Spin at Low x

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work done with Yuri Kovchegov and Daniel Pitonyak

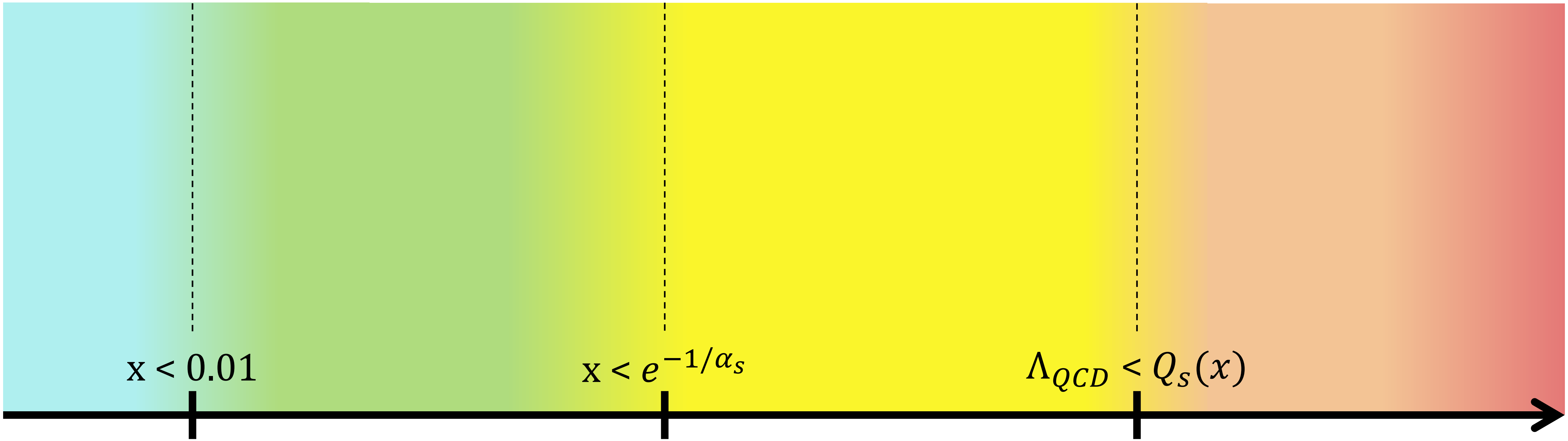
Initial Stages 2019

Fri. June 28, 2019

The Significance of “Low x”

