Nucleon Imaging:
TMD distributions and the CGC

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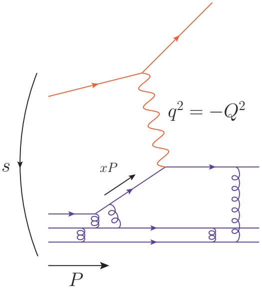
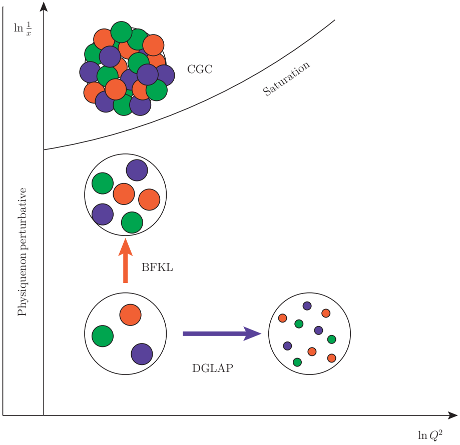
Initial Stages 2019

Accessing the partonic content of hadrons with an electromagnetic probe

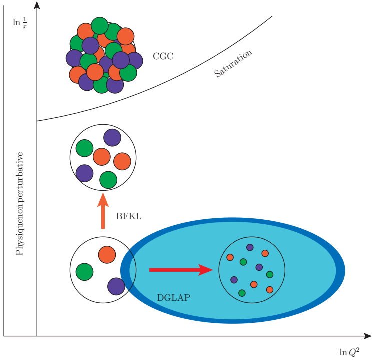


Electron-proton collision (parton model)



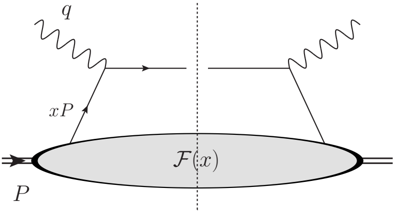
QCD at moderate $x_B = Q^2/s$

$$Q^2 \sim s$$



Collinear factorization:

inclusive processes with a single scale $Q \sim \sqrt{s} \gg \Lambda_{QCD}$



$$\sigma = \mathcal{F}(x, \mu) \otimes \mathcal{H}(\mu)$$

At a scale μ, the process is factorized into:

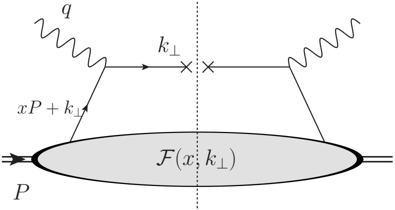
- A hard scattering subamplitude $\mathcal{H}(\mu)$
- A Parton Distribution Function (PDF) $\mathcal{F}(x, \mu)$

μ independence: DGLAP renormalization equation for $\mathcal{F}$

Transverse Momentum Dependent (TMD) factorization:

semi-inclusive processes with one hard and one semihard scale

$$Q \sim \sqrt{s} \gg k_\perp$$



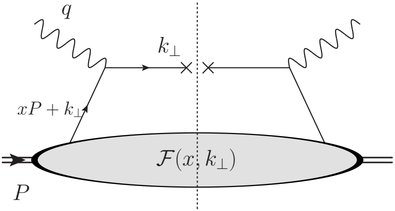
$$\sigma = \mathcal{F}(x, k_\perp, \zeta, \mu) \otimes \mathcal{H}(\mu) \otimes \hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$$

At a scale μ, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(\mu)$
- A TMD PDF $\mathcal{F}(x, k_\perp, \zeta, \mu)$
- A TMD FF $\hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$

$\mu, \zeta, \hat{\zeta}$ independence: TMD evolution for $\mathcal{F}, \hat{\mathcal{F}}$

Another SIDIS order-by-order factorization scheme: Semi-classical effective theory (Shockwaves)[Balitsky, Tarasov]



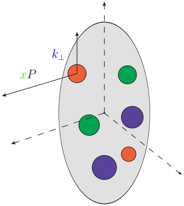
$$\sigma = \mathcal{H}(\sigma, \hat{\sigma}) \otimes \mathcal{F}(x, k_\perp, \sigma) \otimes \hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\sigma})$$

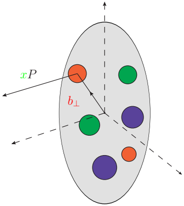
At a scale μ, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(\sigma, \hat{\sigma})$
- A TMD PDF $\mathcal{F}(x, k_\perp, \sigma)$
- A TMD FF $\hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\sigma})$

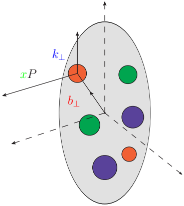
$\sigma, \hat{\sigma}$ independence: Balitsky-Tarasov evolution for $\mathcal{F}, \hat{\mathcal{F}}$

