

# EIC theory overview

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# Electron-Ion Collider (EIC)

A future (2029~) high-luminosity polarized  $ep$ ,  $eA$  collider dedicated to the study of the nucleon and nucleus structure.

Center-of-mass energy  
Luminosity

$$20 \lesssim \sqrt{s} \lesssim 140 \text{ GeV}$$

$$\sim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

Gluons and the quark sea at high energies:  
distributions, polarization, tomography

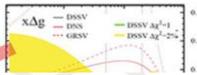
September 13 to November 19, 2010

Report from the INT program "Gluons and the quark sea at high energies: distributions, polarization, tomography"

2010 INT workshop

o small x uncertainty from DSSV

$$\frac{dg_1}{d \log(Q^2)} \propto -\Delta g(x, Q^2)$$



REACHING FOR THE HORIZON

2015 NSAC Long Range Plan

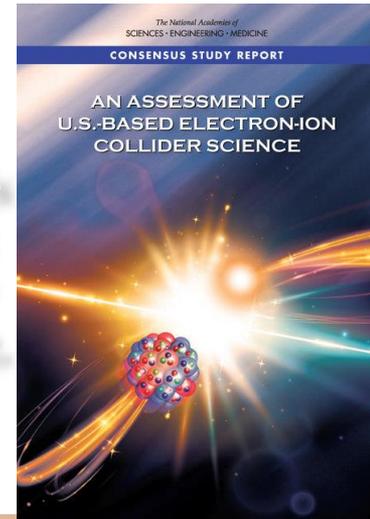


The 2015  
LONG RANGE PLAN  
for NUCLEAR SCIENCE



2018 NAS report

“The committee finds that the science that can be addressed by an EIC is compelling, fundamental and timely.”



2018 INT workshop

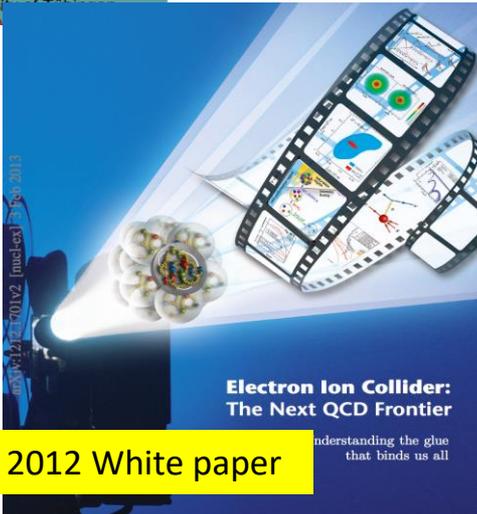
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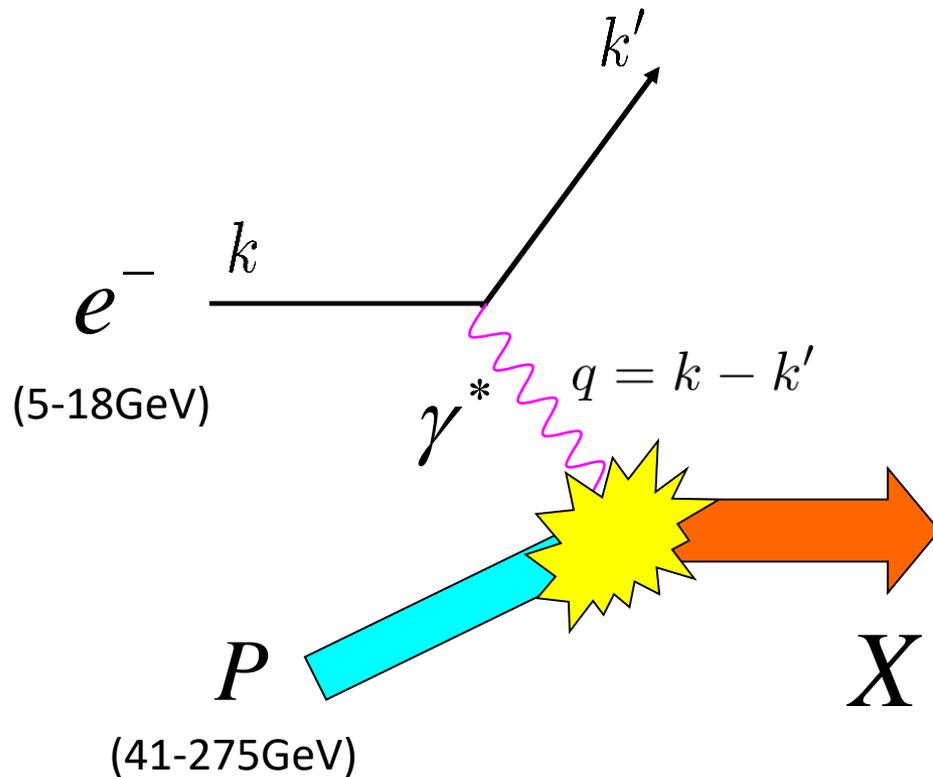
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2012 White paper

# Experiment at EIC: Deep Inelastic Scattering (DIS)



Two most important kinematic variables

$$Q^2 = -q^2 \quad \text{photon virtuality (resolution)}$$

$$x = \frac{Q^2}{2P \cdot q} \quad \text{Bjorken variable (inverse energy)}$$

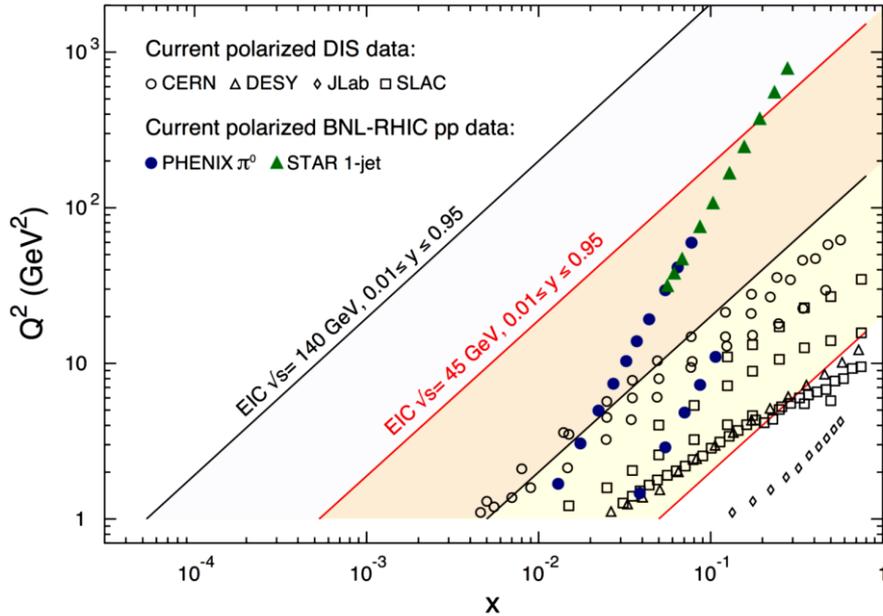
$$\approx \frac{E_{parton}}{E_{proton}}$$

Proton, deuteron, helium, gold...any nucleus of your choice!

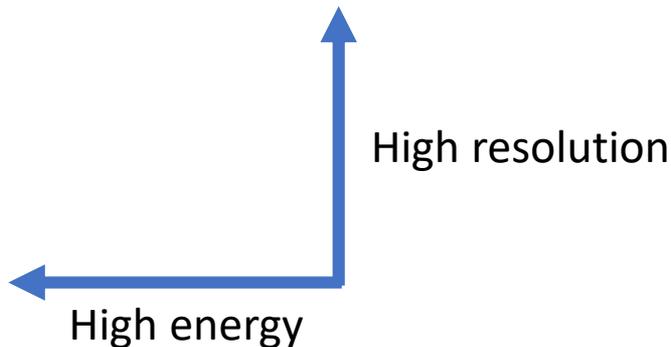
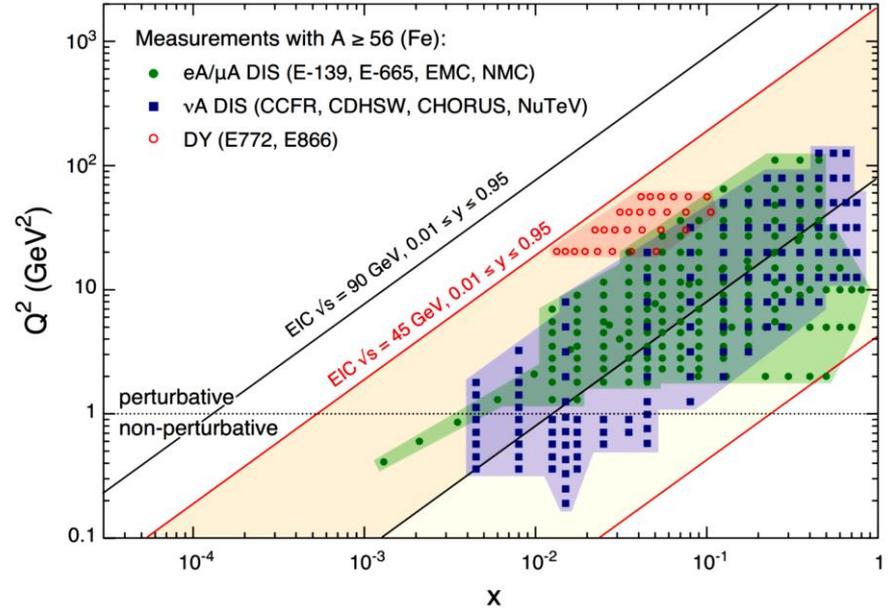
Electron, proton and light nuclei can be polarized.

# EIC Kinematical coverage

## Polarized DIS

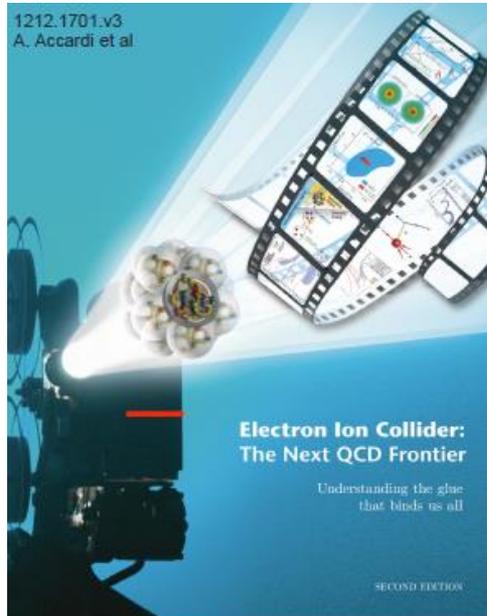


## Nuclear DIS



Unprecedented coverage in kinematics.  
Tremendous physics opportunities!

