Overview of proton PDFs and small-x resummation

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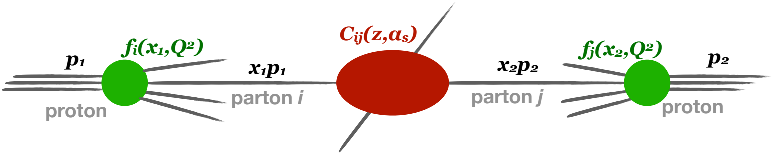
Theoretical predictions with hadrons in the initial state

Collinear factorization theorem in QCD:

$$y = Y - \frac{1}{2}\log\frac{x_1}{x_2}$$

$$\frac{d\sigma}{dQ^2\,dY\,dp_t\ldots} \simeq \sum_{i,j=g,q}\int_0^1 dx_1\,dx_2\, f_i(x_1,Q^2)\, f_j(x_2,Q^2)\, C_{ij}\left(\frac{Q^2}{x_1 x_2 s}, y, p_t, \ldots, \alpha_s(Q^2)\right)$$



- partonic cross sections $C_{ij}(x,\ldots,\alpha_s)$ (observable-dependent, perturbative)
- parton distribution functions (PDFs) $f_i(x,Q^2)$ (universal, non-perturbative)

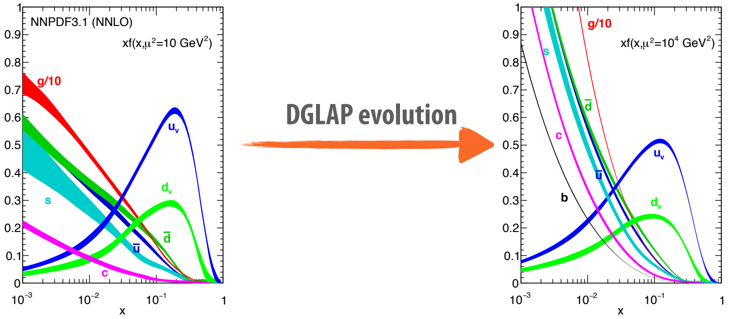
Scale dependence of the PDFs

DGLAP evolution:

$$\mu^2 \frac{d}{d\mu^2} f_i(x,\mu^2) = \sum_{j=g,q} \int_x^1 \frac{dz}{z} P_{ij}(z,\alpha_s(\mu^2)) f_j\left(\frac{x}{z},\mu^2\right)$$

- splitting functions $P_{ij}(x,\alpha_s)$ (universal, perturbative)

PDFs at a given scale μ_0 + DGLAP evolution $\rightarrow$ PDFs at any scale μ



PDF determination from first principles

Non-perturbative problem $\rightarrow$ numerical simulations on a discretized spacetime
However, the field-theoretic definition of PDFs involves light-cone distances:

$$f_q(x, \mu^2) = \int \frac{d\xi}{4\pi} e^{-ix\xi P^+} \langle P | \bar{\psi}_q(\xi) \not{n} U_n(\xi, 0) \psi_q(0) | P \rangle \qquad \xi^2 = 0$$

but in lattice QCD simulation the spacetime is Euclidean, where light-cone separations are only trivial ($\xi = 0$) $\rightarrow$ **PDFs cannot be computed in lattice QCD!**

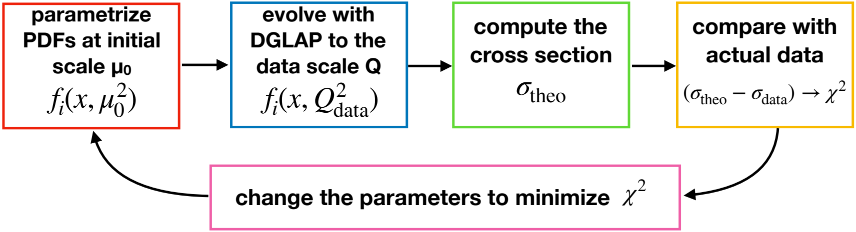
Some possible alternatives:

- compute properties of PDFs (Mellin moments) on the lattice
- compute on the lattice a different object (quasi-PDFs, pseudo-PDFs) which tends to the light-cone PDFs in some limit (some issues though)
- compute scattering amplitudes on the lattice and extract PDFs
- use non-perturbative techniques in the continuum (using e.g. the Bethe-Salpeter equation)

None of these approaches provides sufficient precision for phenomenology today

PDF determination in practice

Strategy: fit $f_i(x, \mu_0^2)$ by comparison with (many) data

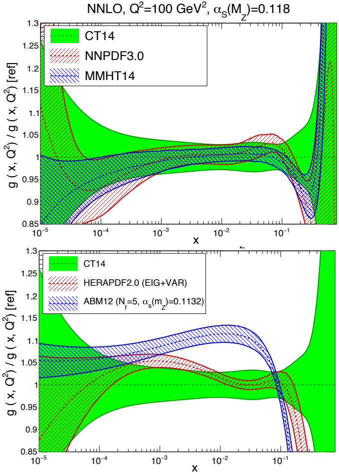


Such fitted PDFs depend unavoidably on the accuracy on the perturbative ingredients $P_{ij}(x, \alpha_s)$, $C_{ij}(x, ..., \alpha_s)$

Various PDF fitting groups (more on LHAPDF):

- CTEQ (**CT** and CJ)
- MRST/MSTW/**MMHT**
- **NNPDF**
- ABM/ABKM/ABMP
- HERAPDF
- xFitter → ATLAS, CMS, ...
- ....

