HYDRO ATTRACTORS

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Based on **1503.07514** with Spaliński, works by many people including some of you (for a partial review, see **1707.02282** with Florkowski and Spaliński) and **1907.xxxxx** with Jefferson, Spaliński and Svensson

See also the upcoming talks by **Noronha (today at 2 PM)**, **Almaalol (today at 3:40 PM)**, **Denicol (today at 5 PM)** and a poster by **Shi**

Introduction

(Relativistic) hydro(dynamics) in HIC	=	a mean for modelling QCD evolution for some observables over a certain time frame

In an ideal world with full theoretical control over QCD one would not NEED to solve hydro eqns etc — we would have the prediction anyway

Since we do not, we should at least understand to the best of our abilities what hydro is, when it works and how to construct hydro eqns

Last decade: ⟶ progress on understanding the emergence of hydro regime in microscopic models ranging from holographic QFTs to kinetic theory

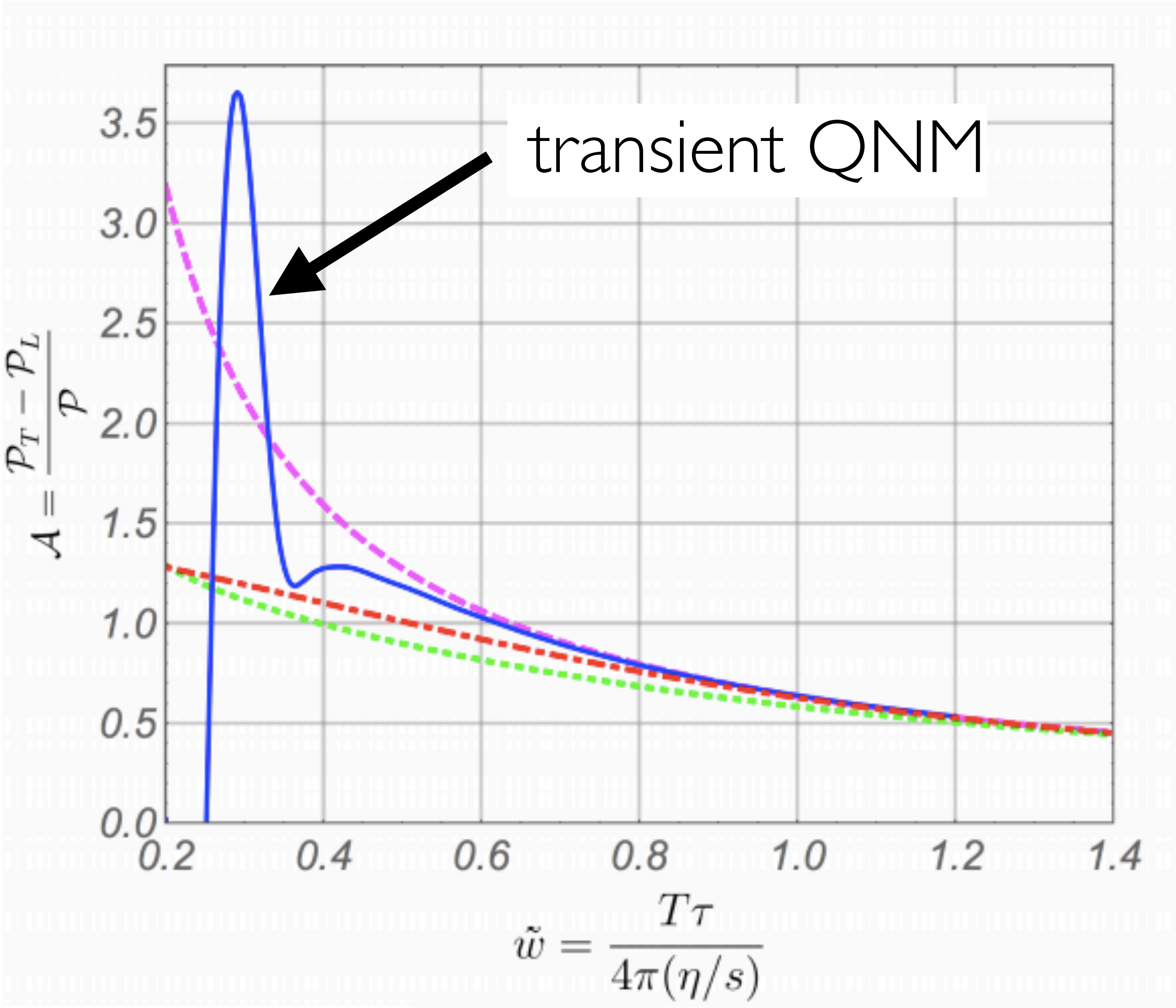
hydro attractors	=	a surprising spin-off of these studies having to do with a discovery that hydro works much better than expected

Hydro far from equilibrium

see **1609.02820** by Romatschke for a viewpoint

Ab initio studies in holography and later studies in other models show that viscous hydro can work even when deviations from local equilibrium are large:

0906.4426, 1011.3562 by Chesler & Yaffe; **1103.3452** with Janik & Witaszczyk



sample n-eq states in:

N=4 SYM (holography)

EKT with $\eta/s = 0.624$

RTA with $\eta/s = 0.624$

viscous hydro prediction:

$$\mathcal{A} = \frac{2}{\pi}\tilde{w}^{-1}$$

plot from **1609.04803v2** with Kurkela, Spalinski & Svensson

C.f. a textbook definition of hydro

hydrodynamics is an EFT of the slow (?) evolution of conserved currents in collective media close to equilibrium (?)

