Initial State Correlations

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Initial Stages, Columbia University, New York

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JYVÄSKYLÄN YLIOPISTO

UNIVERSITY OF JYVÄSKYLÄ

Talks on initial state correlations

How initial state correlations effect final state correlations

Talk by S. Bass

Talk by G. Giacalone

Poster by M. Hippert

Poster by M. Luzum

Initial state from Pythia

Talk by C. Rasmussen

Particle production in CGC

Poster by G. Kapilevich

Poster by K. Roy

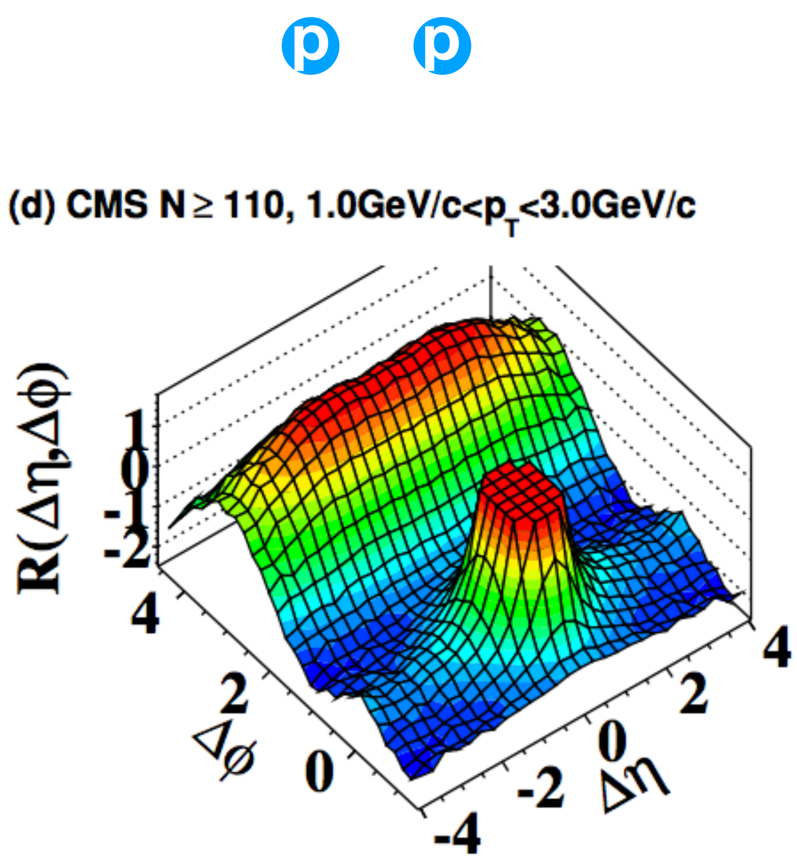
Assessment of physical attributions of initial state correlations

Poster by B. Zajc

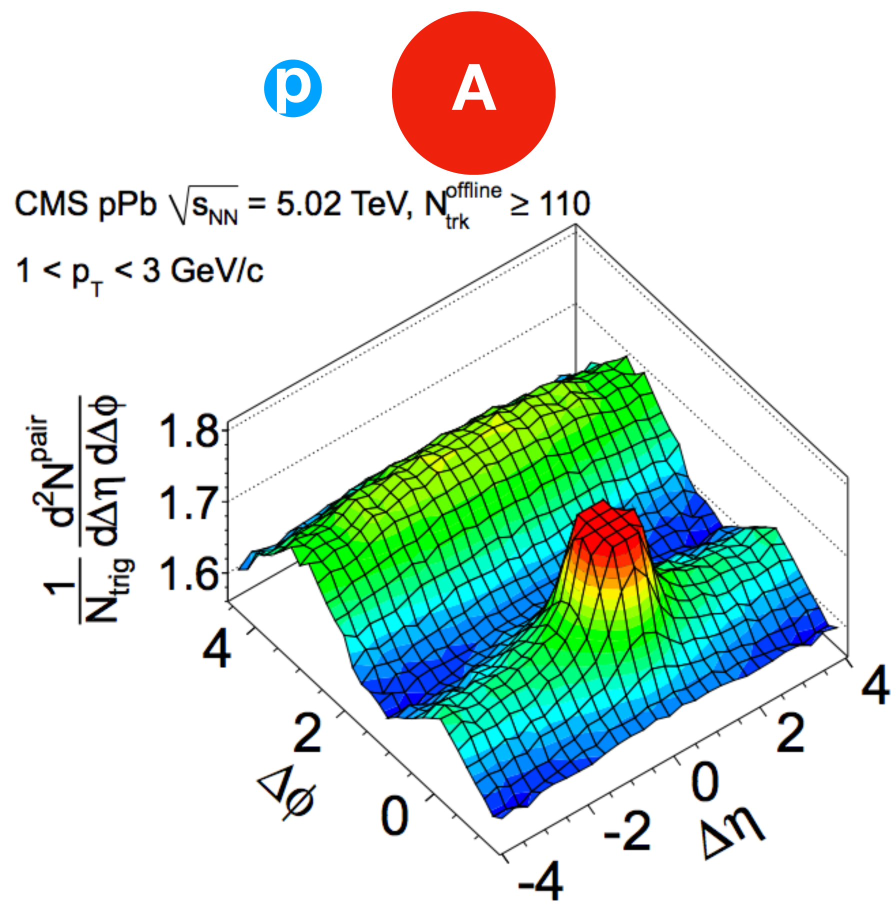
And likely others which I apologize for not mentioning…

Similarity in all systems

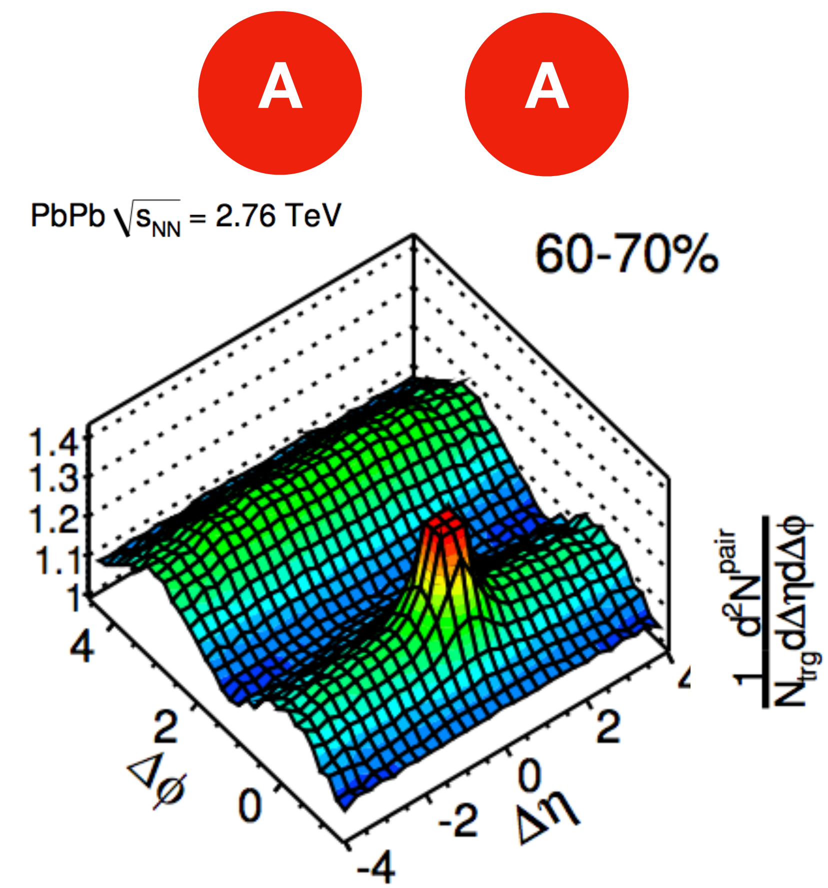
Two particle correlations in high multiplicity events look very similar across all systems



CMS JHEP 1009 (2010)



CMS PLB 718 (2013)

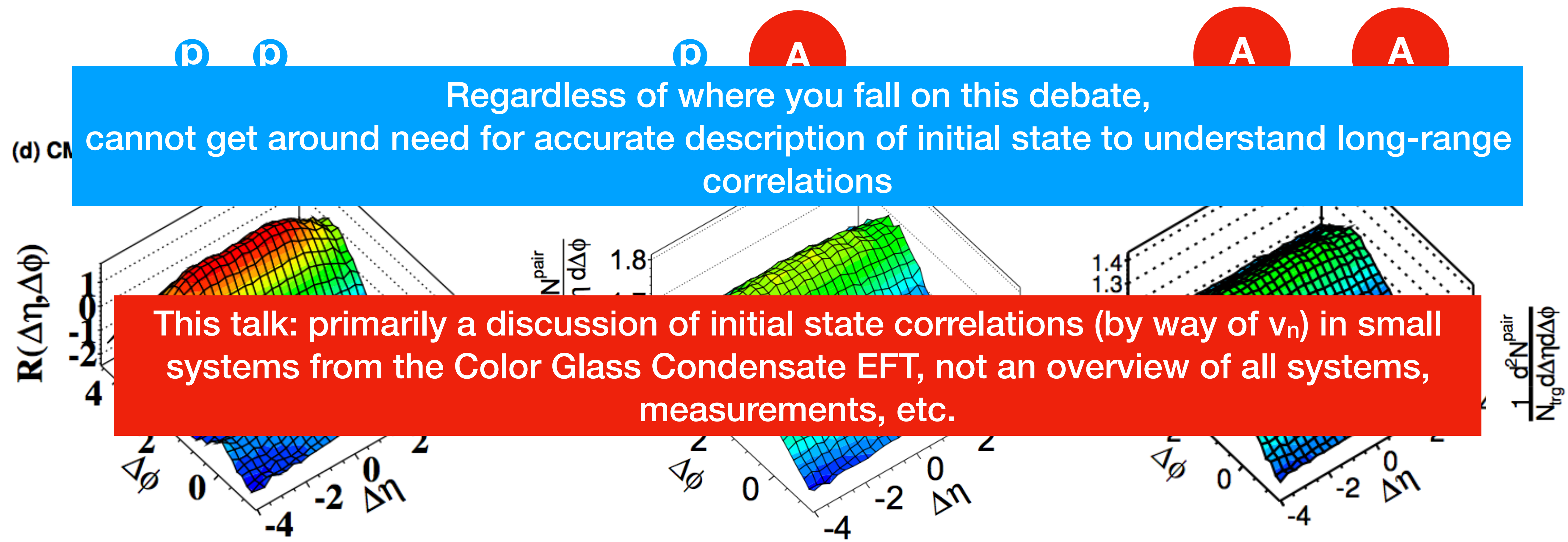


CMS EPC 72 (2012)

Does this hint at a common mechanism?
Or is different physics giving similar macroscopic phenomena?

Similarity in all systems

Two particle correlations in high multiplicity events look very similar across all systems



Regardless of where you fall on this debate,
cannot get around need for accurate description of initial state to understand long-range correlations

This talk: primarily a discussion of initial state correlations (by way of v_n) in small systems from the Color Glass Condensate EFT, not an overview of all systems, measurements, etc.

CMS JHEP 1009 (2010) *CMS PLB 718 (2013)* *CMS EPC 72 (2012)*

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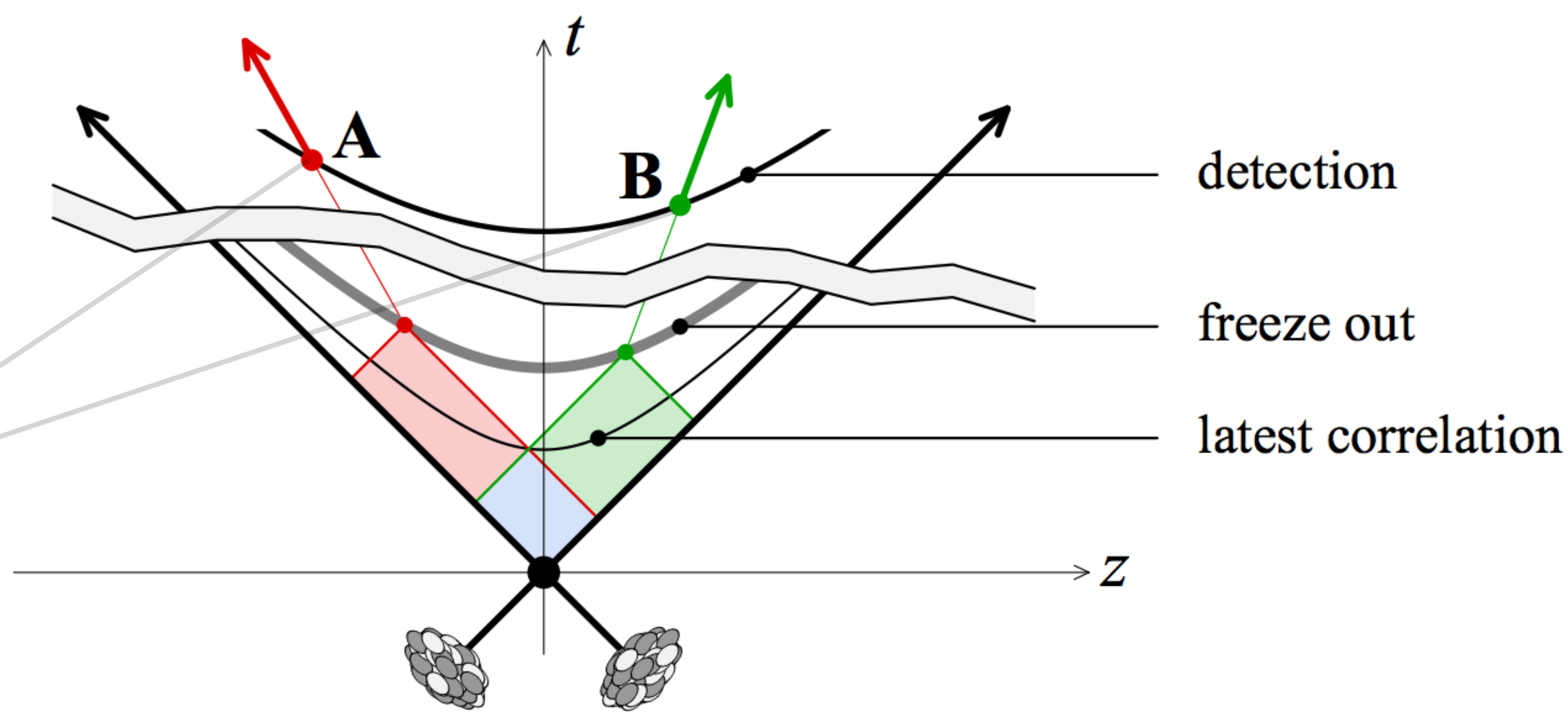
Or is different physics giving similar macroscopic phenomena?

Evidence 1: Long range rapidity correlations as a chronometer

Consider pair of particles correlated rapidity Δy



Dumitru, Gelis, McLerran, Venugopalan
NPA 810 (2008) 91-108

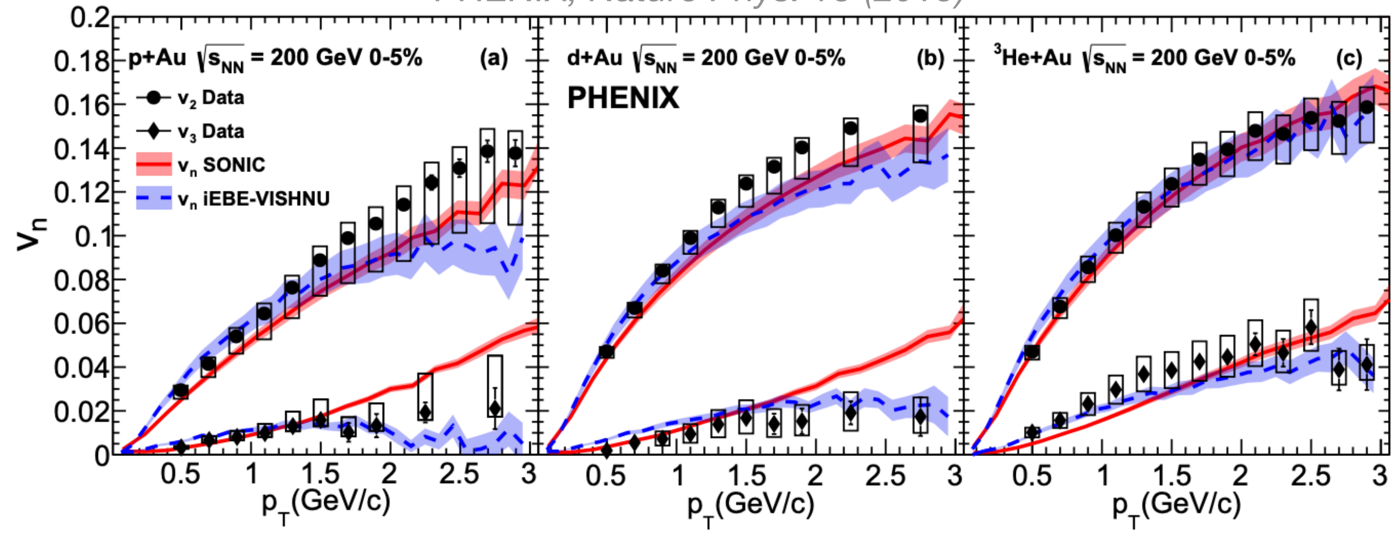
By causality, long-range rapidity correlations sensitive to early time dynamics, $\tau < \tau_{f.o.} e^{-\Delta y/2}$,in collision

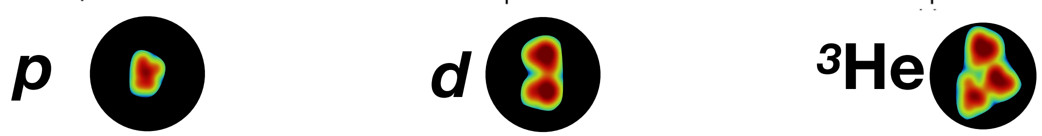
Evidence 2: Hydro dependence on initial conditions

MC-Glauber
+ VISH2+1
+ UrQMD

Shen, Paquet, Denicol, Jeon, Gale PRC 95 (2017)

PHENIX, Nature Phys. 15 (2019)



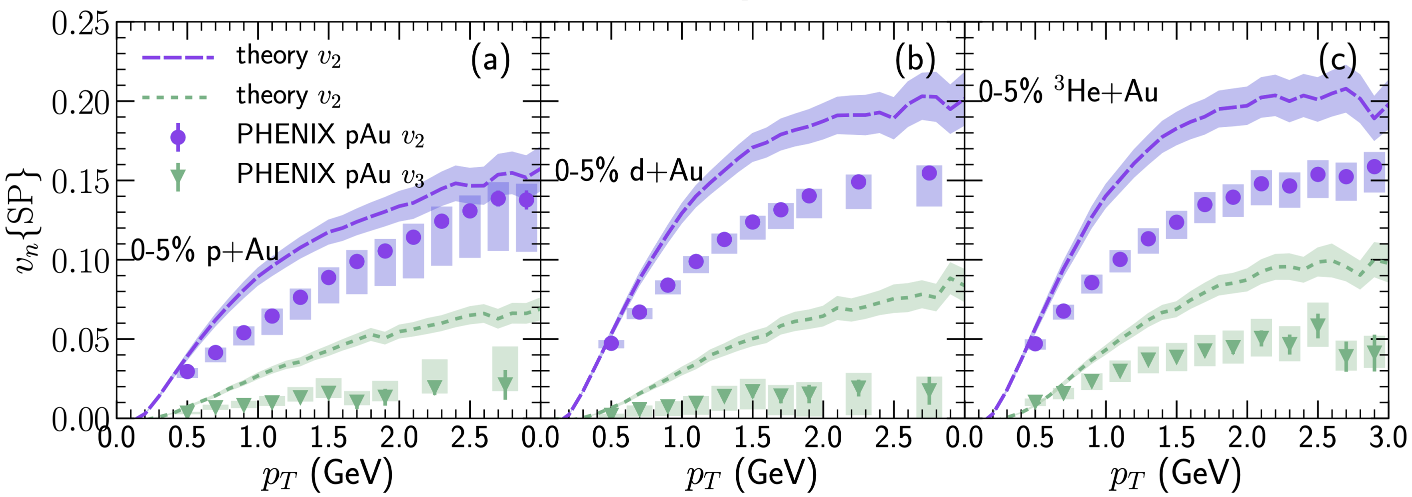


MC-Glauber+AdS
+UVH2+1
+B3D

Habich, Nagle, Romatschke EPJC 75 (2015)

IP-Glasma
+ MUSIC
+ UrQMD

PRELIMINARY
Schenke, Shen, Tribedy, in preparation



B. Schenke, RHIC-AGS Users Meeting June 4,2019

Three (somewhat) different initial states

Initial state important even with dominant final state — need to get dynamics correct

Color Glass Condensate

Color Glass Condensate (CGC) - description of nucleus at high energy (small x) as a highly gluon dense state

McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204

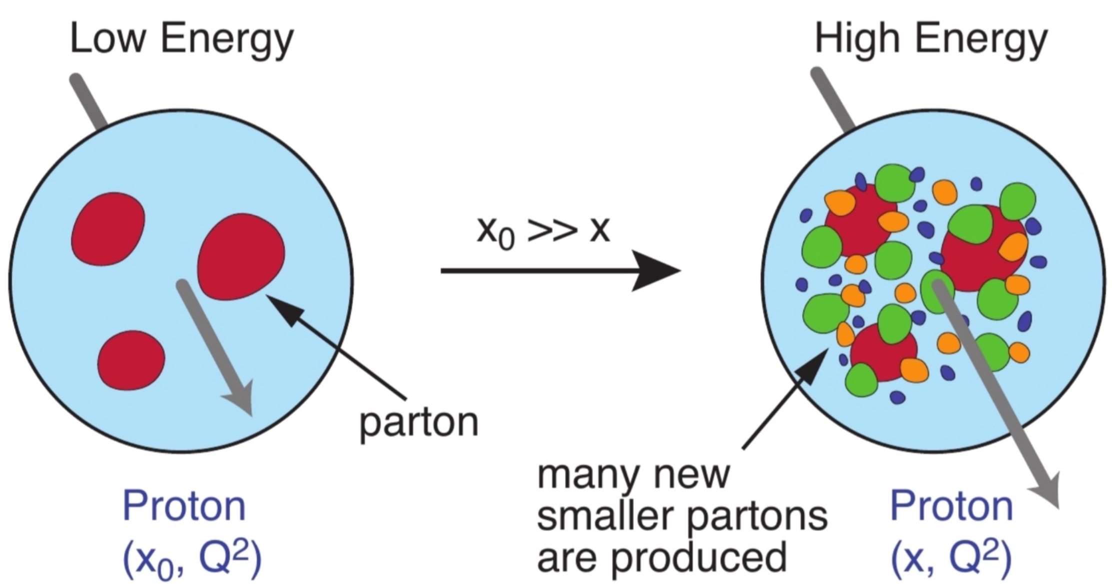
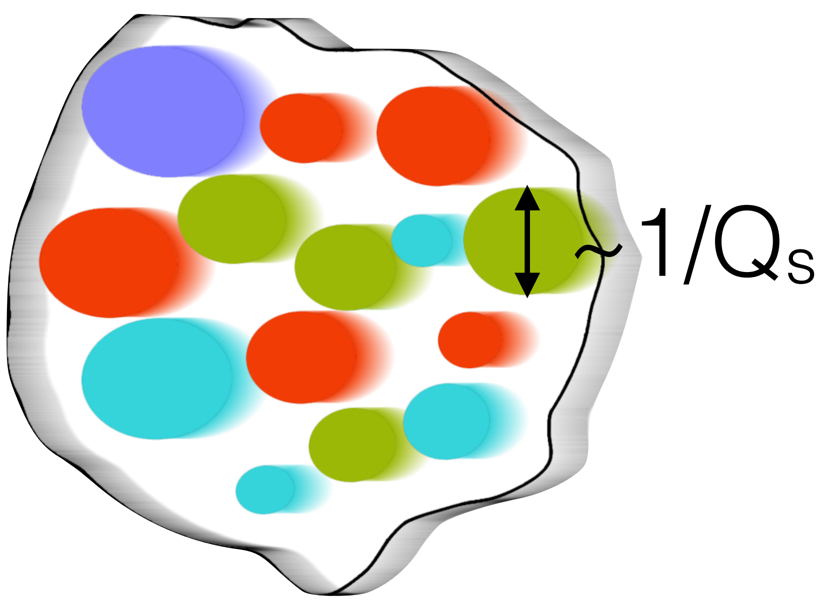


Fig: EIC White Paper



Saturation: recombination competes with gluon splitting, generating scale generate a resolution scale, $Q_s^2 \sim A^{1/3} s^\gamma$

Talk by T. Altinoluk Monday

Weak coupling ($\alpha_s(Q_s) \ll 1$), strong fields ($A^\mu \sim 1/g$), large occupation ($f \sim 1/\alpha_s$) —> *classical limit*

Color Glass Condensate

CGC is an effective field theory in the non-linear regime of QCD ($Q_s^2(x) \gg \Lambda_{QCD}^2$) describing dynamical gluon *fields* (small-x partons) effected by static color *sources* (large-x partons)

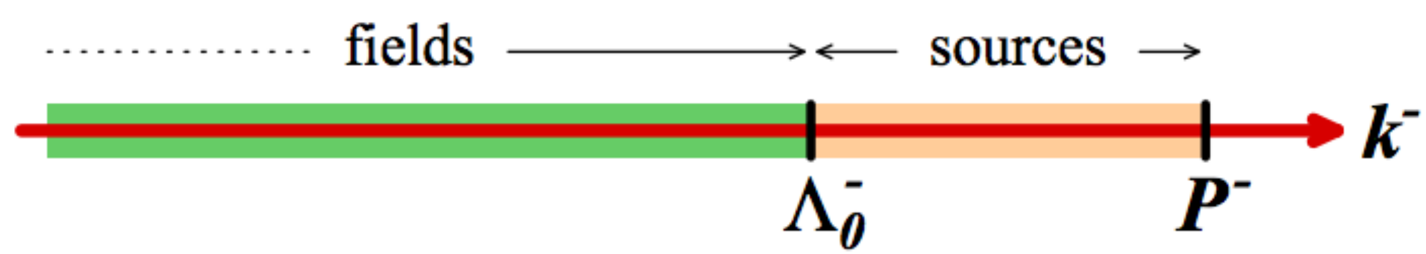
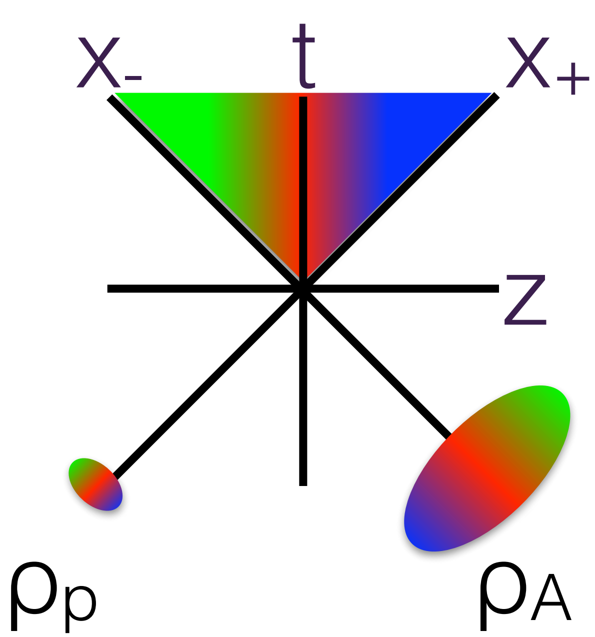


Fig: Gelis, Iancu, Jalilian-Marian, Venugopalan ARNPS. 60 (2010)



Classical gauge field: $[D_\mu, F^{\mu\nu}] = J^\nu$

Static color sources:

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

McLerran-Venugopalan (MV) Model: interactions between nucleons is a Gaussian random walk in color space

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = g^2\delta^{ab}\mu^2\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

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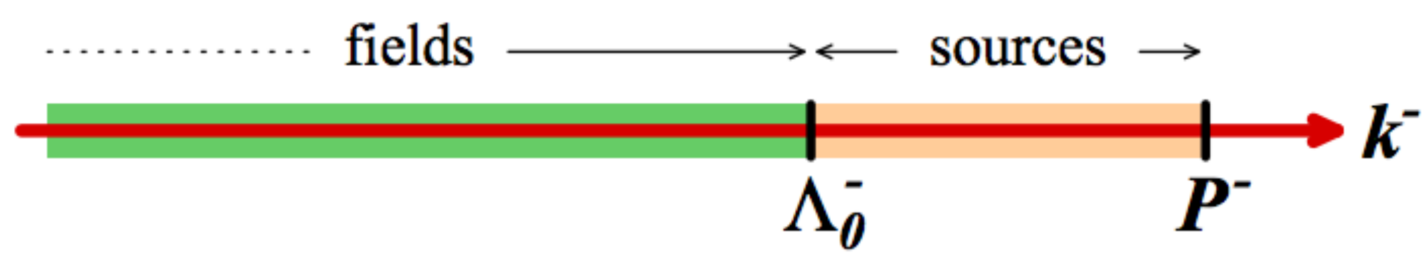
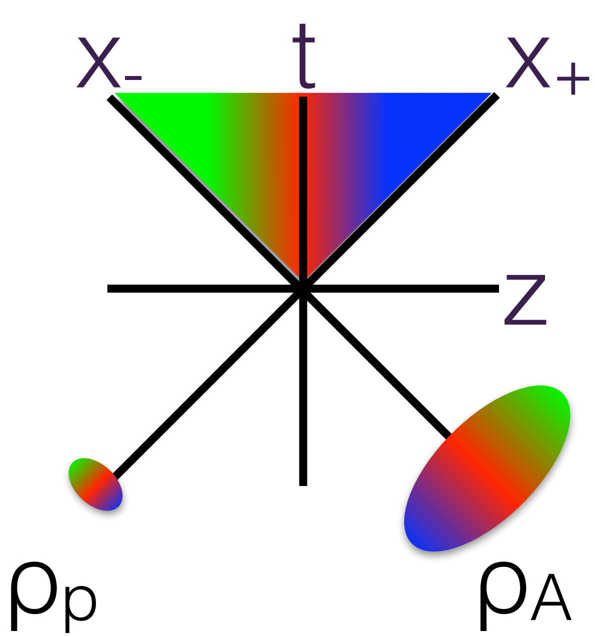


Fig: Gelis, Iancu, Jalilian-Marian, Venugopalan ARNPS. 60 (2010)



CGC is *not* a theory to explain v_ns in small systems
But we can ask how much of an influence it has on the overall signal

CGC provides a first-principles way to calculate for a variety of processes:
DIS, UPCs, initial conditions for hydrodynamics,…
Talk by H. Mäntysaari Wed

Correlation production mechanisms

two extremes

Initial state/CGC

Produced by initial momentum correlations which pre-exist in nuclei before collisions and/or develop at quickly after collision

Contains classical correlations (domains, as well as density gradients)

Contains quantum effects: Bose enhancement in incoming wavefunction, as well as gluon HBT

Final state/hydrodynamics

Produced by conversion of initial spatial (geometry) correlations are converted to final momentum correlations

Develops throughout evolution of the system

Well motivated from A+A, theory questions linger for smaller systems

Next talk by W. Broniowski

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This talk

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Next talk by W. Broniowski

What do these CGC correlations look like?

