# Renormalization of gluon quasi-PDFs in LaMET

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# Introduction

- Parton quantities such as the PDFs are difficult to access on the lattice
  - Defined on the light-cone
  - Example [Collins and Soper, NPB 82'] ($\xi^{\pm} = (t \pm z)/\sqrt{2}$)

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

$$t^2 - x^2 = 0 \rightarrow -\tau^2 - x^2 = 0$$

- However, they were originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

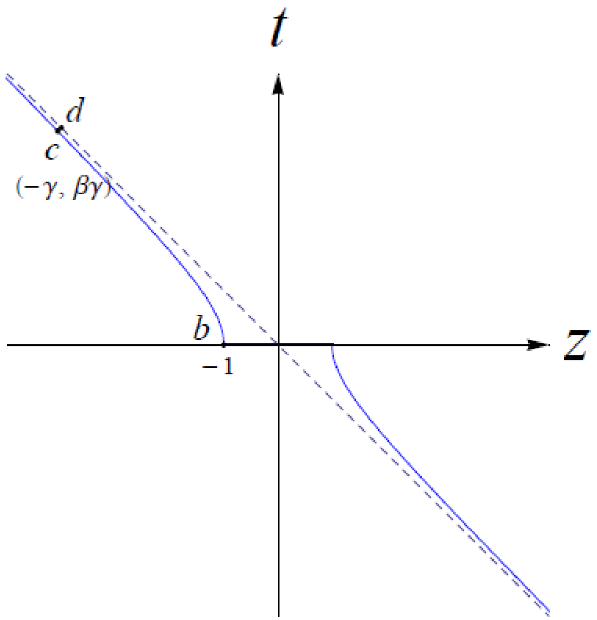
$$q(x) = \lim_{P_z\rightarrow\infty} \tilde{q}(x, P_z)$$

- Boost to infinite momentum leads to light-cone correlations

- If we can construct a $\tilde{q}(x, P_z)$ such that it is calculable on the lattice, and all $P_z$-dependence can be systematically removed

- Then we can calculate $q(x)$!

# Introduction



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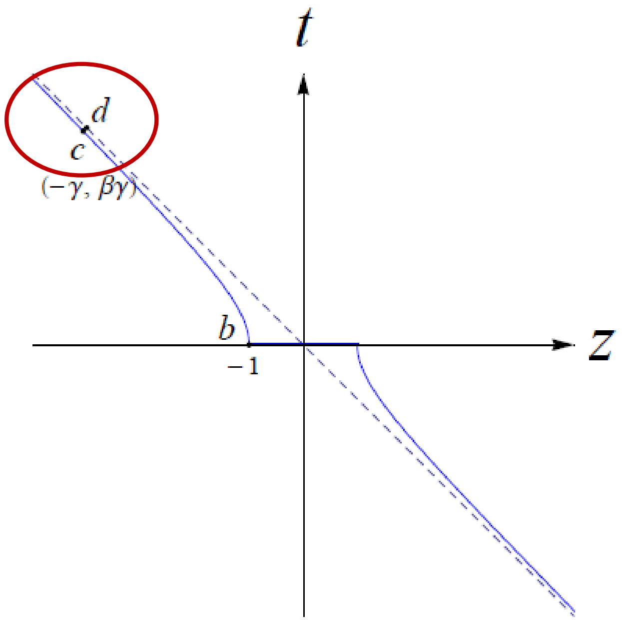

