Understanding p_T-dependent flow fluctuations from initial geometry

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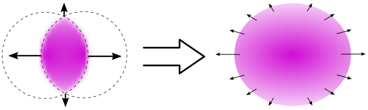
Outline

- What do **final state** observables tell about the **initial state**?
- Conversion of **initial geometry** to **momentum anisotropy**:



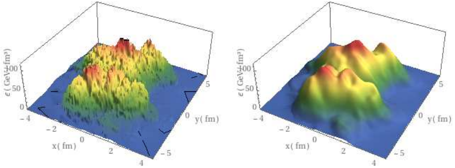
- Captured by **systematic expansion** $V_n \simeq \kappa\, \epsilon_n + \ldots$
- Subleading terms $\Rightarrow$ small-scale structure.
- Probed by p_T-dependent flow fluctuations
  $\Rightarrow$ principal component analysis (**PCA**).

Assumptions

1. Posterior evolution and one-body distribution determined by $T^{\mu\nu}(\tau_0)$, $J^{\mu}(\tau_0)$:

$$E\,\frac{dN}{d^3p} = \mathcal{F}\Big[T^{\mu\nu}(\tau_0),\, J^{\mu}(\tau_0)\Big]$$

2. Hierarchy of scales: large scales dominate



[arXiv:1807.05213]

Mapping hydrodynamic response

- **Eccentricities $\boldsymbol{\epsilon_{n,m}}$** from cumulants of transverse density: $\rho(\boldsymbol{x})$

$$W(\vec{k}) \equiv \ln\left(\int d^2x\, \rho(\boldsymbol{x}) e^{i\,\boldsymbol{k}\cdot\boldsymbol{x}}\right) \equiv \sum_{\substack{m=0\\ n=-\infty}}^{\infty} W_{n,m}\, k^m e^{-in\phi_{\vec{k}}}$$

- Flow V_n can be predicted by the set of all $\epsilon_{n,m}$:

$$V_n = \mathcal{F}_n[\rho(\boldsymbol{x})] = f_n[\{\epsilon_{n',m}\}]$$

- Large scales (small m) dominate $\Rightarrow$ **truncated expansion**

D. Teaney and L. Yan, PRC **83** (2011).
F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, PRC **85** (2012).
See Poster by Matthew Luzum.

Estimators

- **Linear and nonlinear** terms:

$$V_2 \simeq \kappa_0\, \epsilon_{2,2} + \kappa_1\, \epsilon_{2,4} + \kappa_2\, \epsilon_{2,6} + \kappa_3\, \epsilon_{2,8} + \mathcal{O}(m = 10) \\ + \kappa_4\, |\epsilon_{2,2}|^2 \epsilon_{2,2} + \kappa_5\, \epsilon_4 \epsilon_2^* + \kappa_6\, \epsilon_{1,3}^2 + \ldots + \mathcal{O}(\epsilon^3)$$

$$V_3 \simeq \kappa_0\, \epsilon_{3,3} + \kappa_1\, \epsilon_{3,5} + \kappa_2\, \epsilon_{3,7} + \kappa_3\, \epsilon_{3,9} + \mathcal{O}(m = 11) \\ + \kappa_4\, \epsilon_{2,2} \epsilon_{1,3} + \kappa_5 \epsilon_{2,2}^2 \epsilon_{1,3}^* + \kappa_6\, \epsilon_4 \epsilon_{1,3}^* + \ldots + \mathcal{O}(\epsilon^3)$$

Estimators

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$$
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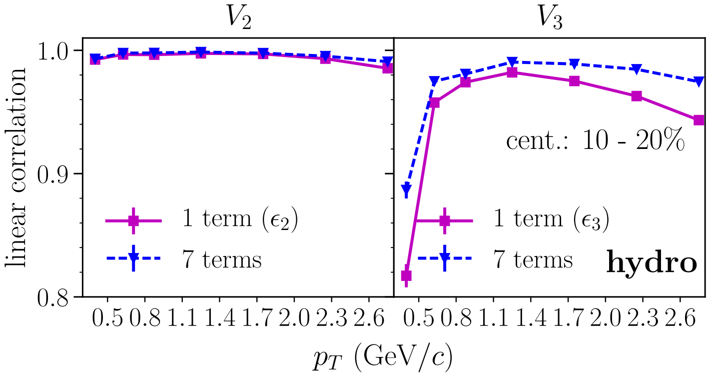
\end{aligned}
$$