Bold: PDF4LHC recommendation



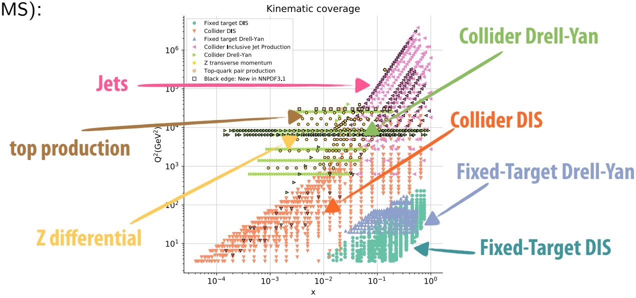
Differences:

- datasets
- theory inputs/assumptions
- PDF parametrization
- fitting methodology
- ....

Consolidated state of the art

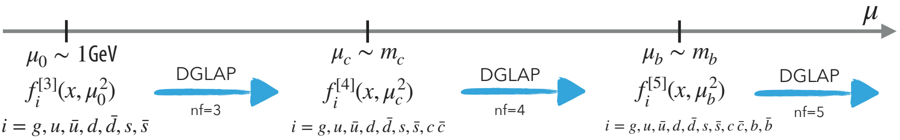
Today most PDF sets are based on the following consolidated ingredients:

- NNLO accuracy in DGLAP evolution and in cross section computation
- variable flavour number scheme (VFNS) with correct mass effects
- flexible PDF parametrizations (with some caveats)
- careful treatment of experimental uncertainties
- a large dataset:
  - HERA (inclusive DIS and heavy flavour production)
  - fixed-target DIS (BCDMS, NMC, NuTeV, ...)
  - fixed-target Drell-Yan (E866, E605)
  - Tevatron Drell-Yan
  - LHC (mostly ATLAS+CMS):
    - Drell-Yan
    - jets
    - $t\bar{t}$ production
    - Z p_t-distribution
    - ....



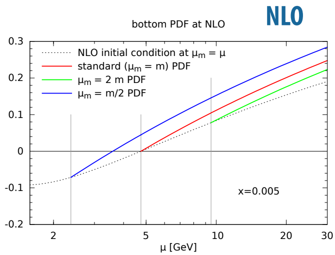
Variable flavour number scheme

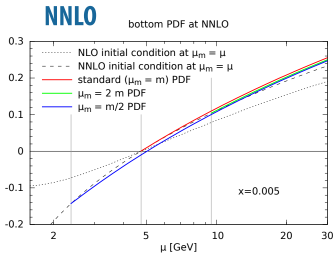
The number n_f of "active" flavours changes during the evolution (factorization scheme choice to resum large collinear logarithms from heavy quark pair production)



Matching relation between PDFs in schemes with different n_f

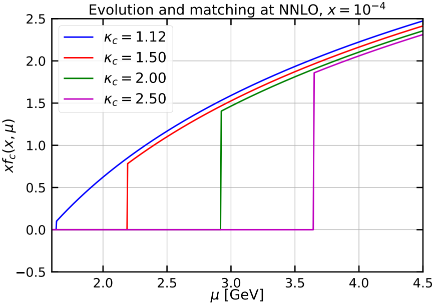
$$f_i^{[n_f+1]}(\mu^2) = \sum_{j=\text{light}} A_{ij}(m^2/\mu^2) \otimes f_j^{[n_f]}(\mu^2) \qquad A_{ij} = \text{perturbative matching coefficients}$$





Matching conditions at the charm threshold

$$\kappa_c = \mu_c/m_c, \qquad \mu_c = \text{charm matching scale (threshold)}$$



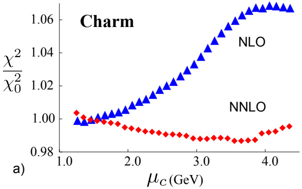
NNLO

The perturbatively generated charm PDF, since the scale is low (thus α_s is large), is affected by large higher order corrections, somehow probed by μ_c variations

recent developments

Improving the charm PDF

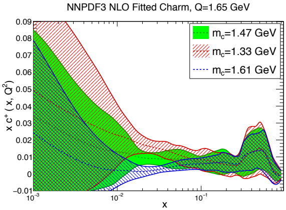
Option 1: use μ_c dependence to improve agreement with data [xFitter 1707.05343]