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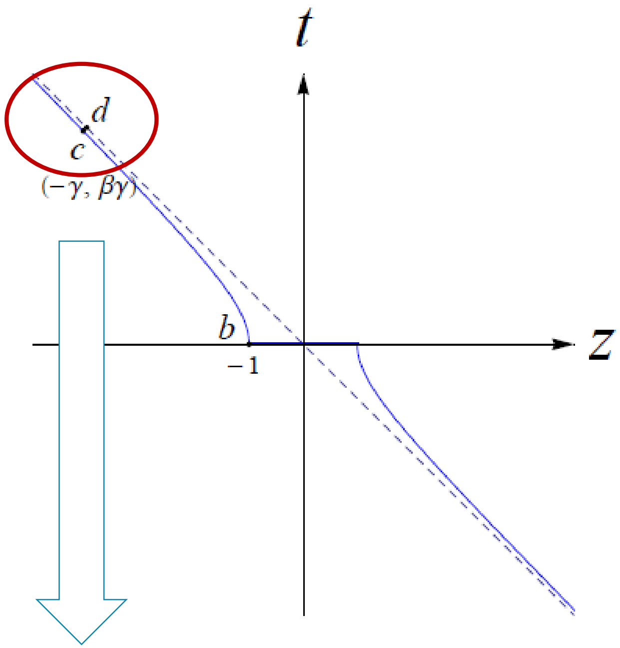


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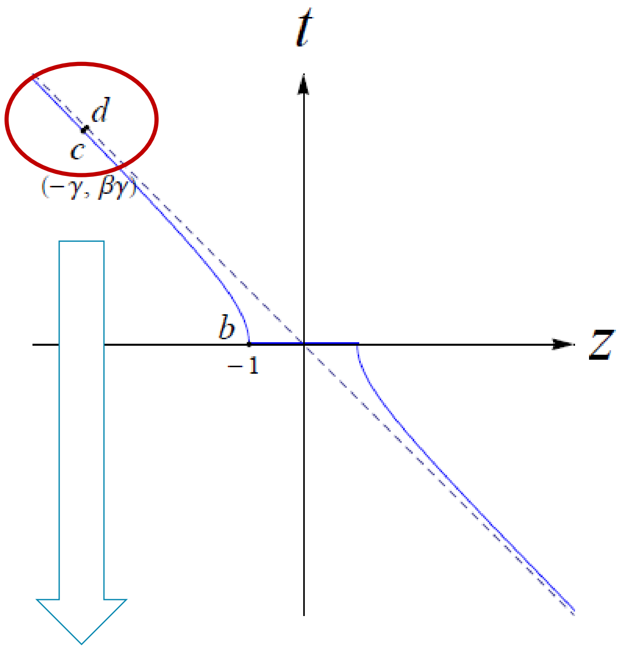


- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
  - Appropriately chosen $\tilde{q}(x, P_z)$ can be calculated on the Euclidean lattice, e.g.

$$\tilde{q}(x, \Lambda, P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0, 0_\perp, z)\gamma^z \exp\left(-ig \int_0^z dz' A^z(0, 0_\perp, z')\right) \psi(0)|P\rangle$$

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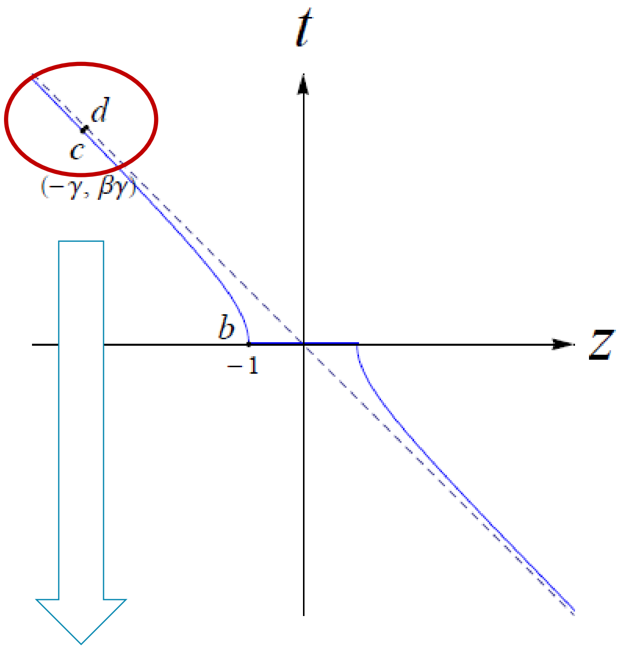