Parton-CGC model

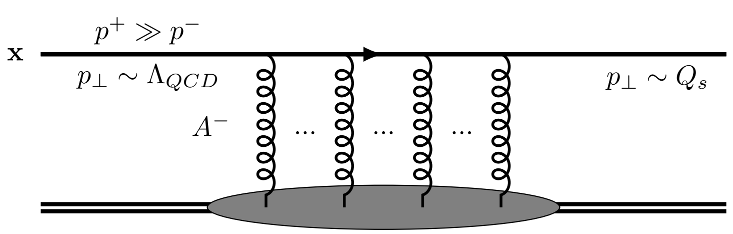
Consider (simple) semi-analytical model of uncorrelated quarks (and/or gluon) scattering off of a CGC

Lappi, PLB 744, 315 (2015); Lappi, Schenke, Schlichting, Venugopalan, JHEP 1601 (2016); Dusling, MM, Venugopalan PRL 120, PRD 97 (2018), Fukushima, Hidaka JHEP 1711 (2017), Davy, Marquet, Shi, Xiao, Zhang NPA 983 (2019)

Parton coherently multiple scatters off target represented by Wilson line phase

Bjorken, Kogut, Soper, PRD (1971), Dumitru, Jalilian-Marian,PRL 89 (2002)

$$U(\mathbf{x}) = \mathcal{P}\exp\Big(-ig\int dz^+ A^{a-}(\mathbf{x},z^+)\,t^a\Big)$$



Single parton inclusive distribution

$$\left\langle \frac{dN_q}{d^2\mathbf{p}} \right\rangle \simeq \int_{\mathbf{b},\mathbf{r},\mathbf{k}} \underbrace{e^{-|\mathbf{b}|^2/B_p} e^{-|\mathbf{k}|^2 B_p}\; e^{i(\mathbf{p}-\mathbf{k})\cdot\mathbf{r}}}_{\text{Projectile: Wigner function}} \underbrace{\left\langle \frac{1}{N_c}\mathrm{Tr}\left(U(\mathbf{b}+\frac{\mathbf{r}}{2})U^\dagger(\mathbf{b}-\frac{\mathbf{r}}{2})\right)\right\rangle}_{\text{Target scattering: Dipole operator } D(x,y)}$$

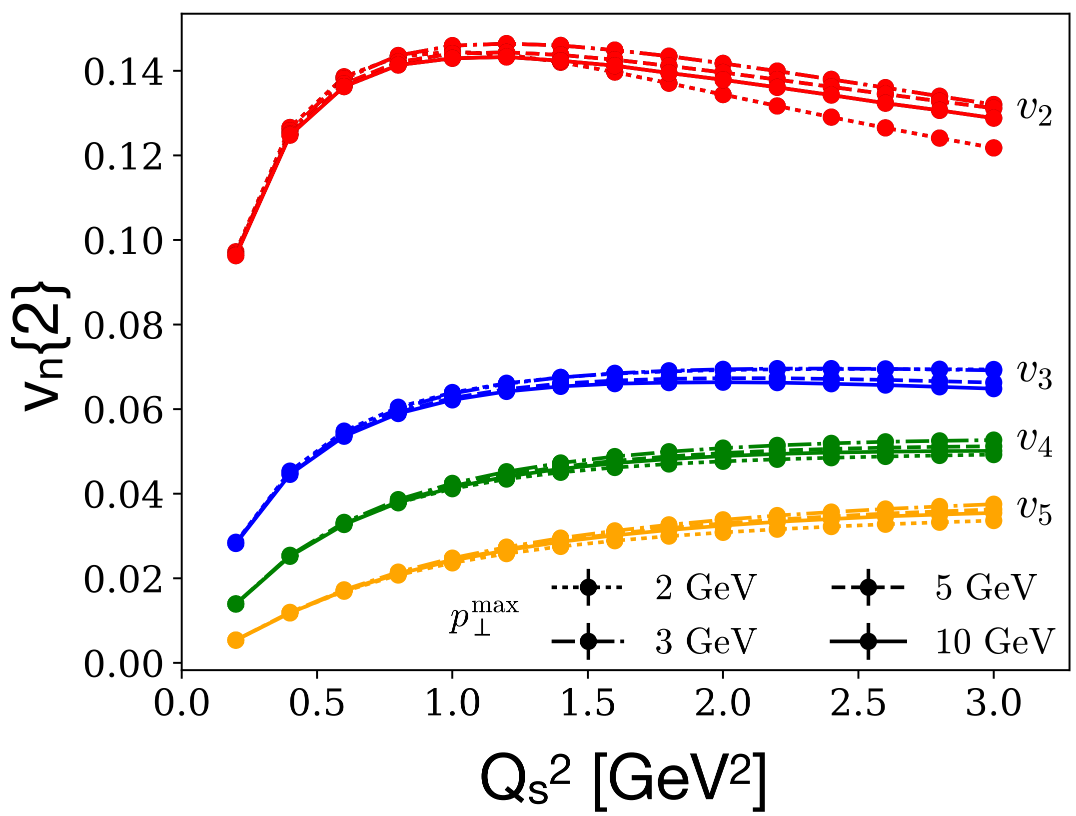
Generalize for multi-particle: $\left\langle \frac{d^m N}{d^2\mathbf{p}_1 \dots d^2\mathbf{p}_m} \right\rangle = \left\langle \frac{dN}{d^2\mathbf{p}_1} \dots \frac{dN}{d^2\mathbf{p}_m} \right\rangle \sim \int \langle D \dots D \rangle$

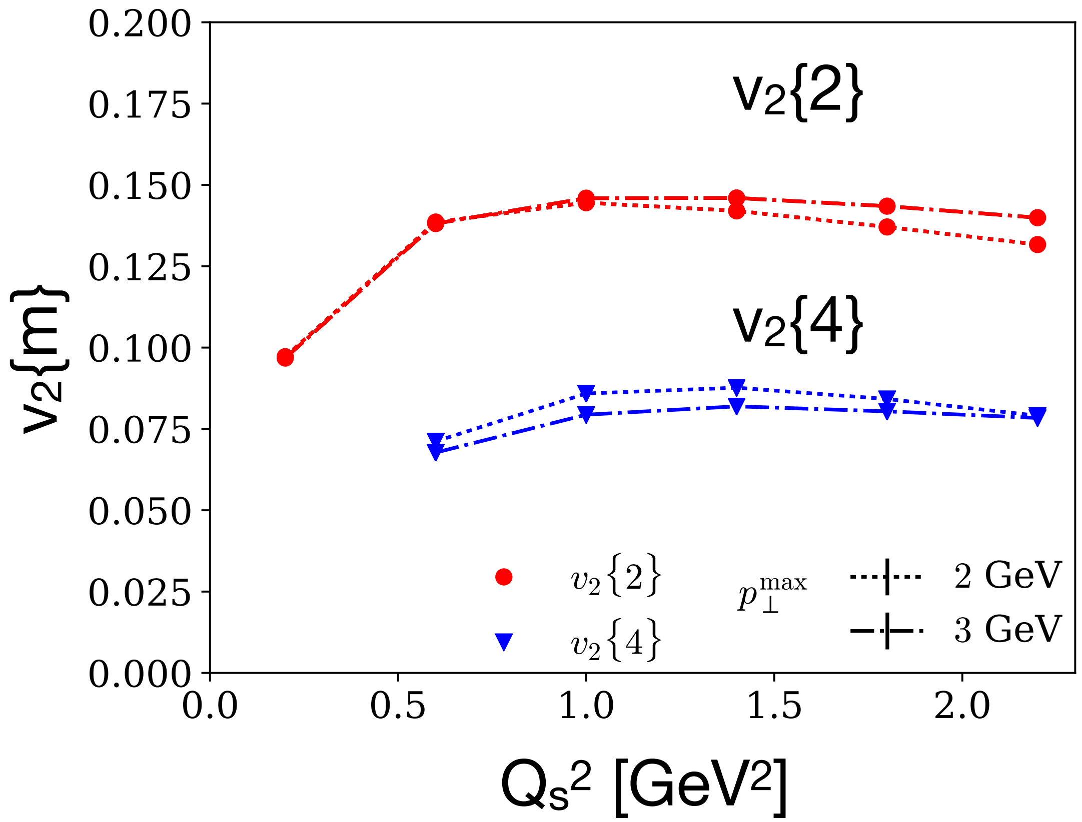
Some of these terms interpreted as Bose, HBT

Multi-particle quark correlations

Ordering in two particle Fourier harmonics similar to data

$c_2\{4\}$ becomes negative for increasing Q_s, giving real $v_2\{4\}$

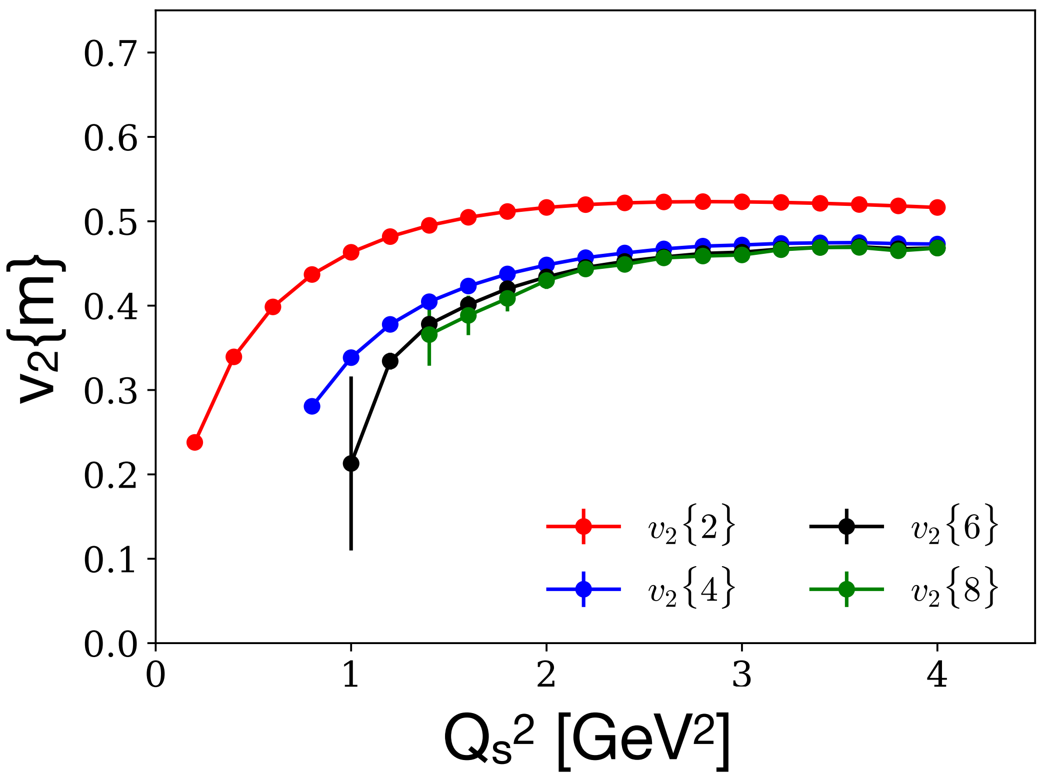




Dusling, MM, Venugopalan PRL 120 (2018)

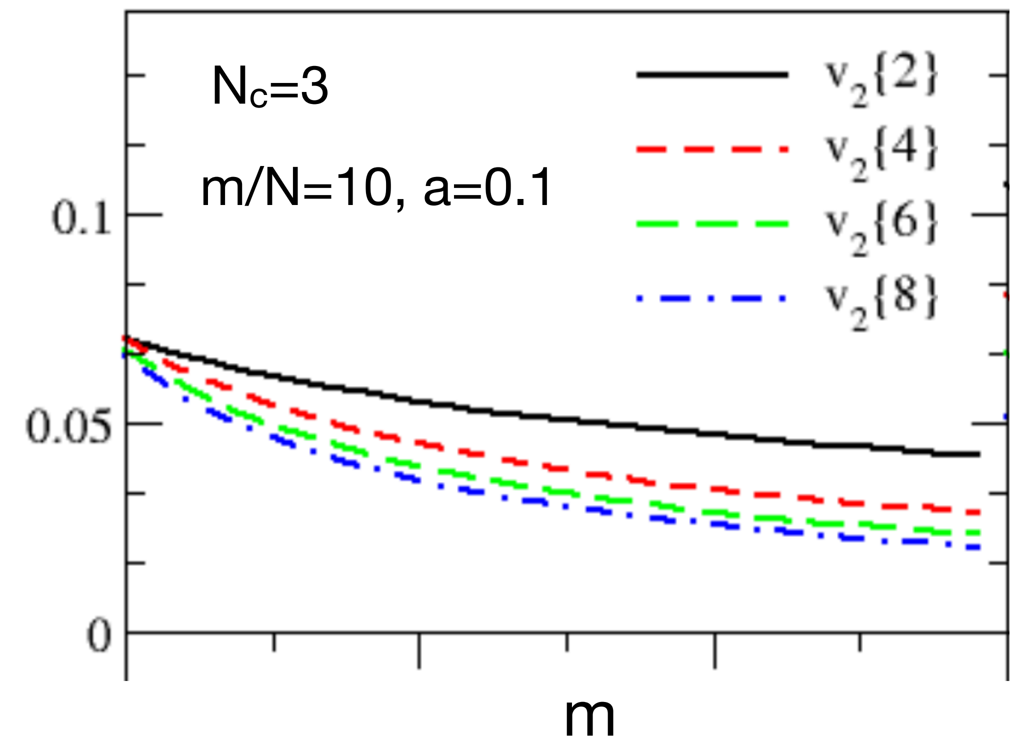
Collectivity

Abelian version of parton-CGC model



Dusling, MM, Venugopalan PRL 120 (2018)

QCD interference diagrams: m gluon emissions from N sources (m>N)



Blok, Wiedemann arXiv:1812.04113

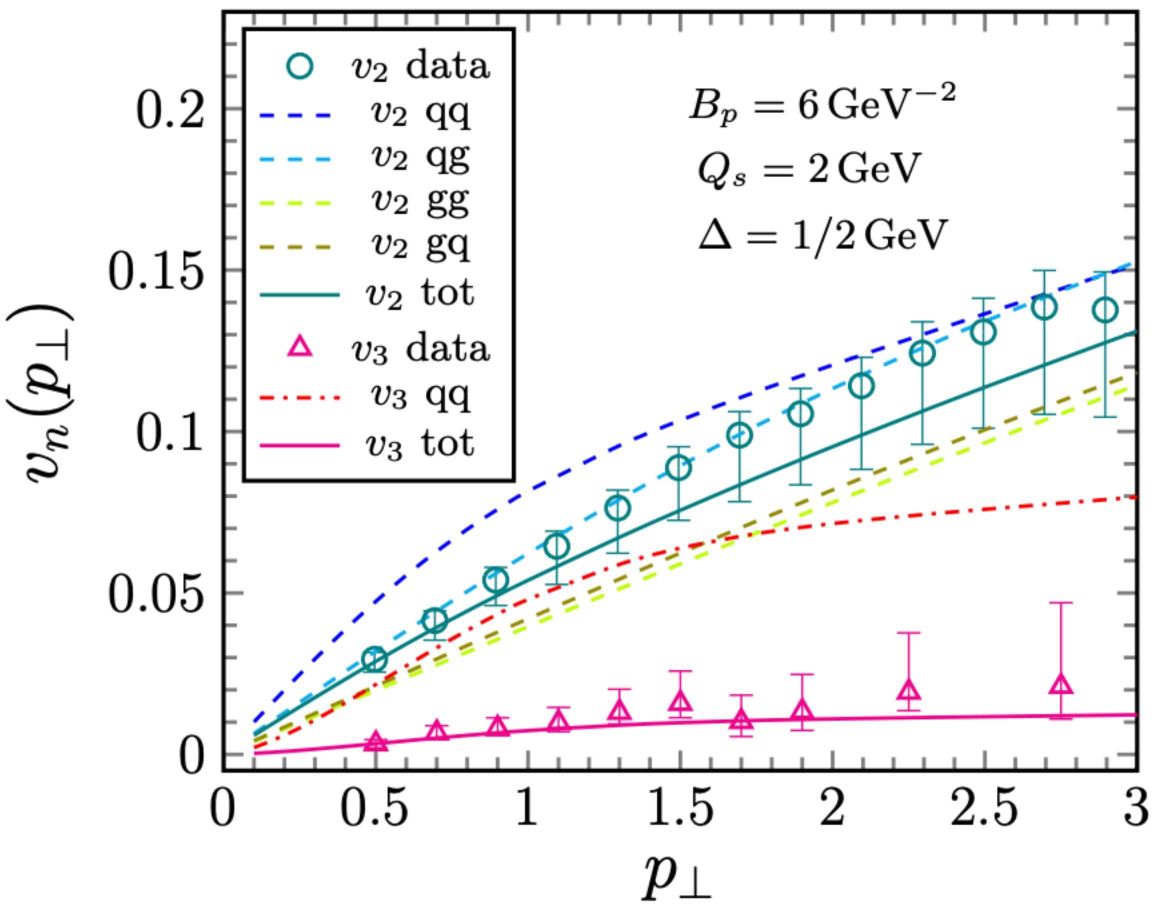
Clear demonstrations that $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ not unique to hydrodynamics

Previously argued in CGC by Dumitru, McLerran, Skokov PLB 743 (2015)

Analytical work in parton-CGC going toward m>4 for N_c=3 Fukushima, Hidaka JHEP 1711 (2017)

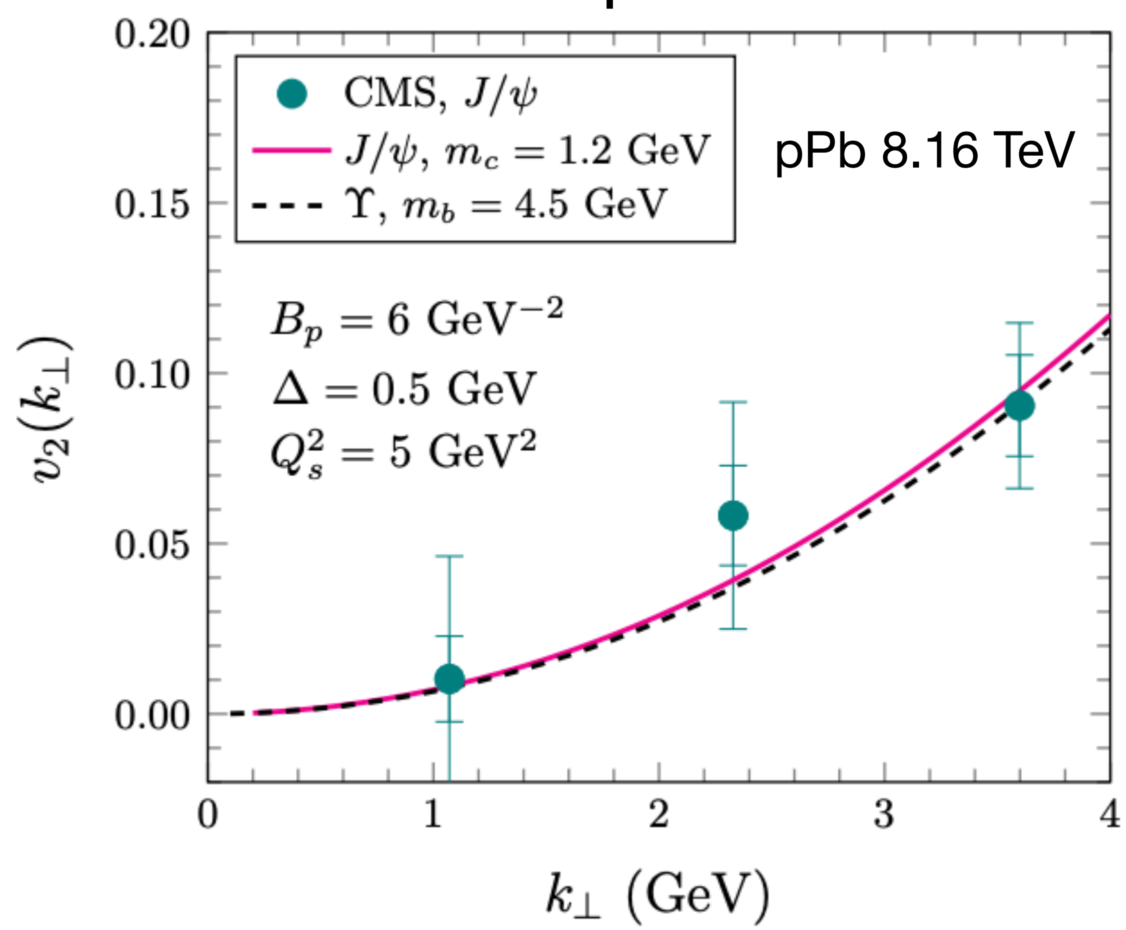
Quark+gluon correlations

Mixture of quark and gluons



Davy, Marquet, Shi, Xiao, Zhang NPA 983 (2019)

Mixture of quarks/gluons
+color evaporation model



Zhang, Marquet, Qin, Shi, Xiao PRL 122 (2019)

Compatible with PHENIX p+Au

PHENIX, Nature Phys. 15 (2019)

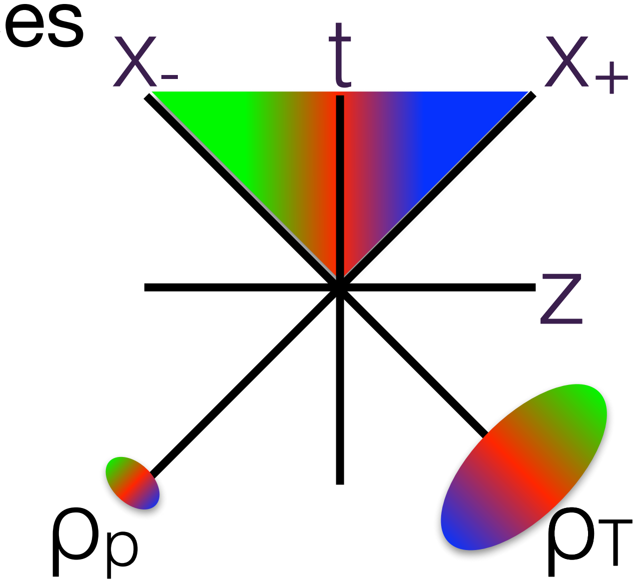
How to model larger projectiles?

Prediction for ϒ

Going beyond the parton model

CGC for gluons

CGC EFT: solve QCD CYM with static color sources



$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\nu = g\delta^{\nu +}\delta(x^-)\rho_p(\mathbf{x}_\perp) + g\delta^{\nu -}\delta(x^+)\rho_T(\mathbf{x}_\perp)$$

Dilute-dense regime: $\rho_T/k_T^2 \gg \rho_p/k_T^2$

Kovchegov, Mueller NPB 529 (1998), Kovner, Wiedemann PRD 64 (2001), Dumitru, McLerran NPA 700 (2002), Blaizot, Gelis, Venugopalan NPA 743 (2004),…

$$\frac{dN}{d^2k}[\rho_p, \rho_T] \sim g^2\rho_p^2 f_{(1)}(p_\perp) + g^4\rho_p^4 f_{(2)} + \dots$$

CGC for gluons

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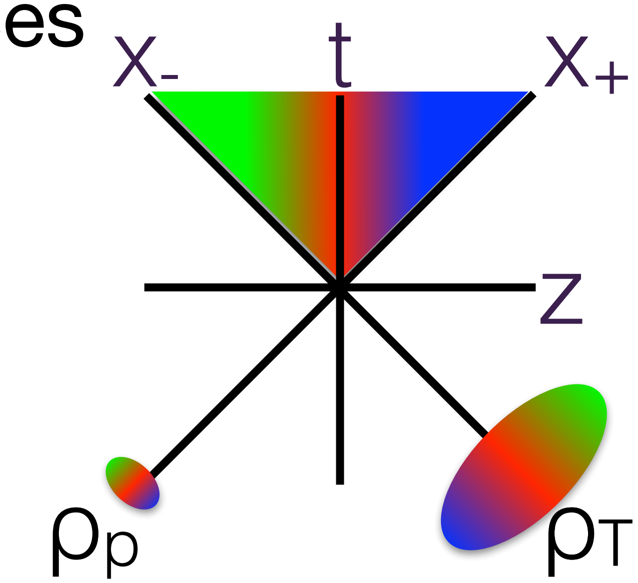

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$$\frac{dN}{d^2k}[\rho_p, \rho_T] \sim g^2\rho_p^2 f_{(1)}(p_\perp) + g^4\rho_p^4 f_{(2)} + \cdots$$

Analytical solution known for ~20 years

All $O(\rho_p^\#)$ is called *dense-dense*, used in IP-Glasma model

Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012)

Keeping only LO in $\rho_{p/T}$ *dilute-dilute* aka glasma graphs

Dumitru, Gelis, McLerran, Venugopalan, NPA 810 (2008), Dusling, Venugopalan PRL 108 (2012), PRD 87 (2013), Dusling, Tribedy, Venugopalan PRD 93 (2016),…

Includes quantum correlations (Bose Enhancement, HBT)

Gelis, Lappi, McLerran NPA 828 (2009), Kovner, Rezaeian, PRD 95, 96 (2017), Altinoluk, Armesto, Beuf, Kovner, Lublinsky, PLB 751 (2015), PRD 95 (2017), Kovner, Skokov PRD 98 (2018), PLB 785 (2018), …

Talk by T. Altinoluk Monday

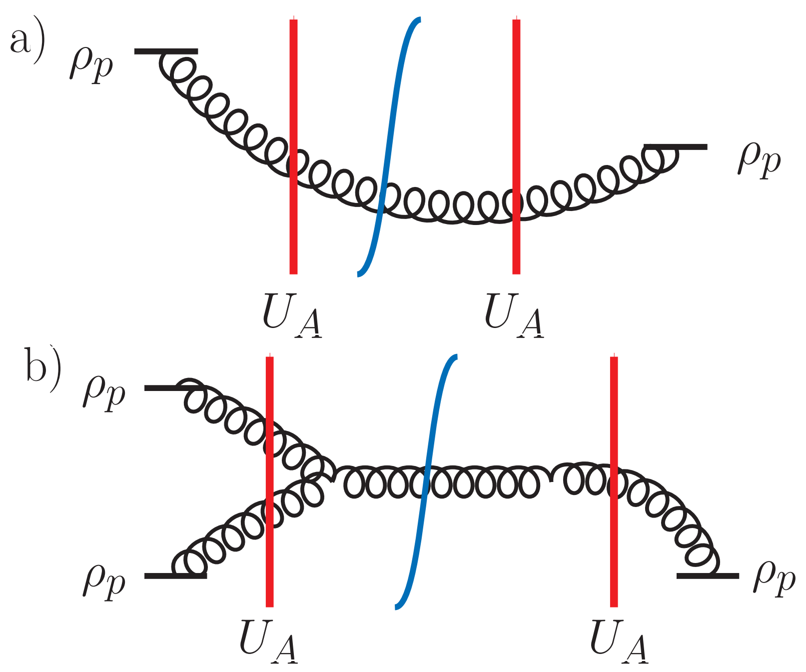
Dilute-dense for gluons

Even harmonics appear at LO dilute-dense

Odd harmonics only non-zero at next to leading order in ρ_p —first saturation correction *(full NLO correction not known)*

McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)

Other sources of v_3 : Kovner, Lublinsky, Skokov PRD 96 (2017), Agostini, Altinoluk, Armesto arXiv: 1902.04483, Schenke, Schlichting, Venugopalan, PLB 747 (2015)



MM, Skokov, Tribedy, Venugopalan, PLB 788 (2019)