TMD



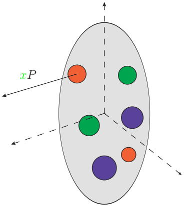


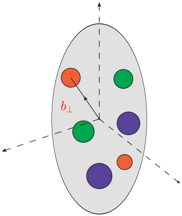
GPD



Wigner

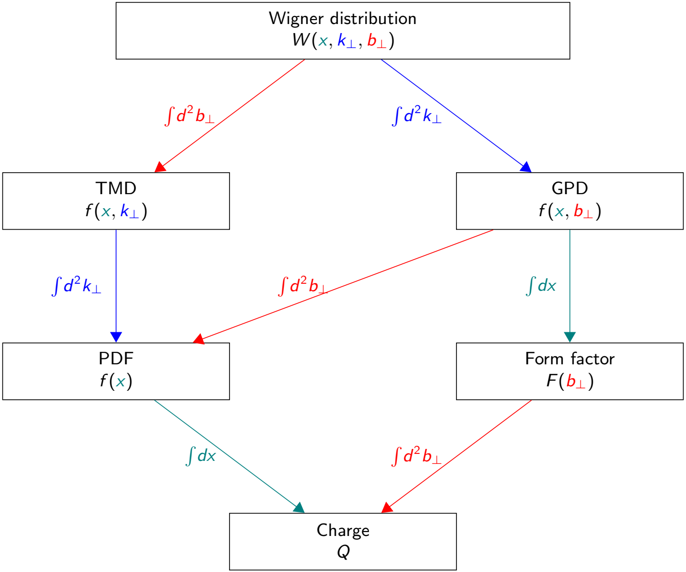
PDF





Form Factor

The family tree of parton distributions



Leading twist quark TMD distributions

Hadron pol. \ Parton	Unpolarized	Chiral	Transverse
Unpolarized	f_1	$\emptyset$	$h_1^\perp$
Longitudinal	$\emptyset$	g_{1L}	$h_{1L}^\perp$
Transverse	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

PDF-spanning

- Unpolarized f_1
- Helicity g_{1L}
- Transversity h_1

Naive T-even pure TMDs

- Worm-gear $h_{1L}^\perp, g_{1T}$
- Pretzelosity $h_{1T}^\perp$
- [Mulders, Tangerman]

Naive T-odd pure TMDs

- Boer-Mulders $h_1^\perp$
- Sivers $f_{1T}^\perp$

Leading twist gluon TMD distributions

Hadron pol. \ Parton	Unpolarized	Circular	Linear
Unpolarized	f_1^g	$\emptyset$	$h_1^{\perp g}$
Longitudinal	$\emptyset$	g_{1L}^g	$h_{1L}^{\perp g}$
Transverse	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

PDF-spanning

- Unpolarized f_1^g
- Helicity g_{1L}^g

Naive T-even pure TMDs

- Worm-gear $h_{1L}^{\perp g}$, g_{1T}^g
- Pretzelosity $h_{1T}^{\perp g}$
- Transversity h_1^g

Naive T-odd pure TMDs

- Boer-Mulders $h_1^{\perp g}$
- Sivers $f_{1T}^{\perp g}$

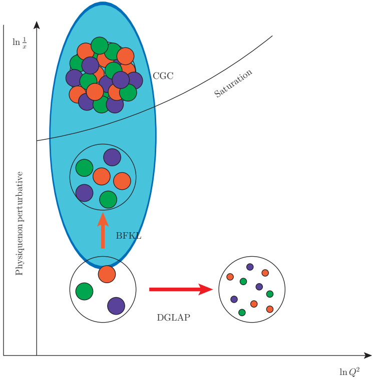
Worth mentioning

Relevant results I will not have time to discuss

- All-order factorization theorems have been derived for Drell-Yan [Collins], [Echevarría, Idilbi, Scimemi], for SIDIS [Ji, Ma, Yuan] and for dihadron production in e^+e^- collisions [Collins]
- A SIDIS and e^+e^- fit for Collins-Sivers and transversity [Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin, Türk] and global TMD fit for data from [HERMES, COMPASS, Tevatron]: [Bacchetta, Delcarro, Pisano, Radici, Signori]
- Single and double spin asymmetries are particularly relevant to probe pure TMD distributions [See M. Sievert's talk?]
- The pretzelosity distribution is a great probe of quark orbital angular momentum [Avakian, Efremov, Schweitzer, Yuan],[Lorcé, Pasquini]
- Processes involving heavy quarks and quarkonia are direct probes of gluon TMD distributions [Boer, Pisano] [Boer, Brodsky, Buffing, Mulders, Pisano], [Boer, Mulders, Pisano, Zhou], [Lansberg, Pisano, Scarpa, Schlegel], [Bacchetta, Boer, Pisano, Taels]
- See Y. Hatta's talk on the gluon Sivers function

QCD at small $x_B = Q^2/s$

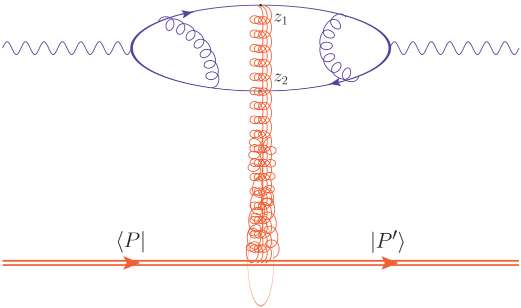
$$Q^2 \ll s$$



Factorized picture

Semi-classical approach to small x physics

[McLerran, Venugopalan], [Balitsky]



$$\mathcal{S} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{Y_c}(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U_{\vec{z}_1}^{Y_c} U_{\vec{z}_2}^{Y_c\,\dagger}) - N_c]|P\rangle$$

Written similarly for any number of Wilson lines in any color representation!