DOFs: always local energy density ϵ and local flow velocity u^μ $\;(u_\nu u^\nu = -1)$

EOMs: conservation eqns $\nabla_\mu \langle T^{\mu\nu}\rangle = 0$ for $\langle T^{\mu\nu}\rangle$ expanded in gradients

$\pi^{\mu\nu}$

$$\langle T^{\mu\nu}\rangle = \epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\} - \eta(\epsilon)\, \sigma^{\mu\nu} - \zeta(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\}(\nabla \cdot u) + \ldots$$

microscopic input:

EoS
$\left(P(\epsilon) = \frac{1}{3}\epsilon\right.$ for CFTs)

shear viscosity contribution

$$\mathcal{A} = \frac{2}{\pi}\tilde{w}^{-1}$$

bulk viscosity
(vanishes for CFTs)

Hydro theories

The crucial subtlety: $\nabla_\mu\Big(\epsilon\, u^\mu u^\nu + P(\epsilon)\{ g^{\mu\nu} + u^\mu u^\nu \} - \eta(\epsilon)\, \sigma^{\mu\nu} + .. \Big) = 0$ does not have a well-posed initial value problem $\longrightarrow$ hydrodynamic theories

Overall idea: make $\pi^{\mu\nu}$ obey an independent PDE ensuring its ↘ to $-\eta\, \sigma^{\mu\nu}$

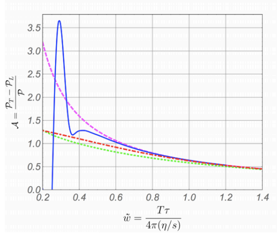
$$(\tau_\pi u^\alpha \mathcal{D}_\alpha + 1)\ [\pi^{\mu\nu} - (-\eta\, \sigma^{\mu\nu})] = 0 \longrightarrow \pi^{\mu\nu} = -\eta\, \sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, \pi^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, (\eta\, \sigma^{\mu\nu})$$

decay timescale

Müller 1967, Israel 1976, Israel & Stewart 1976

New incarnation: **0712.2451** by Baier, Romatschke, Son, Starinets & Stephanov

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} + \lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha}$$

Hydro theories = hydro + transients; interesting to redo  in BRSSS

The first (BRSSS) hydro attractor 1503.07514 with Spaliński

conservation (always the same) $\longrightarrow$ $$\frac{\tau}{w}\frac{dw}{d\tau} = \frac{2}{3} + \frac{1}{18}\mathcal{A}$$

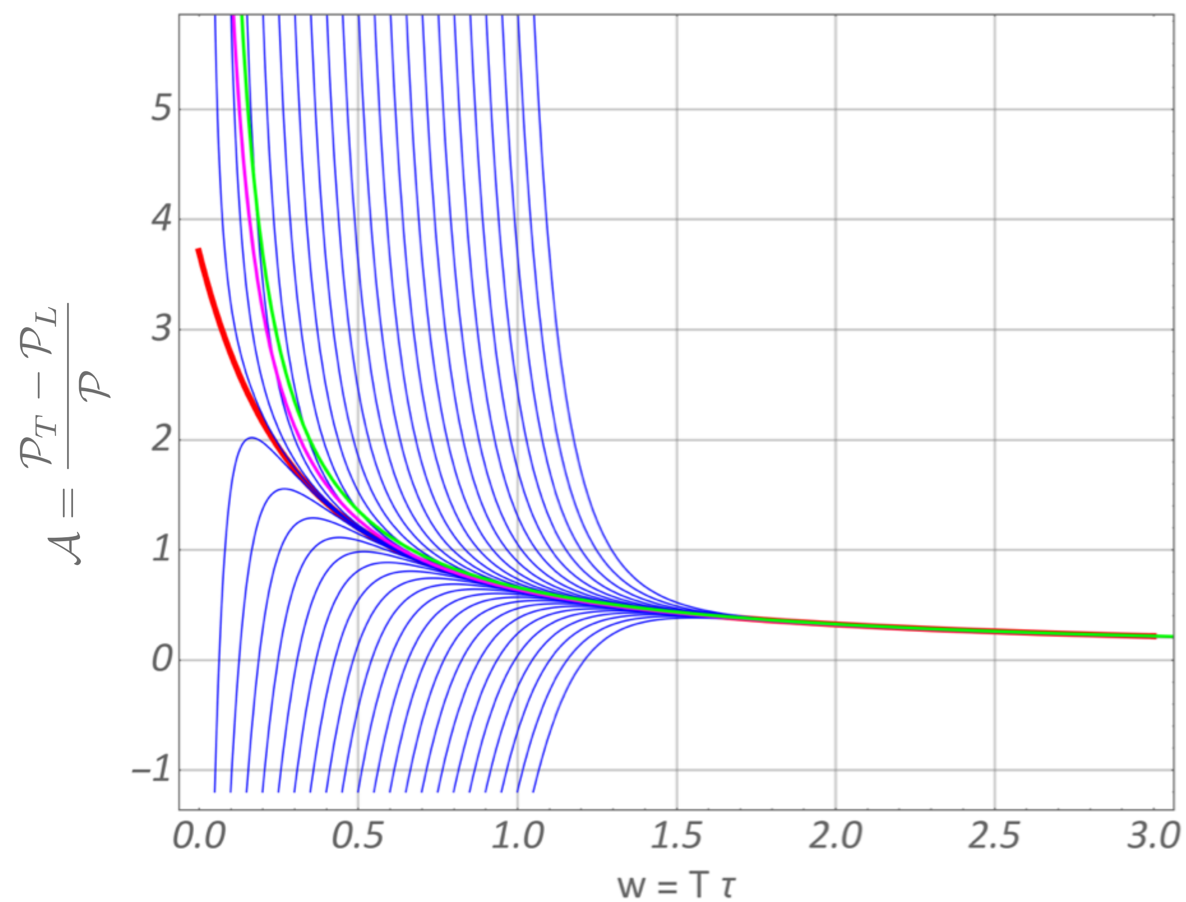
$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} +$$
$$+\,\lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha}$$

