Saturation overview

Tolga Altinoluk

National Centre for Nuclear Research, Warsaw

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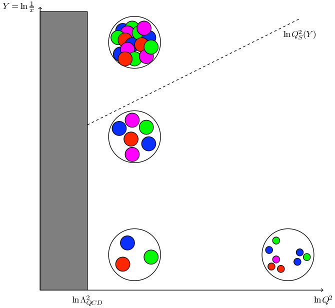
High energy scattering in QCD:



- Regge-Gribov limit : $x \to 0$
- at small $x \to$ **saturation**!
  - $Q_s \equiv$ saturation scale
    $\equiv \alpha_s \times$ (gluon density per unit area)
  - Q_s is a measure of the strength of the gluon interaction processes that may occur when the gluon density becomes large.
  - $Q_s \gg \Lambda_{QCD} \Rightarrow$ weak coupling methods can still be applied !

[McLerran, Venugopalan - hep-ph/9309289 / hep-ph/9311205]
In the saturation regime the prescription of scattering process: Color Glass Condensate (CGC)

CGC description of a process: "effective degrees of freedom" with respect to a cut off Λ^+

- fast partons : $k^+ > \Lambda^+$ $\to$ described by color sources: $J^\mu(x) = \delta^{\mu +}\rho(x^-, x_\perp)$
- slow partons: $k^+ < \Lambda^+$ $\to$ described by color fields $A^\mu(x)$

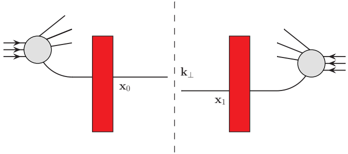
interaction between fast and slow partons: $\int d^4x J^\mu(x) A_\mu(x)$

Forward hadron production

[Dumitru, Hayashigaki, Jalilian-Marian - hep-ph/0506308]:

State-of-the-art calculation framework for forward production in pA collisions: Hybrid factorization

- The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of partons with zero intrinsic transverse momenta)
- Perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes.
- Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)



$$\frac{d\sigma^{pA\to q+X}}{dk^+d^2k_\perp} \propto \int dx_p f_q(x_p,\mu^2) \int e^{ik_\perp(x_0-x_1)} \langle d(x_0,x_1)\rangle$$

NLO calculations (at mid rapidity) in p-space:
[Roy, Venugopalan - arXiv:1802.09550] $\rightarrow$ NLO photon production
[Boussarie, Grabovsky, Szymanowski, Wallon - arXiv:1606.00419] $\rightarrow$ NLO diffractive dijet production

.

Forward hadron production

Does LO "Hybrid" formula take into account all contributions at high $k_\perp$?

[TA, Kovner - arXiv:1102.5327]

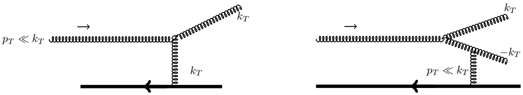
For $k_\perp \gg Q_s$:

$$\frac{d\sigma}{d^2kd\eta} \propto \left[\frac{d\sigma}{d^2kd\eta}\right]_{el.} + \left[\frac{d\sigma}{d^2kd\eta}\right]_{inel.}$$

Real contributions at NLO.

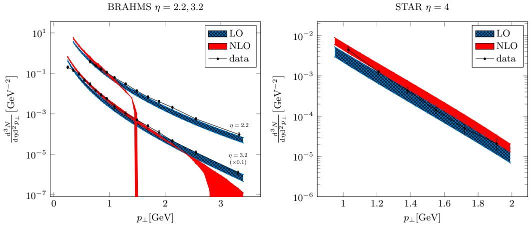
"Elastic Scattering" (LO)

"Inelastic Scattering" (NLO)



[Chirilli, Xiao, Yuan - arXiv:1112.1061 / arXiv:1203.6139] → Full NLO computation.

[Stasto, Xiao, Zaslavsky - arXiv:1307.4057] → Numerical studies of full NLO result.