# Scientific goals of EIC



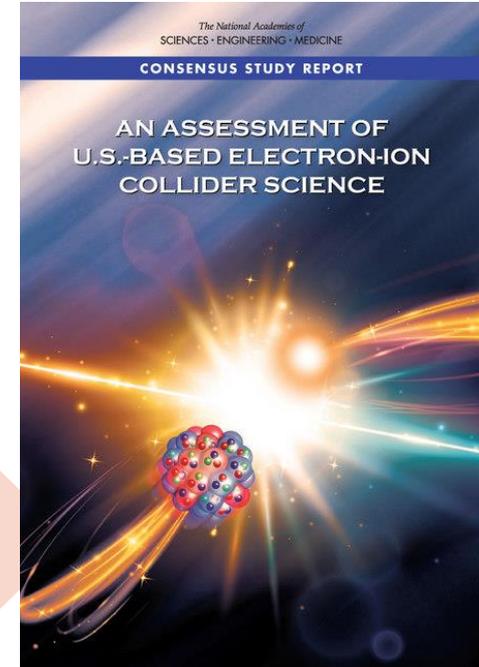
White paper  
arXiv:1212.1701

Origin of  
nucleon  
mass

Origin of  
nucleon  
spin

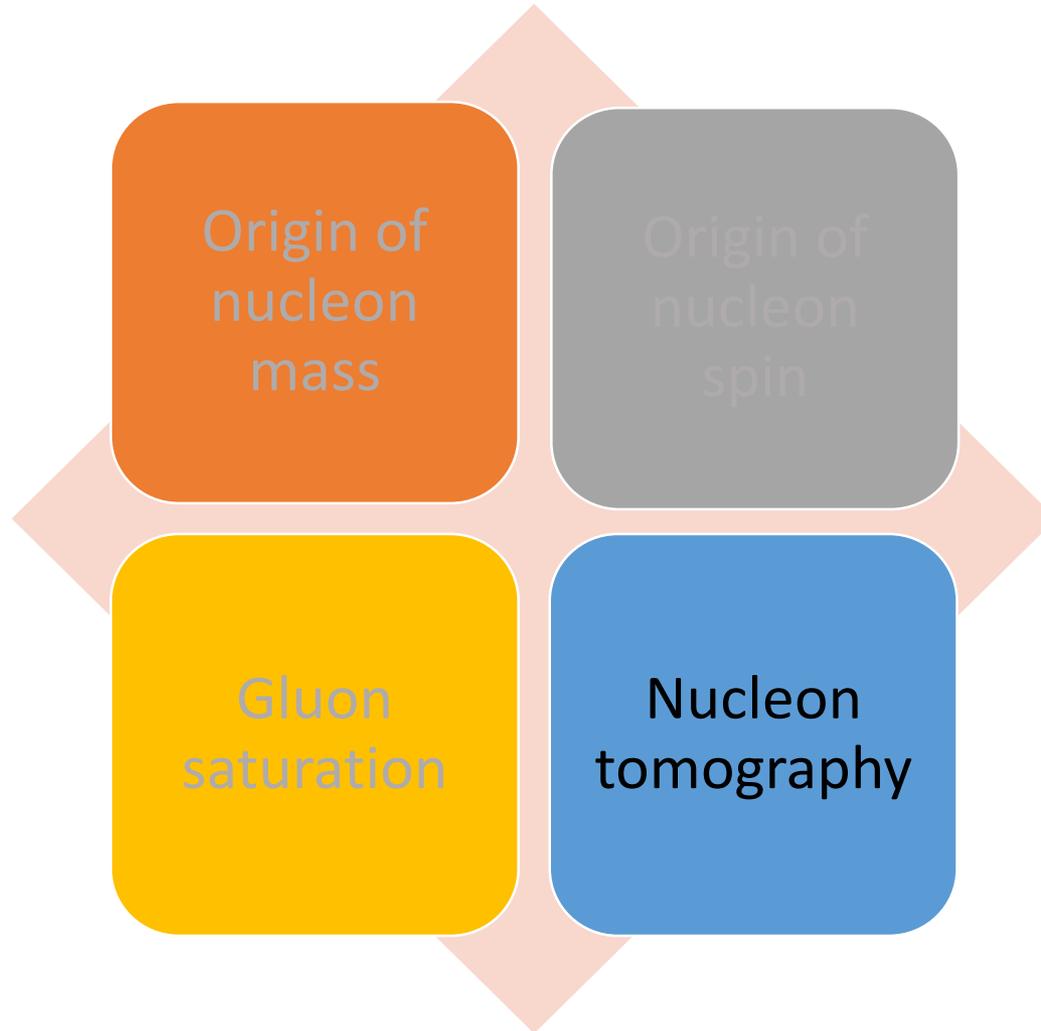
Glue  
saturation

Nucleon  
tomography

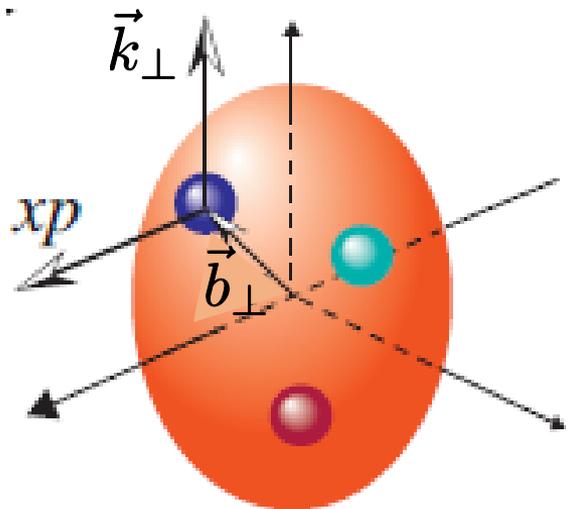


NAS report  
July 2018

# Scientific goals of EIC



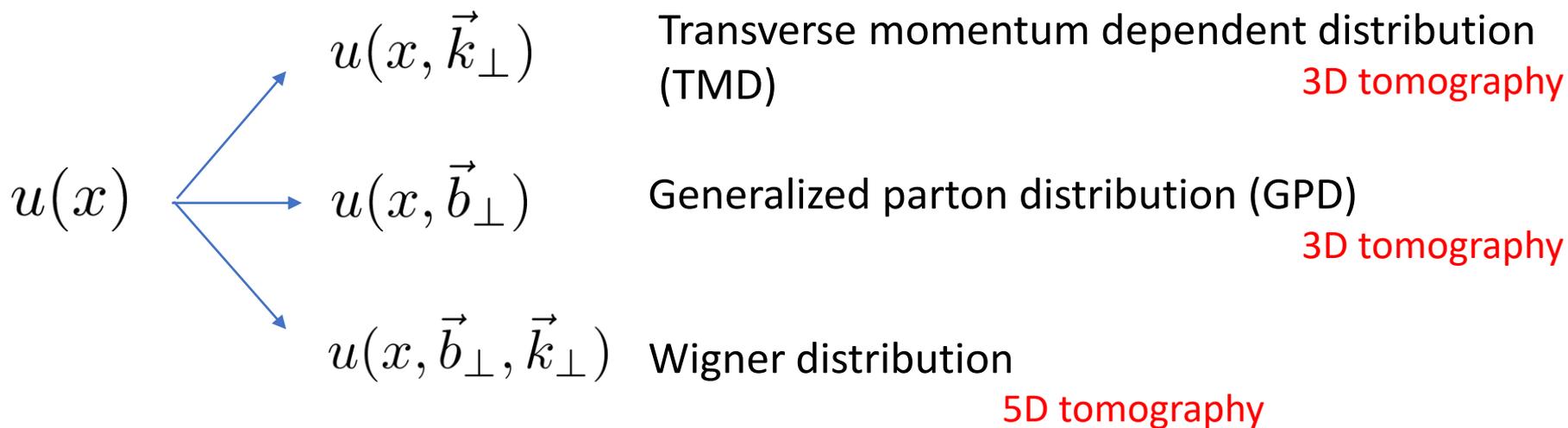
# Multi-dimensional tomography



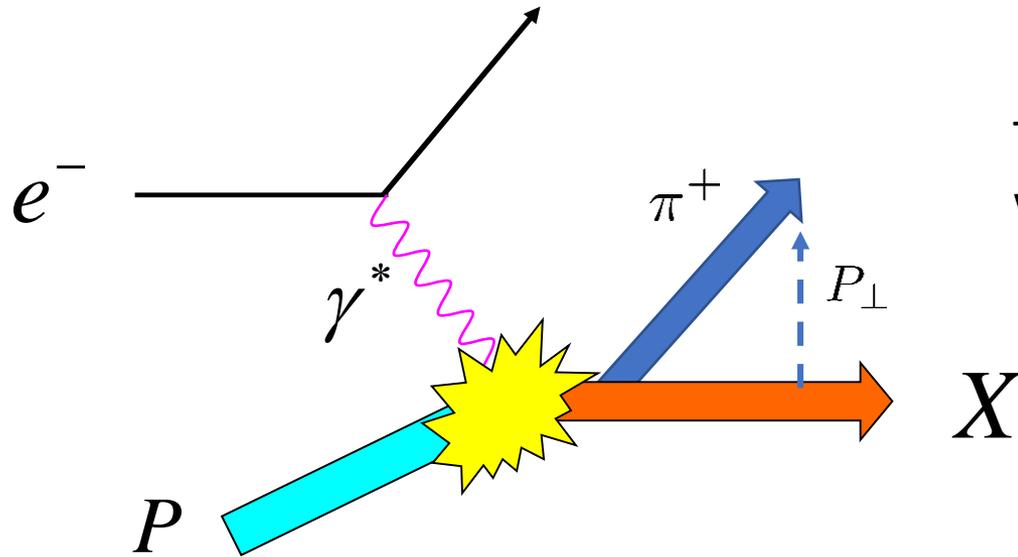
$$u(x) = \int \frac{dz^-}{4\pi} \langle P | \bar{u}(0) \gamma^+ u(z^-) | P \rangle$$

Ordinary parton distribution functions (PDF) can be viewed as the 1D tomographic image of the nucleon

The nucleon is much more complicated!  
Partons also have transverse momentum  $\vec{k}_\perp$   
and are spread in impact parameter space  $\vec{b}_\perp$



# Semi-inclusive DIS



Tag one hadron species  
with fixed transverse momentum  $P_\perp$

When  $P_\perp$  is small, **TMD factorization**

Collins, Soper, Sterman;  
Ji, Ma, Yuan,...

$$\frac{d\sigma}{dP_\perp} = H(\mu) \int d^2q_\perp d^2k_\perp \underbrace{f(x, k_\perp, \mu, \zeta)}_{\text{TMD PDF}} \underbrace{D(z, q_\perp, \mu, Q^2/\zeta)}_{\text{TMD FF}} \delta^{(2)}(zk_\perp + q_\perp - P_\perp) + \dots$$

Open up a new class of observables where perturbative QCD is applicable!

# TMD evolution

Define Fourier transform  $\int d^2 k_{\perp} e^{i k_{\perp} r_{\perp}} f(k_{\perp} \dots) = f(r_{\perp} \dots)$

RG equation

$$\frac{\partial}{\partial \ln \mu} f(x, r_{\perp}, \mu, \zeta) = \gamma_F f(x, r_{\perp}, \mu, \zeta)$$

Known to three loops  
Moch, Vermaseren, Vogt (2005)

Collins-Soper equation

$$\frac{\partial}{\partial \ln \zeta} f(x, r_{\perp}, \mu, \zeta) = -\mathcal{D}(r_{\perp}) f(x, r_{\perp}, \mu, \zeta)$$

Recently computed to three loops!  
Li, Zhu (2017); Vladimirov (2017)

Computable from lattice QCD at large  $r_{\perp}$   
Ebert, Stewart, Zhao (2018)

# TMD global analysis

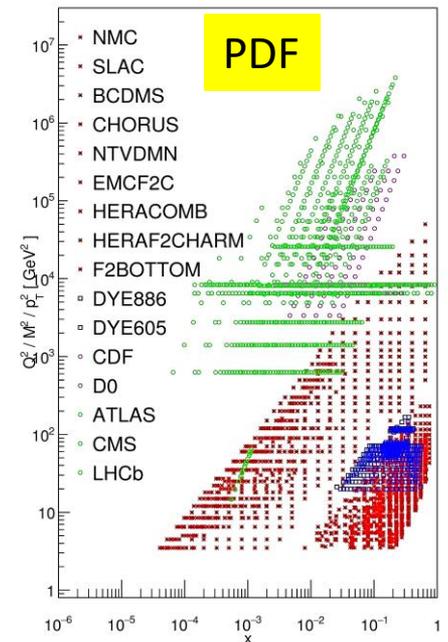
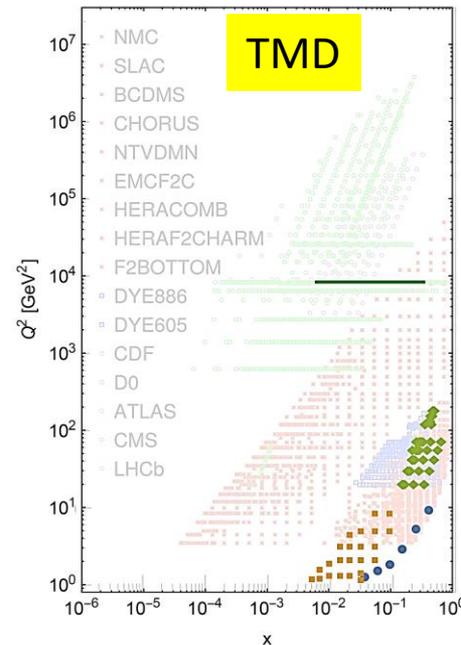
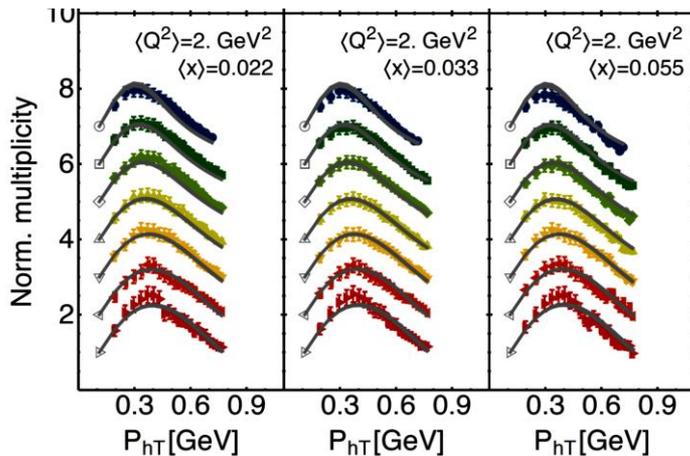
Global analysis of TMD based on  $\sim 8000$  data points from SIDIS, Drell-Yan.

