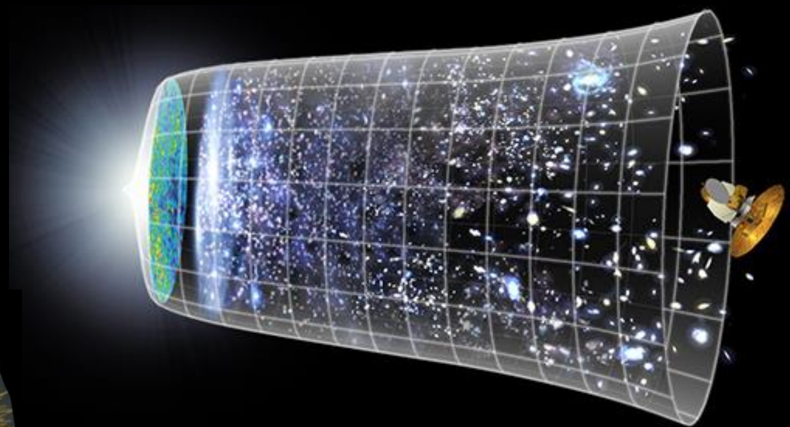
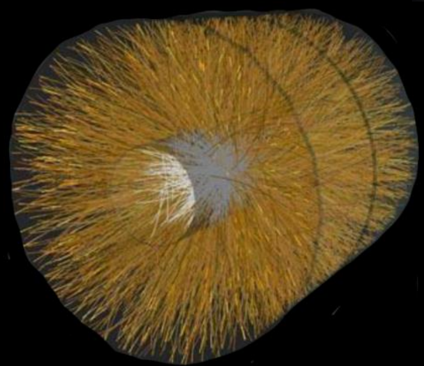
Thermalization across energy scales



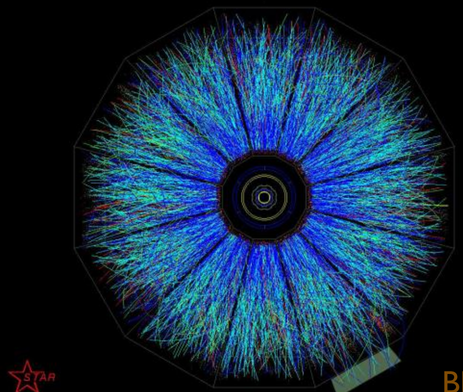
WMAP Science Team



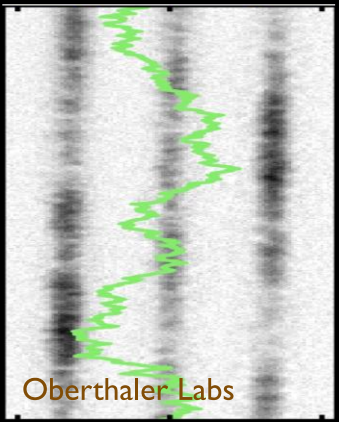
ALICE/CERN

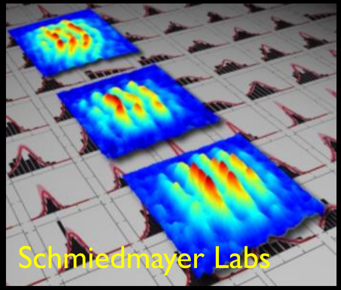
J. Berges

Heidelberg University



BNL





Initial Stages 2019

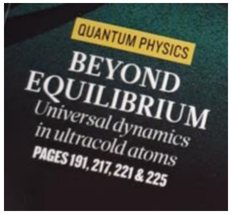
Content

- Thermalization dynamics from **initial over-occupation**
  a) heavy-ion collisions
  b) early-universe reheating
  c) ultracold quantum gases

- Early-time **nonthermal attractors** vs. thermal hydro attractors

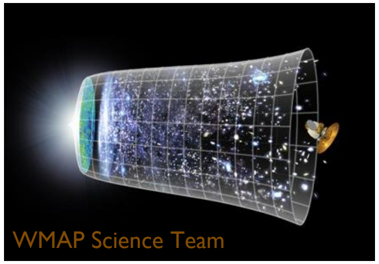
- **(Pre-)scaling** & **Bose condensation**
  a) gauge-invariant order-parameter field for QCD
  b) relativistic scalar inflaton
  c) non-relativistic Bose field

- **Experimental discovery** of nonthermal attractor in cold atomic gases



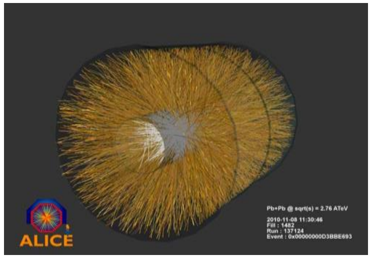
Extreme quantum systems across energy scales

Early-universe inflaton dynamics



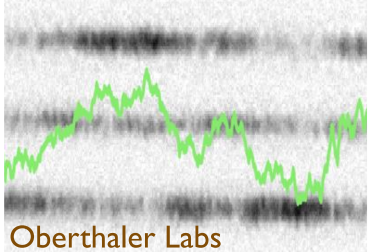
(P)reheating after inflation (~10^{16} GeV)

Heavy-ion collision experiments



Quark-Gluon Plasma (~100 MeV ~10^{12} K)

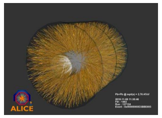
Quenches with ultracold atoms



Quantum gases at nanokelvins

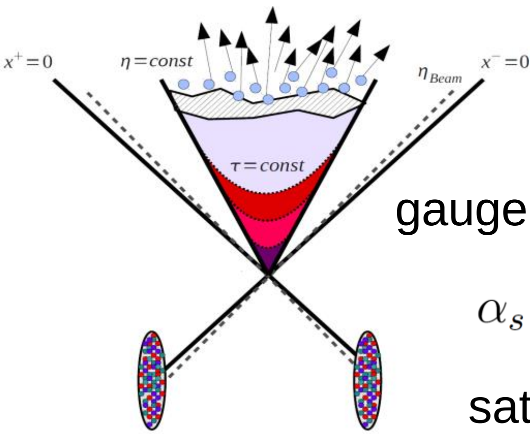
→ Thermalization dynamics with far-from-equilibrium initial conditions

Over-occupation at early times



Heavy-ion collisions in the high-energy limit

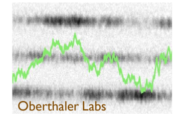
'Color Glass Condensate+Glasma':
Krasnitz, Lappi, McLerran, Venugopalan,…



gauge coupling:

$$\alpha_s(Q_s) \ll 1$$

saturation scale



Quenches in dilute Bose gases

'Bose-Einstein condensation':
Cornell, Ketterle, Wieman,…

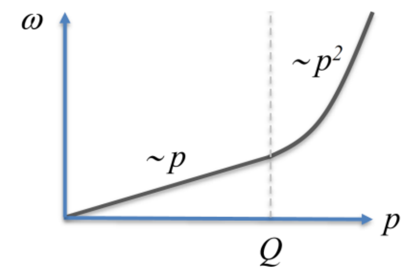
typical scattering length: $a \simeq 5$ nm
typical bulk density: $n \simeq 10^{14}$ cm^{-3}

diluteness: $\zeta \simeq 10^{-3}$

$$\zeta = \sqrt{na^3} \ll 1$$

coherence length^{-1}:

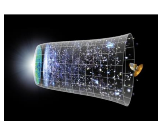
$$Q = \sqrt{16\pi a n}$$



$$\frac{\langle A^2\rangle(Q_s)}{\langle A^2\rangle_{\rm eq}(T)} \sim \frac{1}{\alpha_s}$$

Over-occupation
at time ~ $1/Q_{(s)}$

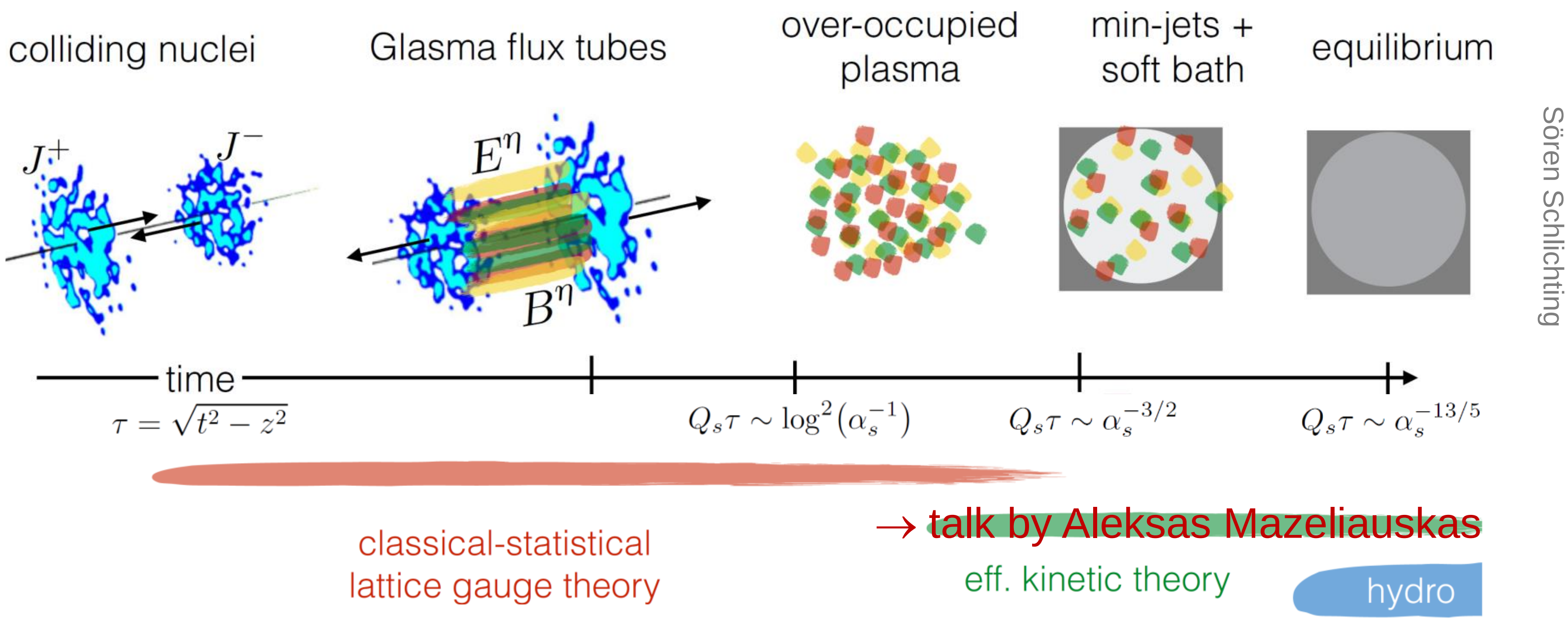
$$\frac{\langle \phi^2\rangle(Q)}{\langle \phi^2\rangle_{\rm eq}(T)} \sim \frac{1}{\zeta}$$



cf. **scalar inflaton over-occupation after preheating**

Kofman, Linde, Starobinsky,…

`Bottom-up´ thermalization for over-occupied gluons



Evolution stages of initially over-occupied gluons: Baier, Muller, Schiff, Son, *PLB* (2001)