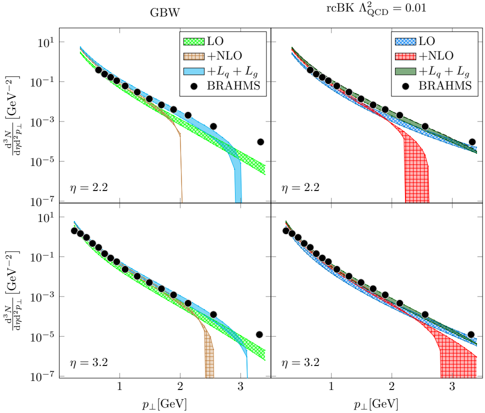
Comparison of BRAHMS (h^-) and STAR (π^0) yields in dAu collisions.

Revisiting NLO hybrid formula

Kinematical constraints are important in the high $k_\perp$ collinear regime!

[TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1411.2869] $\rightarrow$ Ioffe time restriction.

[Watanabe, Xiao, Yuan, Zaslavsky - arXiv:1505.05183] $\rightarrow$ exact kinematical constraint.



BRAHMS data with $\sqrt{s_{NN}} = 200$ GeV.

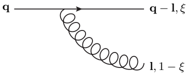
Revisiting NLO hybrid formula - I

Rapidity divergences have to be accounted for properly!

[Ducloue, Lappi, Zhu, - arXiv:1604.00225 / arXiv:1703.04962]

Collinear divergences: absorbed into DGLAP evolution of PDFs and FFs.

Rapidity divergences: absorbed into evolution of the target.



$\xi \to 1$: soft gluon emission collinear to the target.

over subtraction is the cause of the negativity!

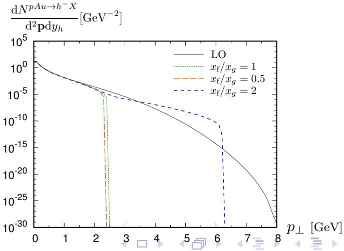
Proposed solution: *implementing k^- ordering in the evolution of the target.*

$$\mathcal{S}(k_\perp) = \mathcal{S}^{(0)}(k_\perp) + 2\alpha_s N_c \int_{\xi_f}^{1} \frac{d\xi}{1-\xi}\Big[\mathcal{I}(k_\perp,1) - \mathcal{I}_\nu(k_\perp,1)\Big]$$

In CXY: $\xi_f \to 0$.

Effectively, solution corresponds to :

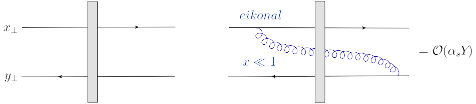
$$\xi_f \to \xi_f(k_\perp) = 1 - min\left(1, \frac{x_g}{x_f}\frac{Q_s^2}{k_\perp^2}\right)$$



[Iancu, Mueller, Triantafyllopoulos - arXiv:1608.05293]

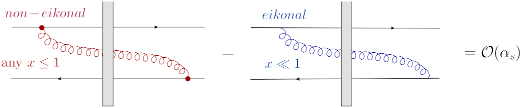
claim: negativity problem is due to approximations adopted in hybrid formalism.

proposed solution: a more general factorization scheme that is non local in rapidity.



"LO dipole S-matrix" *"one step in LO evolution"*

At NLO



$X_g \equiv$ long. mom. frac. of the target gluons
$X_0 \equiv$ initial X when starting the evolution
$X(x) \equiv X_g/x$
$\mathcal{K}(k_\perp, x) \equiv$ non-eikonal emission kernel

$$\frac{dN}{d^2k_\perp d\eta}\bigg|_{NLO} = \mathcal{S}(k_\perp, X_g) + \bar{\alpha}_s \int_{X_g/X_0}^{1} \frac{dx}{x}\big[\mathcal{K}(k_\perp, x) - \mathcal{K}(k_\perp, 0)\big]\mathcal{S}(k_\perp, X(x))$$

to go from NLO (exact calculation of the quark impact factor) $\rightarrow$ CXY:

(i) $X(x) \rightarrow X(1) = X_g$ inside the S-matrix since the integral over x is dominated by $x \sim 1$

(ii) After (i), x integral can be extended down to $x = 0$

Approximations (i) and (ii) are identified to be the source of the negativity problem!!