Bacchetta, Delcarro, Pisano, Radici, Signori (2017)

arTeMiDe state-of-the-art (NNLO+NNLL) implementation

Scimemi, Vladimirov (2017)

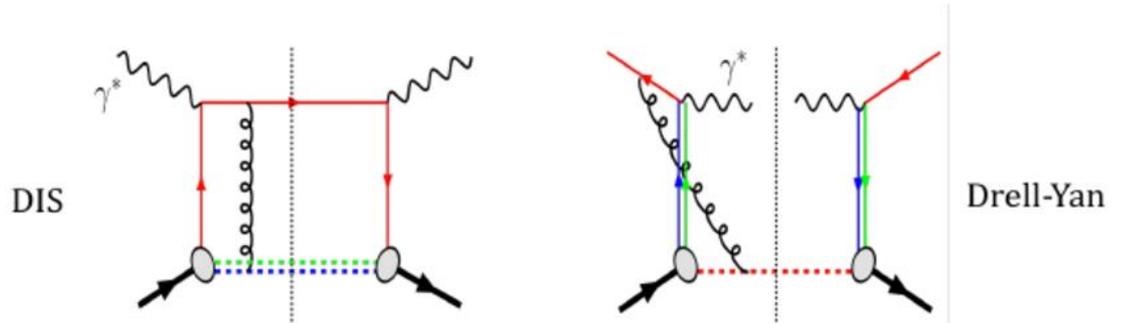
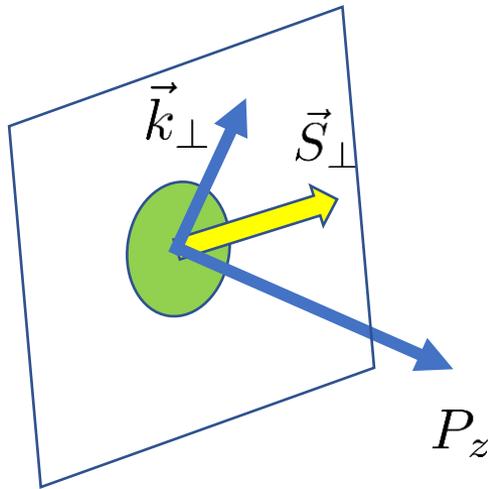
TMDlib public library Hautmann, Jung, Mulders,...



Still in its infancy. Fully blossoms in the EIC era!

# Universality up to a sign

**Sivers function**  $f_{1T}^\perp(x, k_\perp)$  for the transversely polarized nucleon

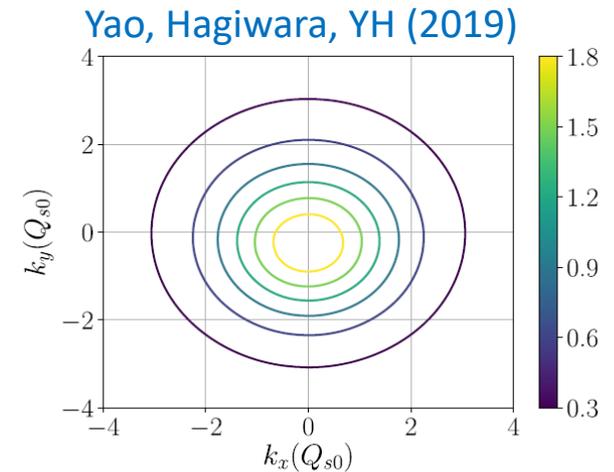


The same function, but with opposite signs in DIS and Drell-Yan. (Collins, 2002)

Experimental tests at RHIC and Compass, to be continued at EIC.

EIC can also probe **gluon** Sivers function from open charm single spin asymmetry

Zheng, Aschenauer, Lee, Xiao, Bao (2018)



# Generalized parton distributions (GPD)

$$P^+ \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \langle P' S' | \bar{\psi}(0) \gamma^\mu \psi(y^-) | PS \rangle$$

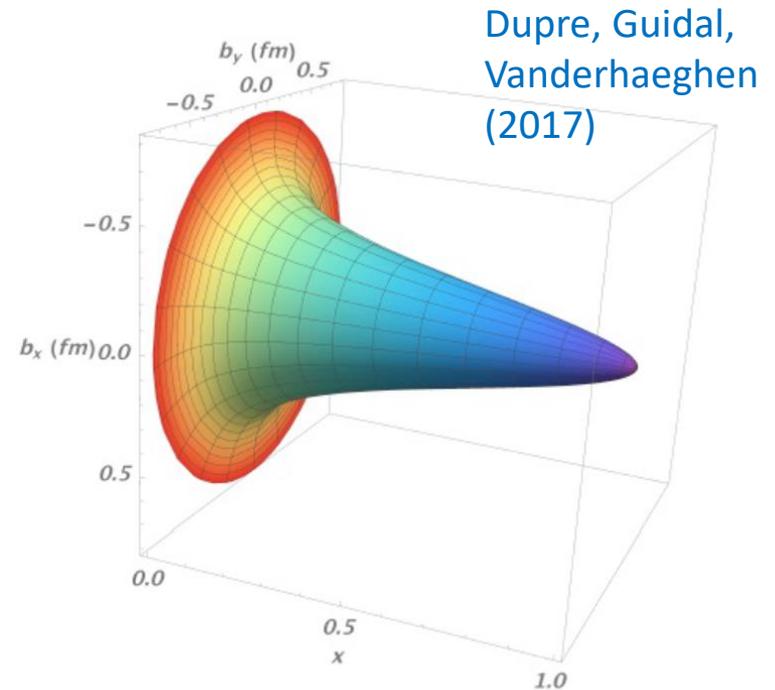
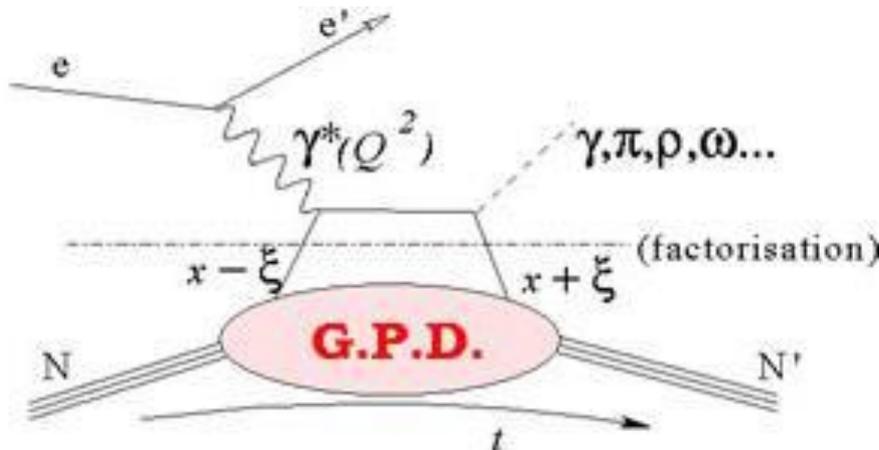
$$= H_q(x, \Delta) \bar{u}(P' S') \gamma^\mu u(PS) + E_q(x, \Delta) \bar{u}(P' S') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m} u(PS) \quad \Delta = P' - P$$



Fourier transform

Distribution of partons in **impact parameter** space  $b_\perp$

Measurable in  
Deeply Virtual Compton Scattering (DVCS)



# Towards measuring GPD $E$ at the EIC

Ji sum rule for proton spin  $\frac{1}{2} = J_q + J_g$

$$J_q = \frac{1}{2} \int dx x (H_q(x) + E_q(x))$$

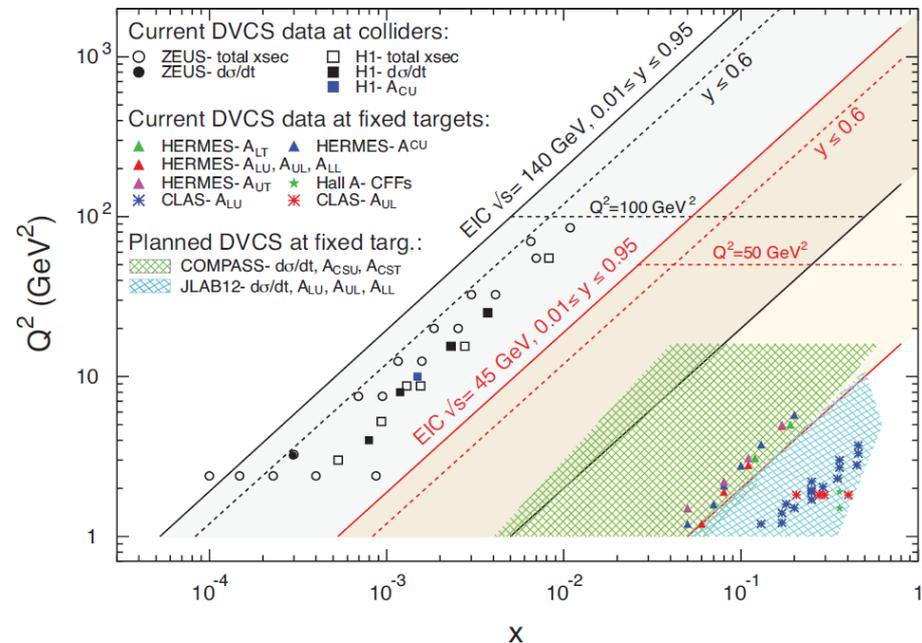
$$J_g = \frac{1}{4} \int dx x (H_g(x) + E_g(x))$$

Currently very little is known about  $E_q$ ,  
nothing about  $E_g$  from experiments.

At EIC, we can get a handle on  $E_q$ .

Aschenauer, Fazio, Kumericki, Muller (2013)

$E_g$  is still challenging, but EIC is the only hope.



# D-term: the last global unknown

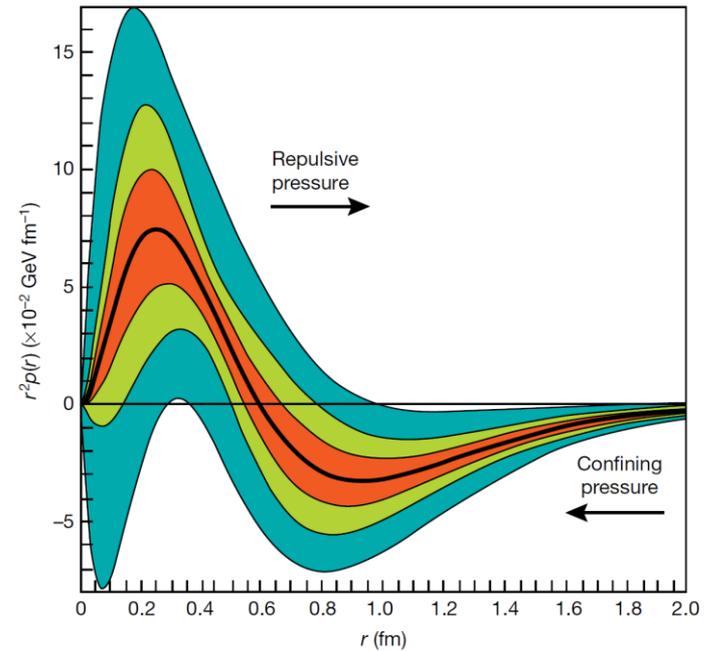
Burkert, Elouadrhiri, Girod (Nature, 2018)

$$\langle P' | T^{ij} | P \rangle \sim (\Delta^i \Delta^k - \delta^{ik} \Delta^2) D(t)$$

$D(t=0)$  is a conserved charge of the nucleon, just like mass and spin!