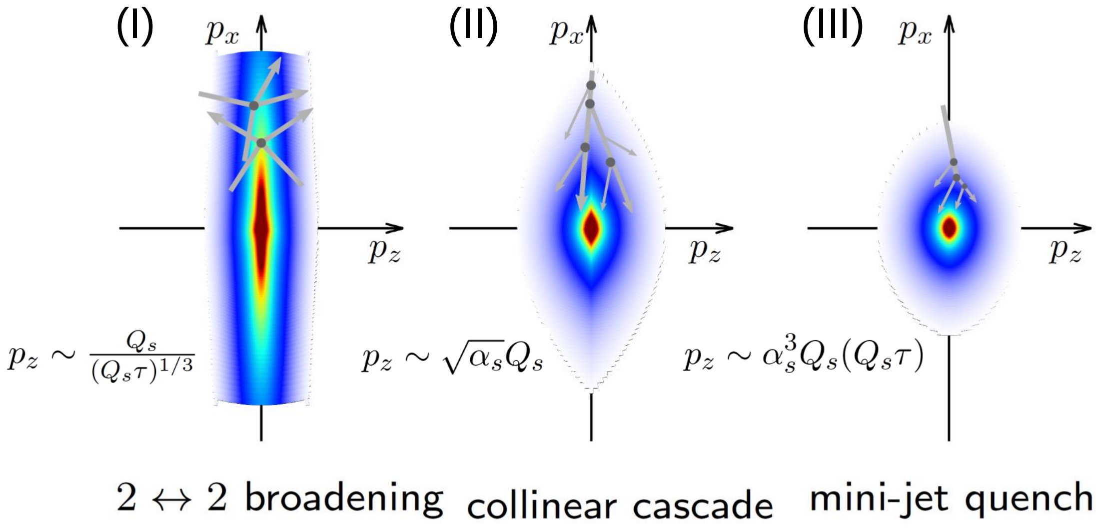
I) over-occupied $\quad p_z \sim \frac{Q_s}{(Q_s\tau)^{1/3}} \quad 1 \ll Q_s\tau \ll \alpha_s^{-3/2}$

II) under-occupied $\quad p_z \sim \sqrt{\alpha_s} Q_s \quad \alpha_s^{-3/2} \ll Q_s\tau \ll \alpha_s^{-5/2}$

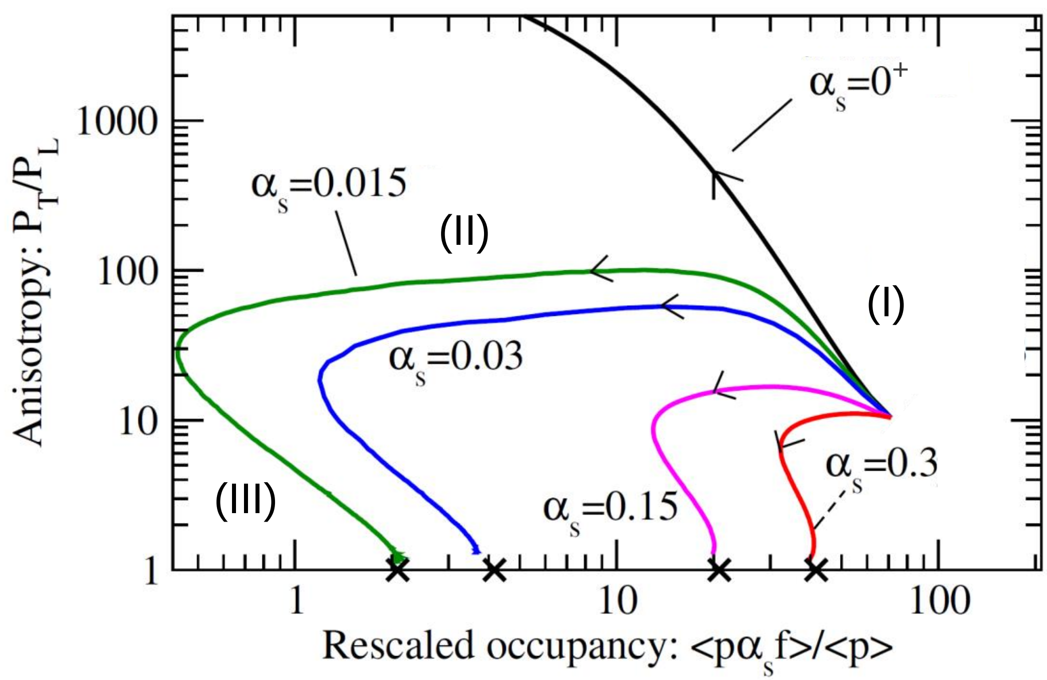
III) mini-jet quenching $\quad p_z \sim \alpha_s^3 Q_s (Q_s\tau) \quad \alpha_s^{-5/2} \ll Q_s\tau \ll \alpha_s^{-13/5}$

Berges, Boguslavski, Schlichting, Venugopalan, *PRD* (2014); Kurkela, Zhu, *PRL* (2015); Keegan, Kurkela, Mazeliauskas, Teaney, *JHEP* (2016); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, *PRL* (2019)

`Bottom-up´ thermalization for over-occupied gluons

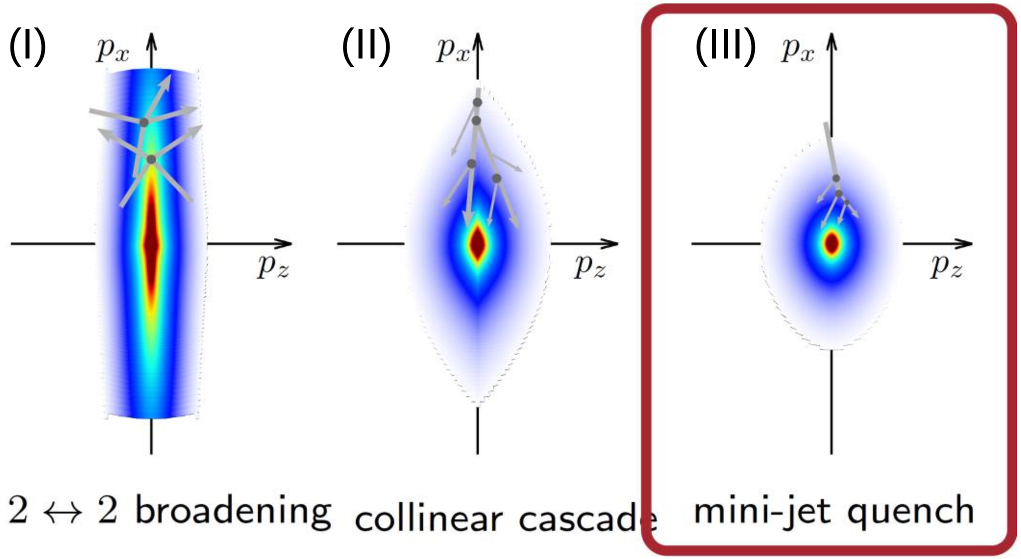


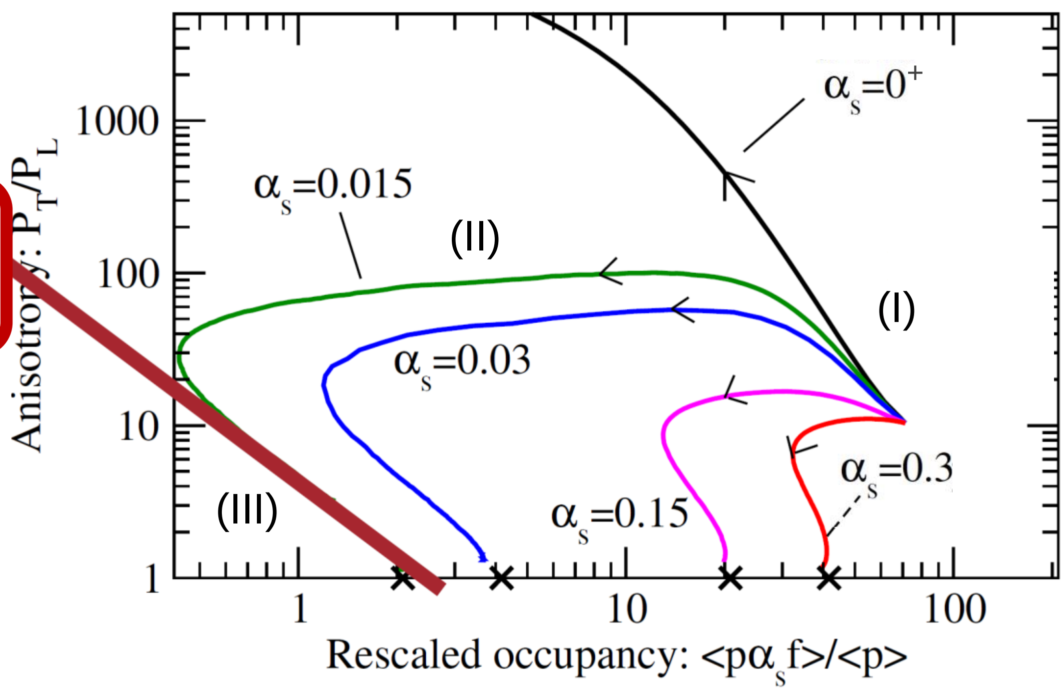
Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, *PRC* (2019)