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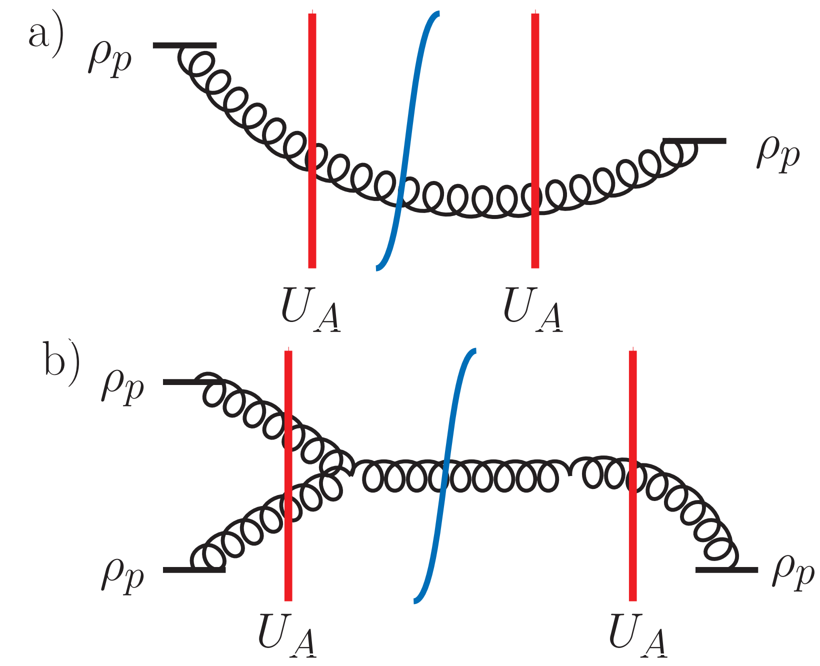

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$$\frac{dN^{\text{even}}(\mathbf{k})}{d^2k dy}\left[\rho_p, \rho_t\right] \sim \rho_p^2 \quad , \qquad \frac{dN^{\text{odd}}(\mathbf{k})}{d^2k dy}\left[\rho_p, \rho_T\right] \sim \rho_p^3$$

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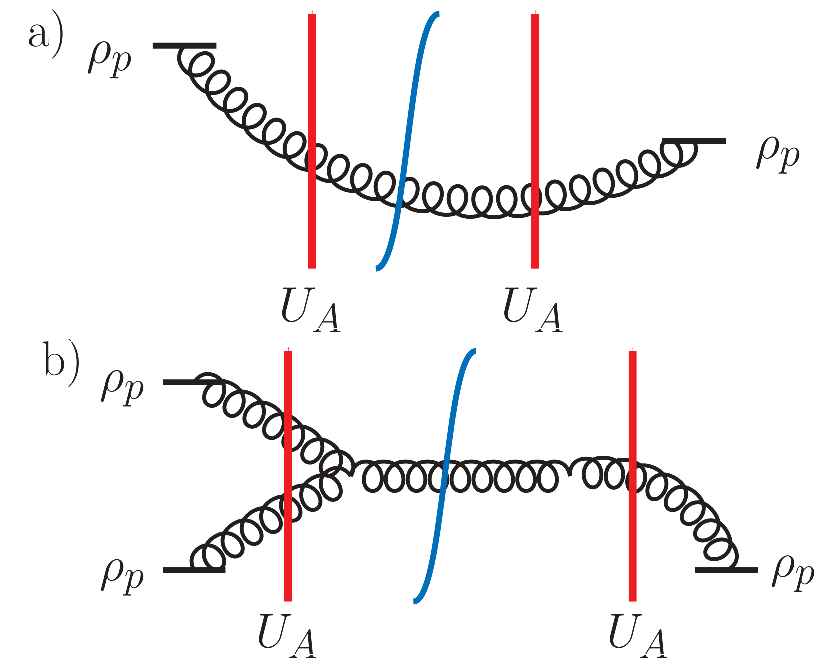

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$$v_{2n}\{2\}^2 = \left\langle \left| \frac{\int_{\mathbf{k}_\perp} e^{2ni\phi} \frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2k_\perp dy}\Big|_{\rho_p,\rho_T}}{\int_{\mathbf{k}_\perp} \frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2k_\perp dy}\Big|_{\rho_p,\rho_T}} \right|^2_{N_{ch}} \right\rangle_{\rho_p,\rho_T} , \quad v_{2n+1}\{2\}^2 = \left\langle \left| \frac{\int_{\mathbf{k}_\perp} e^{(2n+1)i\phi} \frac{dN^{\text{odd}}(\mathbf{k}_\perp)}{d^2k_\perp dy}\Big|_{\rho_p,\rho_T}}{\int_{\mathbf{k}_\perp} \frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2k_\perp dy}\Big|_{\rho_p,\rho_T}} \right|^2_{N_{ch}} \right\rangle_{\rho_p,\rho_T}$$

MM, Skokov, Tribedy, Venugopalan, PLB 788 (2019)

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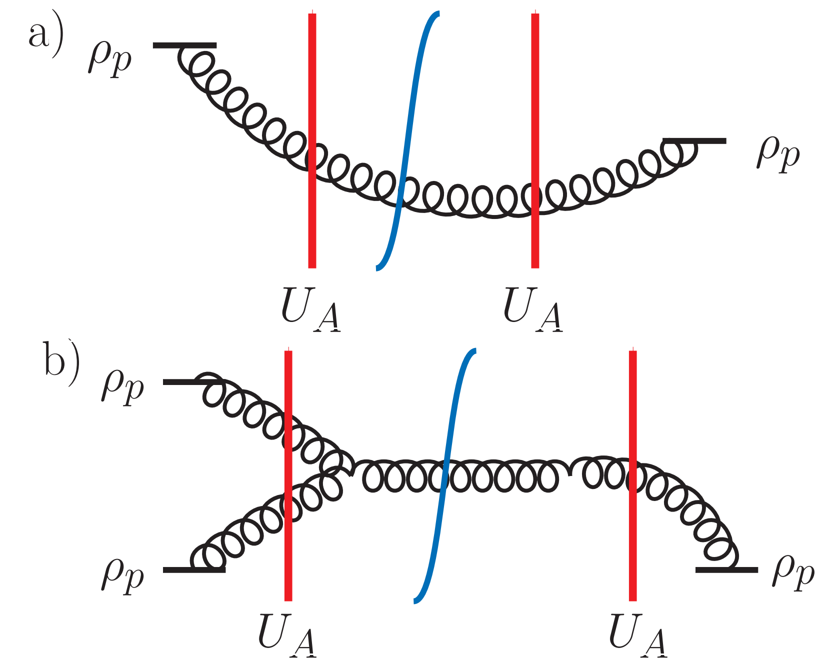

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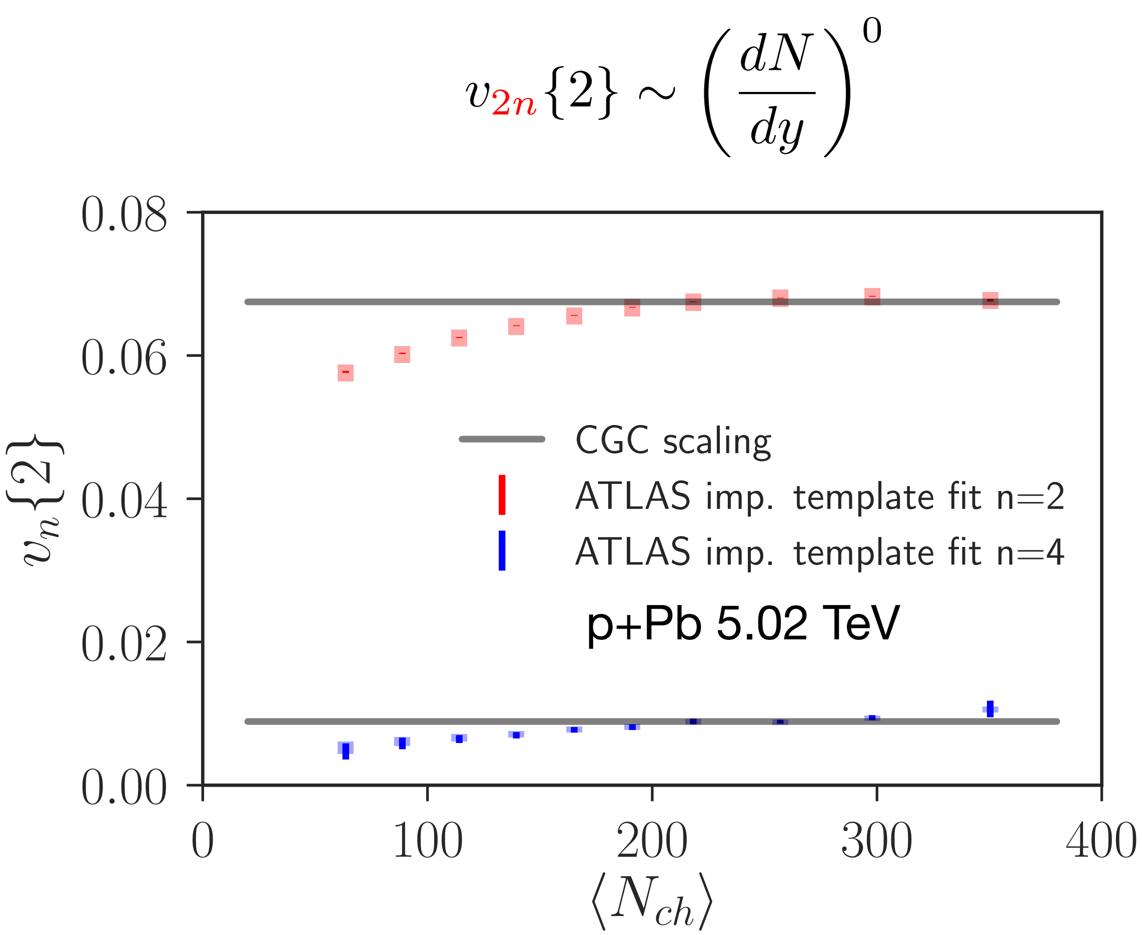
High multiplicity driven by fluctuation in ρ_p

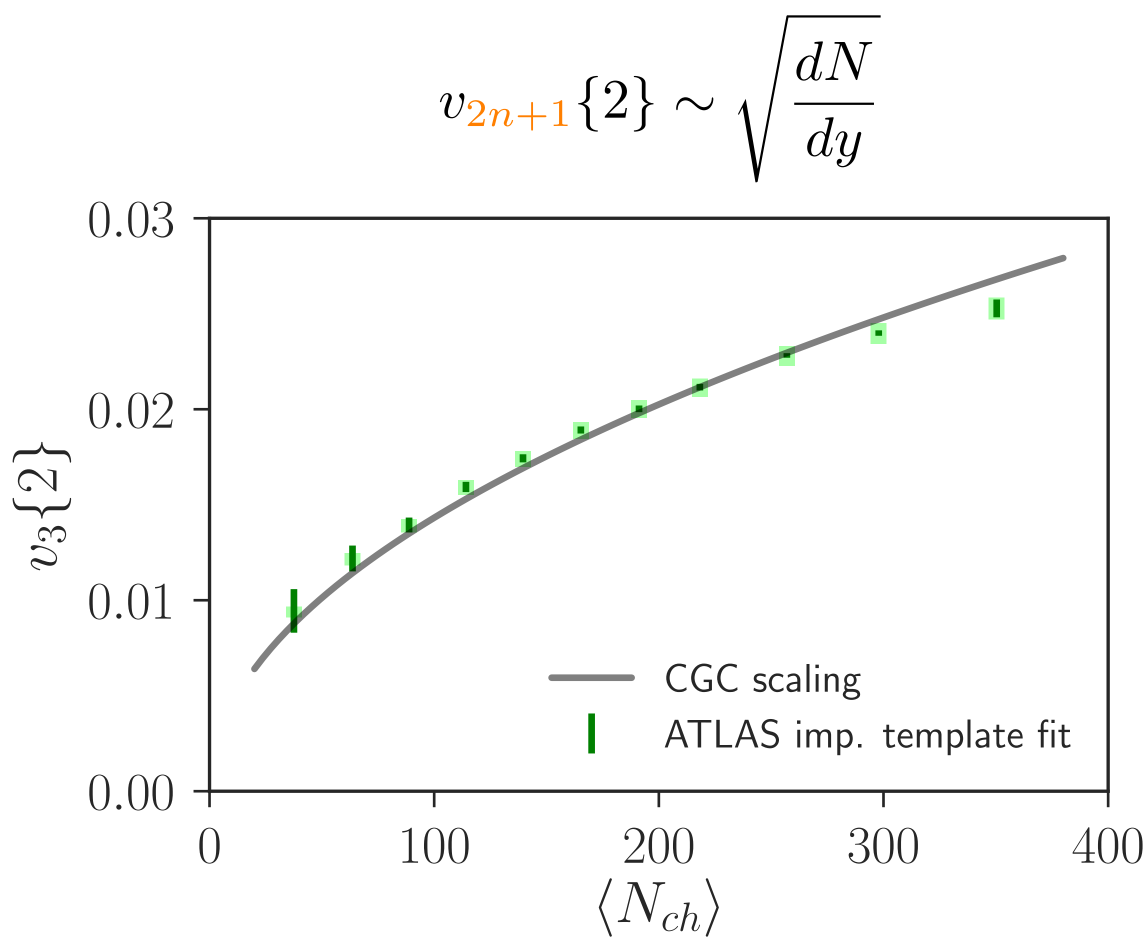
$$\boxed{v_{2n}\{2\} \sim \left(\frac{dN}{dy}\right)^0 , \quad v_{2n+1}\{2\} \sim \sqrt{\frac{dN}{dy}}}$$

MM, Skokov, Tribedy, Venugopalan, PLB 788 (2019)

Dilute-dense CGC scaling

Proportionality coefficient fixed at a single multiplicity for each v_n





Scaling expected by CGC density counting
appears to describe trends in data

MM, Skokov, Tribedy, Venugopalan, PLB 788 (2019)

Discussion of N_{ch} dependence in CGC and other models in many systems
Noronha-Hostler, Paladino, Rao, Sievert, Wertepny arXiv:1905.13323

Modeling with gluons

Initial fields

For initial gluons configurations, standard treatment is that pioneered by IP-Glasma model

Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012), MM, Skokov, Tribedy, Venugopalan, PRL 121 (2018)

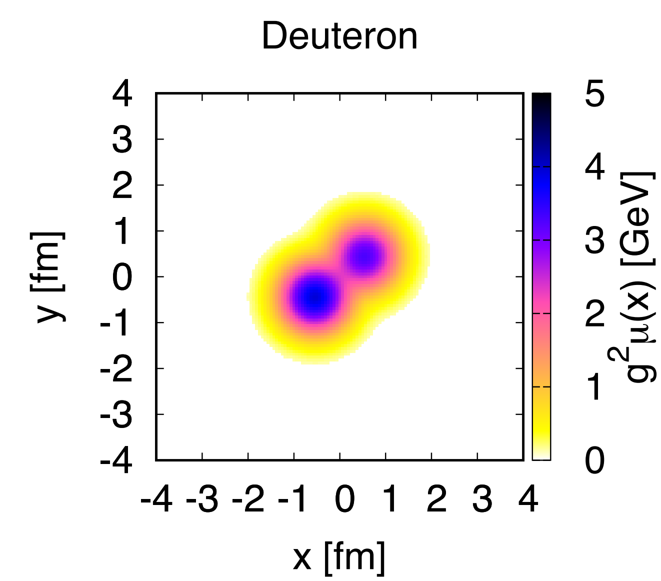
Sample nucleon positions as is done in MC Glauber

IP-Sat model (+fluctuations) provides $Q_s^2(x,\mathbf{b})$ for each nucleus

Kowalski, Teaney, Phys.Rev. D68 (2003) 114005, McLerran, Tribedy NPA 945 (2016)

Color charge fluctuations sampled event-by-event with MV model:

$$\langle \rho^a_{p/T}(\mathbf{x}_\perp)\rho^b_{p/T}(\mathbf{y}_\perp)\rangle = g^2\mu^2(x,\mathbf{b}=(\mathbf{x}_\perp+\mathbf{y}_\perp)/2)\delta^{ab}\delta^2(\mathbf{x}_\perp-\mathbf{y}_\perp)$$



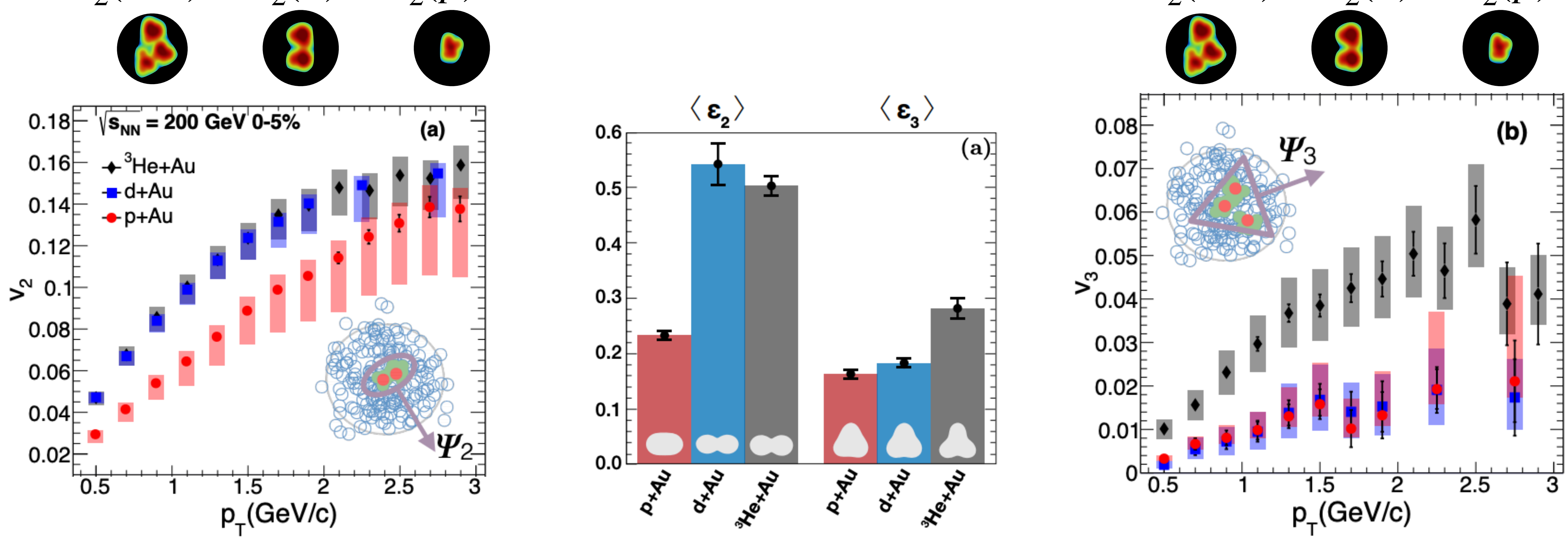
Note that these are (data-driven) choices, not derived from first principles

Possibly other CGC strategies not involving nucleon position sampling **Talk by G. Giacalone Tuesday**

Size hierarchy at RHIC

To test geometry driven hydrodynamic flow hypothesis, consider three systems with different (average) projectile geometries

$v_2(^3\mathrm{He}) \sim v_2(d) > v_2(p)$ $v_2(^3\mathrm{He}) > v_2(d) \sim v_2(p)$



PHENIX, Nature Phys. 15 (2019)

PHENIX results agree with eccentricity driven expectation

For CGC, domain scaling suggests that $v_2(p) > v_2(d) > v_2(^3\mathrm{He})$

Dilute-dense gluons

Previous CGC results showed good description of size hierarchy

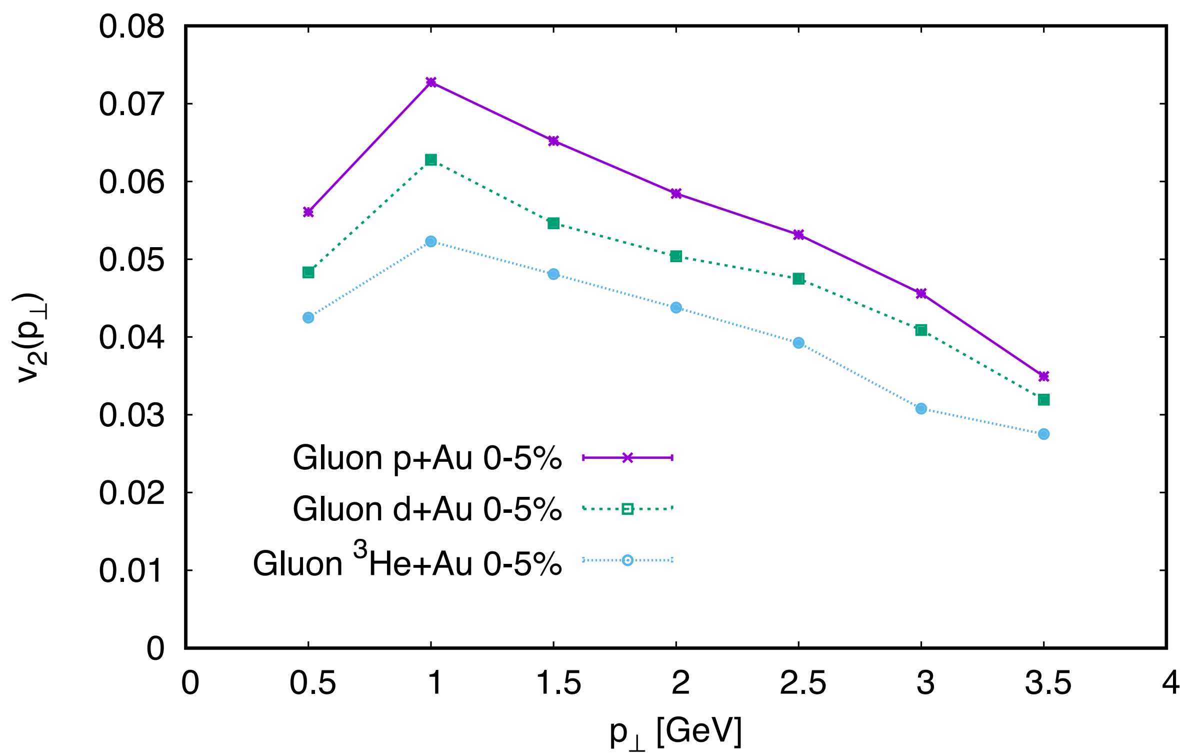
MM, Skokov, Tribedy, Venugopalan, PRL 112 2018

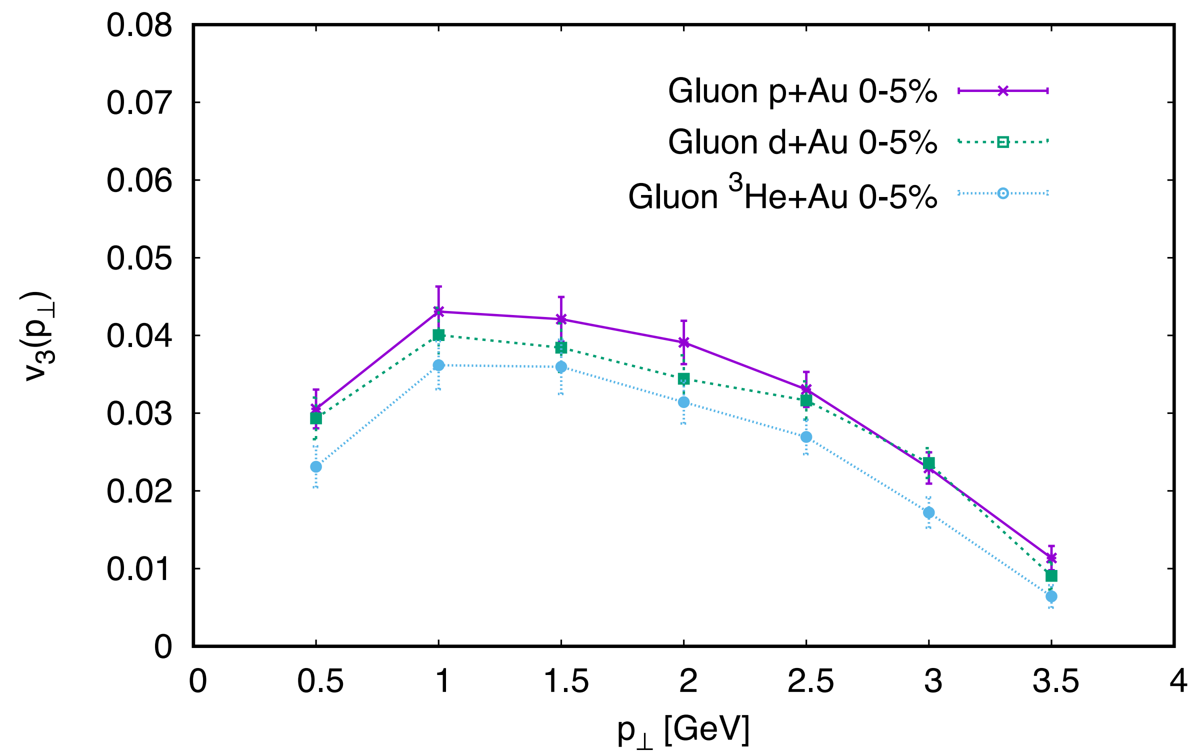
Unit conversion mistake in numerics found, affecting all momenta

Dilute-dense CGC solver publicly available: https://github.com/markfmace/DiluteDenseGluons

Correction to reference bin spoils hierarchy

Previous ordering only exists for $p_T<0.5$ GeV



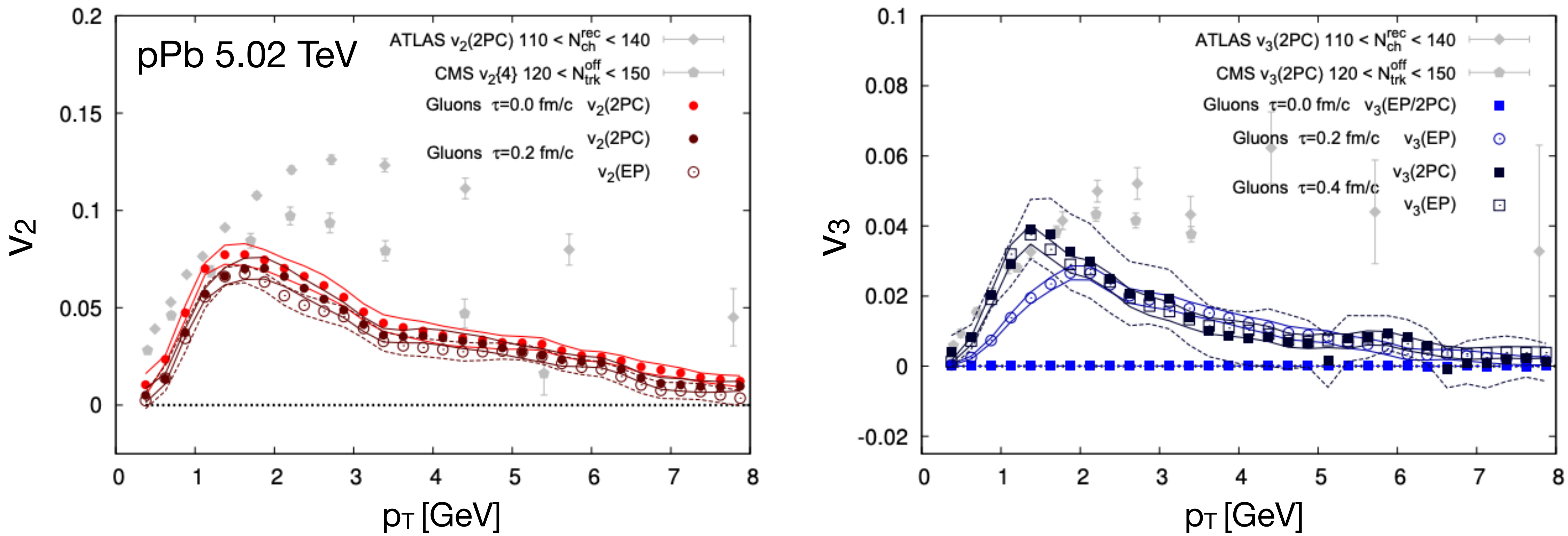


MM, Skokov, Tribedy, Venugopalan, Erratum, PRL, in press

Hierarchy now inverted, magnitudes of v_n can't describe PHENIX results

Dense-dense gluons

(Revised) dilute-dense results qualitatively similar
to dense-dense (IP-Glasma) calculations



Schenke, Schlichting, Venugopalan PLB 747 (2015)

So now what?