Related to the radial pressure distribution inside a nucleon [Polyakov, Schweitzer,...](#)

$$T^{ij}(r) = \left( \frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$



First extraction at Jlab, large model dependence.

Need significant lever-arm in  $Q^2$  to disentangle various moments of GPDs



$D(t) = D_q(t) + D_g(t)$  can be separately computed on a lattice

[Hagler et al. \(2007\); Shanahan, Detmold \(2018\)](#)

# Pressure from quark and gluon subsystems

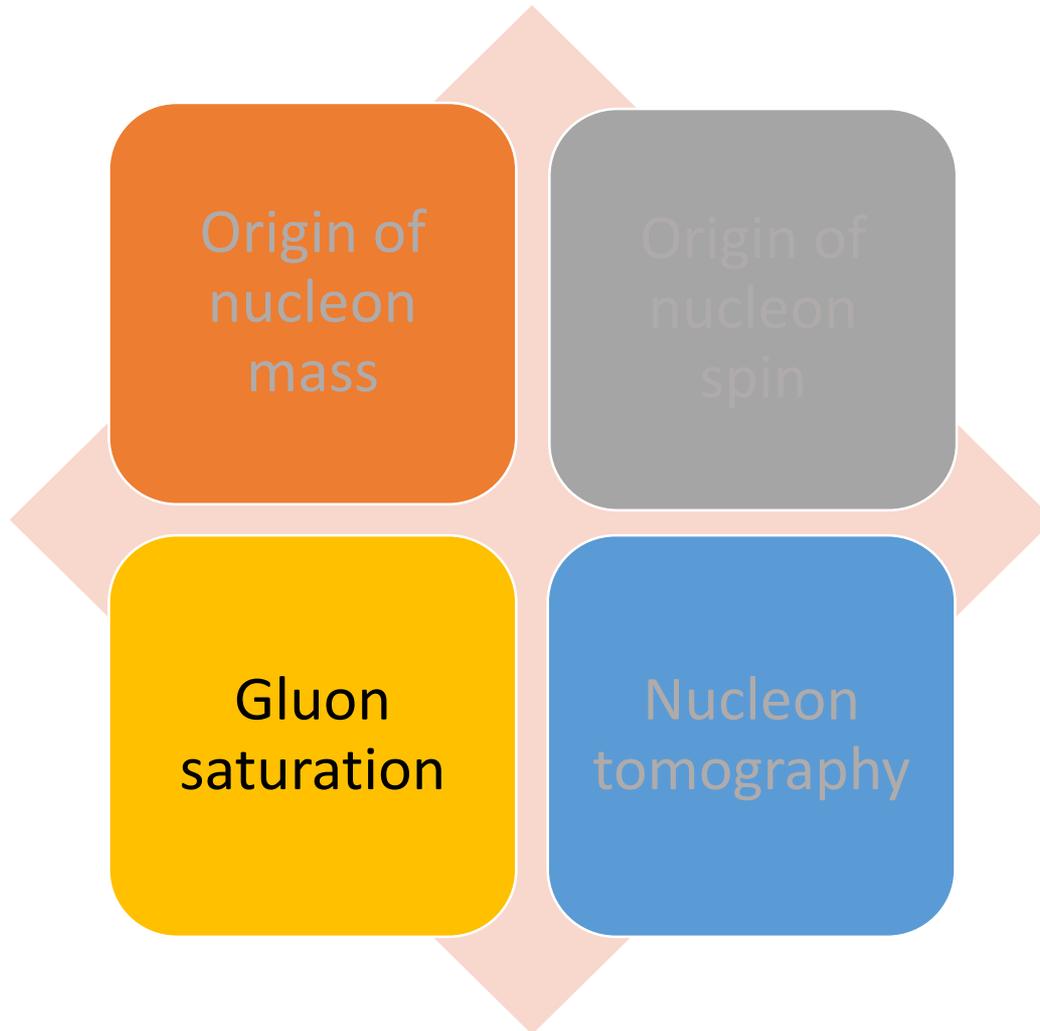
$$p_{q,g}(r) = \frac{1}{6Mr^2} \frac{d}{dr} r^2 \frac{d}{dr} D_{q,g}(r) - M\bar{C}_{q,g}(r)$$

$$\begin{aligned} \bar{C}_q^R(\mu) = & -\frac{1}{4} \left( \frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0} \right) + \frac{1}{4} \left( \frac{2n_f}{3\beta_0} + 1 \right) \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2} \\ & - \frac{4C_F A_q^R(\mu_0) + n_f (A_q^R(\mu_0) - 1)}{4(4C_F + n_f)} \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} \\ & + \frac{\alpha_s(\mu)}{4\pi} \left[ \frac{n_f \left( -\frac{34C_A}{27} - \frac{49C_F}{27} \right)}{4\beta_0} + \frac{\beta_1 n_f}{6\beta_0^2} \right. \\ & \left. + \frac{1}{4} \left( \frac{n_f \left( \frac{34C_A}{27} + \frac{157C_F}{27} \right)}{\beta_0} + \frac{4C_F}{3} - \frac{2\beta_1 n_f}{3\beta_0^2} \right) \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2} \right] + \dots, \\ \simeq & -0.146 - 0.25 (A_q^R(\mu_0) - 0.36) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{50}{81}} - 0.01\alpha_s(\mu) \\ & + (0.306 + 0.08\alpha_s(\mu)) \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2}, \end{aligned}$$

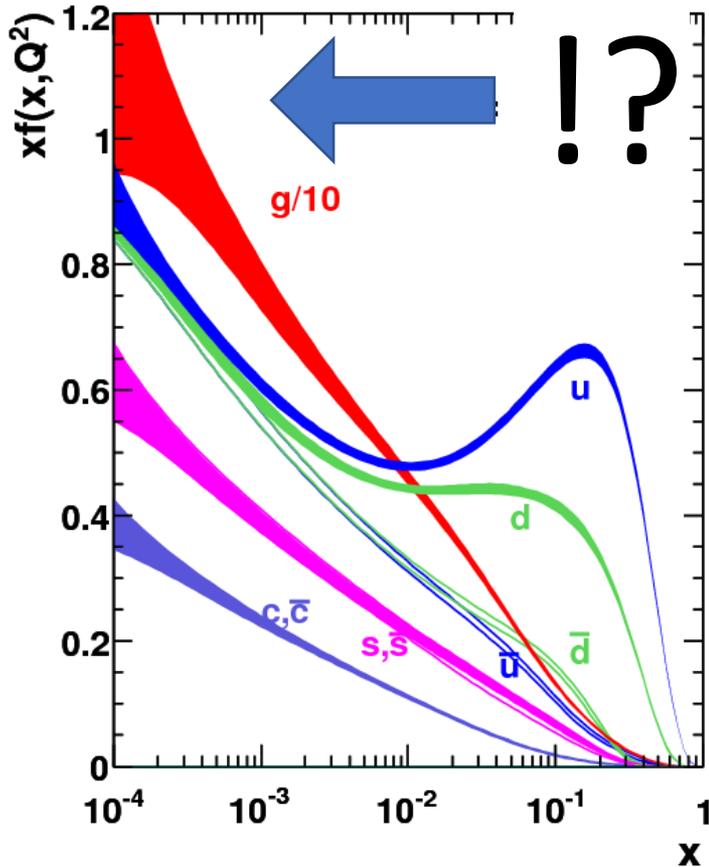
Value of  $\bar{C}_{q,g}(\Delta = 0)$   
completely fixed by the  
trace anomaly

YH, Rajan, Tanaka (2018)

# Scientific goals of EIC

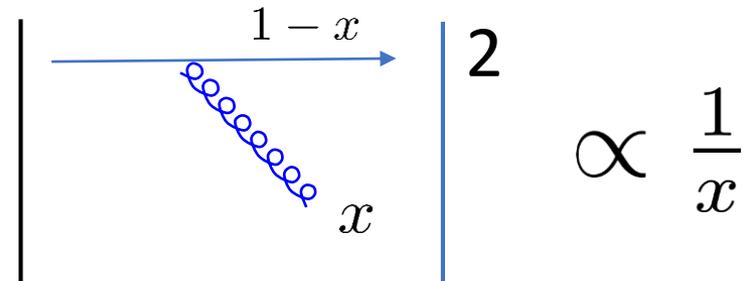


# QCD at small-x



as predicted by BFKL  
(Balitsky-Fadin-Kuraev-Lipatov)

Probability to emit a soft gluon diverges



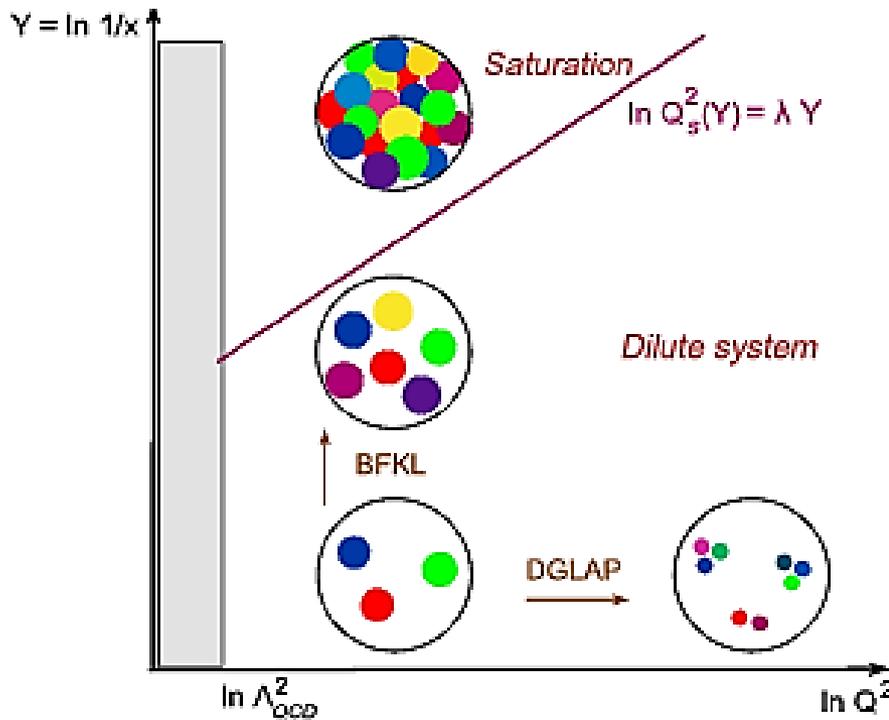
A myriad of small-x gluons  
in a high energy hadron/nucleus!

$$\sum_n \frac{1}{n!} (\alpha_s \ln 1/x)^n \sim \left( \frac{1}{x} \right)^{\alpha_s}$$

# Gluon saturation

The gluon number eventually saturates, forming the universal QCD matter at high energy called the **Color Glass Condensate**.

