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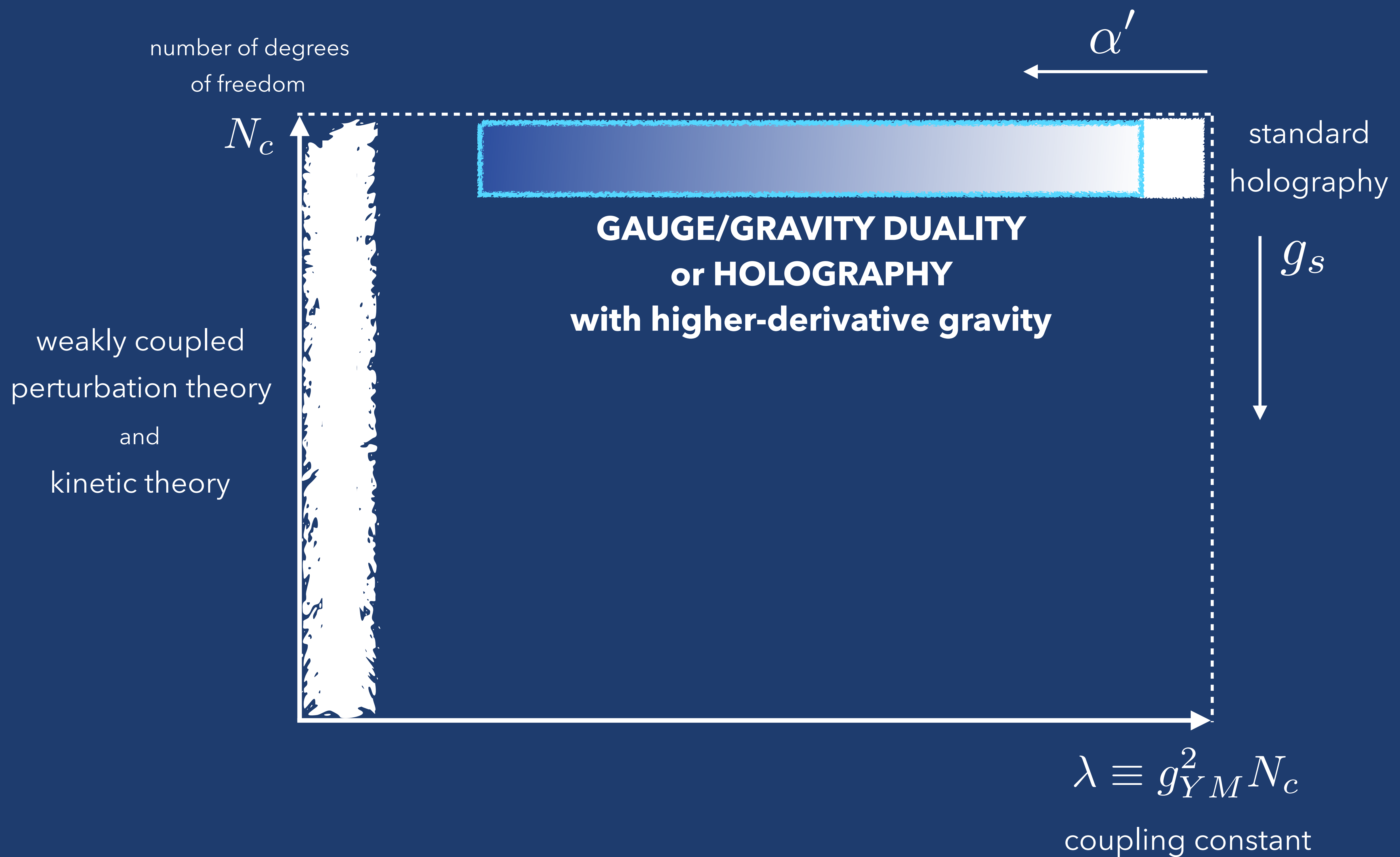
STRONG AND WEAK COUPLING APPROACHES

NEW YORK, 27.6.2019

problem: understand the transition of transport, relaxation, … from strong to weak coupling in non-Abelian gauge theories, *like QCD*



- an example of an exact interpolation from strong to weak coupling (extremely rare, even at zero temperature):

  expectation value of a circular Wilson loop in *SU(N$_c$)*, $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory in the limit $N_c \to \infty$, as a function of the 't Hooft coupling $\lambda \equiv g_{YM}^2 N_c$

  [Erickson, Semenoff, Zarembo (2000)]

exact:

$$\boxed{\langle W_C \rangle = \frac{2}{\sqrt{\lambda}} I_1\left(\sqrt{\lambda}\right)}$$

QFT-type result

perturbative in the coupling: $\lambda \ll 1:$

$$\langle W_C \rangle = 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \cdots$$

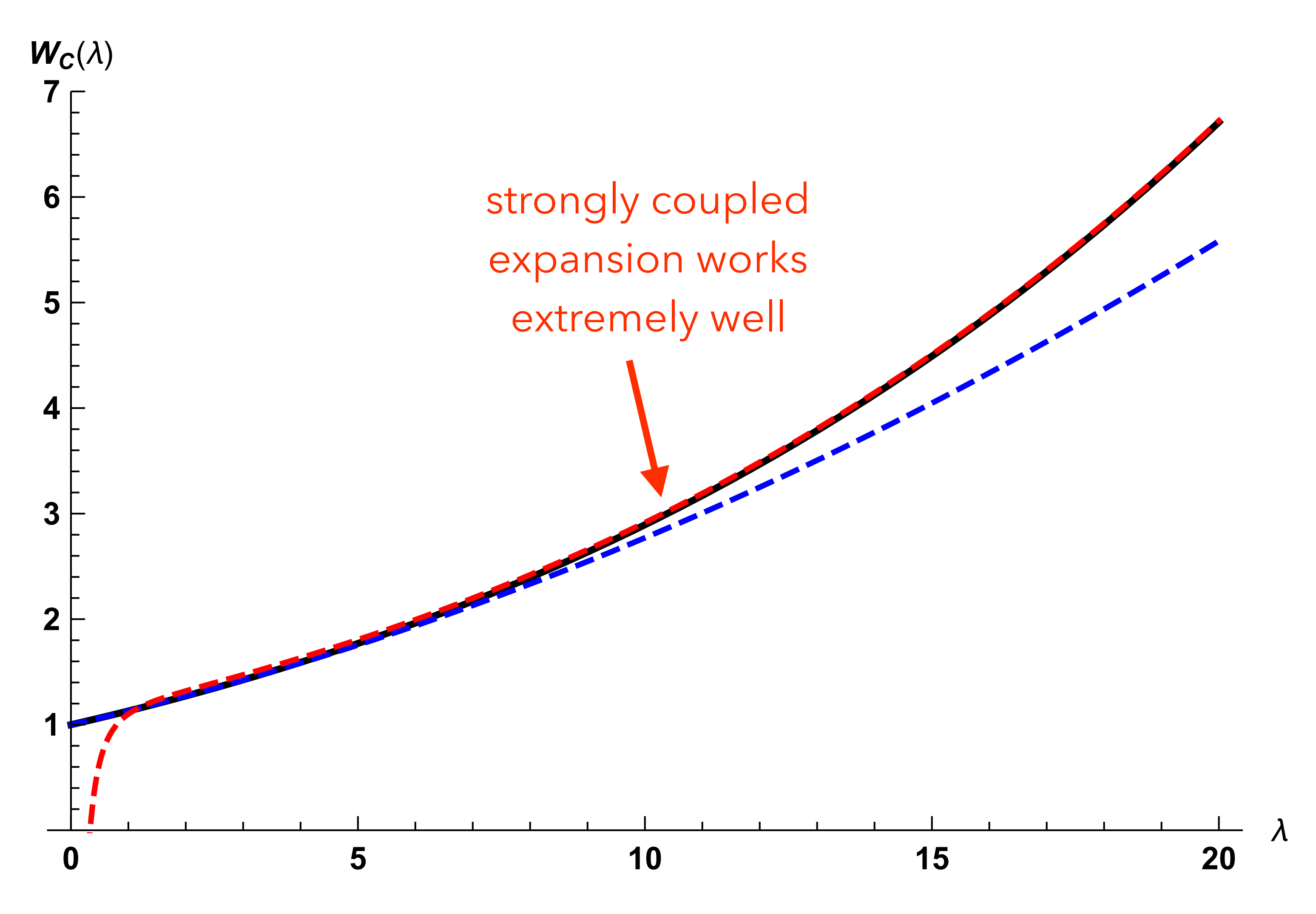
perturbative in the inverse coupling: $\lambda \gg 1:$

$$\langle W_C \rangle \sim \sqrt{\frac{2}{\pi}} \frac{e^{\sqrt{\lambda}}}{\lambda^{3/4}} \left(1 - \frac{3}{8\sqrt{\lambda}} - \frac{15}{128\lambda} + \cdots\right)$$

holography-type result

exact:

$$\langle W_C \rangle = \frac{2}{\sqrt{\lambda}} I_1\left(\sqrt{\lambda}\right)$$



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holography-type result

- entropy density (from free energy) in *SU(N$_c$)*, $\mathcal{N}=4$ SYM theory in the limit $N_c \to \infty$, as a function of the 't Hooft coupling at **finite temperature**

"3/4 problem"

QFT result

perturbative in the coupling: $\lambda \ll 1:$ $$s/s_0 = 1 - \frac{3}{2\pi^2}\lambda + \frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2} + \cdots$$ [Fotopoulus, Taylor (1998)]

perturbative in the inverse coupling: $\lambda \gg 1:$ $$s/s_0 = \frac{3}{4} + \frac{45\zeta(3)}{32}\frac{1}{\lambda^{3/2}} + \cdots$$ [Gubser, Klebanov, Tseytlin (1998)]

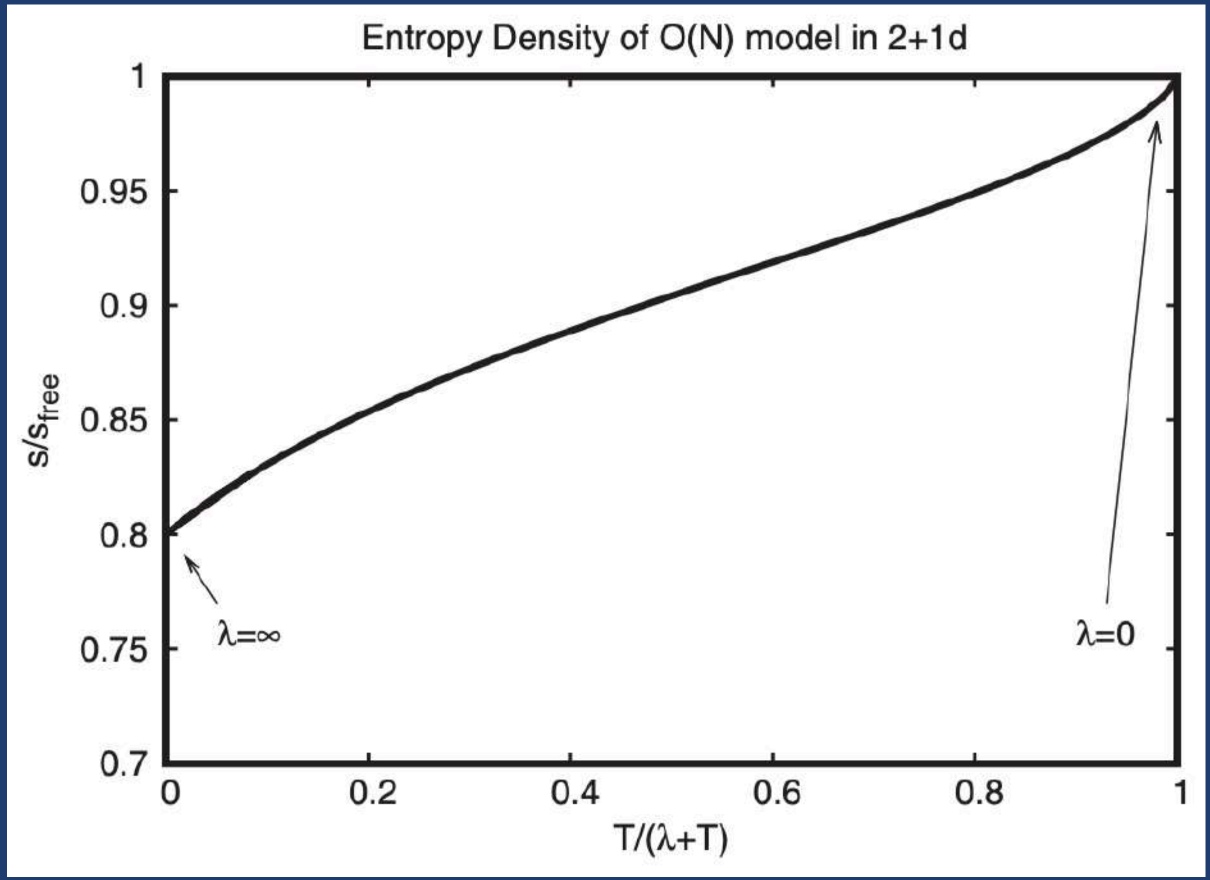
holography result

$$s_0 = \frac{2\pi^2}{3}N_c^2 T^3$$

- monotonicity? [note: historically, an obsession of theorists]
- a new analytic result in the large-*N* vector model [Romatschke, PRL (2019)]