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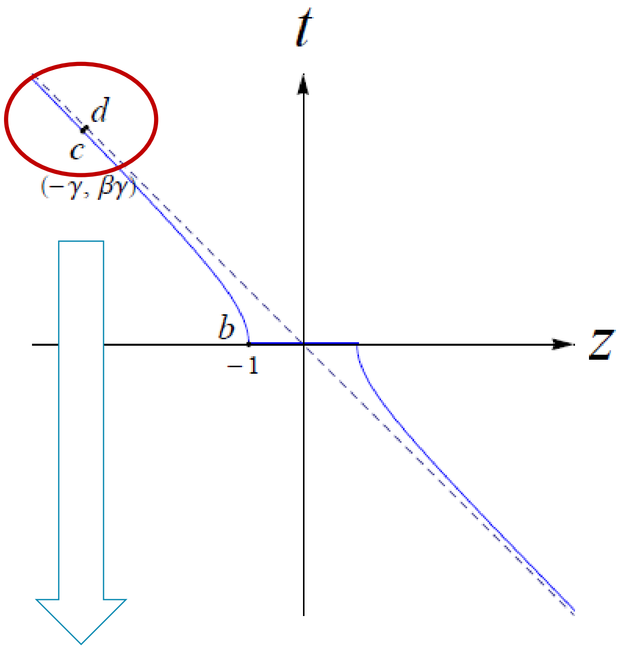
  - A finite but large $P_z$ already offers a good approximation, where (leading) frame-dependence can be removed through a factorization formula

$$\tilde{q}(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left( \frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R} \right) q(y, \mu) + \mathcal{O}\left( \frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2} \right)$$

# Introduction




- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']

- Parton model is an effective theory for the nucleon moving at large momentum

# Other proposals

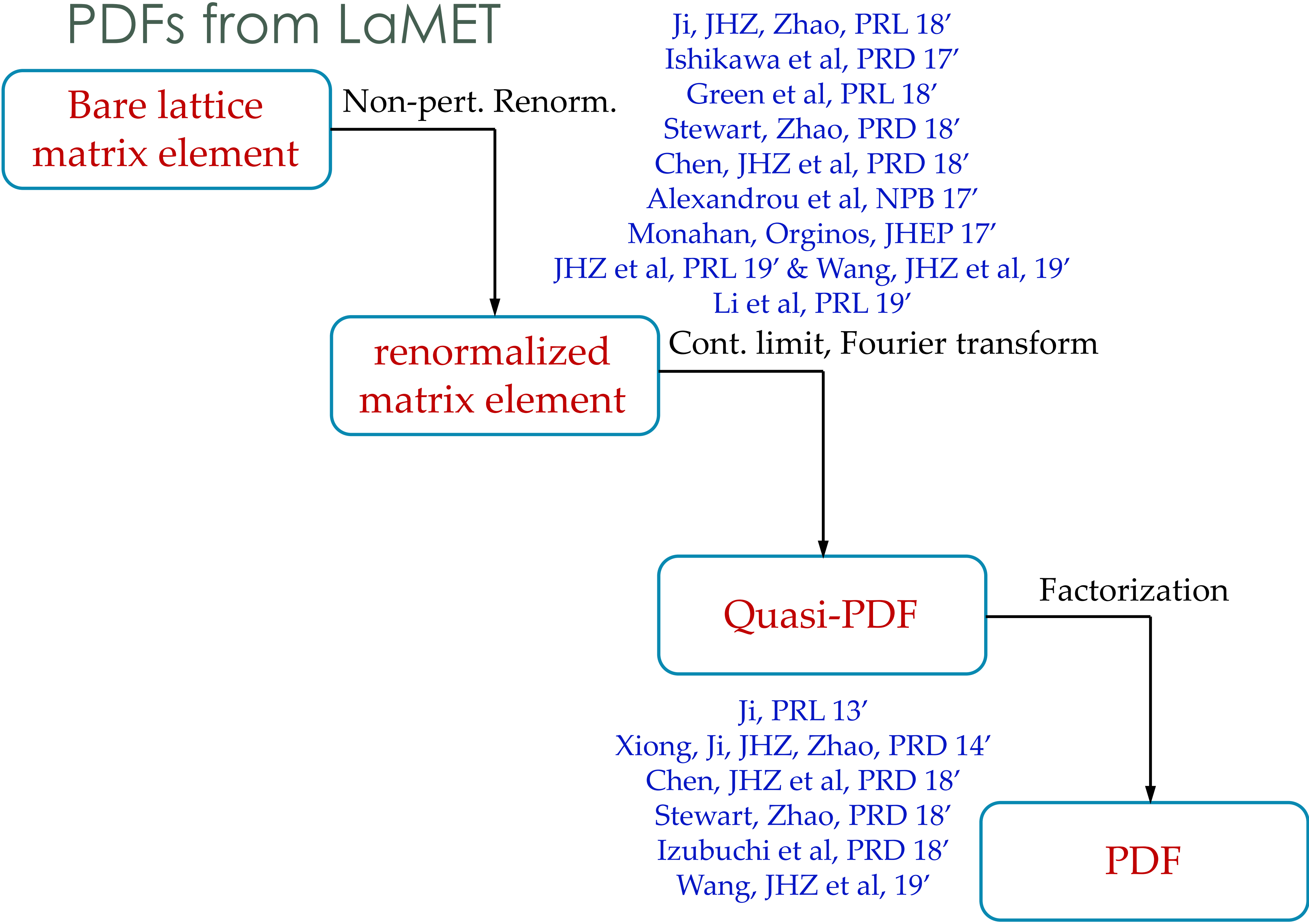
- Current-current correlation functions
- [Liu and Dong, PRL 94']
- [Detmold and Lin, PRD 06']
- [Braun and Müller, EPJC 08']
- [Davoudi and Savage, PRD 12']
- [Chambers et al., PRL 17']