Y_c independence: B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

(So-called) non-universality of TMD distributions:

The importance of gauge links

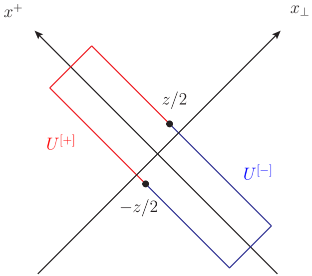
[Collins, Soper, Sterman], [Brodsky, Hwang, Schmidt], [Belitsky, Ji, Yuan], [Bomhof, Mulders, Pijlman], [Boer, Mulders, Pijlman]

[Kharzeev, Kovchegov, Tuchin]

TMD gauge links

"Non-universality" of quark TMD distributions

Gauge links can be future-pointing or past-pointing



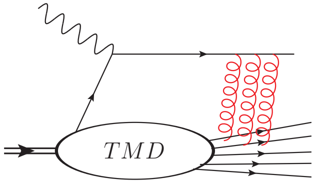
$$q^{[+]}(x,k_\perp) \propto \left\langle P \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P \right\rangle$$

$$q^{[-]}(x,k_\perp) \propto \left\langle P \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[-]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P \right\rangle$$

For naive T-odd distributions, $q^{[+]} = -q^{[-]}$: Sivers effect

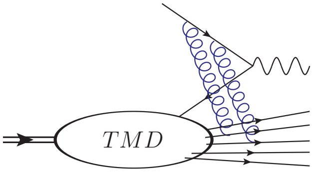
The Sivers effect

SIDIS



Final state interactions: $q^{[+]}$

Drell-Yan

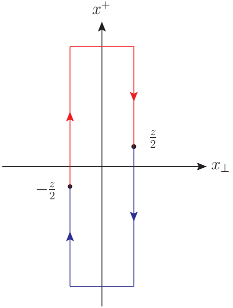


Initial state interactions: $q^{[-]}$

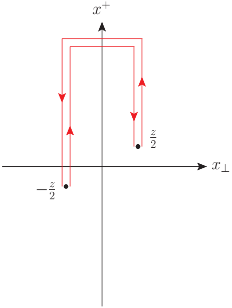
The Sivers distribution comes with a relative $-$ sign between SIDIS and DY: different gauge links for a naive T-odd quantity!

TMD gauge links

"Non-universality" of gluon TMD distributions



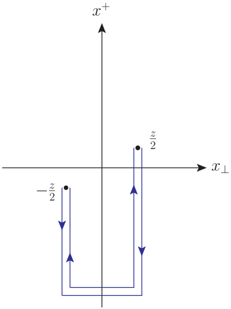
$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$



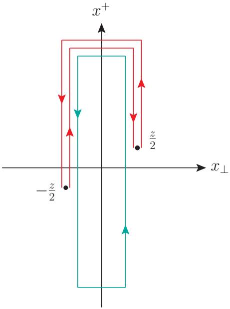
$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

TMD gauge links

"Non-universality" of gluon TMD distributions



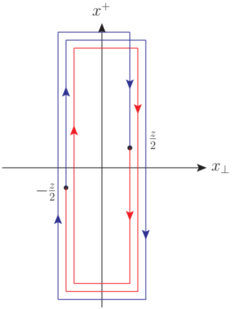
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[-]\dagger}F^{i-}\mathcal{U}^{[-]}\right]$$



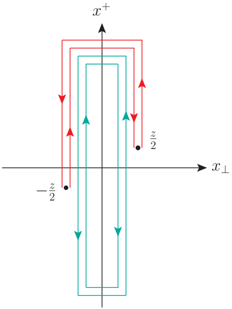
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]$$

TMD gauge links

"Non-universality" of gluon TMD distributions



$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]$$

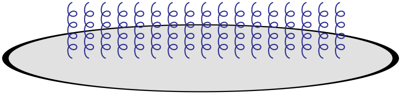


$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]$$

TMD distributions from semiclassical small x physics

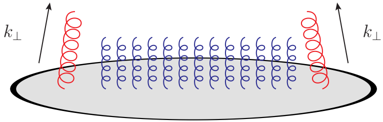
From the CGC to a TMD

From Wilson lines...



$$\left\langle P \left| \mathrm{Tr}\left(U_{\frac{r}{2}} U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

To a parton distribution



$$\left\langle P \left| \mathrm{Tr}\left(\partial^i U_{\frac{r}{2}} \partial^i U^{\dagger}_{-\frac{r}{2}} \right) \right| P \right\rangle$$

From the CGC to a TMD

Staple gauge links from a Wilson line operator

[Dominguez, Marquet, Xiao, Yuan]

Consider the derivative of a path-ordered Wilson line, denoting

$$[x_1^+, x_2^+]_{\vec{x}} \equiv \mathcal{P} \exp\left[ig \int_{x_1^+}^{x_2^+} dx^+ b^-(x^+, \vec{x}) \right]$$