- Extension to **differential flow** $V_n(\boldsymbol{p})$ via $\kappa_i \to \kappa_i(\boldsymbol{p})$:

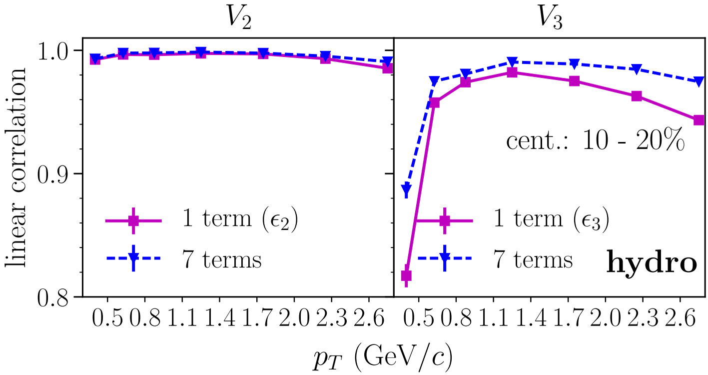
$$
E\,\frac{dN}{d^3p} = \frac{1}{2\pi} N(\boldsymbol{p}) \sum_{n=-\infty}^{\infty} \boldsymbol{V_n(p)}\, e^{-in\varphi}
$$

$V_n(p_T)$ estimator



- **Correlation** between $V_n(p_T)$ and estimator $\Rightarrow$ performance.
- Pearson correlation coefficient (1 $\equiv$ perfect correlation).
- Stricter test?

$V_n(p_T)$ estimator



- **Correlation** between $V_n(p_T)$ and estimator $\Rightarrow$ performance.
- Pearson correlation coefficient ($1 \equiv$ perfect correlation).
- Stricter test $\Rightarrow$ PCA.

Principal Component Analysis

- General statistical method.

[See talk by Ziming Liu.]

- Finds linearly **uncorrelated combinations** of variables.

- Diagonalize flow covariance matrix:

$$V_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) = \langle V_n(\mathbf{p}_1)\, V_n^*(\mathbf{p}_2) \rangle \simeq \sum_{\alpha=1}^{k} V_n^{(\alpha)}(\mathbf{p}_1) V_n^{(\alpha)}(\mathbf{p}_2)\,.$$

- Eigenvalues are **strongly ordered** $\Rightarrow$ truncation.

- Characterization of **two-particle correlations** from few $V_n^{(\alpha)}(\mathbf{p})$.

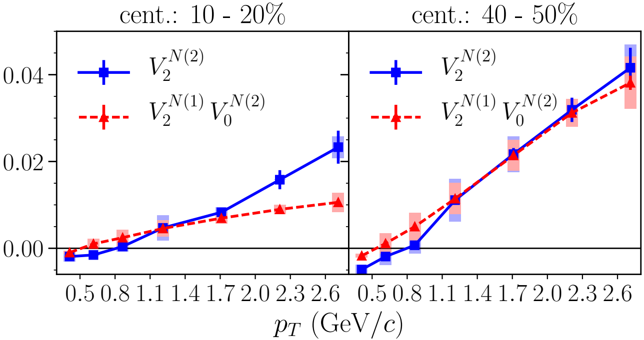
R. S. Bhalerao, J. Y. Ollitrault, S. Pal and D. Teaney, PRL **114** (2015).

Multiplicity fluctuations

- Currently measured PCA from

$$V_{n\Delta}^{N}(\mathbf{p}_1, \mathbf{p}_2) = \langle N(\mathbf{p}_1)\, N(\mathbf{p}_2)\, V_n(\mathbf{p}_1)\, V_n^*(\mathbf{p}_2)\rangle\,.$$

- Anisotropic flow fluctuations obscured by $\langle \Delta N(\boldsymbol{p}_1)\Delta N(\boldsymbol{p}_2)\rangle$:



[Obtained from published CMS data.]