- Lattice cross sections
- [Ma and Qiu, 14' & PRL 17']

- Ioffe-time /pseudo-distribution
- [Radyushkin, PRD 17']

- **They share similar spirit of computing correlations at spacelike separations**

# PDFs from LaMET



# PDFs from LaMET

Bare lattice matrix element

Non-pert. Renorm.

renormalized matrix element

Ji, JHZ, Zhao, PRL 18'
Ishikawa et al, PRD 17'
Green et al, PRL 18'
Stewart, Zhao, PRD 18'
Chen, JHZ et al, PRD 18'
Alexandrou et al, NPB 17'
Monahan, Orginos, JHEP 17'
JHZ et al, PRL 19' & Wang, JHZ et al, 19'
Li et al, PRL 19'

$$\widetilde{h}(z, P_z, a^{-1}) = \frac{1}{2P^0}\langle P|O_{\gamma^t}(z)|P\rangle$$

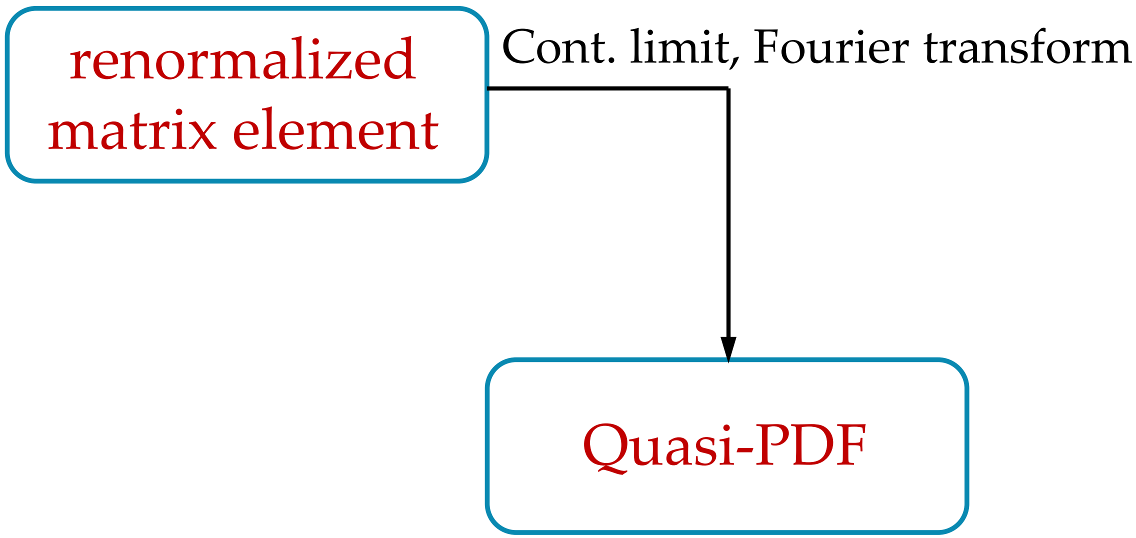
$$O_\Gamma(z) = \bar{\psi}(z)\Gamma U(z,0)\psi(0)$$

