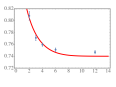

Parton Distributions in Euclidean Space

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BNL Physics Department Seminar
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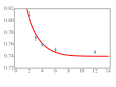
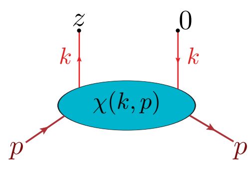


Parton Distributions

- Experimentally, we work with hadrons
- Theoretically, we works with quarks

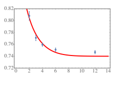
Hadron to Parton Transition



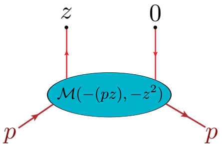
- Can be described in coordinate or momentum space

$$\langle p|\phi(0)\phi(z)|p\rangle = \frac{1}{\pi^2}\int d^4k\, e^{-ikz}\, \chi(k,p)$$

- Concept of PDFs does not rely on spin complications



Ioffe-time distributions



- Basic matrix element (ignoring spin)

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz), -z^2)$$

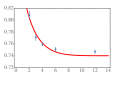
- Lorentz invariance: $\mathcal{M}$ depends on z through $(pz) \equiv -\nu$ and z^2

- Ioffe time ν: $\mathcal{M}(\nu, -z^2)$ = Ioffe-time pseudo-distribution (pseudo-ITD)
- Pseudo $\equiv$ off the light cone
- Using Schwinger's α-representation, it is possible to show that, for any contributing Feynman diagram, for arbitrary z^2 and arbitrary p^2

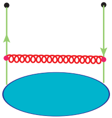
$$\mathcal{M}(\nu, -z^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, \mathcal{P}(x, -z^2)$$

- Limits $-1 \leq x \leq 1$, negative x correspond to anti-particles
- Pseudo-PDF $\mathcal{P}(x, -z^2)$: Fourier transform of pseudo-ITD with respect to ν for fixed z^2
- On the light cone $z_+ = 0$: usual ITD $\mathcal{I}(\nu)$ (with $\nu = p_+ z_-$) and usual PDF $f(x)$

$$\mathcal{I}(\nu) = \int_{-1}^{1} dx\, e^{ix\nu}\, f(x)$$



Logarithmic singularities and evolution



- In QCD $\mathcal{M}(\nu, z^2)$ has logarithmic singularity $\ln(-z^2)$
- At one loop,

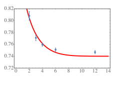
$$\mathcal{M}^{\text{hard}}(\nu, z^2) = -\frac{\alpha_s}{2\pi} C_F \ln(-z^2) \int_0^1 du\, B(u)\, \mathcal{M}^{\text{soft}}(u\nu, 0)$$

- Generates perturbative evolution with Altarelli-Parisi (AP) evolution kernel

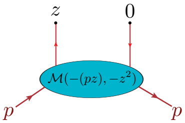
$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

- Since $z^2 \to 0$ limit is singular, regularization (like $\overline{\text{MS}}$) is needed
- Thus, $\mathcal{P}(x, 0) \to f(x, \mu^2)$, and we have $\overline{\text{MS}}$ ITD $\mathcal{I}(\nu, \mu^2)$

$$\mathcal{M}(\nu, 0)|_{\mu^2} \equiv \mathcal{I}(\nu, \mu^2) = \int_{-1}^1 dx\, e^{ix\nu} f(x, \mu^2)$$



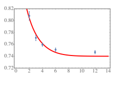
Pseudo-PDF strategy

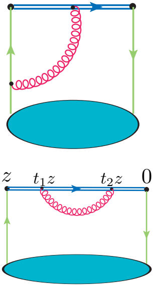


- But we do not have lightlike separations on lattice!
- Take $z = (0, 0, 0, z_3)$ (Ji,2013)
- Then $-(pz) \equiv \nu = p_3 z_3$ and $-z^2 = z_3^2$
- Observation: it does not matter if ν was obtained as $p_+ z_-$ or as $p_3 z_3$