Evolution equations at NLO

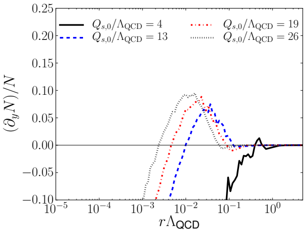
For a fully consistent NLO calculation one needs to use the NLO evolution equations!

[Balitsky, Chirilli - arXiv:0710.4330] $\rightarrow$ the complete set of NLO corrections to the BK equation.

[Kovner, Lublinsky, Mulian - arXiv:1405.0418 / arXiv:1610.03453] $\rightarrow$ NLO JIMWLK equation.

[Balitsky, Grabovsky - arXiv:1405.0443] $\rightarrow$ NLO evolution of three quark Wilson loop operators.

The first numerical solution to NLO BK equation: [Lappi, Mäntysaari - arXiv:1502.02400]



Evolution speed as a func. of r for the MV model ($\gamma = 1$) initial condition.

- *at small $Q_{s,0}/\Lambda_{QCD}$, when α_s and NLO corrections are the largest, the evolution speed is negative at all dipole sizes.*
- *for smaller α_s (larger Q_s), the evolution speed turns negative at $r \ll 1/Q_s$.*

The source of the negativity problem is traced back to the $\ln^2 r$ terms in the NLO BK kernel!

resummation is needed!

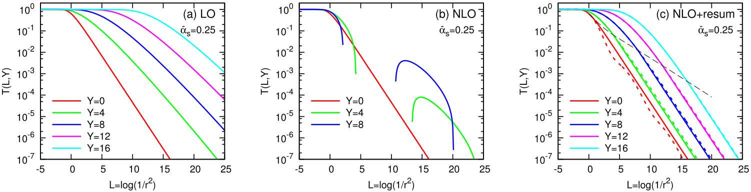
[Beuf - arXiv:1401.0313] $\rightarrow$ *include a kinematical constraint in the BK kernel to account for the finite energy corrections:*

$$\partial_{Y^+}\langle S_{01}\rangle_{Y^+} \propto \int dx_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2}\theta(Y^+ - \Delta_{012})\Big\{\dots\Big\} \quad \text{with} \quad \Delta_{012} = max\left\{0, \log\left(\frac{min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right\}$$

[Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos - arXiv:1502.05642 / arXiv:1507.03651]

Double logarithmic terms are resummed:

$$\partial_Y S_{xy} \propto \int dz\, M_{xyz}\, K_{DLA}\left(\sqrt{\ln\frac{(x-z)^2}{(x-y)^2}\ln\frac{(y-z)^2}{(x-y)^2}}\right)\Big\{\dots\Big\} \quad \text{with} \quad K_{DLA}(\rho) = \frac{J_1\left(2\sqrt{\bar{\alpha}_s\rho^2}\right)}{\sqrt{\bar{\alpha}_s\rho^2}}$$

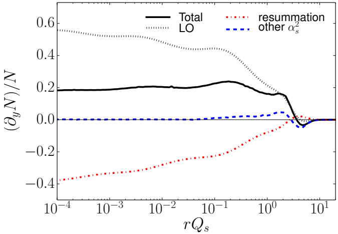


single logs and finite terms appearing in NLO BK have not been included!