Principal Component Analysis (redefined)

- Issue fixed by using **redefined** covariance matrix:

$$V^R_{n\Delta} \equiv \frac{\langle N(\mathbf{p}_1)\, N(\mathbf{p}_2)\, V_n(\mathbf{p}_1)\, V^*_n(\mathbf{p}_2)\rangle}{\langle N(\boldsymbol{p}_1) N(\boldsymbol{p}_2)\rangle} \simeq \langle V_n(\mathbf{p}_1)\, V^*_n(\mathbf{p}_2)\rangle. \qquad (1)$$

MH, D. Dobrigkeit Chinellato, M. Luzum, J. Noronha,
T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

- **Redefined PCA** ⇒ characterizes anisotropic flow only.

Principal Component Analysis (redefined)

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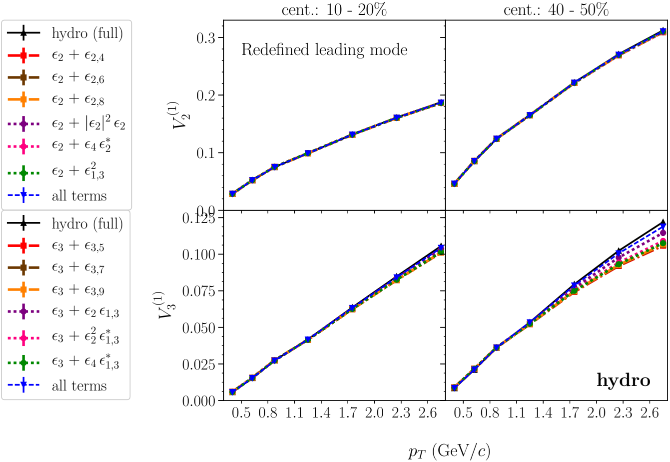
MH, D. Dobrigkeit Chinellato, M. Luzum, J. Noronha, T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

- **Redefined PCA** $\Rightarrow$ characterizes anisotropic flow only.

- Can we understand it from the initial state?

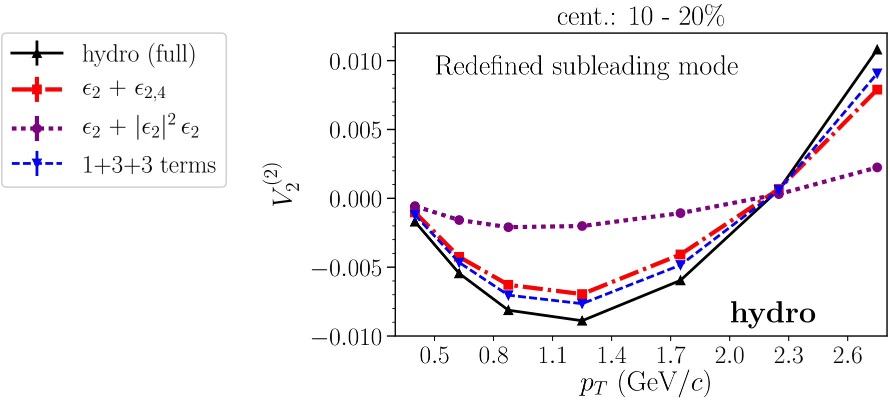
A. Mazeliauskas and D. Teaney, Phys. Rev. C **91** (2015) and **93** (2016).