$$U(z,0) = P\exp\left(-ig\int_0^z dz' A_z(z')\right)$$

$$\widetilde{h}_R(z, P_z, p_z^R, \mu_R) = Z^{-1}(z, p_z^R, a^{-1}, \mu_R)\widetilde{h}(z, P_z, a^{-1})\Big|_{a\to 0}$$

$$Z(z, p_z^R, a^{-1}, \mu_R) = \frac{\sum_s \langle p, s|O_{\gamma^t}(z)|p, s\rangle}{\sum_s \langle p, s|O_{\gamma^t}(z)|p, s\rangle_{\text{tree}}}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

# PDFs from LaMET




$$\tilde{q}_R(x, P_z, p_z^R, \mu_R) = P_z \int \frac{dz}{2\pi}\, e^{ixP_z z}\, \tilde{h}_R(z, P_z, p_z^R, \mu_R)$$

# PDFs from LaMET

$$\tilde{q}_R(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}\right) q(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

Quasi-PDF

Factorization

PDF

Ji, PRL 13′
Xiong, Ji, JHZ, Zhao, PRD 14′
Chen, JHZ et al, PRD 18′
Stewart, Zhao, PRD 18′
Izubuchi et al, PRD 18′
Wang, JHZ et al, 19′

# Gluon quasi-PDFs

- Gluon PDF (unpol.) [Collins, Soper, NPB 82′]

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ixP^+\xi^-} \langle P|F_a^{+i}(\xi^-)\mathcal{W}(\xi^-,0)F_a^{+i}(0)|P\rangle$$

- Naively expected gluon quasi-PDF operators

$$O_g^{\mu\nu}(z,0) = F^{\mu\alpha}(z)\mathcal{W}(z,0)F_\alpha^{\ \nu}(0)$$

  - $\{\mu,\nu\} = \{z,t\}$
  - They mix in general with other operators under renormalization

- Auxiliary field approach [Dorn, Fortsch. Phys. 86′, Ji, JHZ, Zhao, PRL 18′]

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{\mathcal{Q}}(x) i n\cdot D \mathcal{Q}(x)$$

  - For a space like $n$, no dynamical evolution for $\mathcal{Q}$
  - The two-point function of $\mathcal{Q}$ is

$$\int \mathcal{D}\overline{Q}\mathcal{D}Q\, Q(x)\overline{Q}(y) e^{i\int d^4x \mathcal{L}} = S_Q(x,y) e^{i\int d^4x \mathcal{L}_{\rm QCD}}$$

with

$$n\cdot D\, S_Q(x,y) = \delta^{(4)}(x-y)$$

# Gluon quasi-PDFs

- Solution

$$S_Q(x,y) = \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x,y)$$
$$= \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x^z,y^z)$$

  - $\delta$-function ensures that the time and transverse components are equal, and therefore generates a spacelike Wilson line

