Parton Physics from Lattice QCD

Phiala Shanahan, MIT

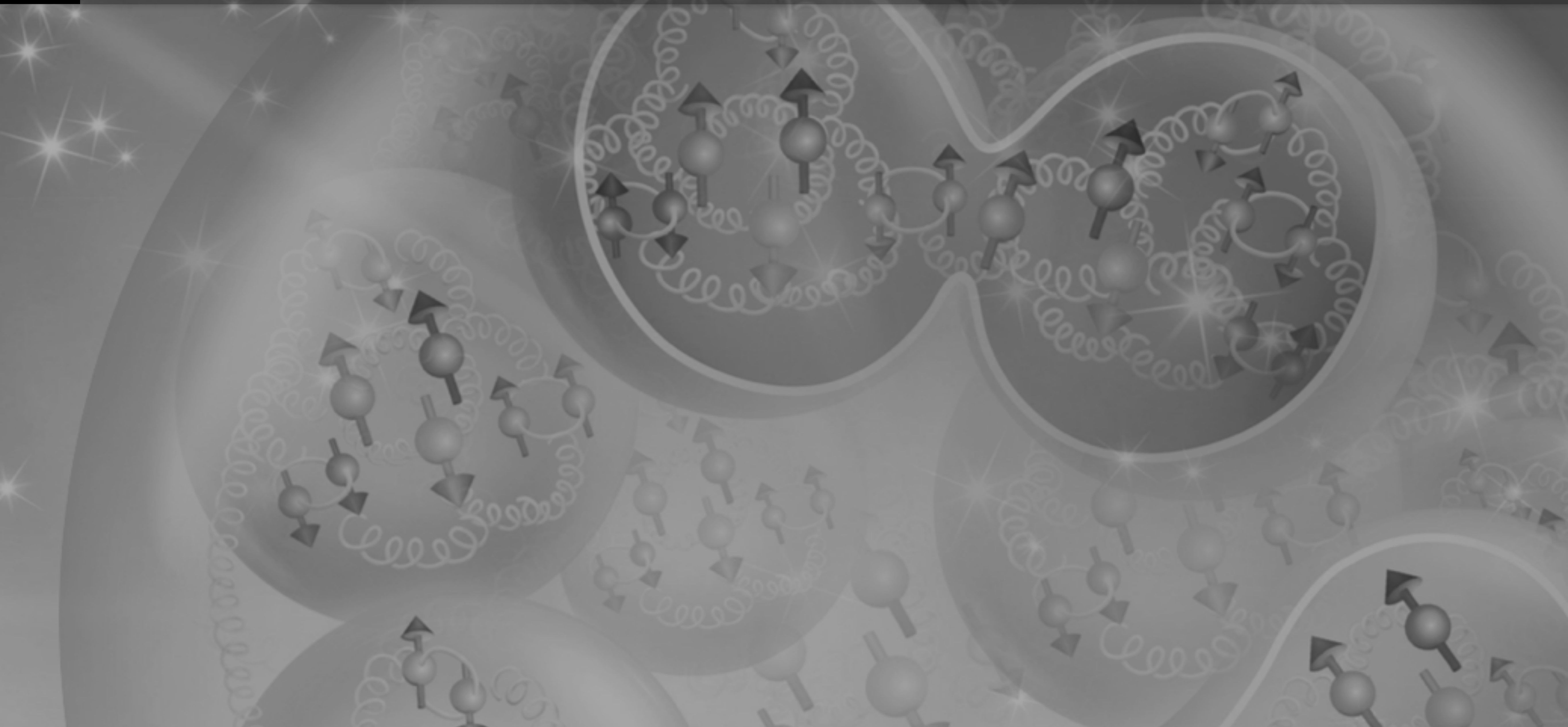
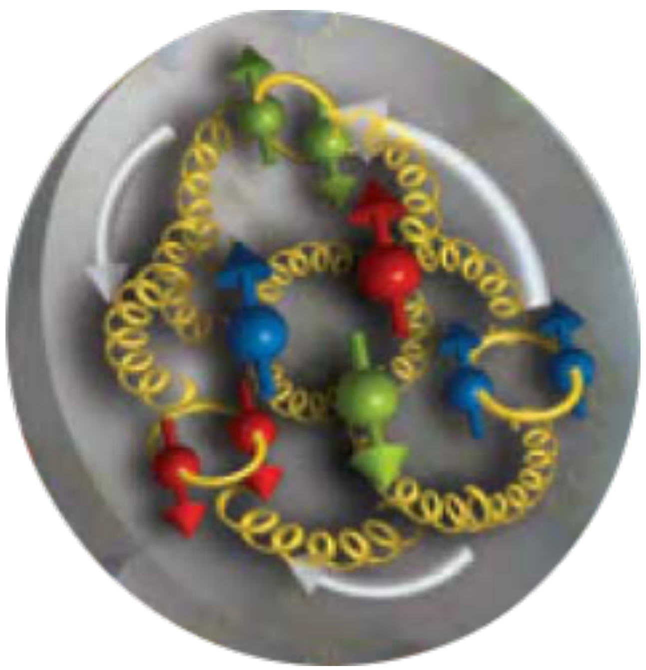


Image Credit: 2018 EIC User's Group Meeting

Massachusetts Institute of Technology

Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter



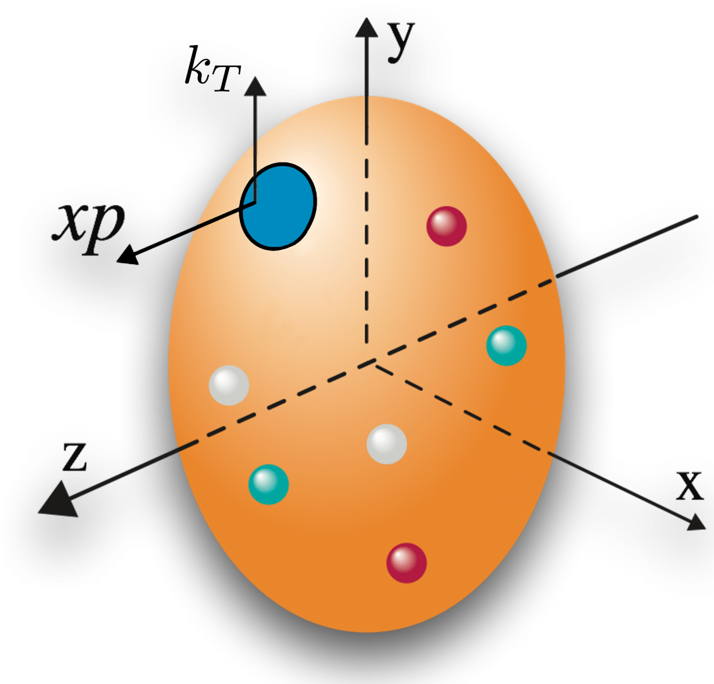
Three-dimensional partonic structure of the proton

How do quarks and gluons carry the proton's

- Mass
- Momentum
- Angular momentum, spin
- Pressure and shear
- ...

Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter



Three-dimensional partonic structure of the proton

PDF
$f_{q/H}(x)$
longitudinal

TMD
$f_{q/H}(x, k_T)$
+ transverse

$f_{q/H}(x, b_T)$
+ impact parameter

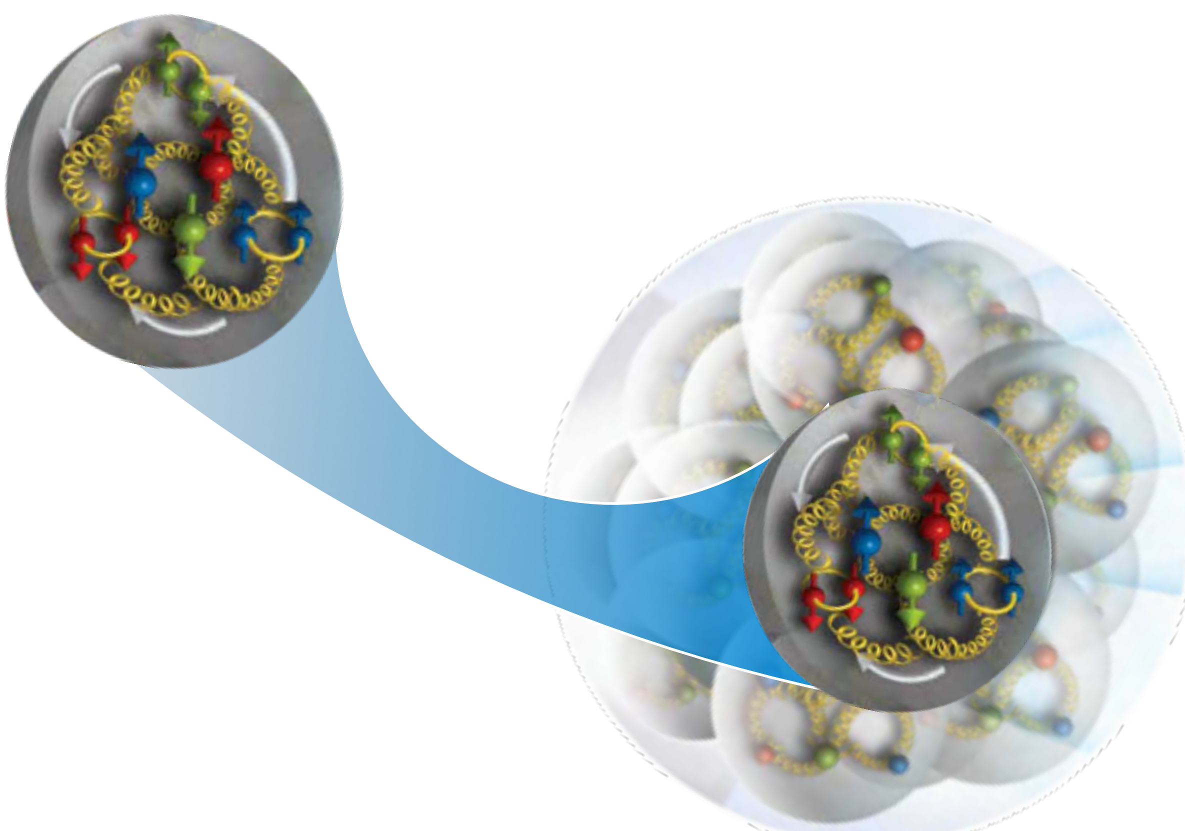
Fourier transform
$\xi = 0$

GPD
$H_{q/H}(x, \xi, \Delta_T^2)$

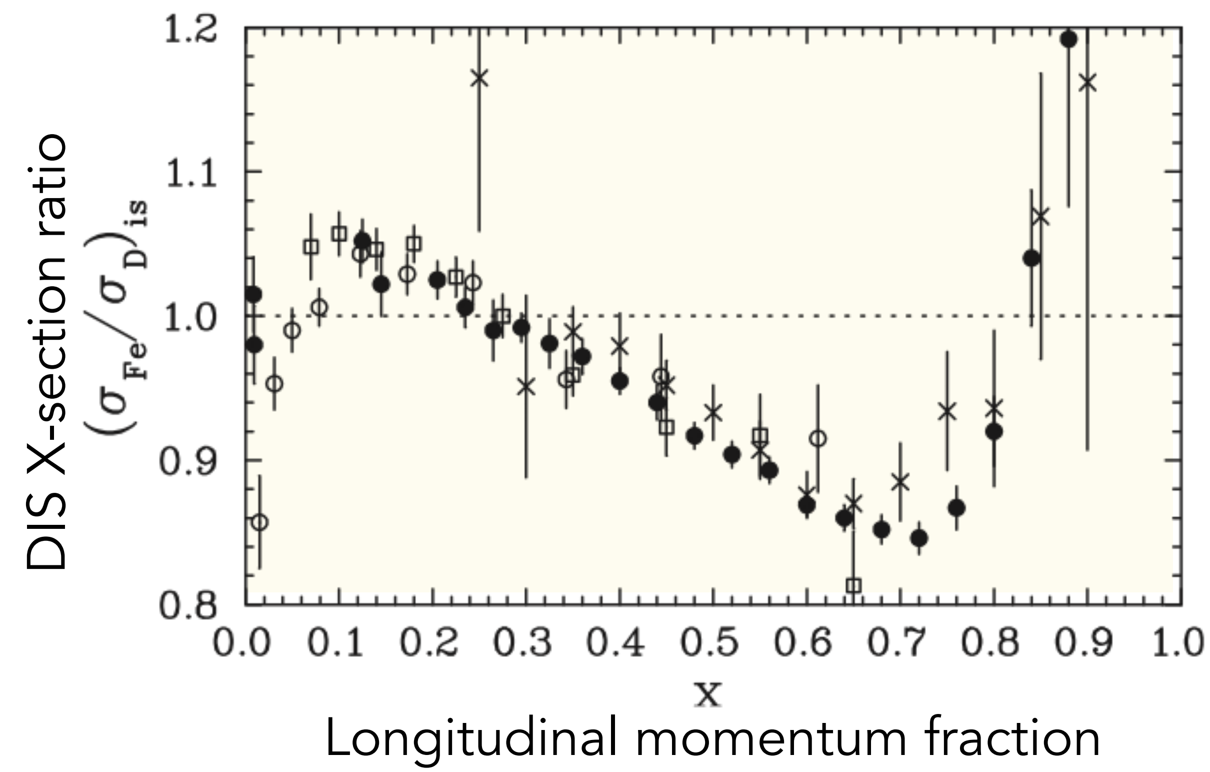
Parton physics from Lattice QCD

Understanding the quark and gluon structure of matter

How is the partonic structure of nucleons modified in nuclei?



Encoded in EMC-type effects

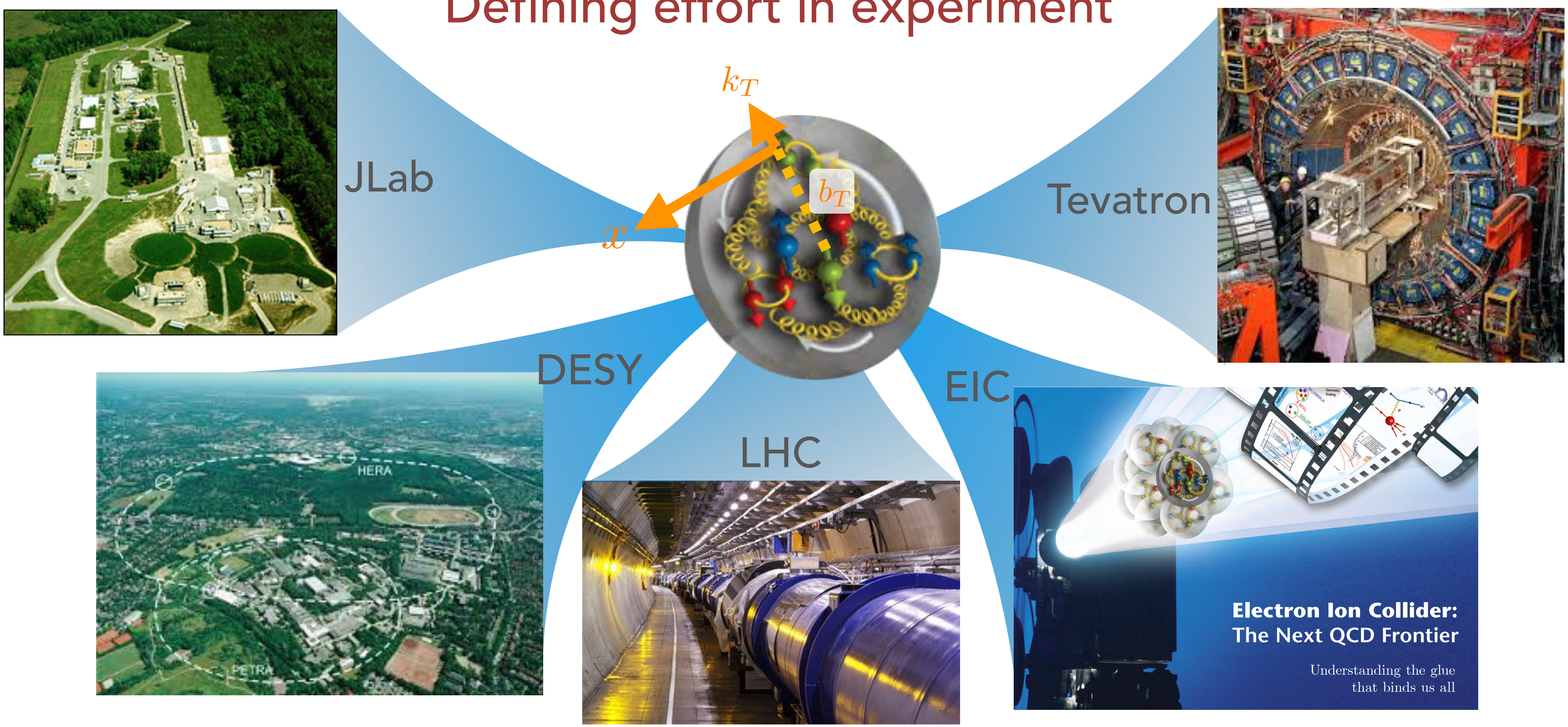


(EMC: Aubert et al., 1983)

Parton physics from Lattice QCD

Understanding the quark and gluon
structure of matter

Defining effort in experiment



Lattice QCD

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Numerical first-principles approach to
non-perturbative QCD
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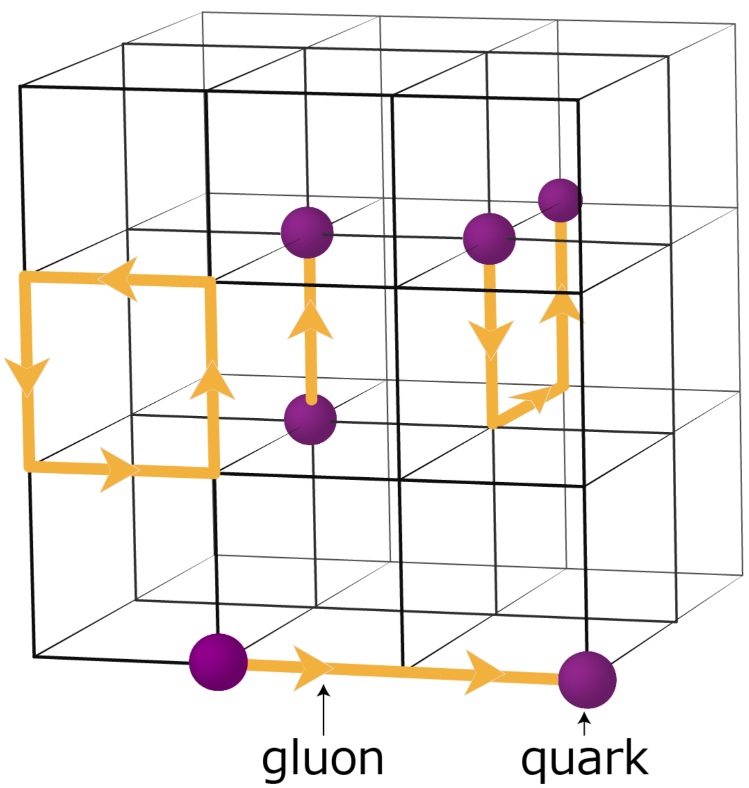
- Discretise QCD onto 4D space-time lattice
- Approximate QCD path integral using Monte-Carlo methods and importance sampling
- Run on supercomputers and dedicated clusters
- Take limit of vanishing discretisation, infinite volume, physical quark masses



Lattice QCD

Numerical first-principles approach to non-perturbative QCD

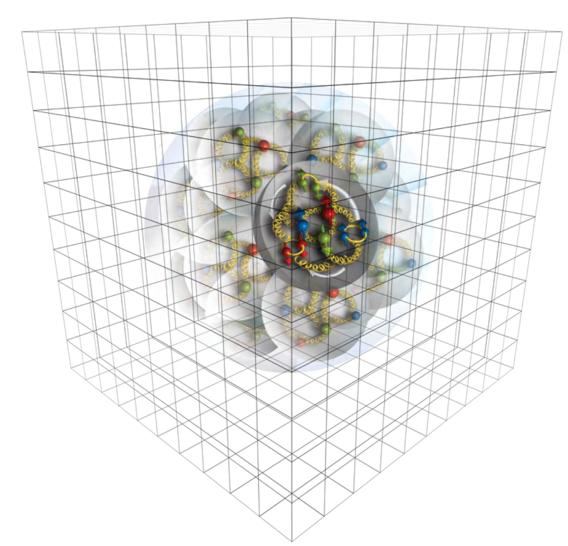
INPUT

Lattice QCD action has same free parameters as QCD: quark masses, α_S

- Fix quark masses by matching to measured hadron masses, e.g., π, K, D_s, B_s for u, d, s, c, b
- One experimental input to fix lattice spacing in GeV (and also α_S), e.g., $2S$-$1S$ splitting in Υ, or f_π or Ω mass

OUTPUT

Calculations of all other quantities are QCD predictions



Lattice QCD

Numerical first-principles approach to non-perturbative QCD

Calculations use world's largest computers

- Many millions of CPU/GPU/KNL hours
- Specifically designed processors for QCD (QCDOC precursor of BlueGene computers)

Parton physics from Lattice QCD

Precision Era

Fully-controlled w/ few-percent errors within ~5y

- Static properties of nucleon incl. spin, flavour decomp.
- Mellin moments of PDFs, GPDs