$$\Big(\eta = C_\eta\, \mathcal{S}, \qquad \tau_\pi = \frac{C_{\tau_\pi}}{T}, \qquad \lambda_1 = C_{\lambda_1}\frac{\eta}{T}$$

$$C_\eta = \frac{1}{4\pi}, \qquad C_{\tau_\pi} = \frac{2-\log 2}{2\pi}, \qquad C_{\lambda_1} = \frac{1}{2\pi}$$

$\rightarrow$ $$C_{\tau_\pi}\, w\,(1 + \frac{1}{12}\mathcal{A})\,\mathcal{A}' + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right)\mathcal{A}^2 + \frac{3}{2}\, w\, \mathcal{A} - 12\, C_\eta = 0$$

The attractor:



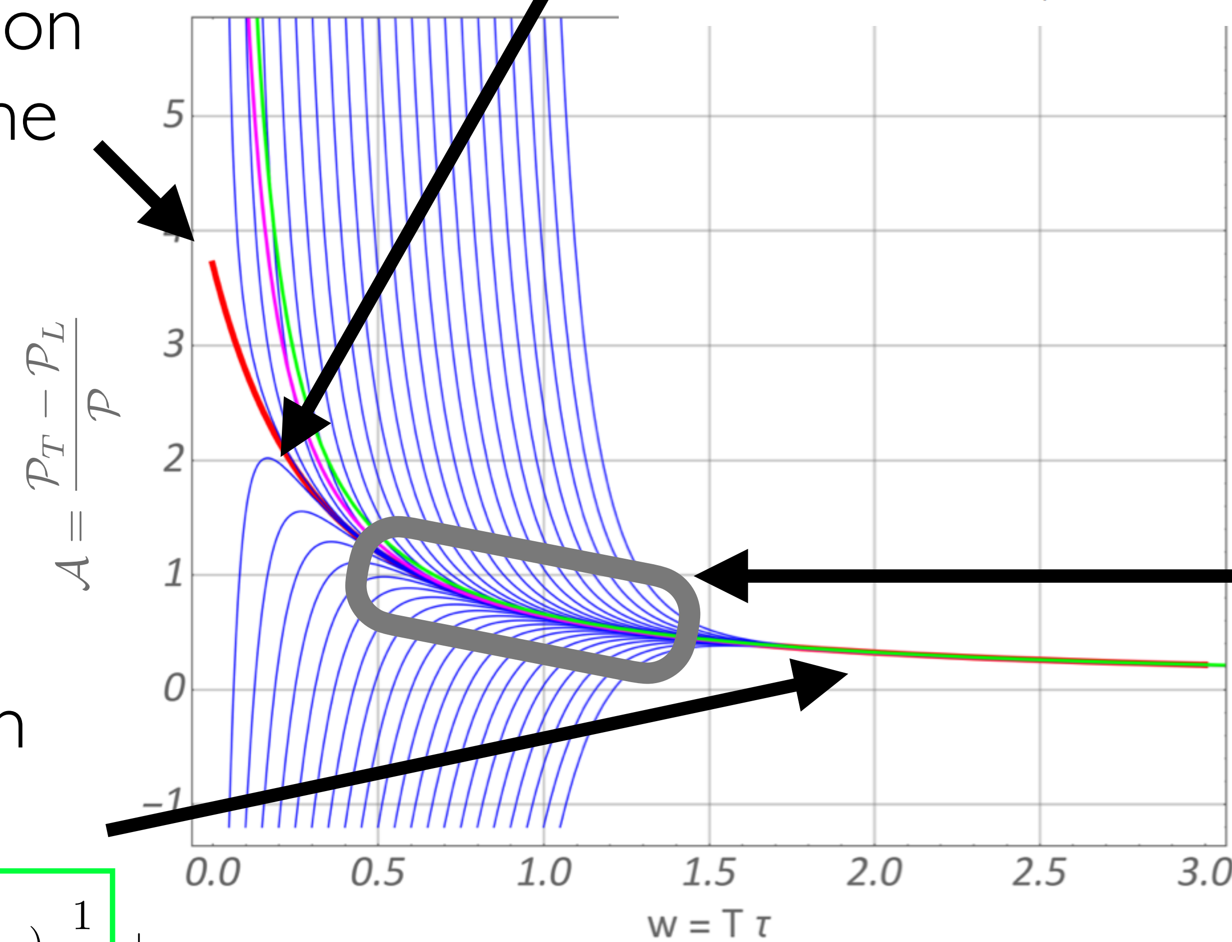
The inner workings of the BRSSS attractor