For a given shockwave operator $U_{\vec{x}} = [-\infty, +\infty]_{\vec{x}}$

$$\partial^i U_{\vec{x}} = ig \int dx^+ [-\infty, x^+]_{\vec{x}} F^{-i}(x^+, \vec{x}) [x^+, +\infty]_{\vec{x}}$$

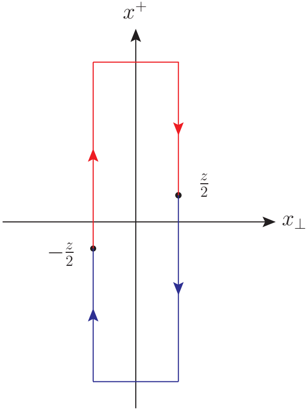
$$\partial^j U^\dagger_{\vec{x}} = -ig \int dx^+ [+\infty, x^+]_{\vec{x}} F^{-j}(x^+, \vec{x}) [x^+, -\infty]_{\vec{x}}$$

$$(\partial^i U^\dagger_{\vec{x}}) U_{\vec{x}} = -ig \int dx^+ [+\infty, x^+]_{\vec{x}} F^{-i}(x^+, \vec{x}) [x^+, +\infty]_{\vec{x}}$$

Taking the derivative of a shockwave operator allows to extract a physical gluon

From the CGC to a TMD

The dipole TMD

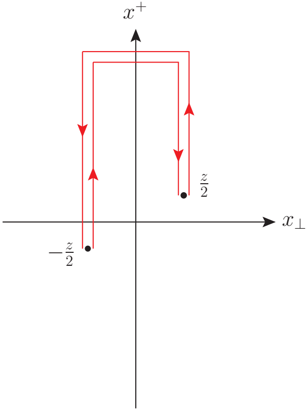


$$\mathcal{F}_{qg}^{(1)}(x,k_\perp) \propto \int d^4z\,\delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right) \mathcal{U}^{[-]\dagger} F^{i-}\left(-\frac{z}{2}\right) \mathcal{U}^{[+]} \right] \right| P \right\rangle$$

$$\to \int d^2z_\perp e^{i(k_\perp\cdot z_\perp)} \left\langle P \left| \mathrm{Tr}\left[\left(\partial^i U^\dagger_{\frac{z}{2}}\right)\left(\partial^i U_{-\frac{z}{2}}\right) \right] \right| P \right\rangle$$

From the CGC to a TMD

The Weizsäcker-Williams TMD



$$\mathcal{F}_{gg}^{(3)}(x, k_\perp) \propto \int d^4z \delta\left(z^+\right) e^{ix(P\cdot z)+i(k_\perp \cdot z_\perp)} \left\langle P \middle| \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger} F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]\middle| P\right\rangle$$

$$\to \int dz_\perp e^{i(k_\perp \cdot z_\perp)} \left\langle P \middle| \mathrm{Tr}\left[\left(\partial^i U_{\frac{z}{2}}\right) U^\dagger_{-\frac{z}{2}} \left(\partial^i U_{-\frac{z}{2}}\right) U^\dagger_{\frac{z}{2}}\right]\middle| P\right\rangle$$

CGC amplitudes and TMD amplitudes

Small dipole "correlation" expansion

[Dominguez, Marquet, Xiao, Yuan]

Taylor expansion of Wilson line operators

$$U_{\boldsymbol{b}+\frac{\boldsymbol{r}}{2}} U_{\boldsymbol{b}-\frac{\boldsymbol{r}}{2}} - 1 = \frac{\boldsymbol{r}^i}{2} \left[\left(\partial^i U_{\boldsymbol{b}} \right) U_{\boldsymbol{b}} - U_{\boldsymbol{b}} \left(\partial^i U_{\boldsymbol{b}} \right) \right] + O\left(\boldsymbol{r}^2 \right)$$

leading twist correspondence:

CGC in the "correlation" limit = TMD in the small x limit

Beyond the correlation limit

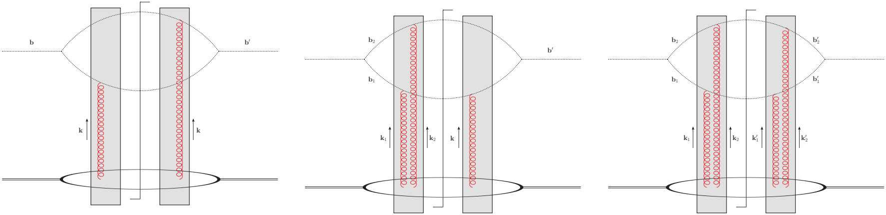
[Altinoluk, RB, Kotko], [Altinoluk, RB]

CGC = infinite twist TMD in the small x limit

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

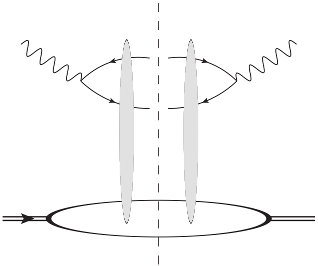
[Altinoluk, RB, Kotko], [Altinoluk, RB]

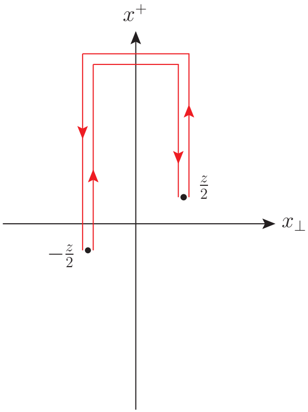


$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$
$$+ \mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$
$$+ \mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