Results: leading flow



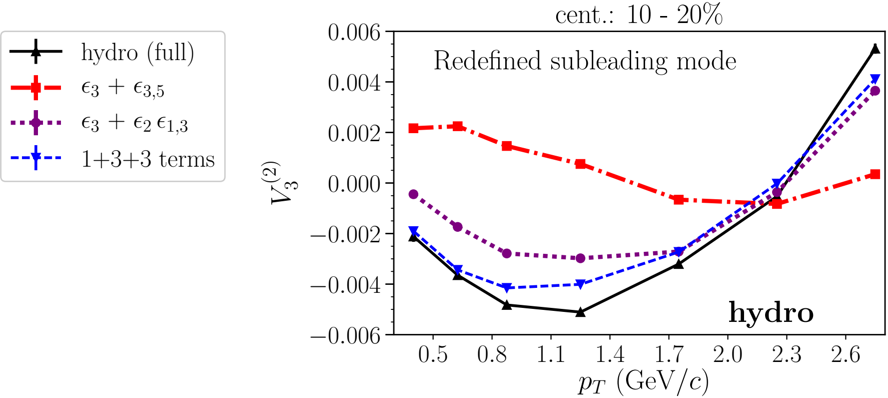
- Leading PCA mode $\Rightarrow$ leading term in eccentricity expansion.
- Are there effects from subleading terms?

Results: Elliptic flow



- **Subleading V_2 mode** is sensitive to **smaller-scale structure**.
- Estimators approach full result ⇒ **validation** of the framework.

Results: Triangular flow



- **Subleading V_3 mode** exhibits **nonlinear response** already at semicentral collisions.
- Estimators approach full result $\Rightarrow$ **validation** of the framework.

Conclusions

- **General framework** to relate final and initial-state fluctuations.
- Works for $\boldsymbol{p_T}$**-dependent** azimuthal flow fluctuations.
- We propose new, improved PCA observables.
- Promising probe of **small-scale structure**.

Acknowledgements

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CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

Backup Slides

Backup slides...

Cumulant examples

$$W_{0,2} = \frac{i^2}{2!}\frac{1}{2}\left[\{r^2\} - \{r\,e^{-i\,\phi}\}\{r\,e^{-i\,\phi}\}\right], \tag{2}$$

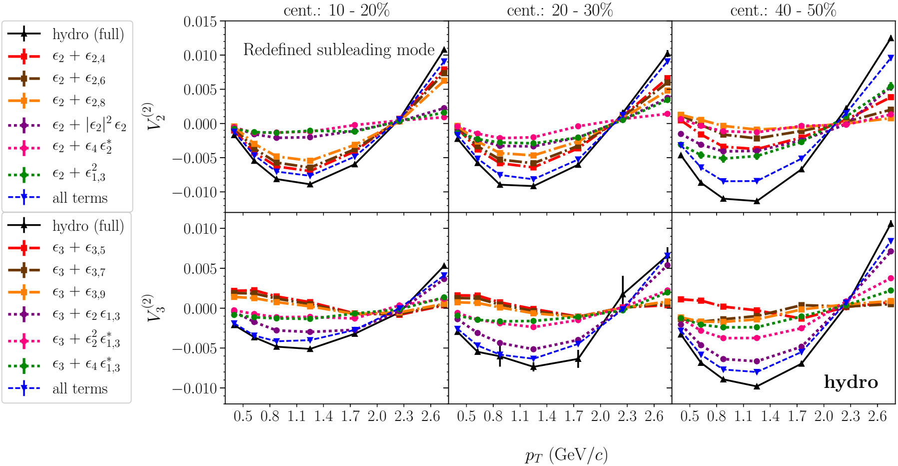
$$W_{2,2} = \frac{i^2}{2!}\frac{1}{4}\left[\{r^2\,e^{2\,i\,\phi}\} - \{r\,e^{i\,\phi}\}^2\right], \tag{3}$$

$$W_{1,3} = \frac{i^3}{3!}\frac{3}{8}\left[\{r^3\,e^{i\,\phi}\} - \{r^2\,e^{2\,i\,\phi}\}\{r\,e^{-i\,\phi}\}\right], \tag{4}$$