Early Era

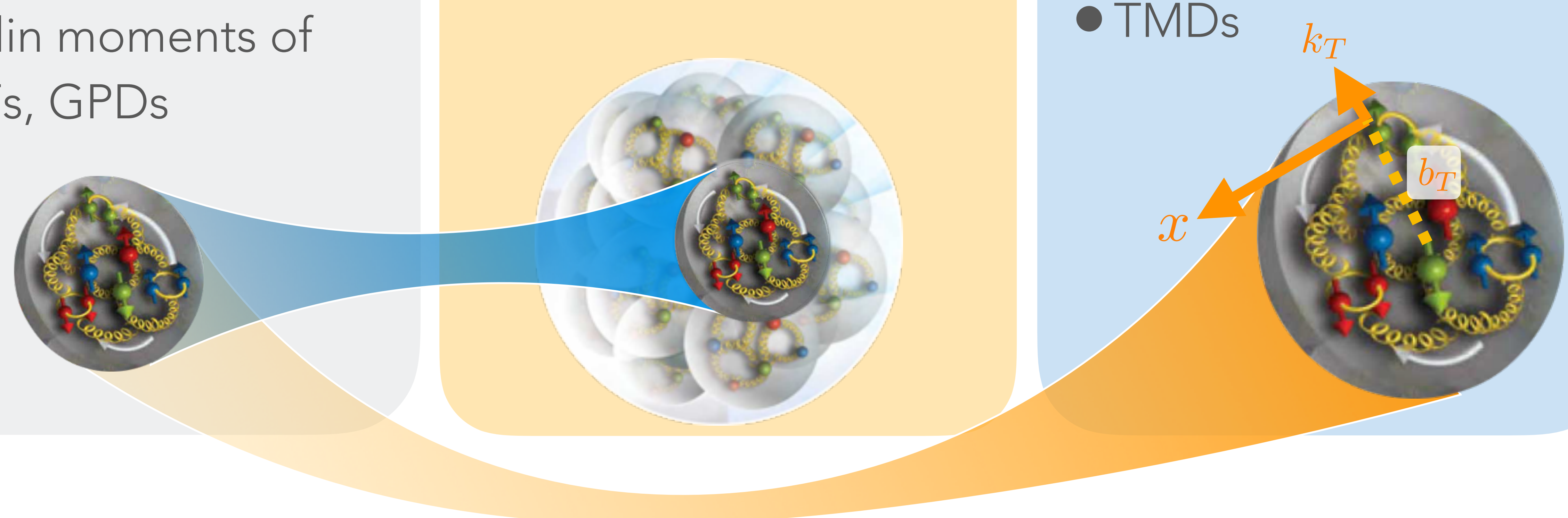
Fully-controlled w/ ~15–percent errors within ~5y

- Nuclear structure A<5
- Spin, flavour decomp. of EMC-type effects

Exploratory Era

First calculations, timeline for controlled calculations unclear

- x-dependence of PDFs
- TMDs



Parton Physics from Lattice QCD

- Mellin moments of PDFs, GPDs
- x-dependence of PDFs
- TMDs and the Collins-Soper evolution kernel
- EMC-type effects in nuclei

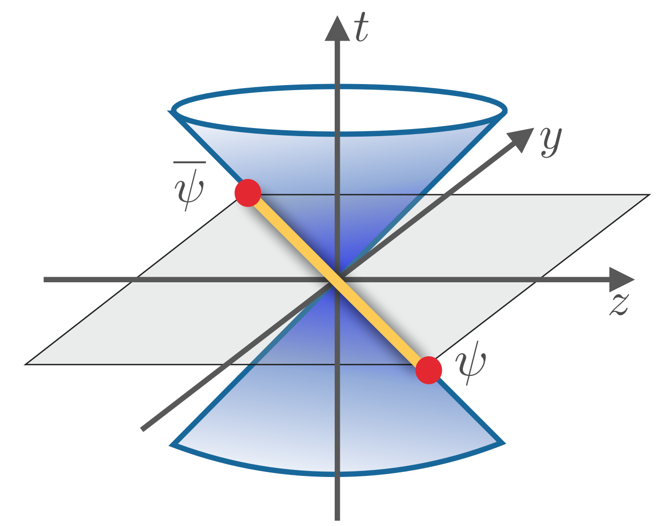
Parton distribution functions

Parton distribution functions $f(x,\mu^2)$

- Non-local light-cone correlations
- Encode non-perturbative physics

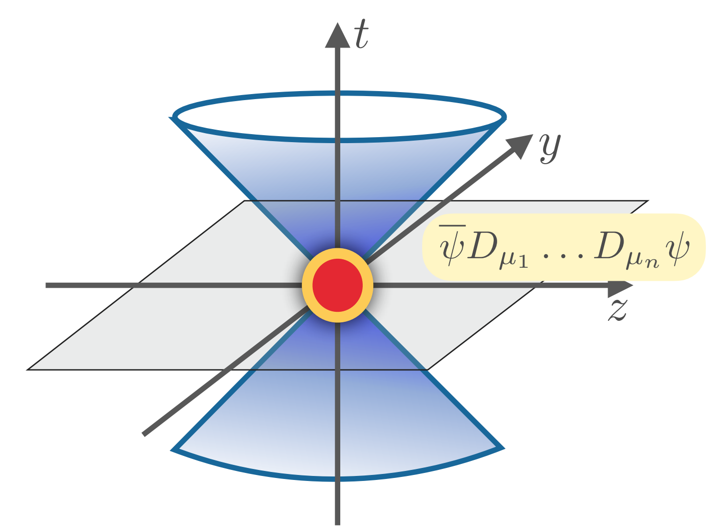
- Correlations at light-like separation not directly accessible in Euclidean-space calculations

$$f(x) = \int \frac{d\xi^-}{2\pi} e^{-2i\xi^-(xP^+)} \langle p|\overline{\psi}_f(\xi^-)\gamma^+ W[\xi^-,-\xi^-]\psi_f(-\xi^-)|p\rangle$$



- Operator Product Expansion relates Mellin moments of PDFs to local operators

$$\langle h|\overline{\psi}_f D_{\mu_1}\dots D_{\mu_n}\psi_f|h\rangle \sim \langle x^n\rangle_f^h = \int_0^1 dx\, x^n f(x)$$



Moments of PDFs

Lattice QCD can cleanly access low moments of PDFs (n ≲ 3)

[work to move beyond: Chambers et al., arXiv:1703.01153, Davoudi & Savage, arXiv:1204.4146]

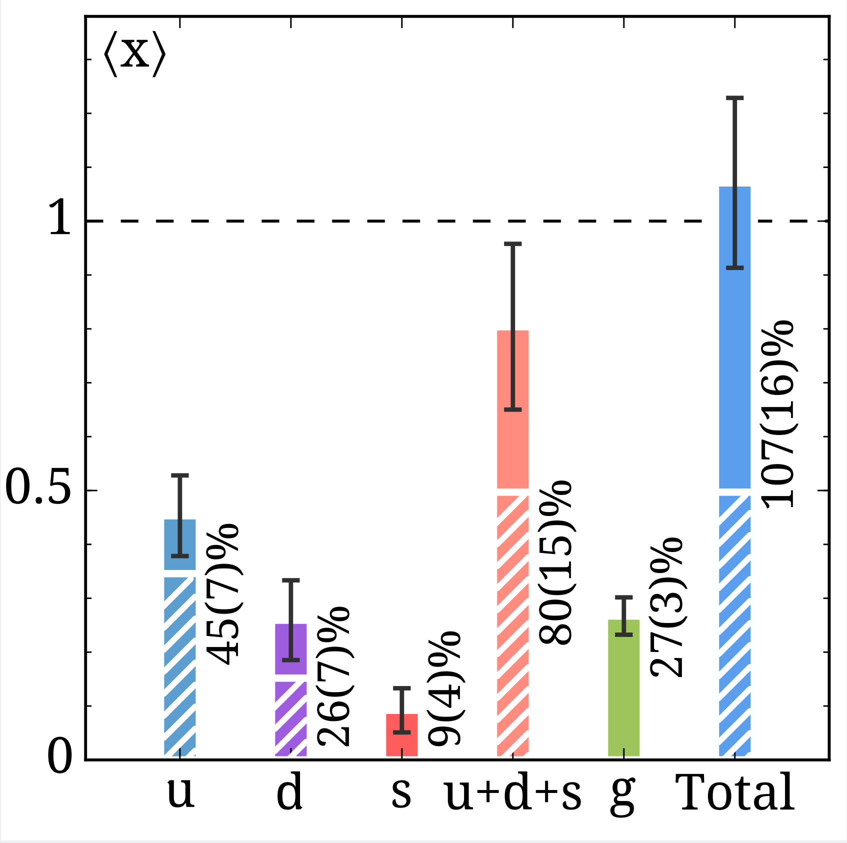
$$\int_0^1 dx\, x^n f(x,\mu^2) = \langle x^n \rangle_f(\mu^2)$$

State-of-the-art calculations have:

- Fully-controlled systematic uncertainties competitive with or better than experiment for some quantities
- Separate contributions from
  - Strangeness and light flavours
  - Charge symmetry violation
  - Gluons

Highlight: All terms of nucleon momentum decomposition calculated with controlled uncertainties

$\overline{\text{MS}}$-scheme at 2 GeV

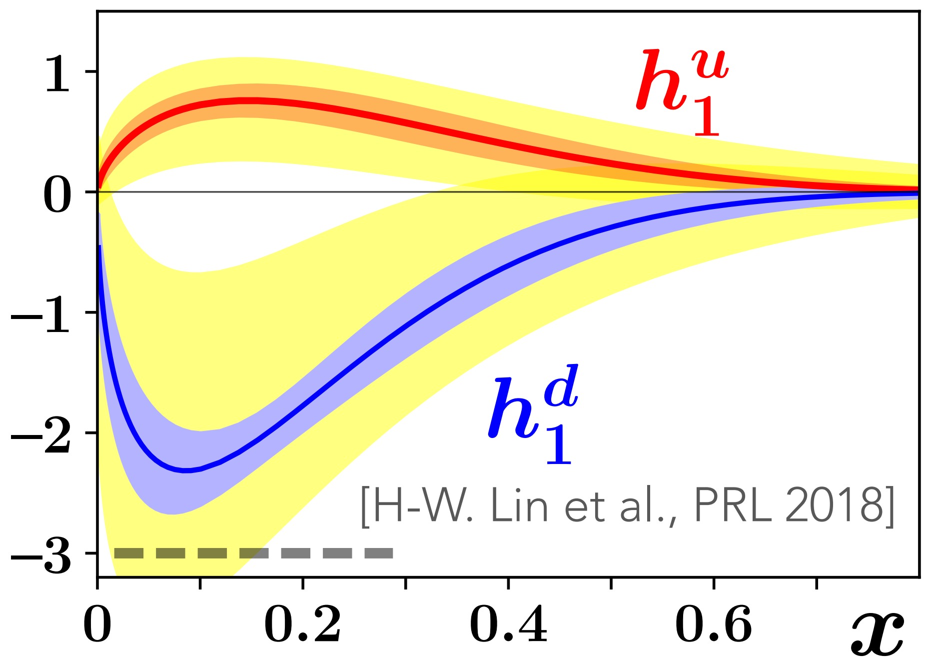


[C. Alexandrou et al., PRL 119 (2017)]