At present, CGC **alone** has trouble describing hierarchy of small system v_ns by PHENIX

Coupling to final state (kinetic theory, hydro, etc) seems necessary

IS Theory To Do's: corrections to classical/infinite energy picture, quark production + correlations, jet graph contributions,…

CGC is an effective description of QCD; a QCD description of the initial state should be the ultimate goal

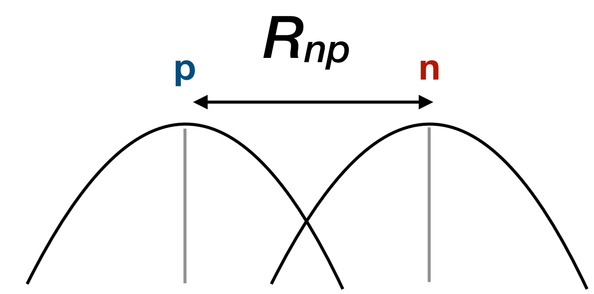
To study nuts and bolts of CGC, consider simplified model:

Projectile 'nucleus' (no IP-Sat) hitting a 'wall' of color charge using the MV-model

Case study

Consider deuteron-MV collisions

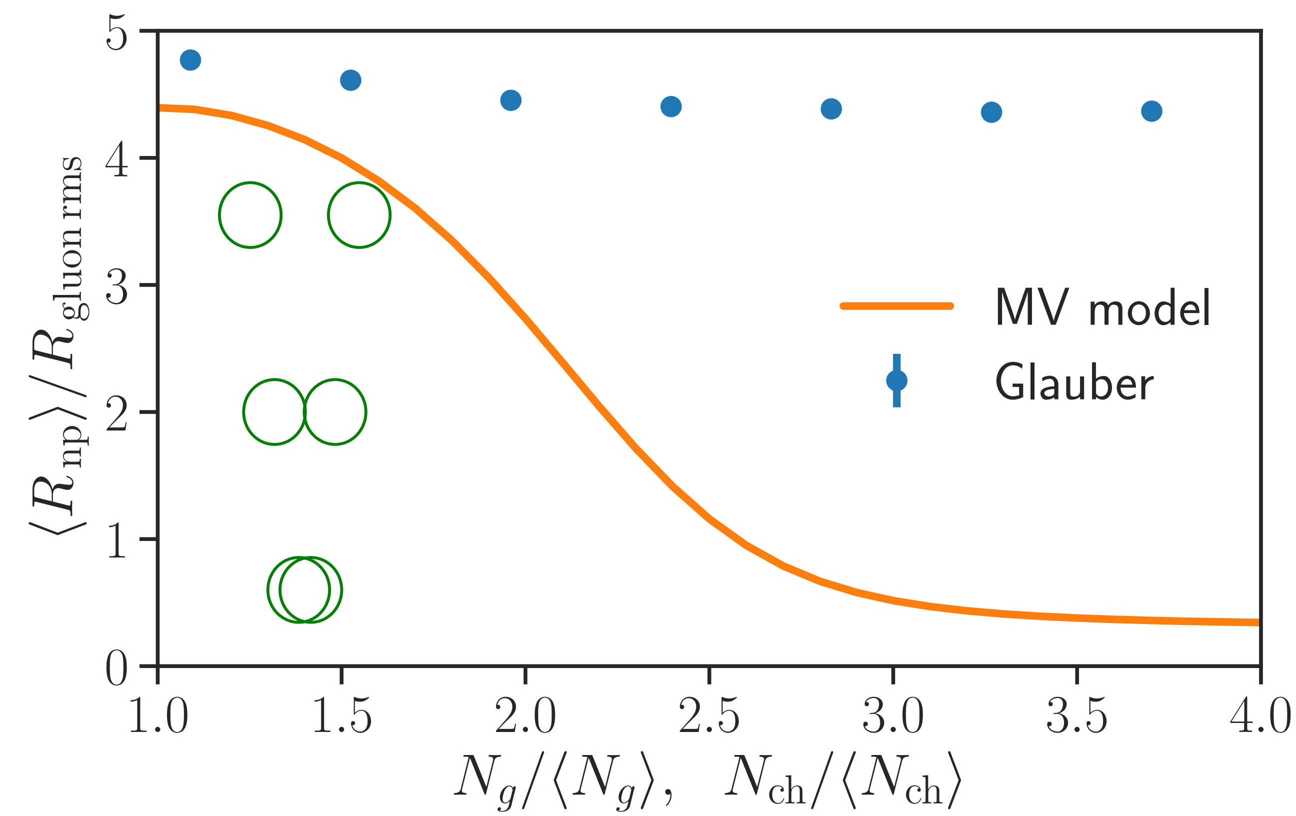
Small transverse R_{np} very rare



In dilute-dense CGC, high-multiplicity events exponentially biased to ‘close’ configurations

n!-growth in higher moments of P(N_g) — consequence of Bose statistics

Kovner, Skokov PRD 98 (2018)

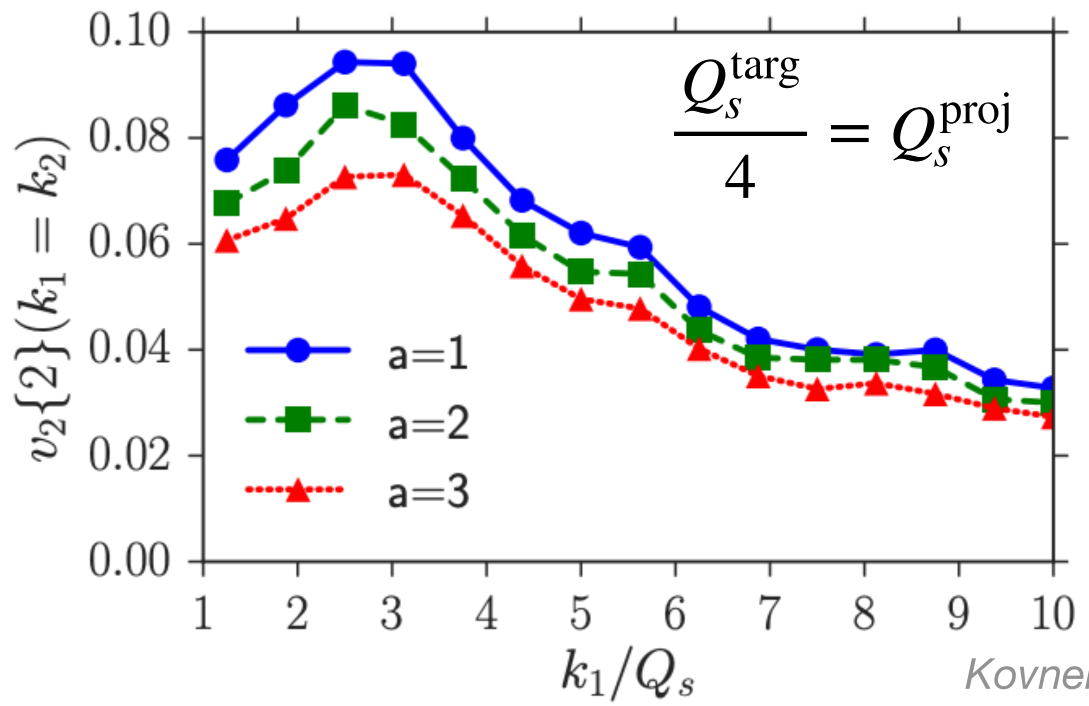


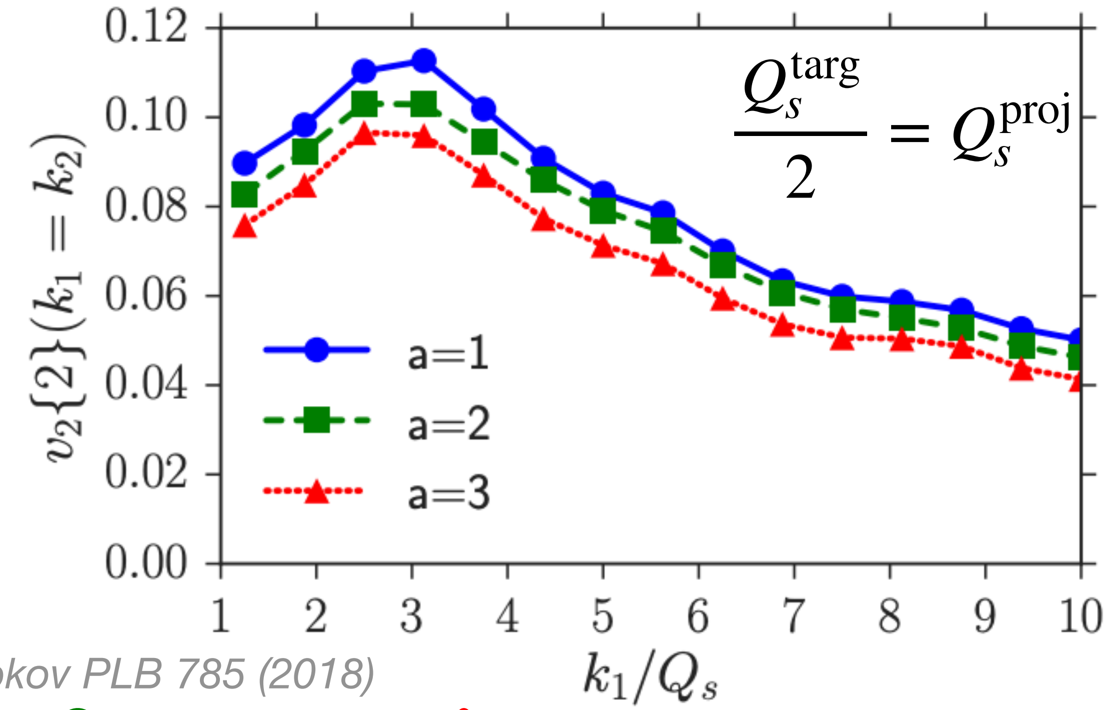
Very different from MC Glauber results — *quantum correlations give different picture!*

‘System scan’ for MV

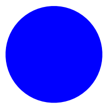
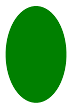
For fixed projectile size, larger Q_s generates larger v_2

$$\xrightarrow{Q_s^{\text{proj}}}$$





Kovner, Skokov PLB 785 (2018)

a=1  **a=2**  **a=3** 

For fixed Q_s, projectile ellipticity and v_2 anti-correlated

At very high multiplicity, full deuteron overlap leads to $Q_s(d)>Q_s(p)$, v_2(deuteron) > v_2(proton)

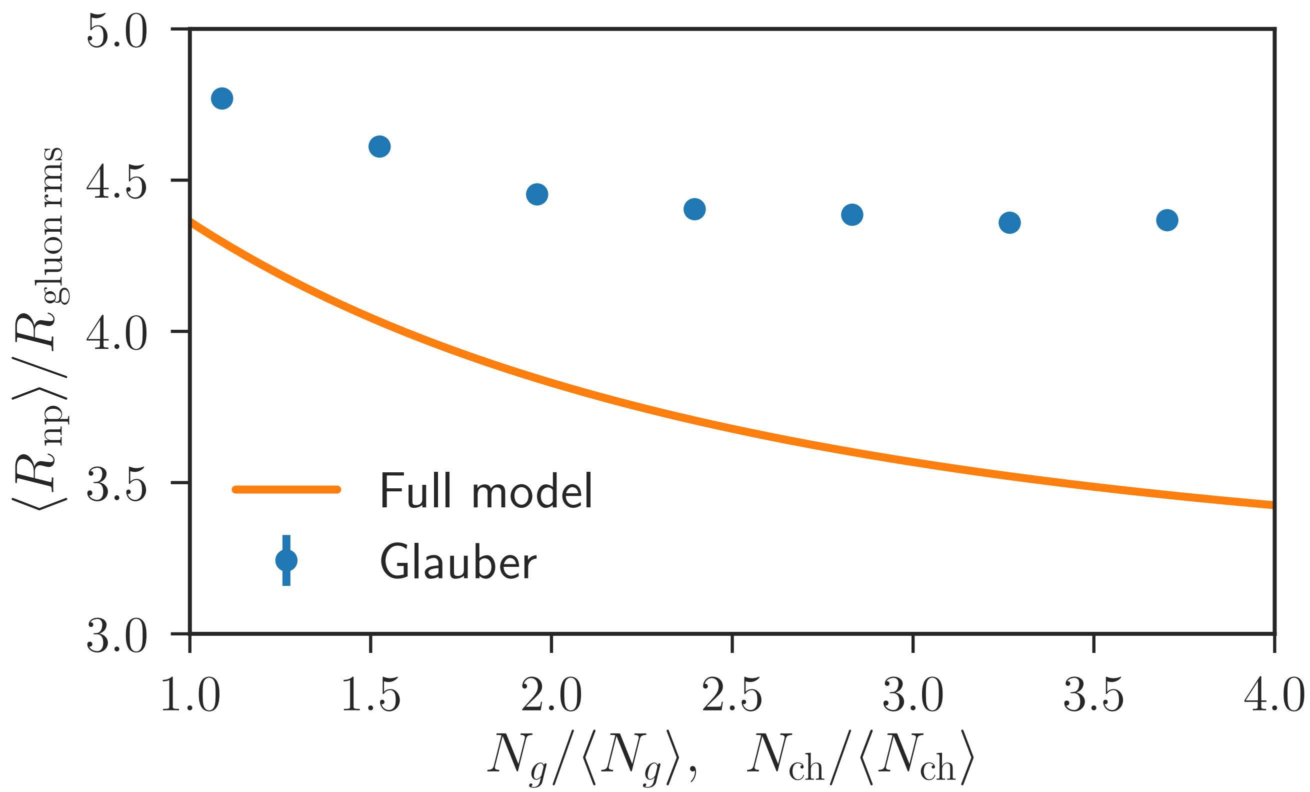
Back to the full model

Returning to full-fluctuating dilute-dense CGC model (MSTV)

Decrease in nucleon separation *not nearly as strong* as with deuteron-MV model

- Apparent subdominance of quantum correlations in MSTV model

Talk by B. Zajc Wednesday



Ultimately high multiplicity events aren't close enough to overcome domain scaling

Where do we stand?

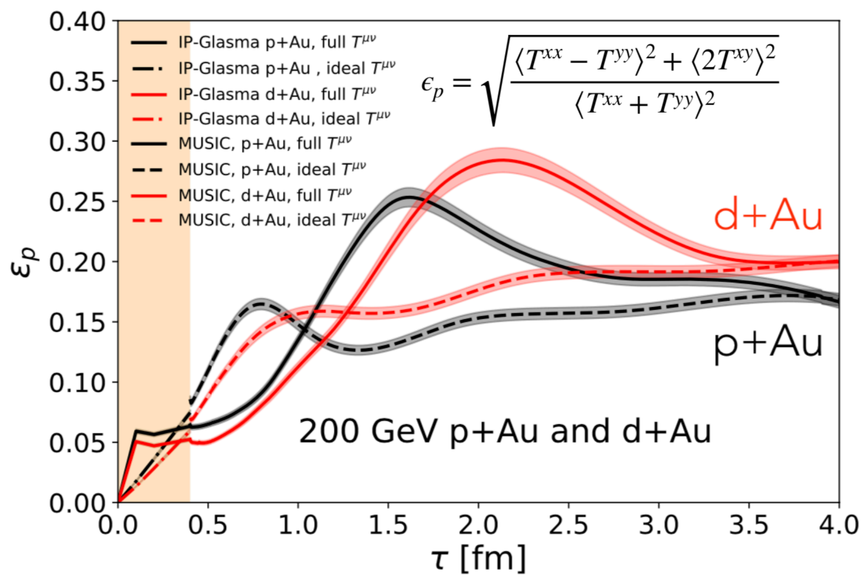
Questions we should all be asking:

What measurements can we make to amplify the effect of the initial state?

How can we discriminate between current models when paired with hydro transport?

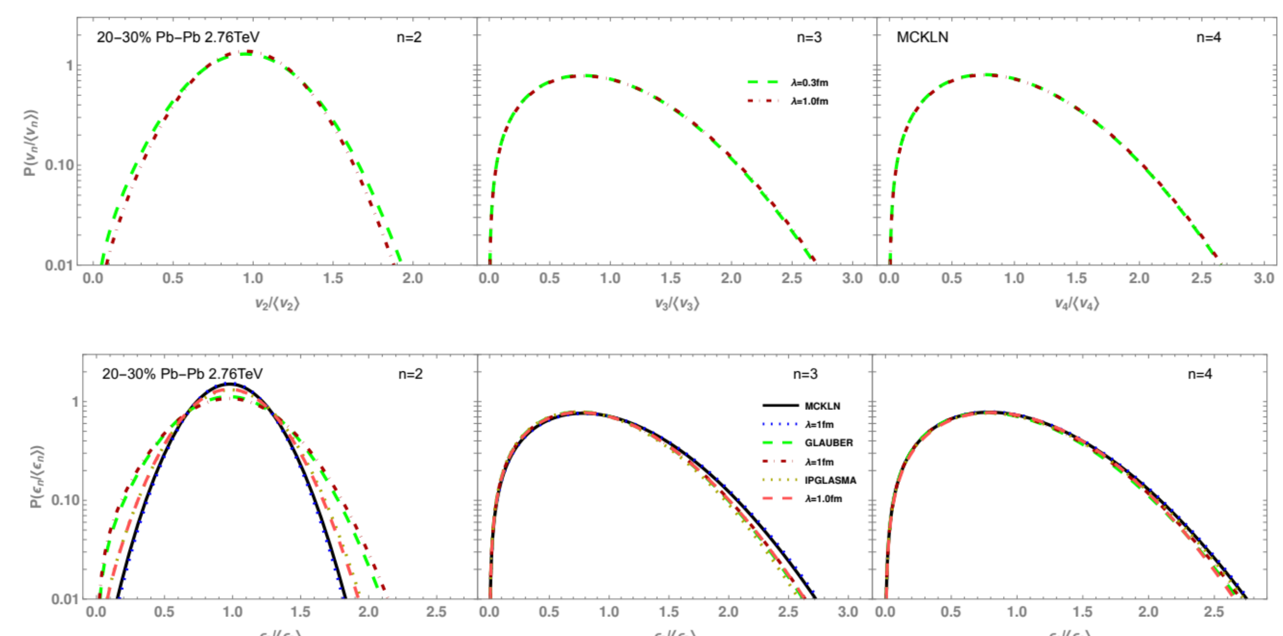
<u>Examples of work in that direction</u>

IP-Glasma+MUSIC+ UrQMD



PRELMINARY Schenke, Shen, Tribedy, in preparation

B. Schenke, RHIC-AGS Users Meeting June 4,2019



+

Gardim, Grassi, Ishida, Luzum, Magalhães, Noronha-Hostler PRC 97 (2018)

Conclusions

Initial state able to produce momentum anisotropies

- Includes both classical and quantum correlations

CGC description *alone* seems unable to describe the PHENIX small system v_n hierarchy

Dilute-dense CGC solver publicly available: https://github.com/markfmace/DiluteDenseGluons

- Important to disentangle how much of disagreement is underlying physics and how much is modeling
- Suggests that final state can not be neglected
- Continued progress in CGC/QCD-based initial state imperative for complete picture

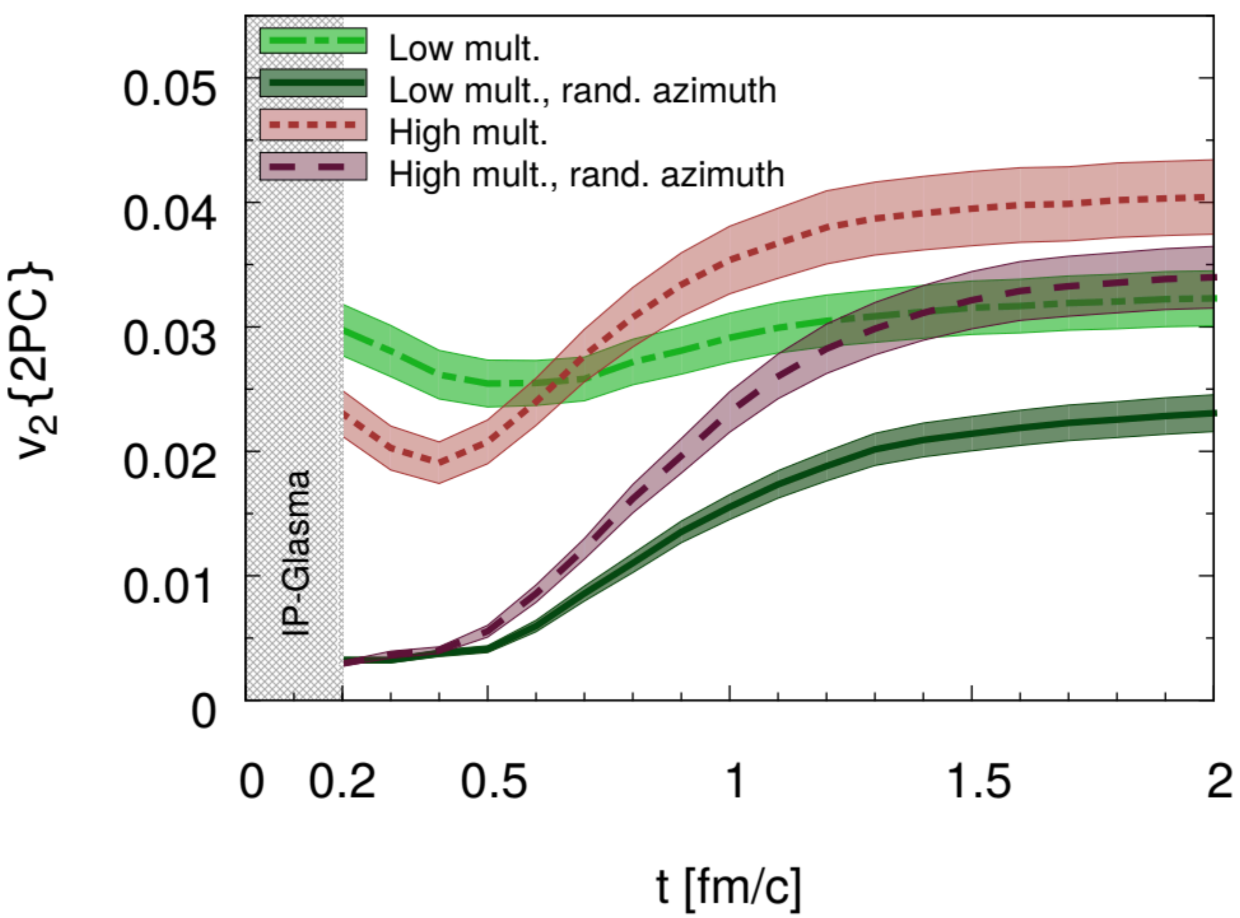
Strong dependence on initial state model used when pairing to hydrodynamic modeling

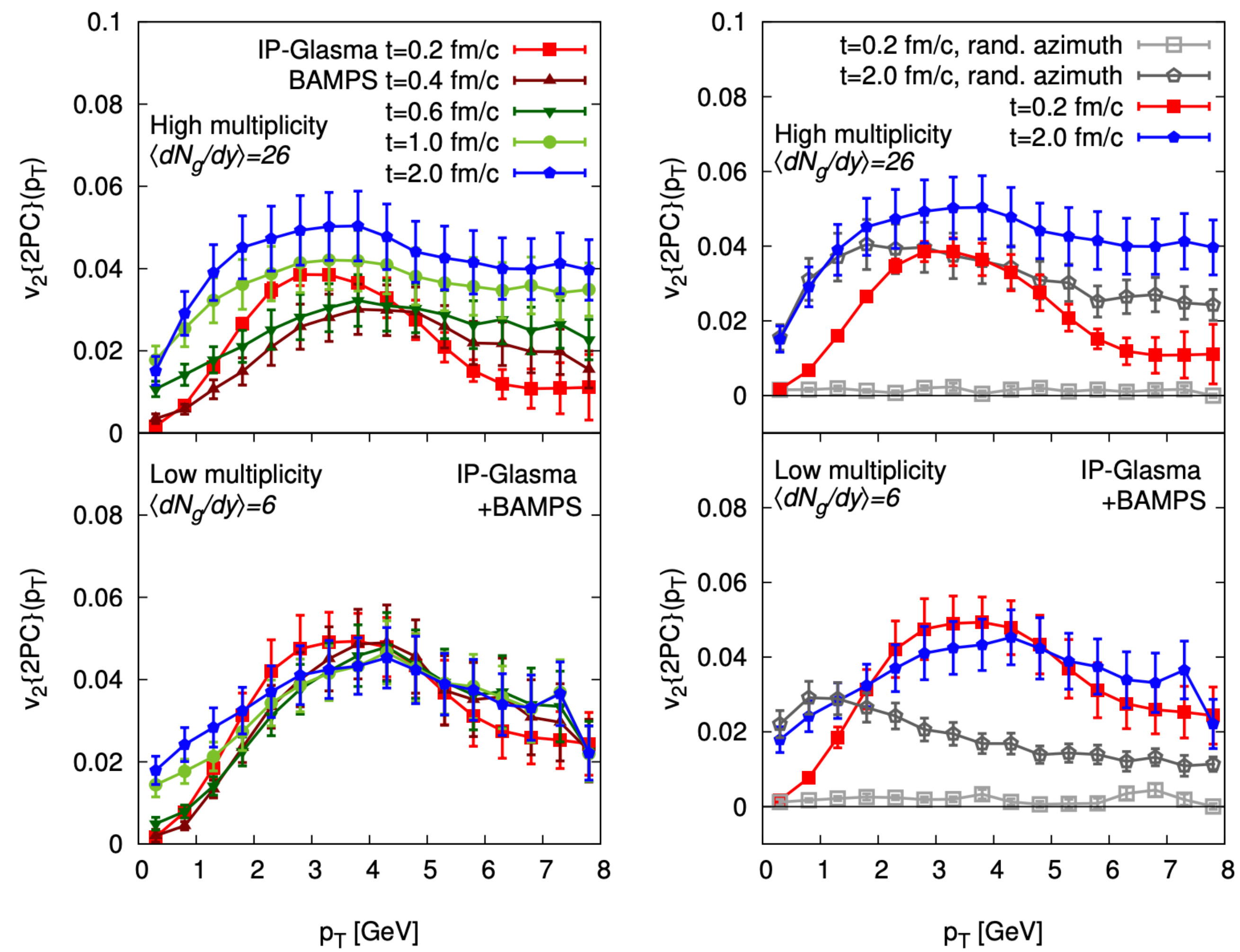
- Important to be able to discriminate between different descriptions
- Improvements when matching to kinetic theory?
  Talk by A. Mazeliauskas Tuesday


Thanks!