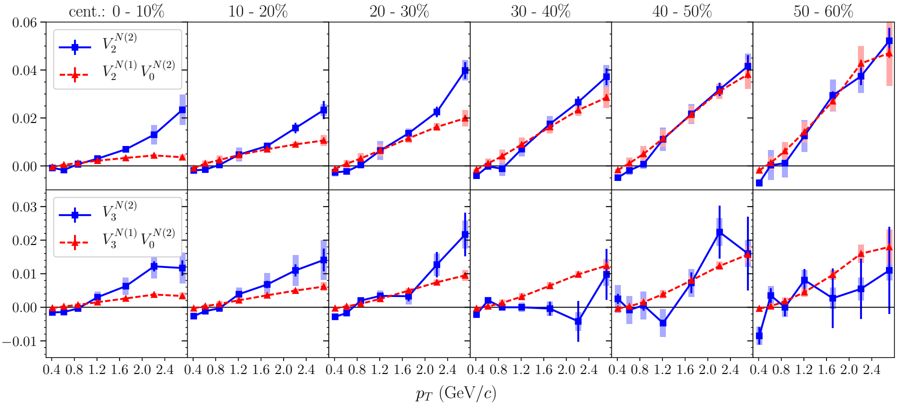
where

$$\{\ldots\} := \frac{\int d^2x\,\rho(\vec{x})\,(\ldots)}{\int d^2x\,\rho(\vec{x})} \tag{5}$$

Subleading modes

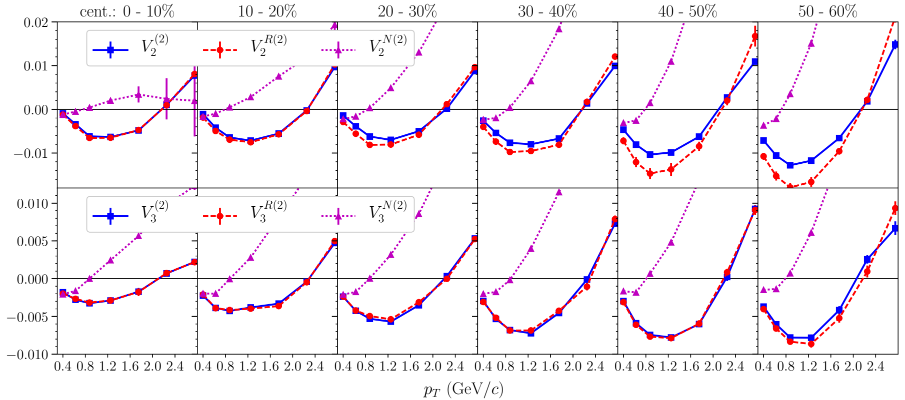


Multiplicity Fluctuations



MH, D. Dobrigkeit Chinellato, M. Luzum, J. Noronha,
T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

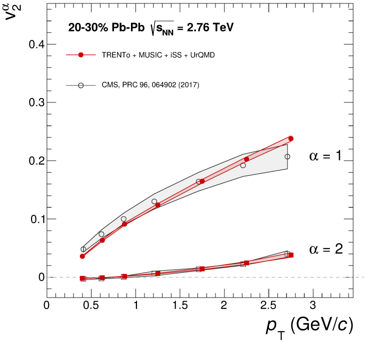
Redefined PCA

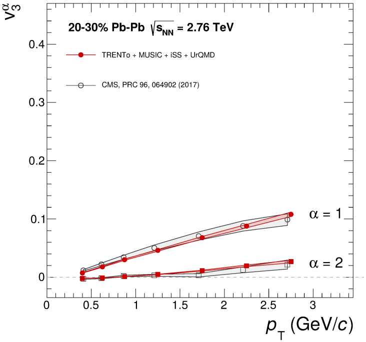


MH, D. Dobrigkeit Chinellato, M. Luzum, J. Noronha, T. Nunes da Silva and J. Takahashi, arXiv:1906.08915

Original PCA

$$V_{n\Delta}(\mathbf{p}_1, \mathbf{p}_2) = \langle N(\mathbf{p}_1)\, V_n(\mathbf{p}_1)\, N(\mathbf{p}_2)\, V_n^*(\mathbf{p}_2) \rangle \,. \tag{6}$$





T. Nunes da Silva, D. Dobrigkeit Chinellato, R. Derradi De Souza, MH, M. Luzum, J. Noronha and J. Takahashi, arXiv:1811.05048