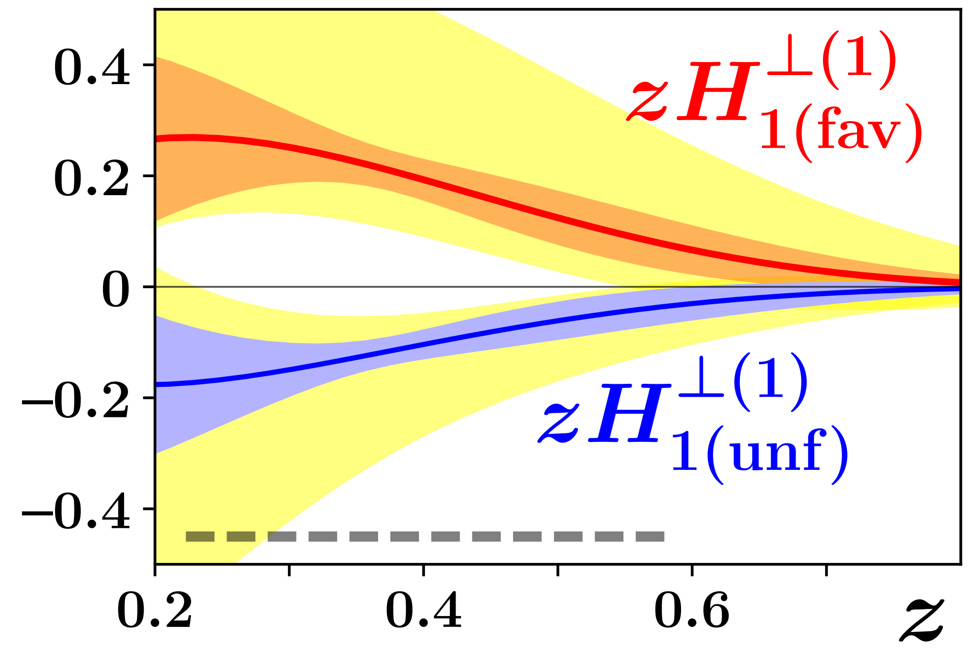
Constraints on global PDF fits

- Including lattice QCD results for moments in global PDF fits can yield significant improvements
- Community white paper (LQCD + phenomenologists) assessed potential impacts [Lin et al., Prog. Part. Nucl. Phys 100 (2018), 107]

Transversity PDFs



Collins fragmentation functions



Yellow: SIDIS data only: direct constraints in region indicated by dashes
Blue/Red: SIDIS + lattice QCD for tensor charge (zeroth moment)

Generalised Parton Distributions

- Quark GPDs: constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV, EIC

- Gluon GPDs: almost unknown from experiment, future constraints central goal of EIC

Leading twist nucleon gluon GPDs:

Gluon field-strength tensor

$$\Delta_\mu = p'_\mu - p_\mu$$
$$P_\mu = (p_\mu + p'_\mu)/2$$
$$t = \Delta^2$$
$$n^2 = 0$$

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p', s' | G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[\mathcal{U}^{(A)}_{[-\frac{\lambda}{2}n, \frac{\lambda}{2}n]}\right]_{ab} G_{b\alpha}^{\ \ \nu\}}(\frac{\lambda}{2}n)|p, s\rangle$$
$$= \frac{1}{2}\left(H_g(x,\xi,t)\bar{U}(p',s')P^{\{\mu}\gamma^{\nu\}}U(p,s) + E_g(x,\xi,t)\bar{U}(p',s')\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M}U(p,s)\right) + \ldots ,$$

GPDs(Bjorken x, skewness, mom transfer)

- Moments of GPDs: Generalised Form Factors (GFFs)

e.g., $\int_0^1 \mathrm{d}x\, H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)\,, \qquad \int_0^1 \mathrm{d}x\, E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$

Energy Momentum Tensor

- Generalised form factors encode e.g., Energy-Momentum Tensor
- Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^a_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s\rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i\, P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D_g(t) \frac{\Delta_{\{\mu}\Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Sum rules of gluon and quark GFFs in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental!
    - $D_a(t)$ GFFs encodes pressure and shear distributions

D-term from JLab DVCS

Experimental determination of DVCS D-term and extraction of proton pressure distribution
[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

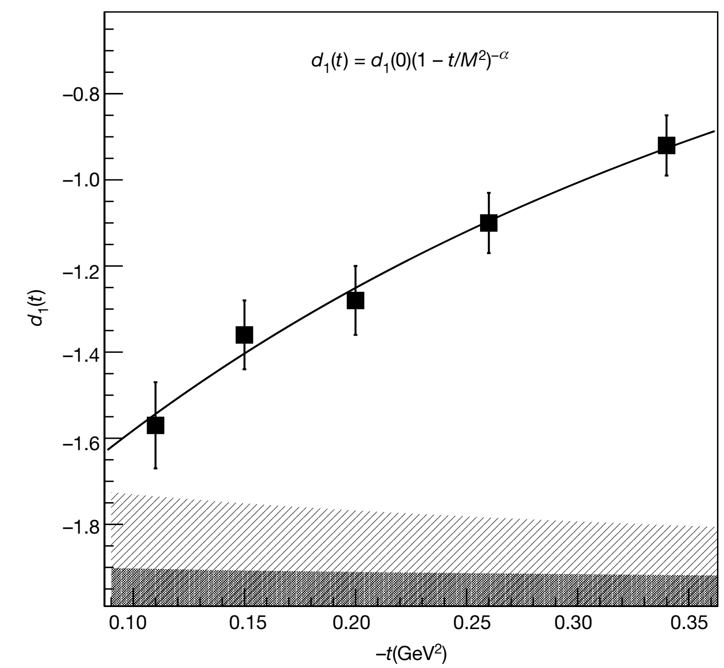
- Peak pressure near centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark term, tripole form factor model, $D_u(t,\mu) = D_d(t,\mu)$

EXP + LQCD

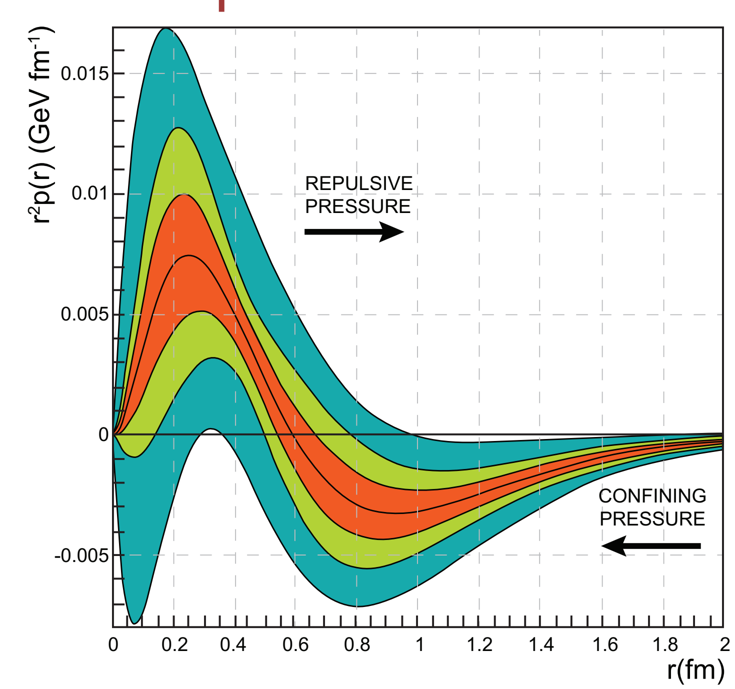
first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

DVCS (quark) D-term



Radial pressure distribution



Nucleon D-term GFFs from LQCD

EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape

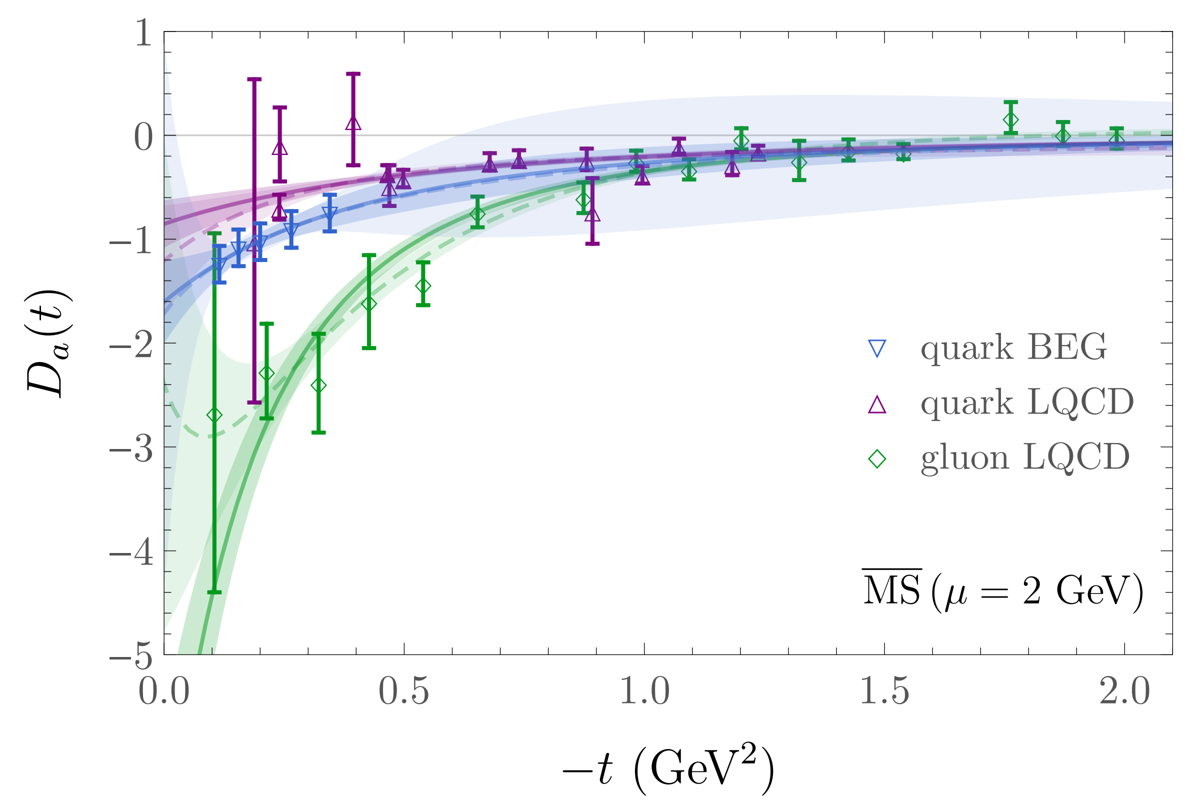
  Factor of ~2 difference in magnitude, somewhat different t-dependence