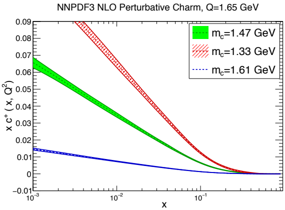


at NNLO $\mu_c \sim 3.5$GeV $\sim 2.5 m_c$ gives the best agreement, but it's not perturbatively stable

one should also vary μ_c to get an uncertainty

Option 2: directly fit the charm PDF from data [NNPDF 1605.06515], default in NNPDF3.1





fitted charm has larger uncertainty but is much less dependent on the charm mass than the perturbatively generated charm

fitted charm may also contain some "intrinsic" component

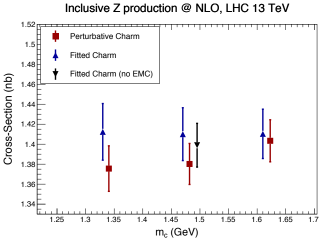
see also [CT 1707.00657]

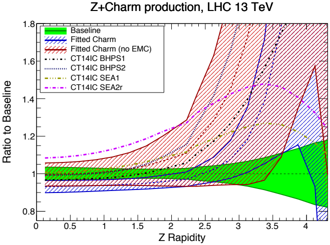
Impact of fitted charm PDF

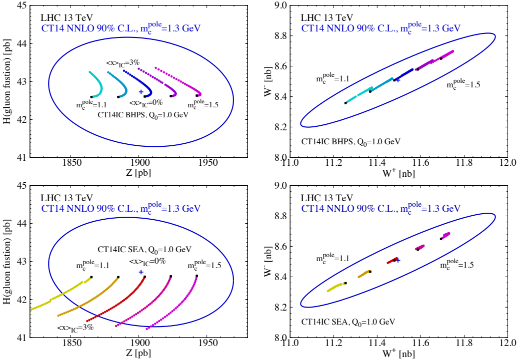
Moderate effect for LHC phenomenology

[CT 1707.00657]

[NNPDF 1605.06515]







processes strongly influenced by the charm PDF in the initial state (like $Z + c$) are more sensitive

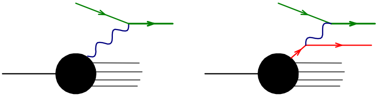
The photon PDF

The proton contains also photons, with probability suppressed by $\alpha_{\rm em}$
However, for percent precision knowing the *photon PDF* is important

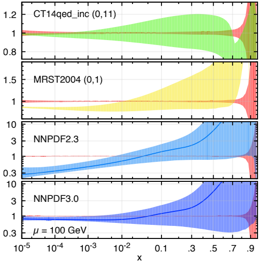
Past: fitting it from data along with the other PDFs (large uncertainties)

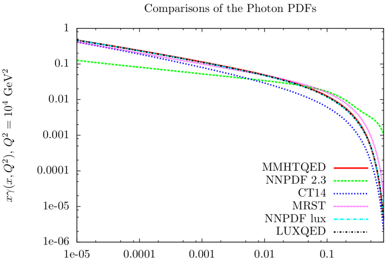
A breakthrough: the LUXqed approach [Manohar,Nason,Salam,Zanderighi 1708.01256]

$$\sigma = C_\gamma \otimes f_\gamma + \alpha_{\rm em} \sum_q C_q \otimes f_q + \ldots$$

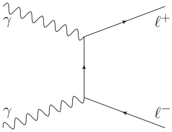


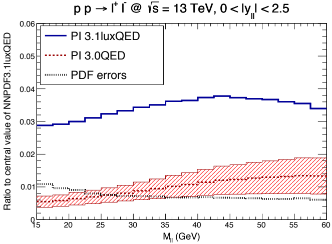
$=$ exact formula in terms of *measured* structure functions

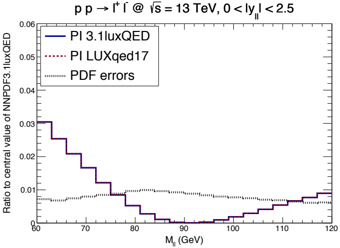




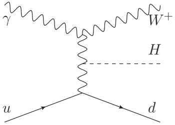
Neutral Drell-Yan production: sizeable effect at low mass

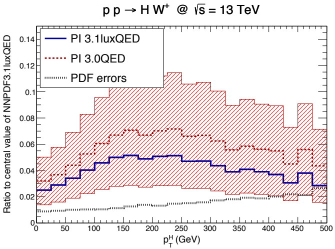






Higgs + $\boldsymbol{W}$ production: sizeable effect up to very high $\boldsymbol{p_t^H}$