$$\frac{s(\lambda\to\infty)}{s_0} = \frac{4}{5}$$

4/5 instead of 3/4 →



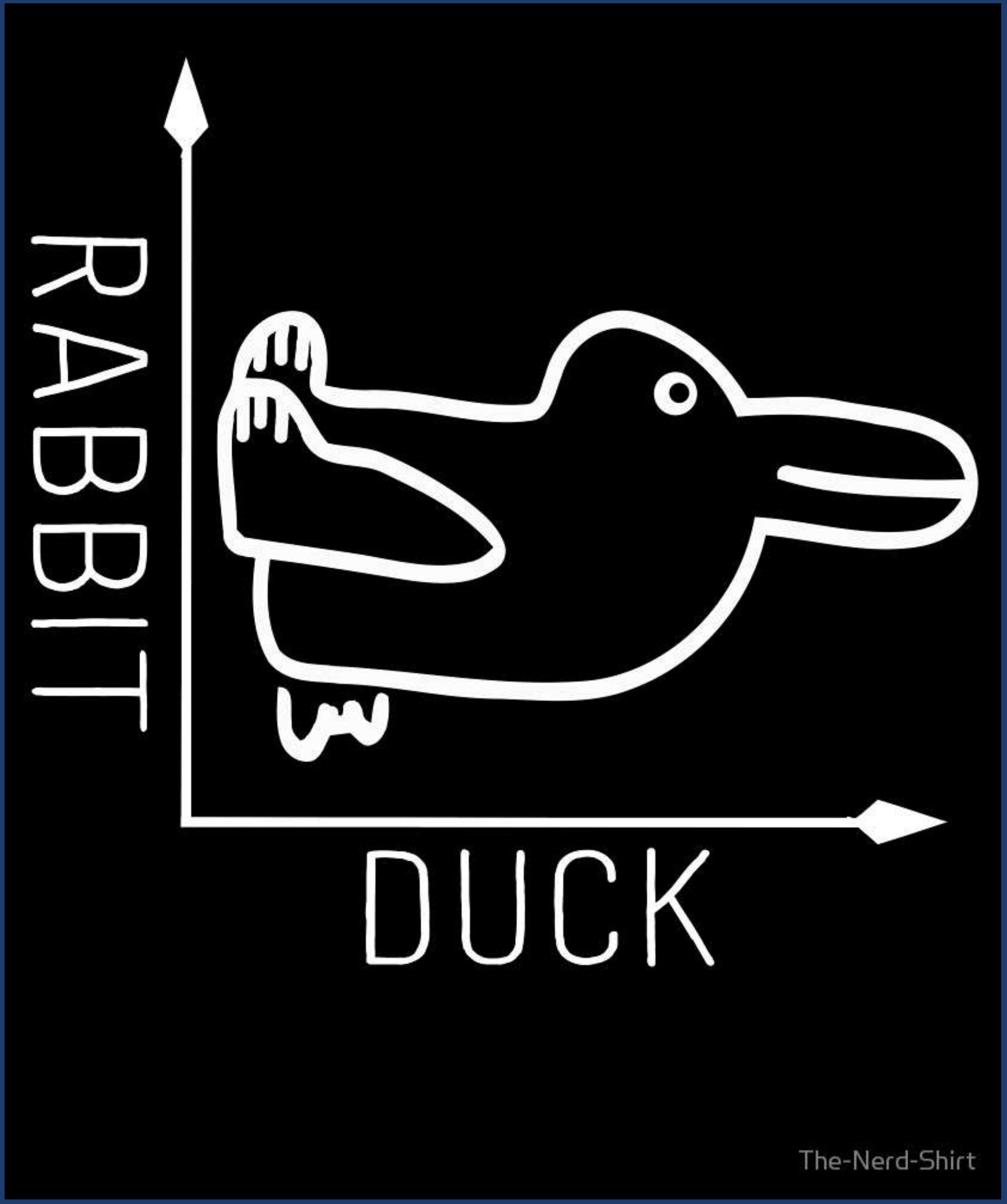
THE REST OF THE TALK

- holographic duality at finite coupling
- hydrodynamics
- hydrodynamics, thermalisation and gapped modes
- holographic models of far-from-equilibrium heavy ion collisions
- future directions

HOLOGRAPHIC DUALITY AT FINITE COUPLING

DUALITY

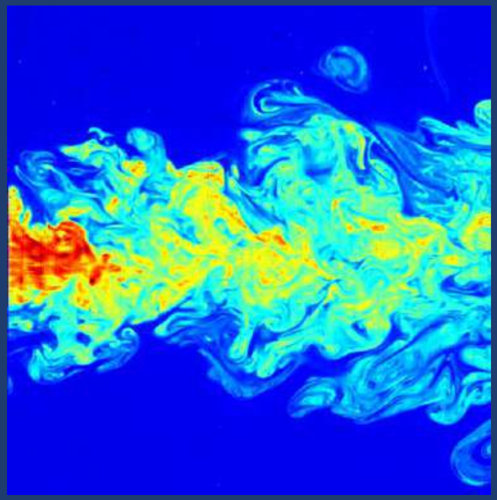
- duality means having two “different” descriptions of the same object (theory)
- Wittgenstein’s view of duality (1953); after Jastrow (1892)
- *duck = rabbit*
- by knowing everything about a duck, we can determine everything about a rabbit, and vice-versa
- extremely useful when, e.g., analysing a duck is difficult, but studying a rabbit is easy



HOLOGRAPHIC DUALITY

- holographic or gauge/gravity duality is a result of string theory, which is a quantum theory of gravity [Maldacena (1997)]

strongly coupled quantum theory	=	*weakly coupled gravity*
(extremely hard)	=	(much easier)



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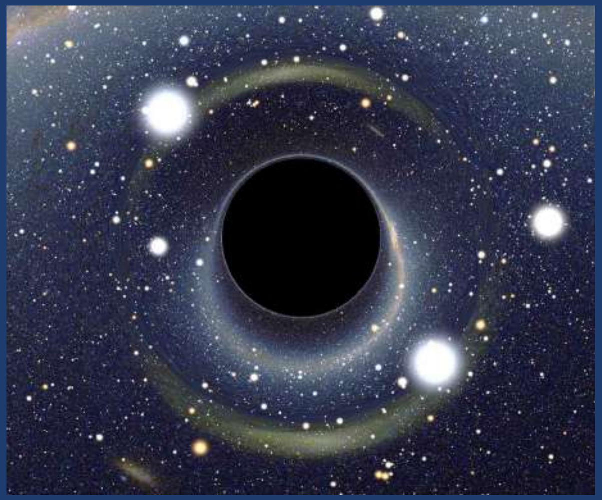
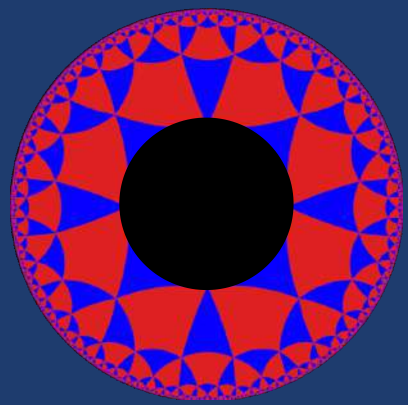


- weakly interacting gravity allows to analyse strongly coupled microscopic QFTs
- it is not known how to construct exact duals of realistic QFTs
- we hope to learn general properties of physics in strongly coupled states

COUPLING DEPENDENT HOLOGRAPHY

- *SU(N_c)*, $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory is dual to type IIB string theory on $AdS_5 \times S^5$ – **a toy model for deconfined QCD**
- the energy-momentum dynamics with $\lambda \equiv g_{YM}^2 N_c \gg 1,\ N_c \to \infty$ from R^4 theory

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\,(R - 2\Lambda + \gamma\mathcal{W})$$



$$\Lambda = -6/L^2,\ \kappa_5 = 2\pi/N_c,\ \gamma = \alpha'^3\zeta(3)/8,\ \alpha'/L^2 = 1/\sqrt{\lambda}$$

$$\mathcal{W} = C^{\alpha\beta\gamma\delta}C_{\mu\beta\gamma\nu}C_{\alpha}{}^{\rho\sigma\mu}C^{\nu}{}_{\rho\sigma\delta} + \frac{1}{2}C^{\alpha\delta\beta\gamma}C_{\mu\nu\beta\gamma}C_{\alpha}{}^{\rho\sigma\mu}C^{\nu}{}_{\rho\sigma\delta}$$

- perturbative R^2 theory and non-perturbative Einstein-Gauss-Bonnet theory

$$S_{R^2} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R + 12 + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)$$

$|\alpha_i| \ll 1$

↓ field-redefinition at perturbative level

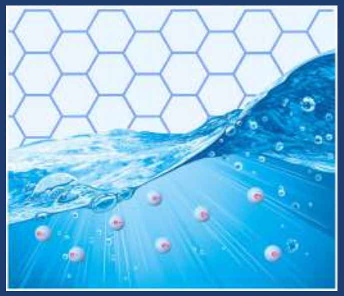
$$S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R + 12 + \frac{\lambda_{GB}}{2}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

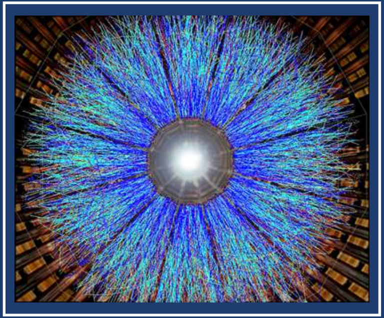
$\lambda_{GB} \in [-1/4, \infty)$

HYDRODYNAMICS

HYDRODYNAMICS

- collective dynamics: liquids, graphene, neutron stars, **quark-gluon plasma**



- low-energy limit of QFTs—a Schwinger-Keldysh effective field theory [Grozdanov, Polonyi, PRD 91 (2015) 10, 105031, arXiv:1305.3670; Crossley, Glorioso, Liu, arXiv:1511.03646; Haehl, Loganayagam, Rangamani. arXiv:1511.07809; …]
- conservation laws (equations of motion) of globally conserved operators

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^\mu = 0 \qquad \dots \nabla_\mu J^{\mu\nu_1\dots\nu_n} = 0$$

- tensor structures (symmetries and phenomenological gradient expansions) with transport coefficients (microscopic)

$$\partial u^\mu \sim \partial T \sim \sim \partial \mu \sim \epsilon$$

$$T^{\mu\nu}\left(u^\lambda, T, \mu\right) = \left(\varepsilon + P\right) u^\mu u^\nu + P g^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta \nabla\cdot u \Delta^{\mu\nu} + \dots$$

$$J^\mu(u^\lambda, T, \mu) = n u^\mu - \sigma T \Delta^{\mu\nu}\nabla_\nu(\mu/T) + \dots$$

ALL-ORDER HYDRODYNAMICS

- infinite, all-order hydrodynamic expansion

$$T^{\mu\nu} = \sum_{n=0}^{\infty} \left[\sum_{i}^{N} \lambda_i^{(n)} \mathcal{T}_{(n)}^{\mu\nu} \right] \xrightarrow[u^\mu \sim T \sim e^{-i\omega t + iqz}]{\nabla_\mu T^{\mu\nu} = 0} \boxed{\omega(q) = \sum_{n=0}^{\infty} \alpha_{n+1} q^{n+1}}$$

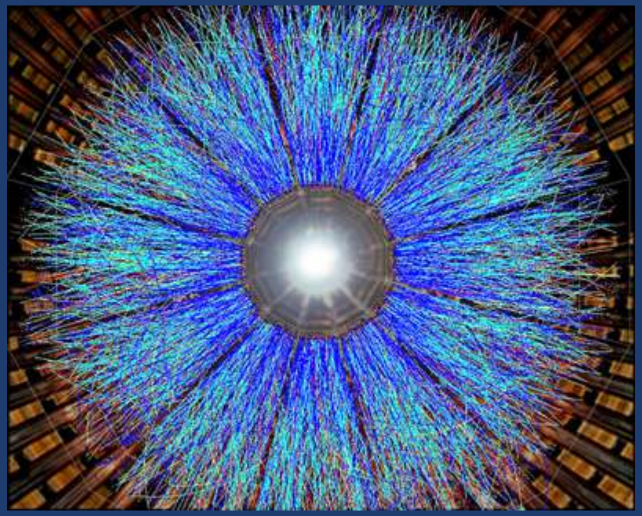
- conformal symmetry (scale invariance) constrains the series

CFT:
Weyl covariance
$T^{\mu}_{\ \mu} = 0$

- state of the art for relativistic neutral hydrodynamics

	max N	max N in CFT	
first order	*2*	*1*	Navier-Stokes (1821)
second order	*15*	*5*	BRSSS (2007)
third order	*68*	*20*	Grozdanov, Kaplis, PRD 93 (2016) 6, 066012