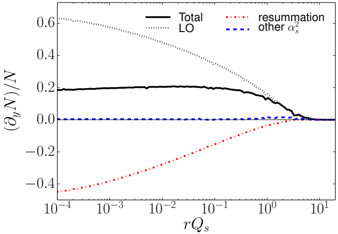
Collinearly improved BK evolution - II

[Lappi, Mäntysaari - arXiv:1601.06598]

Numerical solution of NLO BK with resummation of large single and double logs.



Evolution speed at initial condition.



Evolution speed at $y = 10$.

- *NLO BK eqn. becomes stable when single and & double logs are resummed.*
- *Negative evolution speed problem is fixed.*
- *Evolution speed is reduced compared to LO BK with running coupling.*

[Ducloue, Iancu, Mueller, Soyez, Triantafyllopoulos - arXiv:1902.06637]

Resummed NLO BK: evolution from projectile to target requires fine tuning.

claim: inverted evolution (from target to projectile), ordered in k^+, gives more stable/reliable results.

see talk by Ducloue

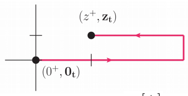
Hybrid factorization and gluon TMDs at small-x

[Collins - hep-ph/0204004] / [Belitsky, Ji, Yuan - hep-ph/0208038] / [Ji,Yuan - arXiv: 0206057]

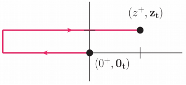
The unpolarized TMDs are defined as the FT of forward matrix elements of bilocal products gluon field strength tensor:

$$\mathcal{F}(x_2,k_t) = 2\int \frac{dz^+ d^2 z_\perp}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- z^+ - ik_t\cdot z_\perp} \langle p_A|\mathrm{tr}\big[F_0^{i-} U^{[C]}_{(0,z)} F_z^{i-} U^{[C']}_{(z,0)}\big]|p_A\rangle$$

$U^{[C]}_{(0,z)}$: gauge staples connecting the points $(0^+,0_\perp)$ and $(z^+,z_\perp)$ to ensure gauge invariance.



$U^{[+]}$



$U^{[-]}$

• different choices to connect the points! $\rightarrow$ different TMDs enter different processes!

[Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren - arXiv:1503.03421]

TMD factorization formula for dijet production in pA collisions

$$\frac{d\sigma^{pA\to dijets+X}}{dy_1 d^2p_1 dy_2 d^2p_2} \propto x_1 f_{a/p}(x_1) \sum_i H^{(i)}_{ag\to cd}\, \mathcal{F}^{(i)}_{ag}(x_2,k_t)$$

$x_1 f_{a/p}(x_1)$: parton distribution functions

$H^{(i)}_{ag\to cd}$: hard factors

$\mathcal{F}^{(i)}_{ag}(x_2,k_t)$: several different TMDs

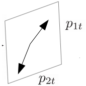
see talk by Boussarie

Forward dijet production in pA collisions

Correlation limit in the CGC:

production of two hard jets: $|p_1| \sim |p_2| \gg Q_s$

total momenta of the produced jets come from target: $|p_1 + p_2| \sim Q_s$



Two typical transverse scale that appears:

$P_T = p_1 + p_2$: *total momentum of the produced jets*
$Q_T = p_1 - p_2$: *momentum imbalance of the two jets*

$P_T \ll Q_T$: *jets fly almost back-to-back (correlation limit).*

[Dominguez, Marquet, Xiao, Yuan - arXiv: 1101.0715] / [Petreska - arXiv:1804.04981] / [Marquet, Petreska, Roiesnel - arXiv:1608.02577]

small-x limit of TMD factorization $\equiv$ *correlation limit of the Hybrid factorization*

in the small-x limit: phase drops - only longitudinal dependence is in staple gauge links.
in the correlation limit: expansion around small dipole size!

$$F^{(1)}_{qg}(x_2, P_T) \propto \int_{r\bar{r}b\bar{b}} r^i \bar{r}^j e^{iP_T(b-\bar{b})} \langle \mathrm{tr}[(\partial^i U_b^\dagger)(\partial^j U_{\bar{b}})] \rangle$$

[Marquet, Roiesnel, Taels - arXiv:1710.05698] Forward heavy quark production
probes not only the unpolarized gluon TMDs but also their linearly polarized partners.