Dijet electro- or photoproduction

Weizsäcker-Williams TMD

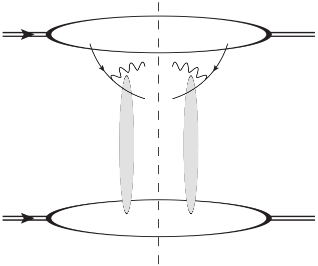


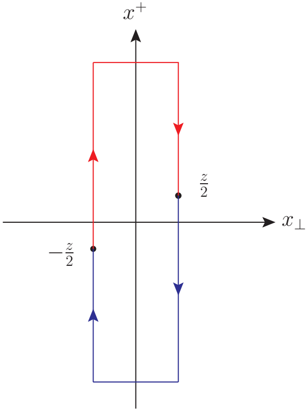


$$\mathcal{F}_{gg}^{(3)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P | \mathrm{Tr}(\partial^i U^\dagger_{\frac{z}{2}}) U_{\frac{z}{2}} (\partial^i U^\dagger_{-\frac{z}{2}}) U_{-\frac{z}{2}} | P \rangle$$

Jet+photon production in pA collisions

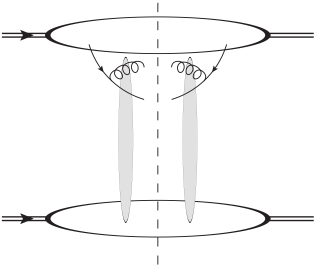
Dipole TMD

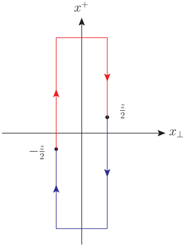




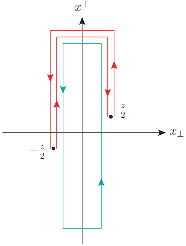
$$\mathcal{F}_{gg}^{(1)}(x \sim 0, k_\perp) \propto \int d^2 z_\perp e^{-i(k_\perp \cdot z_\perp)} \langle P|\mathrm{tr}(\partial^i U_{\frac{z}{2}})(\partial^i U^\dagger_{-\frac{z}{2}})|P\rangle$$

Forward dijet production in *pA* collisions



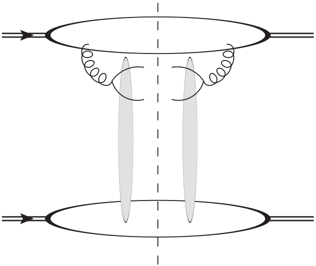


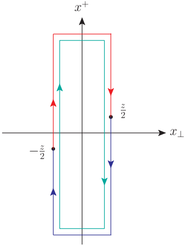
$\mathcal{F}_{qg}^{(1)}$



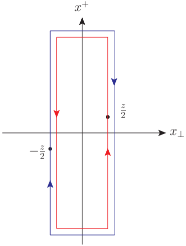
$\mathcal{F}_{qg}^{(2)}$

Forward dijet production in *pA* collisions

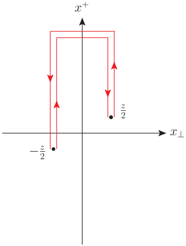




$\mathcal{F}_{gg}^{(1)}$

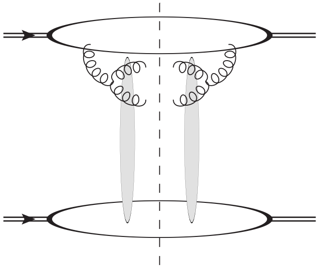


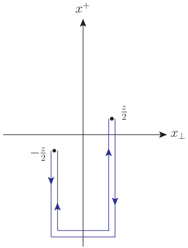
$\mathcal{F}_{gg}^{(2)}$



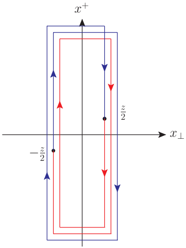
$\mathcal{F}_{gg}^{(3)}$

Forward dijet production in pA collisions

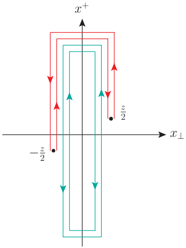




$\mathcal{F}_{gg}^{(4)}$



$\mathcal{F}_{gg}^{(5)}$



$\mathcal{F}_{gg}^{(6)}$

Common tools

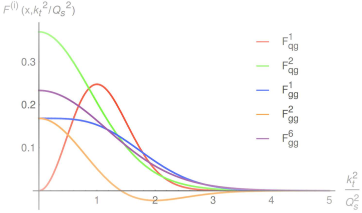
The CGC/TMD equivalence allows to use some TMD tools for the CGC:

- Target Sudakov log resummation for small x processes [Mueller, Xiao, Yuan], [Xiao, Yuan, Zhou]
- Phenomenological Sudakov log simulation [Kotko, Kutak, Sapeta, Stasto, Strikman], [Van Hameren, Kotko, Kutak, Sapeta]