backup

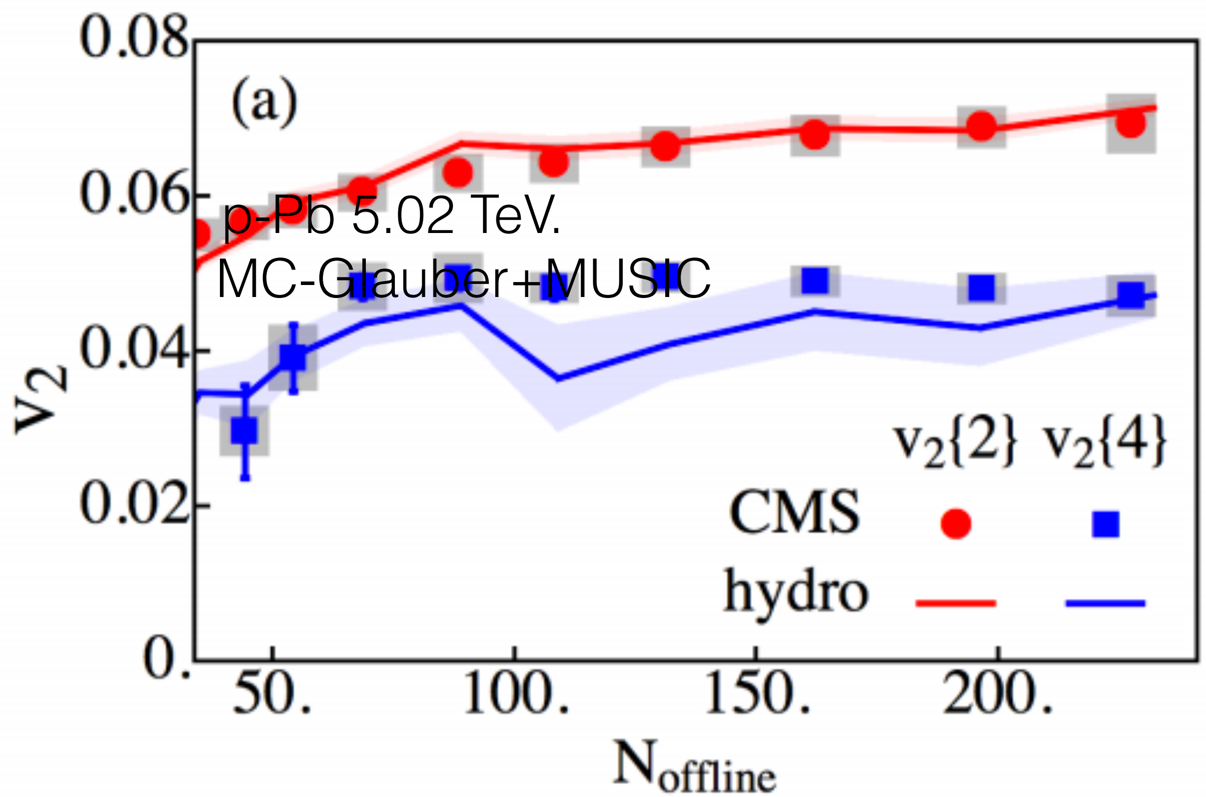
IP-Glasma+BAMPS

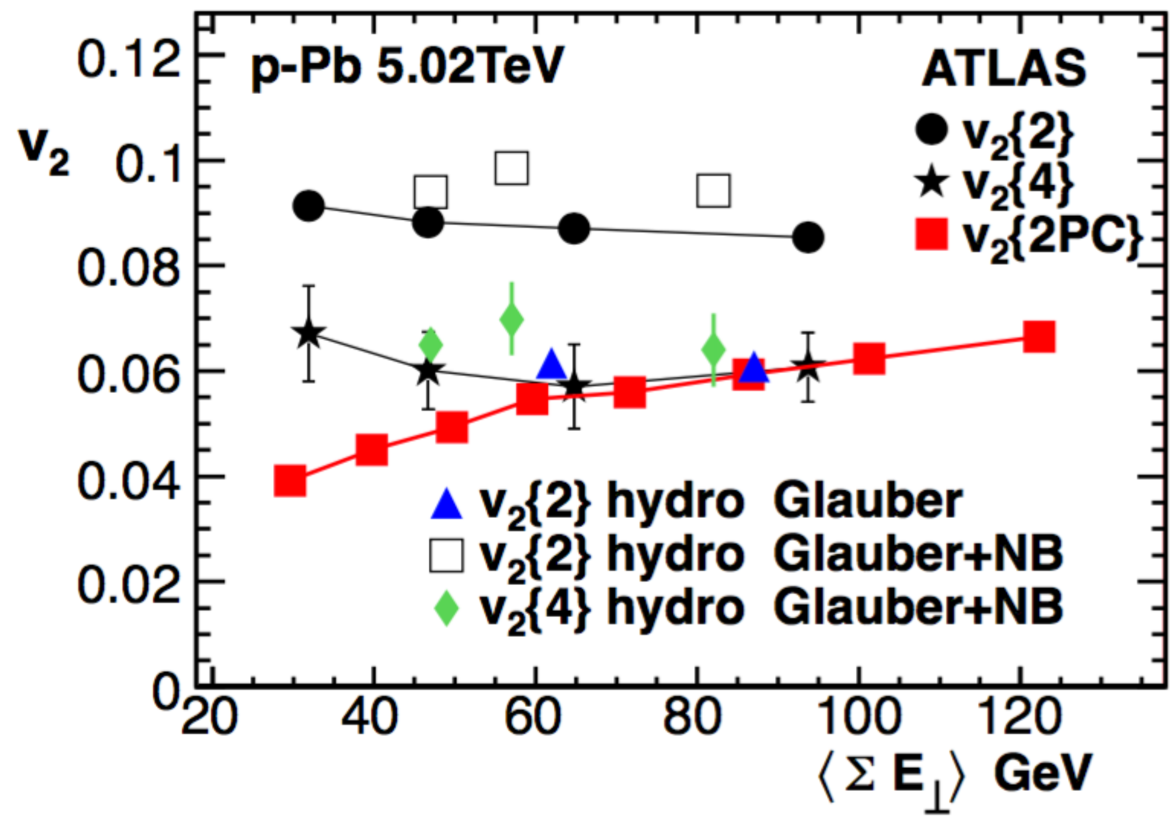




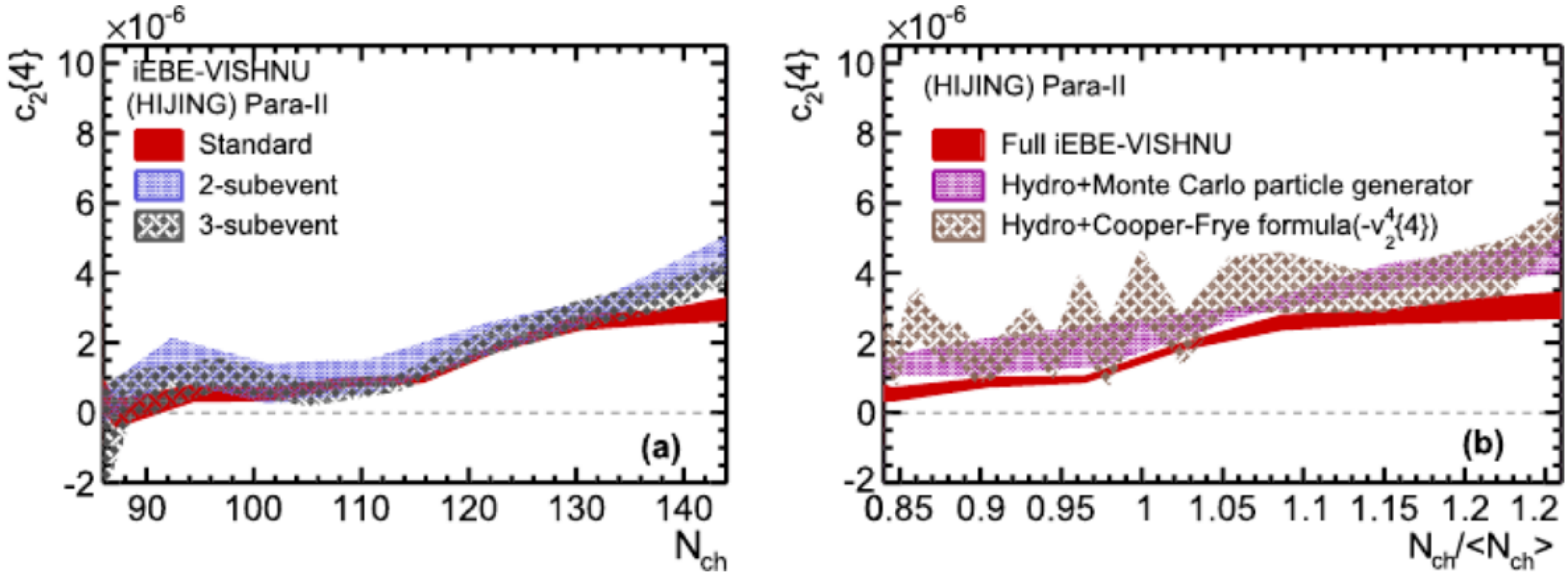
Initial CGC gives smaller v_2 for larger multiplicity system, but quickly reverse by kinetic theory

Greif, Greiner, Schenke, Schlichting, Xu PRD 96 (2017)





Kozlov, Denicol, Luzum, Jeon, Gale
NPA 931 (2014) 1045-1050



Zhao, Zhou, Xu, Deng, Song PLB 780 (2018)

Absence of four-particle v_2 from hydro in pp

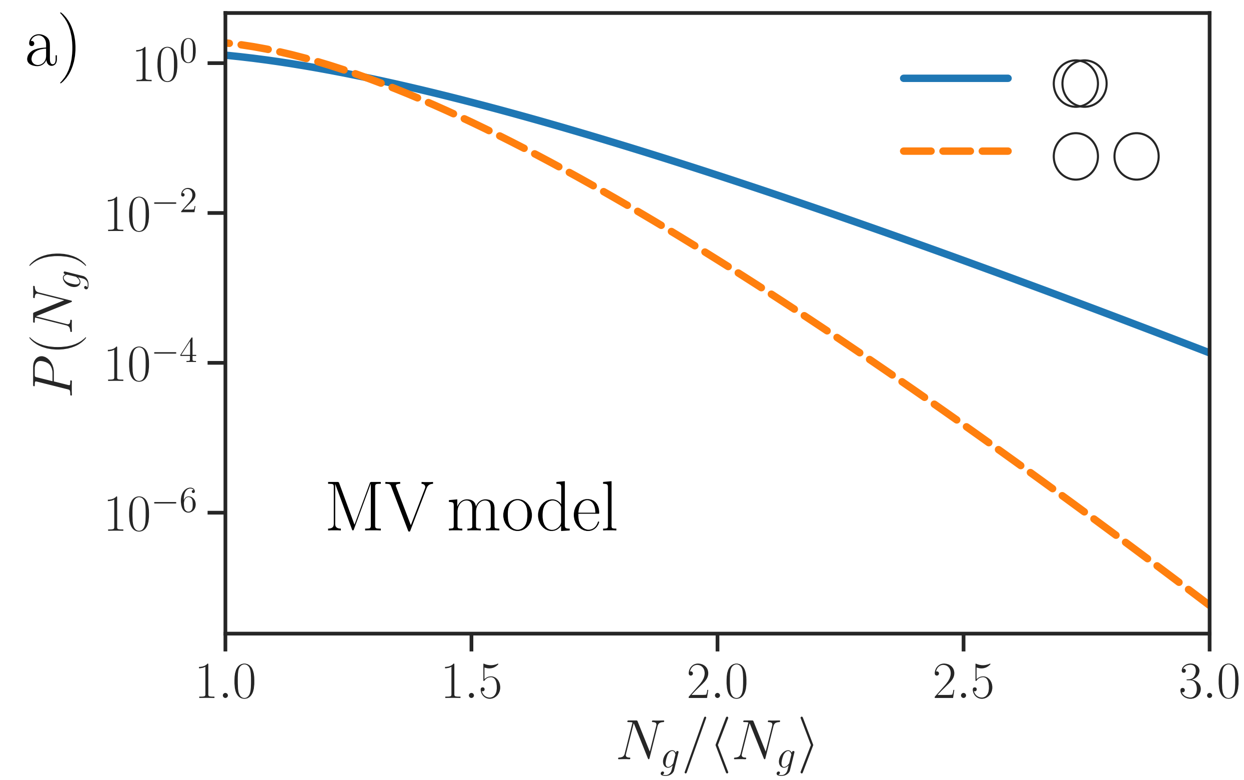
Case study: deuteron-MV

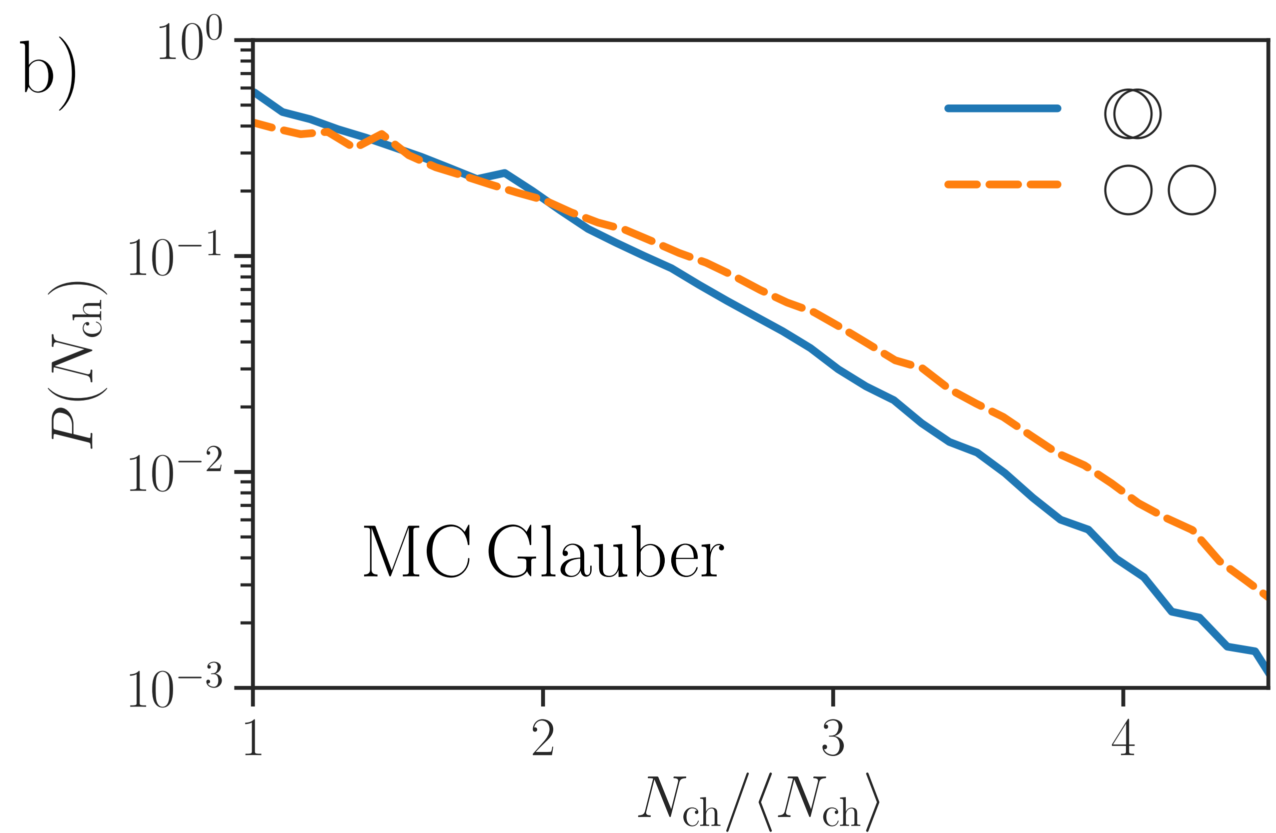
Close configurations dominate high-multiplicity events, very different from MC Glauber results — *quantum correlations give different picture!*

Parametrically in dilute-dense CGC

$N_{ch} \sim Q^2_{s,\mathrm{proj}} S_\perp$

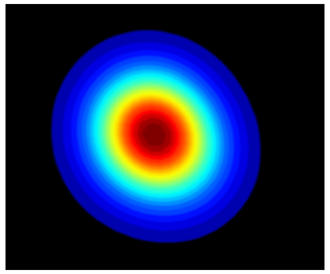
close configurations have greater *effective* $Q_{s,proj}$





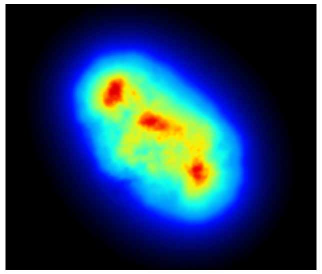
Hydro in small systems

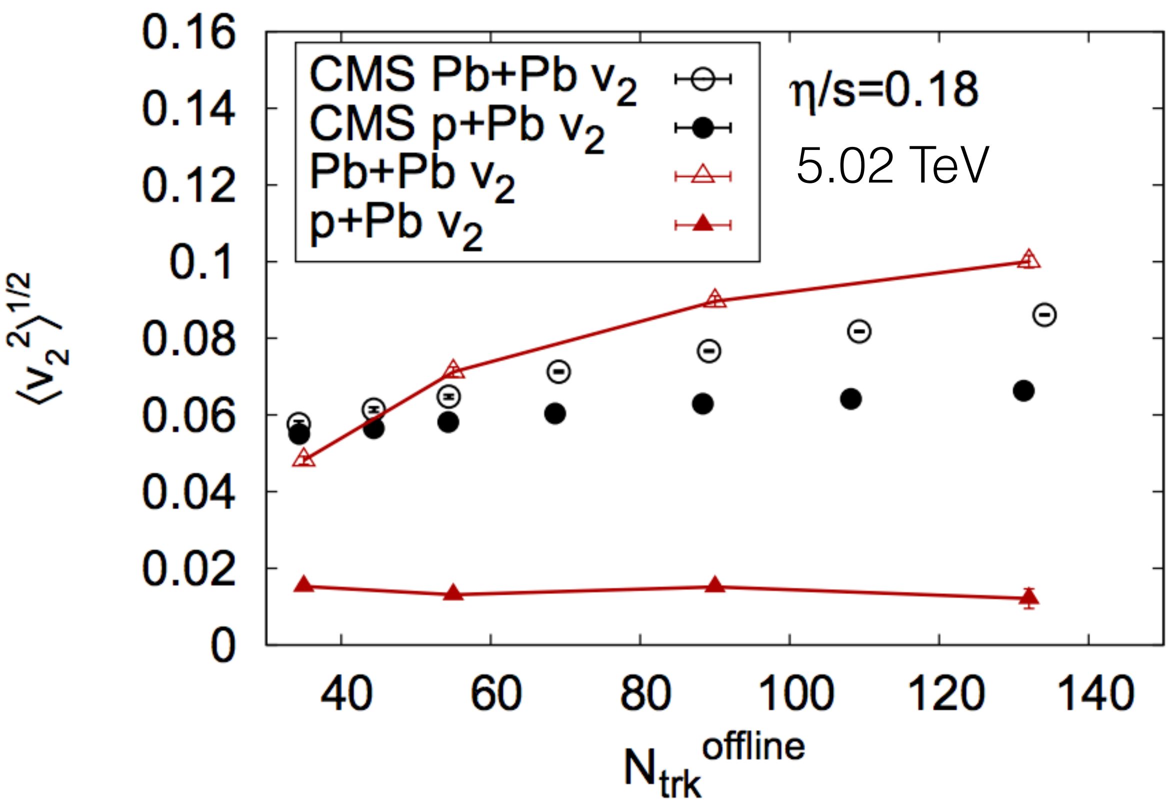
Round proton



Constrain proton shape fluctuations using exclusive J/Ψ production (HERA)

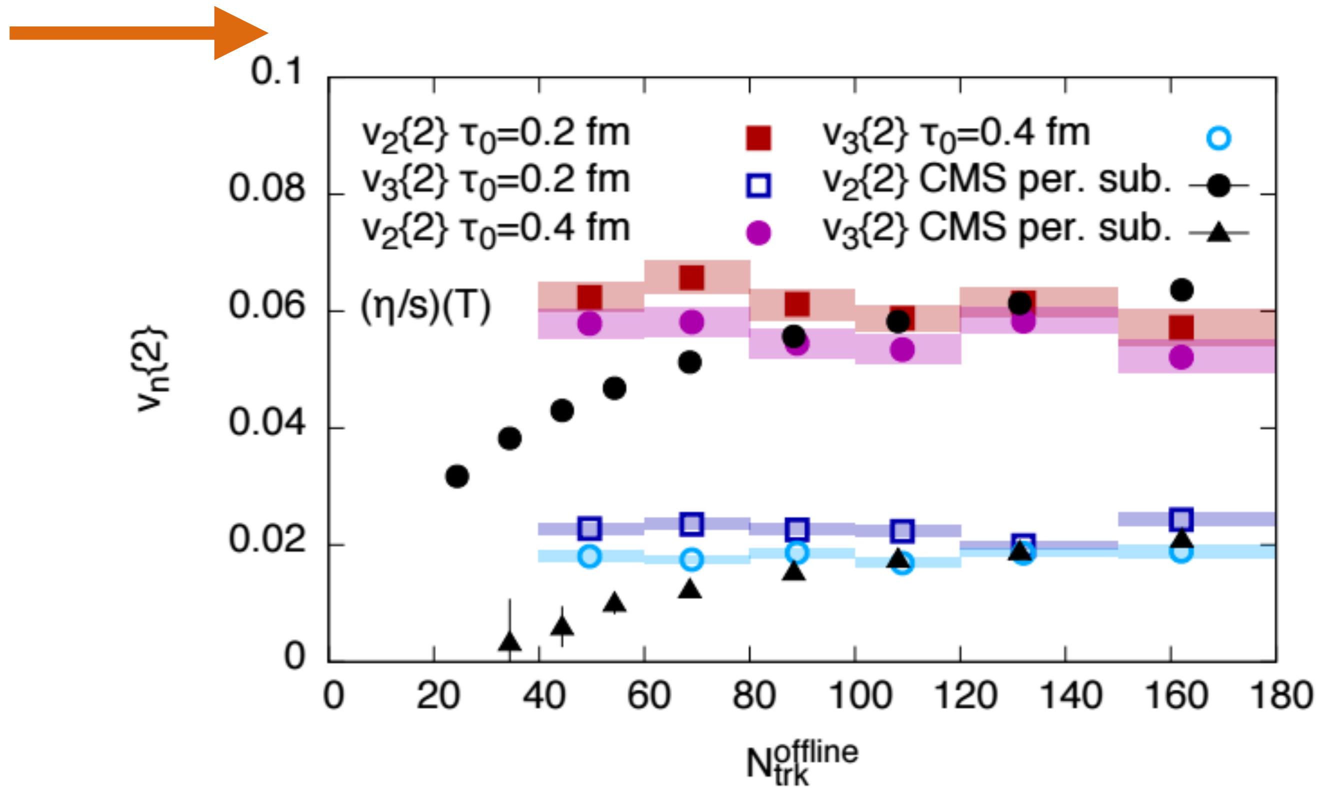
Fluctuating proton





Schenke, Venugopalan PRL 113 (2014) 102301

IP-Glasma+Hydro



Mäntysaari, Schenke, Shen, Tribedy arXiv: 1705.03177

IP-Glasma+**Fluct. proton** +Hydro+UrQMD

At face value, hydro calculations do okay

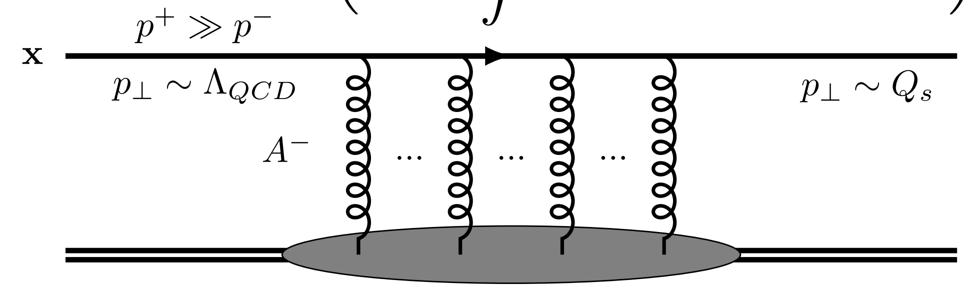
Dipole correlators

First, need to be able to compute correlation functions expectation values of dipoles

Consider dipole scattering matrix

$$\langle D(\mathbf{x},\mathbf{y})\rangle_U = \left\langle \frac{1}{N_c}\mathrm{tr}(U(\mathbf{x})U^\dagger(\mathbf{y}))\right\rangle$$

$$U(\mathbf{x}) = \mathcal{P}\mathrm{exp}\Big(- ig\int dz^+ A^{a-}(\mathbf{x},z^+)\, t^a\Big)$$



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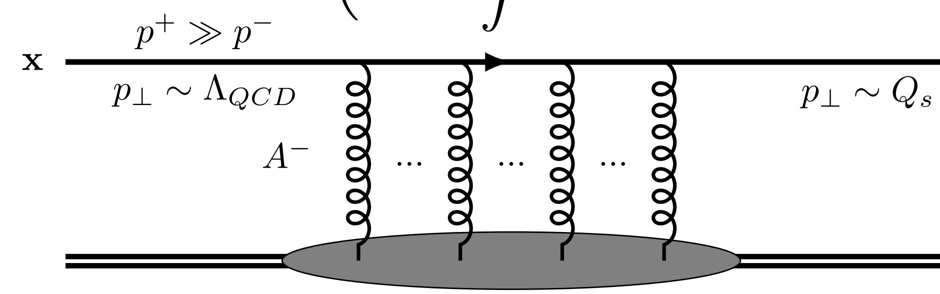

$$\langle D(\mathbf{x}, \mathbf{y}) \rangle_U = \left\langle \frac{1}{N_c} \mathrm{tr}(U(\mathbf{x}) U^\dagger(\mathbf{y})) \right\rangle$$

Expand out Wilson line in slices in rapidity

$$U(\mathbf{x}) = \mathcal{P}\mathrm{exp}\left(-ig \int dx^+ A^{a-}(\mathbf{x}, x^+) t^a \right) \simeq V(\mathbf{x})[1 - igA^{a-}(\zeta, \mathbf{x}) t^a + ...]$$

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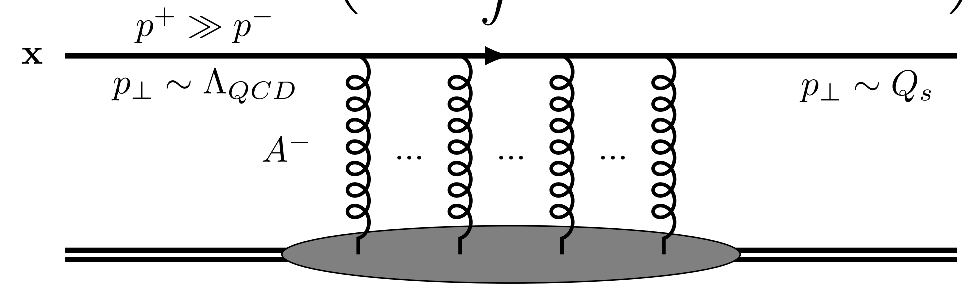

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Then gluons emissions with MV model

$$g^2 \langle A_a^-(x^+, \mathbf{x}_\perp) A_b^-(y^+, \mathbf{y}_\perp) \rangle = \delta_{ab} \delta(x^+ - y^+) L_{\mathbf{xy}}$$

where $L_{\mathbf{x}_\perp, \mathbf{y}_\perp} = -\dfrac{(g^2\mu)^2}{16\pi^2} |\mathbf{x} - \mathbf{y}|^2 \log \left(\dfrac{1}{|\mathbf{x}_\perp - \mathbf{y}_\perp| \Lambda} + e \right)$

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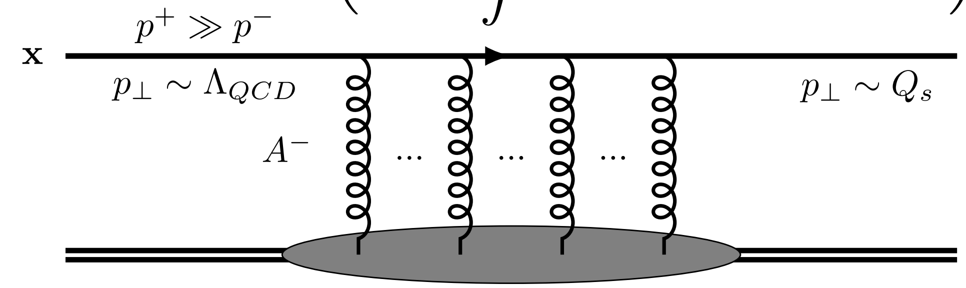

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We can re-exponentiate

$$\langle D(\mathbf{x},\mathbf{y})\rangle_U = \exp(C_F L(\mathbf{x},\mathbf{y}))$$

See arXiv:1706.06260 for details

Dipole correlators

Multiple dipole correlation functions encode projectile nucleus scattering, depends on scale Q_s

Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dominguez, Marquet, Wu NPA 823 (2009), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018), Fukushima, Hidaka JHEP (2007,2017),…

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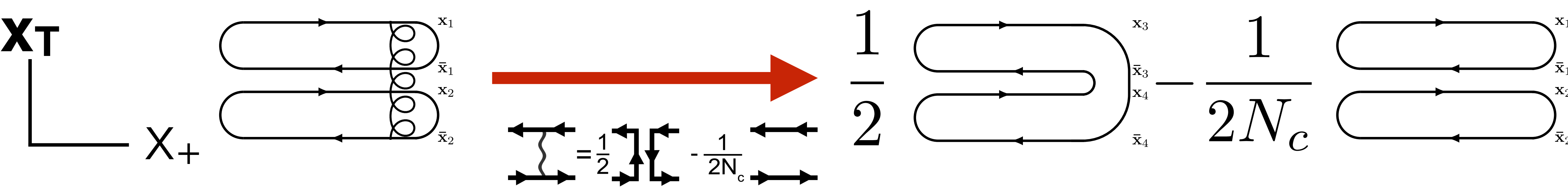
Then we can obtain $\langle DD \rangle$ similarly, first considering single gluon exchange, given by Fierz identity



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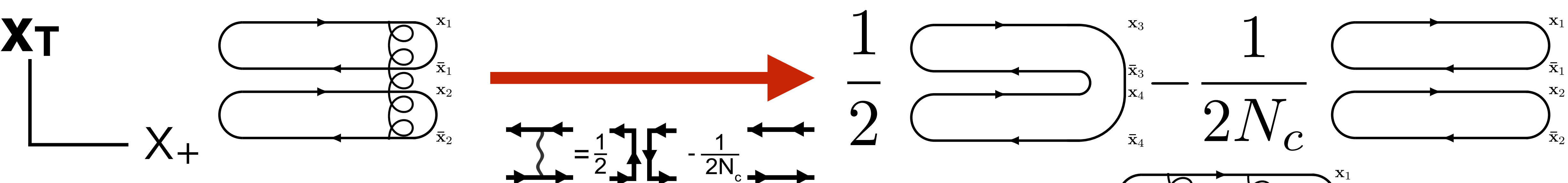
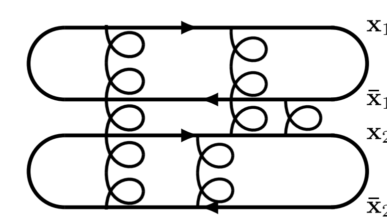


Doing this for all possible exchanges:



$$\begin{pmatrix} \langle D_{x_1\bar{x}_1} D_{x_2\bar{x}_2} \rangle \\ \langle Q_{x_1\bar{x}_2 x_2\bar{x}_1} \rangle \end{pmatrix}_U = \begin{pmatrix} \alpha_{x_1\bar{x}_1x_2\bar{x}_2} & \beta_{x_1\bar{x}_2x_2\bar{x}_1} \\ \beta_{x_1\bar{x}_1x_2\bar{x}_2} & \alpha_{x_1\bar{x}_2x_2\bar{x}_1} \end{pmatrix} \begin{pmatrix} \langle D_{x_1\bar{x}_1} D_{x_2\bar{x}_2} \rangle \\ \langle Q_{x_1\bar{x}_2 x_2\bar{x}_1} \rangle \end{pmatrix}_V$$

Which can be solved to all orders in gluon exchanges

Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dominguez, Marquet, Wu NPA 823 (2009), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018), Fukushima, Hidaka JHEP (2007,2017),…

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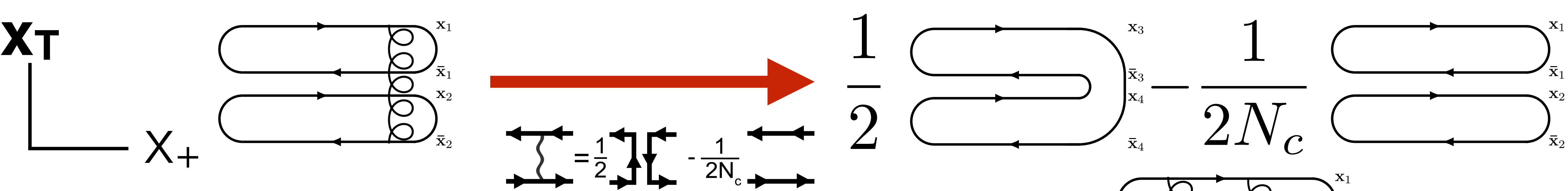

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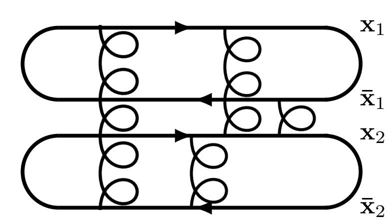