3 Meanings of “Low x”

Low-x Kinematics **Linear Evolution** **Nonlinear Evolution**

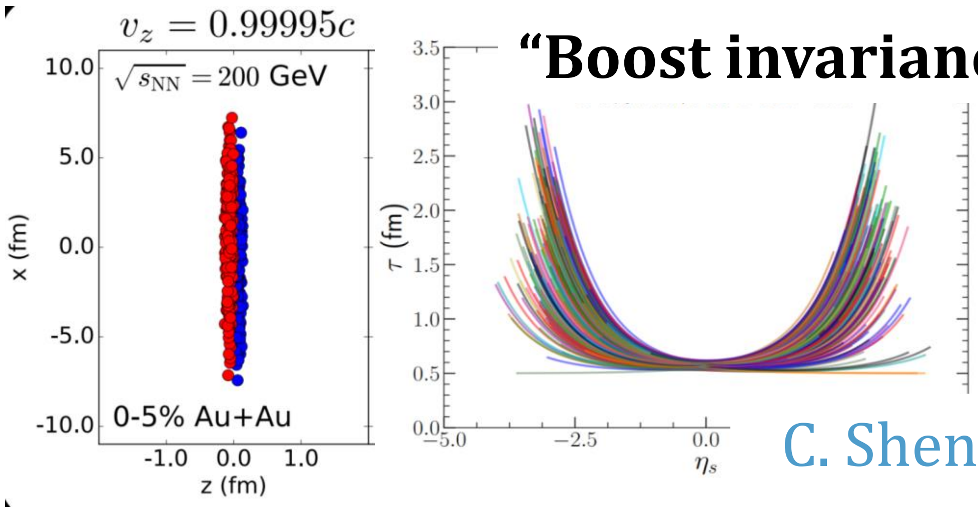


Low-x Kinematics

- Lightlike: $p^2 \approx 0$
- Soft gluons
- $\mu_B \approx 0$

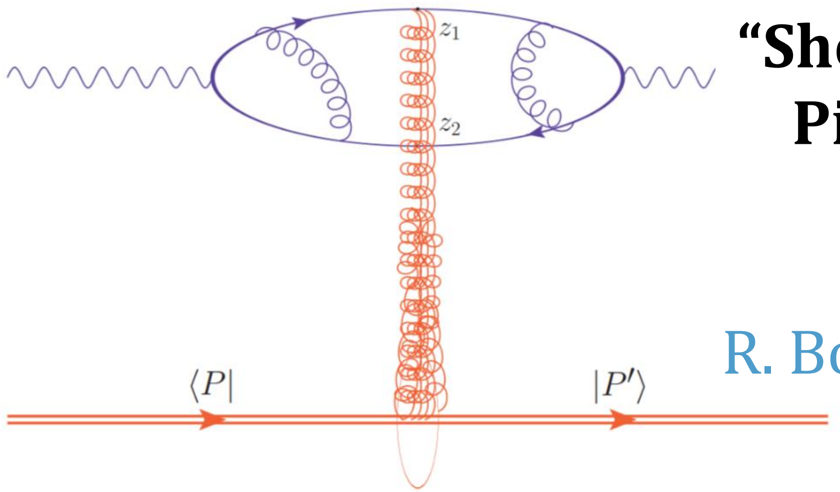
x < 0.01

x < $e^{-1/}$



“Boost invariance”

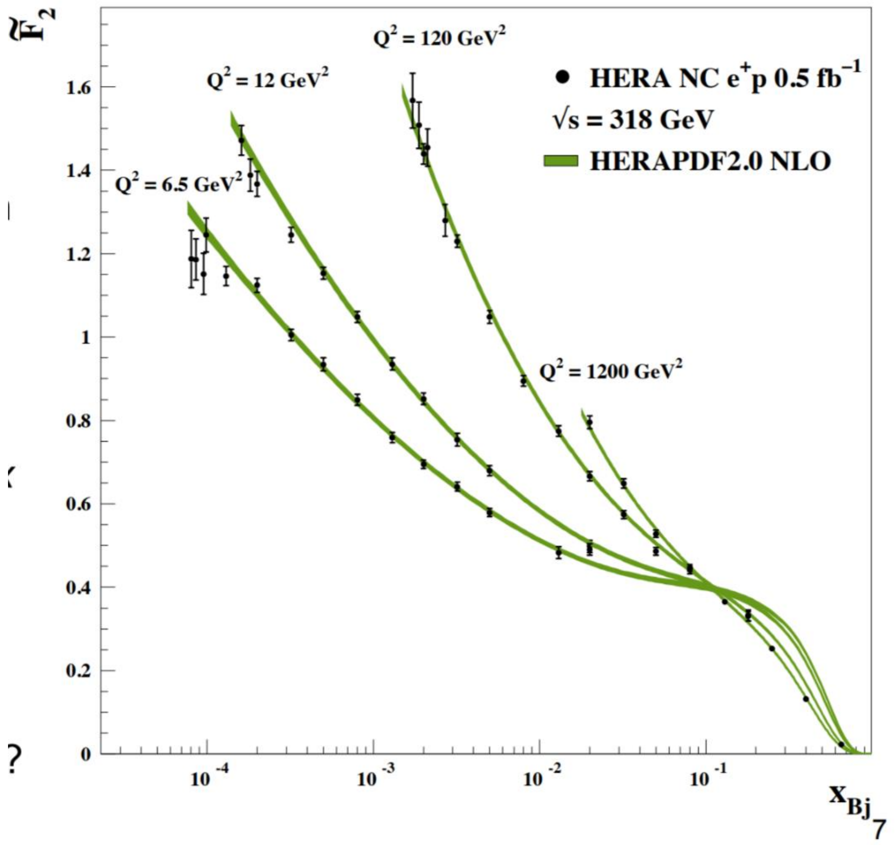
C. Shen talk



“Shockwave Picture”