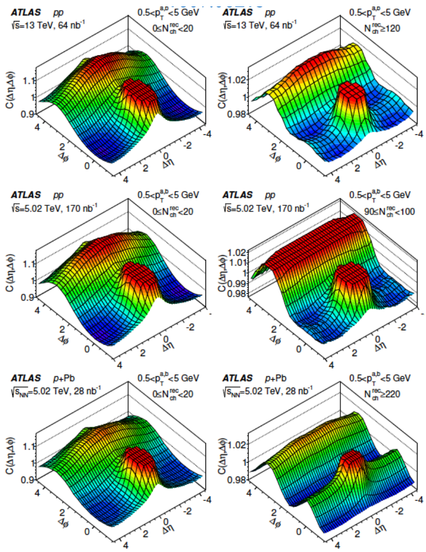
[TA, Armesto, Kovner, Lublinsky, Petreska - arXiv:1802.01398]
[TA, Boussarie, Marquet,Taels - arXiv:1810.11273]
Extension to three final state particles: forward dijet+ photon production

first step to study the equivalence between the TMD and CGC frameworks at NLO.

Two particle correlations

Motivation: Ridge structure

[ATLAS Collaboration - arXiv:1609.06213]



• *correlations between particles over large intervals of rapidity peaking at zero and π relative azimuthal angle.*

• observed first at RHIC in Au-Au collisions.

• observed at LHC for high multiplicity pp and pA collisions.

Correlations within the CGC framework

Ridge in HICs ↔ collective flow due to strong final state interactions

(good description of the data in the framework of relativistic viscous hydrodynamics)

Ridge in small size systems: similar reasoning looks tenuous but hydro describes the data very well.

see talks by Wiedemann & Broniowski

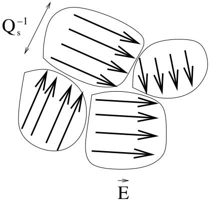
Can it be initial state effect?

idea: *final state particles carry the imprint of the partonic correlations that exist in the initial state.*

Several mechanisms have been suggested to explain the ridge correlations in the CGC framework.

[Kovner, Lublinsky - arXiv:1012.3398 / arXiv:1109.0347 / arXiv:1211.1928]

Local anisotropy of the target fields → rotational symmetry is broken.



particles correlated in the incoming w.f.

transverse separation $\ll 1/Q_s$

scatter through the same domain.

initial state correlations → final state correlations

Numerical studies based on local anisotropy of the target:

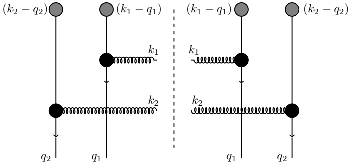
[Dumitru, Skokov - arXiv:1411.6030] / [Dumitru, McLerran, Skokov - arXiv:1410.4844] / [Dumitru, Giannini - arXiv:1406.5781]

Correlations within the CGC framework -II

[Dumitru, Gelis, McLerran, Venugopalan - arXiv:0804.3858]

[Dumitru, Dusling, Gelis, Jalilian-Marian, Lappi, Venugopalan - arXiv:1009.5295]

Glasma graph approach to two gluon production



Glasma graph calculation contains two physical effects:

- Bose enhancement of the gluons in projectile/target wave function
  [TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1503.07126]
  $\sigma|_{BE,P} \propto \left\{\delta^{(2)}\left[(k_1-q_1)-(k_2-q_2)\right]+\delta^{(2)}\left[(k_1-q_1)+(k_2-q_2)\right]\right\}$
  $\sigma|_{BE,T} \propto \left\{\delta^{(2)}(q_1-q_2)+\delta^{(2)}(q_1+q_2)\right\}$
- Hanbury-Brown-Twiss (HBT) correlations between gluons far separated in rapidity.
  $\sigma|_{HBT} \propto \left\{\delta^{(2)}(k_1-k_2)+\delta^{(2)}(k_1+k_2)\right\}$
  [Kovchegov,Wertepny - arXiv:1212.1195 / arXiv:1310.6701] $\rightarrow$ $k_\perp$-factorized approach
  [TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1509.03223] $\rightarrow$ Glasma graph approach