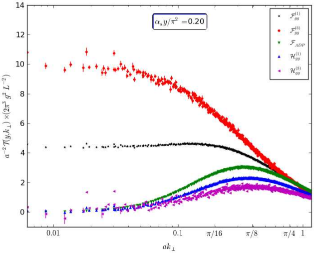
and some CGC tools for small x TMD distributions:

- Golec-Biernat Wüsthoff model for a TMD [Van Hameren, Kotko, Kutak, Marquet, Petreska, Sapeta]
- McLerran-Venugopalan model for a TMD, with JIMWLK evolution [Marquet, Petreska, Roiesnel], [Marquet, Petreska, Taels]

TMD from the CGC



TMD in the GBW model
[Van Hameren, Kotko, Kutak, Marquet, Petreska, Sapeta]



TMD in the MV model with JIMWLK evolution
[Marquet, Petreska, Roiesnel], [Marquet, Roiesnel, Taels]

Linearly polarized gluons in the CGC

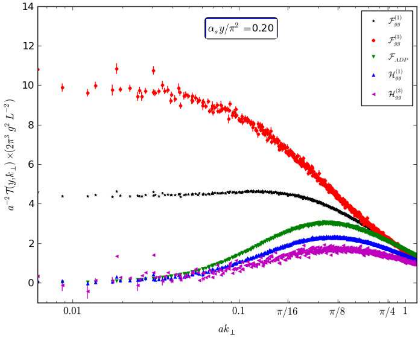
Polarized TMD in the CGC

Wilson line operators also contain linearly polarized gluon TMDs

$$\left\langle P \left| \partial^i U \partial^j U \right| P \right\rangle \to \frac{\delta^{ij}}{2} \mathcal{F}\left(k_\perp\right) + \left(\frac{k_\perp^i k_\perp^j}{k_\perp^2} - \frac{\delta^{ij}}{2} \right) \mathcal{H}\left(k_\perp\right)$$

- They can be computed in the MV model [Metz, Zhou]
- They can be observed in processes with massive quarks [Marquet, Roiesnel, Taels]
- Or in processes with 3 body final states (requires an extension of the notion of the correlation limit) [Altinoluk, RB, Marquet, Taels]
- Can also be seen from loop corrections to 2-body observables, for example prompt photon+jet production in *pA* collisions [Benić, Dumitru], based on a computation by [Benić, Fukushima, Garcia-Montero, Venugopalan]

Polarized TMD in the CGC



In the large $k_\perp \sim Q$ limit (BFKL limit), all TMDs are equal:

$$\mathcal{F}(k_\perp) = \mathcal{H}(k_\perp), \text{ then } \left\langle P \left| \partial^i U \partial^j U \right| P \right\rangle \to \frac{k_\perp^i k_\perp^j}{k_\perp^2} \mathcal{F}(k_\perp)$$

We can recognize the so-called *non-sense polarization* in lightcone gauge: $\frac{k_\perp^i}{|k_\perp|}$. BFKL contains as many linearly polarized gluon pairs as unpolarized ones. At large $k_\perp$, the CGC is very polarized [Boer, Mulders, Zhou, Zhou]

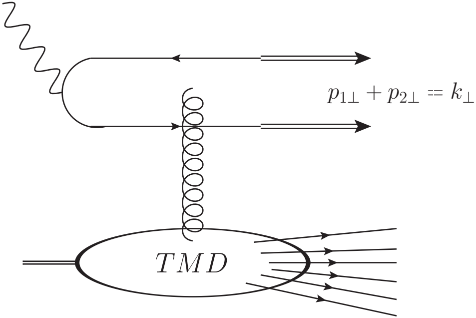
Azimuthal harmonics from TMDs

Azimuthal harmonics in inclusive processes can arise from polarized TMDs

[Boer, Mulders, Pisano], [Metz, Zhou], [Dominguez, Qiu, Xiao, Yuan] , [Dumitru, Skokov]

$$\left\langle P \left| F^{-i}(z) W_{z,0}^{[\pm]} F^{-j}(0) W_{z,0}^{[\pm]} \right| P \right\rangle_{z^-=0} \to \frac{\delta^{ij}}{2}\mathcal{F}(k_\perp) + \left(\frac{k^i k^j}{k^2} - \frac{\delta^{ij}}{2}\right)\mathcal{H}(k_\perp)$$

$$\langle P | FWFW | P \rangle \times \mathcal{H} \Rightarrow v_0 \mathcal{F}(k_\perp) + v_2 \cos(2\phi) \mathcal{H}(k_\perp)$$

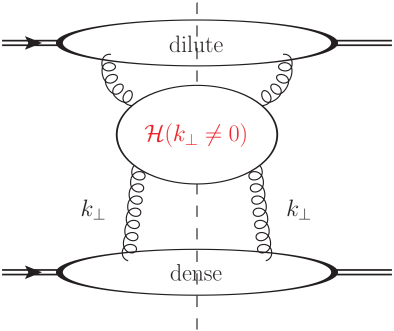


Saturation in terms of TMD distributions

Small x improved TMD framework (iTMD)

A hybrid framework with off-shell gluons from the target

[Kotko, Kutak, Marquet, Petreska, Sapeta, Van Hameren]