AMY gluon kinetic theory: Kurkela, Zhu, *PRL* (2015)

Late-time hydrodynamic attractor

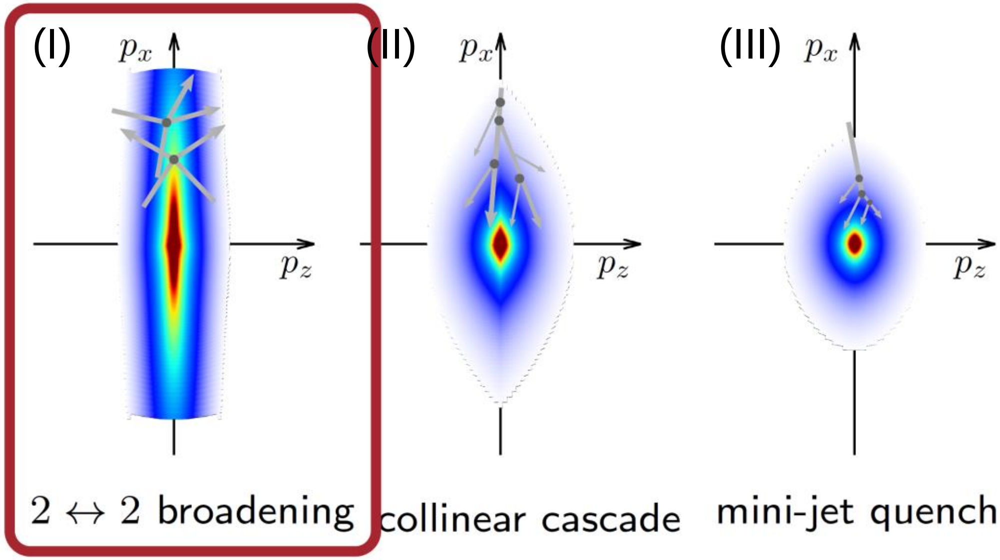


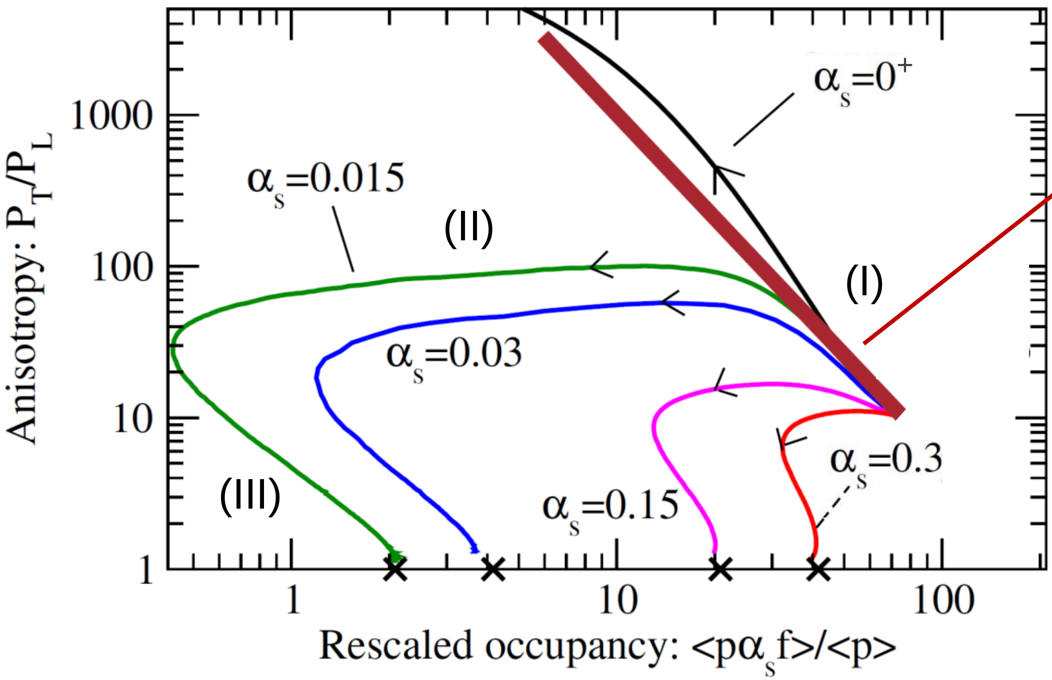


hydro attractor

→ talk by Michal Heller

Early-time nonthermal attractor





nonthermal attractor

Cl.-stat.:
Berges, Boguslavski, Schlichting, Venugopalan, *PRD* (2014) …
Gluon kinetic:
Kurkela, Zhu, *PRL* (2015)
Full QCD kinetic:
Mazeliauskas, Berges, *PRL* (2019)

(Pre-)scaling & condensation from nonthermal attractors

Inflaton/rel. scalar dynamics

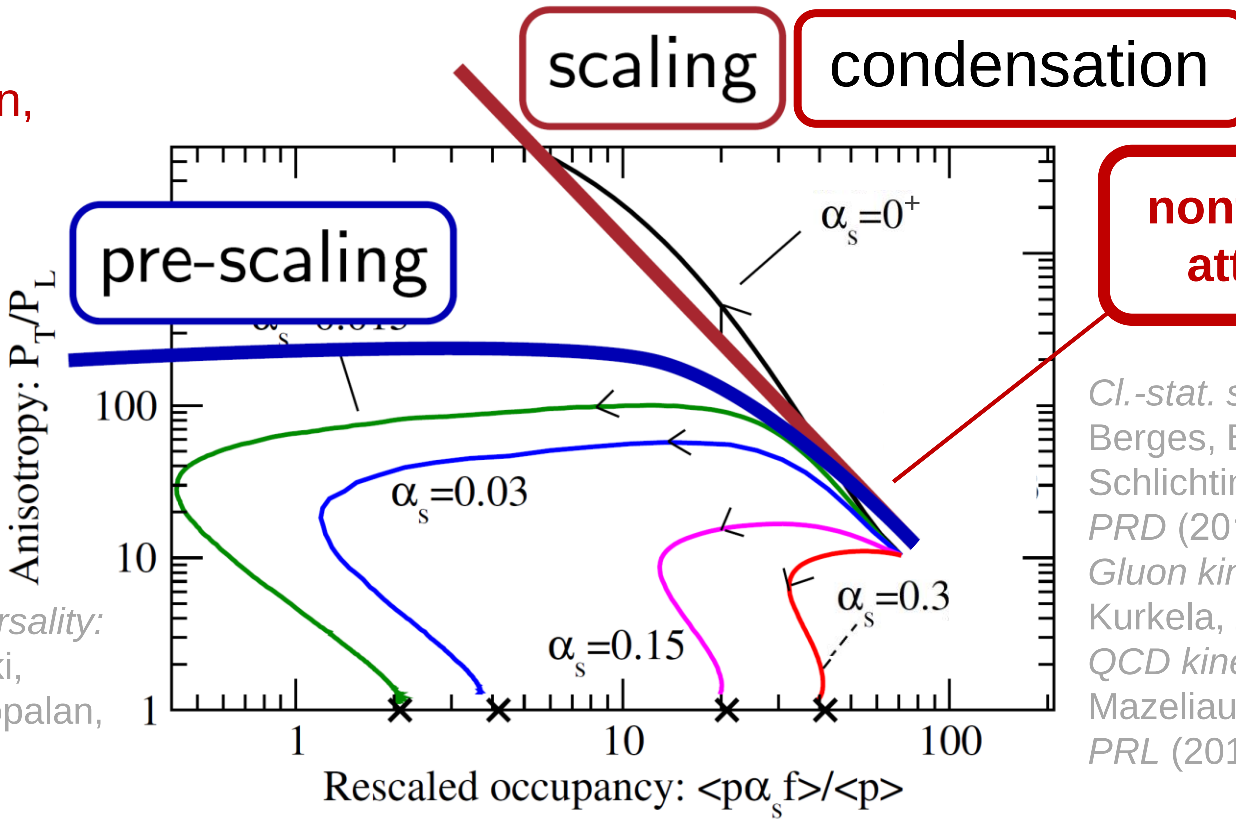
UV scaling: Micha, Tkatchev, *PRL* (2003); *IR scaling:* Berges, Rothkopf, *PRL* (2008); *Condensation:* Berges, Sexty, *PRL* (2012); Berges, Boguslavski, Schlichting Venugopalan, *PRD, JHEP* (2014); Moore, *PRD* (2016); Deng, Schlichting, Venugopalan, Wang, *PRA* (2018); Walz, Boguslavski, Berges, *PRD* (2018) …

Ultracold quantum gases

Theory: Pineiro Orioli, Boguslavski, Berges, *PRD* (2015); Karl, Gasenzer, *NJP* (2017); Chantesana, Pineiro Orioli, Gasenzer, *PRA* (2019); Pineiro Orioli, Berges, *PRL* (2019); Schmied, Mikheev, Gasenzer, *PRL* (2019) … *Experiment:* Prüfer et al., *Nature* (2018); Erne et al., *Nature* (2018)