R. Boussarie talk

“Growth of F_2”



M. Wing talk

Linear Evolution

- Soft gluon cascade
- Growth of F_2 (HERA)
- BFKL equation

e^{-1/α_s}

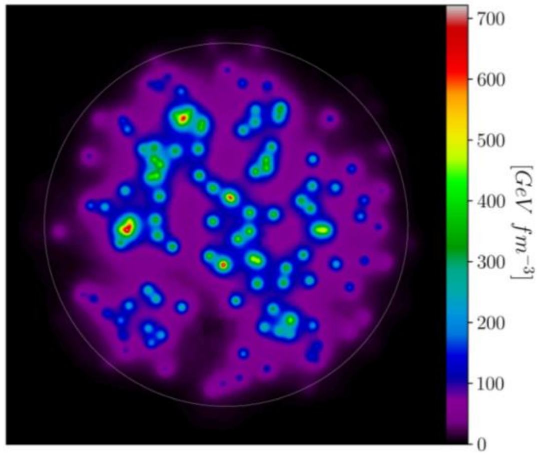
$\Lambda_{QCD} < Q_s(x)$

“Leading Log Resummation”

$$L = \log \frac{1}{x}$$

$$\begin{aligned} & a_0 \\ &+ \alpha_s [a_1 L + b_1] \\ &+ \alpha_s^2 [a_2 L^2 + b_2 L + c_2] \\ &+ \alpha_s^3 [a_3 L^3 + b_3 L^2 + c_3 L + d_3] \\ &+ \alpha_s^4 [a_4 L^4 + b_4 L^3 + c_4 L^2 + d_4 L + e_4] \\ &+ \dots \end{aligned}$$

M. Bonvini talk

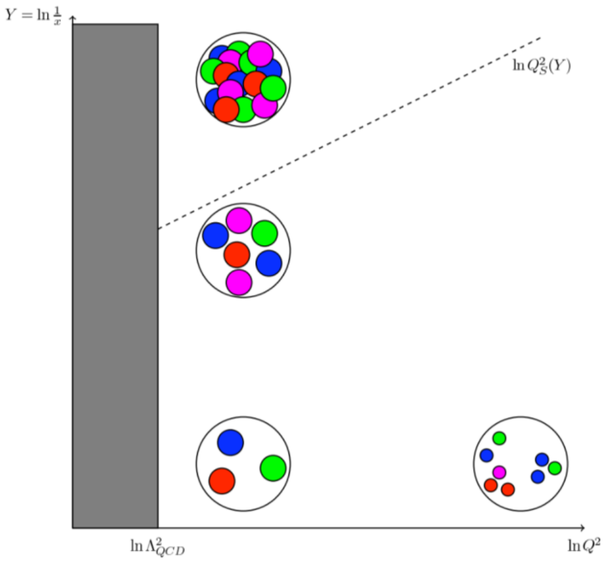


G. Giacalone talk

“Classical Gluon Fields”

“Yang-Mills”

“Color-Glass Condensate”