- "Pseudo-PDF strategy: extract $\mathcal{M}(\nu, z_3^2)$ from lattice and "extrapolate" to $z_3^2 \to 0$ limit
- Implemented by "matching" between "z_3^2" and $\overline{\rm MS}$ schemes
- Basically, matching converts $\mathcal{M}(\nu, z_3^2)$ into $\mathcal{I}(\nu, \mu^2)$, i.e. z_3^2-dependence of $\mathcal{M}(\nu, z_3^2)$ into μ^2-dependence of $\mathcal{I}(\nu, \mu^2)$
- Last step: get PDF $f(x, \mu^2)$ by Fourier transformation

$$f(x, \mu^2) = \int_{-1}^{1} dx\, e^{-ix\nu}\, \mathcal{I}(\nu, \mu^2)$$

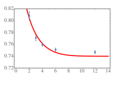




- In QCD, there is one more source of the z^2-dependence: gauge link $\hat{E}(0, z; A)$
- It has specific ultraviolet divergences $\sim z_3/a$, $\ln(1 + z_3^2/a^2)$, with $a \sim$ UV cut-off
- Use Polyakov regularization $1/z^2 \to 1/(z^2 - a^2)$ for gluon propagator in coordinate space
- Effect of the UV cut-off a is similar to that of the lattice spacing
- At one loop, UV singular terms from link self-energy have the structure (Ji et al., 2016)

$$\Gamma_{\rm UV}(z_3, a) \sim -\frac{\alpha_s}{2\pi} C_F \left[2 \frac{|z_3|}{a} \tan^{-1}\left(\frac{|z_3|}{a}\right) - 2 \ln\left(1 + \frac{z_3^2}{a^2}\right)\right]$$

- Vertex corrections produce $\ln\left(1 + z_3^2/a^2\right)$ terms only
- For fixed a, these terms vanish when $z_3 \to 0$
- No correction to local current

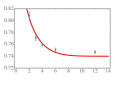


Renormalization

- Link-related UV divergences have the same structure as in HQET
- They are multiplicatively renormalizable
(Qiu et al. , Ji et al. , Green et al. 2017)
- UV regulator a appears only in the combination z_3/a
- UV-sensitive terms form a factor $Z(z_3^2/a^2)$
- This factor is an artifact of having a non-lightlike z
- It has nothing to do with the lightcone PDFs
- We should build modified function $Z^{-1}(z_3^2/a^2)\mathcal{M}(\nu, z_3^2; a)$
- To do this, one should know the $Z(z_3^2/a^2)$ factor
- Easier way out: consider reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)} = \lim_{a \to 0} \frac{\mathcal{M}(\nu, z_3^2; a)}{\mathcal{M}(0, z_3^2; a)}$$

- $Z(z_3^2/a^2)$ factors cancel, and $\mathfrak{M}(\nu, z_3^2)$ has finite $a \to 0$ limit



Reduced Ioffe-time pseudo-distribution

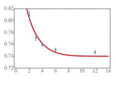
- Reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ is a “physical observable” (like, say, DIS structure functions $F(x, Q^2)$)
- No need to specify renormalization scheme, scale, etc.
- Note: $\nu = 0$ with $z_3 \neq 0$ is obtained by taking $p_3 = 0$
- Still, $\mathcal{M}(0, z_3^2)$ is in perturbative regime as far as z_3^2 is small
- $\mathcal{M}(0, z_3^2)$ is just the zeroth moment of the pseudo-PDF $\mathcal{P}(x, z_3^2)$

$$\mathcal{M}(\nu, z_3^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, \mathcal{P}(x, z_3^2)$$

$$\Rightarrow \quad \mathcal{M}(0, z_3^2) = \int_{-1}^{1} dx\, \mathcal{P}(x, z_3^2)$$

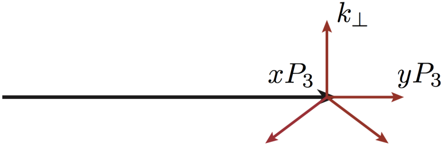
- Matching condition for $\overline{\mathrm{MS}}$ ITD in terms of reduced pseudo-ITD (Y. Zhao 2017, A.R. 2017)

$$\mathcal{I}(\nu, \mu^2) = \mathfrak{M}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{M}(w\nu, z_3^2) \times \left\{ B(w) \left[\ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4\frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}$$