→ talks by Jarkko Peuron, Qun Wang

Spectral function: Boguslavski, Kurkela, Lappi, Peuron, *PRD* (2018) … *Plasmon mass:* Lappi, Peuron, *PRD* (2017,2018) *Gauge/scalar universality:* Berges, Boguslavski, Schlichting, Venugopalan, *PRL* (2015)…

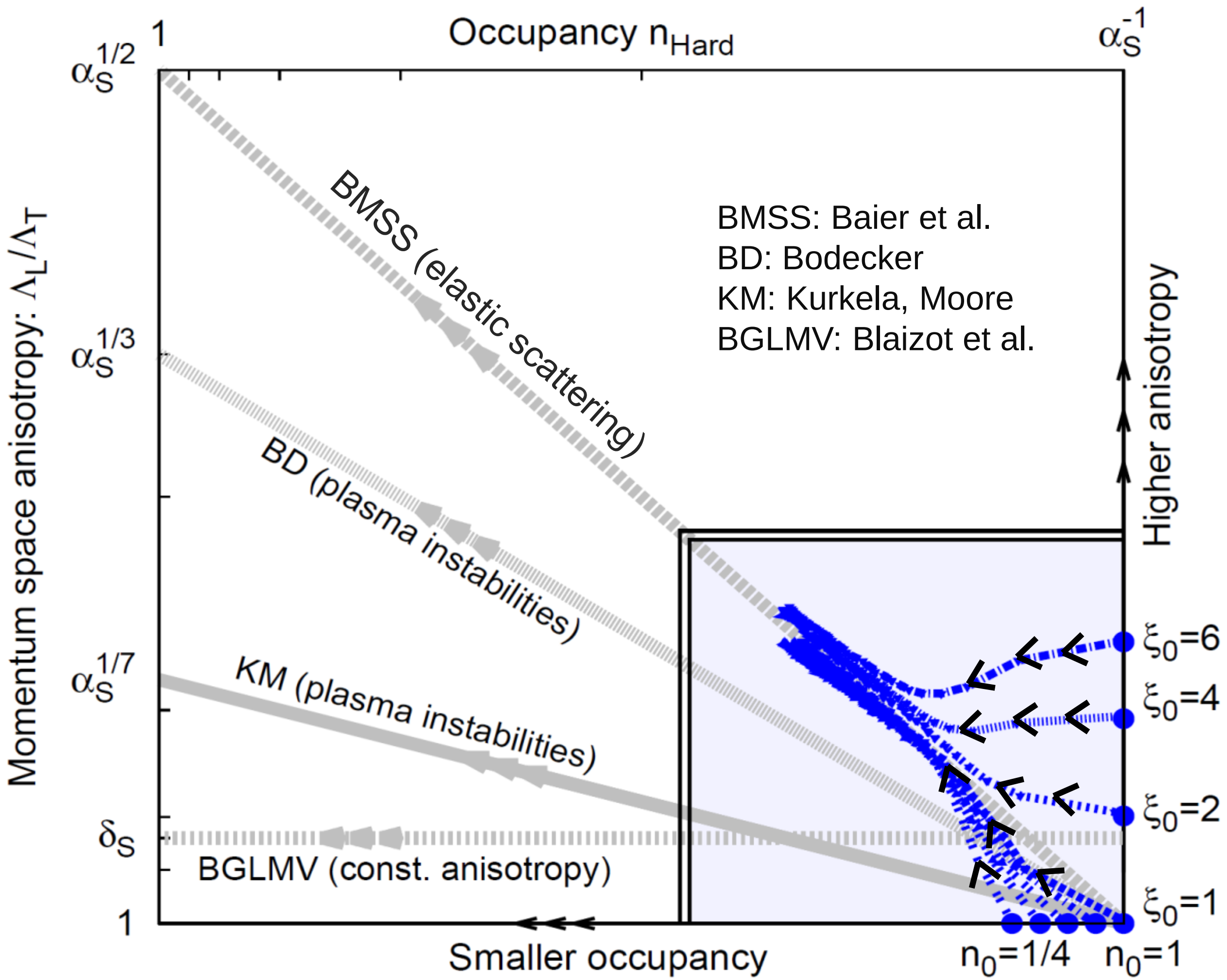


nonthermal attractor

Cl.-stat. scaling: Berges, Boguslavski, Schlichting, Venugopalan, *PRD* (2014) … *Gluon kinetic scaling:* Kurkela, Zhu, *PRL* (2015) *QCD kinetic (pre-)scaling:* Mazeliauskas, Berges, *PRL* (2019)

Nonthermal attractor for over-occupied gluons

Evolution in the `anisotropy-occupancy plane´



Lattice data
($256^2\times4096$)

Berges, Boguslavski, Schlichting, Venugopalan, *PRD* (2014)

→ Early stage of `bottom-up´ scaling emerges as a consequence of the attractor

(Pre-)scaling around nonthermal fixed-point distribution

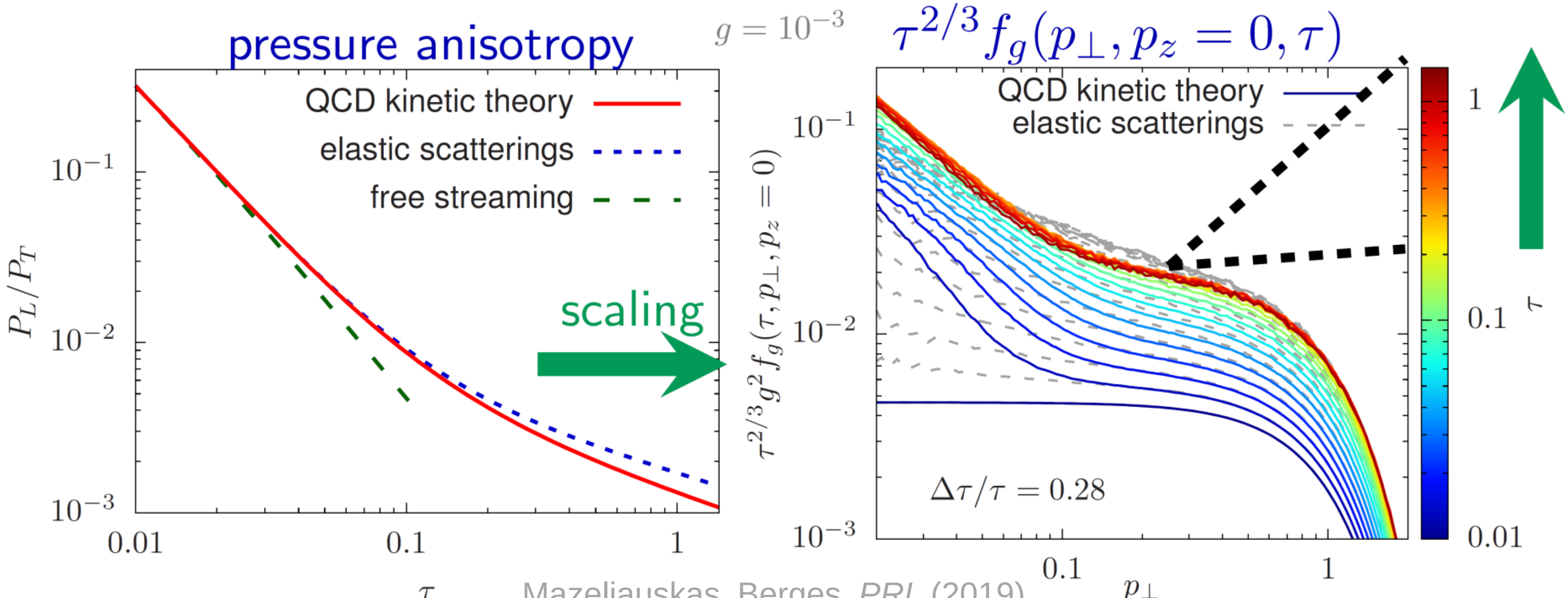
Effective loss of information due to nonthermal attractor:

gluon distribution function — *(pre-)scaling exponents*

$$f_g(p_\perp, p_z, \tau) \quad = \quad \tau^{\alpha(\tau)} f_S\left(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z\right)$$

universal fixed-point distribution

→ ***Scaling limit at weak couplings: universal exponents*** $\alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$



Mazeliauskas, Berges, *PRL* (2019)

→ ***Approach to nonthermal fixed point in full leading-order QCD kinetic theory***