  - The non-local gluon quasi-PDF operator can be replaced by a product of two local composite operators

$$\mathcal{O}_g^{(3)}(z_2,z_1) = J_1^{ti}(z_2)\overline{J}_{1,i}{}^{z}(z_1)$$

$$J_1^{ti}(z_2) = F_a^{ti}(z_2)\mathcal{Q}_a(z_2), \quad \overline{J}_{1,i}{}^{z}(z_1) = \overline{\mathcal{Q}}_b(z_1)F_{b,i}{}^{z}(z_1)$$

  - After integrating out $\mathcal{Q}$, the gluon quasi-PDF is recovered

# Gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76', Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - For $J_1^{\mu\nu}$, the operators allowed to mix are

$$J_2^{\mu\nu} = n_\rho (F_a^{\mu\rho} n^\nu - F_a^{\nu\rho} n^\mu) \mathcal{Q}_a / n^2,$$
$$J_3^{\mu\nu} = (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in \cdot D - m)\mathcal{Q})_a / n^2,$$

  - The mass term might be absent in DR, but can be generated by radiative corrections in a cutoff regularization such as lattice regularization

  - General mixing pattern

$$\begin{pmatrix} J_{1,R}^{\mu\nu} \\ J_{2,R}^{\mu\nu} \\ J_{3,R}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{\mu\nu} \\ J_2^{\mu\nu} \\ J_3^{\mu\nu} \end{pmatrix},$$

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  - The mass term might be absent in DR, but can be generated by radiative corrections in a cutoff regularization such as lattice regularization

  - Renormalization constants are not all independent

$$\begin{pmatrix} J_{1,R}^{z\mu} \\ J_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{z\mu} \\ J_3^{z\mu} \end{pmatrix}, \qquad J_{1,R}^{ti} = Z_{11} J_1^{ti}, \quad J_{1,R}^{ij} = Z_{11} J_1^{ij}$$

  - Different components have different renormalization due to Lorentz symmetry breaking

# Gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76', Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - We can identify building blocks that can be used to construct multiplicatively renormalizable gluon quasi-PDFs, e.g.

$$\mathcal{O}_R^1(z_2, z_1) \equiv J^{ti}_{1,R}(z_2)\overline{J}^{ti}_{1,R}(z_1)$$

  - After integrating out $\mathcal{Q}$,

$$O_R^1(z_2, z_1) = (F^{ti}(z_2)L(z_2, z_1)F^{ti}(z_1))_R = Z_{11}^2 e^{\overline{\delta m}|z_2 - z_1|} F^{ti}(z_2)L(z_2, z_1)F^{ti}(z_1)$$

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    - The only linear divergence comes from the Wilson line self energy

  - Four such operators have been identified [JHZ et al, PRL 19']

$$O_g^{(1)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ t}(0), \quad O_g^{(2)}(z,0) \equiv F^{zi}(z)\mathcal{W}(z,0)F_i^{\ z}(0),$$
$$O_g^{(3)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ z}(0), \quad O_g^{(4)}(z,0) \equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu^{\ z}(0),$$

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- Local operator mixing [Joglekar, Lee, Annals Phys. 76′, Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - Early studies on gluon quasi-PDFs [Wang et al, EPJC 18′, JHEP 18′] do not seem to be consistent with the renormalization behavior
    - To extract genuine power divergences, we need a UV regulator compatible with gauge symmetry
    - DR, keep track of linear divergences by expanding around $d = 3$

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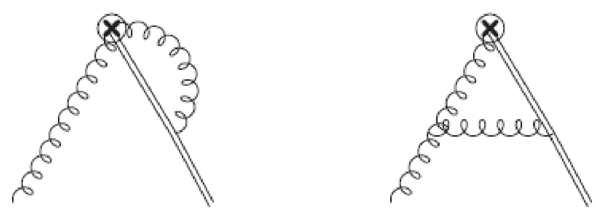

$$I_1^{\rho\nu} = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{d-4}(A_a^\nu n^\rho - A_a^\rho n^\nu) n\cdot\partial\mathcal{Q}_a/n^2 + \frac{\pi\mu}{d-3}(n^\rho A_a^\nu - n^\nu A_a^\rho)\,\mathcal{Q}_a + reg.\right\},$$