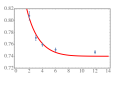


Parton Densities and $P_3 \to \infty$ limit

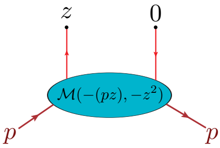
- Original Feynman approach to PDFs $f(x)$: infinite momentum $P_3 \to \infty$ limit of $k_3 = xP_3$ momentum distributions ($\sim$ quasi-PDFs $Q(x, P_3)$)
- $f(x)$ were treated as $k_\perp$-integrated $f(x, k_\perp)$ distributions
- Understood from the start: $Q(x, P_3 \to \infty) \to f(x)$ limit exists only if $f(x, k_\perp)$ rapidly decreases with $k_\perp$
- "Transverse momentum cut-off", $\langle k_\perp^2 \rangle \sim 1/R_{\text{hadr}}^2$
- Question 1: why $Q(x, P_3)$ differs from $f(x)$?
- Question 2: how does $Q(x, P_3)$ convert into $f(x)$ when $P_3 \to \infty$?
- Qualitative answer: yP_3 comes from two sources:
  from the motion of the hadron (xP_3) and
  from Fermi motion of quarks inside the hadron $(y - x)P_3 \sim 1/R_{\text{hadr}}$



- The role of $(y - x)$ part $\sim 1/P_3 R_{\text{hadr}}$ diminishes as P_3 increases



Quasi-PDFs



- Basic matrix element (ignoring spin)

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz), -z^2)$$

- Take $z = (0, 0, 0, z_3)$, then $-(pz) \equiv \nu = Pz_3$ and $-z^2 = z_3^2$
- Introduce quasi-PDF (Ji,2013)

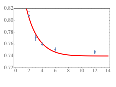
$$Q(y, P) = \frac{P}{2\pi} \int_{-\infty}^{\infty} dz_3\, e^{-iyPz_3}\, \mathcal{M}(Pz_3, z_3^2)$$

- Combine with pseudo-PDF definition

$$\mathcal{M}(Pz_3, z_3^2) = \int_{-1}^{1} dx\, e^{ixPz_3}\, \mathcal{P}(x, z_3^2)$$

- Get quasi-PDF/ pseudo-PDF relation

$$Q(y, P) = \frac{P}{2\pi} \int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3\, e^{-i(y-x)Pz_3}\, \mathcal{P}(x, z_3^2)$$



Factorized model

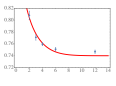
$$Q(y,P) = \frac{P}{2\pi}\int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3\, e^{-i(y-x)Pz_3}\, \mathcal{P}(x, z_3^2)$$

- Observation: qPDF $Q(y,P)$ has $-\infty < y < \infty$ support
- To get a feeling about qPDF shape, try factorized model

$$\mathcal{P}^{\rm fact}(x, z_3^2) = f(x) I(z_3^2)$$

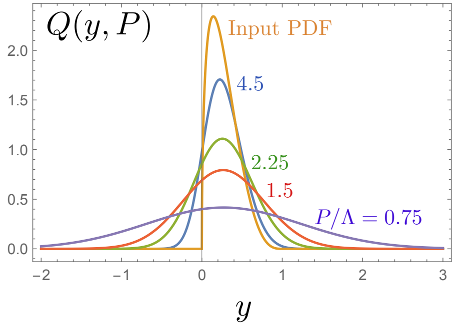
- Popular idea: Gaussian dependence $I(z_3^2) = e^{-z_3^2\Lambda^2/4}$

$$Q_G^{\rm fact}(y,P) = \frac{P}{\Lambda\sqrt{\pi}} \int_{-1}^{1} dx\, f(x)\, e^{-(y-x)P^2/\Lambda^2}$$

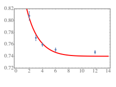


Numerical results for Gaussian model

- Take PDF $f(x) = u_v(x) - d_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3\theta(0 \le x \le 1)$ suggested by results of pseudo-PDF method (Orginos et al. 2017)

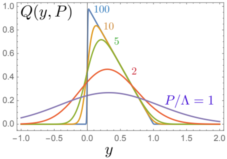


- Curves for $P/\Lambda = 0.75, 1.5, 2.25$ are close to qPDFs obtained by Lin et al (2016), upper momentum $P = 1.3$ GeV, effective $\Lambda \approx 600$ MeV
- Need $P \sim 4.5\,\Lambda \approx 2.7$ GeV to get reasonably close to input PDF
- Note a lot of dirt for negative y, even for $P/\Lambda = 4.5$



Rate of approach

- How do qPDF curves approach limiting PDF curve point by point in y?