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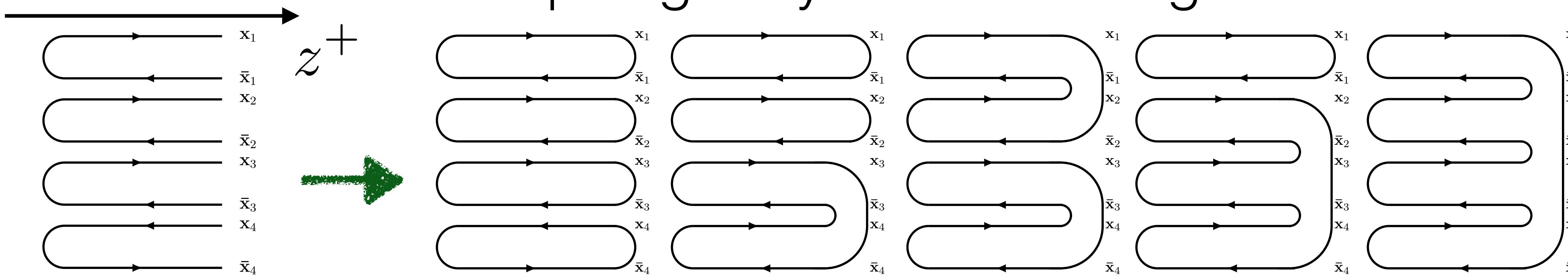
Which can be solved to all orders in gluon exchanges

Straightforward to generalize $\dfrac{d^4 N}{d^2\mathbf{p}_1 \cdots d^2\mathbf{p}_4} \simeq \int \langle DDDD \rangle$

Kovner, Wiedemann, PRD 64 (2001), Blaizot, Gelis, Venugopalan. NPA 743 (2004), Dominguez, Marquet, Wu NPA 823 (2009), Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018), Fukushima, Hidaka JHEP (2007,2017),…

Four dipole correlators

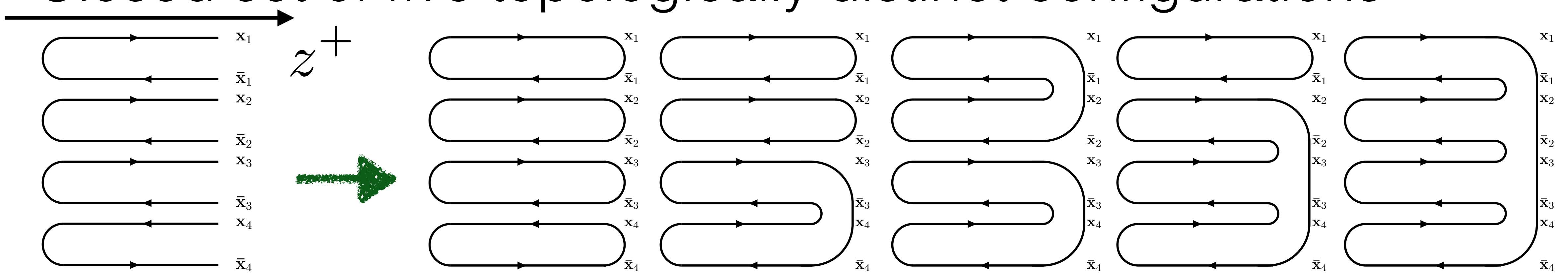
Closed set of five topologically distinct configurations



Permutations for each topology for closing on $z^+ = +\infty$

Four dipole correlators

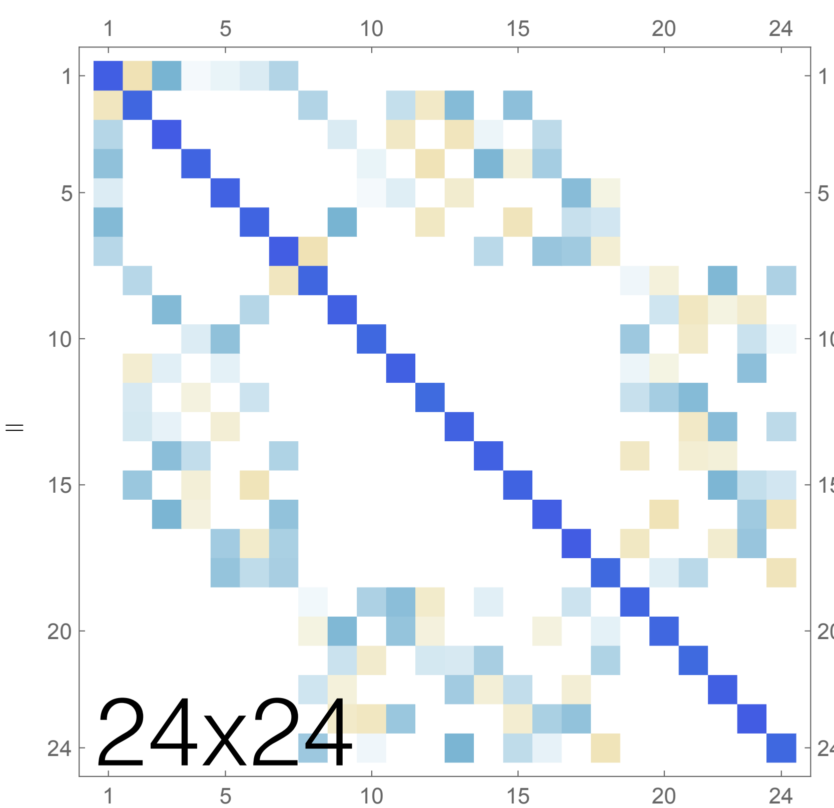
Closed set of five topologically distinct configurations



Permutations for each topology for closing on $z^+ = +\infty$

Define single gluon exchange matrix in terms of $L_{\mathbf{x},\mathbf{y}}$

$$
\begin{bmatrix}
\langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\
\langle Q_{w\bar{w}x\bar{x}} D_{y\bar{z}} D_{z\bar{y}} \rangle \\
\langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\
\langle Q_{w\bar{w}x\bar{y}} D_{y\bar{z}} D_{z\bar{x}} \rangle \\
\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{y}} \rangle \\
\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle \\
\langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\
\langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle \\
\langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{w}z\bar{z}} \rangle \\
\langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle \\
\langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle \\
\langle S_{w\bar{x}x\bar{z}y\bar{y}} D_{z\bar{w}} \rangle \\
\langle S_{w\bar{y}x\bar{w}y\bar{x}} D_{z\bar{z}} \rangle \\
\langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle \\
\langle S_{w\bar{y}x\bar{x}y\bar{w}} D_{z\bar{z}} \rangle \\
\langle S_{w\bar{y}x\bar{x}y\bar{z}} D_{z\bar{w}} \rangle \\
\langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle \\
\langle S_{w\bar{y}x\bar{z}y\bar{x}} D_{z\bar{w}} \rangle \\
\langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}} \rangle \\
\langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}} \rangle \\
\langle O_{w\bar{z}x\bar{x}y\bar{w}z\bar{y}} \rangle \\
\langle O_{w\bar{z}x\bar{x}y\bar{y}z\bar{w}} \rangle \\
\langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}} \rangle \\
\langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}} \rangle
\end{bmatrix}_U =
$$



$$
\begin{bmatrix}
\langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\
\langle Q_{w\bar{w}x\bar{x}} D_{y\bar{z}} D_{z\bar{y}} \rangle \\
\langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\
\langle Q_{w\bar{w}x\bar{y}} D_{y\bar{z}} D_{z\bar{x}} \rangle \\
\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{y}} \rangle \\
\langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle \\
\langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\
\langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle \\
\langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{w}z\bar{z}} \rangle \\
\langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle \\
\langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle \\
\langle S_{w\bar{x}x\bar{z}y\bar{y}} D_{z\bar{w}} \rangle \\
\langle S_{w\bar{y}x\bar{w}y\bar{x}} D_{z\bar{z}} \rangle \\
\langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle \\
\langle S_{w\bar{y}x\bar{x}y\bar{w}} D_{z\bar{z}} \rangle \\
\langle S_{w\bar{y}x\bar{x}y\bar{z}} D_{z\bar{w}} \rangle \\
\langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle \\
\langle S_{w\bar{y}x\bar{z}y\bar{x}} D_{z\bar{w}} \rangle \\
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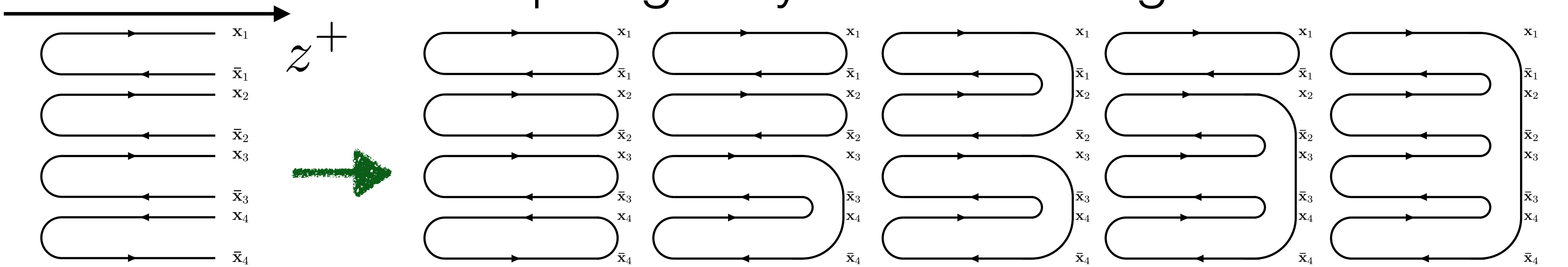
\langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}} \rangle \\
\langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}} \rangle
\end{bmatrix}_V
$$

Four dipole correlators

Closed set of five topologically distinct configurations



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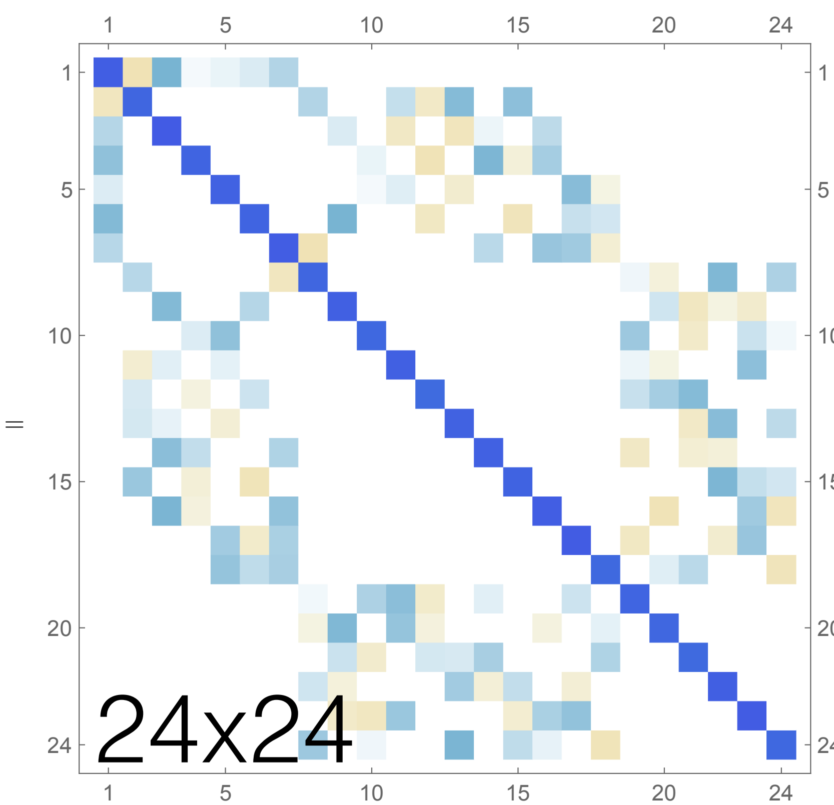
$$\left\langle \frac{d^4N}{d^2\mathbf{p}_1 ... d^2\mathbf{p}_4} \right\rangle \simeq \int \langle DDDD \rangle \sim e^{\square}$$

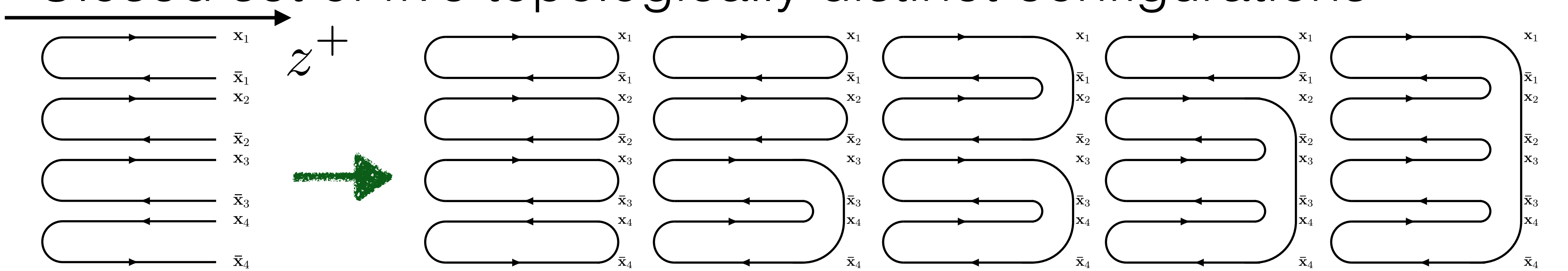
$$\begin{bmatrix} \langle D_{w\bar{w}}D_{x\bar{x}}D_{y\bar{y}}D_{z\bar{z}}\rangle \\ \langle Q_{w\bar{w}x\bar{x}}D_{y\bar{z}}D_{z\bar{y}}\rangle \\ \langle Q_{w\bar{w}x\bar{y}}D_{y\bar{x}}D_{z\bar{z}}\rangle \\ \langle Q_{w\bar{w}x\bar{y}}D_{y\bar{z}}D_{z\bar{x}}\rangle \\ \langle Q_{w\bar{w}x\bar{z}}D_{y\bar{x}}D_{z\bar{y}}\rangle \\ \langle Q_{w\bar{w}x\bar{z}}D_{y\bar{y}}D_{z\bar{x}}\rangle \\ \langle Q_{w\bar{x}x\bar{w}}D_{y\bar{y}}D_{z\bar{z}}\rangle \\ \langle Q_{w\bar{x}x\bar{w}}Q_{y\bar{z}z\bar{y}}\rangle \\ \langle Q_{w\bar{x}x\bar{y}}Q_{y\bar{w}z\bar{z}}\rangle \\ \langle Q_{w\bar{x}x\bar{y}}Q_{y\bar{z}z\bar{w}}\rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{w}}D_{z\bar{y}}\rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{y}}D_{z\bar{w}}\rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{x}}D_{z\bar{z}}\rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}}D_{z\bar{x}}\rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{w}}D_{z\bar{z}}\rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{z}}D_{z\bar{w}}\rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{w}}D_{z\bar{x}}\rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{x}}D_{z\bar{w}}\rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}}\rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}}\rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{w}z\bar{y}}\rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{y}z\bar{w}}\rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}}\rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}}\rangle \end{bmatrix}_U =$$



$$\begin{bmatrix} \langle D_{w\bar{w}}D_{x\bar{x}}D_{y\bar{y}}D_{z\bar{z}}\rangle \\ \langle Q_{w\bar{w}x\bar{x}}D_{y\bar{z}}D_{z\bar{y}}\rangle \\ \langle Q_{w\bar{w}x\bar{y}}D_{y\bar{x}}D_{z\bar{z}}\rangle \\ \langle Q_{w\bar{w}x\bar{y}}D_{y\bar{z}}D_{z\bar{x}}\rangle \\ \langle Q_{w\bar{w}x\bar{z}}D_{y\bar{x}}D_{z\bar{y}}\rangle \\ \langle Q_{w\bar{w}x\bar{z}}D_{y\bar{y}}D_{z\bar{x}}\rangle \\ \langle Q_{w\bar{x}x\bar{w}}D_{y\bar{y}}D_{z\bar{z}}\rangle \\ \langle Q_{w\bar{x}x\bar{w}}Q_{y\bar{z}z\bar{y}}\rangle \\ \langle Q_{w\bar{x}x\bar{y}}Q_{y\bar{w}z\bar{z}}\rangle \\ \langle Q_{w\bar{x}x\bar{y}}Q_{y\bar{z}z\bar{w}}\rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{w}}D_{z\bar{y}}\rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{y}}D_{z\bar{w}}\rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{x}}D_{z\bar{z}}\rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}}D_{z\bar{x}}\rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{w}}D_{z\bar{z}}\rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{z}}D_{z\bar{w}}\rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{w}}D_{z\bar{x}}\rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{x}}D_{z\bar{w}}\rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}}\rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}}\rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{w}z\bar{y}}\rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{y}z\bar{w}}\rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}}\rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}}\rangle \end{bmatrix}_V$$

Four dipole correlators

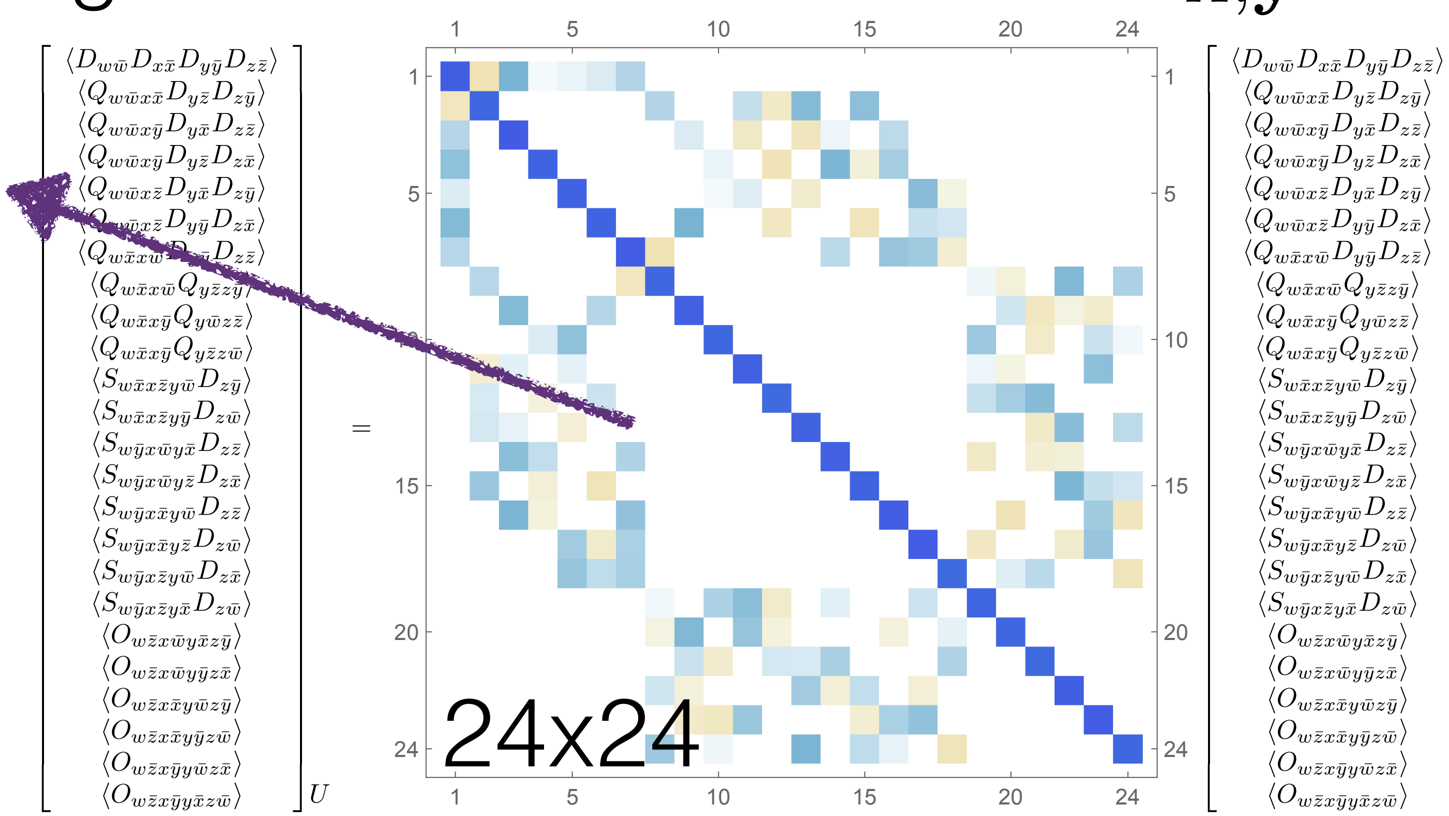
Closed set of five topologically distinct configurations



Permutations for each topology for closing on $z^+ = +\infty$

Define single gluon exchange matrix in terms of $L_{\mathbf{x},\mathbf{y}}$

$$\left\langle \frac{d^4 N}{d^2\mathbf{p}_1 \ldots d^2\mathbf{p}_4} \right\rangle \simeq \int \langle DDDD \rangle \sim e^{\square}$$



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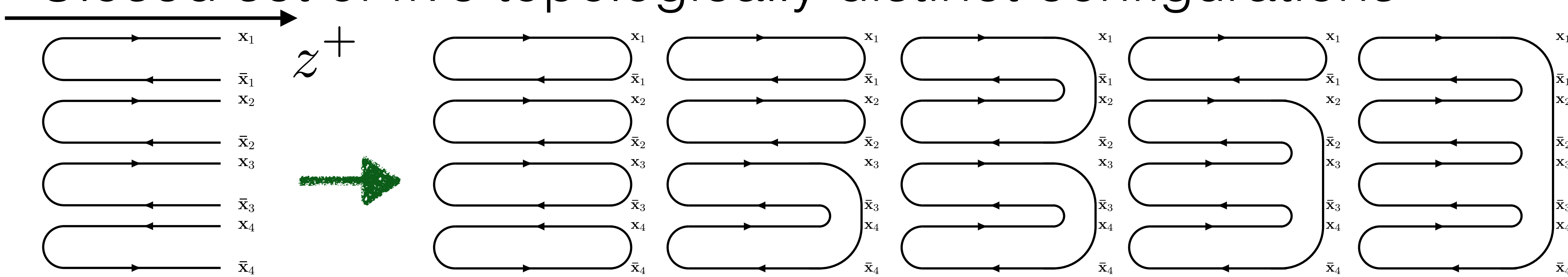

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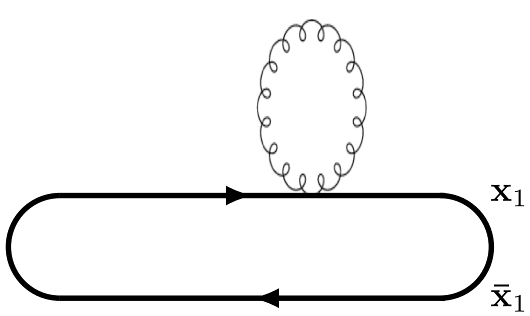
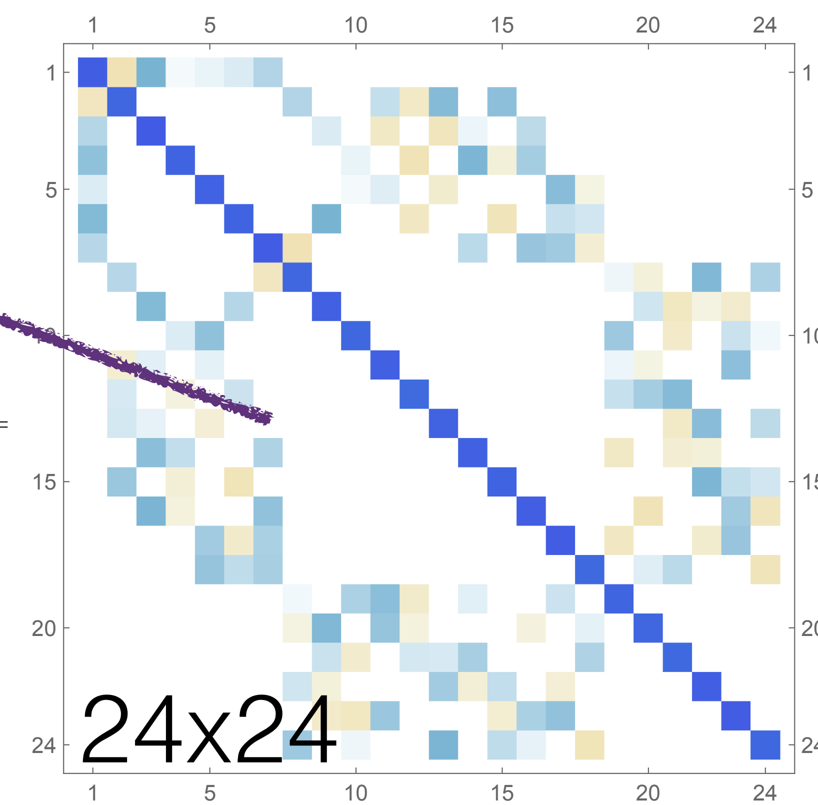


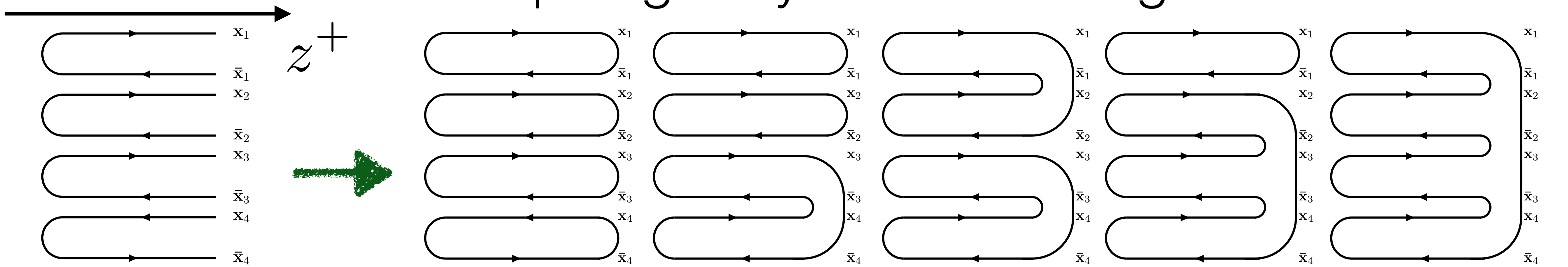
$$\begin{bmatrix} \langle D_{w\bar w} D_{x\bar x} D_{y\bar y} D_{z\bar z}\rangle \\ \langle Q_{w\bar w x\bar x} D_{y\bar z} D_{z\bar y}\rangle \\ \langle Q_{w\bar w x\bar y} D_{y\bar x} D_{z\bar z}\rangle \\ \langle Q_{w\bar w x\bar y} D_{y\bar z} D_{z\bar x}\rangle \\ \langle Q_{w\bar w x\bar z} D_{y\bar x} D_{z\bar y}\rangle \\ \langle Q_{w\bar w x\bar z} D_{y\bar y} D_{z\bar x}\rangle \\ \langle Q_{w\bar x x\bar w} D_{y\bar y} D_{z\bar z}\rangle \\ \langle Q_{w\bar x x\bar w} Q_{y\bar z z\bar y}\rangle \\ \langle Q_{w\bar x x\bar y} Q_{y\bar w z\bar z}\rangle \\ \langle Q_{w\bar x x\bar y} Q_{y\bar z z\bar w}\rangle \\ \langle S_{w\bar x x\bar z y\bar w} D_{z\bar y}\rangle \\ \langle S_{w\bar x x\bar z y\bar y} D_{z\bar w}\rangle \\ \langle S_{w\bar y x\bar w y\bar x} D_{z\bar z}\rangle \\ \langle S_{w\bar y x\bar w y\bar z} D_{z\bar x}\rangle \\ \langle S_{w\bar y x\bar x y\bar w} D_{z\bar z}\rangle \\ \langle S_{w\bar y x\bar x y\bar z} D_{z\bar w}\rangle \\ \langle S_{w\bar y x\bar z y\bar w} D_{z\bar x}\rangle \\ \langle S_{w\bar y x\bar z y\bar x} D_{z\bar w}\rangle \\ \langle O_{w\bar z x\bar w y\bar x z\bar y}\rangle \\ \langle O_{w\bar z x\bar w y\bar y z\bar x}\rangle \\ \langle O_{w\bar z x\bar x y\bar w z\bar y}\rangle \\ \langle O_{w\bar z x\bar x y\bar y z\bar w}\rangle \\ \langle O_{w\bar z x\bar y y\bar w z\bar x}\rangle \\ \langle O_{w\bar z x\bar y y\bar x z\bar w}\rangle \end{bmatrix}_U =$$



$$\begin{bmatrix} \langle D_{w\bar w} D_{x\bar x} D_{y\bar y} D_{z\bar z}\rangle \\ \langle Q_{w\bar w x\bar x} D_{y\bar z} D_{z\bar y}\rangle \\ \langle Q_{w\bar w x\bar y} D_{y\bar x} D_{z\bar z}\rangle \\ \langle Q_{w\bar w x\bar y} D_{y\bar z} D_{z\bar x}\rangle \\ \langle Q_{w\bar w x\bar z} D_{y\bar x} D_{z\bar y}\rangle \\ \langle Q_{w\bar w x\bar z} D_{y\bar y} D_{z\bar x}\rangle \\ \langle Q_{w\bar x x\bar w} D_{y\bar y} D_{z\bar z}\rangle \\ \langle Q_{w\bar x x\bar w} Q_{y\bar z z\bar y}\rangle \\ \langle Q_{w\bar x x\bar y} Q_{y\bar w z\bar z}\rangle \\ \langle Q_{w\bar x x\bar y} Q_{y\bar z z\bar w}\rangle \\ \langle S_{w\bar x x\bar z y\bar w} D_{z\bar y}\rangle \\ \langle S_{w\bar x x\bar z y\bar y} D_{z\bar w}\rangle \\ \langle S_{w\bar y x\bar w y\bar x} D_{z\bar z}\rangle \\ \langle S_{w\bar y x\bar w y\bar z} D_{z\bar x}\rangle \\ \langle S_{w\bar y x\bar x y\bar w} D_{z\bar z}\rangle \\ \langle S_{w\bar y x\bar x y\bar z} D_{z\bar w}\rangle \\ \langle S_{w\bar y x\bar z y\bar w} D_{z\bar x}\rangle \\ \langle S_{w\bar y x\bar z y\bar x} D_{z\bar w}\rangle \\ \langle O_{w\bar z x\bar w y\bar x z\bar y}\rangle \\ \langle O_{w\bar z x\bar w y\bar y z\bar x}\rangle \\ \langle O_{w\bar z x\bar x y\bar w z\bar y}\rangle \\ \langle O_{w\bar z x\bar x y\bar y z\bar w}\rangle \\ \langle O_{w\bar z x\bar y y\bar w z\bar x}\rangle \\ \langle O_{w\bar z x\bar y y\bar x z\bar w}\rangle \end{bmatrix}_V$$