Correlations within the CGC framework -III

Two particle correlations beyond the glasma graph approach: 2 gluon production in pA collisions

[TA, Armesto, Wertepny - arXiv:1804.02910] $\rightarrow$ $k_\perp$-factorized approach

[TA, Armesto, Kovner, Lublinsky - arXiv:1805.07739] $\rightarrow$ Glasma graph approach .

scattering on a dense target $\rightarrow$ dipole and quadrupole operators. Factorization assumption:

$$\begin{aligned}
\langle Q(x,y,z,v)\rangle_T &\rightarrow d(x,y)d(z,v) + d(x,v)d(z,y) + \frac{1}{N_c^2-1} d(x,z)d(y,v) \\
\langle D(x,y)D(z,v)\rangle_T &\rightarrow d(x,y)d(z,v) + \frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z) + d(x,z)d(v,y)\right]
\end{aligned}$$

double inclusive X-section:

$$\frac{d\sigma}{d^3k_1 d^3k_2} \propto \int_{q_1 q_2} \left\{ d(q_1)d(q_2)\left[I_0 + \frac{1}{N_c^2-1} I_1 + \frac{1}{(N_c^2-1)^2} I_2\right] + (k_2 \rightarrow -k_2) \right\} + O\left(\frac{1}{Q_s S_\perp}\right)$$

symmetry under $(k_2 \rightarrow -k_2)$: "**accidental symmetry of the CGC**"

$I_0 \propto \delta^{(2)}(0) \rightarrow$ uncorrelated contribution.

$$I_1 \propto \left\{ \underbrace{f_1 \delta^{(2)}\left[(k_1 - q_1) - (k_2 - q_2)\right]}_{\text{BE. proj.}} + \underbrace{f_2 \delta^{(2)}(k_1 - k_2)}_{\text{HBT}} \right\}$$

$$I_2 \propto \left\{ \underbrace{g_1 \delta^{(2)}(q_1 - q_2)}_{\text{BE. target}} + \underbrace{g_2 \delta^{(2)}\left[(k_1 - q_1) - (k_2 - q_2)\right]}_{\text{BE. proj.}} \right\}$$

[Martinez, Sievert, Wertepny - arXiv:1801.08986/arXiv:1808.04896]$\rightarrow$$(gg), (qq), (q\bar{q})$ correlations in pA

Accidental symmetry in the CGC

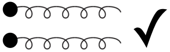
"accidental symmetry in CGC" $\Rightarrow$ vanishing odd harmonics

• *breaking the accidental symmetry with nonlinear Gaussian approximation for dipole-dipole correlator:*
[Lappi, Schenke, Schlichting, Venugopalan - arXiv:1509.03499]

$$\langle D(x,y)D(u,v)\rangle = d_1 + \frac{1}{N_c^2}\left[\frac{\ln(d_3/d_2)}{\ln(d_1/d_2)}\right]^2\left\{d_1 + d_2\left[\ln(d_1/d_2) - 1\right]\right\}$$

$$d_1 \equiv D(x-y)D(u-v)$$
$$d_2 \equiv D(x-v)D(u-y)$$
$$d_3 \equiv D(x-u)D(y-v)$$

• *breaking the accidental symmetry with the density corrections to the projectile:*
[Kovner, Lublinsky, Skokov - arXiv:1612.07790] / [Kovchegov, Skokov - arXiv:1802.08166]



$$\frac{dN^{\text{even, odd}}(\mathbf{k}_\perp)}{d^2k dy} = \frac{1}{2}\left(\frac{dN(\mathbf{k}_\perp)}{d^2k dy}\left[\rho_p,\rho_t\right] \pm \frac{dN(-\mathbf{k}_\perp)}{d^2k dy}\left[\rho_p,\rho_t\right]\right)$$ $\Rightarrow$ non-vanishing odd harmonics.