INTERLUDE: UNREASONABLE EFFECTIVENESS OF HYDRODYNAMICS



"**unreasonable**": hydro works for large ∂

hydrodynamic modes are complex spectral curves; dispersion relations are infinite Puiseux series with a finite radius of convergence
[Grozdanov, Kovtun, Starinets, Tadić, PRL (2019)]

$$\omega(q) = \sum_{n=1}^{\infty} \alpha_n q^n$$

radius of convergence in *N*=4 SYM at infinite coupling →

$$q/T \sim O(10)$$

orders of magnitude larger radius of convergence than naive $q/T \ll 1$ – if this fact is generically true in neutral hydrodynamic theories, this could provide an explanation for the **"unreasonable effectiveness of hydrodynamics"**

ALL-ORDER HYDRODYNAMICS

- holography is extremely useful for studying all-order (*precision*) hydrodynamics
- in $\mathcal{N}=4$ supersymmetric Yang-Mills theory at $N_c \to \infty$:

first order (1/1):	$\eta = \lambda_1^{(1)} = \# + \#/\lambda^{3/2} + \cdots$	Buchel, Liu, Starinets (2004)
second order (5/5):	$\lambda_i^{(2)} = \#_i + \#_i/\lambda^{3/2} + \cdots, \quad i = \{1,\dots,5\}$	Grozdanov, Starinets (2014)
third order (5/20):	$\lambda_i^{(3)} = \#_i + \cdots, \quad i = \{1,\dots,5\}$	Grozdanov, Kaplis (2016)

- universality of *1st*-order hydro (*2*-pt functions)

  [Kovtun, Son, Starinets (2004)]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \cdots\right)$$

weak coupling:
$$\frac{\eta}{s} \sim \frac{T^3}{\lambda^4 \ln(1/\lambda)}$$
[Huot, Jeon, Moore (2006)]

- *2nd*-order hydro and its, more general, universality (*3*-pt functions)

$$\tau_\Pi = \frac{2-\ln 2}{2\pi T} + \frac{375\zeta(3)}{32\pi T\lambda^{3/2}} + \cdots$$
$$\lambda_1 = \frac{N_c^2 T^2}{16}\left(1 + \frac{175\zeta(3)}{4\lambda^{3/2}} + \cdots\right)$$
$$\vdots$$

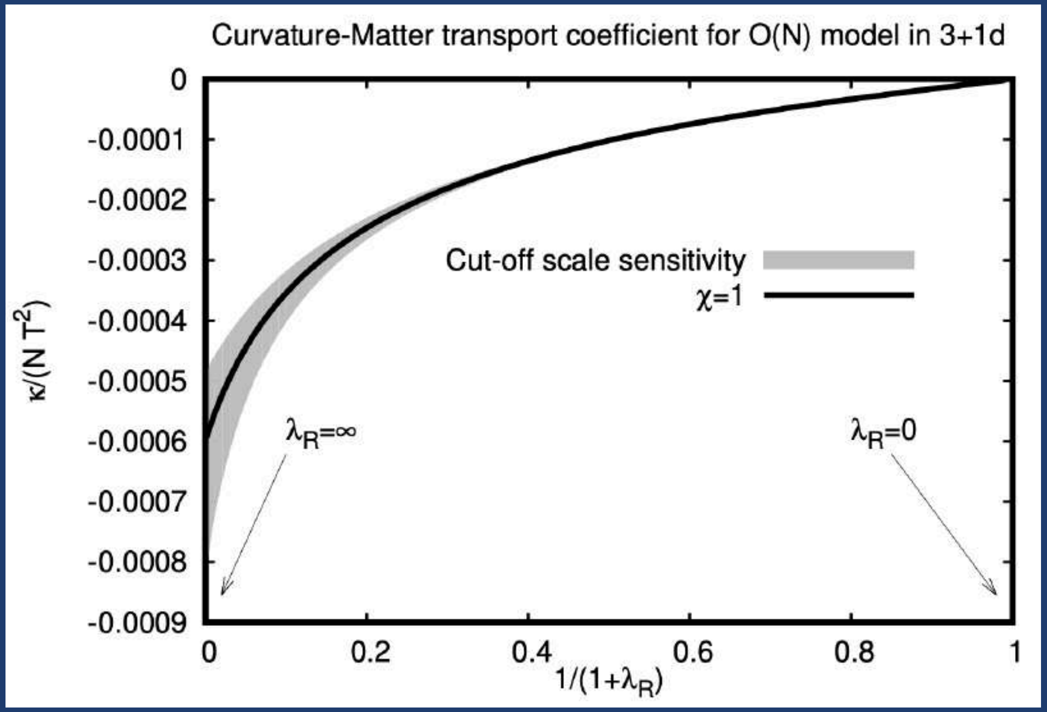
weak coupling:
$$\lambda_1 \sim \frac{T^2}{\lambda^4 \ln^2(1/\lambda)}$$
[York, Moore (2009)]

$$2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0 + \frac{0}{\lambda^{3/2}} + O(1/\lambda^2)$$

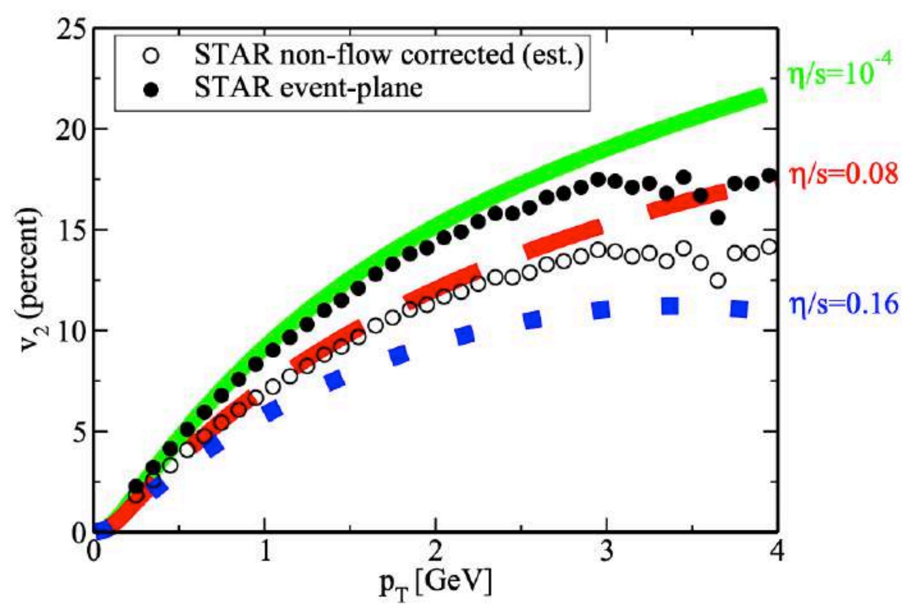
[Haack, Yarom (2008); Grozdanov, Starinets (2014)]

ALL-ORDER HYDRODYNAMICS

- a recent calculation of the *2nd*-order transport coefficient κ in the *O(N)* vector model for all values of the coupling [Romatschke, arXiv:1905.09290]



- can *2nd*-order transport coefficients be measured or estimated in QGP, like η/s ?



[from Luzum, Romatschke, PRC (2018)]

HYDRODYNAMICS, THERMALISATION AND GAPPED MODES

WEAK COUPLING, KINETIC THEORY AND PERTURBATIVE SPECTRUM

- coupling constant dependence of non-hydrodynamic transport
  [Grozdanov, Kaplis, Starinets, JHEP (2016)]
- weakly coupled kinetic theory and the concept of quasi-particles
- Boltzmann equation (truncated BBGKY)

1-particle distribution

the collision integral

$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = C[F]$$

- close to equilibrium

$$F(t, \mathbf{r}, \mathbf{p}) = F_0(\mathbf{r}, \mathbf{p})\left[1 + \varphi(t, \mathbf{r}, \mathbf{p})\right]$$

- assuming homogeneous eq. distribution gives a spectrum with a hierarchy of relaxation times

$$\varphi(t, \mathbf{p}) = \sum_n C_n e^{-\nu_n t} h_n(\mathbf{p})$$

dominant: $\boxed{\tau_R = 1/\nu_{min}}$

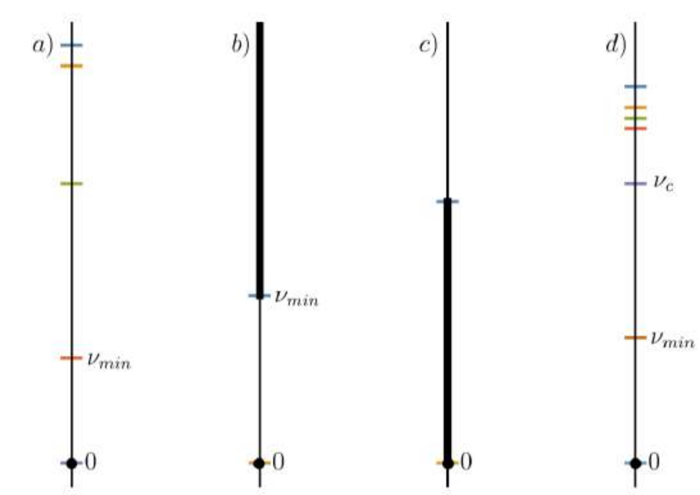


Figure 1: The spectrum of a linear collision operator: a) discrete spectrum, b) continuous spectrum with a gap, realized for the interaction potential $U = \alpha/r^n$, $n > 4$, c) gapless continuous spectrum, realized for the interaction potential $U = \alpha/r^n$, $n < 4$, d) Hod spectrum (see text): $0 \leq \nu_{min} \leq \nu_c$. In all cases, $\nu = 0$ is a degenerate eigenvalue corresponding to hydrodynamic modes (at zero spatial momentum).

WEAK COUPLING, KINETIC THEORY AND PERTURBATIVE SPECTRUM

- RTA equation

$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = -\frac{F - F_0}{\tau_R}$$

- kinetic theory predicts

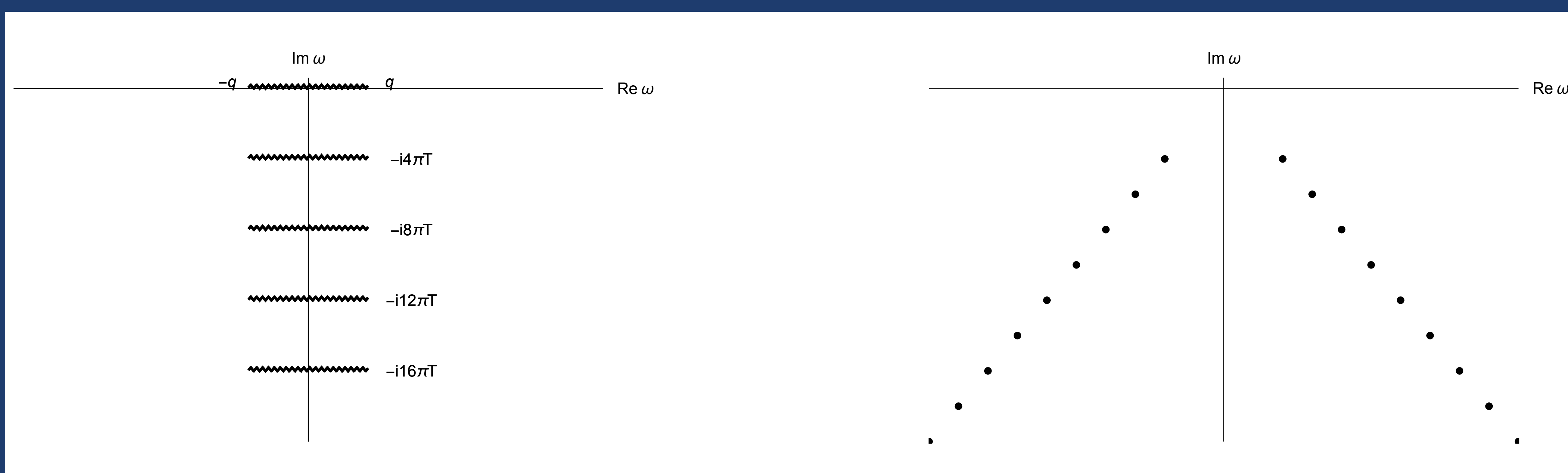
$$\boxed{\eta = \text{const.} \times \tau_R s T}$$

- relaxation time bound [Sachdev]

$$\tau_R \geq \mathcal{C}\,\frac{\hbar}{k_B T}$$

- full QFT spectrum of $\langle T_{\mu\nu}(\omega), T_{\rho\sigma}(-\omega)\rangle_R$ at extreme coupling [Hartnoll, Kumar (2005)]