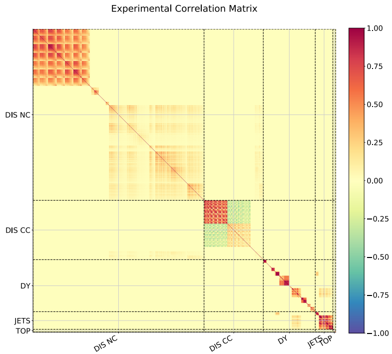


$$\chi^2 = \sum_{i,j=1}^{N_{\text{data}}} (T_i - D_i)(\mathbf{cov}^{-1})_{ij}(T_j - D_j)$$

the covariance matrix $\mathbf{cov}$ usually contains ONLY experimental uncertainties

Theoretical predictions are perturbative $\rightarrow$ **uncertainty from missing higher orders**

Usually estimated through (unphysical) scale variation: μ_R, μ_F
Include such theory uncertainties in the covariance matrix [NNPDF 1905.04311]



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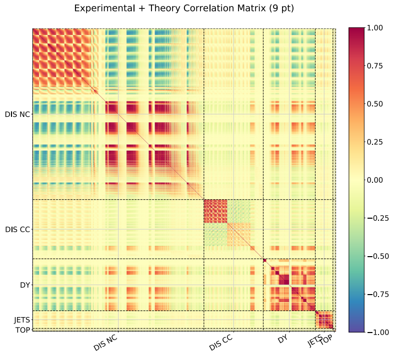
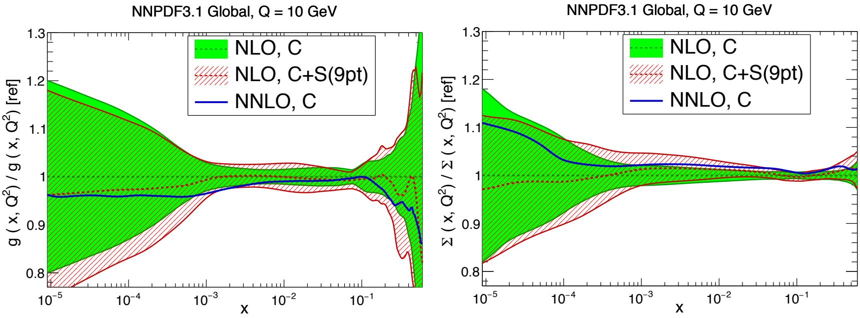


PDFs with theory uncertainties



Including theory uncertainties in the fit leads to (slightly) larger PDF uncertainties

The fitted PDF at next order is contained within the band: good!

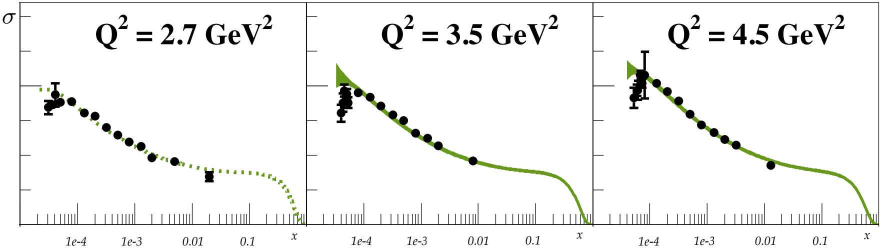
However, varying again the scale when making a prediction using these PDFs can lead to a double counting of the scale variation effect [Harland-Lang,Thorne 1811.08434]

small-x resummation

Low x at HERA: importance of resummation in PDF fits

Deep-inelastic scattering (DIS) data from HERA extend down to $x \sim 3 \times 10^{-5}$

Tension between HERA data at low Q^2 and low x with fixed-order theory



Also leads to a deterioration of the χ^2 when including low-Q^2 data

Attempts to explain this deviation with higher twists, phenomenological models, ...

Successful description of this region when including small-x resummation!

[Ball,Bertone,MB,Marzani,Rojo,Rottoli 1710.05935] [xFitter+MB 1802.00064] [MB,Giuli 1902.11125]

Logarithmic enhancements → all-order resummation

Structure of logarithmically enhanced contributions

$$\text{pert. coeff. } (P, A, C) = a_0 \qquad\qquad L = \log\frac{1}{x}$$
$$+ \alpha_s \left[a_1 L + b_1\right]$$
$$+ \alpha_s^2 \left[a_2 L^2 + b_2 L + c_2\right]$$
$$+ \alpha_s^3 \left[a_3 L^3 + b_3 L^2 + c_3 L + d_3\right]$$
$$+ \alpha_s^4 \left[a_4 L^4 + b_4 L^3 + c_4 L^2 + d_4 L + e_4\right]$$
$$+ \dots$$

If/when $\alpha_s L \sim 1$ the fixed-order expansion is no longer predictive!