- Take a simple input PDF $f(x) = 1 - x$ (and Gaussian dependence on z_3)
- Analytic form. for. quasi-PDF:

$$Q(y,P) = \frac{1}{2}(1-y)\Big[\text{erf}\left[(1-y)P/\Lambda\right] + \text{erf}\left[yP/\Lambda\right]\Big] + \frac{\Lambda}{2\sqrt{\pi}P}\left[e^{-(1-y)^2P^2/\Lambda^2} - e^{-y^2P^2/\Lambda^2}\right]$$

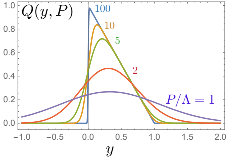
- P-dependence reflects the z_3-dependence of pseudo-PDF
- In the middle of the $0 \le y \le 1$ interval

$$Q(1/2,P) = \frac{1}{2} - \frac{\Lambda e^{-P^2/4\Lambda^2}}{\sqrt{\pi}P}\left[1 - \frac{2\Lambda^2}{P^2} - \dots\right]$$

- The approach to the limiting value is $\sim e^{-P^2/4\Lambda^2}$ rather than a powerlike
- For $y = 1$, the approach is like $\sqrt{\Lambda^2/P^2}$

$$Q(1,P) = \frac{\Lambda}{2\sqrt{\pi}P}\left[1 - e^{-P^2/\Lambda^2}\right]$$

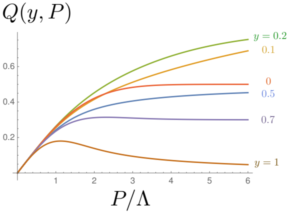
rather than like Λ^2/P^2



Non-analytic behavior with respect to Λ^2/P^2 is present at another end-point $y = 0$ as well

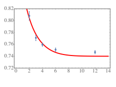
$$Q(0,P) = \frac{1}{2} + \frac{\Lambda}{2\sqrt{\pi}P}\left[1 - 2e^{-P^2/\Lambda^2}\left(1 - \frac{\Lambda^2}{4P^2} - \dots\right)\right]$$

- Quasi-PDF approaches 1/2, average of its 0_+ and 0_- limits of the input PDF



- Curves illustrating P-dependence of quasi-PDFs for particular values of y
- With just three points, at $P/\Lambda = 0.75, 1.5$ and 2.25, it is rather difficult to make an accurate extrapolation to correct $P = \infty$ values

- z_3^2-dependence effects generate a very nontrivial pattern of nonperturbative evolution of the quasi-PDFs $Q(y,P)$
- It cannot be described by a $\mathcal{O}(\Lambda^2/P^2)$ correction on the point-by-point basis in y-variable



$1/P^2$ Expansion

- Look at $1/P^2$ terms using the quasi-PDF to pseudo-PDF relation

$$Q(y,P) = \frac{|P|}{2\pi} \int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3 \, e^{-i(y-x)Pz_3} \, \mathcal{P}(x, z_3^2)$$

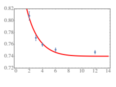
- Expand $\mathcal{P}(x, z_3^2)$ in z_3^2

$$\mathcal{P}(x, z_3^2) = \sum_{l=0}^{\infty} (z_3^2 \Lambda^2)^l \, \mathcal{P}_l(x)$$

- $Q(y,P)$ approaches $f(y)$ like

$$Q(y,P) = f(y) + \sum_{l=1}^{\infty} \left(\frac{\Lambda^2}{P^2} \right)^l \frac{\partial^{2l}}{\partial y^{2l}} \mathcal{P}_l(y)$$