Nonthermal hydrdynamics from pre-scaling

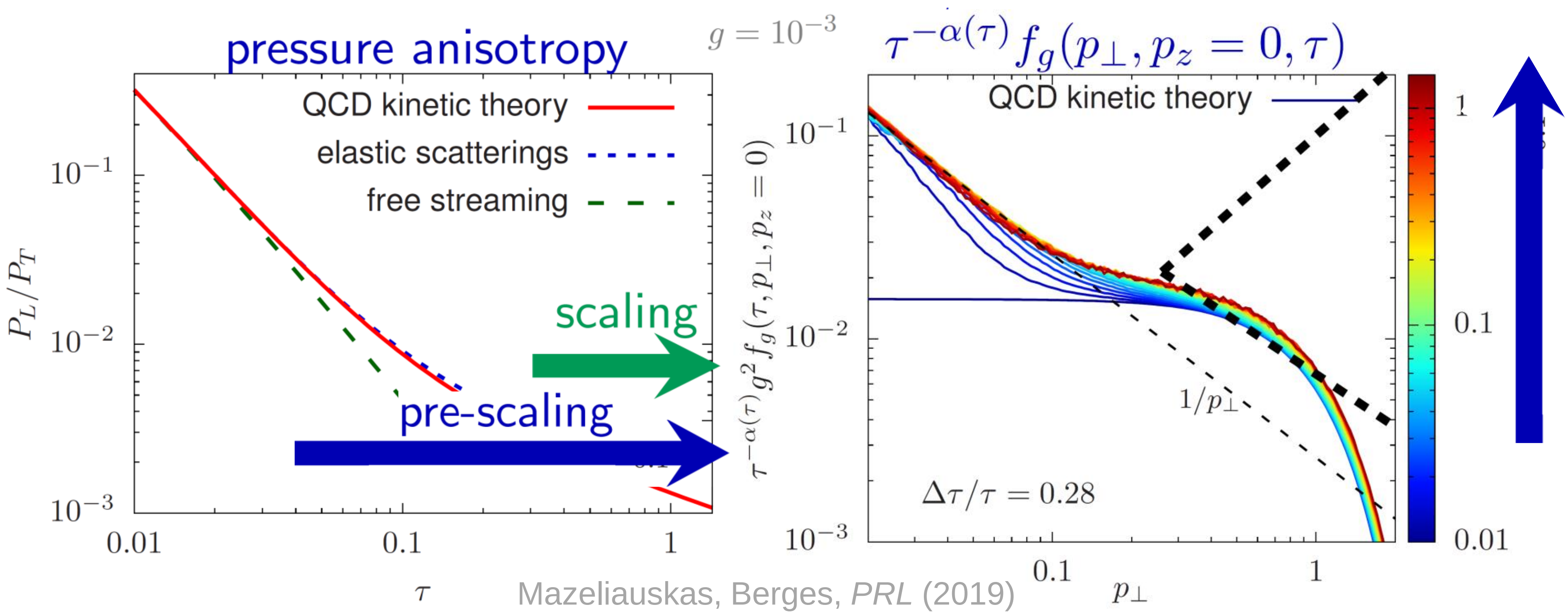
Effective loss of information due to nonthermal attractor:

gluon distribution function — (pre-)scaling exponents

$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S\left(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z\right)$$

universal fixed-point distribution

→ ***Pre-scaling: non-universal time-dependent exponents = hydrodynamic modes***



Mazeliauskas, Berges, *PRL* (2019)

→ ***Universal fixed-point distribution established already in pre-scaling regime***

Beyond the first stage of `bottom-up´ thermalization

Effective loss of information due to nonthermal attractor:

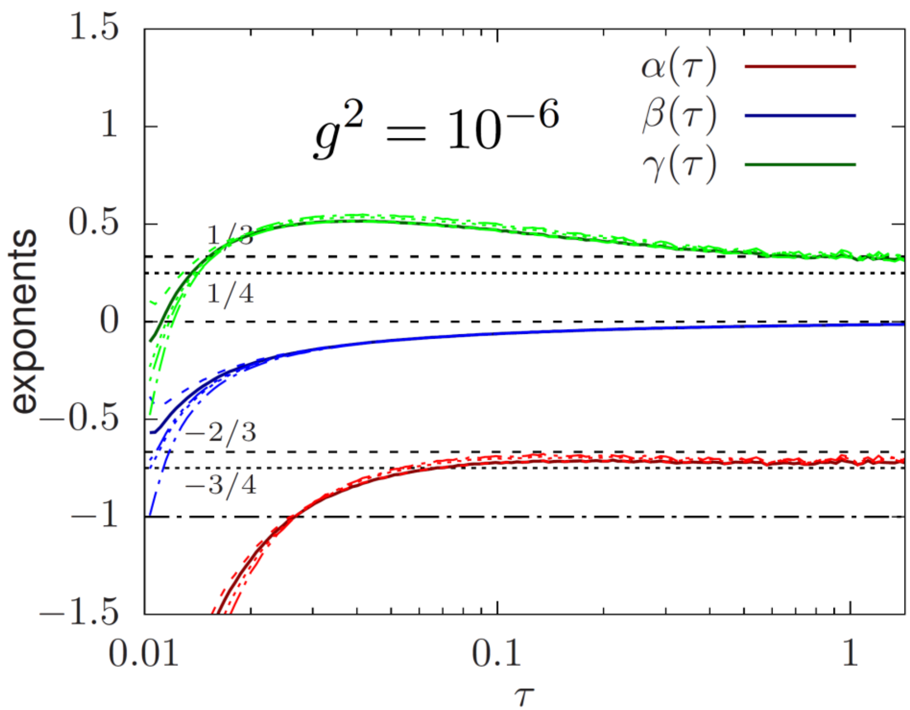
gluon distribution function — *(pre-)scaling exponents*

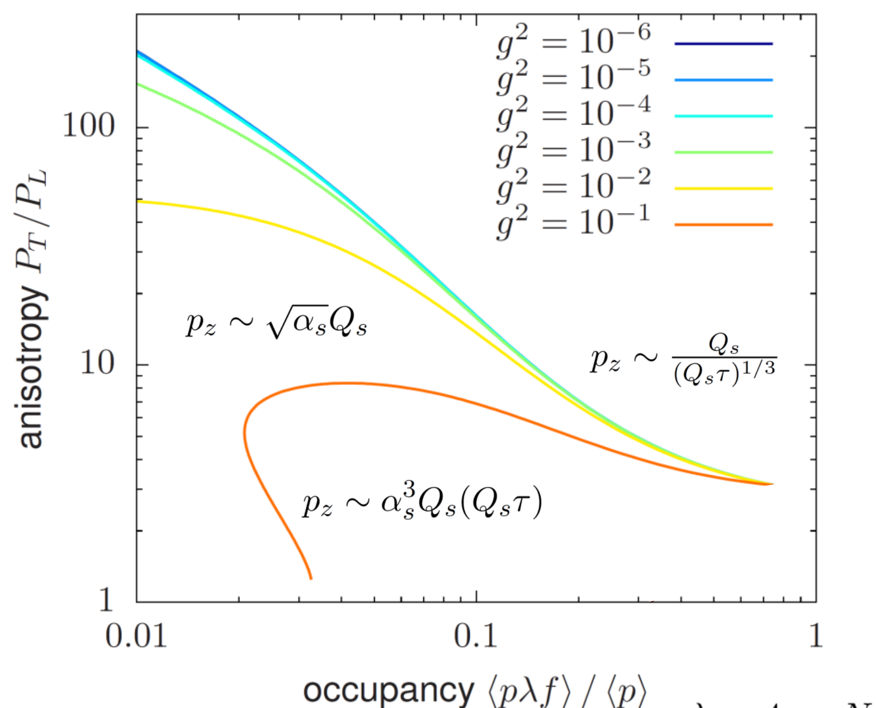
$$f_g(p_\perp, p_z, \tau) = \tau^{\alpha(\tau)} f_S\left(\tau^{\beta(\tau)} p_\perp, \tau^{\gamma(\tau)} p_z\right)$$

universal fixed-point distribution

Vary the coupling constant $\alpha_s = g^2/(4\pi)$

Berges, Mikheev, Mazeliauskas, *in prep.*





$\lambda = 4\pi\alpha_s N_c$

Beyond the first stage of `bottom-up´ thermalization

Effective loss of information due to nonthermal attractor:

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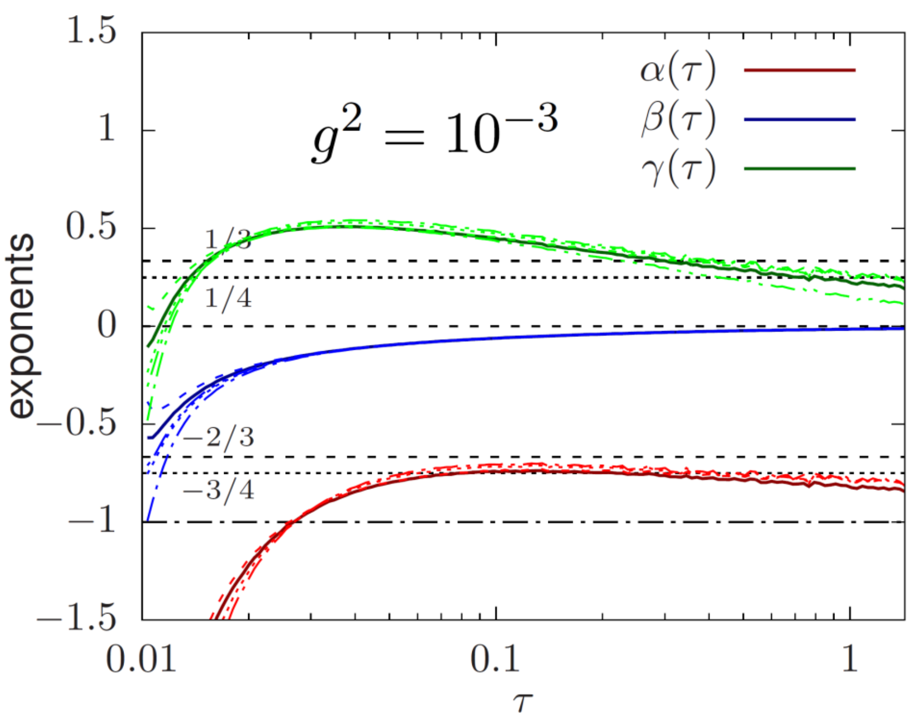