Gribov, Levin, Ryskin (1980); Mueller, Qiu (1986); McLerran, Venugopalan (1993)



Gluons overlap when

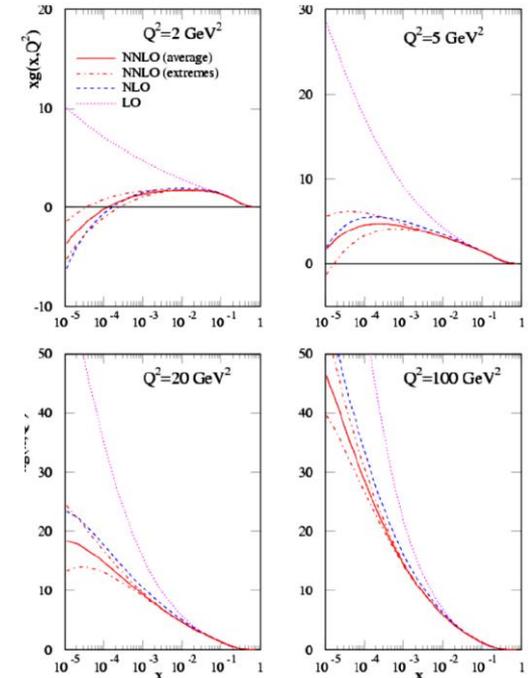
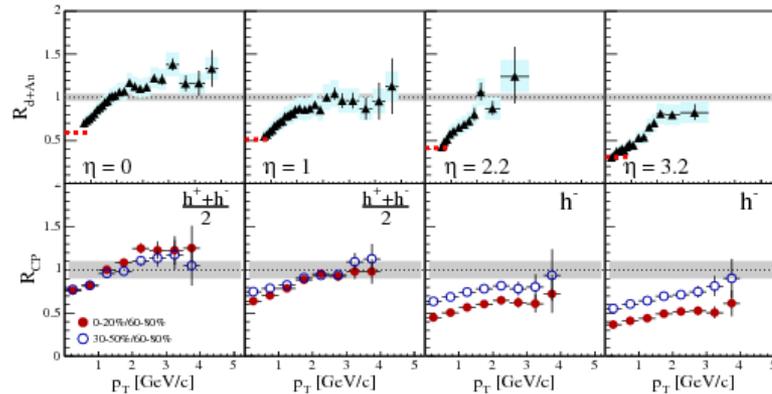
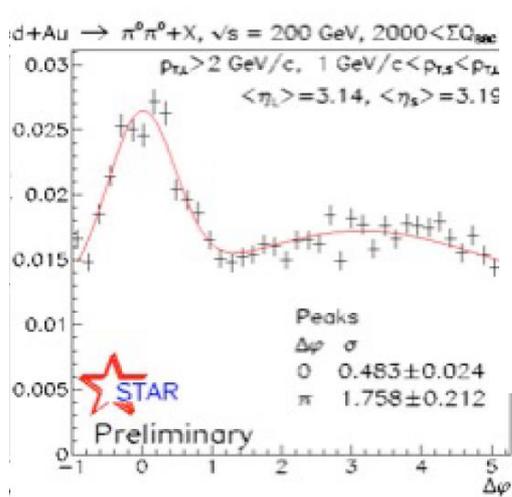
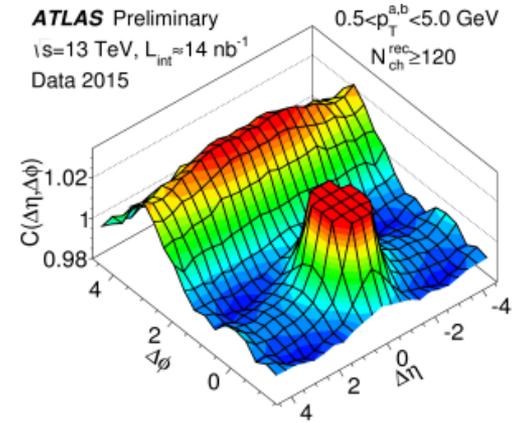
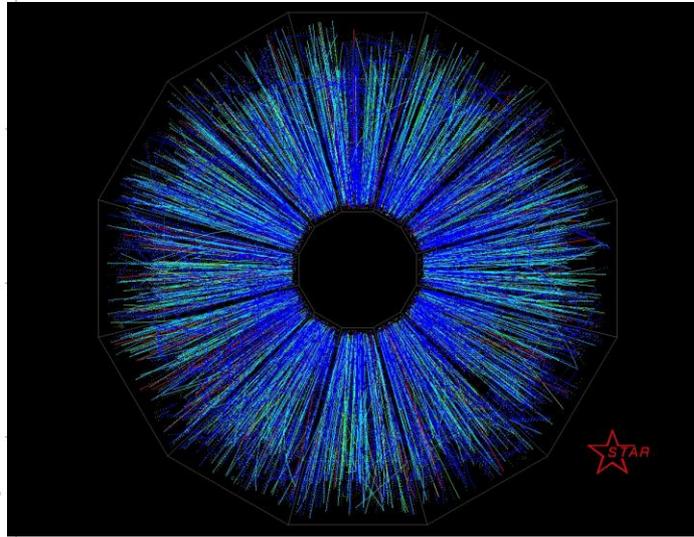
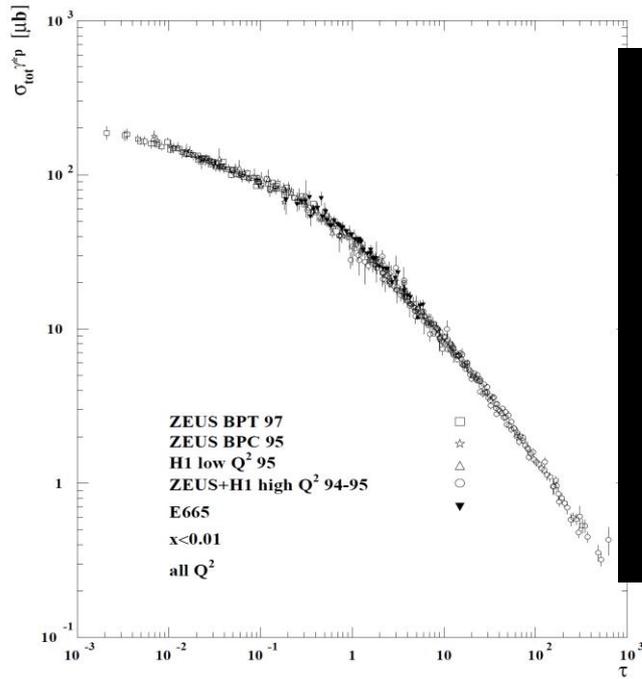
$$\frac{\alpha_s}{Q^2} x G(x, Q^2) = \pi R_p^2$$

The saturation momentum

$$Q = Q_s(x) \gg \Lambda_{QCD}$$

High density, but weakly coupled many-body problem

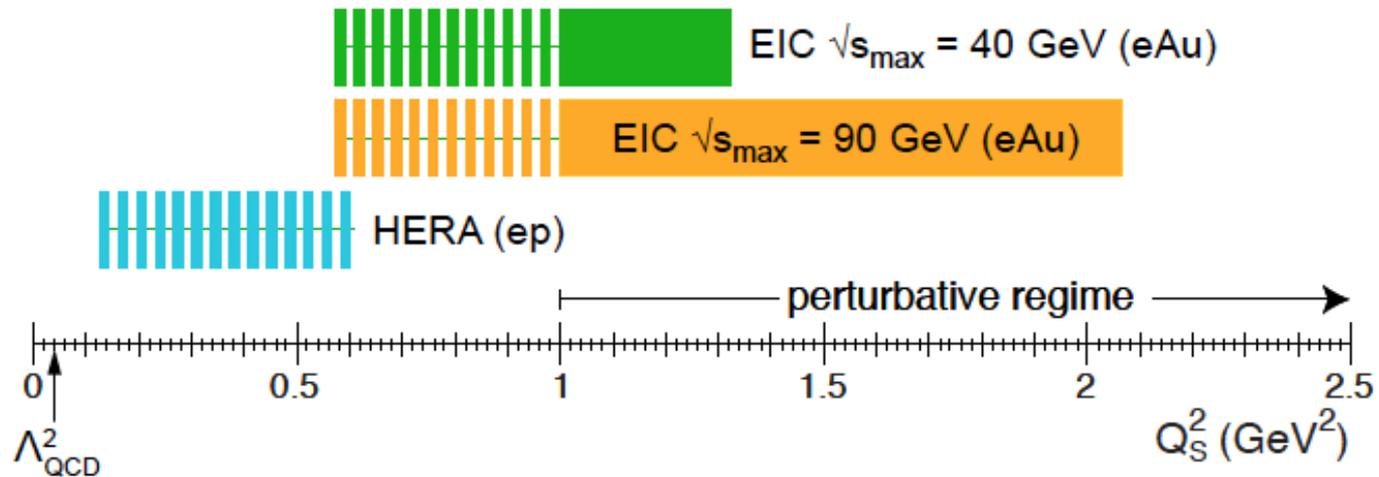
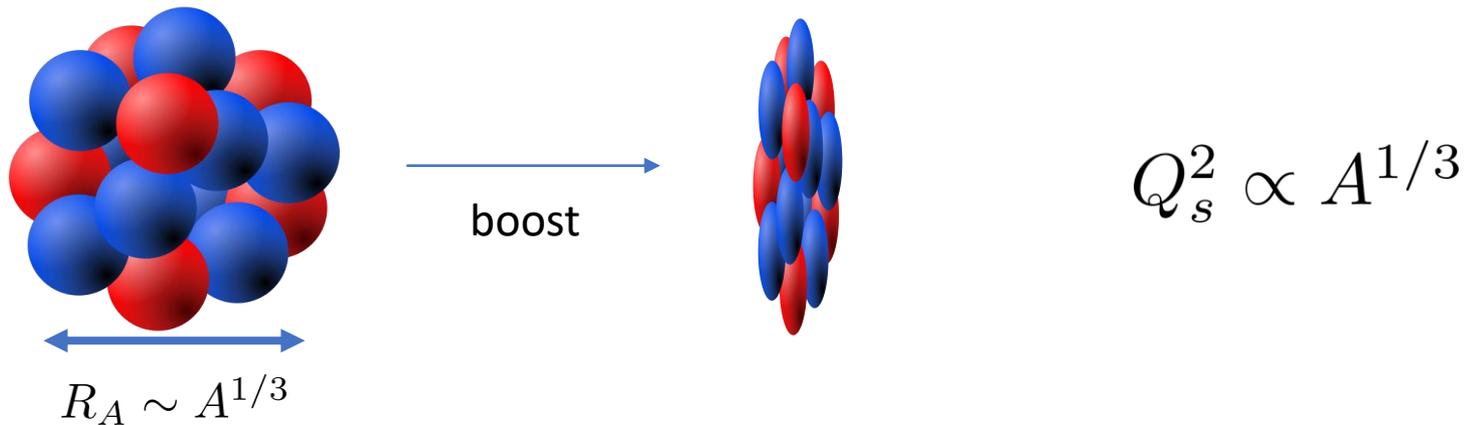
# Has saturation been observed at HERA, RHIC, LHC?



# eA collision at EIC : ideal place to study saturation

No initial state interactions (advantage over LHC, RHIC)

Nuclear enhancement of the saturation momentum (advantage over HERA)

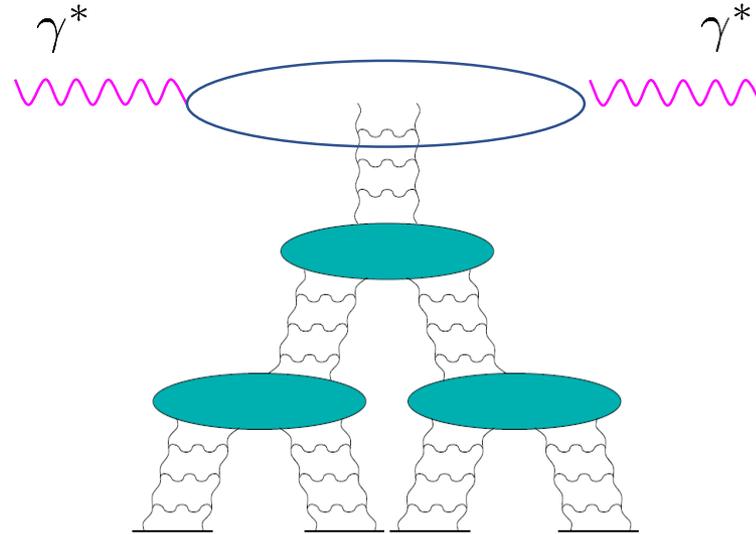


# BK-JIMWLK equation

Balitsky  
Kovchegov

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Photon-nucleus scattering  
at high energy