1503.07514 with Spaliński

At w = 0 the only finite value of $\mathcal{A}$ leading to a sensible solution is the attractor one

"slow roll" approximates this attractor:

$$C_{\tau_\pi}\, w \left(1 + \frac{1}{12}\mathcal{A}\right)\mathcal{A}' + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta}w\right)\mathcal{A}^2 + \frac{3}{2}\, w\, \mathcal{A} - 12\, C_\eta = 0$$



exponential approach through a transient QNM

1st or 2nd order gradient expansion works well:

$$\mathcal{A}_H(w) = 8\, C_\eta \frac{1}{w} + \frac{16}{3} C_\eta \left(C_{\tau_\pi} - C_{\lambda_1}\right) \frac{1}{w^2} + \ldots$$

divergent series

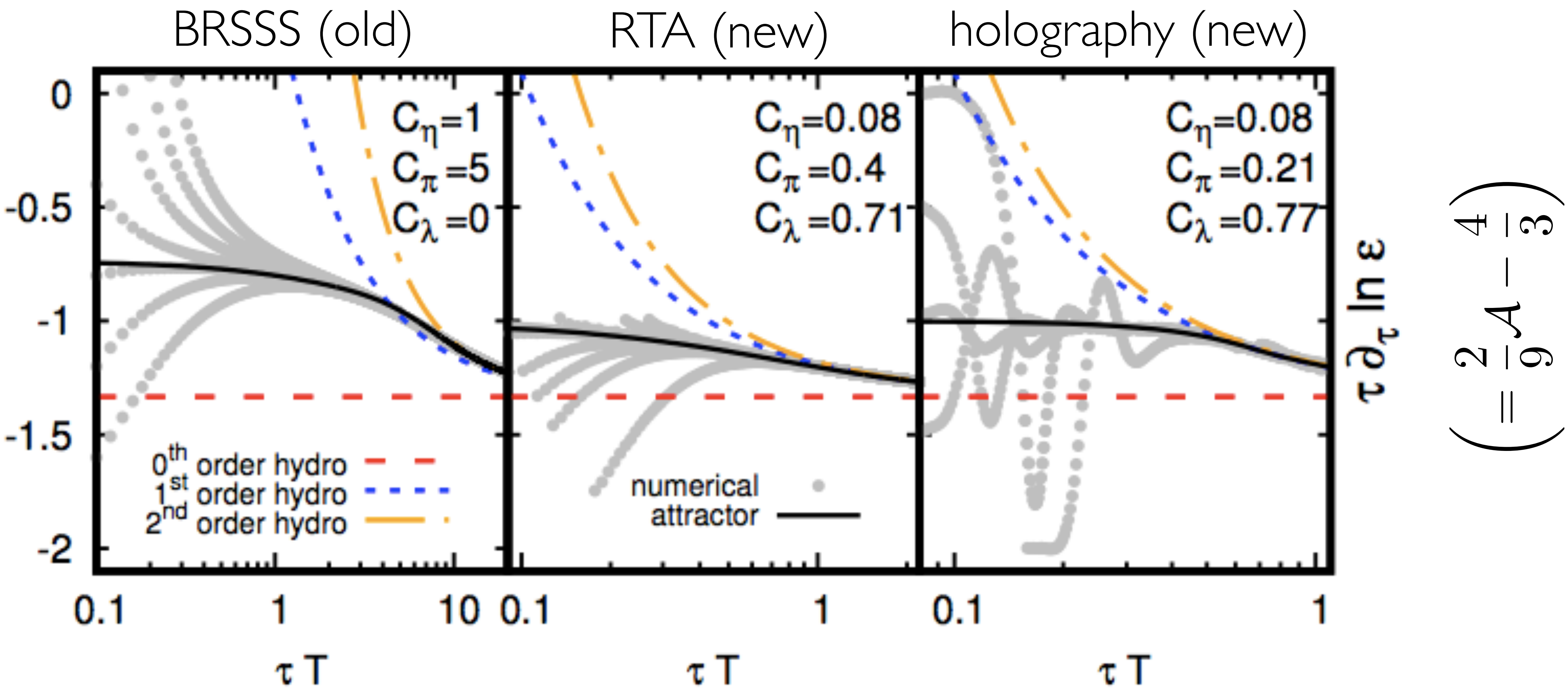
attractor = a natural resummation

Attractors in hydrodynamics include also **1511.06358** (HJSW), **1709.06644** (DNMR&aHydro), **1711.01657** (DNMR), **1801.10173** (aHydro), **1804.04771** (MIS) and **1808.07038** (aHydro)