QFT　　　　　　holography

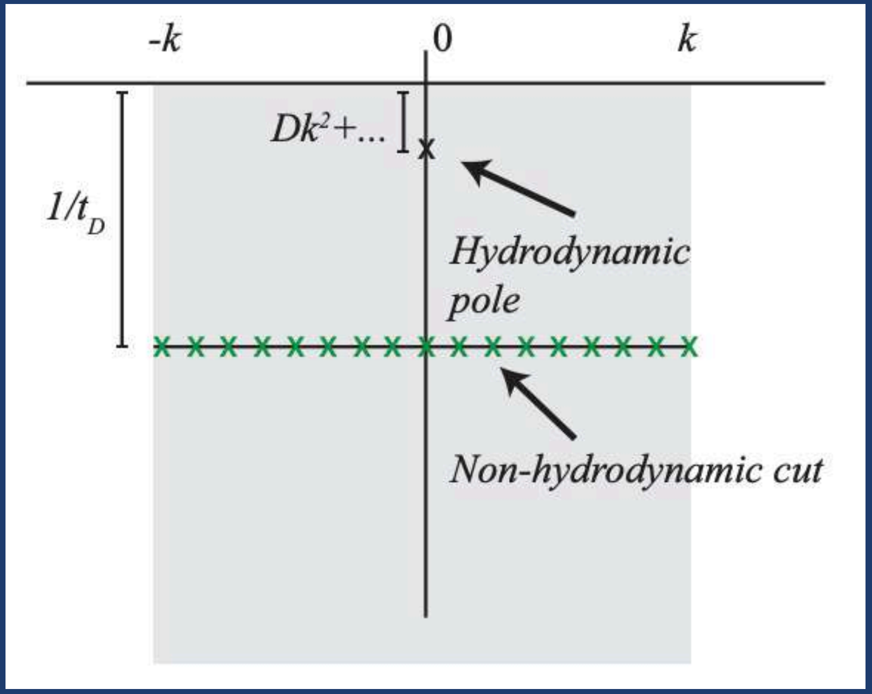


$\lambda \to 0$　　　　　　$\lambda \to \infty$

WEAK COUPLING, KINETIC THEORY AND PERTURBATIVE SPECTRUM

- what happens in between these two extreme regimes? cuts vs. poles?
- resummations of the type of Arnold-Moore-Yaffe (AMY) exist, which determine transport form the spectrum of the kinetic operator
- perturbation theory at zero momentum creates a cut at one loop [Moore (2018); Grozdanov, Schalm, Scopelliti, (2018)]
- at non-zero momentum, structure depends on truncation of kinetic theory, resummations (choice of relaxation time), but cuts seem to be preferred [Romatschke (2015); Kurkela, Wiedemann (2017) Kurkela, Wiedemann, Wu (2019)]

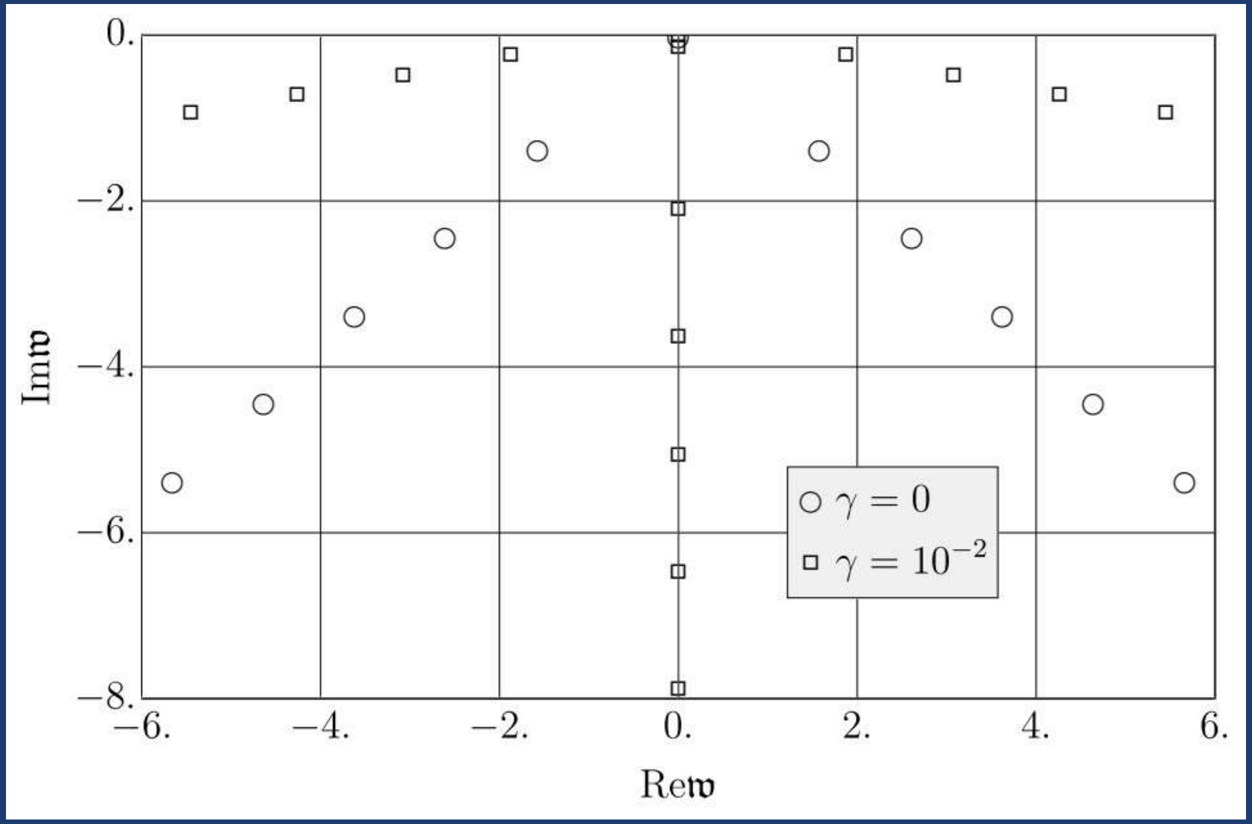
[see talks by Wiedemann (Wednesday) and Wu (Thursday)]



APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

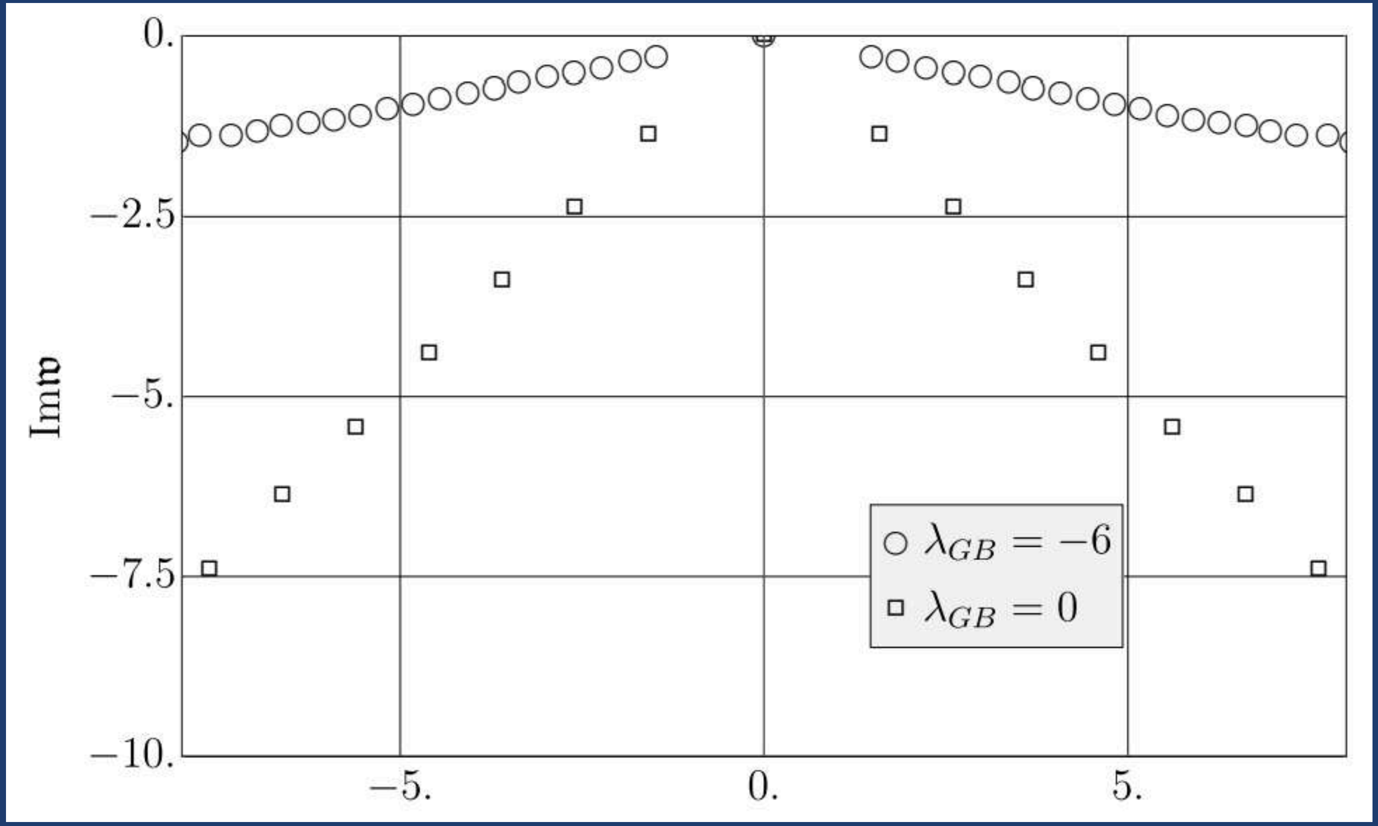
- universal behaviour across different higher-derivative gravities
  [Grozdanov, Kaplis, Starinets, JHEP (2016)]
- as the coupling increases, $\eta/s > \hbar/4\pi k_B$
- poles become denser, forming a branch cut which starts at $\omega = \pm q$
- new gapped excitations on imaginary axis

N=4 SYM / R^4 gravity



$\gamma \sim 1/\lambda^{3/2}$

R^2 (Gauss-Bonnet) gravity

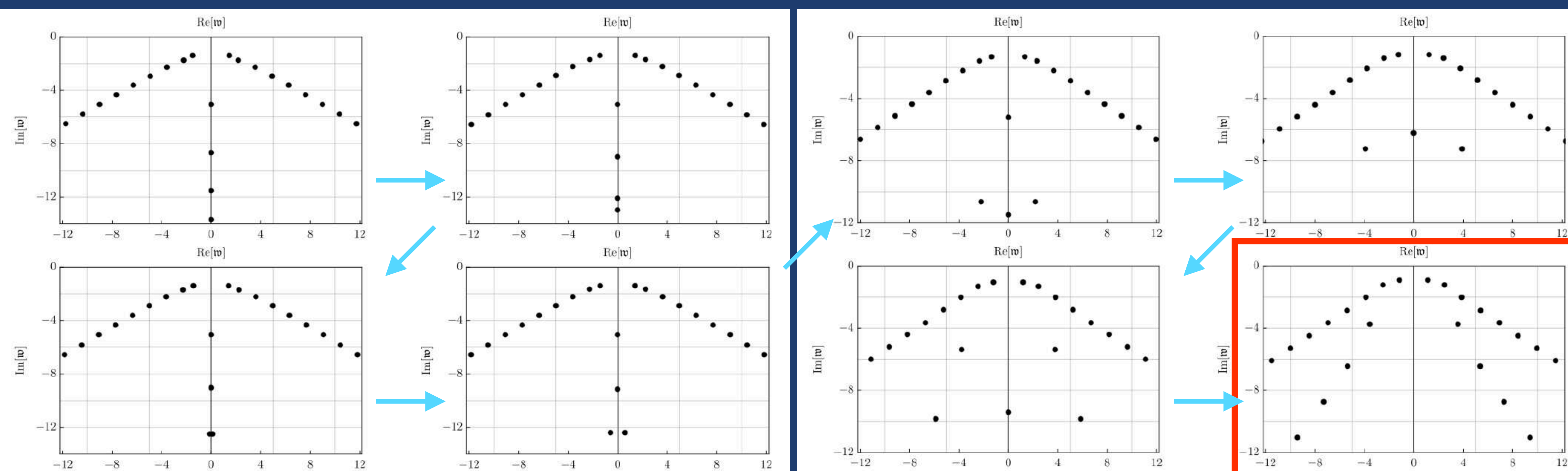


APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

$\lambda \to 0$ $\lambda \to \infty$

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda - \alpha\gamma_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \alpha^2\gamma_2\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)^2\right]$$