$\equiv S$

Leading Logarithmic (LL) evolution of the scattering amplitude with energy

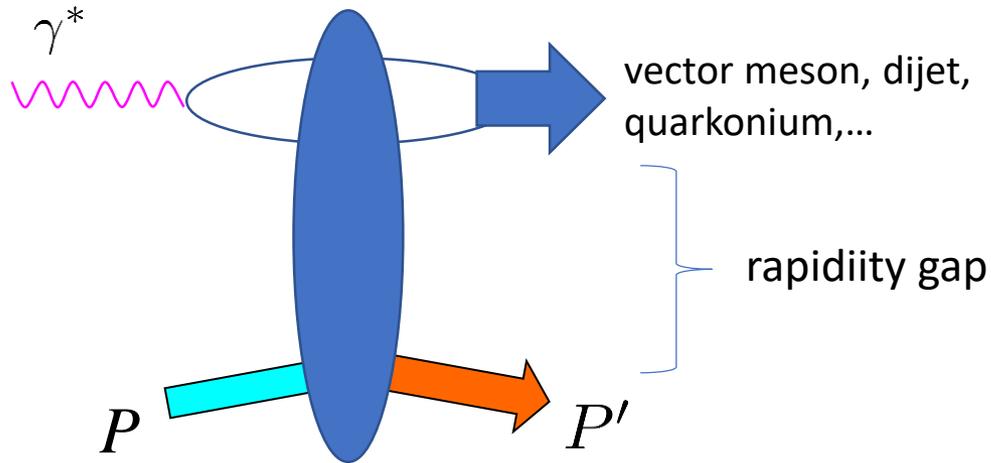
$$\frac{\partial}{\partial \ln 1/x} S(r_{\perp}) = \frac{N_c \alpha_s}{2\pi} \int d^2 r_{\perp} \frac{r_{\perp}^2}{z_{\perp}^2 (r_{\perp} - z_{\perp})^2} (S(z_{\perp}) S(z_{\perp} - r_{\perp}) - S(r_{\perp}))$$

Extension to NLL [Balitsky, Chirilli \(2008\)](#)

Even to NNLL? [Caron-Huot \(2016\)](#)

State-of-the-art: **NLL' + NLO**

# Golden channel for saturation: Diffraction



Cross sections proportional to the **square** of the gluon distribution

→ More sensitive to saturation

'Day 1 prediction'

[Kowalski, Lappi, Marquet, Venugopalan \(2008\)](#)

$$\left. \frac{\sigma_{diff}}{\sigma_{tot}} \right|_{eA} \approx 20\% > \left. \frac{\sigma_{diff}}{\sigma_{tot}} \right|_{ep}$$

Nucleus stays intact in every 1 out of 5 events!

Recently extended to NLO for dijet, vector meson...

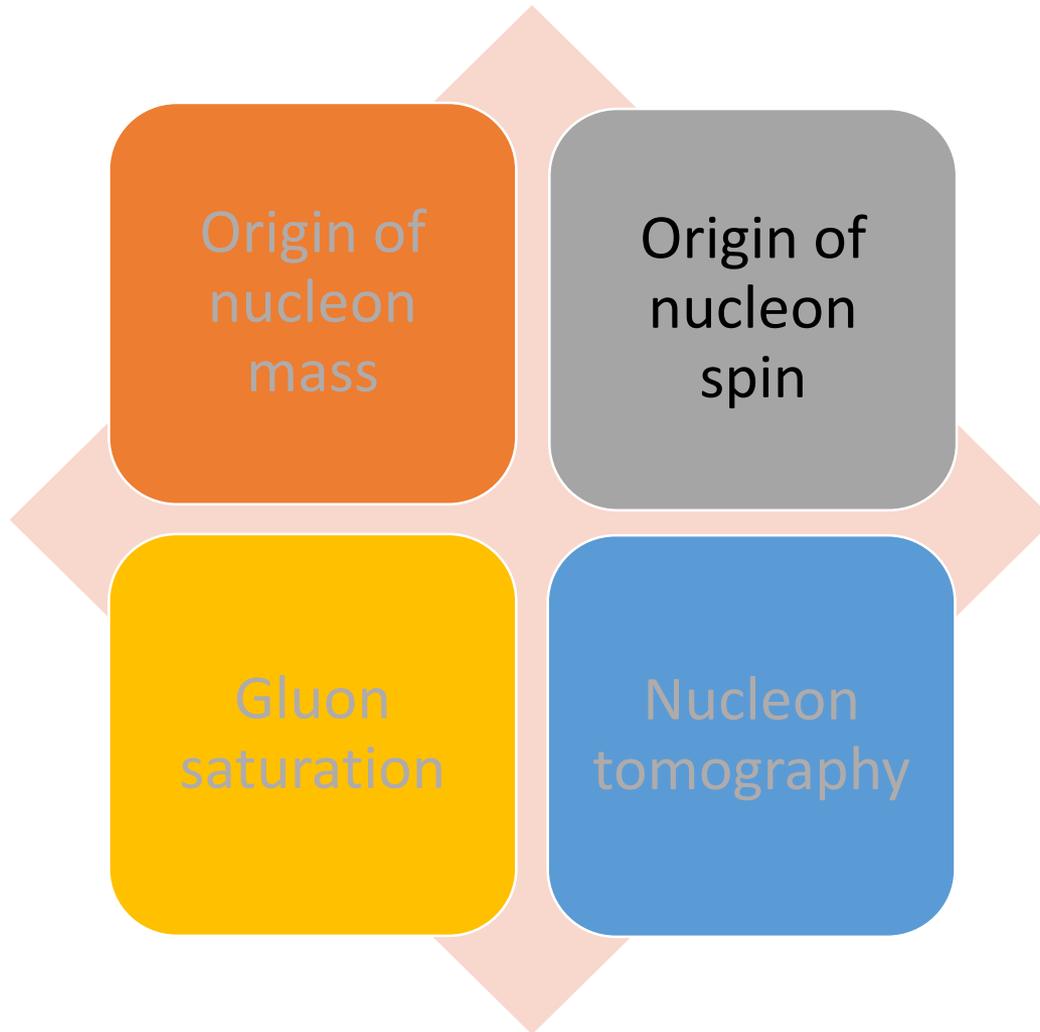
[Boussarie, Grabovsky, Szymanowski, Wallon \(2016\)](#)

Can access also the Wigner distribution

[YH, Xiao, Yuan \(2016\)](#)

[Mantysaari, Mueller, Schenke 1902.05087](#)

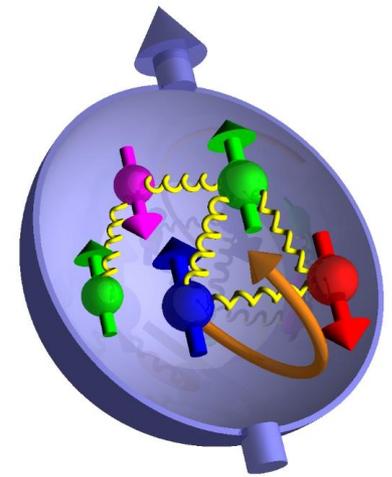
# Scientific goals of EIC



# Proton spin decomposition

The proton has spin  $\frac{1}{2}$ .

The proton is not an elementary particle.



➔ Jaffe-Manohar sum rule

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity      Gluons' helicity      Orbital angular Momentum (OAM)

$$\Delta\Sigma = 1 \text{ in the quark model}$$

# Spin crisis

In 1987, EMC (European Muon Collaboration) announced a very small value of the quark helicity contribution

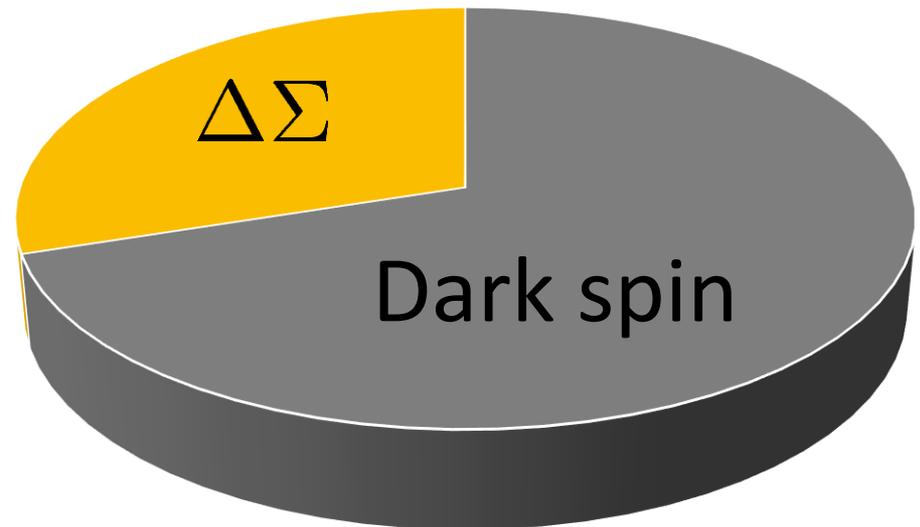
$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \quad !?$$

Recent values from NLO global analysis

$$\Delta\Sigma = 0.25 \sim 0.3$$

$$\int_{0.05}^1 dx \Delta G(x, Q^2) \approx 0.2 \pm_{0.07}^{0.06}$$

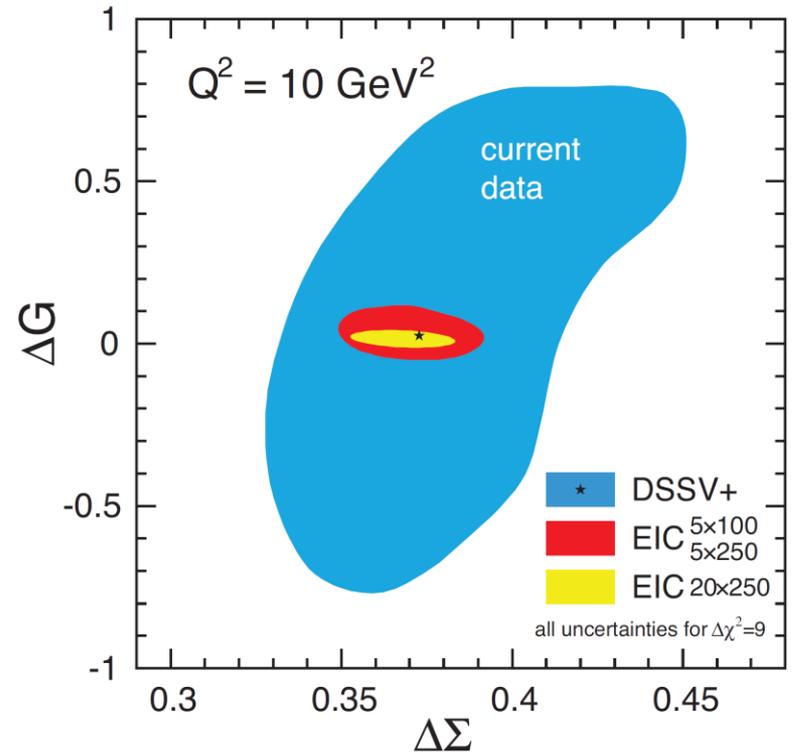
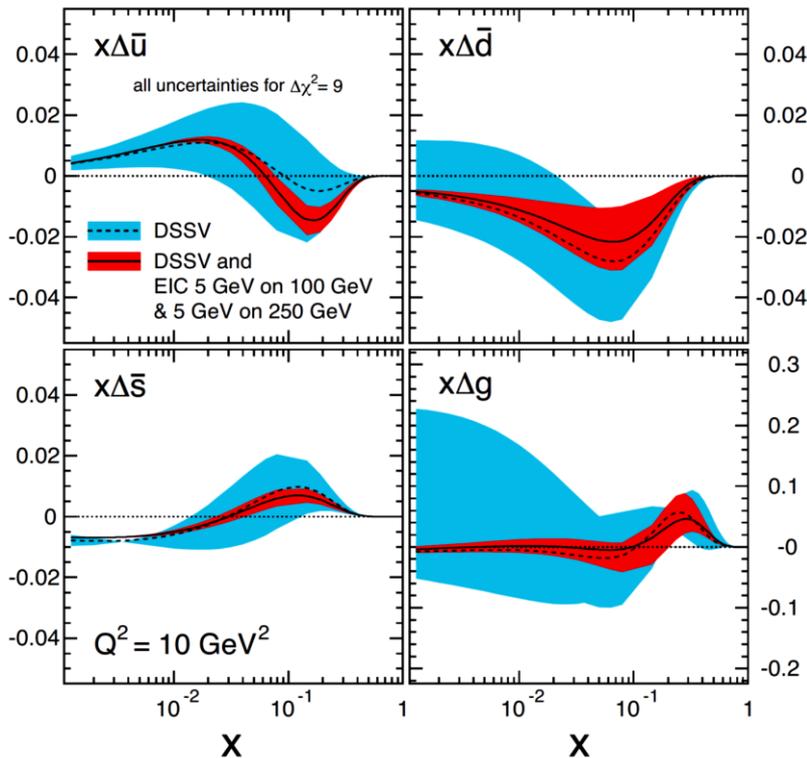
DeFlorian, Sassot, Stratmann, Vogelsang (2014)