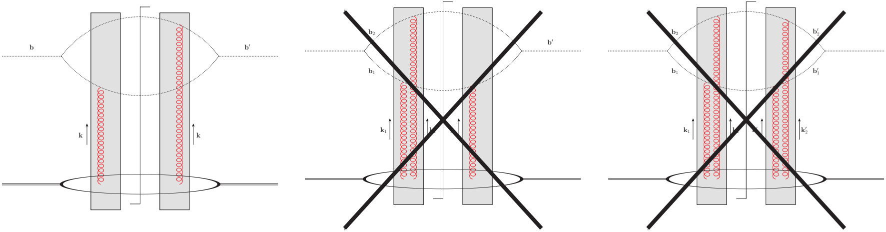


- QCD gauge invariance for multileg amplitudes with an off-shell leg restored with target counterterms [Kotko]
- TMD gauge links are built from the [Bomhof, Mulders, Pijlman] techniques
- Eventually, looks like BFKL, but with distinct TMD distributions for different color flow structures. Interpolates between the TMD regime $|k_\perp| \ll Q$ and the BFKL regime $|k_\perp| \sim Q$

Inclusive low x cross section

Inclusive low x cross section + WW = iTMD cross section

[Altinoluk, RB, Kotko], [Altinoluk, RB]



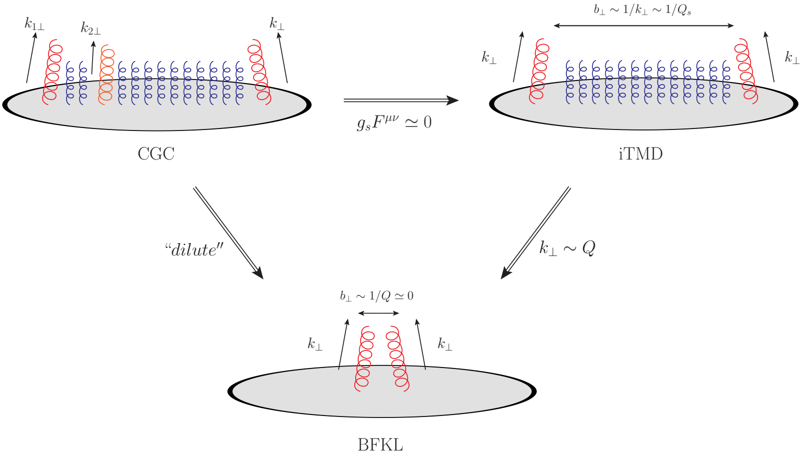
$$\sigma = \mathcal{H}_2^{ij}(k_\perp) \otimes \left\langle P \left| F^{-i} W F^{-j} W \right| P \right\rangle$$

$$+\mathcal{H}_3^{ijk}(k_\perp, k_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W F^{-k} W \right| P \right\rangle$$

$$+\mathcal{H}_4^{ijkl}(k_\perp, k_{1\perp}, k'_{1\perp}) \otimes \left\langle P \left| F^{-i} W g_s F^{-j} W g_s F^{-k} W F^{-l} W \right| P \right\rangle$$

The dilute limit

The dilute limit in terms of TMD distributions



Two kinds of multiple scattering effects for small-ish $Q \to Q_s$:
higher genuine twists and higher kinematic twists

Conclusions

- TMD distributions are what allows to match standard parton distributions and semi-classical descriptions of small x physics
- Color Glass Condensate models can give insights on TMDs at small x
- The reformulation of the CGC in terms of TMD distributions allows to access polarized gluons in the CGC
- Two distinct kinds of multiple scattering effects must be distinguished to understand gluonic saturation

Backup slides

Light ray twist expansion for TMD observables: expansion in powers of $k_\perp/Q$

Suppose we managed to build gauge-invariant hard subamplitudes with non-zero transverse momenta. The amplitude would read:

$$\mathcal{H}_1^i(\boldsymbol{k}) \otimes \int d^2\boldsymbol{x}_1 e^{-i(\boldsymbol{k}\cdot\boldsymbol{x}_1)} [\pm\infty, x_1] F^{i-}(x_1) [x_1, \pm\infty]$$

$$+ \mathcal{H}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \int d^2\boldsymbol{x}_1 d^2\boldsymbol{x}_2 e^{-i(\boldsymbol{k}_1\cdot\boldsymbol{x}_1)-i(\boldsymbol{k}_2\cdot\boldsymbol{x}_2)} [\pm\infty, x_1] F^{i-}(x_1) [x_1, x_2] F^{j-}(x_2) [x_2, \pm\infty]$$

$$+ \dots$$

$$= \mathcal{H}_1^i(\boldsymbol{k}) \otimes \mathcal{O}_1^i(\boldsymbol{k}) + \mathcal{H}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes \mathcal{O}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) + \dots$$

Light ray twist expansion for TMD observables: expansion in powers of $k_\perp/Q$

Leading twist amplitude

$$\mathcal{A}_{LT} = \mathcal{H}_1^i(\mathbf{0}) \otimes \mathcal{O}_1^i(\boldsymbol{k})$$

Next-to-leading twist amplitude

$$\mathcal{A}_{NLT} = \boldsymbol{k} \cdot \left(\partial_{\boldsymbol{k}} \mathcal{H}_1^i\right)(\mathbf{0}) \otimes \mathcal{O}_1^i(\boldsymbol{k}) + \mathcal{H}_2^{ij}(\mathbf{0}, \mathbf{0}) \otimes \mathcal{O}_2^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