Four dipole correlators

Closed set of five topologically distinct configurations



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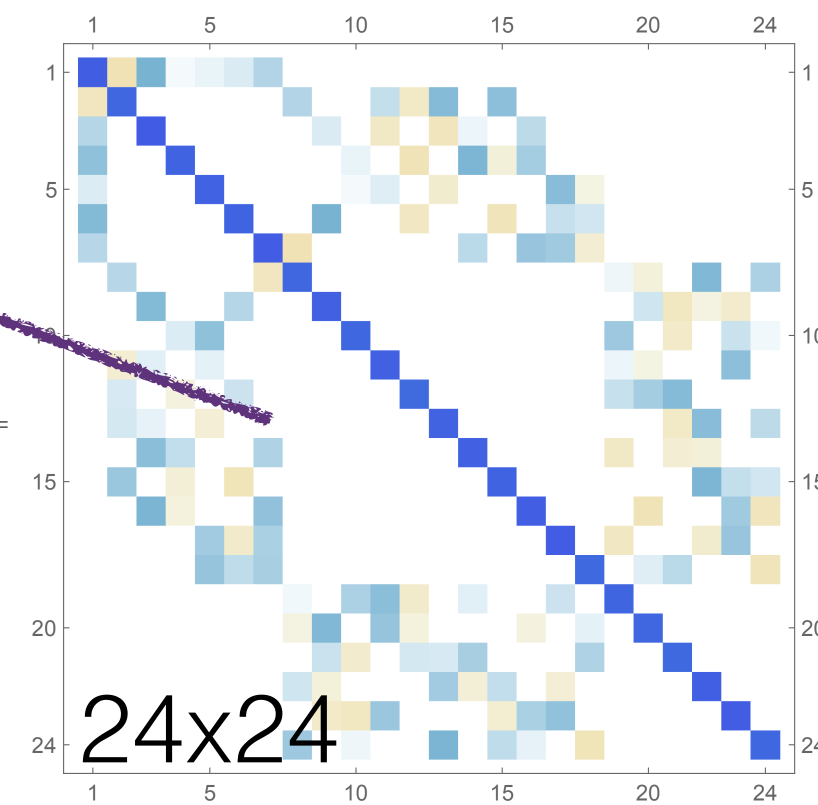

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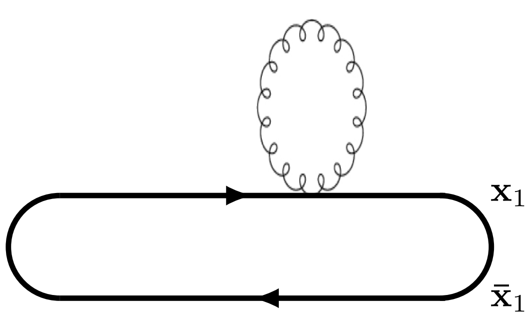
Also includes tadpole contribution

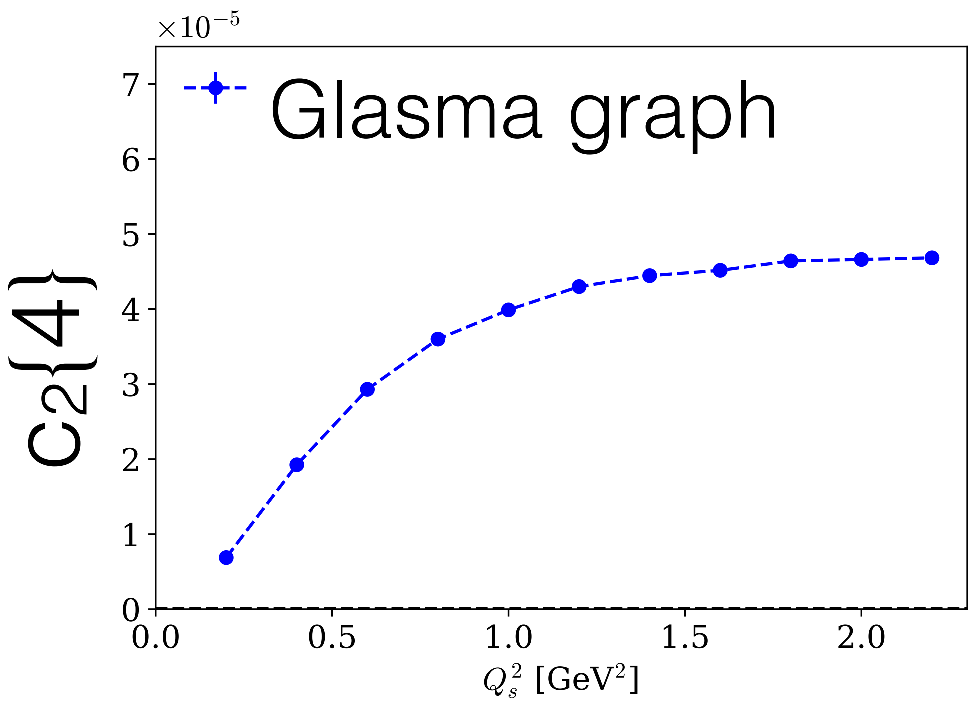


Algorithm can be used to compute other configurations, arbitrary number of Wilson lines

Collectivity from initial state

For higher particle cumulants and harmonics, Glasma graph gives $c_2\{4\}>0$

Dusling, MM, Venugopalan, arXiv:1706.06260

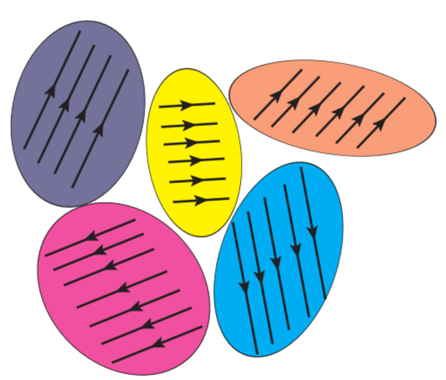


Adding multiple scattering, qualitatively

A. Dumitru, L. McLerran, V. Skokov, Phys.Lett. B743 (2015) 134-137

$$c_2\{4\} \equiv -(v_2\{4\})^4 = -\frac{1}{N_D^3}\left(\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3}\right)$$

"Glasma graph" Always positive



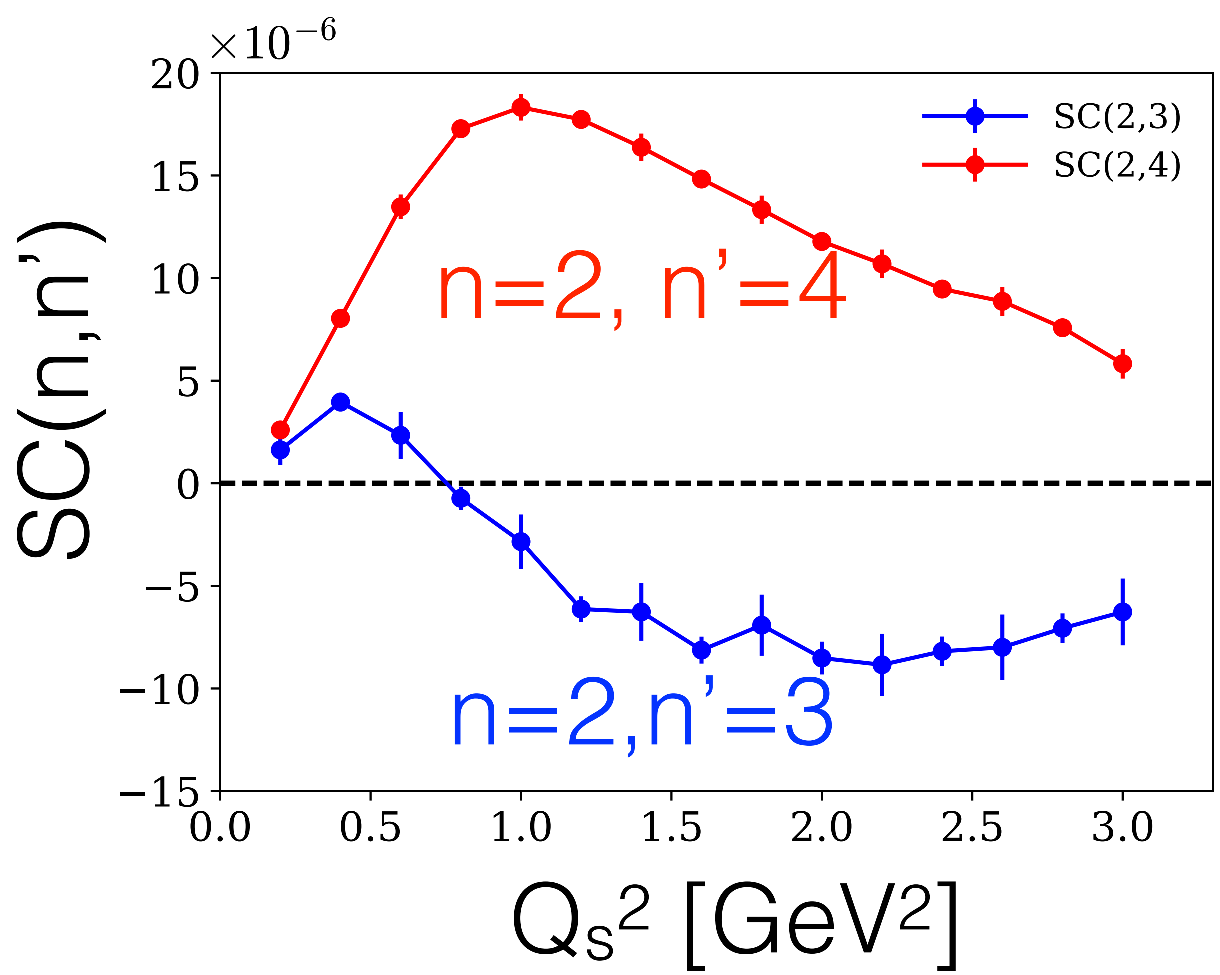
Negative contribution: Non-linear, non-Gaussian from color domain

Symmetric Quark Cumulants

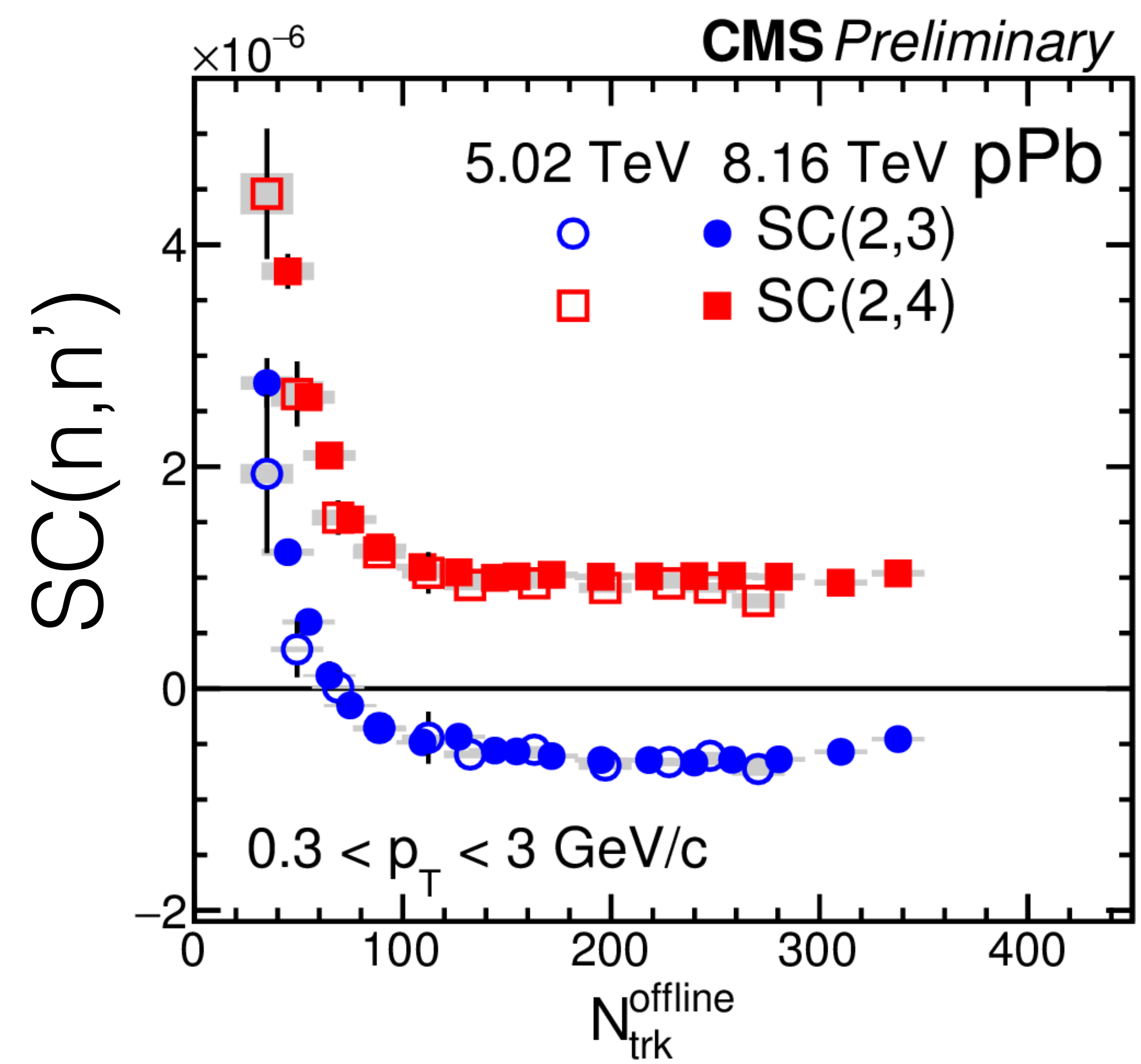
Symmetric cumulants: mixed harmonic cumulants

$$\mathrm{SC}(n,n') = \langle e^{i(n(\phi_1-\phi_3)-n'(\phi_2-\phi_4))}\rangle - \langle e^{in(\phi_1-\phi_3)}\rangle\langle e^{in'(\phi_2-\phi_4)}\rangle$$

Bilandzic et al, PRC 89, no. 6, 064904 (2014)



Dusling, MM, Venugopalan PRD 97 (2018)



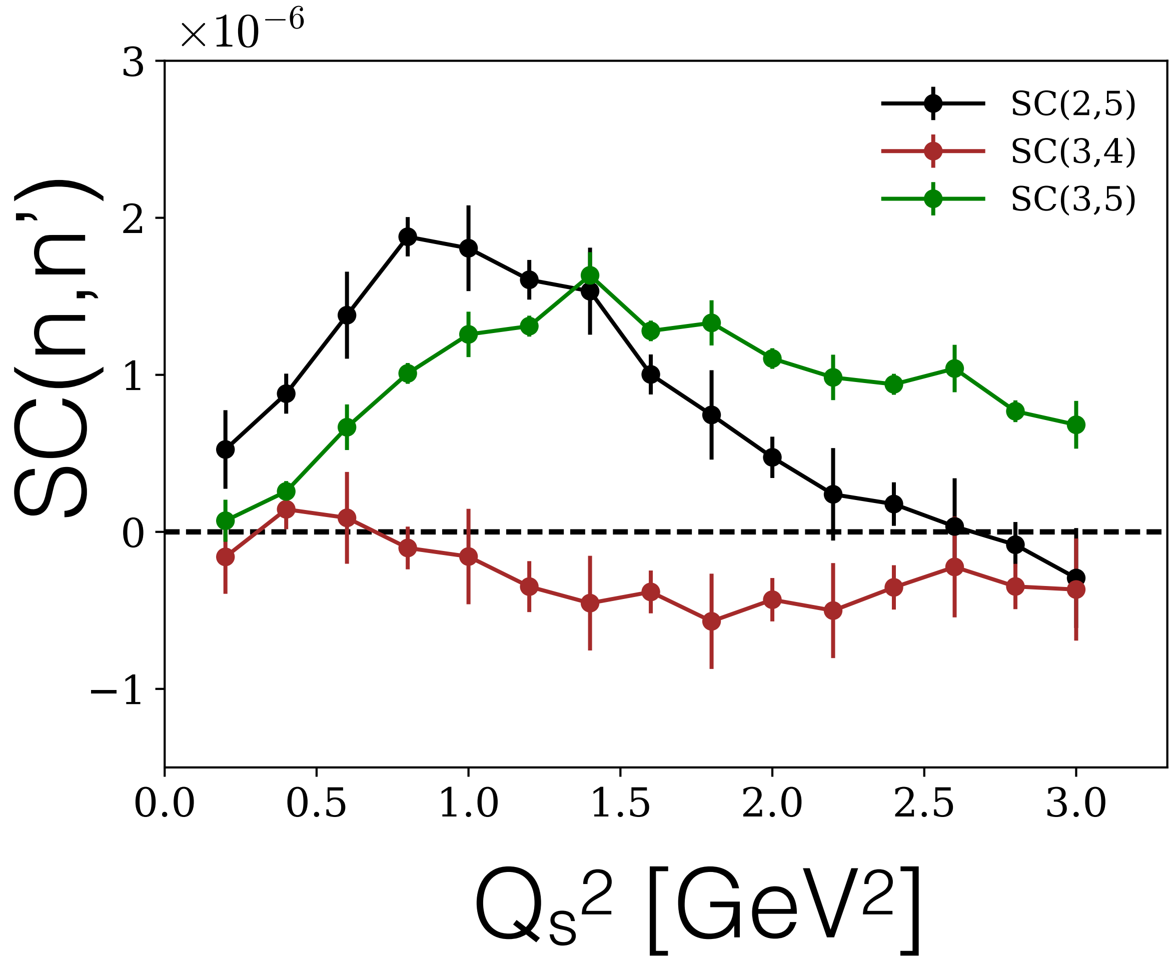
CMS-PAS-HIN-16-022

Symmetric Cumulants

$$\mathrm{SC}(n,n') = \langle e^{i(n(\phi_1-\phi_3)-n'(\phi_2-\phi_4))}\rangle - \langle e^{in(\phi_1-\phi_3)}\rangle\langle e^{in'(\phi_2-\phi_4)}\rangle$$

Bilandzic et al, PRC 89, no. 6, 064904 (2014)

Prediction for higher moments in small systems



n=2,n'=5

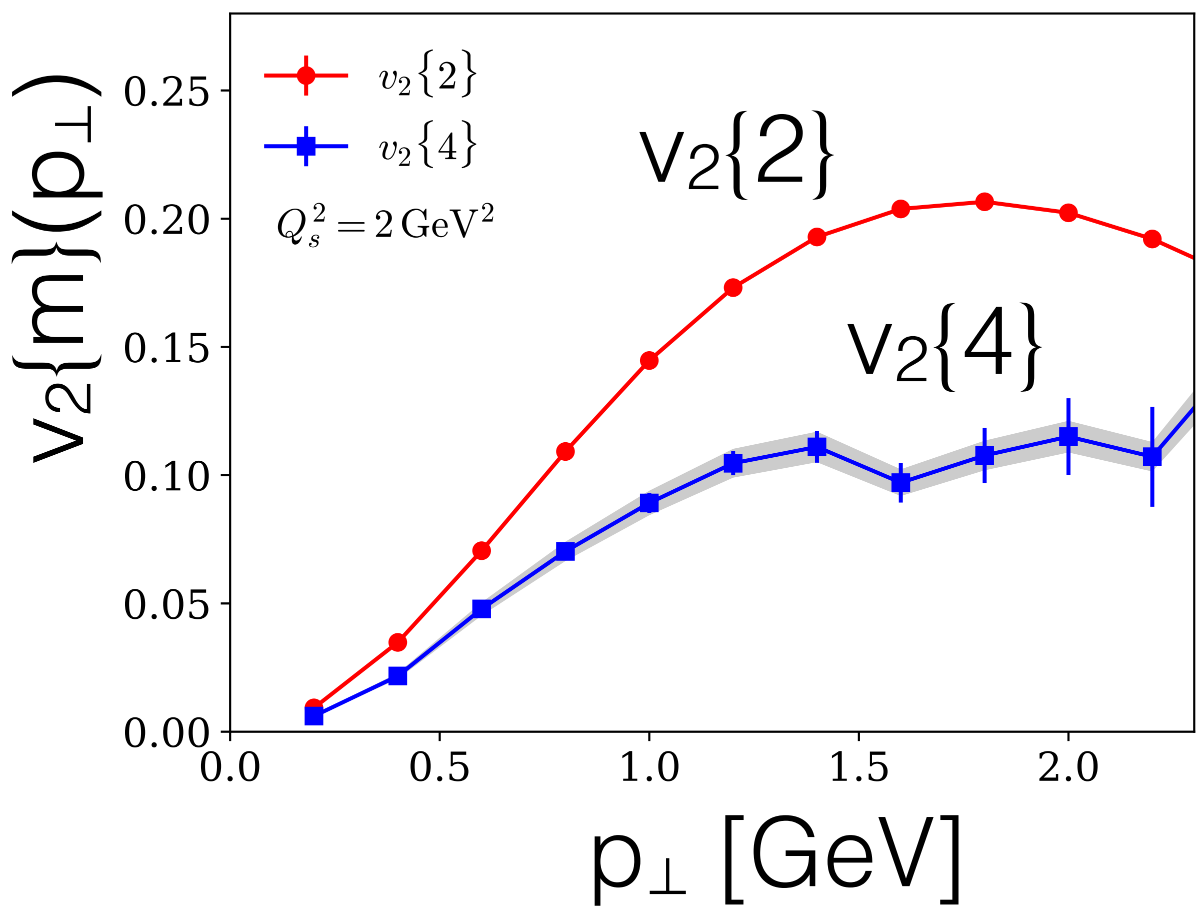
n=3,n'=4

n=3,n'=5

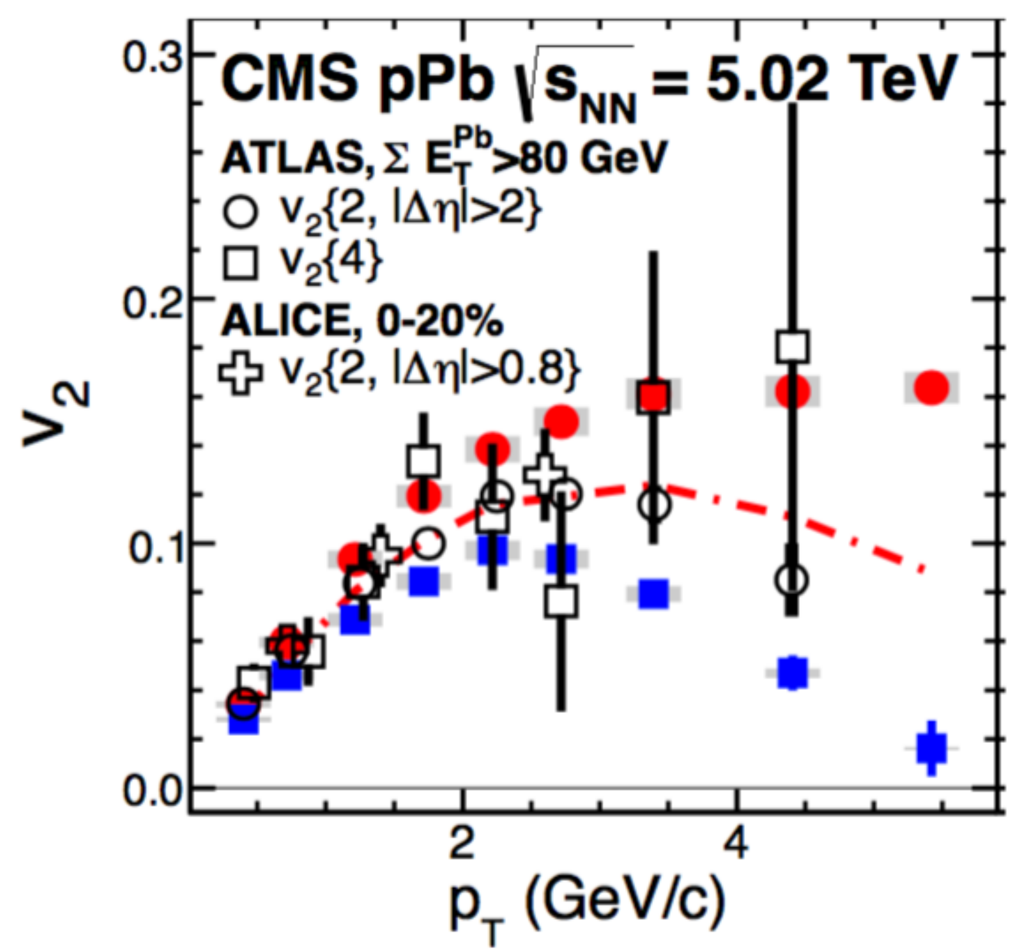
Dusling, MM, Venugopalan PRD 97 (2018) arXiv:1706.06260

Multiparticle correlations

Integrating momentum of m-1 particles



Dusling, MM, Venugopalan PRD 97 (2018)

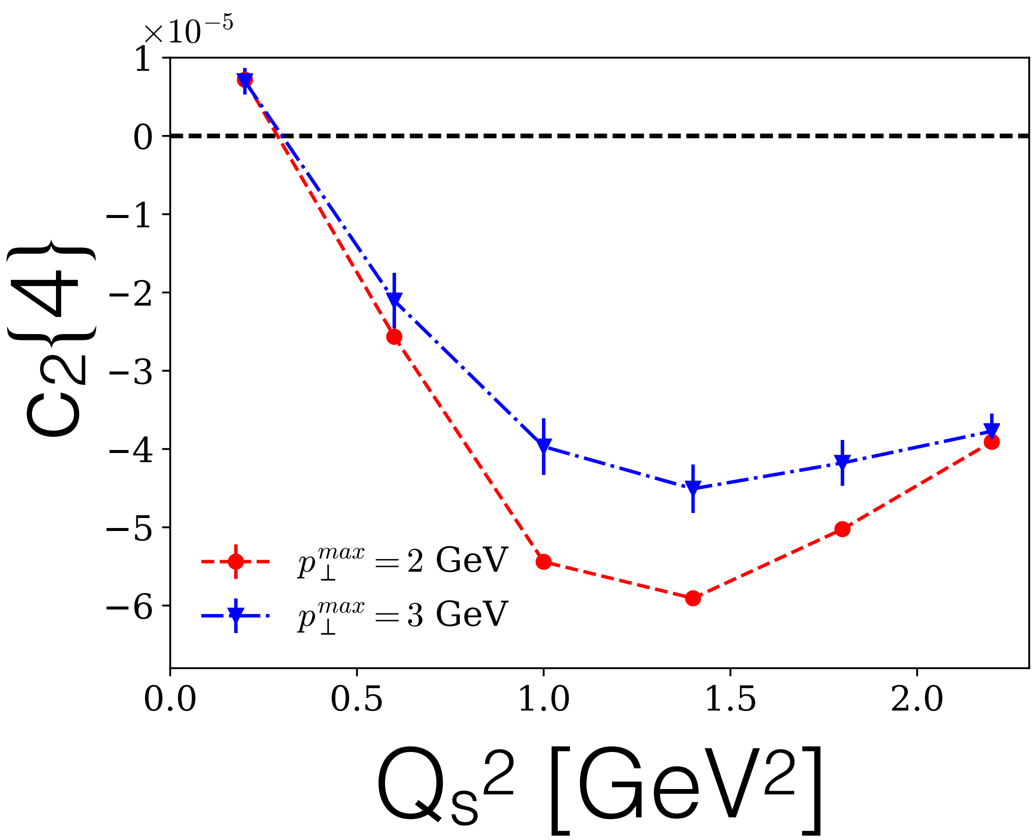


CMS PLB 724 (2013) 213

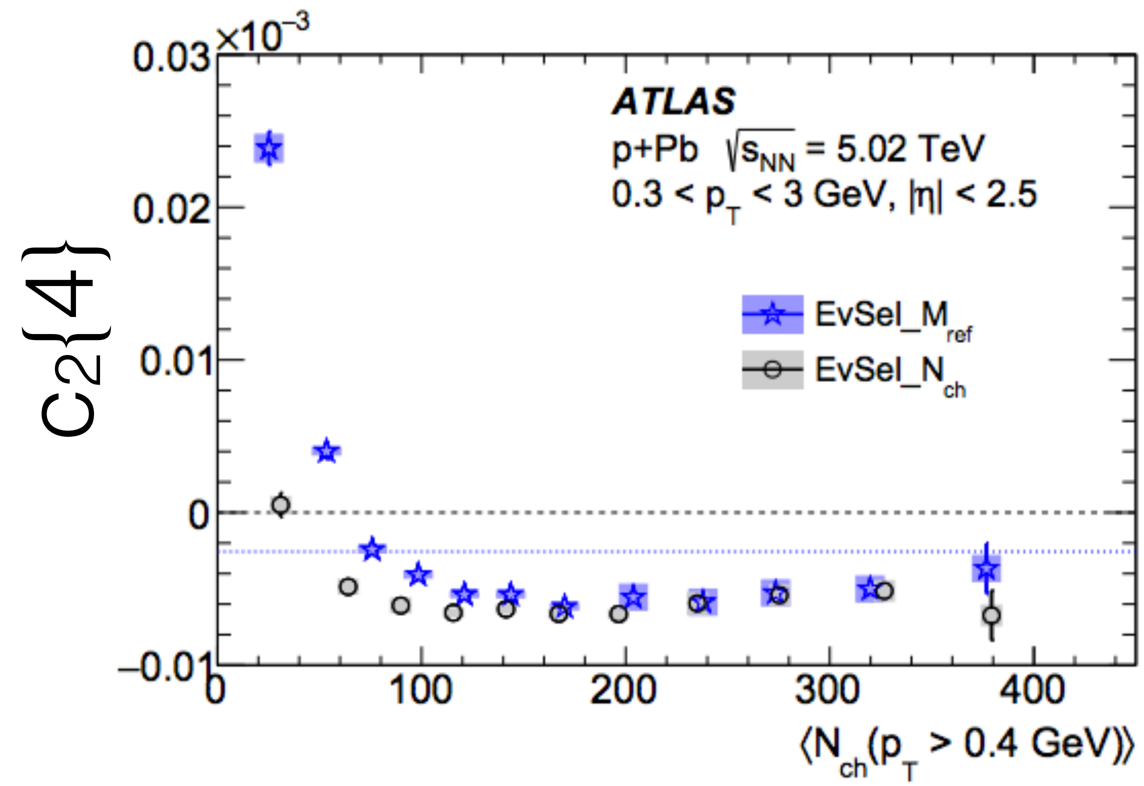
Similar characteristic shape

Multi-particle quark correlations

$c_2\{4\}$ becomes negative for increasing Q_s



Dusling, MM, Venugopalan PRD 97 (2018)