**Warning:** Huge uncertainties from the small-x region

# Helicity measurements at EIC

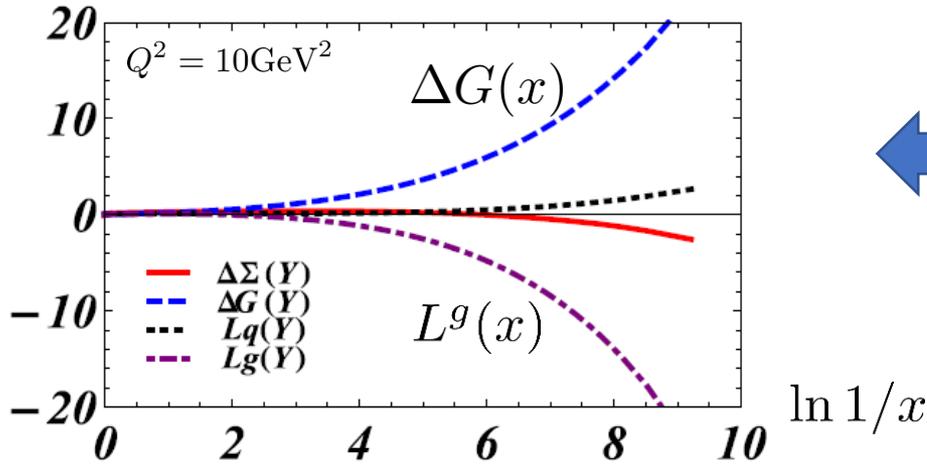
After one-year of data taking at EIC...



Wider coverage in  $x$  and  $Q^2$  ... finally solve the spin puzzle?

**No!**

# Don't forget Orbital Angular Momentum. It's there!



Significant cancellation at small-x  
from one-loop DGLAP  
YH, Yang (2018)

All-loop resummation of small-x double logarithms  
( $\alpha_s \ln^2 1/x$ )<sup>n</sup> by **I**nfra**R**ed **E**volution **E**quation

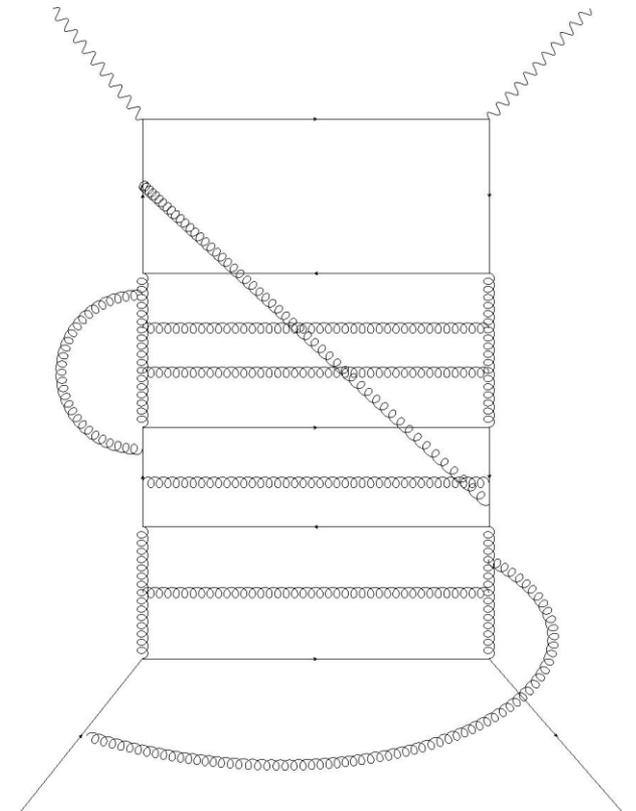
Kirschner, Lipatov (1983)

Bartels, Ermolaev, Ryskin (1996)

Generalized to OAM

$$L_g(x) \approx -2\Delta G(x)$$

Boussarie, YH, Yuan (2019)



# Measuring OAM at EIC

Ji, Yuan, Zhao (2016)

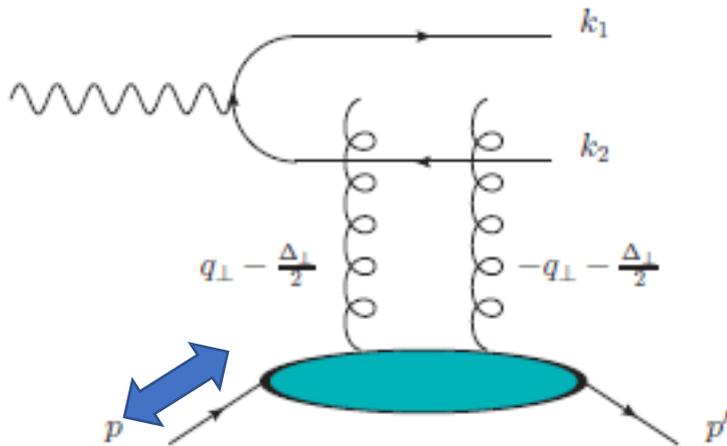
YH, Nakagawa, Xiao, Yuan, Zhao (2016)

Bhattacharya, Metz, Zhou (2017)

Exploit the connection between OAM and the **Wigner distribution**

$$L^{q,g} = \int dx \int d^2b_{\perp} d^2k_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z W^{q,g}(x, \vec{b}_{\perp}, \vec{k}_{\perp})$$

Longitudinal single spin asymmetry in diffractive dijet production



$$\sigma^{\rightarrow} - \sigma^{\leftarrow} \propto \sin(\phi_P - \phi_{\Delta})$$

proton recoil momentum

dijet relative momentum

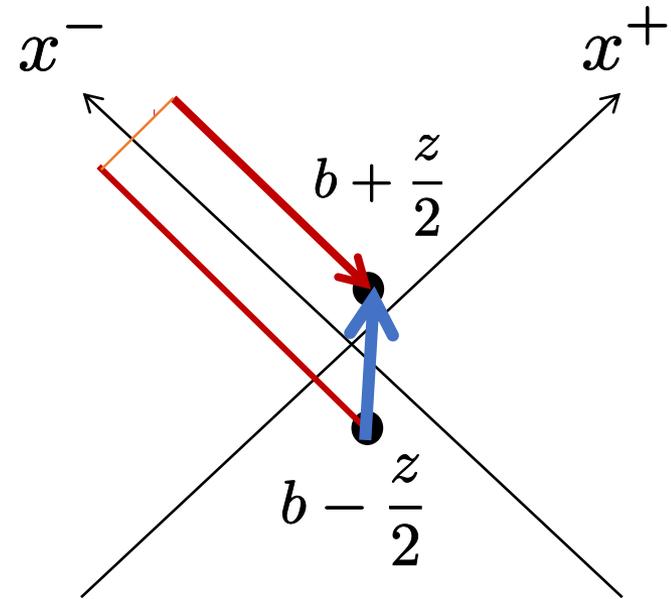
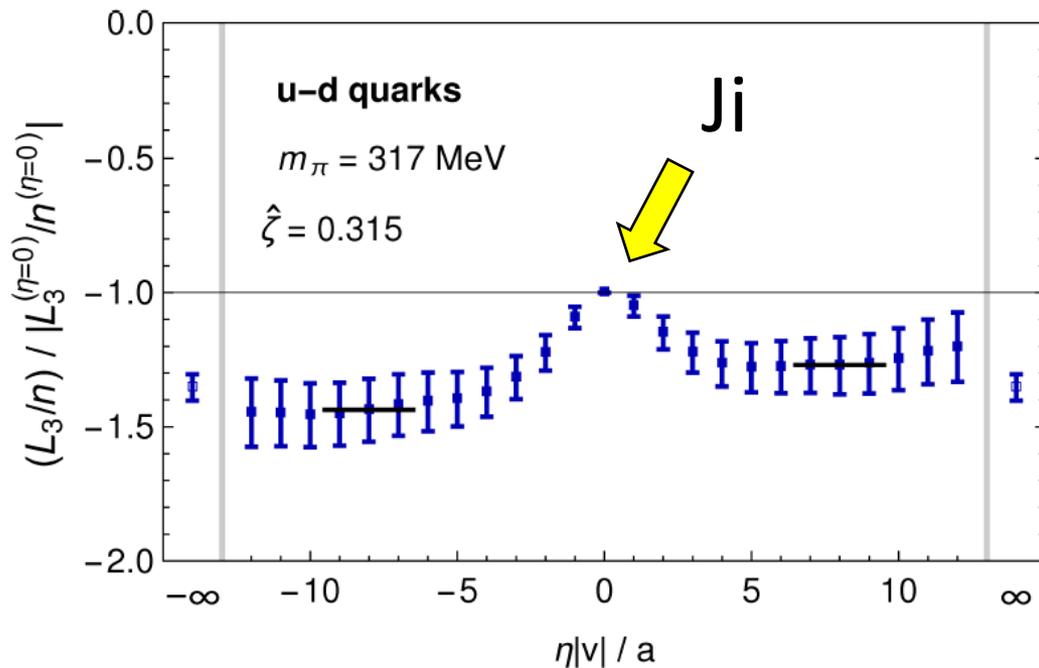
Need more work, more new ideas!

# OAM from lattice QCD

Engelhardt (2017)

Jaffe-Manohar OAM from staple Wilson line YH (2011)

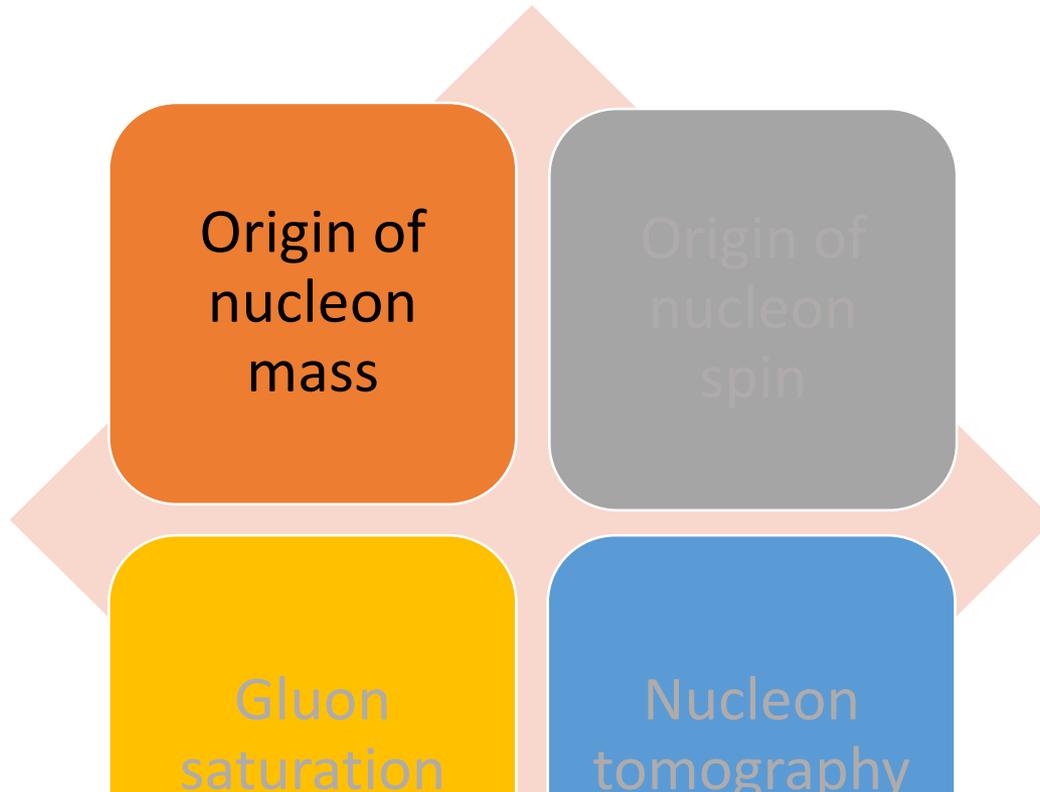
Ji's OAM from straight Wilson line Ji, Xiong, Yuan (2012)