$$I_2^{\rho\nu} = \frac{\alpha_s C_A}{\pi}\left\{\frac{1}{d-4}\left[\frac{1}{4}F_a^{\rho\nu}\mathcal{Q}_a + \frac{1}{2}(F_a^{\rho\sigma}n_\nu n_\sigma - F_a^{\nu\sigma}n_\rho n_\sigma)\,\mathcal{Q}_a/n^2 + \frac{1}{2}(A_a^\rho n^\nu - A_a^\nu n^\rho) n\cdot\partial\mathcal{Q}_a/n^2\right]\right.$$
$$\left. - \frac{\pi\mu}{d-3}(n^\rho A_a^\nu - n^\nu A_a^\rho)\,\mathcal{Q}_a + reg.\right\}, \qquad (2.29)$$

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  - Early studies on gluon quasi-PDFs [Wang et al, EPJC 18′, JHEP 18′] do not seem to be consistent with the renormalization behavior
    - To extract genuine power divergences, we need a UV regulator compatible with gauge symmetry
    - DR, keep track of linear divergences by expanding around $d = 3$

  - The same conclusion is reached in Feynman diagram calculations [Wang et al, EPJC 18′, JHEP 18′] if the above regulator is used

# Gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76', Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - In principle, any linear combination of the operators

$$O_g^{(1)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ t}(0), \quad O_g^{(2)}(z,0) \equiv F^{zi}(z)\mathcal{W}(z,0)F_i^{\ z}(0),$$
$$O_g^{(3)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ z}(0), \quad O_g^{(4)}(z,0) \equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu^{\ z}(0),$$

can be used to study gluon quasi-PDFs, but usually they are not multiplicatively renormalizable

  - For example [Fan et al, PRL 18']

$$O_{g,R}^{(5)}(z_2,z_1) \equiv (F^{t\mu}(z_2)\mathcal{W}(z_2,z_1)F^t{}_\mu(z_1))_R = -O_{g,R}^{(1)}(z_2,z_1) - O_{g,R}^{(2)}(z_2,z_1) - O_{g,R}^{(4)}(z_2,z_1)$$

is not multiplicatively renormalizable

# RI/MOM scheme

- Nonlocal quasi-PDF operators at different *z* do not mix under renormalization. Two ways to perform renormalization:
  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each *z* (RI/MOM)

- Inserting gluon (quark) quasi-PDF operators into a quark (gluon) state yields finite mixing

- Taking it into account in RI/MOM renormalization helps improve convergence in the implementation of the matching

$$\begin{pmatrix} O_g^{(n)}(z,0) \\ O_q^s(z,0) \end{pmatrix} = \begin{pmatrix} Z_{11}(z) & Z_{12}(z)/z \\ zZ_{21}(z) & Z_{22}(z) \end{pmatrix} \begin{pmatrix} O_{g,R}^{(n)}(z,0) \\ O_{q,R}^s(z,0) \end{pmatrix},$$

with

$$O_q^s(z_1,z_2) = 1/2[\bar{q}_i(z_1)\Gamma W(z_1,z_2)q_i(z_2) - (z_1 \leftrightarrow z_2)]$$

# RI/MOM scheme

- Nonlocal quasi-PDF operators at different $z$ do not mix under renormalization. Two ways to perform renormalization:
  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each $z$ (RI/MOM)

- RI/MOM renormalization condition [Wang, JHZ et al, 19']

$$\frac{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_R}{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_{\mathrm{tree}}}\Bigg|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=1, \qquad \frac{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_R}{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_{\mathrm{tree}}}\Bigg|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=1,$$

$$\mathrm{Tr}[\Lambda_{12}(p,z)\mathcal{P}]_R\Big|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=0, \qquad [P_{ij}^{ab}\Lambda_{21}^{ab,ij}(p,z)]_R\Big|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=0,$$