T. Altinoluk talk

Nonlinear Evolution

- BK / JIMWLK eqns.
- Multiple scattering
- Gluon saturation

$\Lambda_{QCD} < Q_s(x)$

3 Meanings of “Low x”

Low-x Kinematics

- Lightlike: $p^2 \approx 0$
- Soft gluon emission
- $\mu_B \approx 0$

Linear Evolution

- Soft gluon cascade
- Growth of F_2 (HERA)
- BFKL equation

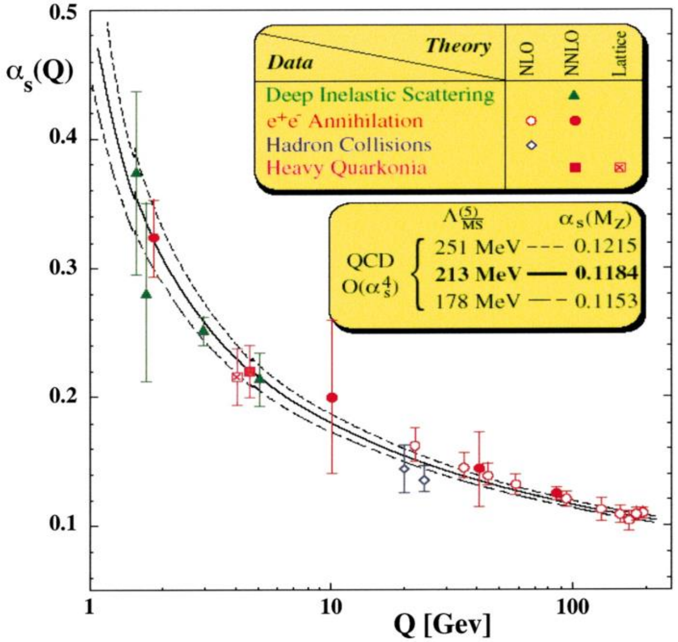
Nonlinear Evolution

- BK / JIMWLK eqns.
- Multiple scattering
- Gluon saturation

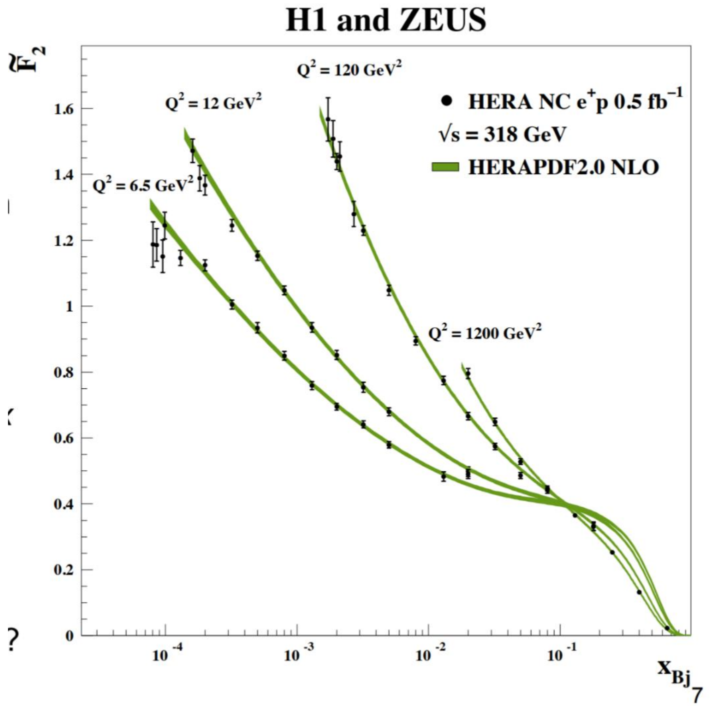
x < 0.01   x < e^{-1/α_s} $\Lambda_{QCD} < Q_s(x)$

Low-x Saturation: Bigger than QCD

Asymptotic Freedom



Density Explosion



Saturation + Unitarization

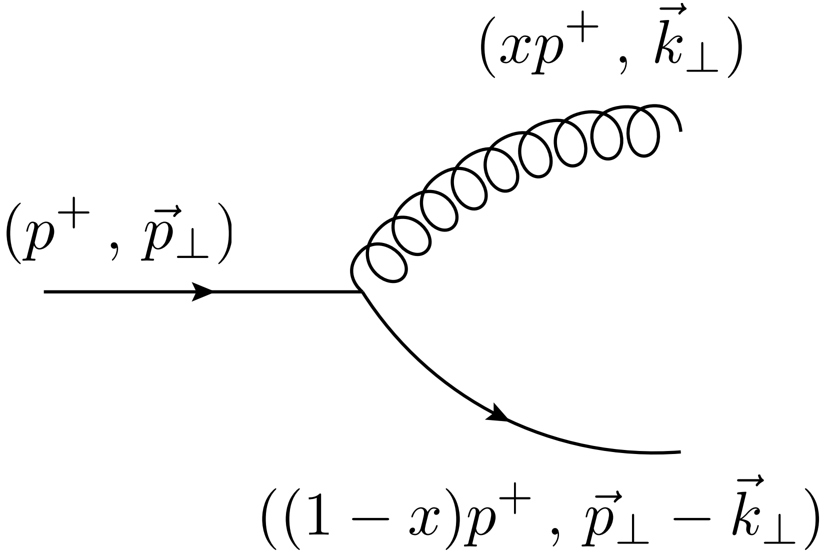
- *"Saturation is a non-negotiable consequence of QCD."* *- N. Armesto*
- A **fundamental consistency test** of UV completeness in the S.M.