- Tripole form factor model

  LQCD results consistent with ansatz, but more general form is less well constrained

- Isovector quark D-term vanishes

  $D_{u-d}(t) \sim 0$ *from other LQCD studies*



Gluon GFFs: Shanahan, Detmold, PRD 99, 014511

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

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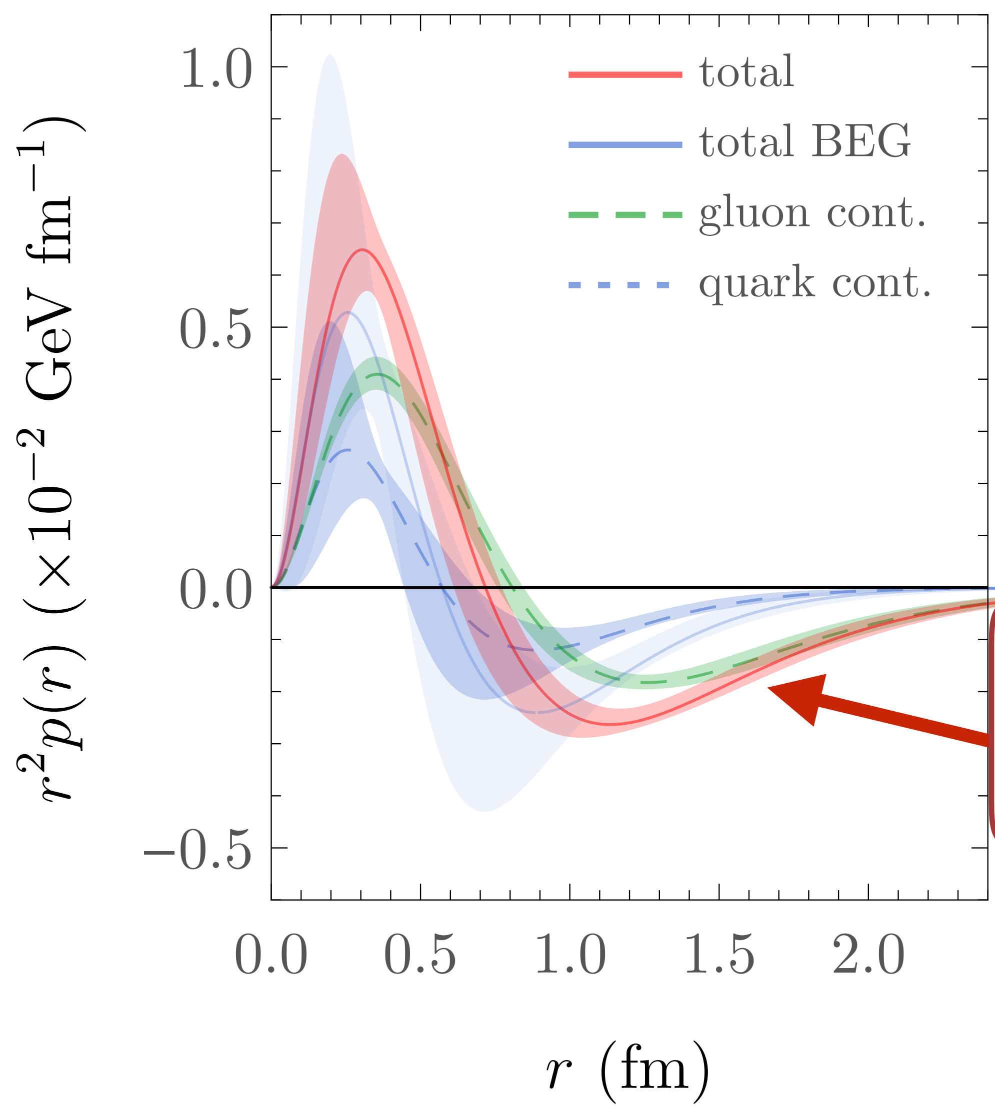
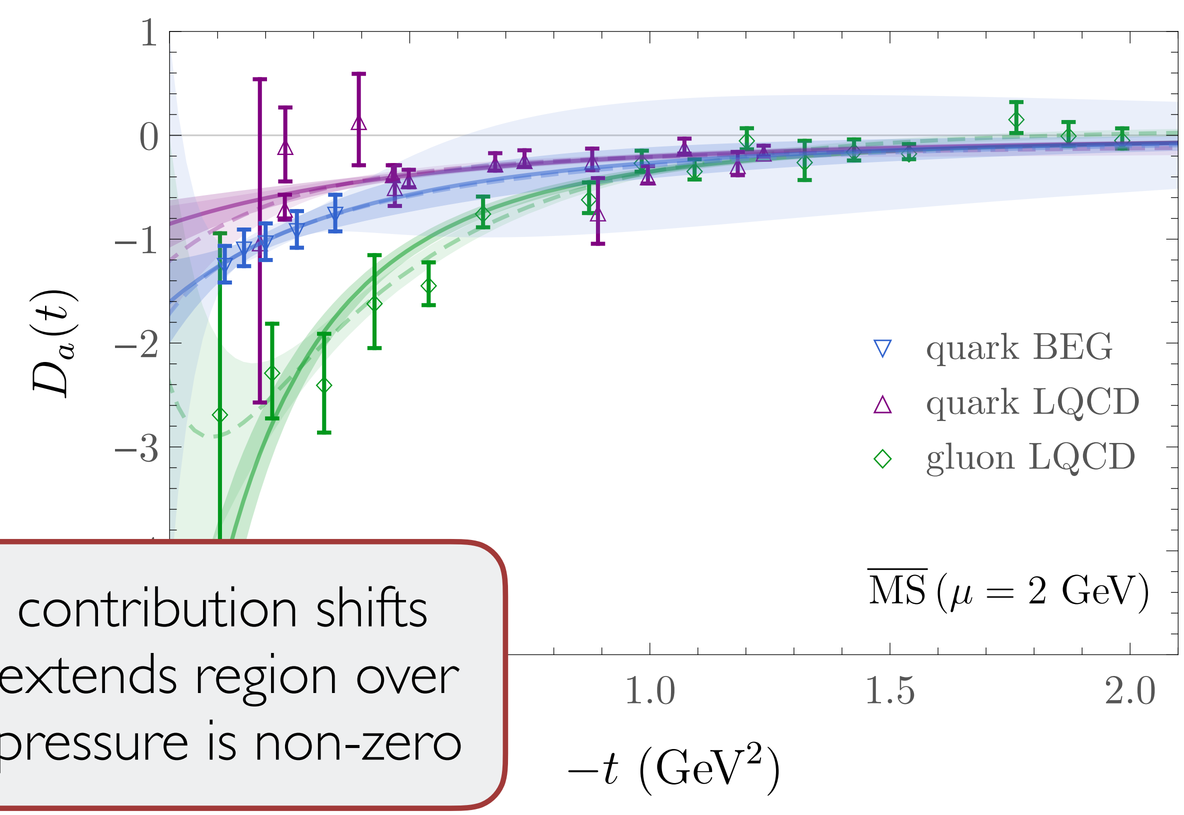
EXP + LQCD

first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]





gluon contribution shifts peaks, extends region over which pressure is non-zero

Gluon GFFs: Shanahan, Detmold, PRD 99, 014511

Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)

Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Parton Physics from Lattice QCD

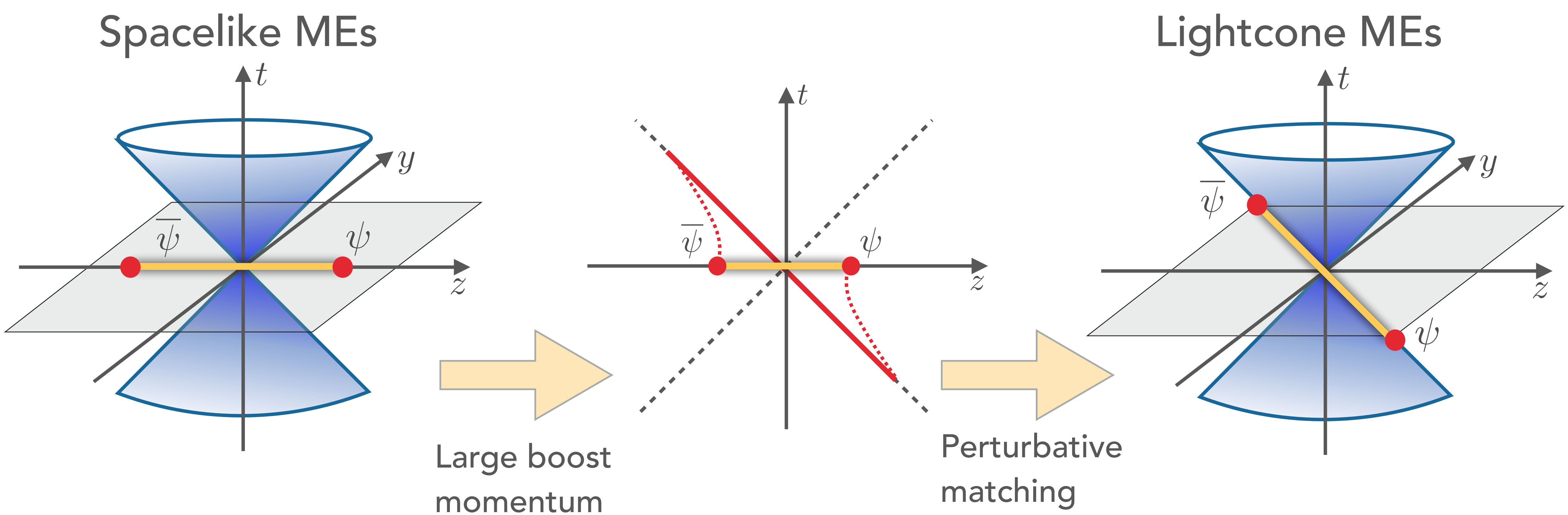
- Mellin moments of PDFs, GPDs
- x-dependence of PDFs
- TMDs and the Collins-Soper evolution kernel
- EMC-type effects in nuclei

x-dependence of PDFs

Parton distribution functions $f(x, \mu^2)$

- Non-local light-cone correlations
- Encode non-perturbative physics

- Access x-dependence in Euclidean calculation by relating spacelike non-local operator matrix elements to lightlike



