z) \int db^z e^{ib^z x p_2^z} \widetilde{B}_q^{\overline{\mathrm{MS}}}(b^z, b_T, \eta, \mu, p_2^z)}$$

Detailed study of Fourier transform systematics important

Next-to-next: dynamical sea quarks, continuum extrapolation, finite volume, …

Stay tuned