Resum the logs, and convert to a "logarithmic-order" expansion:

$$g_{\mathrm{LL}}(\alpha_s L) + \alpha_s\, g_{\mathrm{NLL}}(\alpha_s L) + \alpha_s^2\, g_{\mathrm{NNLL}}(\alpha_s L) + \dots$$

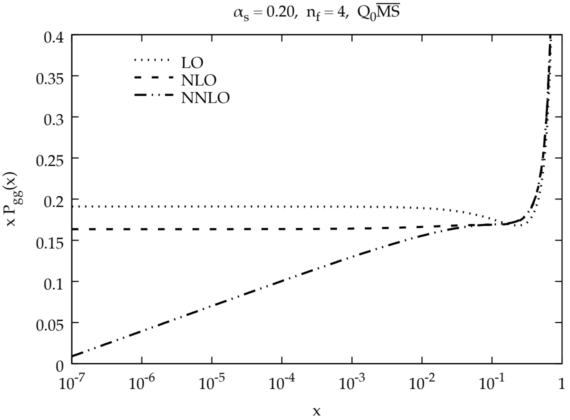
Leading log (LLx), next-to-leading log (NLLx), next-to-next-to-leading log (NNLLx)...

Small-x resummation formalism developed in the 90s-00s
Known at LLx for partonic cross sections and NLLx for DGLAP evolution

[Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto] [Thorne,White] [Altarelli,Ball,Forte]

Example: small-x logarithms in gluon-gluon splitting function

$P_{gg}(x, \alpha_s)$ splitting function at fixed order



Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability**

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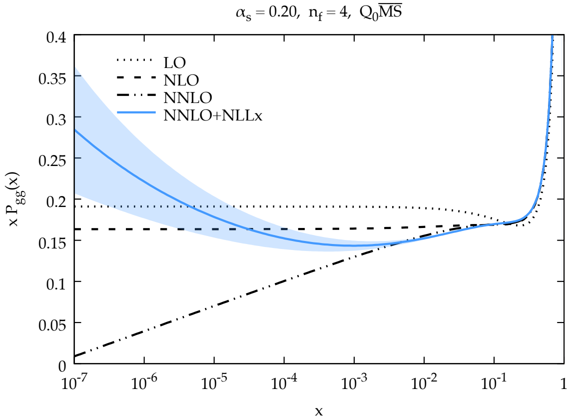
$P_{gg}(x, \alpha_s)$ splitting function at fixed order



Logarithms start to grow for $x \lesssim 10^{-2}$ → **perturbative instability**

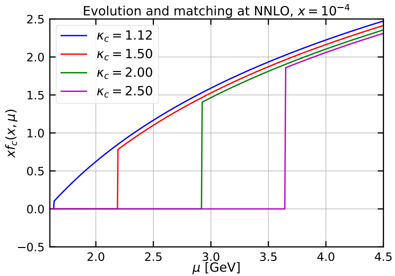
Resummation obtained with the HELL public code

[MB,Marzani,Peraro 1607.02153] [MB,Marzani,Muselli 1708.07510] [MB,Marzani 1805.06460]

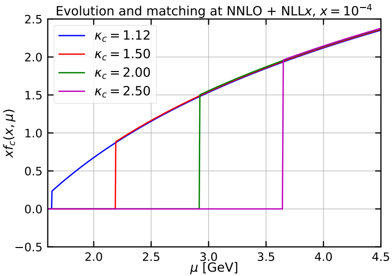
Another example: matching conditions at the charm threshold

$$\kappa_c = \mu_c/m_c, \qquad \mu_c = \text{charm matching scale (threshold)}$$

NNLO



NNLO+NLLx



The perturbatively generated charm PDF is much less dependent on the (unphysical) matching scale when small-x resummation is included!

The first two PDF fits with small-x resummation

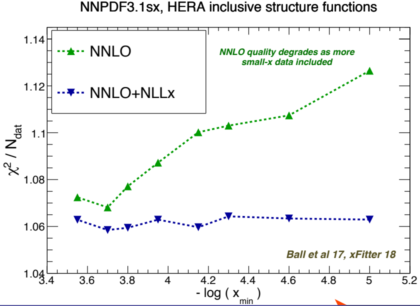
HELL $\rightarrow$ makes possible a PDF fit with small-x resummation

NNPDF3.1sx [1710.05935]	xFitter [1802.00064]
NeuralNet parametrization of PDFs	polynomial paramterization
MonteCarlo uncertainty	Hessian uncertainty
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC ($\sim$ **4000** datapoints)	only HERA data ($\sim$ **1200** datapoints)
NLO, NLO+NLLx, NNLO, NNLO+NLLx	NNLO, NNLO+NLLx

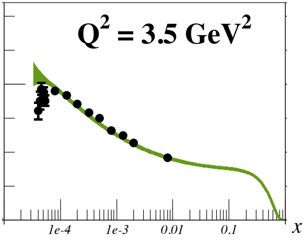
The quality of the fit improves substantially including small-x resummation

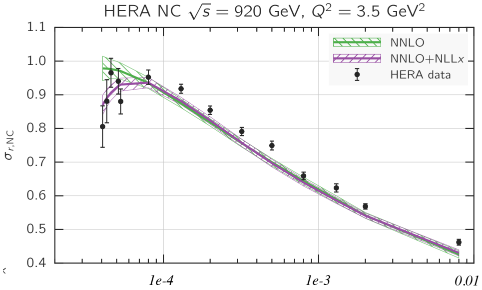
χ^2/N_{dat}	NNLO	NNLO+NLLx
xFitter	**1.23**	**1.17**
NNPDF3.1sx	**1.130**	**1.100**
		smaller!