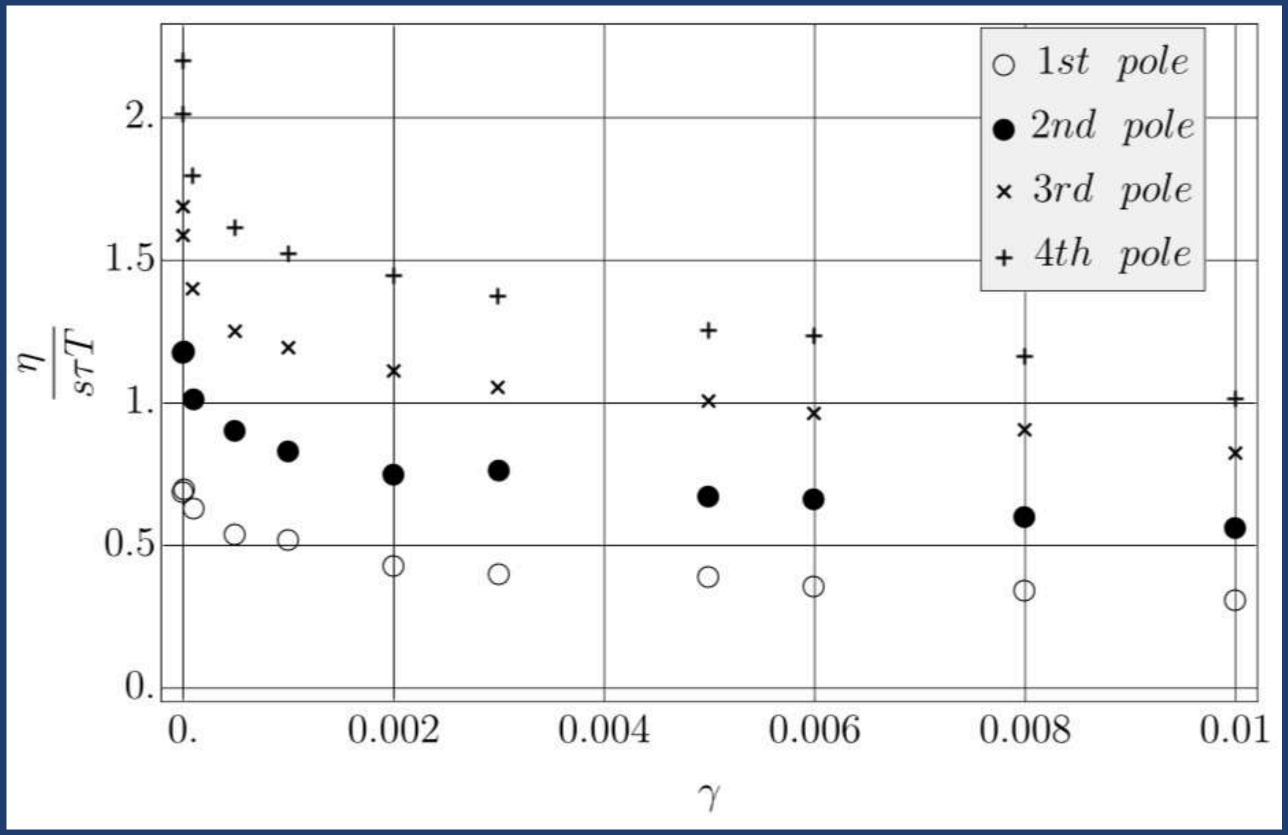
[Grozdanov, Starinets, JHEP (2019)]



APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

- kinetic theory prediction of $\eta/s \sim \text{const.} \times \tau_R T$ at weak coupling is recovered, where $\tau_R^{(n)} = -\dfrac{1}{\text{Im}\,\omega_n}$

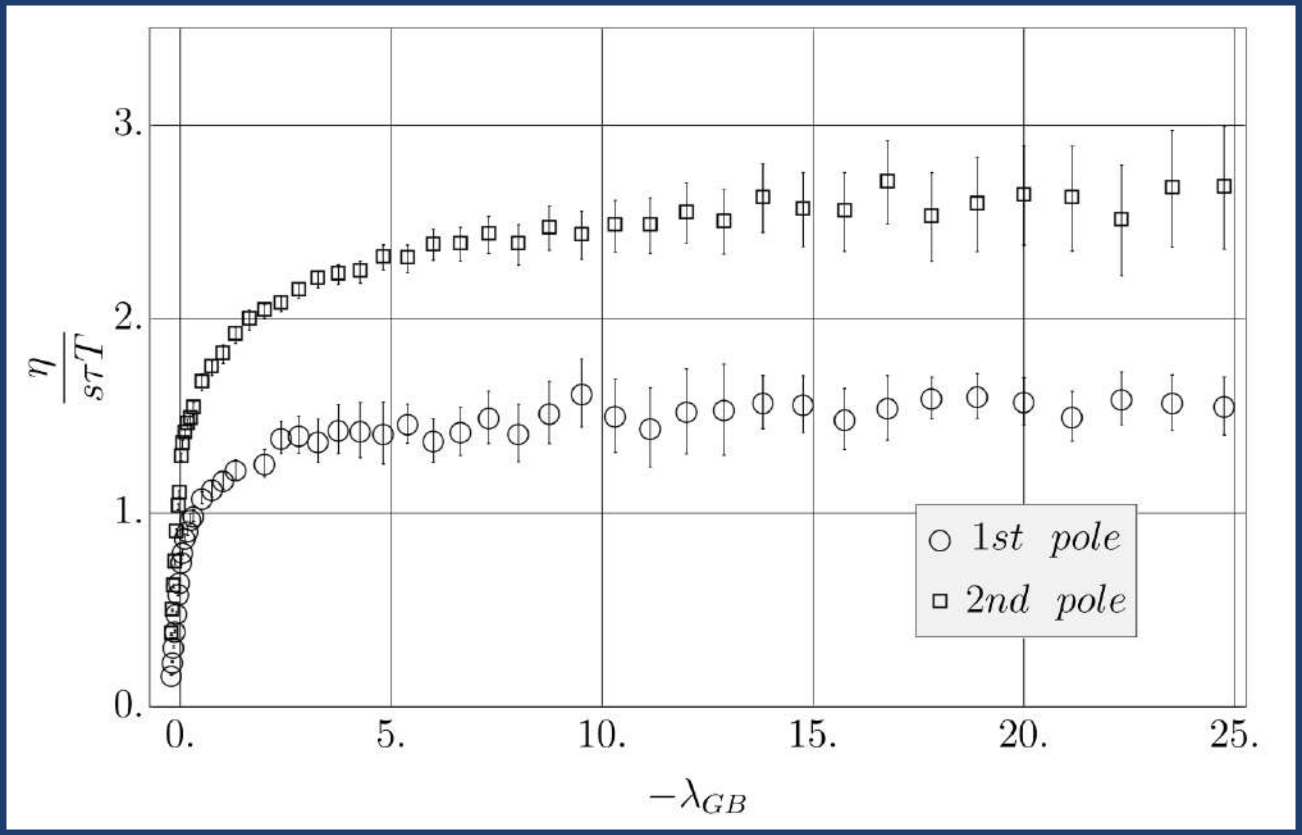
N=4 SYM / R^4 gravity



$\gamma \sim 1/\lambda^{3/2}$

strong coupling → weak coupling

R^2 (Gauss-Bonnet) gravity

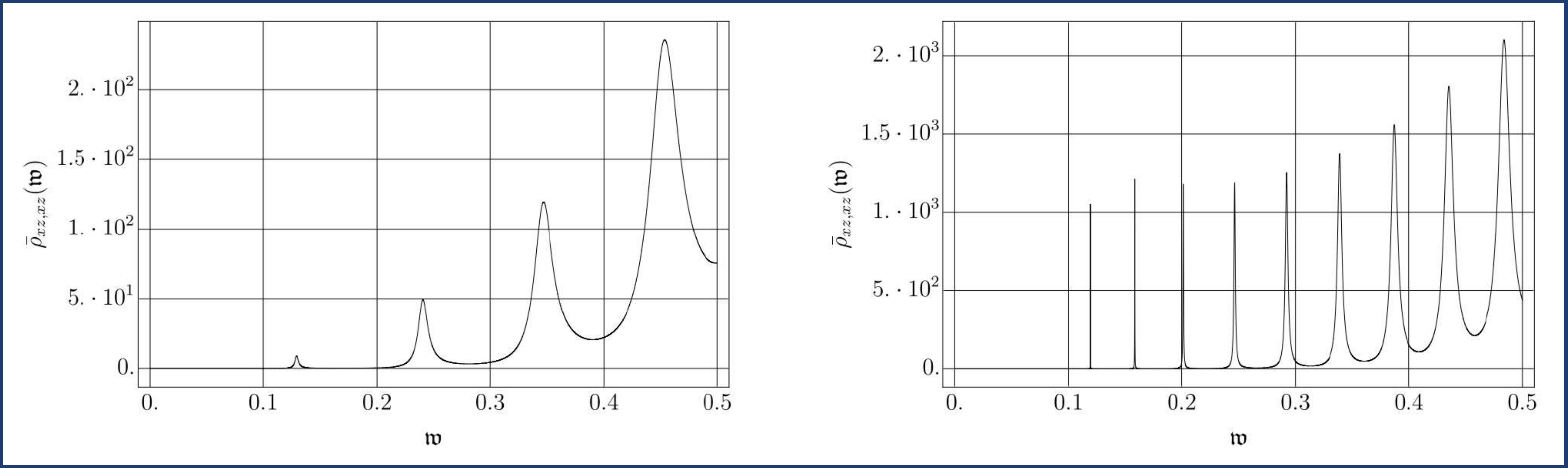


strong coupling → weak coupling

APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

- before the branch cut is formed, well-pronounced quasiparticle excitations appear in the spectrum

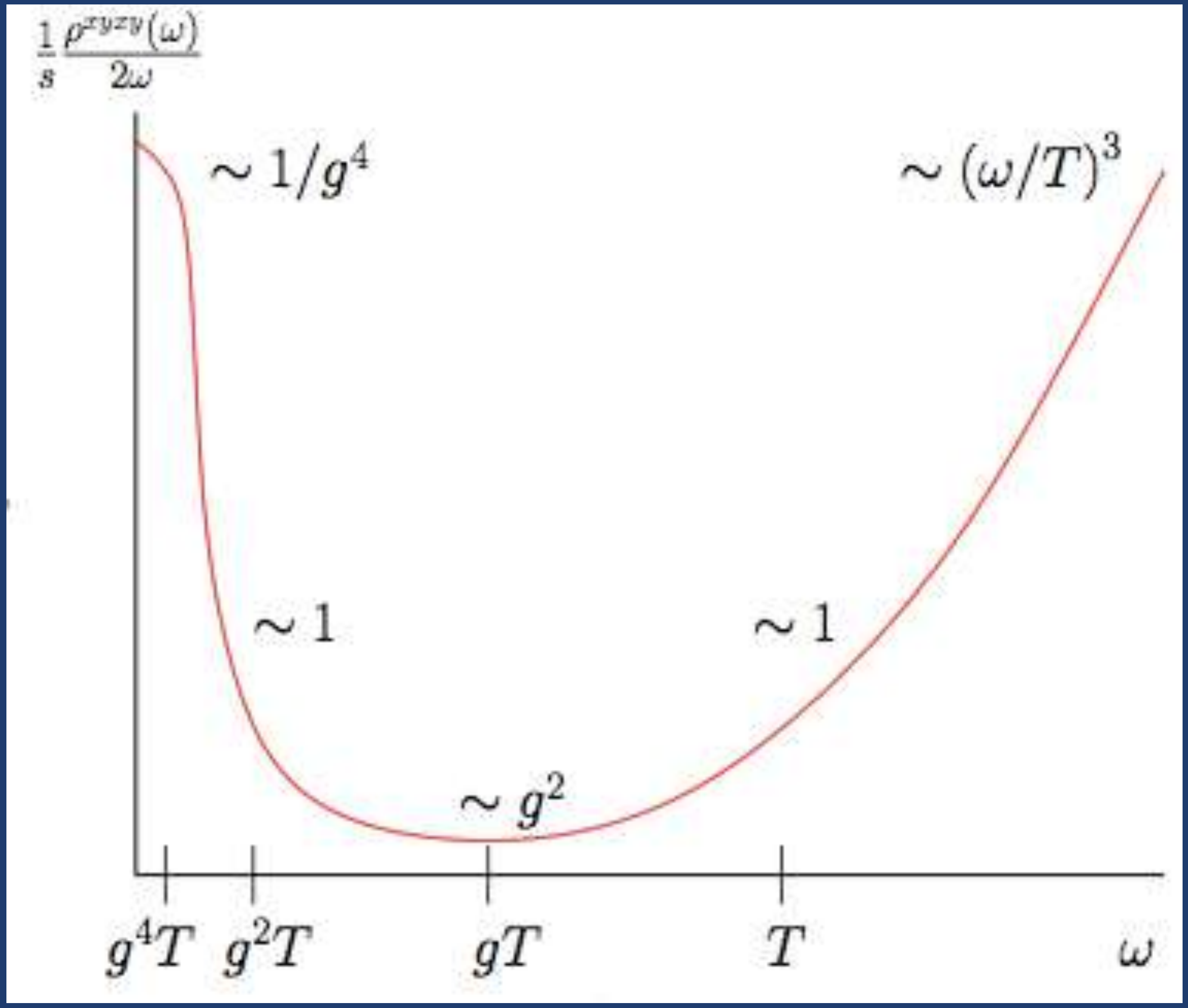
$$\rho = -2\,\mathrm{Im}\,\langle T_{\mu\nu}(\omega, q), T_{\rho\sigma}(-\omega, -q)\rangle_R$$