First term: kinematic twist correction, second term: genuine twist corrections

Match without an expansion

Trick: rewrite operators in terms of their derivatives

$$U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}} = -ir^{\mu}_{\perp}\int\frac{d^2\boldsymbol{k}_1}{(2\pi)^2}\int d^2\boldsymbol{b}_1 e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{b}_1-\boldsymbol{b})}\frac{e^{i\bar{z}(\boldsymbol{k}_1\cdot\boldsymbol{r})}-1}{(\boldsymbol{k}_1\cdot\boldsymbol{r})}\left(\partial_{\mu}U^{R_1}_{\boldsymbol{b}}\right)$$

Rewrite the amplitude

$$\mathcal{A} = (2\pi)\,\delta\left(p_1^+ + p_2^+ - p_0^+\right)\int d^2\boldsymbol{b}\,d^2\boldsymbol{r}\,e^{-i(\boldsymbol{q}\cdot\boldsymbol{r})-i(\boldsymbol{k}\cdot\boldsymbol{b})}\mathcal{H}(\boldsymbol{r})$$
$$\times\left[\left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}}\right)T^{R_0}\left(U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}} - U^{R_2}_{\boldsymbol{b}}\right) + \left(U^{R_1}_{\boldsymbol{b}+\bar{z}\boldsymbol{r}} - U^{R_1}_{\boldsymbol{b}}\right)T^{R_0}U^{R_2}_{\boldsymbol{b}} + U^{R_1}_{\boldsymbol{b}}T^{R_0}\left(U^{R_2}_{\boldsymbol{b}-z\boldsymbol{r}} - U^{R_2}_{\boldsymbol{b}}\right)\right]$$

genuine twist | kinematic + genuine twists

Extracting genuine twists: Taylor, IbP, resummation.

Why are BFKL distributions "dilute"?

- Wandzura-Wilczek approximation: low gluon occupancy
- No multiple scattering from the gauge links

TMD with staple gauge links

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, \pm\infty\right]_{\boldsymbol{x}} \left[\pm\infty, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, \pm\infty\right]_{\boldsymbol{0}} \left[\pm\infty, x^+\right]_{\boldsymbol{x}} \right| P \right\rangle$$

Large $k_\perp \sim Q \Rightarrow$ small transverse distance $x_\perp$

$$\left[x^+, \pm\infty\right]_{\boldsymbol{x}} \left[\pm\infty, y^+\right]_{\boldsymbol{0}} \sim \left[x^+, y^+\right]_{\boldsymbol{x}\sim\boldsymbol{0}}.$$

All TMD distributions shrink into the unintegrated PDF

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, x^+\right]_{\boldsymbol{0}} \right| P \right\rangle \Big|_{x^-=0}$$

iTMD becomes BFKL

$$d\sigma = \sum_i d\sigma^{(i)}_{\boldsymbol{k}=\boldsymbol{0}} \otimes \Phi^{(i)}(x, \boldsymbol{k}) = \left(\sum_i d\sigma^{(i)}_{\boldsymbol{k}=\boldsymbol{0}}\right) \otimes \Phi(x, \boldsymbol{k})$$

BFKL distributions and genuine twist corrections

Unintegrated PDF = 2-Reggeon matrix element

$$\int d^2\boldsymbol{x} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \int dx^+ \left\langle P \left| F^{i-}(x) \left[x^+, 0^+\right]_{\boldsymbol{0}} F^{j-}(0) \left[0^+, x^+\right]_{\boldsymbol{0}} \right| P \right\rangle \Big|_{x^-=0}$$

Integration by parts

$$\int dx^+ \int d^2\boldsymbol{x} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \boldsymbol{k}^i \boldsymbol{k}^j \langle P | \left[-\infty, x^+\right]_{\boldsymbol{0}} A^-(x) \left[x^+, +\infty\right]_{\boldsymbol{0}} \left[+\infty, 0^+\right]_{\boldsymbol{0}} A^-(0) \left[0^+, -\infty\right]_{\boldsymbol{0}} | P \rangle$$

We recognize the nonsense polarizations in axial gauge. We could identify the Reggeon operator:

$$R(x) = \int dx^+ \left[-\infty, x^+\right]_{\boldsymbol{0}} A^-(x) \left[x^+, +\infty\right]_{\boldsymbol{0}}$$

and rewrite the unintegrated PDF as

$$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2} e^{-i(\boldsymbol{k}\cdot\boldsymbol{x})} \frac{\boldsymbol{k}^i \boldsymbol{k}^j}{\boldsymbol{k}^2} \boldsymbol{k}^2 \left\langle P \left| R(\boldsymbol{x}) R^\dagger(0) \right| P \right\rangle$$

BFKL distributions and genuine twist corrections

What is missing in BFKL: 3- and 4-Reggeon matrix elements.

$$\langle P\,|RR|\,P\rangle\,, \quad \langle P\,|R(g_s R)R|\,P\rangle\,, \quad \langle P\,|R(g_s R)(g_s R)R|\,P\rangle$$

They are not perturbatively suppressed.

Suppression = Wandzura Wilczek approximation
(unquantifiable) = no genuine saturation