x-dependence of PDFs

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- Non-local light-cone correlations
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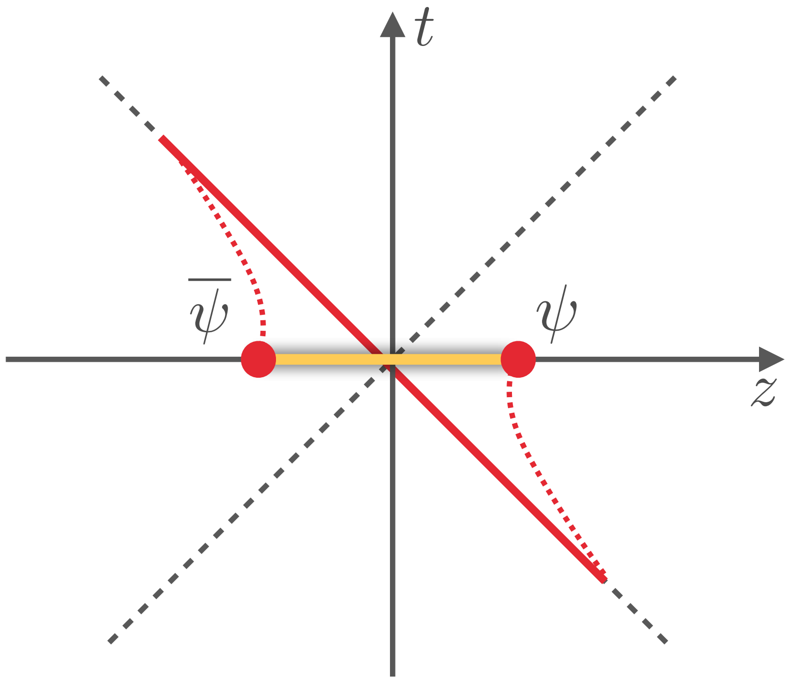
- Access x-dependence in Euclidean calculation by relating spacelike non-local operator matrix elements to lightlike

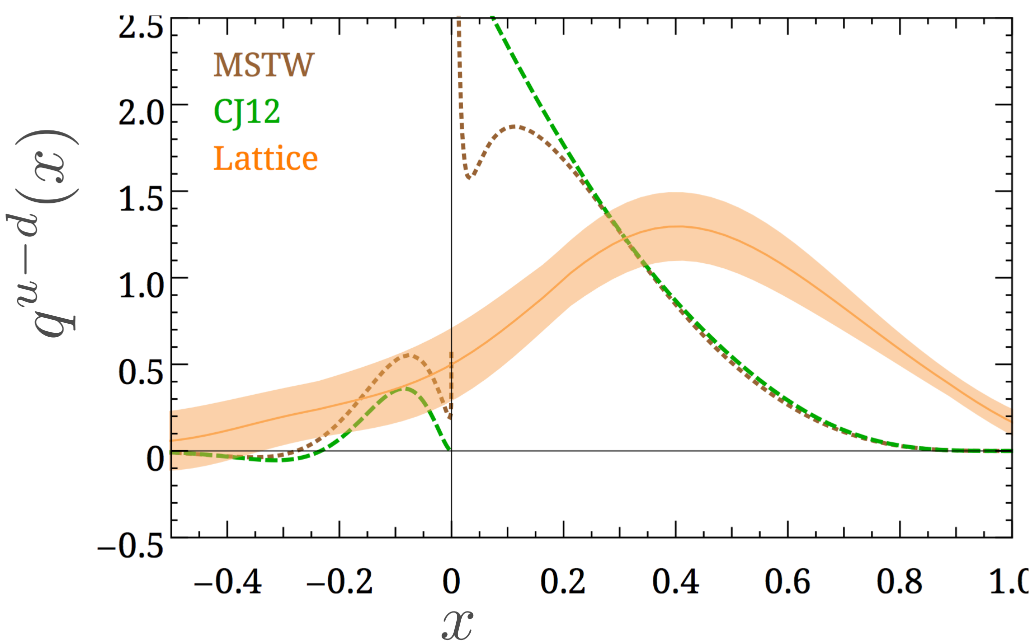
- Quasi-PDFs [Ji, PRL 110 (2013) 262002]
- Pseudo-PDFs [Radyushkin, PRD 96 (2017) 034025]
- Factorisable matrix elements
  [Ma & Qiu, PRL 120 (2018) 022003]
- (Heavy quark) Compton tensor
  [Braun & Müller, EPJ C55 (2008) 349; Chambers et al., PRL 118 (2017) 242001, Detmold & Lin, PRD 73 (2006) 014501, Liu & Dong, PRL 72 (1994) 1790]



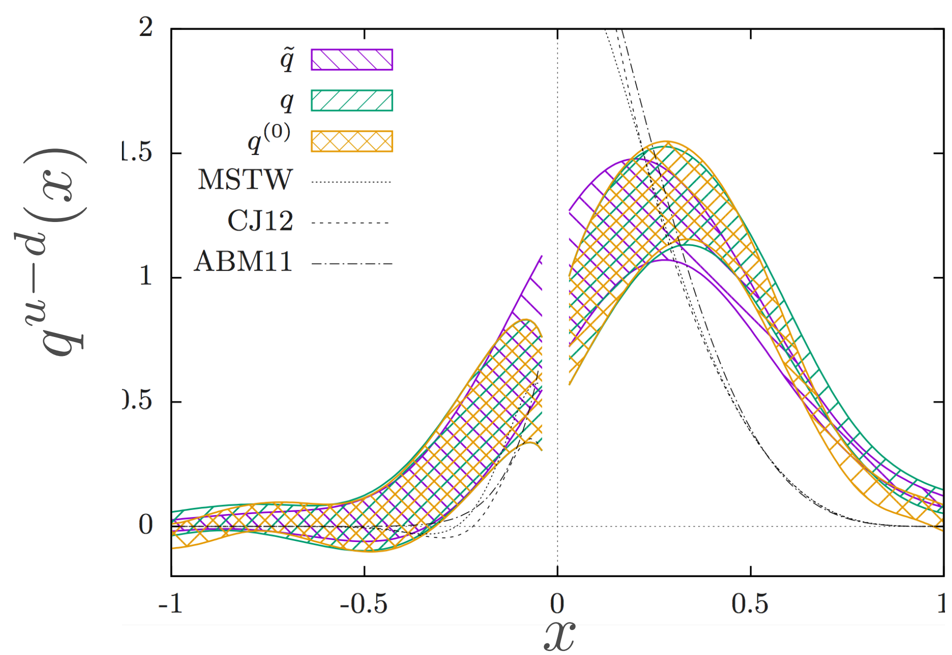
x-dependence of PDFs

- Rapid progress since first calculations in ~2014 **BUT** many systematics
- Renormalisation and perturbative matching understood (~5 year effort)
- Low-x, high-x, regions particularly challenging (lattice systematics)
- Flavour separation is relatively straightforward

Nucleon (u-d) PDFs [quasi-PDF approach]
First exploratory attempts 2015



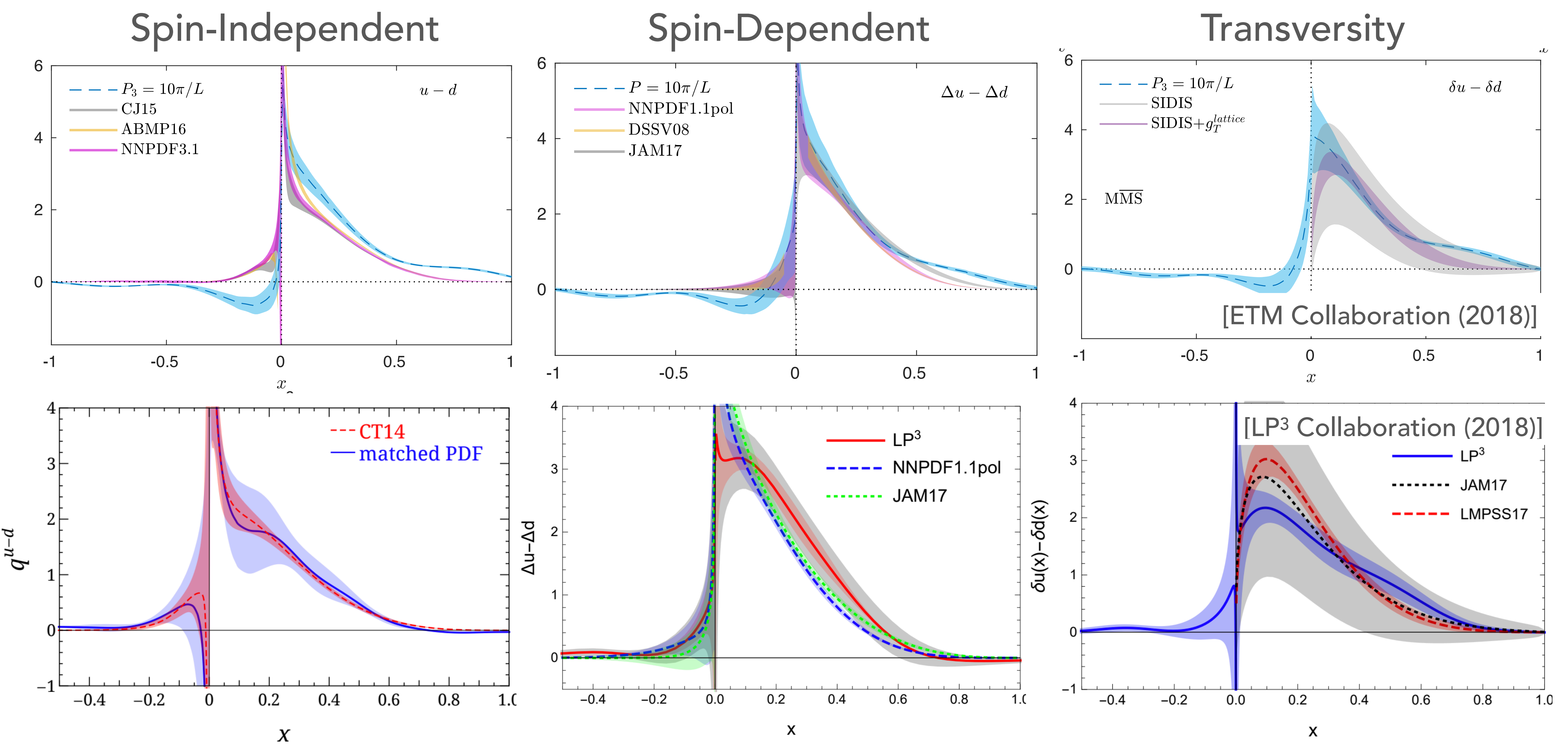
[H.W. Lin et al., Phys. Rev. D 91, 054510 (2015)]