- Support mismatch: $-\infty < y < \infty$ for qPDF $Q(y,P)$, while $\mathcal{P}_l(y)$'s vanish outside $-1 \le y \le 1$
- Do not take this expansion too literally!
- Innocently-looking derivatives of $\mathcal{P}_l(y)$ generate infinite tower of singular functions like $\delta(y)$, $\delta(y \pm 1)$ and their derivatives



Support mismatch

- Recall: even if a function $f(y)$ has a nontrivial support Ω (say, $-1 \leq y \leq 1$), one may formally represent it by a series

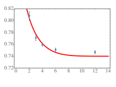
$$f(y) = \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} M_N \, \delta^{(N)}(y)$$

over the functions $\delta^{(N)}(y)$ with an apparent support at one point $y = 0$ only

- M_N are moments of $f(y)$

$$M_N = \int_\Omega dy \, y^N \, f(y)$$

- Lesson: while the difference between $Q(y, P)$ and $f(y)$ is formally given by a series in powers of $1/P^2$, its coefficients are not ordinary functions of y
- Still, one may hope that qPDF converts into lightcone PDF in the $P \to \infty$ limit
- Is this expectation correct?



Hard term and DGLAP evolution of qPDFs

- At one loop in QCD, one has logarithmic singularity in pseudo-ITD $\mathcal{M}(\nu, z_3^2)$

$$\mathcal{M}^{\rm hard}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi} C_F \ln(z_3^2) \int_0^1 du\, B(u)\, \mathcal{M}^{\rm soft}(u\nu, 0)$$

- $B(u) =$ Altarelli-Parisi (AP) evolution kernel
- Recall quasi-PDF definition and use $z_3 = \nu/P$

$$Q(y,P) = \frac{P}{2\pi} \int_{-\infty}^{\infty} dz_3\, e^{-iyPz_3}\, \mathcal{M}(Pz_3, z_3^2) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi}\, e^{-iy\nu}\, \mathcal{M}(\nu, \nu^2/P^2)$$

- Hard part of the function $\mathcal{M}(\nu, \nu^2/P^2)$ that generates the quasi-PDF is

$$\mathcal{M}^{\rm hard}(\nu, \nu^2/P^2) = -\frac{\alpha_s}{2\pi} C_F \ln(\nu^2/P^2) \int_0^1 du\, B(u) \int_{-1}^1 dx\, e^{-iux\nu}\, f^{\rm soft}(x)$$

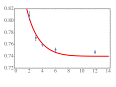
- Hard part of the quasi-PDF $Q(y,P)$ has a $\ln P^2$ term

$$Q^{\rm hard}(y,P) = \ln(P^2)\, \Delta(y) + \ldots$$

- It is nonzero in the $-1 \le y \le 1$ region only

$$\Delta(y) = \frac{\alpha_s}{2\pi} C_F \int_0^1 \frac{du}{u} B(u) f^{\rm soft}(y/u)$$

- The $\ln z_3^2$ singularity of the ITD leads to a logarithmic perturbative evolution of the quasi-PDF $Q(y,P)$ for large P



Hard part of quasi-PDF

- $\ln z_3^2$ asks to be written as $\ln(z_3^2 m^2)$, with m some regulator mass
- Better way is to use, say, $K_0(z_3 m)$ that vanishes for large z_3. Then

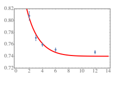
$$Q^{\text{hard}}(y,P) = \int_{-1}^{1} dx \, \frac{\Delta(x)}{\sqrt{(x-y)^2 + m^2/P^2}}$$

- In terms of generating soft PDF

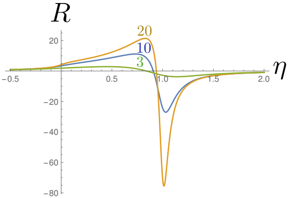
$$Q^{\text{hard}}(y,P) = C_F \frac{\alpha_s}{2\pi} \int_{-1}^{1} \frac{d\xi}{|\xi|} R(y/\xi, m^2/\xi^2 P^2) \, f^{\text{soft}}(\xi)$$