Generalized eccentricities

Anisotropy and orientation of the initial density profile $\rho(\vec{x})$ given by

$$W(\vec{k}) := \log\left(\int d^2x\,\rho(\vec{x})\,e^{i\,\vec{k}\cdot\vec{x}}\right) := \sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty} W_{n,m}\,k^m\,e^{-i\,n\,\phi_{\vec{k}}}\,. \qquad (7)$$

Dimensionless eccentricities from cumulants:

$$\epsilon_{n,m} := W_{n,m}/(W_{0,2})^{m/2}\,. \qquad (8)$$

General hydrodynamic estimators

$$V_n^{\text{est}}(p_T, y) = \sum_{m=\max[n,2]}^{m_{\max}} \kappa_{n,m}(p_T, y)\, \epsilon_{n,m} + \\ + \sum_{p=2}^{p_{\max}} \sum_{\substack{\{n_i, m_i\} \\ \sum n_i = n \\ m < m_{\max}}} \kappa^{(p)}_{\{n_i, m_i\}}(p_T, y) \prod_{i=1}^{p} \epsilon_{n_i, m_i}\,, \tag{9}$$

The framework

- Flow harmonics V_n can be predicted by $\epsilon_{n,m}$.
- The κ's are found by minimizing

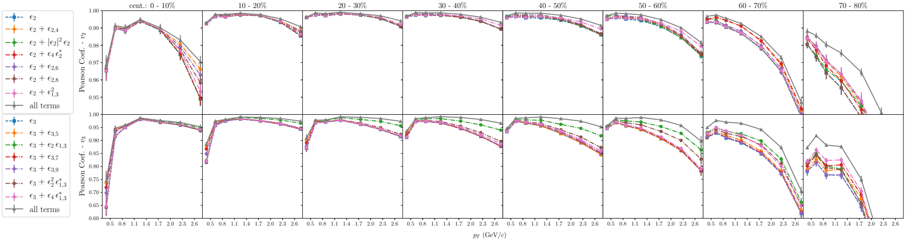
$$\langle|\delta_{\text{residual}}|\rangle^2 := \langle|V_n^{\text{est.}} - V_n|^2\rangle \,. \tag{10}$$

- The quality of the predictor can be measured by the Pearson coefficient

$$\frac{\langle V_n^* \, V_n^{\text{est.}}\rangle}{\sqrt{\langle|V_n|^2\rangle \, \langle|V_n^{\text{est.}}|^2\rangle}} \leq 1 \,. \tag{11}$$

- To predict differential flow, $V_n(p_T)$, find $\kappa(p_T)$.

Pearson coefficients



Bin-independent orthonormality condition

Naive diagonalization of $V_{n\Delta}^{ab}$ with conventional algorithms will result in

$$\sum_a \psi_n^{(\alpha)a}\,\psi_n^{(\beta)a} = \delta_{\alpha\beta}\,, \tag{12}$$

not compatible with

$$\int dp_T\,\psi_n^{(\alpha)}(p_T)\,\psi^{(\beta)}(p_T) = \delta_{\alpha\beta}\,. \tag{13}$$

We instead diagonalize

$$\sqrt{\Delta p_T^a \Delta p_T^b}\,V_{n\Delta}^{ab} = \sum_i \lambda^{(\alpha)}\,\tilde{\psi}_n^{(\alpha)a}\,\tilde{\psi}_n^{(\alpha)b} \tag{14}$$

$$\sum_a \tilde{\psi}_n^{(\alpha)a}\,\tilde{\psi}_n^{(\beta)a} = \delta_{\alpha\beta}\,. \tag{15}$$

Replacing $\tilde{\psi}^{(\alpha)a} \to \sqrt{\Delta p_T^a}\psi^{(\alpha)a}$:

$$V_{n\Delta}^{ab} = \sum_\alpha \lambda^{(\alpha)}\,\psi_n^{(\alpha)a}\,\psi_n^{(\alpha)b}\,, \qquad \sum_a \Delta p_T^a\,\psi_n^{(\alpha)a}\,\psi_n^{(\beta)a} = \delta_{\alpha\beta}\,. \tag{16}$$