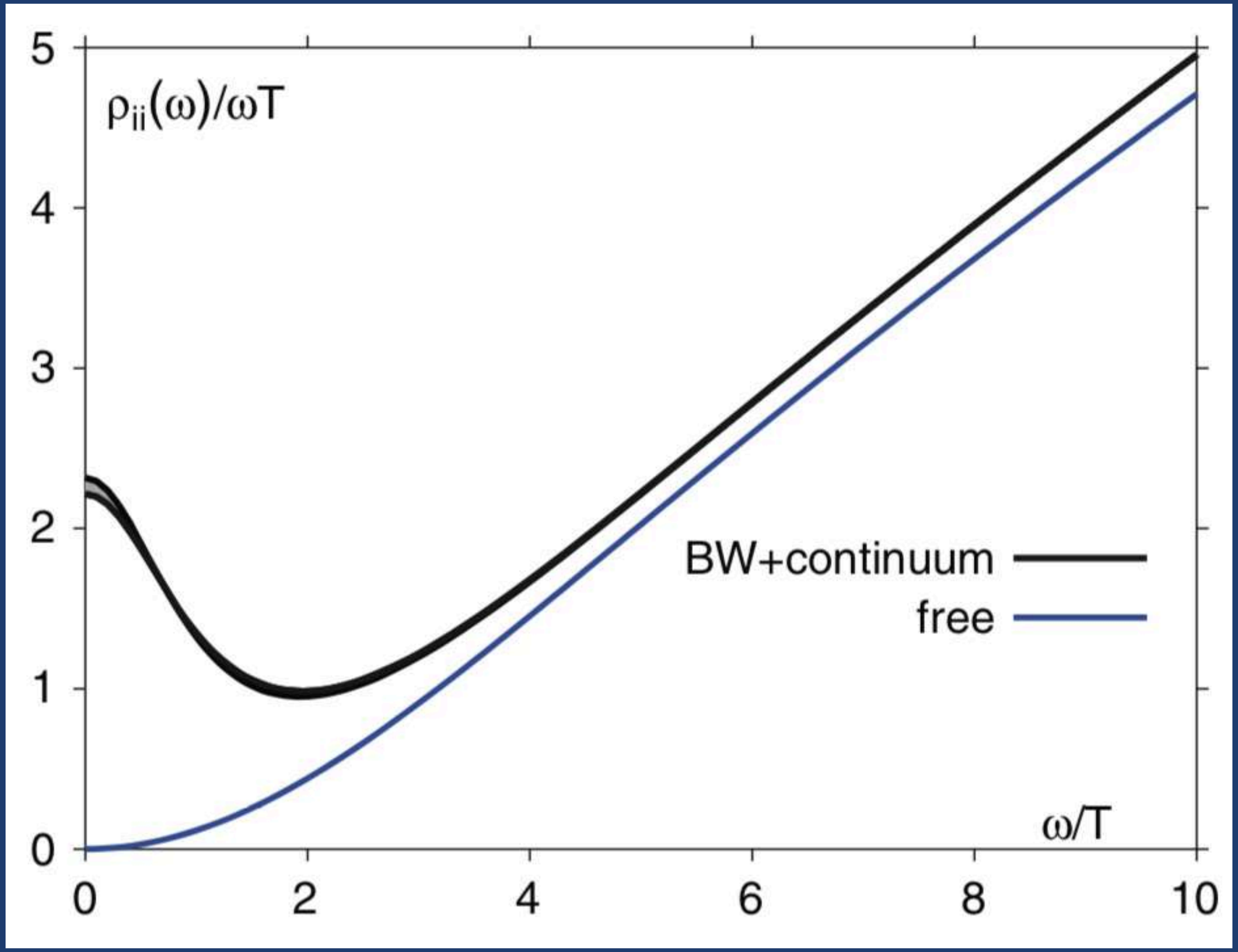
strong coupling → weak coupling

APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

- as a result of quasiparticles, a transport peak appears in the spectral functions computed from kinetic theory



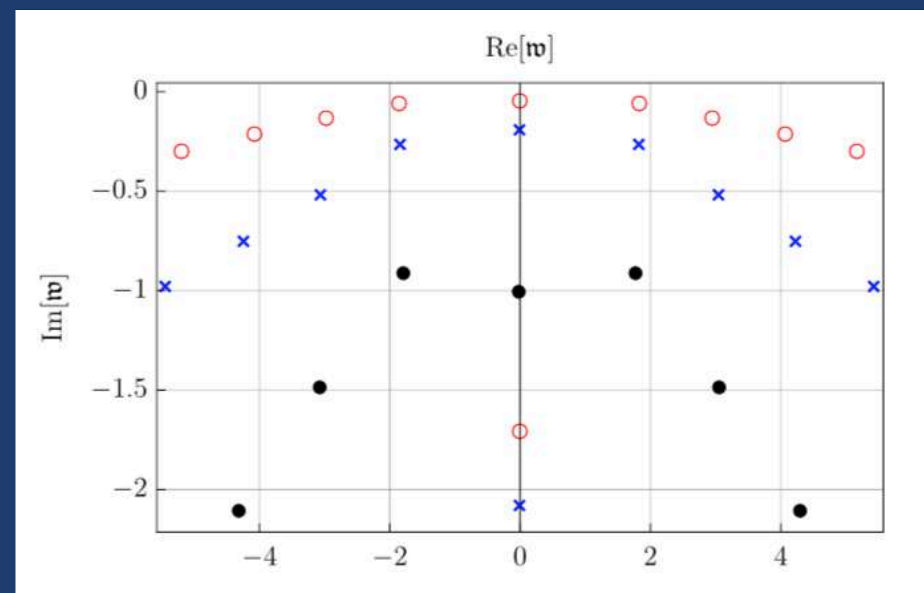
expectation from QCD—a sketch;
see Casalderrey-Solana, Liu, Mateos, Rajagopal, Wiedemann, ``Gauge/String Duality, Hot QCD and Heavy Ion Collisions", CUP (2014)

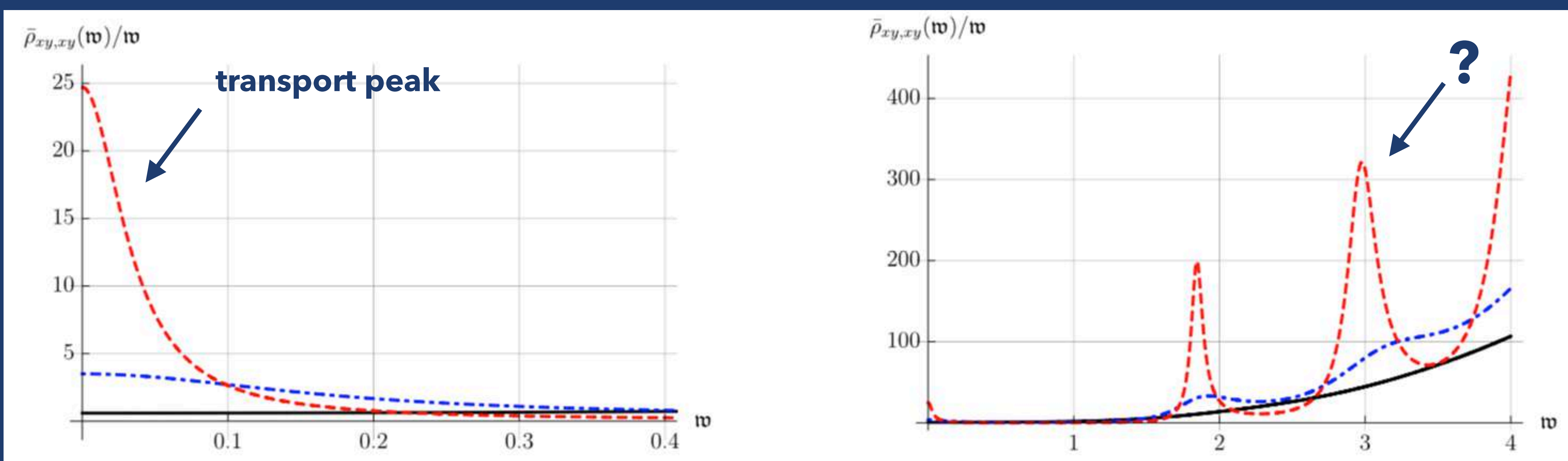


electromagnetic current-current spectral function in QCD at $T=1.45T_c$ from lattice data of Ding, Francis, Kaczmarek, Karsch, et. al., PRD (2011)

APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

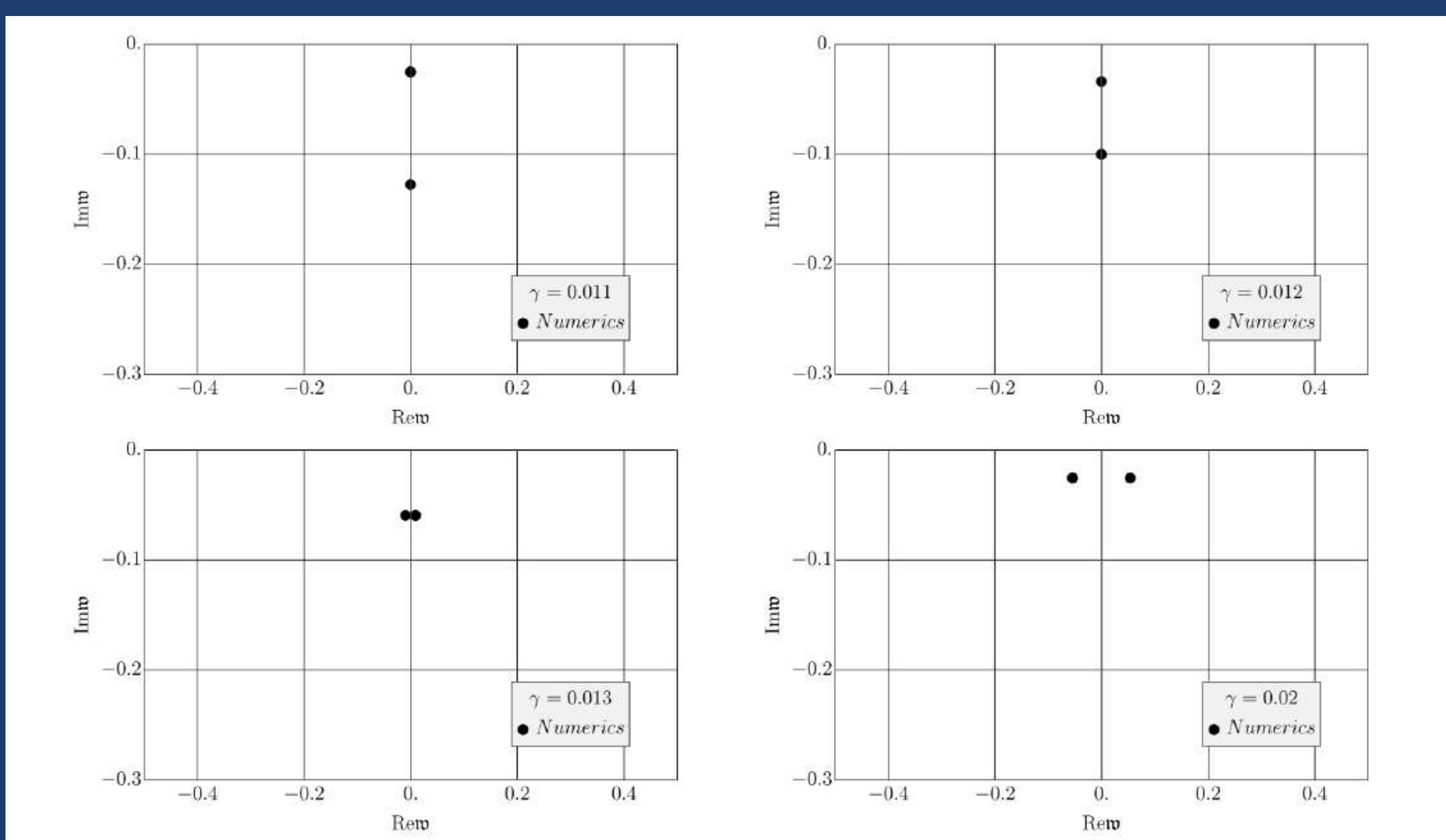
- in *N=4* SYM, holography explicitly reproduces the transport peak ($q = 0$) [Casalderey-Solana, Grozdanov, Starinets, PRL (2018)]

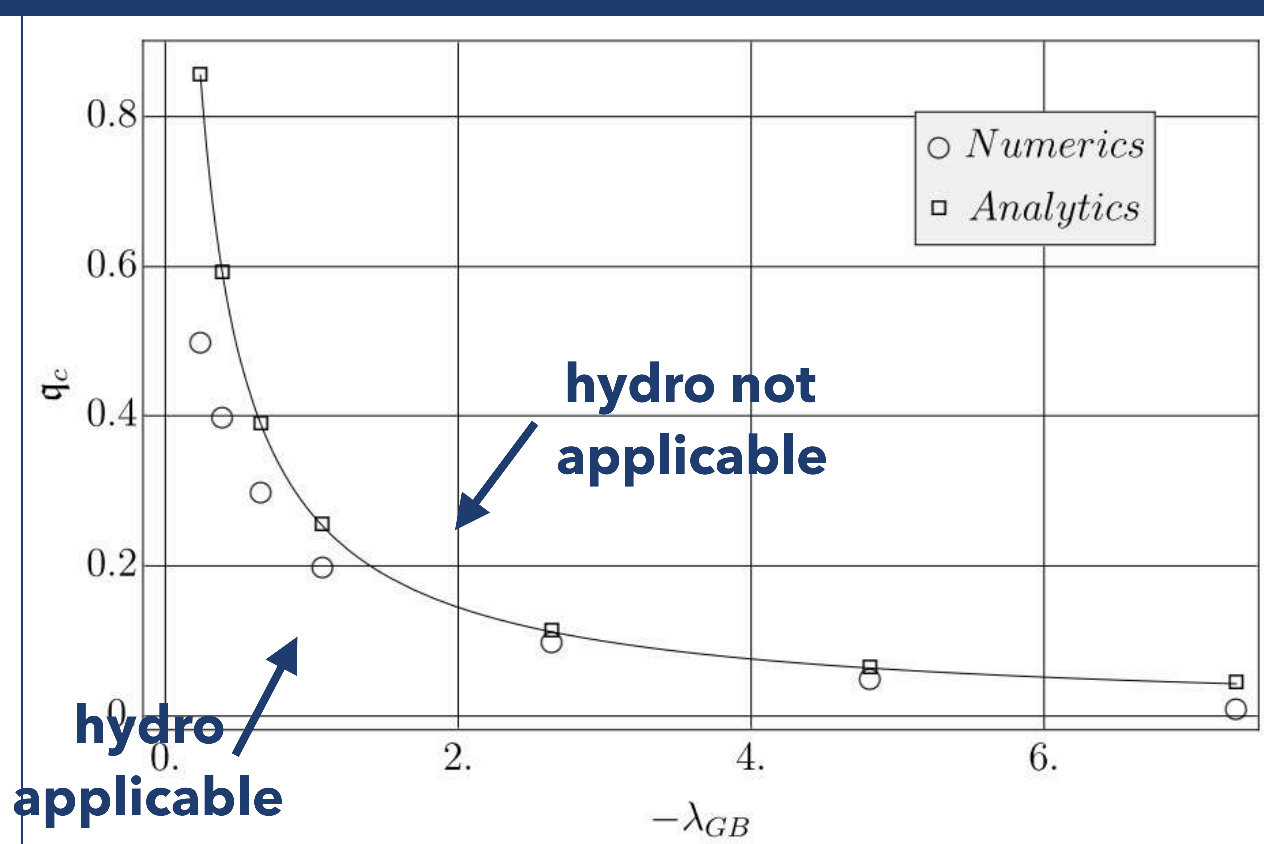




APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

- as weak coupling is approached, in spectra with hydro modes, the lowest imaginary mode (the one responsible for the transport peak) becomes long-lived and enters the hydro regime
- the mode collides with the hydro mode—explicit mechanism for destruction of hydrodynamics: hydrodynamics is worse at weak coupling