Attractors beyond BRSSS: conformal bif

1704.08699 by Romatschke (figure adapted from arXiv)

Idea: use ~slow roll approximation to generate attractors in other theories



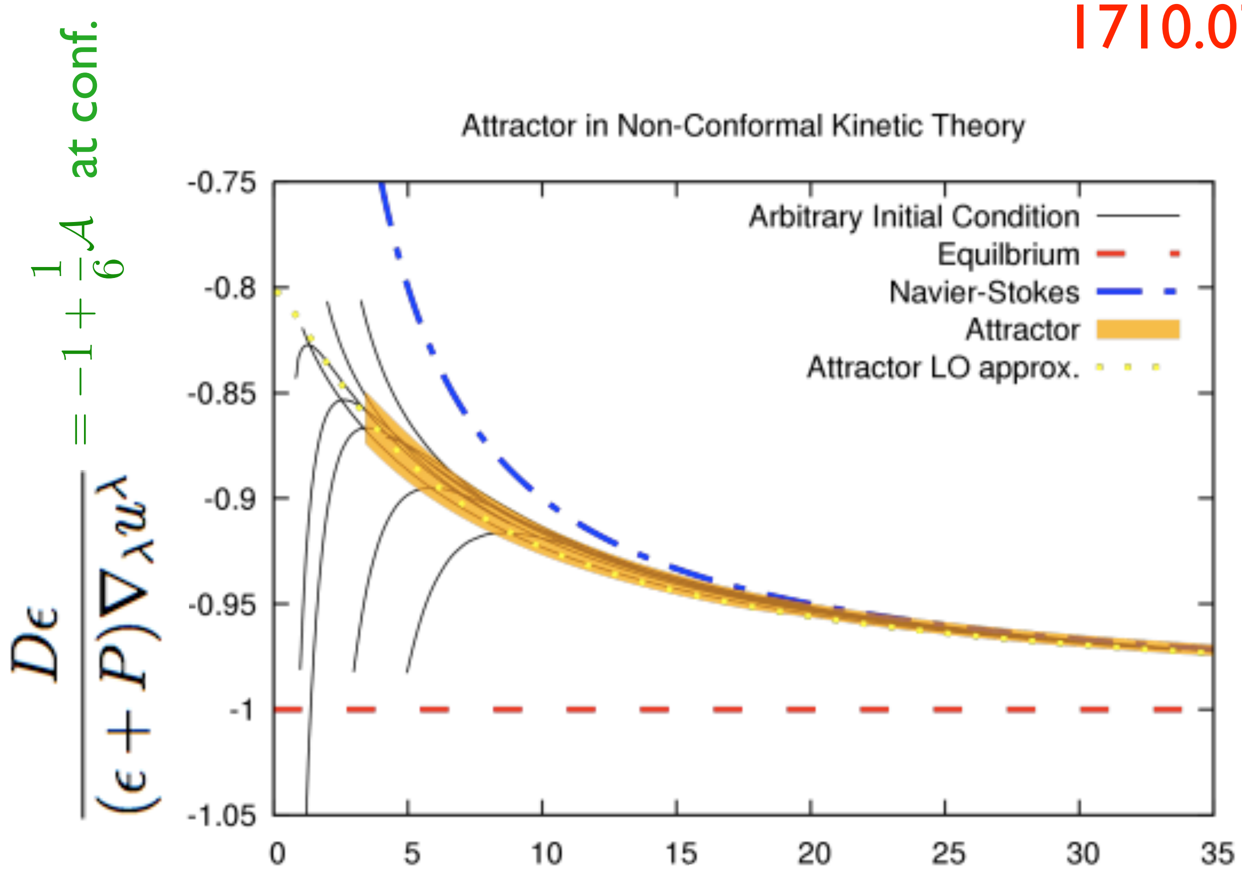
Note: centre and right are projections from infinitely-dimensional phase space