- The kernel $R(\eta, m^2/P^2)$ is generated from the AP kernel $B(u)$

$$R(\eta; m^2/P^2) = \int_0^1 du \, \frac{B(u)}{\sqrt{(\eta-u)^2 + m^2/P^2}}$$



Structure of kernel



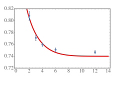
- Kernel for several values of P/m
- Understand m as IR cut-off $\sim 1/R_{\text{hadr}} \sim 0.5$ GeV
- In the $m/P \to 0$ limit

$$\left.\frac{1}{\sqrt{(x-y)^2+m^2/P^2}}\right|_{m^2/P^2\to 0} = \left(\frac{1}{|x-y|}\right)_+ + \delta(x-y)\ln\left[4y(1-y)\frac{P^2}{m^2}\right]$$

- $\delta(x-y)$ gives $\ln P^2$ evolution in $-1 \leq y \leq 1$ region
- Outside $|\eta| \leq 1$ region, limit $m/P \to 0$ is finite

$$R(\eta; 0) = \int_0^1 \frac{du}{|\eta - u|} B(u)$$

- For $|\eta > 1$, kernel can be written as a series in $1/\eta$



Kernel outside central region

$$R(\eta;0)|_{\eta>1} = -\sum_{n=1}^{\infty} \frac{\gamma_n}{\eta^{n+1}} = \frac{1+\eta^2}{\eta-1} \ln\left(\frac{\eta-1}{\eta}\right) + \frac{3}{2(\eta-1)} + 1$$

$$R(\eta;0)|_{\eta<-1} = \sum_{n=1}^{\infty} \frac{\gamma_n}{\eta^{n+1}}$$

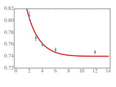
- γ_n are proportional to anomalous dimensions of operators with n derivatives

$$\gamma_n = \int_0^1 du\, u^n\, B(u) = 2\sum_{j=2}^{n+1} \frac{1}{j} + \frac{1}{2} - \frac{1}{(n+1)(n+2)} \,. \qquad (1)$$

- $\gamma_0 = 0$, hence the asymptotic behavior for large $|\eta|$ is

$$R(\eta;0)|_{|\eta|\gg 1} = -\frac{4}{3}\frac{\text{sgn}(\eta)}{\eta^2} + \mathcal{O}(1/\eta^3)$$

- Results in $\sim 1/y^2$ behavior of $Q(y,P)$ for large $|y|$



Subtlety of $P \to \infty$ limit

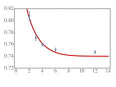
- For $|\eta| > 1$, the kernel has finite $P \to \infty$ limit

$$R(\eta;0)|_{\eta>1} = \frac{1+\eta^2}{\eta-1} \ln\left(\frac{\eta-1}{\eta}\right) + \frac{3}{2(\eta-1)} + 1$$

- Structure of the hard part for large P

$$Q^{\text{hard}}(y,P) = C_F \frac{\alpha_s}{2\pi} \int_{-1}^{1} \frac{d\xi}{|\xi|} R(y/\xi, m^2/\xi^2 P^2)\, f^{\text{soft}}(\xi)$$

- Even when powers of Λ^2/P^2 may be neglected, quasi-PDFs $Q(y,P)$ have P-independent contributions outside the “canonical” $|y| \le 1$ region
- Shape of $Q(y,P)$ for $y > 1$ is calculable (if PDF is known)
- One should see that lattice gives it, and subtract (this is automatically provided by matching conditions)
- Then one gets PDF with $|x| \le 1$ support



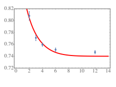
Pseudo-PDF strategy in action

- Exploratory lattice study of reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ for the valence $u_v - d_v$ parton distribution in the nucleon [Orginos et al. 2017]
- Lattice QCD calculations in quenched approximation
- $32^3 \times 64$ lattices, lattice spacing $a = 0.093$ fm
- Pion mass $601(1)$ MeV and nucleon mass $1411(4)$MeV
- Six lattice momenta $p_i\ (2\pi/L)$, with 2.5 GeV maximal momentum
- Real part of lightcone ITD $\mathcal{I}(\nu)$ corresponds to cosine Fourier transform of $q_v(x) = u_v(x) - d_v(x)$

$$\mathcal{R}(\nu) \equiv \mathrm{Re}\,\mathcal{I}(\nu) = \int_0^1 dx\ \cos(\nu x)\, q_v(x)$$