• *numerical studies and comparison with data:*

[Mace, Skokov, Tribedy, Venugopalan - arXiv:1805.09342 / arXiv:1807.00825 / arXiv:1901.10506]

see talk by Mace

Subeikonal corrections in the CGC

Eikonal approximation amounts dropping the energy suppressed terms!

For realistic values of energy one should go beyond eikonal approximation.

Subeikonal studies for evolution equations:

- [Balitsky, Tarasov - arXiv:1505.02151/arXiv:1603.06548/arXiv:1712.09389]
  Rapidity evolution of gluon TMDs from low to moderate x.
- [Kovchegov, Pitonyak, Sievert - arXiv:1511.06737/arXiv:1610.06188/arXiv:1610.06197/arXiv:1703.05809/arXiv:1706.04236]
  Helicity evolution of quark and gluon distributions at small x.
- [Chirilli - arXiv:1807.11435]
  Rapidity evolution for flavour singlet and non-singlet polarized structure functions.

see talk by Sievert

Subeikonal corrections in particle production:

[TA, Armesto, Beuf, Martnez, Salgado - arXiv:1404.2219]
[TA, Armesto, Beuf, Moscoso - arXiv:1505.01400]

finite-width-target corrections in single inclusive gluon production in pA collisions.

• dense target is defined by $\mathcal{A}^\mu(x)$ and eikonal approximation amounts to:

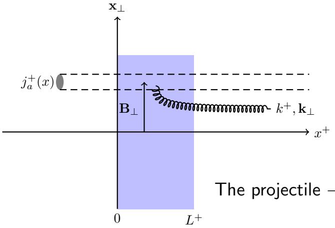
1. $\mathcal{A}^\mu_a(x) \simeq \delta^{\mu-}\mathcal{A}^-_a(x)$
2. $\mathcal{A}^\mu_a(x) \simeq \mathcal{A}^\mu_a(x^+,\mathbf{x})$
3. $\mathcal{A}^\mu_a(x) \propto \delta(x^+)$

1. other components of the target background field $\mathcal{A}^\mu_a(x)$
2. dynamics of the target : x^- dependence of $\mathcal{A}^\mu_a(x)$
3. Finite width L^+ of the target along x^+

$$\boxed{\mathcal{A}^\mu = \delta^{\mu-}\delta(x^+)\mathcal{A}^-(\mathbf{x}) \to \mathcal{A}^\mu = \delta^{\mu-}\mathcal{A}^-(x^+,\mathbf{x})}$$

direct relation with jet quenching (BDMPS-Z formulation)!

[TA, Armesto, Beuf, Moscoso - arXiv:1505.01400]



The target $\rightarrow \mathcal{A}^\mu(x) \equiv \delta^{\mu -} \mathcal{A}_a^-(x^+, \mathbf{x})$

The projectile $\rightarrow$ $j_a^\mu(x) \propto \delta^{\mu +} \delta(x^-)\, \rho^b(\mathbf{x} - \mathbf{B})$

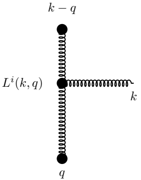
Prod. Amp. $\mathcal{M} \propto$ scalar background propagator $\rightarrow$ *eikonal expansion (in powers of L^+/k^+)*

eikonal order: standard Wilson line / higher orders: new operators (decorated Wilson lines)

[TA, Dumitru - arXiv:1512.00279] $\rightarrow$ *corrections to the Lipatov vertex.*

from pA to pp: expand the standard & decorated Wilson lines to first order in the background field.

$$\boxed{\mathcal{M} \propto \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right] \left\{1 + i\frac{k^2}{2k^+}x^+ - \frac{1}{2}\left(\frac{k^2}{2k^+}x^+\right)^2\right\}}$$



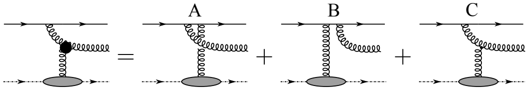
$O(1)$ term eikonal Lipatov vertex.

the form of the corrections suggests exponentiation.