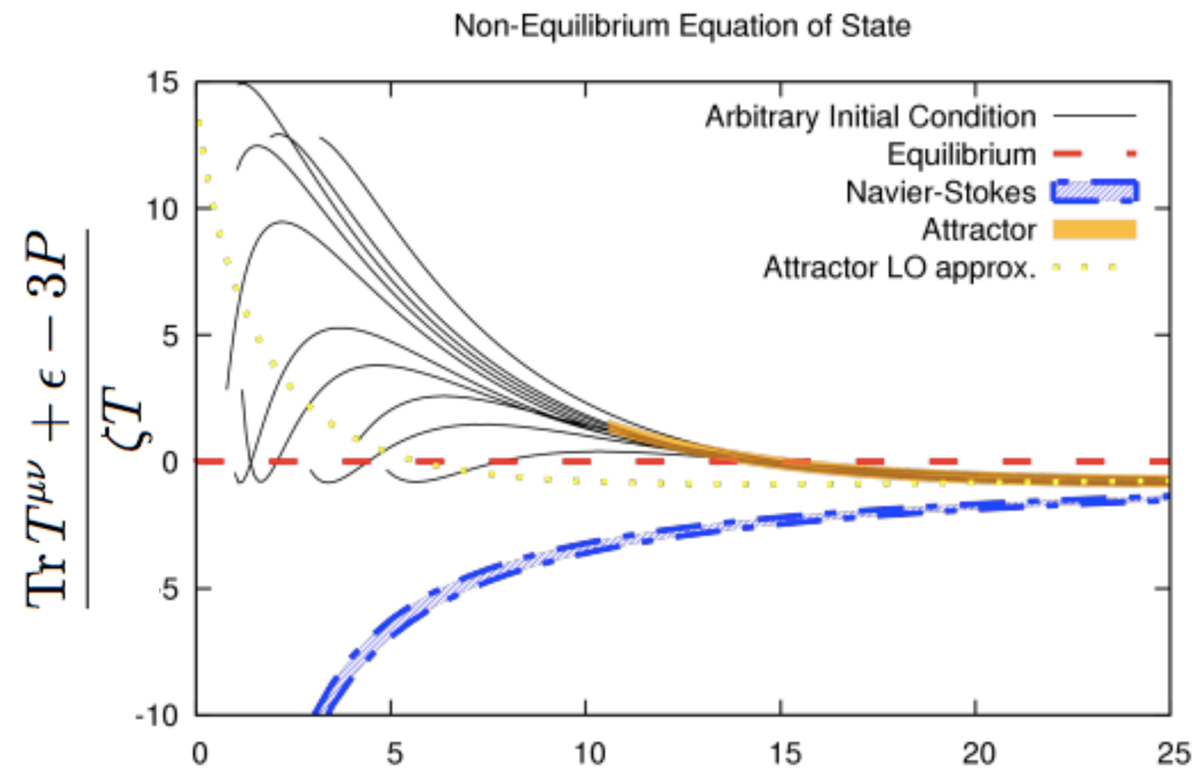
Attractors in holography include also **1708.01921** & **1805.11689** by Spaliński, as well as **1712.02772**' & **1810.02314*** by Casalderrey-Solana, Gushterov', Herzog* & Meiring

Attractors beyond BRSSS: non-conformal bif

1710.03234 by Romatschke (figure adapted from arXiv)

1710.07095 by Florkowski, Maksymiuk, Ryblewski





$$\left[\frac{\eta}{2s}\frac{\sigma^{\mu\nu}\sigma_{\mu\nu}}{T\nabla_\lambda u^\lambda}+\frac{\zeta}{s}\frac{\nabla_\lambda u^\lambda}{T}\right]^{-1} \qquad \left[\frac{\eta}{2s}\frac{\sigma^{\mu\nu}\sigma_{\mu\nu}}{T\nabla_\lambda u^\lambda}+\frac{\zeta}{s}\frac{\nabla_\lambda u^\lambda}{T}\right]^{-1}$$