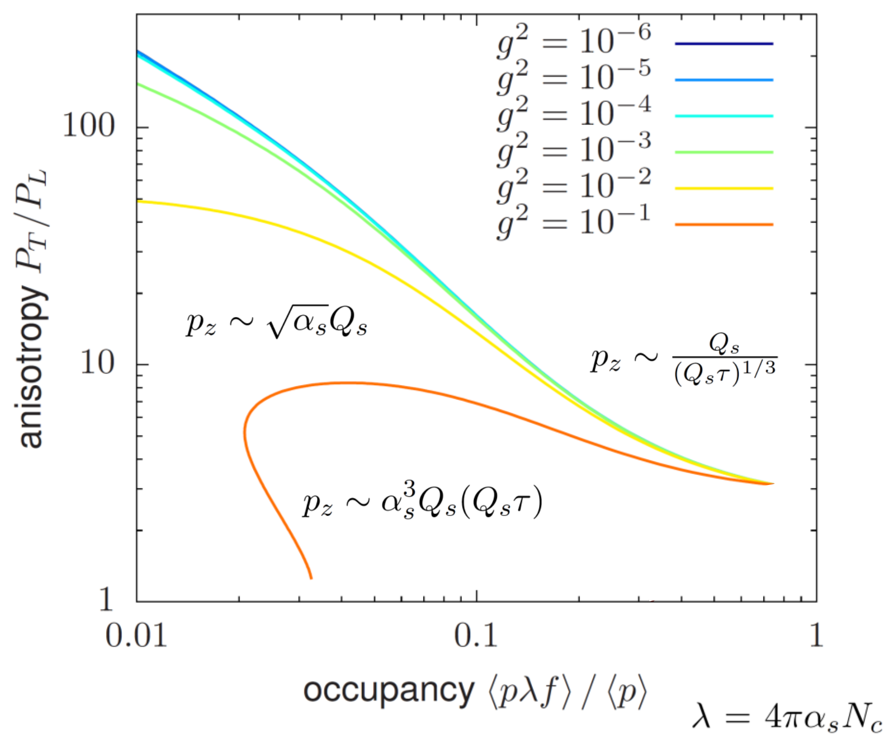

Beyond the first stage of ´bottom-up´ thermalization

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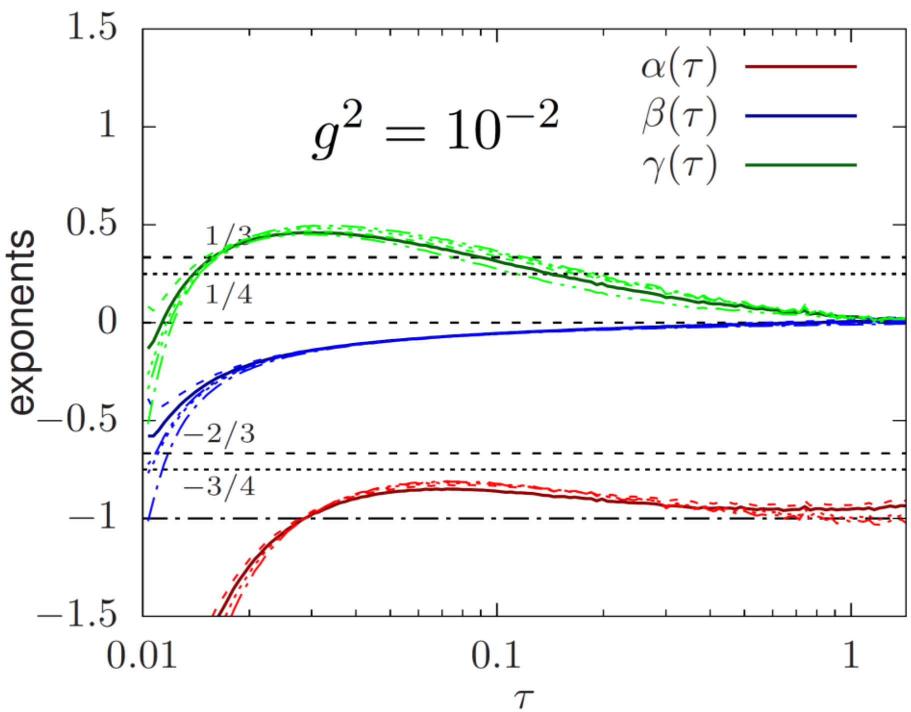

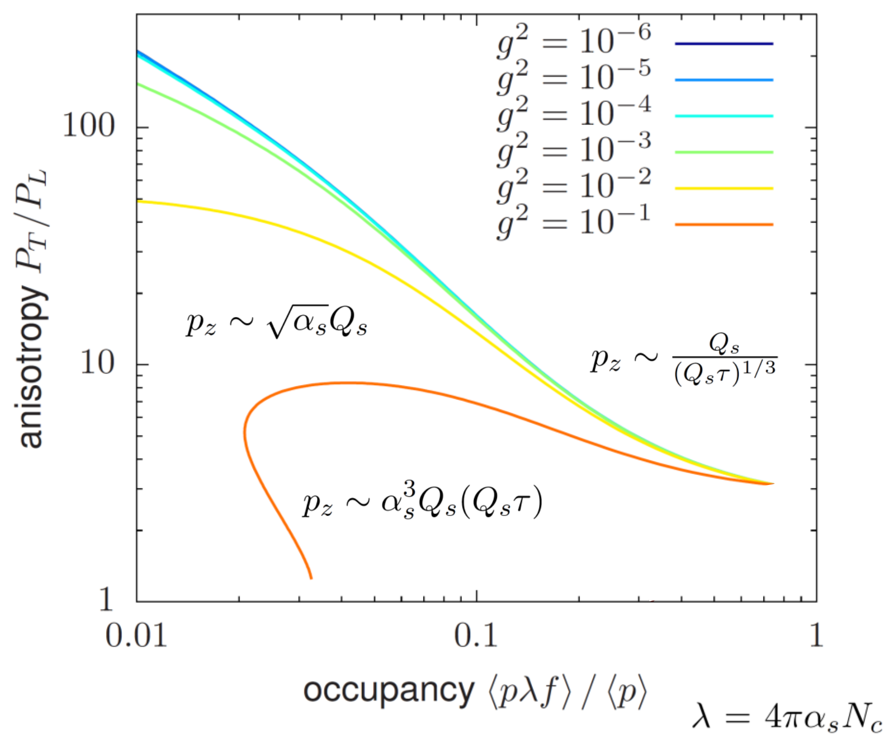

Beyond the first stage of `bottom-up´ thermalization

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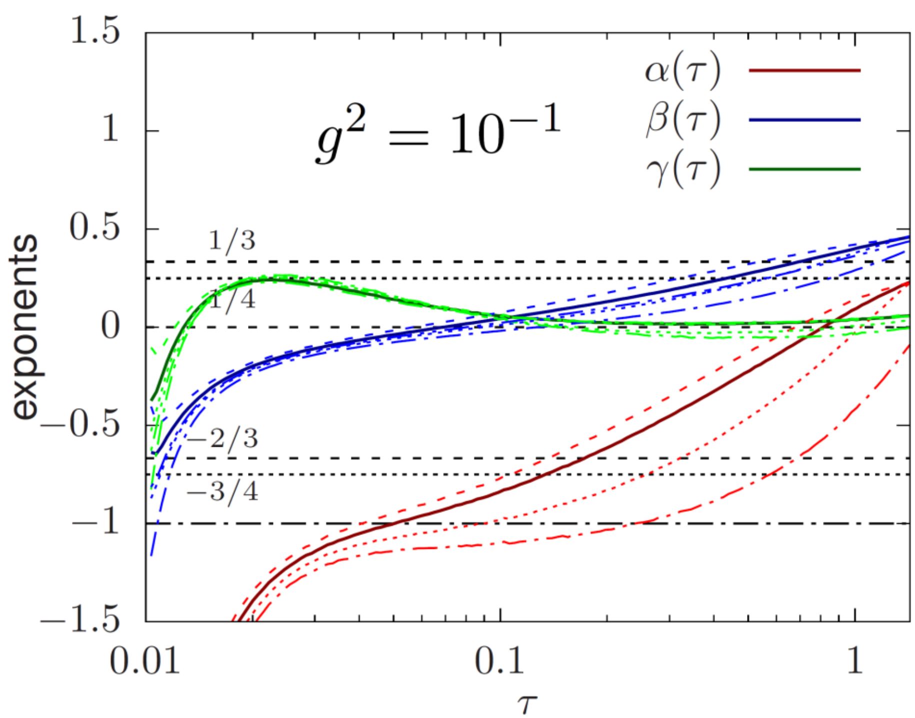
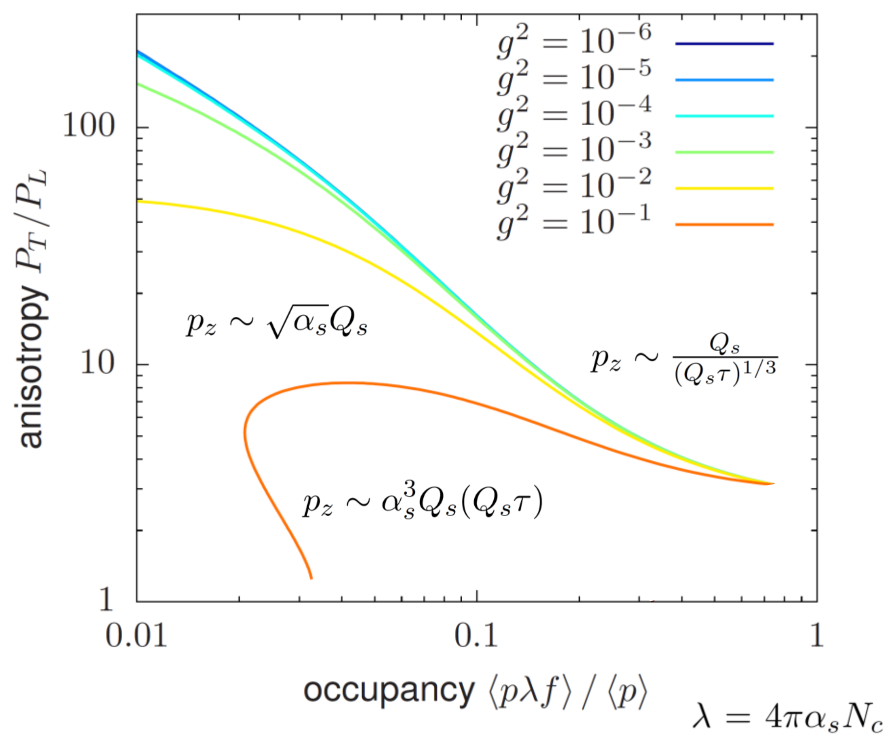