Subeikonal corrections in the CGC

[Agostini, TA, Armesto - arXiv:1902.04483]

- calculate the diagrams by keeping the phase $e^{ik^- x^+}$ which is taken to be 1 in the eikonal limit.



$$\boxed{L^i_{\rm NE}(\underline{k}, q; x^+) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right] e^{ik^- x^+}}$$

$$\underline{k} \equiv (k^+, k)$$
$$k^- = k^2/2k^+$$

Double inclusive cross section with Non-Eik Lipatov vertex

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2}\bigg|^{\rm NE}_{\rm dilute} \propto \int_{q_1 q_2} \Big\{ \Big[f(k_1,q_1,k_2,q_2) + \mathcal{G}_2^{\rm NE}(k_1^-,k_2^-;L^+)\, g(k_1,q_1,k_2,q_2) \Big] + (\underline{k}_2 \to -\underline{k}_2) \Big\}$$

all non-eikonal effects are encoded in

$$\boxed{\mathcal{G}_2^{\rm NE}(k_1^-,k_2^-;L^+) = \left\{ \frac{2}{(k_1^- - k_2^-)L^+} \sin\left[\frac{(k_1^- - k_2^-)}{2} L^+\right] \right\}^2}$$

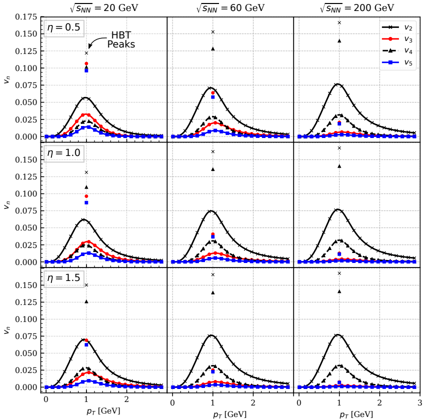
$\mathcal{G}_2^{\rm NE}(k_1^-,k_2^-;L^+)$ is not symmetric under $(\underline{k}_2 \to -\underline{k}_2)$

$\Rightarrow$non-eikonal corrections seem to be breaking the accidental symmetry!!

odd-harmonics from the non-eikonal corrections?

[P. Agostini, T.A., N. Armesto - in preparation]

Can we generate non-zero odd harmonics from the non-eikonal corrections?



$$V_{n\Delta}(k_1, k_2) = \frac{\int_0^\pi N(k_1, k_2, \Delta\phi)\cos(n\Delta\phi)\, d\Delta\phi}{\int_0^\pi N(k_1, k_2, \Delta\phi)\, d\Delta\phi}$$

$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

- $L^+ = 6$ fm in the rest frame and we scale it with the γ factor for different energies.
- $\mu_T = 0.4$ GeV and $\mu_P = 0.2$ GeV (these are the values that maximize v_3).
- $\eta_1 = \eta_2$ & $p_t^{ref} = 1$ GeV.

Non-eikonal effects alone can not explain the odd-harmonics HOWEVER there is a contribution originating from these effects for certain kinematic region.

Summary

- We have covered the recent advances from NLO observables/evolution to non-eikonal corrections.
- The field is progressing well and we would like to keep it that way.
- Recent works that study the relation between TMDs and the CGC: helpful for communication between two communities.
- Subeikonal studies might be also important for EIC since it will not probe very high energies.

Apologies from the people whose works have not been presented due to constraint on time!