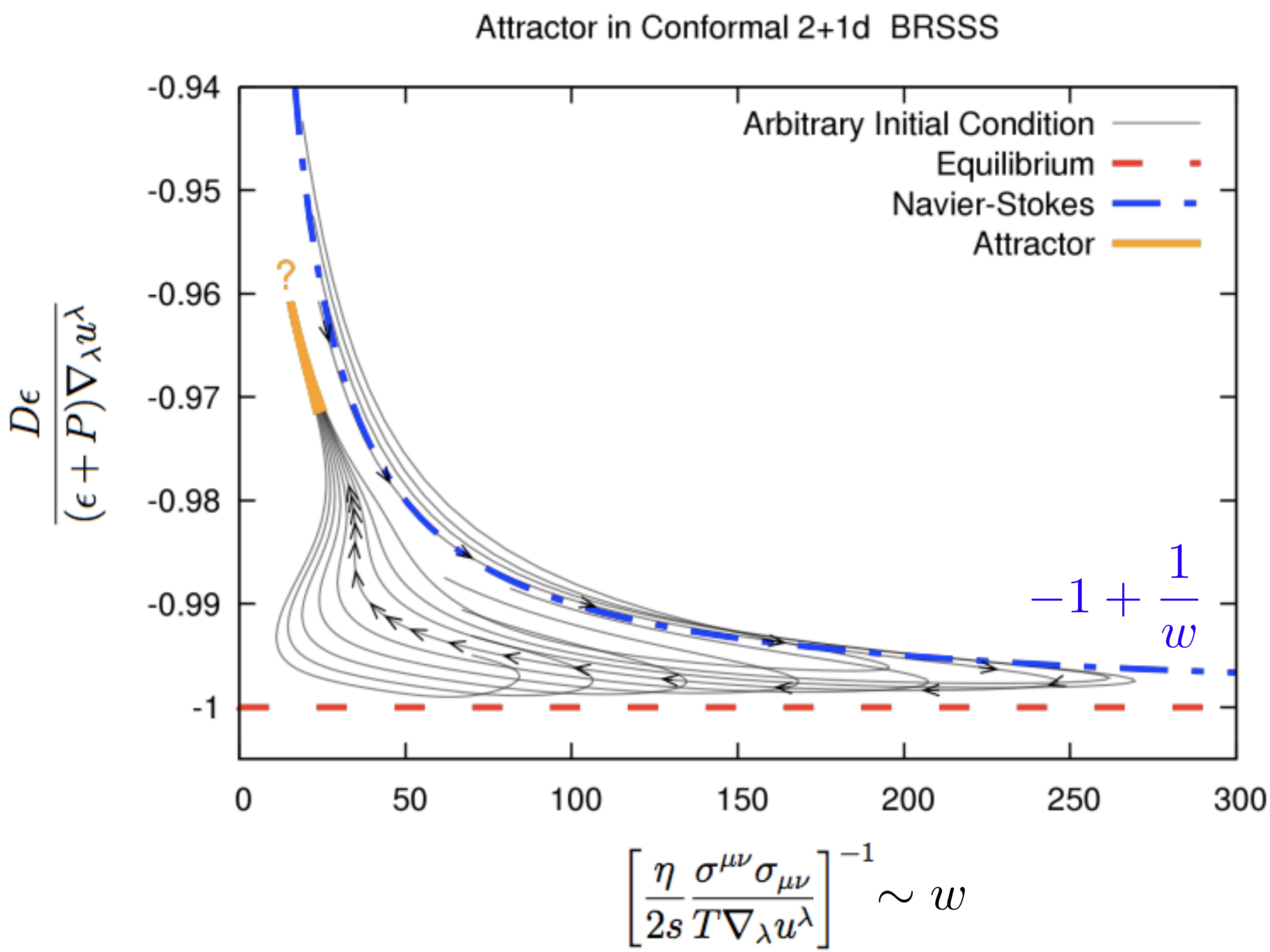
$\sim w^{-1}$ at conformality

Results for RTA kinetic theory for massive particles and $\tau_{rel} \sim \frac{1}{T}$

Attractors in kinetic theory include also **1712.03856** + **1904.08677** by Blaizot & Yan, **1809.01200** + **1903.03145*** by Strickland (+ Tantary*), **1901.08632** by Behtash, Kamata, Martinez & Shi,