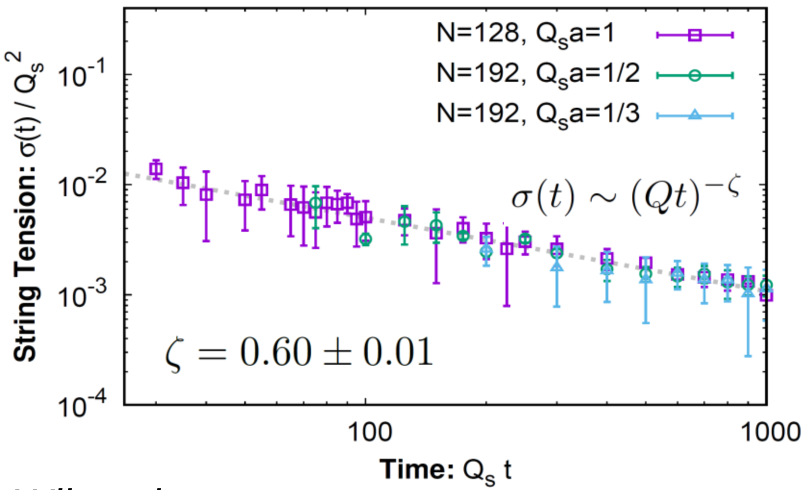


Separation of scales far from equilibrium

Consider for a moment initial over-occupation in fixed box (no expansion):

Soft scale evolution

Berges, Scheffler, Sexty, *PRD* (2008); Mace, Schlichting, Venugopalan, *PRD* (2016); Berges, Mace, Schlichting, *PRL* (2017)



Wilson loop:

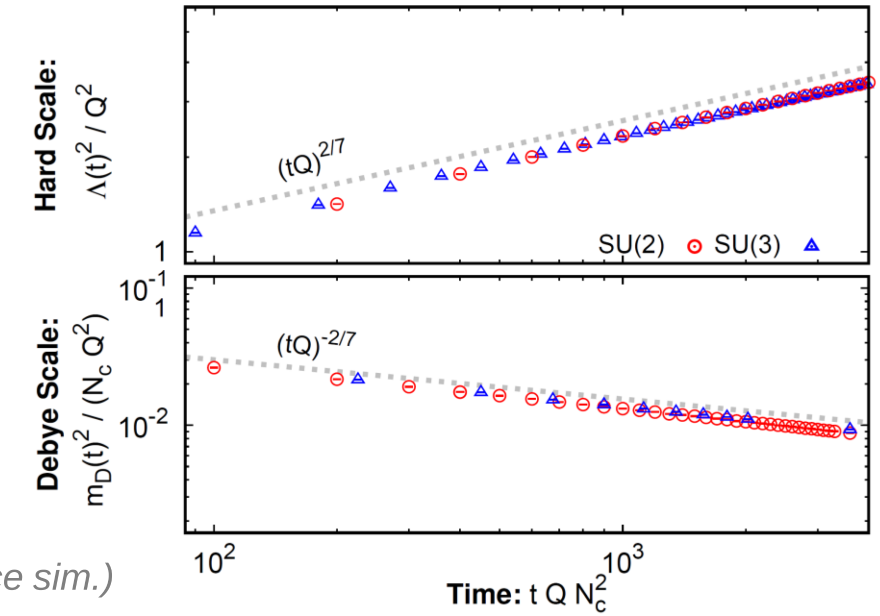
(cl.-stat. lattice sim.)

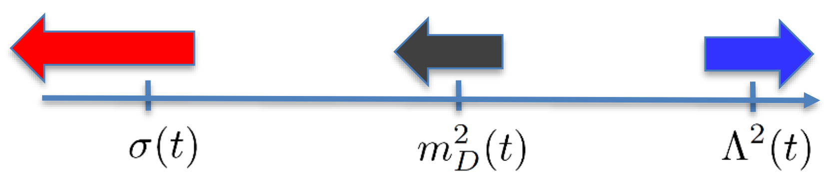
$$W = \frac{1}{N_c} \mathrm{Tr}\, \mathcal{P} e^{ig \oint_C dx^\mu A_\mu(x)}$$

$$\sigma(t) = -\frac{\partial \log \langle W \rangle (t, A)}{\partial A}$$

Hard scale evolution

Schlichting, *PRD* (2012); Kurkela, Moore, *PRD* (2012);…





→ ***Dynamical separation of soft non-perturbative scale and perturbative scales***

Bose condensation from initial over-occupation

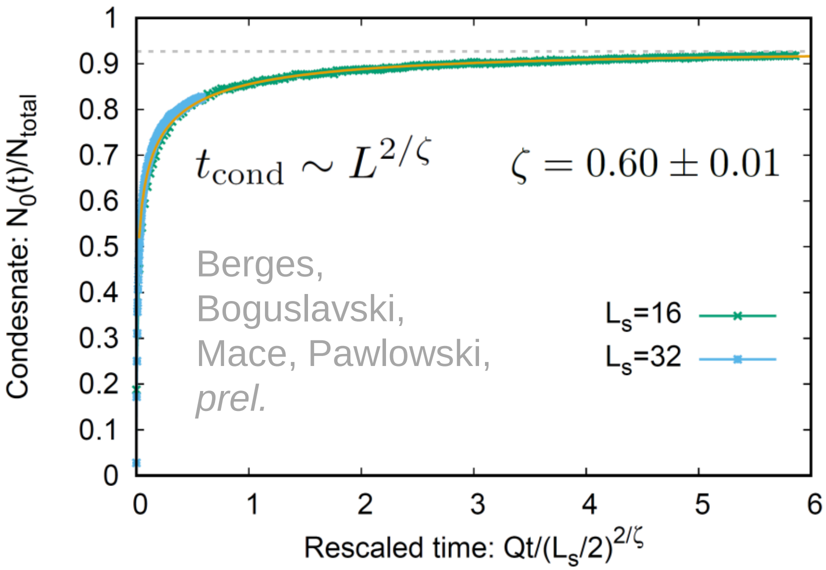
Gauge field theory

closed spatial Wilson lines

$$\langle W(\Delta x, L, t)\rangle = \langle P(\mathbf{x}_1, L, t)\, P^\dagger(\mathbf{x}_2, L, t)\rangle$$

$$\frac{N_0(t,L)}{N_{\rm total}} = \frac{1}{V}\int_0^L d^d x\, \langle W(\Delta x, L, t)\rangle$$

$$\overset{\text{scaling}}{=} \frac{1}{V}\int_0^L d^d \Delta x\, \omega_S(\Delta x\, L/t^\zeta)$$



→ ***Gauge-invariant condensation!***

Cf. proposed & disputed vector field condensation:

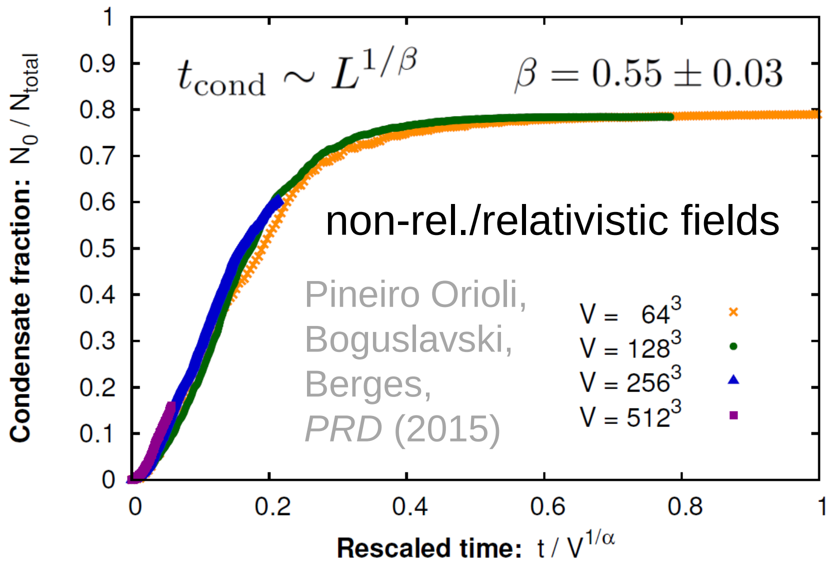
Blaizot, Gelis, Liao, McLerran, Venugopalan, *NPB* (2012);
Kurkela, Moore, *PRD* (2012); Floerchinger, Wetterich *JHEP* (2014)

Scalar field theory

$$N^\phi_{\rm total} = \langle \phi(\mathbf{x},t)\, \phi^\dagger(\mathbf{x},t)\rangle$$

$$\frac{N^\phi_0(t)}{N^\phi_{\rm total}} = \frac{1}{N^\phi_{\rm total} V}\int_0^L d^d x\, \langle \phi(x)\phi^\dagger(0)\rangle$$

$$\overset{\text{scaling}}{=} \frac{1}{N^\phi_{\rm total} V}\int_0^L d^d \Delta x\, f_S(\Delta x/t^\beta)$$