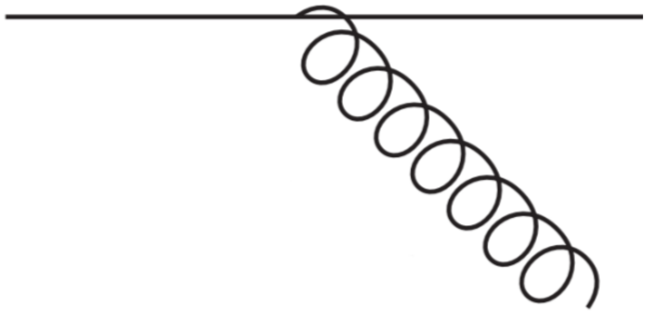
What Causes
the Small-x Explosion?

Phase Space for Soft Radiation

- Elementary splitting wave functions:

$x \ll 1$

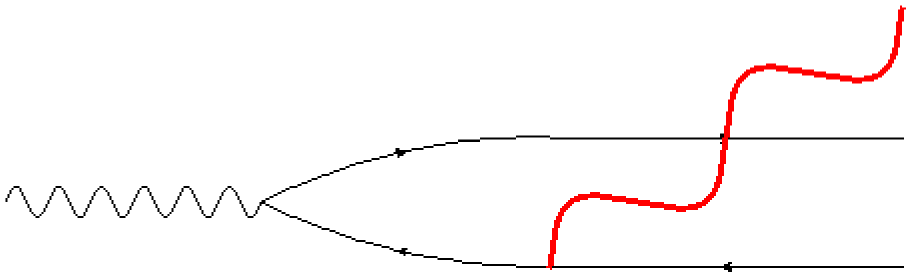




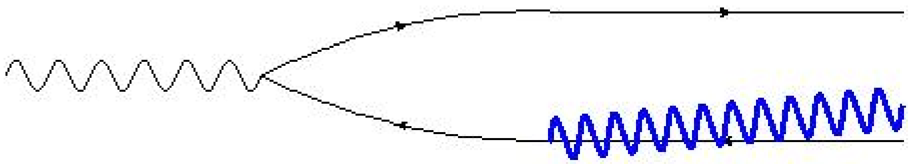
- Logarithmic phase space enhancement:
  - **Soft gluons $x \ll 1$**
  - **Transverse momentum** both in **UV** and **IR**

$$dP \sim \alpha_s \frac{dx}{x} \frac{d^2k}{k_T^2}$$

Screening of the Transverse Phase Space



- (Color) charge **neutrality**:
  - ➢ **Long wavelength** radiation is screened
  - ➢ Cancellation of quark + antiquark radiation