ATLAS EPJC 77 (2017)

Mild dependence on maximum integrated $p_\perp$

Moments of multiplicity

From generating functional for gluon distribution in MV model at leading order in N_c:

Kovner, Skokov PRD 98 (2018)

Average multiplicity: $\kappa_1 \sim S_\perp^{\mathrm{pr}} \mu_{\mathrm{pr}}^2$

Higher moments: $\kappa_{n\geq 2} \sim (n-2)!\, S_\perp^{\mathrm{pr}} \Lambda^2 \left(\frac{\mu_{\mathrm{pr}}^2}{\Lambda^2}\right)^n$

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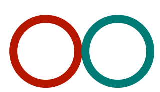
Case 1:  $\mu_{\mathrm{pr}} = \mu_{\mathrm{p}}$, $S_\perp^{\mathrm{pr}} = 2 S_\perp^{\mathrm{p}}$

$$\kappa_1^{\mathrm{typ}} \sim 2 S_\perp^{\mathrm{p}} \mu_{\mathrm{p}}^2$$

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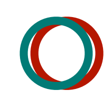
Case 2:  $\mu_{\mathrm{pr}} = 2\mu_{\mathrm{p}}$, $S_\perp^{\mathrm{pr}} = S_\perp^{\mathrm{p}}$

$$\kappa_1^{\mathrm{rare}} \sim 2 S_\perp^{\mathrm{p}} \mu_{\mathrm{p}}^2$$

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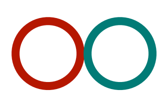
Case 1: 

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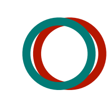
Case 2: 

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Close configurations contribute dominantly to high multiplicity tail

$$\boxed{\kappa_{n\geq 2}^{\mathrm{rare}} = 2^{n-1} \kappa_{n\geq 2}^{\mathrm{typ}}}$$

“Nucleus”-MV model

Remove many sources of fluctuations and study only the essential components of CGC with simplification of MV target — projectile ‘nucleus’ hitting a ‘wall’ of color charge

McLerran, Venugopalan, PRD 49 (1994)

Considering Gaussian nucleons, no IP-Sat

Color charge fluctuations sampled event-by-event with MV model: $\langle \rho^a_{p/T}(\mathbf{x}_\perp)\rho^b_{p/T}(\mathbf{y}_\perp)\rangle = g^2\mu^2_{p/T}\delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)$

Domain where effects of quantum correlations (BE,HBT) likely play significant role

Gelis, Lappi, McLerran NPA 828 (2009), Kovner, Rezaeian, PRD 95, 96 (2017), Altinoluk, Armesto, Beuf, Kovner, Lublinsky, PLB 751 (2015), PRD 95 (2017), Kovner, Skokov PRD 98 (2018), PLB 785 (2018), …

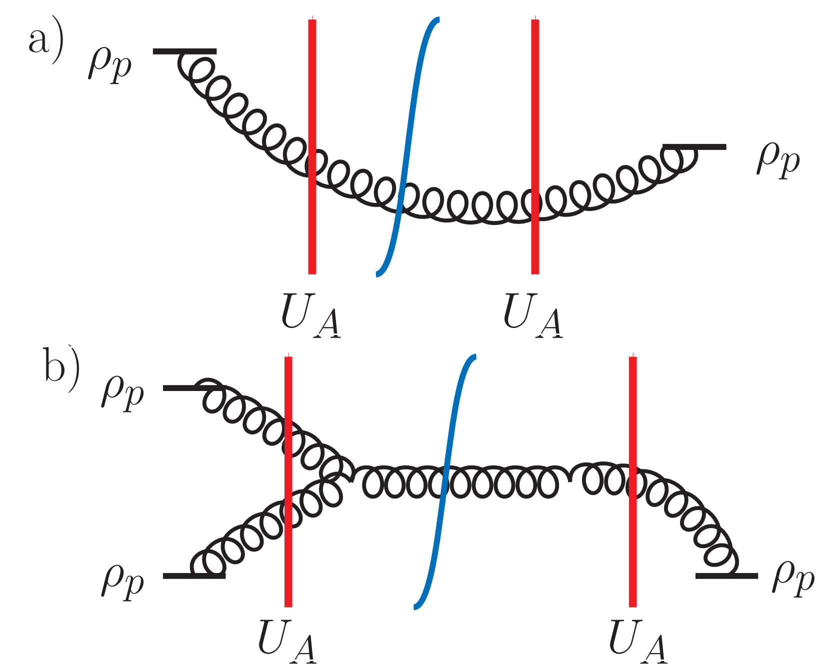
Applicable to both numerical *and analytical* studies

Dilute-dense for gluons

Even harmonics appear at LO dilute-dense

Odd harmonics only non-zero at next to leading order in ρ_p —first saturation correction *(full NLO correction not known)*

McLerran, Skokov NPA 959 (2017)



MSTV PLB 788 (2019)

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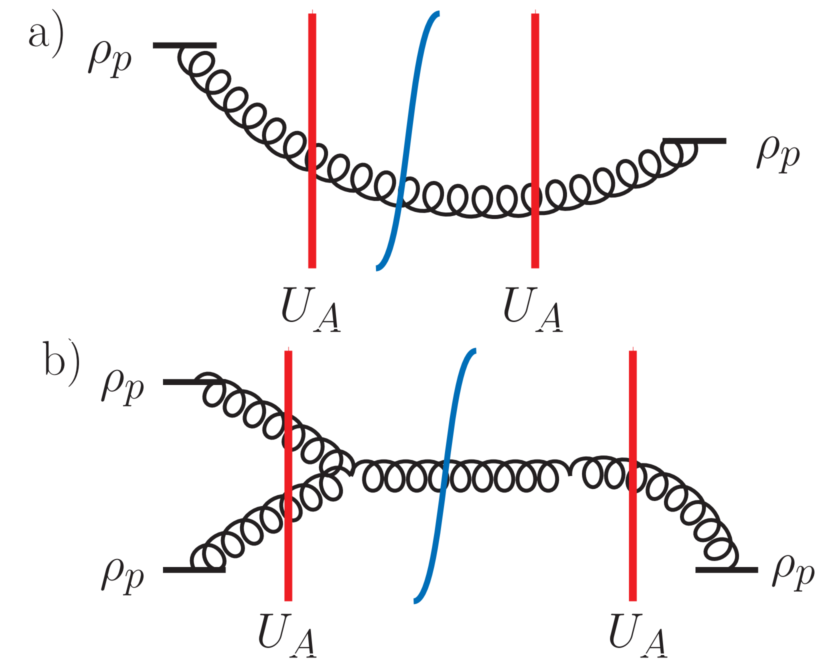

MSTV PLB 788 (2019)

Then in Fock-Schwinger gauge (A_τ=0)

$$\frac{dN^{\text{even}}(\mathbf{k})}{d^2kdy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\mathbf{k})\left[\Omega^a_{lm}(\mathbf{k})\right]^\star$$

$$\frac{dN^{\text{odd}}(\mathbf{k})}{d^2kdy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}\text{Im}\left\{\frac{g}{\mathbf{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{\text{Sign}(\mathbf{k}\times\mathbf{l})}{l^2|\mathbf{k}-\mathbf{l}|^2}f^{abc}\Omega^a_{ij}(\mathbf{l})\Omega^b_{mn}(\mathbf{k}-\mathbf{l})\left[\Omega^c_{rp}(\mathbf{k})\right]^\star\times\right.$$
$$\left.\left[\left(\mathbf{k}^2\epsilon^{ij}\epsilon^{mn}-\mathbf{l}\cdot(\mathbf{k}-\mathbf{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\mathbf{k}\cdot(\mathbf{k}-\mathbf{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Projectile Target

In terms of: $$\Omega^a_{ij}(\mathbf{x}) = g\left[\frac{\partial_i}{\partial^2}\rho^b_p(\mathbf{x})\right]\partial_j U^{ab}(\mathbf{x})$$

Valence sources rotated by target

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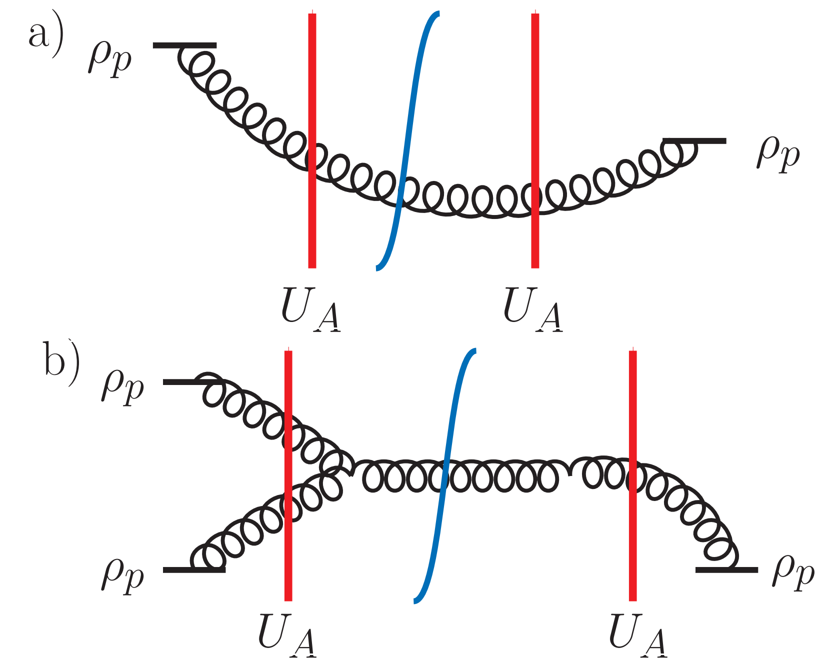

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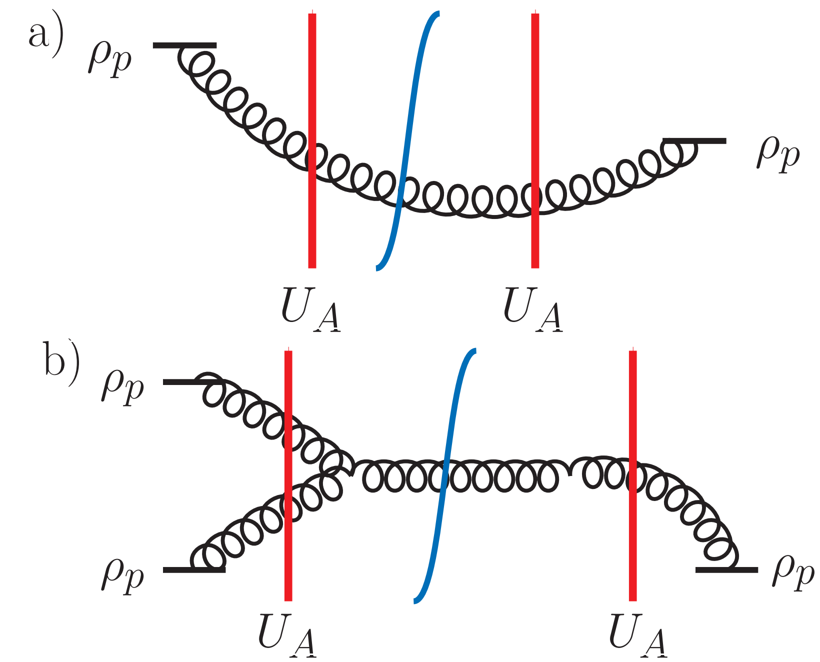

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Valence sources rotated by target

Same results in LC gauge (A^+=0)

Kovchegov, Skokov PRD 97 (2018)

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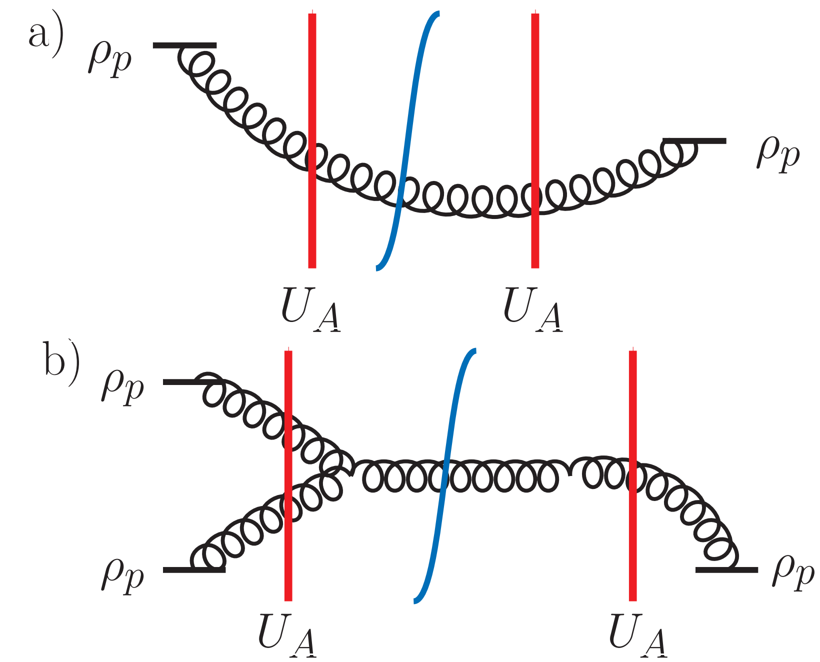

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$$\frac{d^2N}{d^2k_1dy_1\ldots d^2k_ndy_n} = \left\langle\left\langle\frac{dN}{d^2k_1dy_1}\Big|_{\rho_p,\rho_T}\cdots\frac{dN}{d^2k_ndy_n}\Big|_{\rho_p,\rho_T}\right\rangle_p\right\rangle_T$$

Armesto, McLerran, Para NPA 781 (2006), Gelis, Lappi, Venugopalan PRD 78 (2008)

Only well defined for ensemble $W[\rho_T,\rho_p]$

The v_3 Problem

Leading order dilute-dense limit highly amenable to numerics

Lappi EPJC 55 (2008)

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Leading order dilute-dense limit highly amenable to numerics

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For double inclusive, $\frac{d^2N}{d^3k_1 d^3k_2}$, leading order is also known

Kovner. Lublinsky, IJMPE 22 (2013), Kovchegov, Wertepny, NPA 906, (2013),

$$\frac{d^2N}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{d^2N}{k_1 dk_1 dy_1 k_2 dk_2 dy_2}$$
$$\times \left(1 + 2v_2^2\{2\}\cos 2(\phi_1 - \phi_2) + 2v_3^2\{2\}\cos 3(\phi_1 - \phi_2) + \cdots\right)$$

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For a non-zero v_3

McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)

$$\int_0^{2\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\delta\phi\right) = \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\delta\phi\right) - \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\delta\phi + \pi\right)$$
$$= \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \left[\frac{d^2N}{d^2k_1 d^2k_2}\left(\mathbf{k}_1, \mathbf{k}_2\right) - \frac{d^2N}{d^2k_1 d^2k_2}\left(\mathbf{k}_1, -\mathbf{k}_2\right)\right]$$

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McLerran, Skokov NPA 959 (2017), Kovchegov, Skokov PRD 97 (2018)

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However, at leading order (ρ_p^4) it is exactly zero, but not in dense-dense

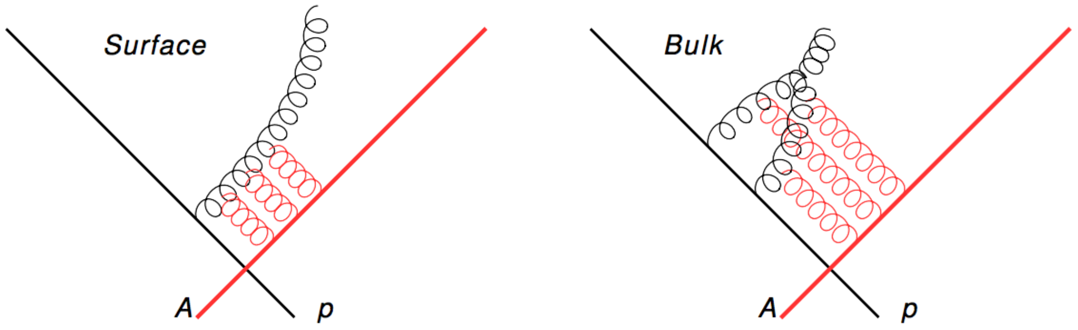
Kovner, Lublinsky, PRD 83 (2011), Kovchegov, Wertepny, NPA 906 (2013), Kovchegov, Skokov PRD 97 (2018)
Lappi, Srednyak, Venugopalan JHEP 1001 (2010), Schenke, Schlichting, Venugopalan PLB 747 (2015)

Dilute dense for gluons

Issue resolved at next order in ρ_p

Symmetry broken in $\frac{d^2N}{d^3k_1 d^3k_2}$ by first saturation correction $O(\rho_p^6)$

McLerran, Skokov NPA 959 (2017)



McLerran, Skokov NPA 959 (2017)

Final state matters!

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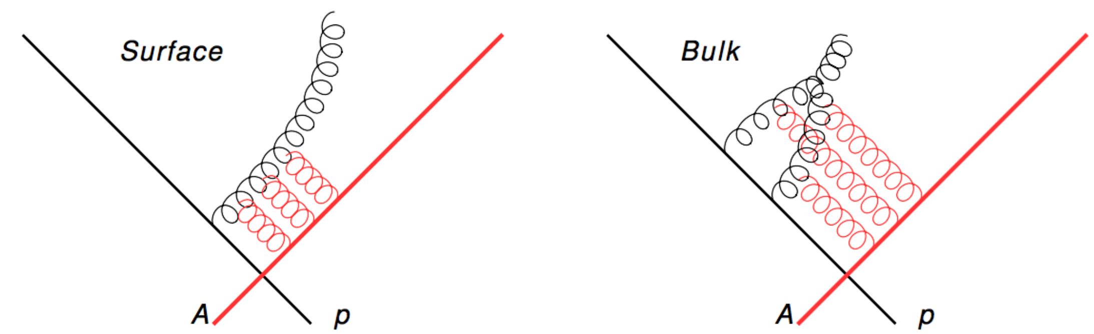

McLerran, Skokov NPA 959 (2017)

Then in Fock-Schwinger gauge (A_τ=0)

Final state matters!

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$$\left[\left(\mathbf{k}^2\epsilon^{ij}\epsilon^{mn}-\mathbf{l}\cdot(\mathbf{k}-\mathbf{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\mathbf{k}\cdot(\mathbf{k}-\mathbf{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\Bigg\}$$

Projectile Target

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Valence sources rotated by target

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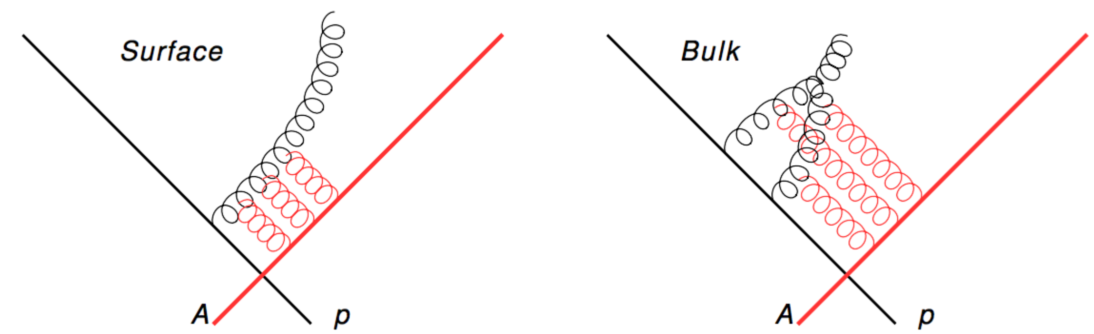
McLerran, Skokov NPA 959 (2017)



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Final state matters!

$$\frac{dN^{\rm even}(\mathbf{k})}{d^2k dy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\mathbf{k})\left[\Omega^a_{lm}(\mathbf{k})\right]^\star$$

$$\frac{dN^{\rm odd}(\mathbf{k})}{d^2k dy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}{\rm Im}\Bigg\{\frac{g}{\mathbf{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{{\rm Sign}(\mathbf{k}\times\mathbf{l})}{l^2|\mathbf{k}-\mathbf{l}|^2}f^{abc}\Omega^a_{ij}(\mathbf{l})\Omega^b_{mn}(\mathbf{k}-\mathbf{l})\left[\Omega^c_{rp}(\mathbf{k})\right]^\star\times$$
$$\left[\left(\mathbf{k}^2\epsilon^{ij}\epsilon^{mn}-\mathbf{l}\cdot(\mathbf{k}-\mathbf{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\mathbf{k}\cdot(\mathbf{k}-\mathbf{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\Bigg\}$$

Projectile Target

In terms of:
$$\Omega^a_{ij}(\mathbf{x}) = g\left[\frac{\partial_i}{\partial^2}\rho^b_p(\mathbf{x})\right]\partial_j U^{ab}(\mathbf{x})$$

Valence sources rotated by target

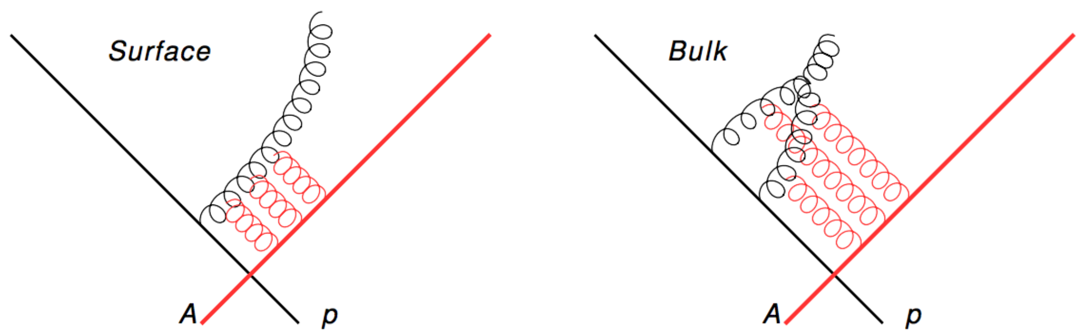
Same results in LC gauge (A^+=0), resolution similar to STSA

Kovchegov, Skokov PRD 97 (2018), Brodsky, Hwang, Kovchegov, Schmidt, Sievert, PRD 88 (2013)

Dilute dense for gluons

Issue resolved at next order in ρ$_p$
Symmetry broken in $\frac{d^2N}{d^3k_1 d^3k_2}$ by first saturation correction O(ρ$_p^6$)

McLerran, Skokov NPA 959 (2017)



McLerran, Skokov NPA 959 (2017)

Then in Fock-Schwinger gauge (A_τ=0)

Final state matters!

$$\frac{dN^{\rm even}(\mathbf{k})}{d^2k dy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\mathbf{k})\left[\Omega^a_{lm}(\mathbf{k})\right]^\star$$

$$\frac{dN^{\rm odd}(\mathbf{k})}{d^2k dy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}\mathrm{Im}\left\{\frac{g}{\mathbf{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{\mathrm{Sign}(\mathbf{k}\times\mathbf{l})}{l^2|\mathbf{k}-\mathbf{l}|^2}f^{abc}\Omega^a_{ij}(\mathbf{l})\Omega^b_{mn}(\mathbf{k}-\mathbf{l})\left[\Omega^c_{rp}(\mathbf{k})\right]^\star\times\right.$$
$$\left.\left[\left(\mathbf{k}^2\epsilon^{ij}\epsilon^{mn}-\mathbf{l}\cdot(\mathbf{k}-\mathbf{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\mathbf{k}\cdot(\mathbf{k}-\mathbf{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Projectile Target

In terms of: $$\Omega^a_{ij}(\mathbf{x}) = g\left[\frac{\partial_i}{\partial^2}\rho^b_p(\mathbf{x})\right]\partial_j U^{ab}(\mathbf{x})$$

Valence sources rotated by target

Also non-zero contribution to v_3 from proj. JIMWLK evolution

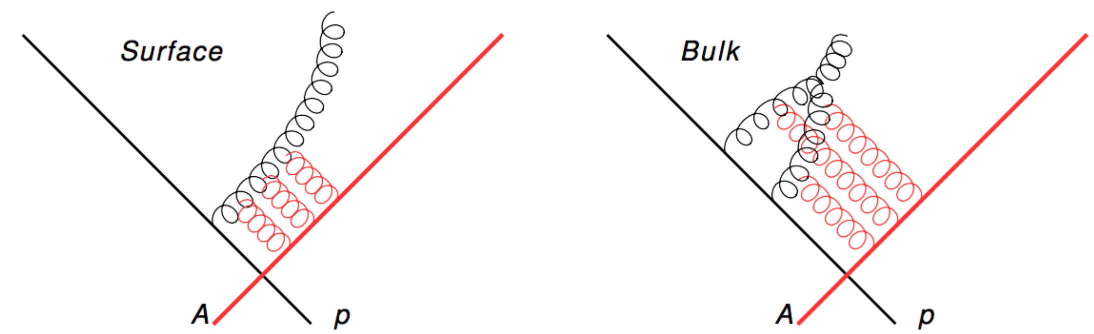
Kovner, Lublinsky, Skokov PRD 96 (2017)

Dilute dense for gluons

Issue resolved at next order in ρ_p

Symmetry broken in $\frac{d^2N}{d^3k_1 d^3k_2}$ by first saturation correction $O(\rho_p{}^6)$

McLerran, Skokov NPA 959 (2017)



McLerran, Skokov NPA 959 (2017)

Then in Fock-Schwinger gauge (A_τ=0)

Final state matters!

$$\frac{dN^{\rm even}(\mathbf{k})}{d^2kdy}\Big[\rho_p,\rho_t\Big] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\mathbf{k})\,[\Omega^a_{lm}(\mathbf{k})]^\star$$

$$\frac{dN^{\rm odd}(\mathbf{k})}{d^2kdy}\Big[\rho_p,\rho_T\Big] = \frac{2}{(2\pi)^3}{\rm Im}\Bigg\{\frac{g}{\mathbf{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{{\rm Sign}(\mathbf{k}\times\mathbf{l})}{l^2|\mathbf{k}-\mathbf{l}|^2}f^{abc}\Omega^a_{ij}(\mathbf{l})\Omega^b_{mn}(\mathbf{k}-\mathbf{l})\,\Big[\Omega^c_{rp}(\mathbf{k})\Big]^\star\times$$

$$\Big[\Big(\mathbf{k}^2\epsilon^{ij}\epsilon^{mn}-\mathbf{l}\cdot(\mathbf{k}-\mathbf{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\Big)\,\epsilon^{rp}+2\mathbf{k}\cdot(\mathbf{k}-\mathbf{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\Big]\Bigg\}$$

Multi-particle distributions then defined as

$$\frac{d^2N}{d^2k_1dy_1\ldots d^2k_ndy_n} = \left\langle\left\langle\frac{dN}{d^2k_1dy_1}\bigg|_{\rho_p,\rho_T}\cdots\frac{dN}{d^2k_ndy_n}\bigg|_{\rho_p,\rho_T}\right\rangle_p\right\rangle_T$$

Only well defined for ensemble over $W[\rho_T,\rho_p]$