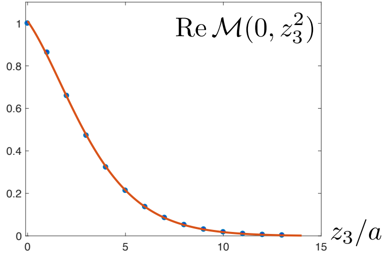
- On the lattice, we extract the reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$$

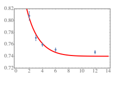


Rest-frame density and Z factor

- Rest-frame density $\mathcal{M}(0, z_3^2)$ is produced by data at $P = 0$
- Results for imaginary part are compatible with zero, as required

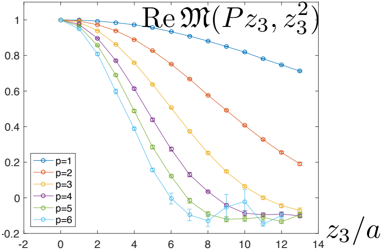


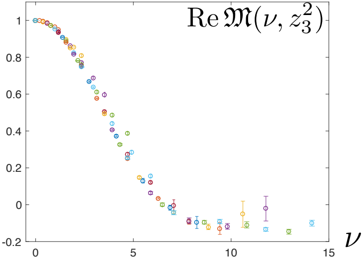
- Visible linear component in $|z_3|$ for small and middle values of $|z_3|$
- Linear exponential factor $Z(z_3^2) \sim e^{-c|z_3|/a}$ is expected
- Generated by straight-line gauge link renormalization



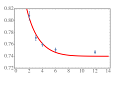
Reduced Ioffe-time distributions

- Left: Real part of the ratio $\mathcal{M}(Pz_3, z_3^2)/\mathcal{M}(0, z_3^2)$ as a function of z_3
- Taken at six values of P $\Rightarrow$ curves have Gaussian-like shape
- $\Rightarrow$ $Z(z_3^2)$ link factor cancels in the ratio





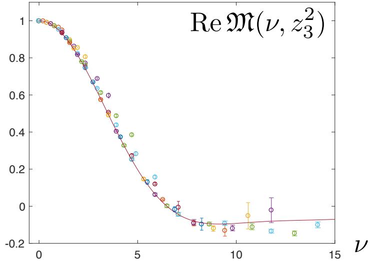
- Right: Same data, as functions of $\nu = Pz_3$ (z_3^2 varies from point to poiint)
- Data practically fall on the same curve



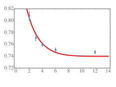
Fitting Real Part

- Data for real part are well described by function

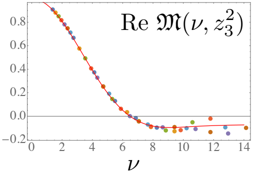
$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$



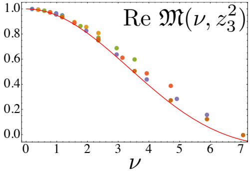
- Obtained by forming cosine Fourier transforms of $x^a(1-x)^b$-type functions and fitting a, b
- Shape is dominated by points with smaller values of Re $\mathfrak{M}(\nu, z_3^2)$



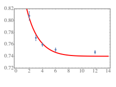
Structure of lattice data



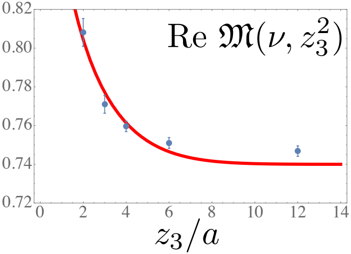
- Points corresponding to $7a \leq z_3 \leq 13a$ values
- Some scatter for points with $\nu \gtrsim 10$
- Otherwise, practically all the points lie on the universal curve based on $f(x)$.
- No z_3-evolution visible in large-z_3 data



- Points in $a \leq z_3 \leq 6a$ region
- All points lie higher than universal curve
- Perturbative evolution increases real part of the pseudo-ITD when z_3 decreases
- Conjecture that the observed higher values of $\mathfrak{R}$ for smaller-z_3 points may be a consequence of evolution