$$h_{g,R}^{(n)}(z,P^z,\mu_R,p_z^R) = \bar{Z}_{11}(z,\mu_R,p_z^R,1/a)h_g^{(n)}(z,P^z,1/a)+\bar{Z}_{12}(z,\mu_R,p_z^R,1/a)/z\ h_q^s(z,P^z,1/a),$$
$$h_{q,R}^{s}(z,P^z,\mu_R,p_z^R) = \bar{Z}_{22}(z,\mu_R,p_z^R,1/a)h_q^{s}(z,P^z,1/a)+z\bar{Z}_{21}(z,\mu_R,p_z^R,1/a)\ h_g^{(n)}(z,P^z,1/a).$$

with

$$\bar{\mathcal{Z}}=\begin{pmatrix}\bar{Z}_{11}(z) & \bar{Z}_{12}(z)/z\\ z\bar{Z}_{21}(z) & \bar{Z}_{22}(z)\end{pmatrix}=\begin{pmatrix}Z_{11}(z) & Z_{12}(z)/z\\ zZ_{21}(z) & Z_{22}(z)\end{pmatrix}^{-1}$$

# Factorization and matching

- Coordinate space

$$\tilde{h}_{q_i,R}(z,P^z,\mu) = \int_{-1}^{1} du\, \mathcal{C}_{q_iq_j}(u,\mu^2z^2) h_{q_j}(u\nu,\mu) + \int_{-1}^{1} du\, \mathcal{C}_{qg}(u,\mu^2z^2) h_g(u\nu,\mu).$$

$$\tilde{h}_{g,R}(z,P^z,\mu) = \int_{-1}^{1} du\, \frac{\mathcal{C}_{gg}(u,\mu^2z^2)}{\nu} h_g(u\nu,\mu) + \int_{-1}^{1} du \frac{\mathcal{C}_{gq}(u,\mu^2z^2)}{\nu} h_{q_i}(u\nu,\mu).$$

- Momentum space

$$\begin{aligned}
\tilde{f}^{(n)}_{g/H}(x,P^z,p_z^R,\mu_R) &= \int_{-1}^{1} \frac{dy}{|y|} \left[ C_{gg}\left(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\right) f_{g/H}(y,\mu) + C_{gq}\left(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\right) f_{q_j/H}(y,\mu)\right] \\
&\quad + \mathcal{O}\left(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right), \\
\tilde{f}_{q_i/H}(x,P^z,p_z^R,\mu_R) &= \int_{-1}^{1} \frac{dy}{|y|} \left[ C_{q_iq_j}\left(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\right) f_{q_j/H}(y,\mu) + C_{qg}\left(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\right) f_{g/H}(y,\mu)\right] \\
&\quad + \mathcal{O}\left(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right), \qquad (2.53)
\end{aligned}$$

- Perturbative matching coefficients have been available at one-loop

# Polarized gluon PDF

- For

$$\Delta f_{g/H}(x,\mu) = i\epsilon_{\perp ij}\int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- x P^+}\langle P|F^{+i}(\xi^- n_+)\mathcal{W}(\xi^- n_+, 0; L_{n_+})F^{j+}(0)|P\rangle$$

- We have identified three multiplicatively renormalizable quasi-PDF operators [JHZ et al, PRL 19']

$$\Delta O_g^1(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{tj}(z_1),$$
$$\Delta O_g^2(z,0) = i\epsilon_{\perp,ij}F^{zi}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$
$$\Delta O_g^3(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$

- Renormalization, factorization and matching are similar to the unpolarized case [Wang, JHZ et al, 19']

- Perturbative matching coefficients also available at one-loop

# Summary and outlook

- Rapid progress has been achieved in the past few years on direct computations of x-dependence of hadron structure from lattice QCD

- Applications to nucleon PDFs have yielded encouraging results, but so far only to isovector quark combinations which do not mix with gluons

- Flavor-singlet quark PDF and Gluon PDF
  - Appropriate gluon quasi-PDF operators identified
  - Renormalization and factorization understood
  - Perturbative matching available at 1-loop
  - Can now be studied on the lattice

- Generalization to GPDs …