Stable upon inclusion of low-x data $\rightarrow$



Significantly improved description of the HERA data

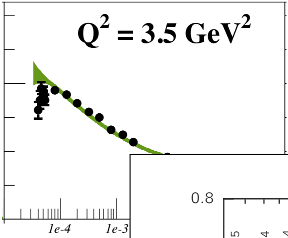




The better description mostly comes from a larger resummed F_L

$$\sigma_{r,\mathrm{NC}} = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2) \qquad y = \frac{Q^2}{x\,s}$$

Significantly improved description of the HERA data



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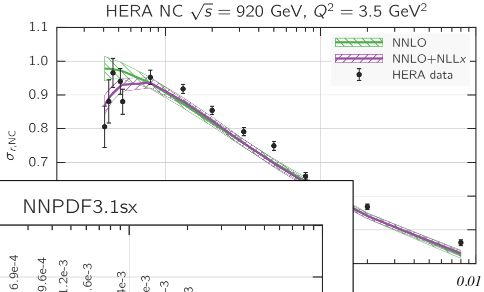
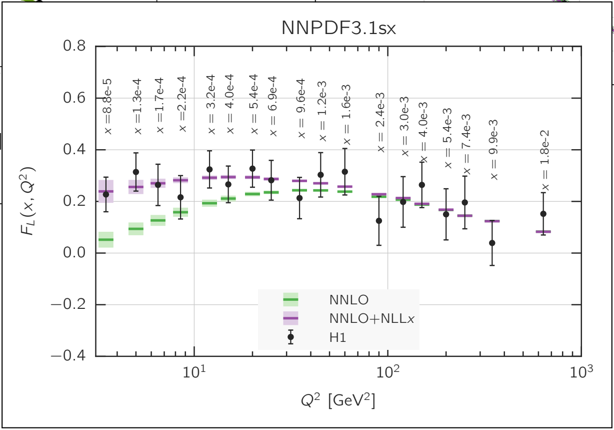




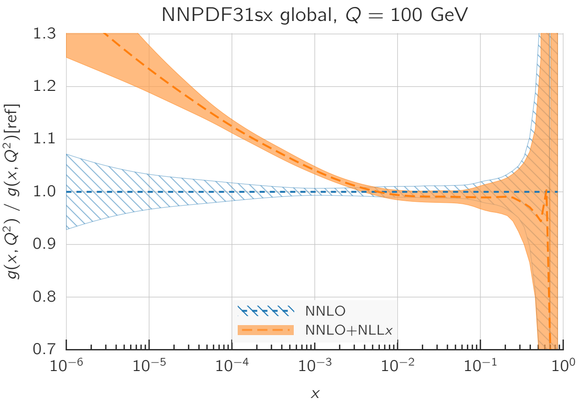
The better d

$\sigma_{r,\mathrm{N}}$

$\frac{Q^2}{x\, s}$

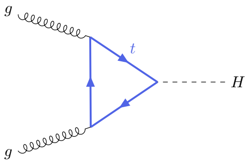
Impact of small-x resummation on PDFs: the gluon

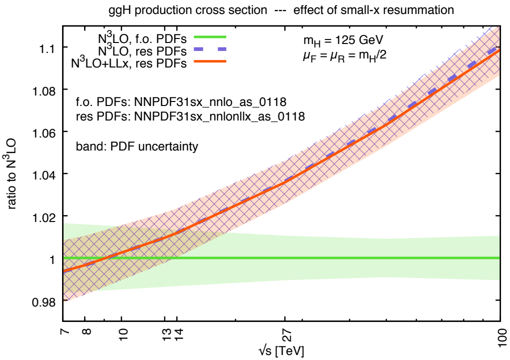
Small-$\boldsymbol{x}$ resummation mostly affects the gluon PDF (and the total quark singlet)



Note: future higher energy colliders will probe smaller values of $\boldsymbol{x}$ $\quad (x_{\min} \sim Q^2/s)$
$\rightarrow$ small-$\boldsymbol{x}$ resummation will be even more important in future!

$gg \to H$ inclusive cross section: [MB,Marzani 1802.07758] [MB 1805.08785]





ggH cross section at FCC-hh $\sim 10\%$ larger than expected!