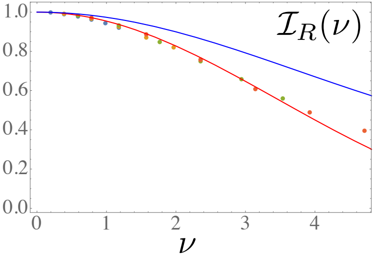


Evolution in lattice data

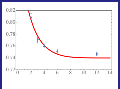


- z_3-dependence of the lattice points for "magic" Ioffe-time value $\nu = 3\pi/4 = 12\frac{\pi}{16}$
- Shape of eye-ball fit line is $\mathrm{const} + \Gamma(0, z_3^2/30a^2)$
- $\Gamma(0, z_3^2/30a^2)$ has "perturbative" $\ln(1/z_3^2)$ behavior for small z_3, rapidly vanishes for $z_3 > 6a$
- $\mathfrak{R}(\nu, z_3^2)$ decreases when z_3 increases
- Data show a logarithmic evolution behavior in small z_3 region
- Starts to visibly deviate from a pure logarithmic $\ln z_3^2$ pattern for $z_3 \gtrsim 5a$
- This sets the boundary $z_3 \leq 4a$ on the "logarithmic region"

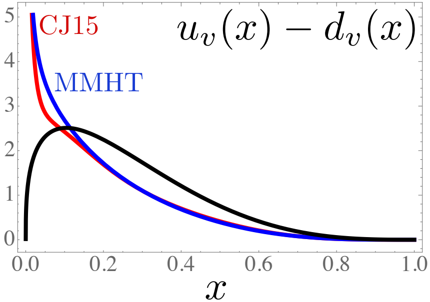
Building $\overline{\text{MS}}$ ITD



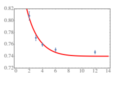
- We choose $\mu = 1/a$ which, at lattice spacing of 0.093 fm is $\approx$ 2.15 GeV
- Using $\alpha_s/\pi = 0.1$ and $z_3 \leq 4a$ data,
  we generate the points for $\mathcal{I}_R(\nu, (1/a)^2)$
- Evolved points are close to some universal curve with a rather small scatter
- The curve itself corresponds to the cosine transform of a normalized $\sim x^a(1-x)^b$ distribution with $a = 0.35$ and $b = 3$
- Upper curve corresponds to the ITD
  of the CJ15 global fit PDF for μ =2.15 GeV



Curves in x space

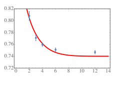


- $\sim x^{0.35}(1-x)^3$ PDF compared to CJ15 and MMHT global fits for $\mu = 2.15$ GeV
- Unable to reproduce $\sim x^{-0.5}$ Regge behavior
- Possible reasons: quenched approximation, large pion mass
- Maybe, a crude way used for doing Fourier transformation



Summary: quasi-PDFs

- Analyzed nonperturbative structure of quasi-PDFs $Q(y, P)$ using their relation to pseudo-ITDs
- Using factorized models for TMDs, studied rate of approach of quasi-PDFs $Q(y, P)$ to PDFs $f(y)$ when $P \to \infty$
- Shown that $(\Lambda^2/P^2)^n$ expansion for $Q(y, P)$ involves generalized functions
- Analyzed perturbative structure of quasi-PDFs using their relation to pseudo-ITDs and TMDs
- Shown that evolution log $\ln z_3^2$ gives $\sim 1/y^2$ behavior of qPDFs $Q(y, P)$ for large y



Summary: pseudo-PDFs

- Formulated algorithm for extraction of light-cone PDF from lattice data
- First step: measure reduced ITD $\mathfrak{M}(\nu, z_3^2) = \mathcal{M}(\nu, z_3^2)/\mathcal{M}(0, z_3^2)$
- No UV renormalization needed
- UV-generated z_3^2-dependence also cancels
- Second step: check that small-z_3^2 behavior has $\ln z_3^2$ structure
- Check the DGLAP evolution equation
- Build $\overline{\rm MS}$ ITD $\mathcal{I}(\nu, \mu^2)$
- Last step: invert Fourier to get $\overline{\rm MS}$ PDF
- Due to the limited range of ν, this is a rather dirty exersize
- Instead, one can skip the last step and just compare the lattice ITD with that obtained from phenomenological fits

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