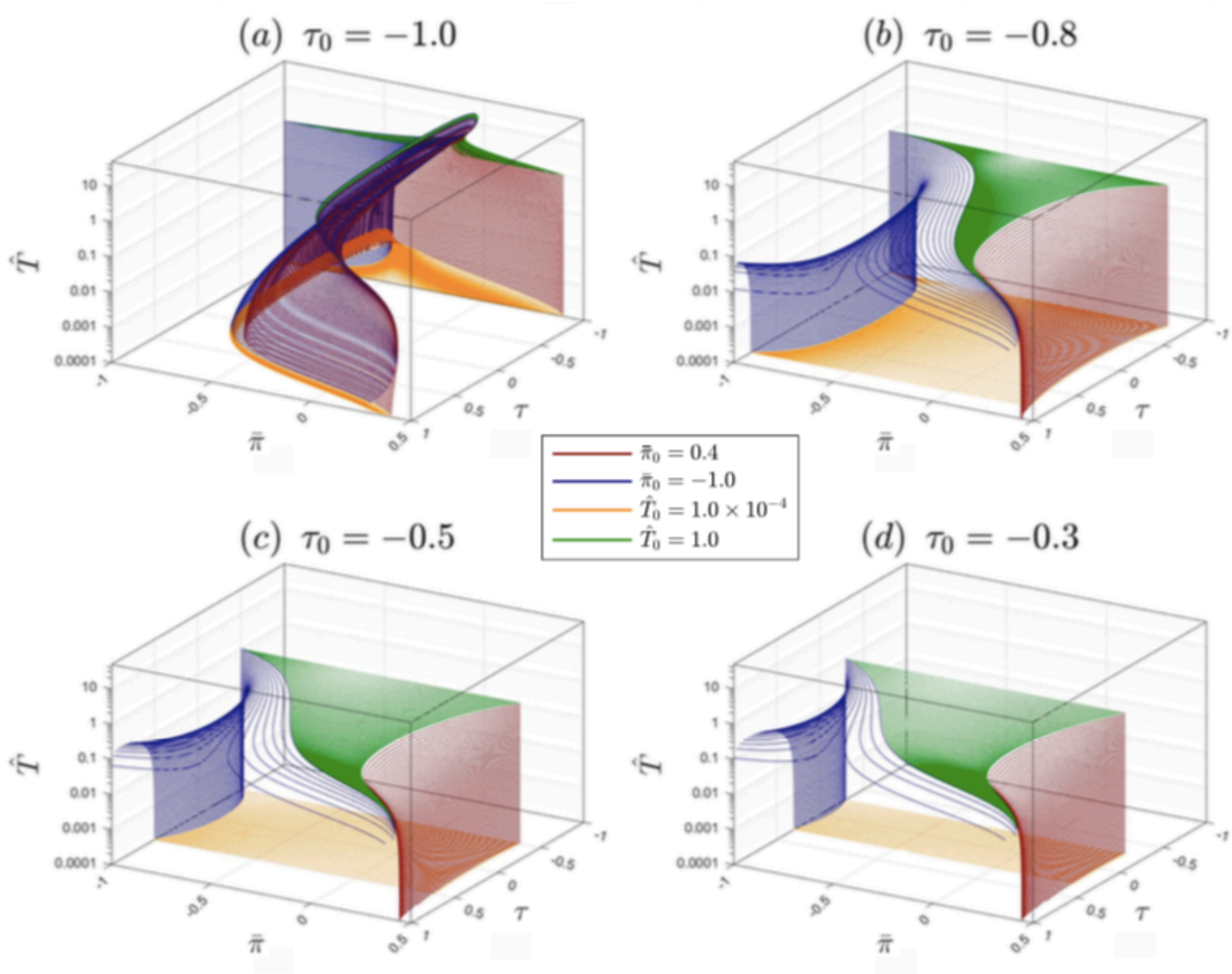
Attractors beyond bif: (1+2)D BRSSS

1710.03234 by Romatschke (figure adapted from arXiv)



Attractors beyond pure bif: the Gubser flow

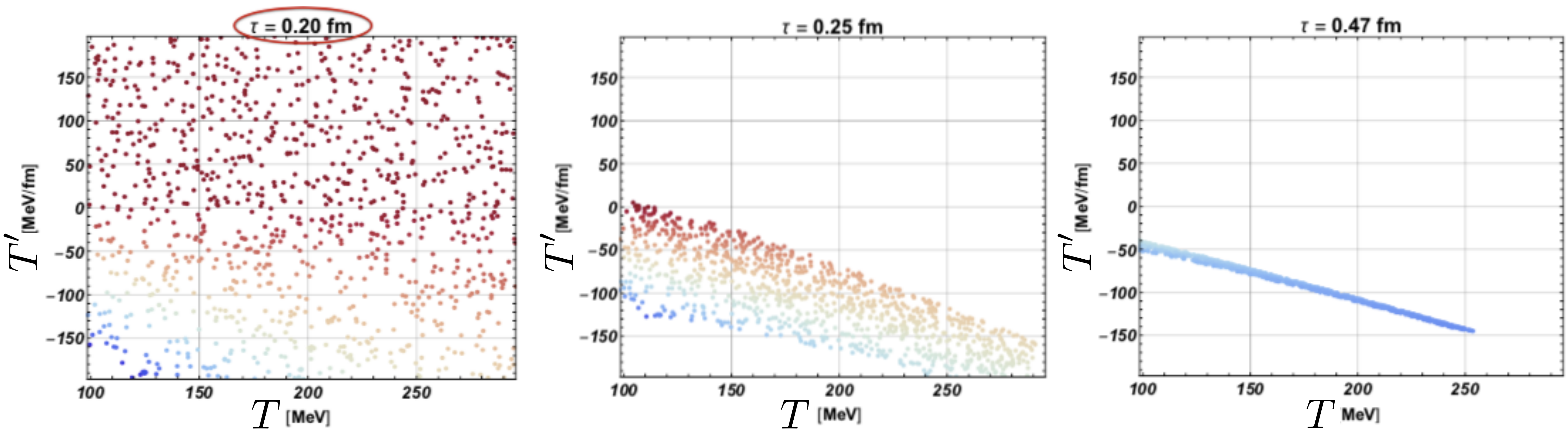
1711.01745 by Behtash, Cruz-Camacho & Martinez (figure adapted from arXiv)



Attractors as dimensionality reduction

1907.xxxxx with Jefferson, Spaliński & Svensson

In more general situations we may not find w and $\mathcal{A}$, but we always have phase space variables which we can use. For bif in BRSSS: $(\tau, T, \underbrace{T'}_{\sim \pi_{yy}})$



A new look at attractors: fix a set of initial conditions in phase space; expansion + dissipation drives it to become lower dimensional; the attractor is the locus where the dimensionality reduction occurred the most.

Vision: use ML techniques to tackle the existence of attractors beyond (1+0)D

Outlook

define far from equilibrium hydro
in a class of expanding plasma setups

have become an
interesting research
field on its own

hydro attractors

what are the conditions for
their existence, e.g. should
we expect them in full QCD
or outside expanding plasmas?

if they turn out generic,
can we observe them?