[C. Alexandrou, Phys. Rev. D 92, 014502 (2015)]

x-dependence of PDFs

Nucleon (u-d) PDFs [quasi-PDF approach]
State-of-the-art 2018-2019

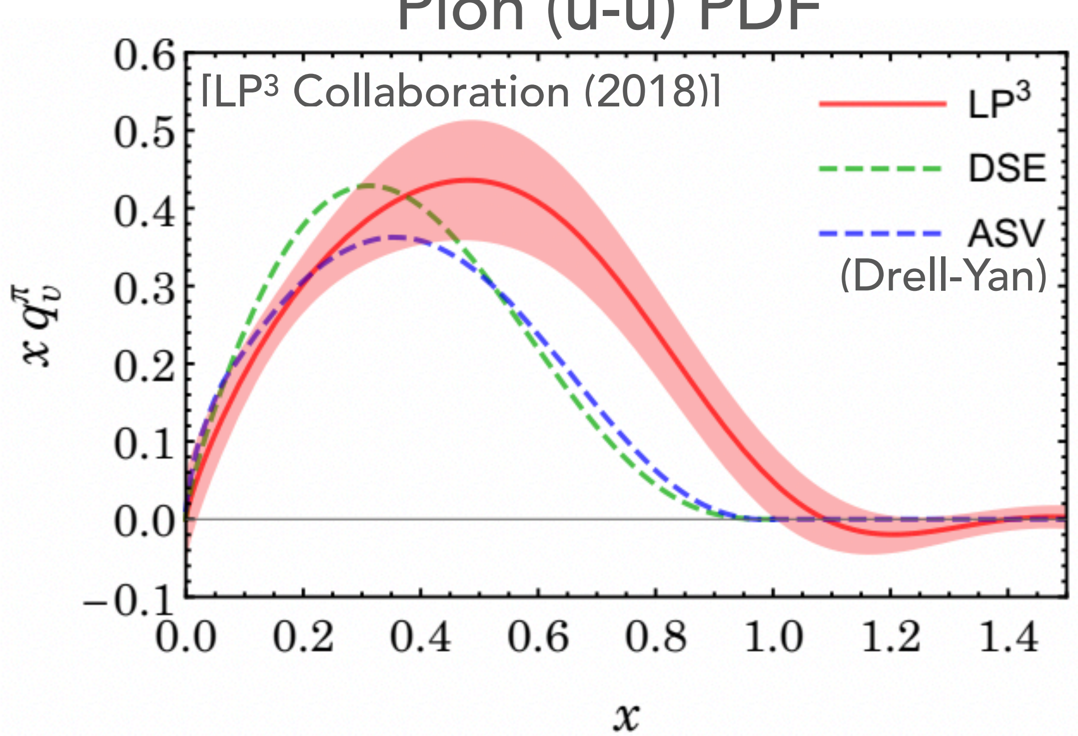


x-dependence of PDFs

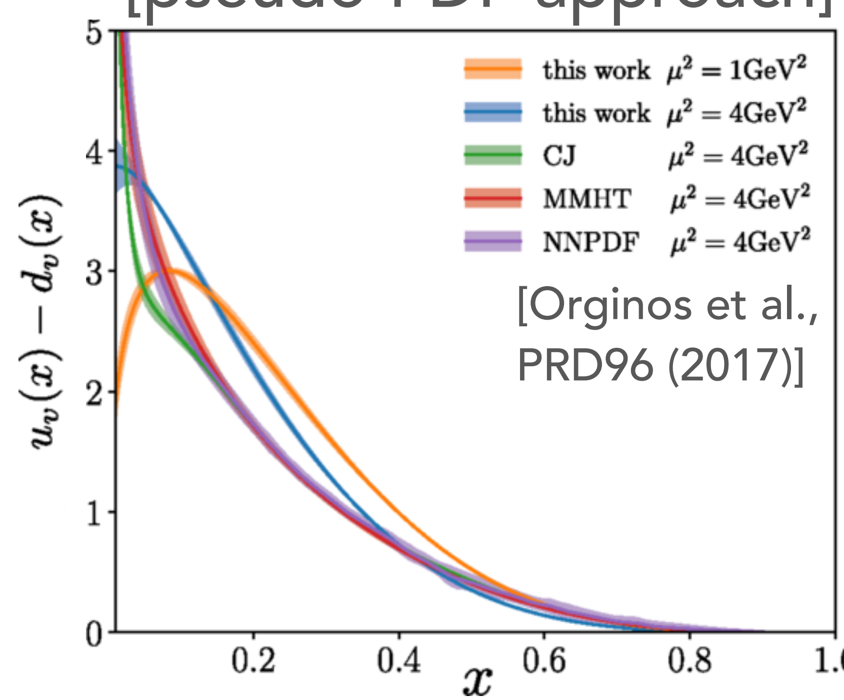
Also first results in 2017-2019 for

- Gluon quasi-PDFs [Z. Y. Fan, et al., PRL 121, no. 24 (2018) 242001]
- Pion quasi-PDFs [J. Chen, et al., arXiv:1804.01483]
- Quasi-GPDs of nucleon and pion, matching to GPDs available [Bhattacharya, Cocuzza and Metz, PLB 788 (2019) 453, Chen, Lin and Zhang, arXiv: 1904.12376, Y.-S. Liu et al., arXiv:1902.00307]
- Nucleon pseudo-PDFs [Orginos et al., PRD96 (2017)]

Pion (u-ū) PDF



Nucleon (u-d) PDF
[pseudo-PDF approach]



Parton Physics from Lattice QCD

- Mellin moments of PDFs, GPDs
- x-dependence of PDFs
- TMDs and the Collins-Soper evolution kernel
- EMC-type effects in nuclei

Transverse Momentum Dependent PDFs

- More detailed picture of nucleon structure

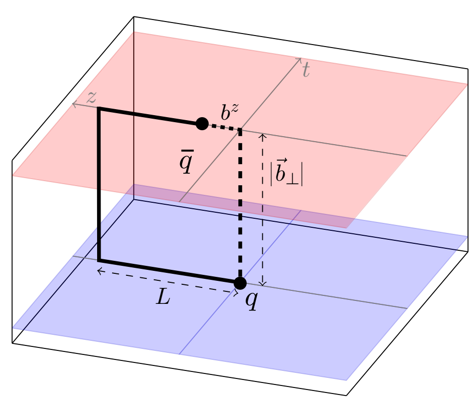
PDF
$f_{q/H}(x)$
longitudinal

→

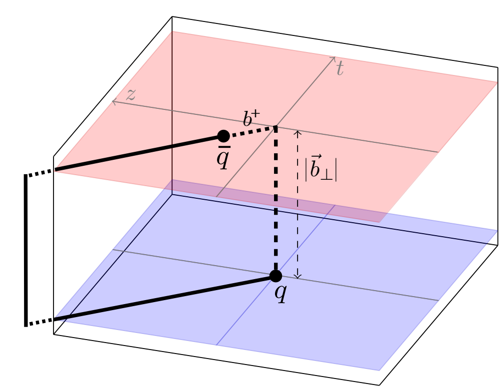
TMD
$f_{q/H}(x, k_T)$
+ transverse

- TMDPDF defined by matrix element of non-local light-cone quark bilinear operator
  - Calculate via quasi-TMD prescription analogous to PDF case
    [Ji, Sun, Xiong,Yuan '14, Ji, Jin, Yuan, Zhang, Zhao, 1801.05930, Ebert, Stewart, Zhao, PRD99 (2019) + 1901.03685]

Spatial staple calculable in LQCD



Large boost + perturbative matching

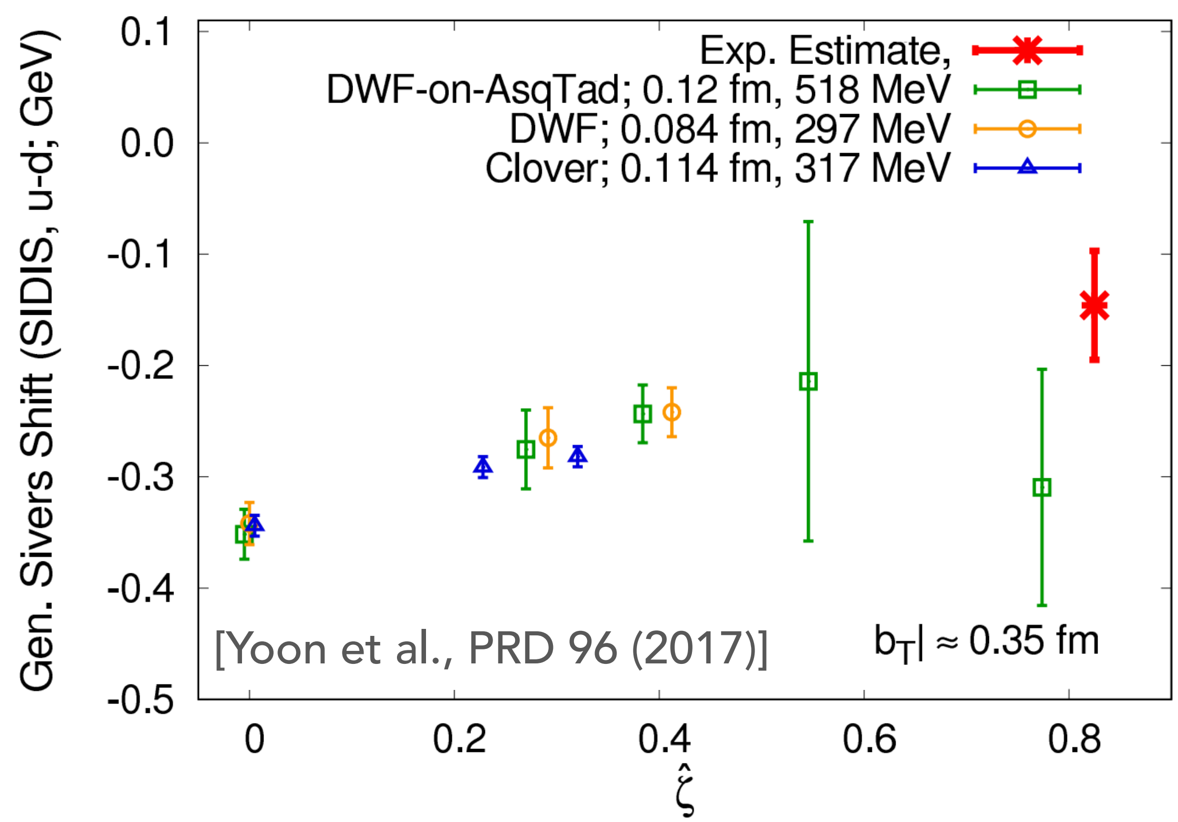


$$\tilde{f}_{u\text{-}d}(x, \vec{b}_T, \mu, P^z) = C^{\mathrm{TMD}}_{u\text{-}d}(\mu, xP^z)\exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP^z)^2}{\zeta}\right] f_{u\text{-}d}(x, \vec{b}_T, \mu, \zeta)$$

quasi-TMD from LQCD | **perturbative matching** | **Collins-Soper evolution kernel** | **desired TMDPDF**