At LHC $+1\%$ effect; larger effect expected at differential level

Other processes (Drell-Yan, $c\bar{c}$, ...): work in progress (at multi-differential level)

Conclusions

Our study of the structure of the proton keeps progressing and producing very interesting new results

Stay tuned

Backup slides

A digression on the theory of small-x resummation

Small-x resummation formalism based on k_t-factorization and BFKL [Altarelli,Ball,Forte]
Developed in the 90s-00s [Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto] [Thorne,White]

Known at LLx and NLLx since many years, but very limited number of applications until very recently, because small-x resummation is a hell!

Recent developments: [MB,Marzani,Peraro 1607.02153][MB,Marzani,Muselli 1708.07510]

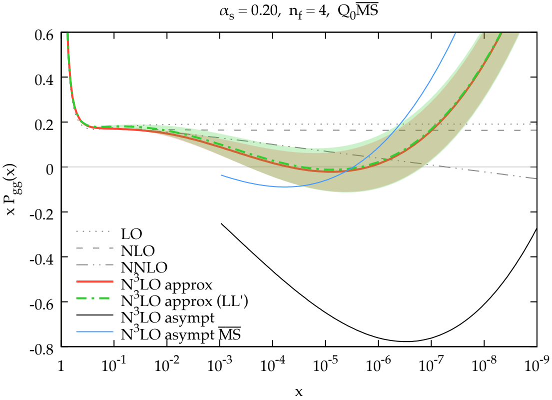
- improved ABF [Altarelli,Ball,Forte 1995,...,2008] procedure to resum splitting functions and new formalism for coefficient functions
- all the ingredients for describing DIS process at small x, including mass effects and heavy flavour matching conditions in DGLAP evolution
- match resummation to NNLO, allowing **NNLO+NLLx phenomenology**
- we released (and keep developing) a public code `HELL: High-Energy Large Logarithms` `www.ge.infn.it/~bonvini/hell` which delivers resummed splitting functions and coefficient functions
- `HELL` interfaced to `APFEL` (`apfel.hepforge.org`) → PDF fits
- matching to N^{3}LO also available [MB,Marzani 1805.06460]
- resummation of LHC observables (Higgs in gluon fusion) [MB 1805.08785]

Towards N^{3}LO evolution

Recent impressive progress towards N^{3}LO splitting functions

[Davies,Vogt,Ruijl,Ueda,Vermaseren 1610.07477] [Moch,Ruijl,Ueda,Vermaseren,Vogt 1707.08315]

At small x, approximate predictions from NLLx resummation [MB,Marzani 1805.06460]

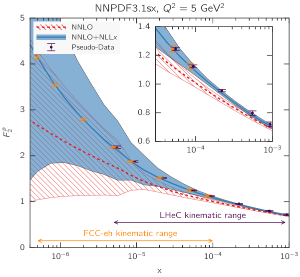


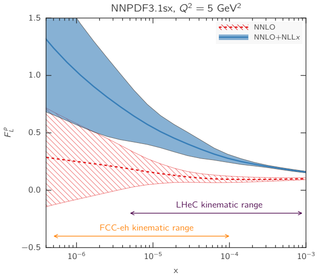
Large uncertainties from subleading logs

N^{3}LO splitting functions are much more unstable at small x → need resummation!

The role of FCC-eh (and LHeC)

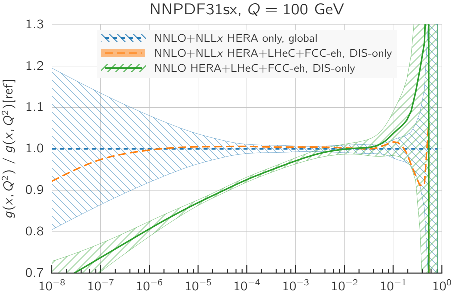
Prediction in the LHeC and FCC-eh kinematic regions for F_2 and F_L





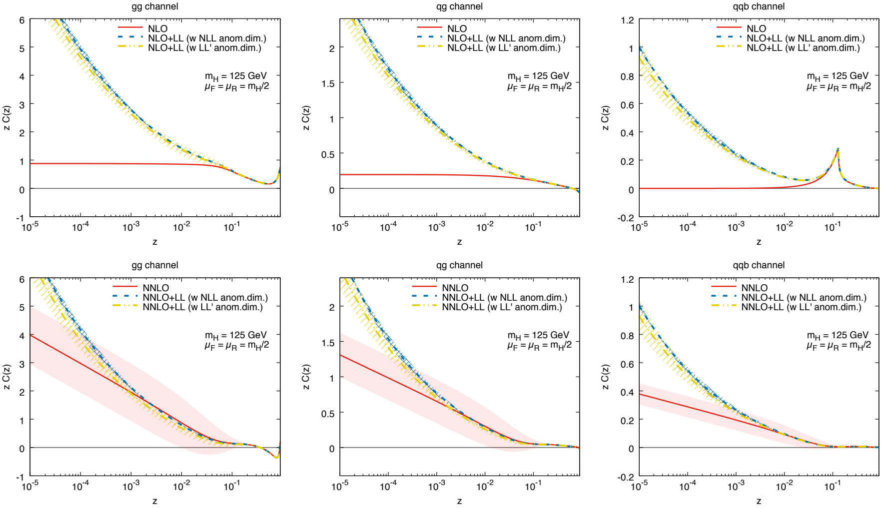
Pseudo data show a small errors: significant constraining power!