→ ***Scalar condensation***

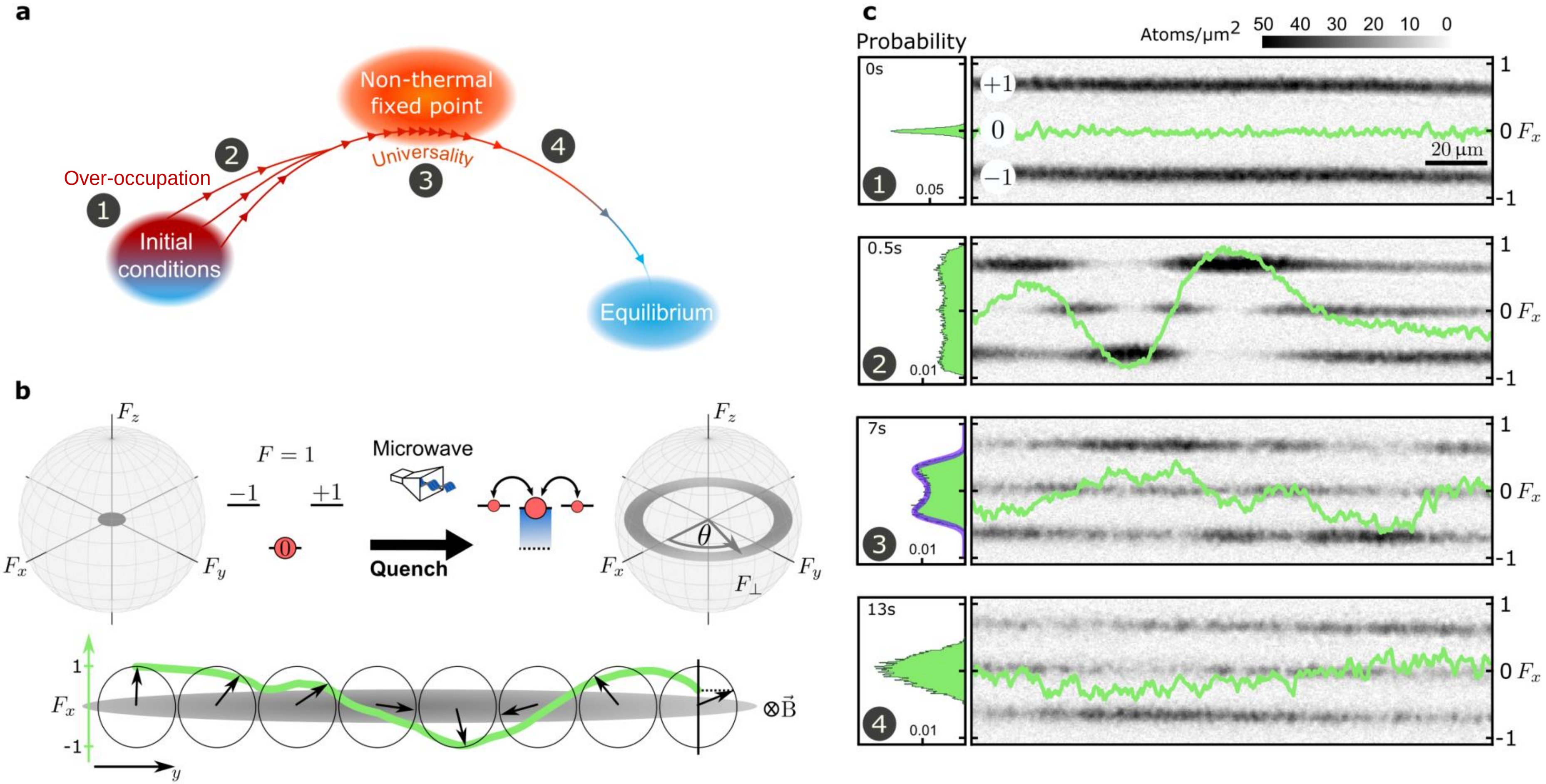
Also with longitudinal expansion:

Berges, Boguslavski, Schlichting, Venugopalan, *PRD* (2015)

Observing nonthermal attractor scaling with cold atoms

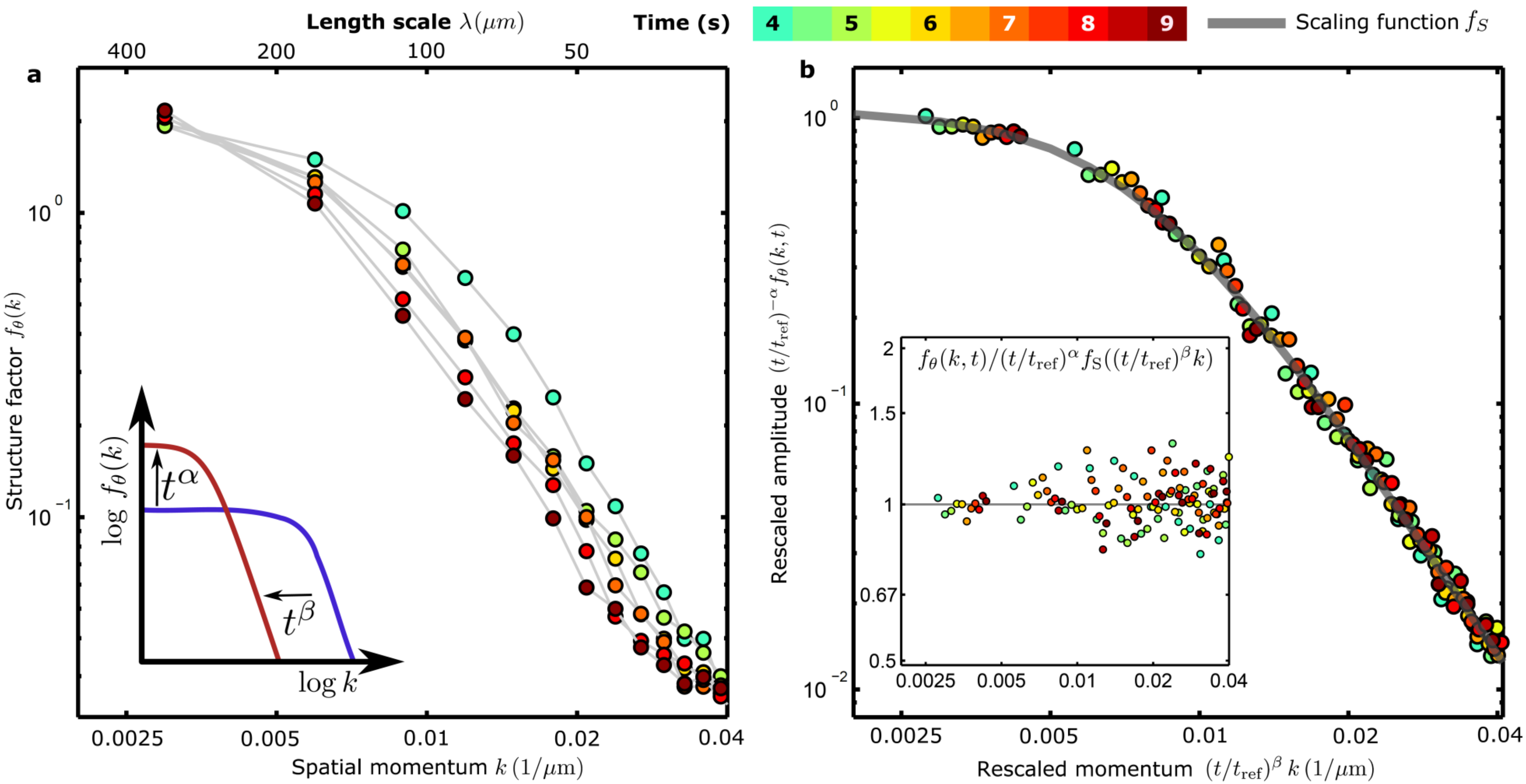
^{87}Rb BEC in a quasi 1D optical trap: Oberthaler BEC labs @ SynQS

Spin-1 BEC (F=1 hyperfine state with magnetic sublevels m_F = 0, ±1)



Prüfer, Kunkel, Strobel, Lanning, Linnemann, Schmied, Berges, Gasenzer, Oberthaler, *Nature* (2018)
See also Erne, Bücker, Gasenzer, Berges, Schmiedmayer, *Nature* (2018)

Scaling exponents and fixed-point distribution



(d=1) $\alpha = 0.33 \pm 0.08\,, \quad \beta = 0.54 \pm 0.06$

$$f_\theta(k,t) = t^\alpha f_S\left(t^\beta k\right)$$

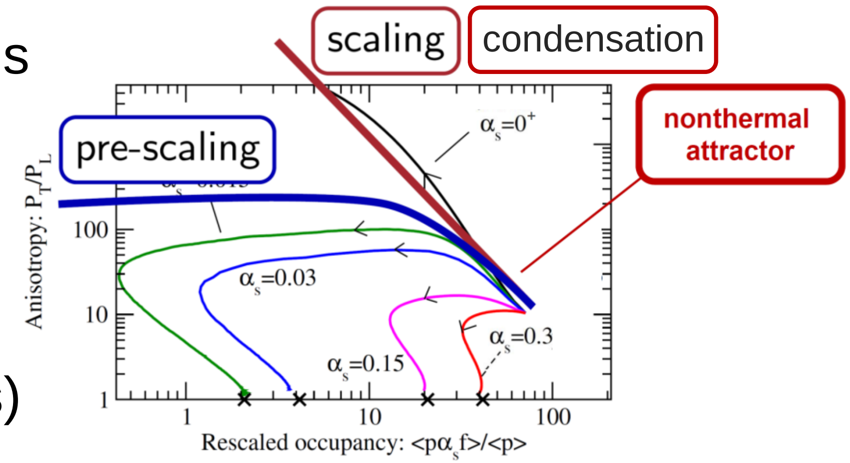
Prüfer, Kunkel, Strobel, Lanning, Linnemann, Schmied, Berges, Gasenzer, Oberthaler, *Nature* (2018)

Conclusions

Thermalization across energy scales starting from over-occupation:

- ***Nonthermal attractors***

→ early effective loss of microscopic details



- ***Universal scaling at weak coupling***

→ new links between very different systems (heavy-ion collisions/early-universe reheating/cold-atom quenches)

- ***Pre-scaling***

→ early hydrodynamic evolution with nonthermal fixed-point distribution (rather than thermal distribution in conventional hydrodynamics)

- ***Bose condensation***

→ far-from-equilibrium formation of a (gauge-invariant) Bose condensate