Jaffe-Manohar

[arXiv.1901.00843](https://arxiv.org/abs/1901.00843)

# Scientific goals of EIC



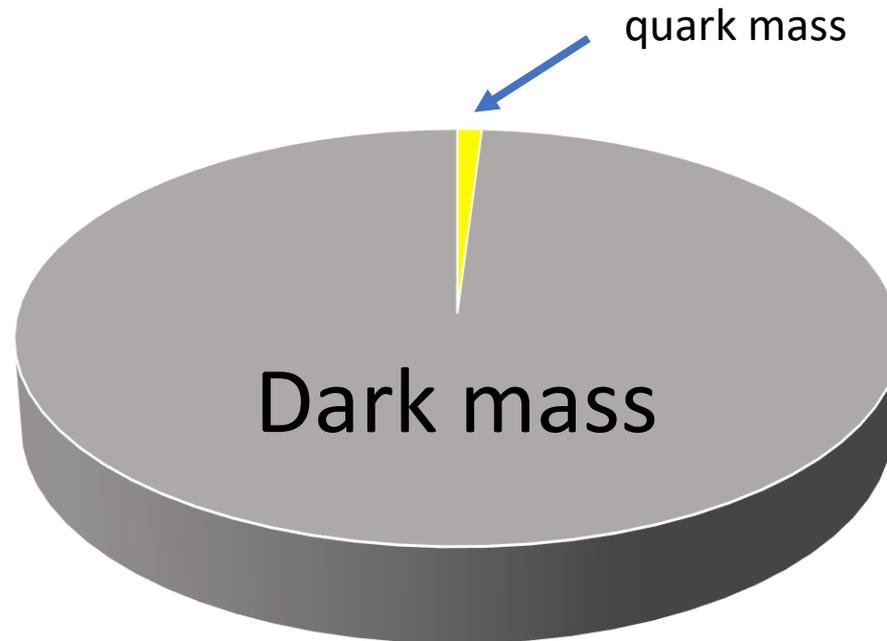
**Finding 1:** An EIC can uniquely address three profound questions about nucleons—protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

NAS report  
(2018/07)

# Proton mass crisis

u,d quark masses add up to  $\sim 10\text{MeV}$ , only 1 % of the proton mass!



Higgs mechanism explains quark masses, but not hadron masses!

# The trace anomaly

QCD Lagrangian approximately scale (conformal) invariant.

Why is the proton mass nonvanishing in the first place?

Conformal symmetry is explicitly broken by the **trace anomaly**.

QCD energy-momentum tensor

$$T^{\mu\nu} = -F^{\mu\lambda}F^\nu{}_\lambda + \frac{\eta^{\mu\nu}}{4}F^2 + i\bar{q}\gamma^{(\mu}D^{\nu)}q$$

$$T^\mu{}_\mu = \frac{\beta(g)}{2g}F^2 + m(1 + \gamma_m(g))\bar{q}q$$

$$\langle P|T^\mu{}_\mu|P\rangle = 2M^2$$

Can we measure the trace anomaly  $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$  ?

The operator  $F^{\mu\nu}F_{\mu\nu}$  is twist-**four**,  
highly suppressed in high energy scattering.

Purely gluonic operator, very difficult to compute in lattice QCD

Instead, we should look at **low**-energy scattering.

Purely gluonic operator. Use **quarkonium** as a probe.

Luke-Manohar-Savage (1992)

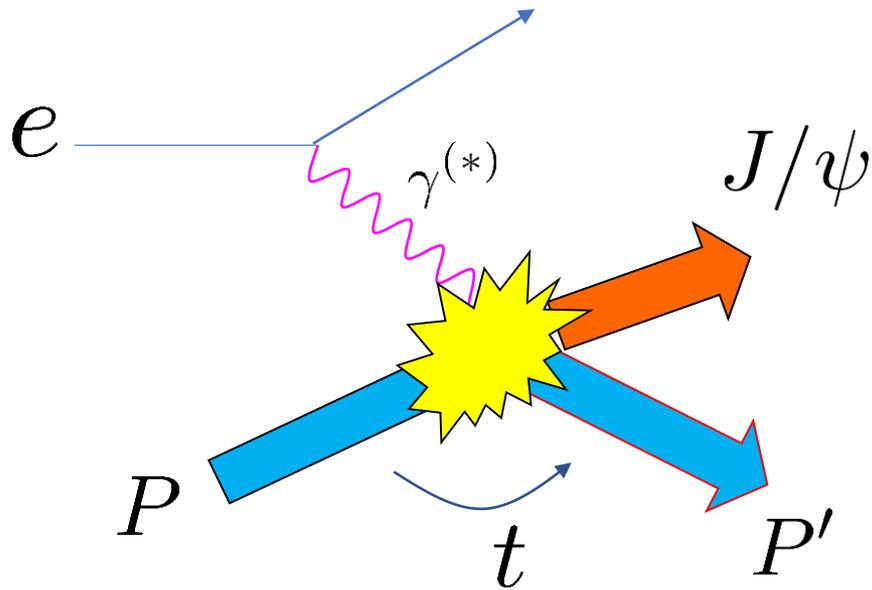
→  $J/\psi$  photo-production near threshold.

# Photo-production of $J/\psi$ near threshold

Kharzeev, Satz, Syamtomov, Zinovjev (1998)

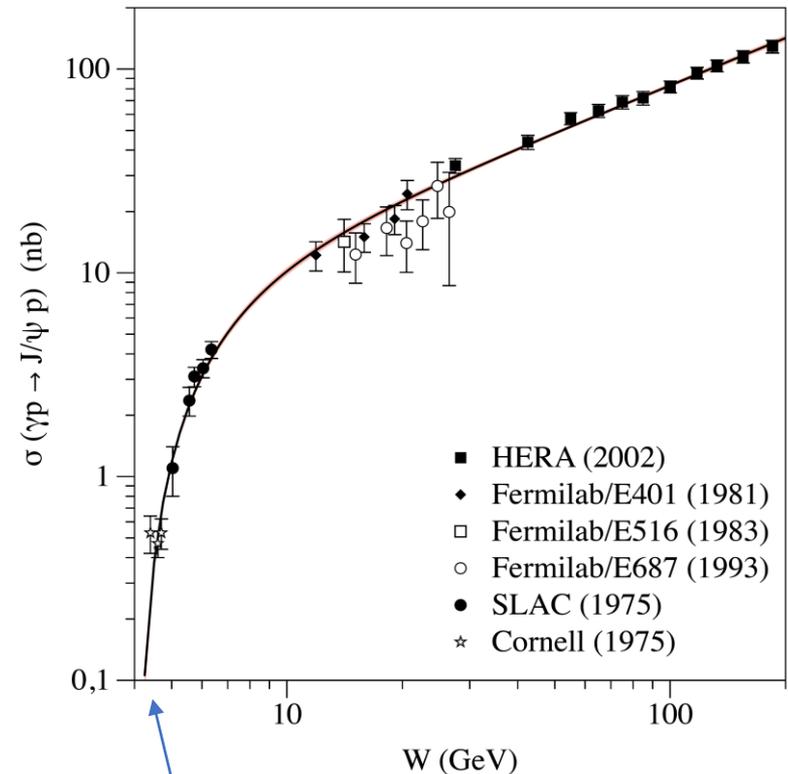
Brodsky, Chudakov, Hoyer, Laget (2000)

Sensitive to the matrix element  $\langle P' | F^{\mu\nu} F_{\mu\nu} | P \rangle$



Straightforward to measure.  
Ongoing experiments at Jlab.

Difficult to compute from first principles  
(need nonperturbative approaches)

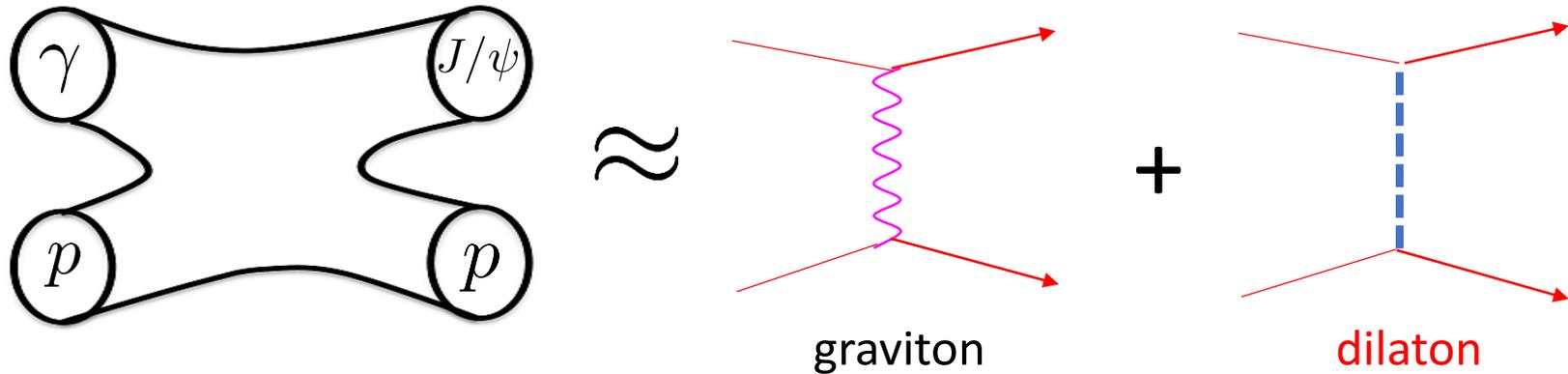


$W_{th} \approx 4.04 \text{ GeV}$

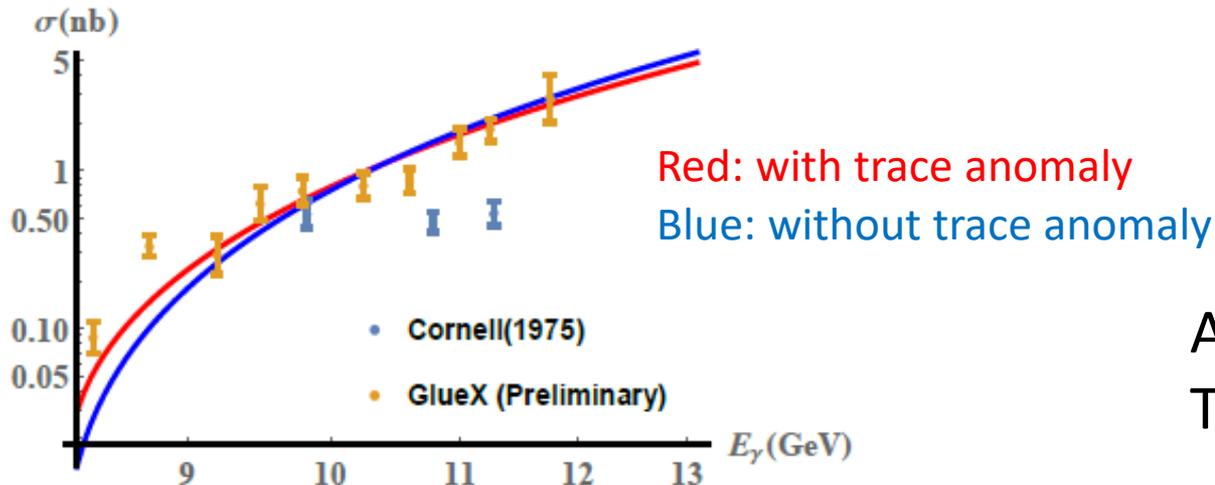
# Holographic approach

YH, Yang (2018)

The operator  $F^{\mu\nu} F_{\mu\nu}$  is dual to a massless string called **dilaton**



Suppressed compared to graviton exchange at high energy, but **not** at very low energy!



At EIC, use  $\Upsilon$  instead.  
The heavier, the better.

# Conclusion

- EIC will significantly advance our knowledge of the nucleon/nuclei, the fundamental building blocks of the universe.
- Topics not covered include: jets, EMC, short-range correlation, UPC, heavy-quark, etc. etc.
- Great opportunity for lattice QCD. PDF, TMD, GPD, Wigner, DA, form factors,...tons of observables to calculate!