Transverse Momentum Dependent PDFs

- Most robust results for ratios of TMDPDFs: cancellation of renormalisation ambiguities and soft factors
  - e.g., **Generalised Sivers shift** (~ratio of Sivers TMD over unpolarised TMD)
  - Encouraging comparison with expt: global fit to HERMES, COMPASS, JLab
    Light cone: $\hat{\zeta} \to \infty$



- First study of Generalized Transverse Momentum-Dependent Distributions (GTMDs) to obtain quark orbital angular momentum (OAM) in proton [Engelhardt, PRD 95 (2017), USQCD 1904.09512]

- First results for x-dependence of TMDs [Engelhardt, Lattice 2018]

TMD Evolution

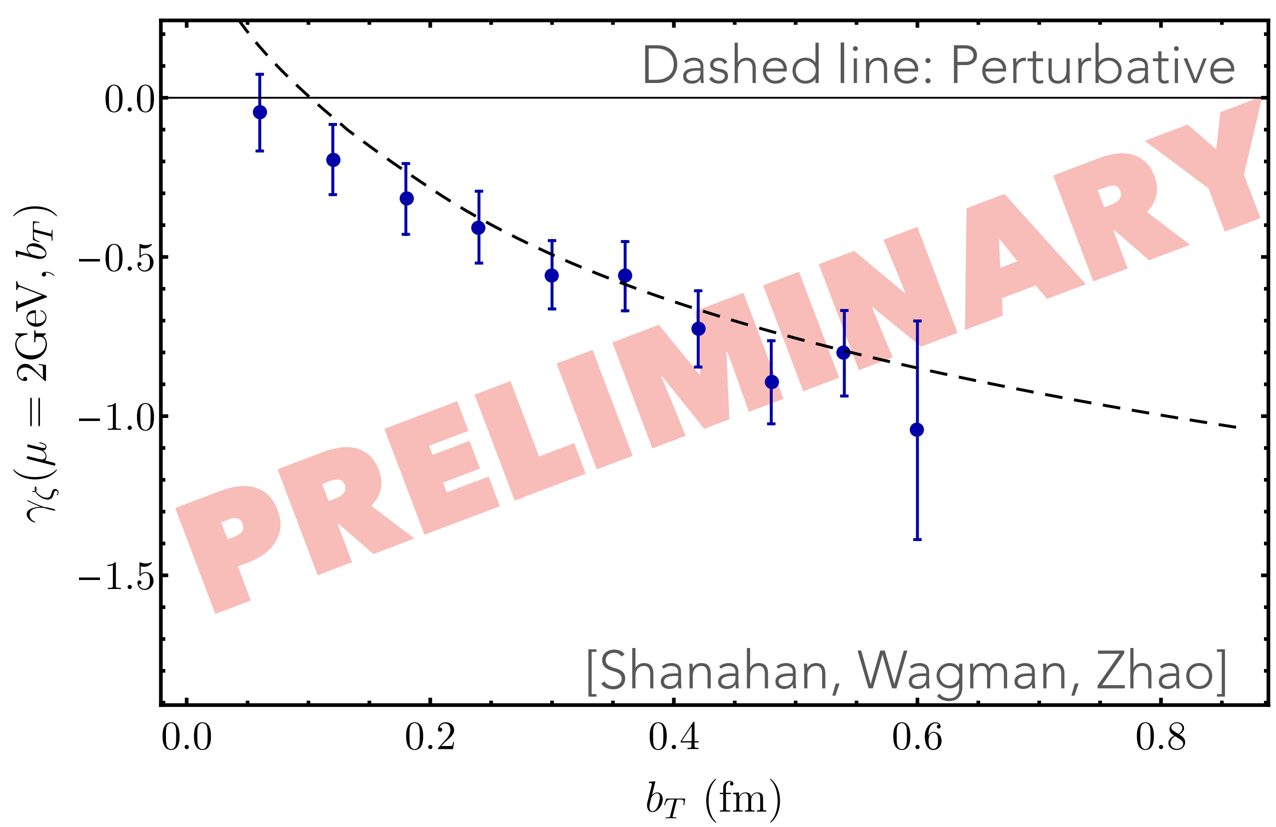
Collins-Soper Evolution Kernel

$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD evolution
- Needed to match quasi-TMD to physical TMD

- Perturbative at short distances $\mu, b_T^{-1} \gg \Lambda_{\rm QCD}$
- Non-perturbative for $b_T^{-1} \lesssim \Lambda_{\rm QCD}$
  Can be accessed via ratio of non-local MEs in LQCD [Ebert, Stewart, Zhao, PRD99 (2019)]

- First calculation in progress [PES, Wagman, Zhao]
- CS-kernel independent of state: study unphysically-heavy pion with no systematic bias
- 5x statistics, 1.5x b_T range will constrain $\gamma_\zeta^q(\mu, b_T)$ in non-perturbative region



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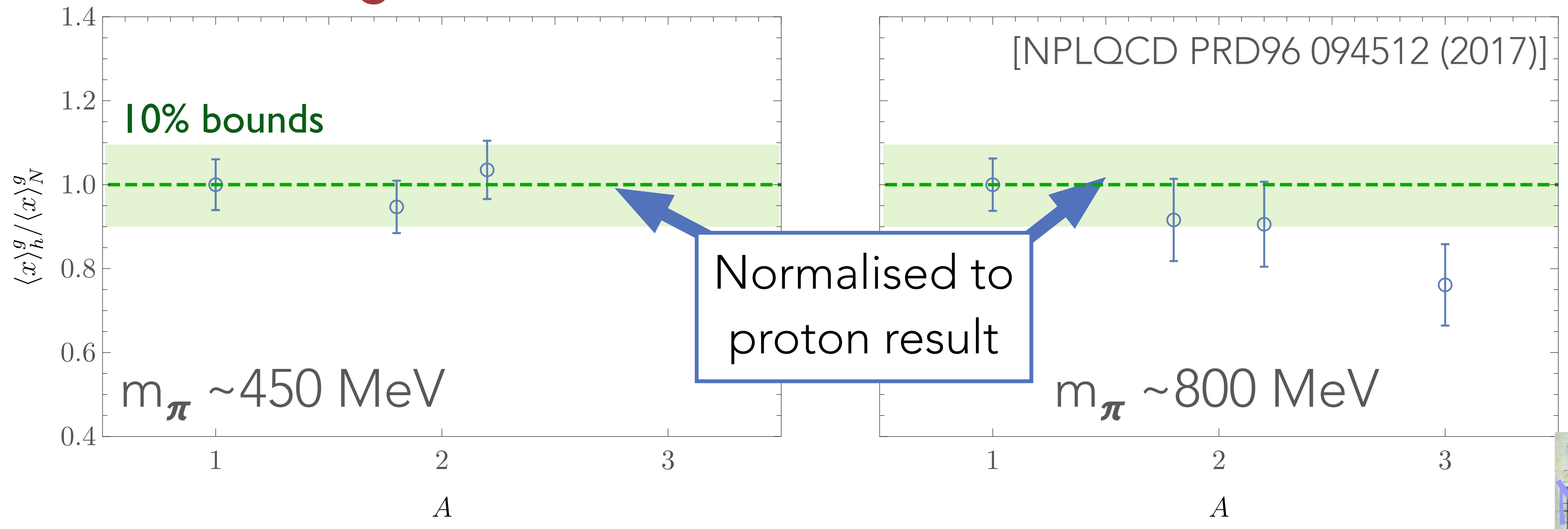
Gluon momentum fraction of nuclei

Matrix elements of the spin-independent gluon operator in nucleon + light nuclei [NPLQCD PRD96 094512 (2017)]

→ **first determination of gluon momentum fraction of nuclei**

- Present statistics: can't distinguish from no-EMC effect scenario

Ratio of gluon momentum fraction in nucleus to nucleon



Non-nucleonic glue in deuteron

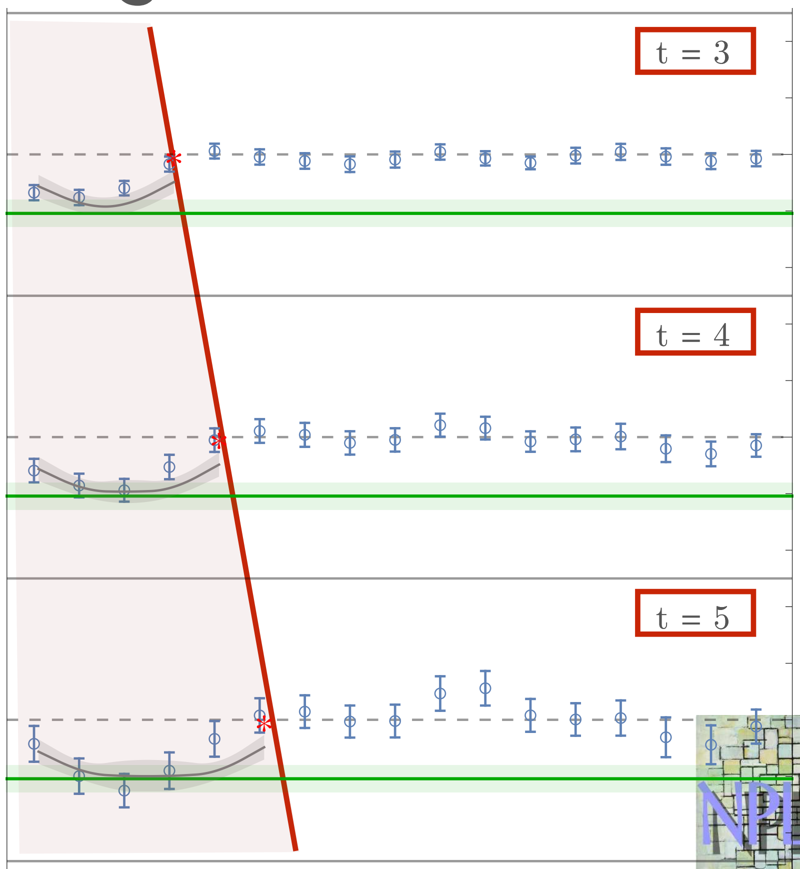
Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

nucleon: $\langle p|\mathcal{O}|p\rangle = 0$

nucleus: $\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$

- First moment of gluon transversity distribution in the deuteron [Jaffe, Manohar PLB223 (1989) 218]
- First evidence for non-nucleonic gluon contributions to nuclear structure: LQCD with m_π ~800 MeV [NPLQCD PRD96 (2017)]
- Magnitude relative to momentum fraction as expected from large-N_c

Signal in LQCD data



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