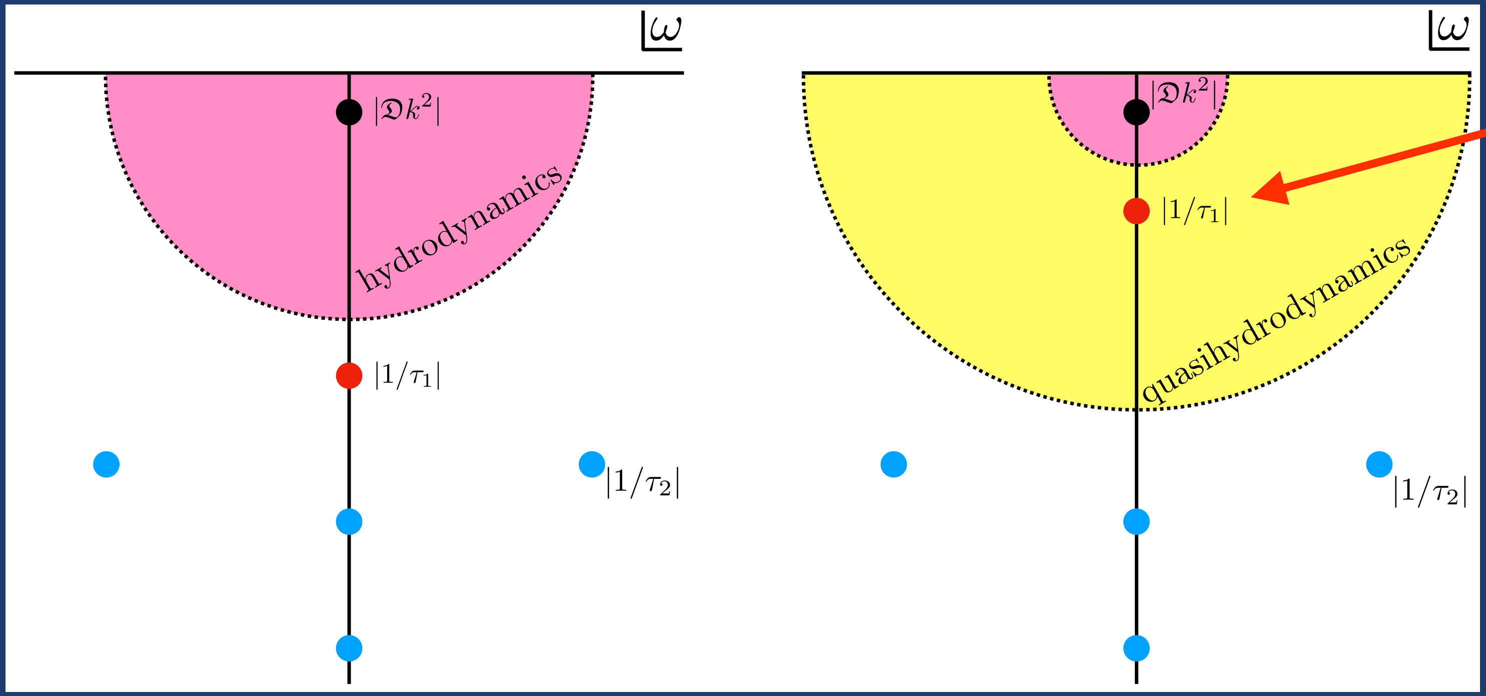


$$q_c/T \sim \lambda^{3/2}$$

strong coupling → weak coupling

APPROACH TO WEAK COUPLING FROM HOLOGRAPHY

- **quasihydrodynamics**—hydrodynamics with approximately conserved symmetries [Grozdanov, Lucas, Poovuttikul, PRD (2019)]
- a systematic way to derive such theories from holography
- linearised (phenomenological) **Müller-Israel-Stewart theory**, used to cure causality problems by introducing extra degrees of freedom, can be derived systematically from higher-derivative holographic theories



MIS theory's $\Pi^{\mu\nu}$ correctly describes the leading extra gapped mode in all 3 channels at linear order in the fluctuations

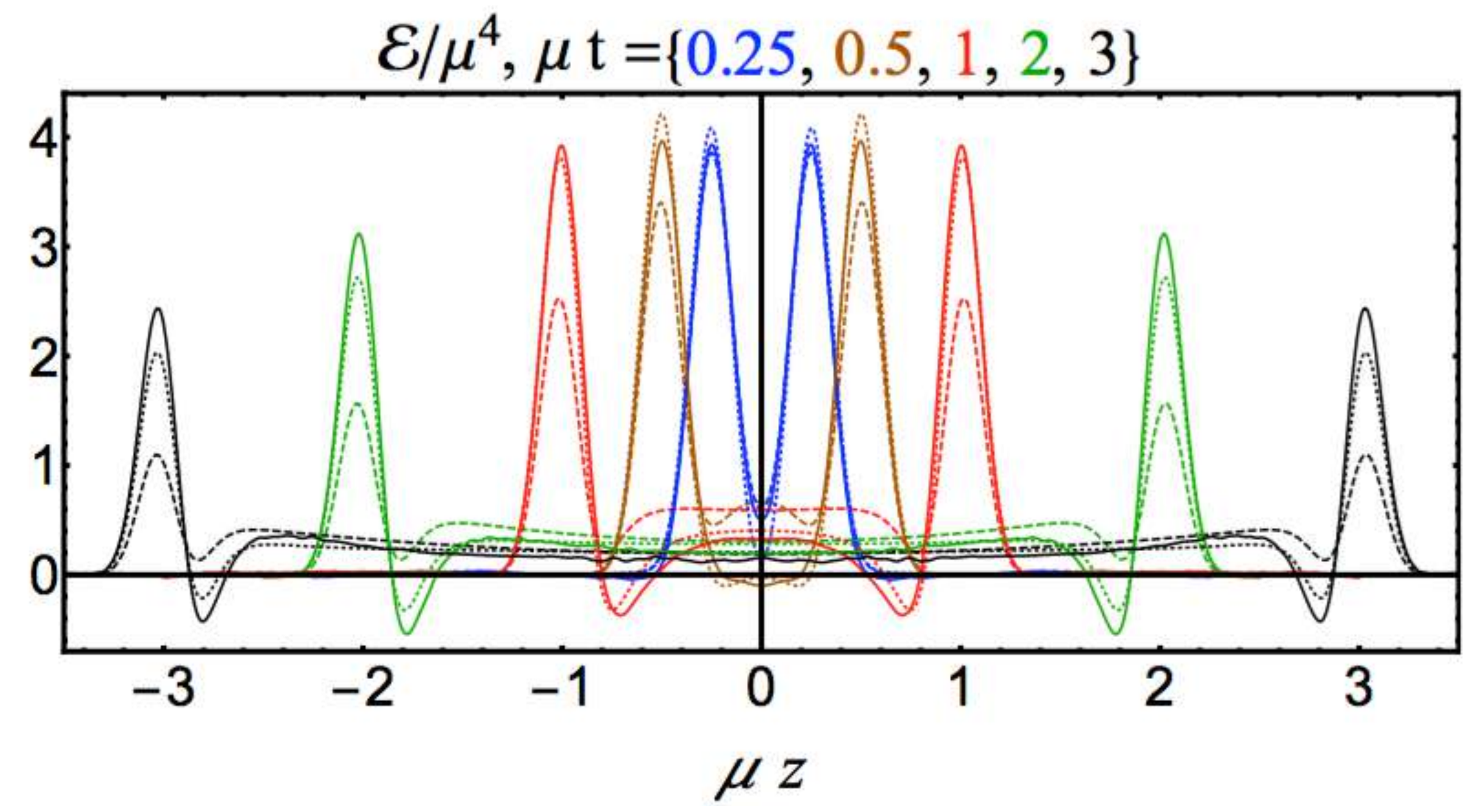
$$\tau_1 \approx \frac{373\zeta(3)}{32\pi T \lambda^{3/2}} \approx \frac{4.5}{T\lambda^{3/2}}$$

leading-order analytical approximation to the ``relaxation time” in *N=4* SYM [Grozdanov, Kaplis, Starinets, JHEP (2016)]

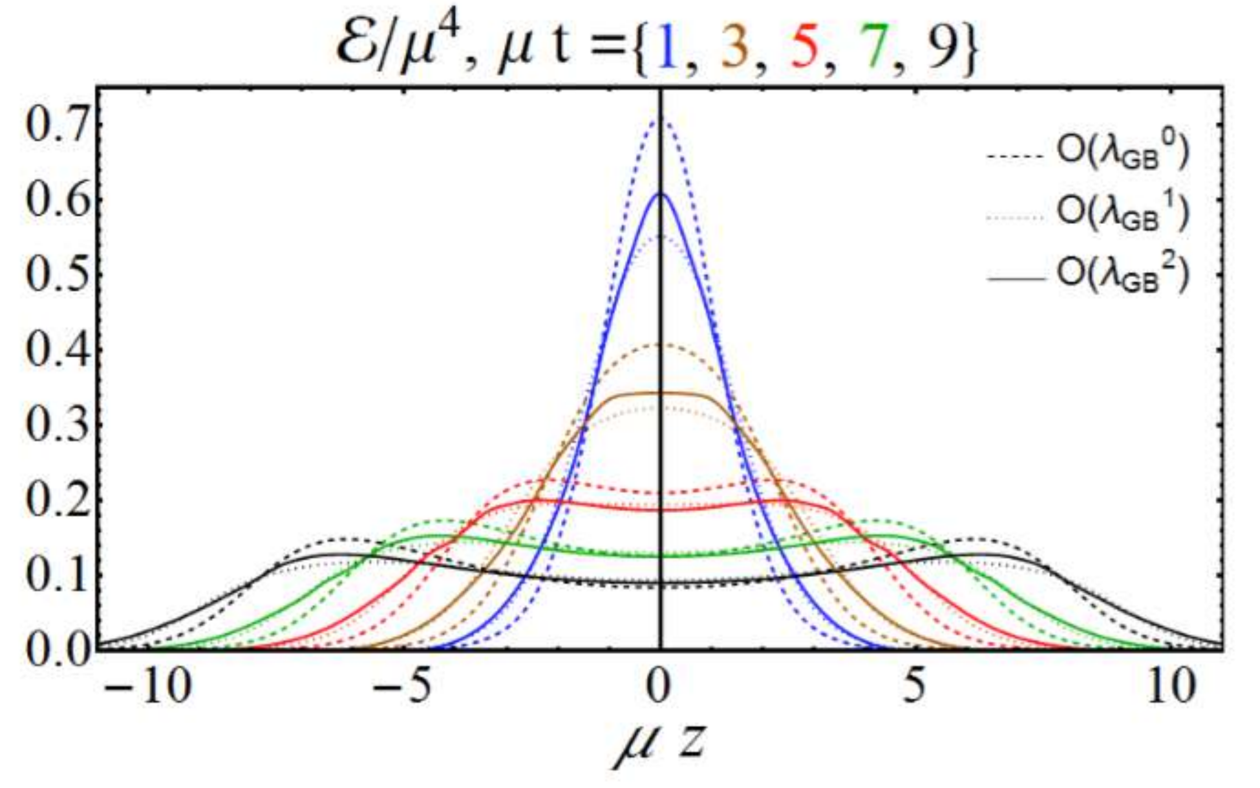
HOLOGRAPHIC MODEL OF FAR-FROM-EQUILIBRIUM HEAVY ION COLLISIONS

HOLOGRAPHIC HEAVY-ION COLLISIONS

- initial study of holographic heavy-ion collisions by Chesler and Yaffe in 2011
- coupling dependence from R^2 (Einstein-Gauss-Bonnet) theory [Grozdanov, van der Schee, PRL (2016)]
- at intermediate coupling, ``nuclei” experience less stopping and have more energy deposited near the light cone
- see also poster by Attems on collisions near a critical point



narrow shocks



wide shocks

HOLOGRAPHIC HEAVY-ION COLLISIONS

- delayed hydrodynamisation– a quantifiable prediction of holography
- consistent with worse applicability of hydrodynamics at finite coupling

$$t_{\rm hyd} T_{\rm hyd} = \{0.41 - 0.52\lambda_{GB},\; 0.43 - 6.3\lambda_{GB}\}$$