Fit to pseudo data shows a significantly reduced uncertainty, and a huge effect of small-x resummation

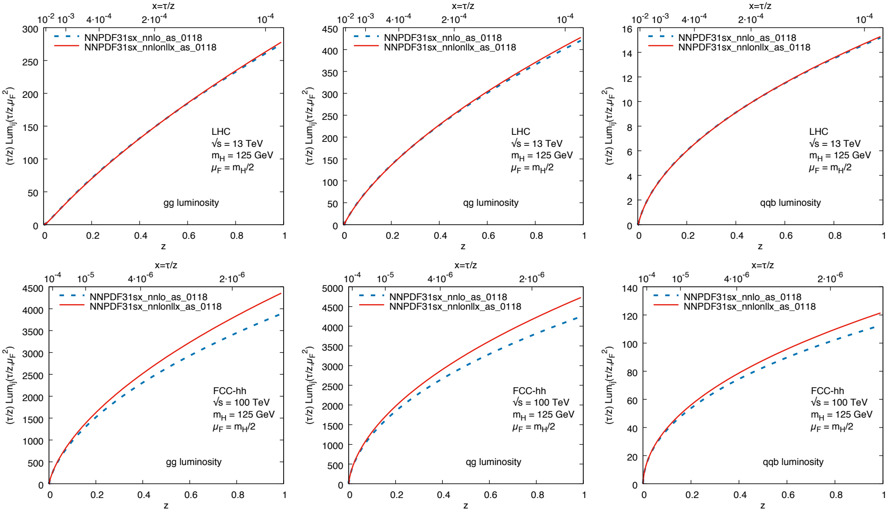


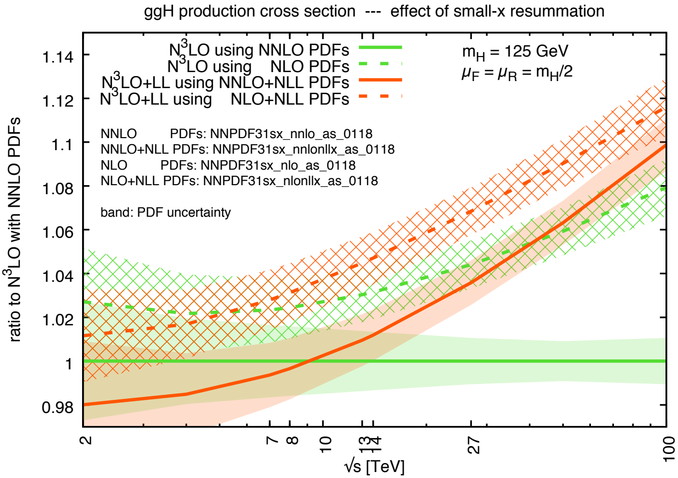
Higgs production: parton-level results

$$\sigma(m_H^2, s) = \sigma_0(m_H^2) \sum_{i,j=g,q} \int_\tau^1 \frac{dz}{z}\, C_{ij}\big(z, \alpha_s(m_H^2)\big)\, \mathscr{L}_{ij}\left(\frac{\tau}{z}, m_H^2\right) \qquad \tau = \frac{m_h^2}{s}$$



$$\sigma(m_H^2, s) = \sigma_0(m_H^2) \sum_{i,j=g,q} \int_\tau^1 \frac{dz}{z} C_{ij}\left(z, \alpha_s(m_H^2)\right) \mathscr{L}_{ij}\left(\frac{\tau}{z}, m_H^2\right) \qquad \tau = \frac{m_h^2}{s}$$





The large effect of the resummation is due to the NNLO being perturbatively unstable at small x

New parametrization in xFitter

Default xFitter parametrization $xf(x) = A\,x^B(1-x)^C\left[1 + Dx + Ex^2\right]$
Not flexible enough at small x! May lead to bias.

Newly proposed parametrization [MB,Giuli 1902.11125]

$$xf(x) = A\,x^B(1-x)^C\left[1 + Dx + Ex^2 + F\log x + G\log^2 x + H\log^3 x\right]$$

much more flexible at small x!

Reduction of the χ^2, both at fixed order and with resummation
Slightly different PDF shapes, well compatible with the (more flexible) NNPDF fit