→

at $\lambda_{GB} = -0.2$:

η/s increases by 80%

⇊

$t_{\rm hyd} T_{\rm hyd}$ increases by

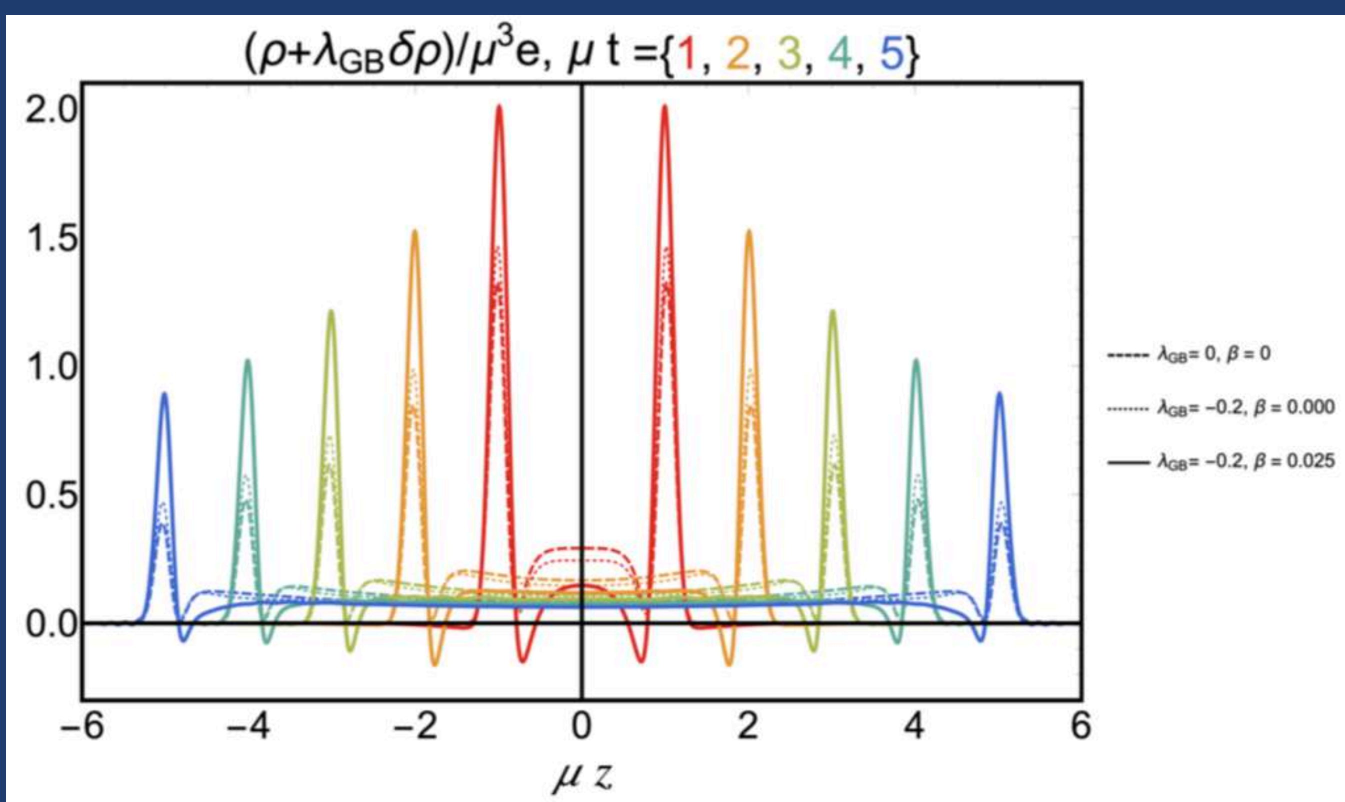
$\{25\%, 290\%\}$

↑ narrow shocks

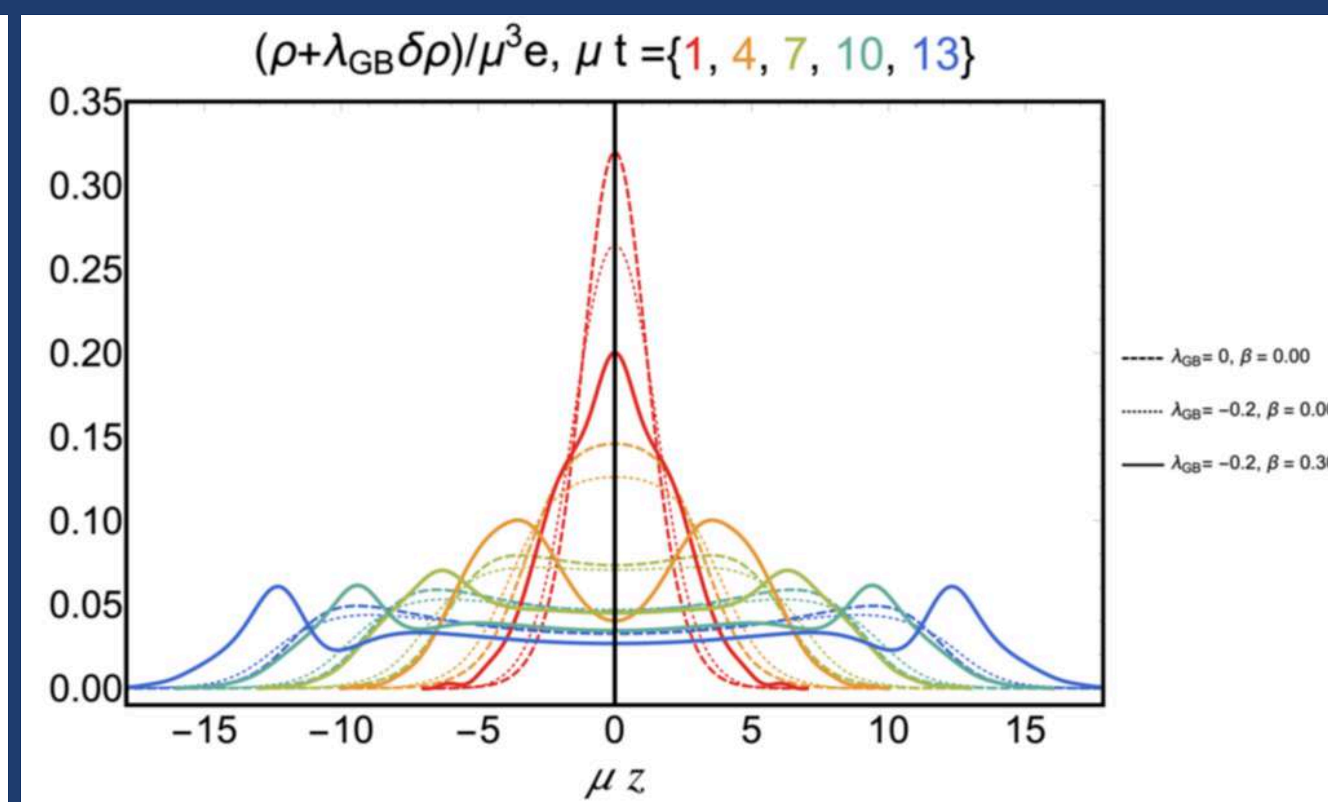
↑ wide shocks

HOLOGRAPHIC HEAVY-ION COLLISIONS

- coupling dependence in holographic heavy-ion collisions in the presence of finite baryon number density from the most general Einstein-Maxwell R^2-type theory [Folkestad, Grozdanov, Rajagopal, van der Schee, *to appear.*, also, talk on Tuesday by van der Schee]
- at finite coupling, significantly less stopping of the baryon charge



narrow shocks

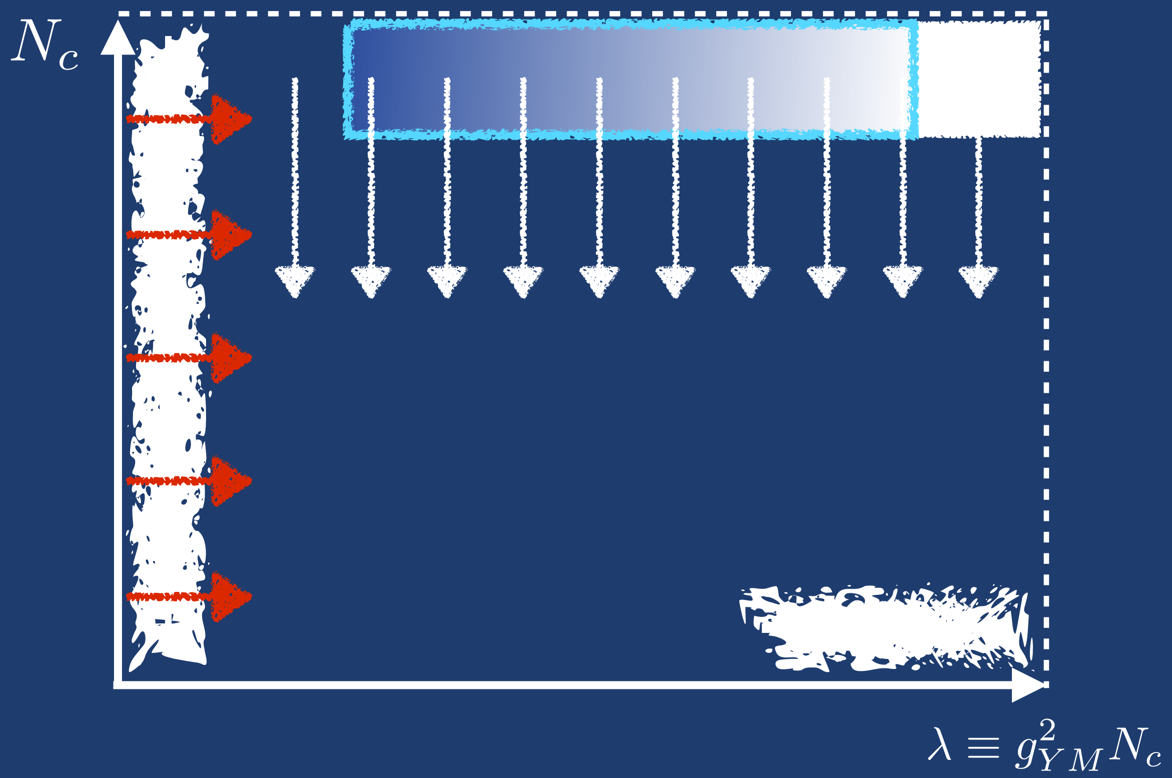


wide shocks

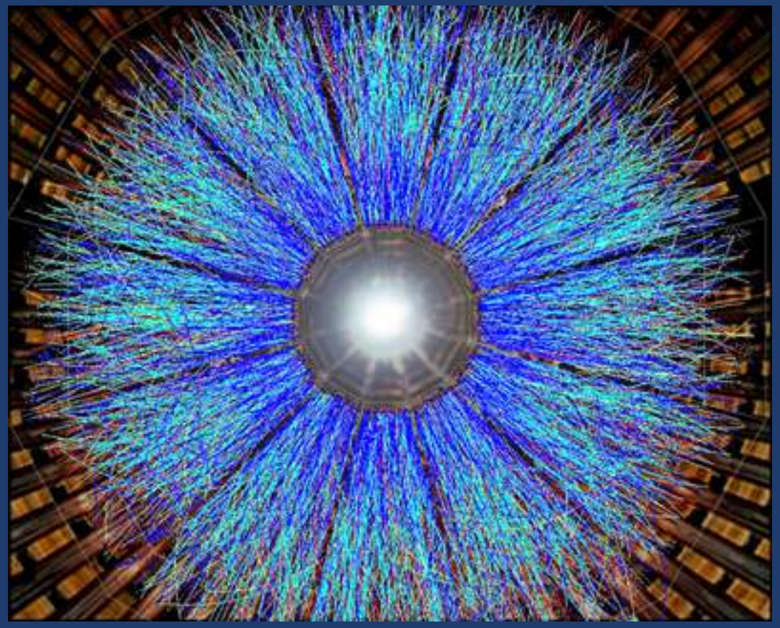
FUTURE DIRECTIONS

SUMMARY AND FUTURE DIRECTIONS

- holography is a useful tool for studying coupling constant dependence
- results are highly universal across different higher-derivative theories
- transition to ``weakly coupled physics” is extremely quick!



- connection to perturbative, weakly coupled QFTs and kinetic theory
- away from the infinite N_c limit = quantum gravity
- experimental tests of **quantitative** holographic predictions in QGP



THANK YOU!