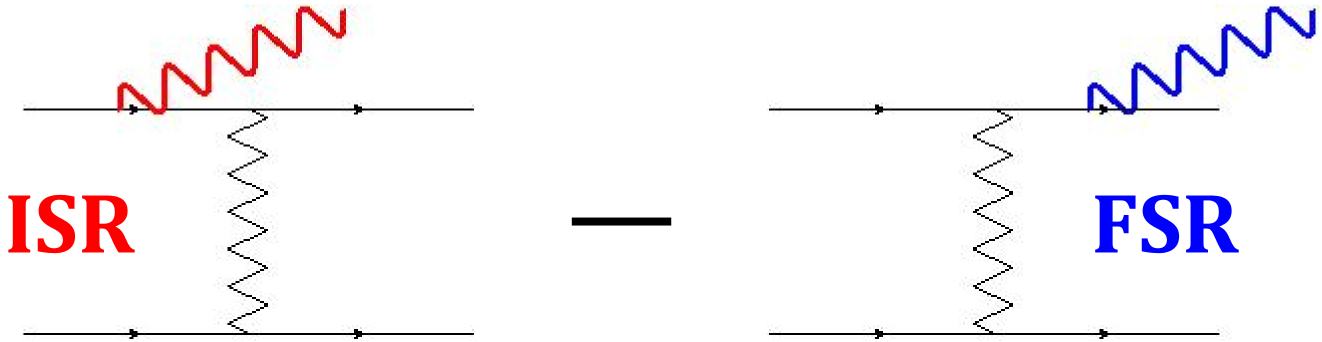


- Color **transparency**:
  - ➢ **Short wavelength** radiation doesn't interact
  - ➢ Unitarity: real / virtual cancellations

$$dP \sim \alpha_s \frac{dx}{x} \cancel{\frac{d^2k}{k_T^2}}$$

- Transverse logarithms are screened
  - ➢ No preferred transverse ordering

Small-x Enhancement: A Non-Abelian Phenomenon

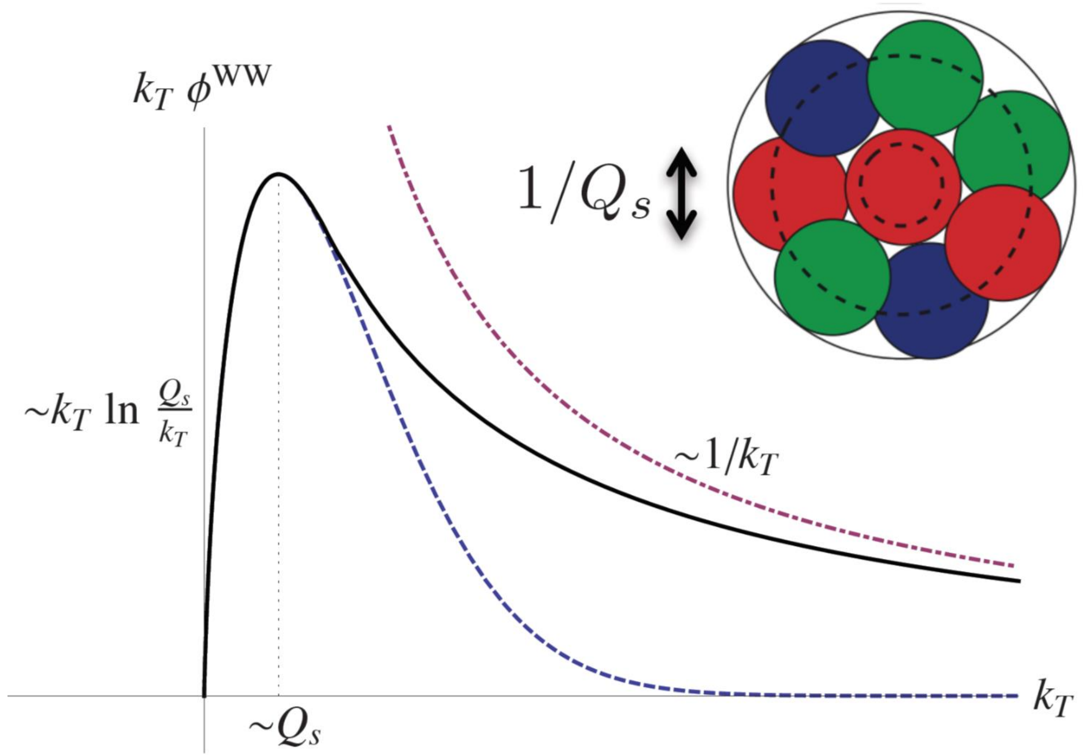


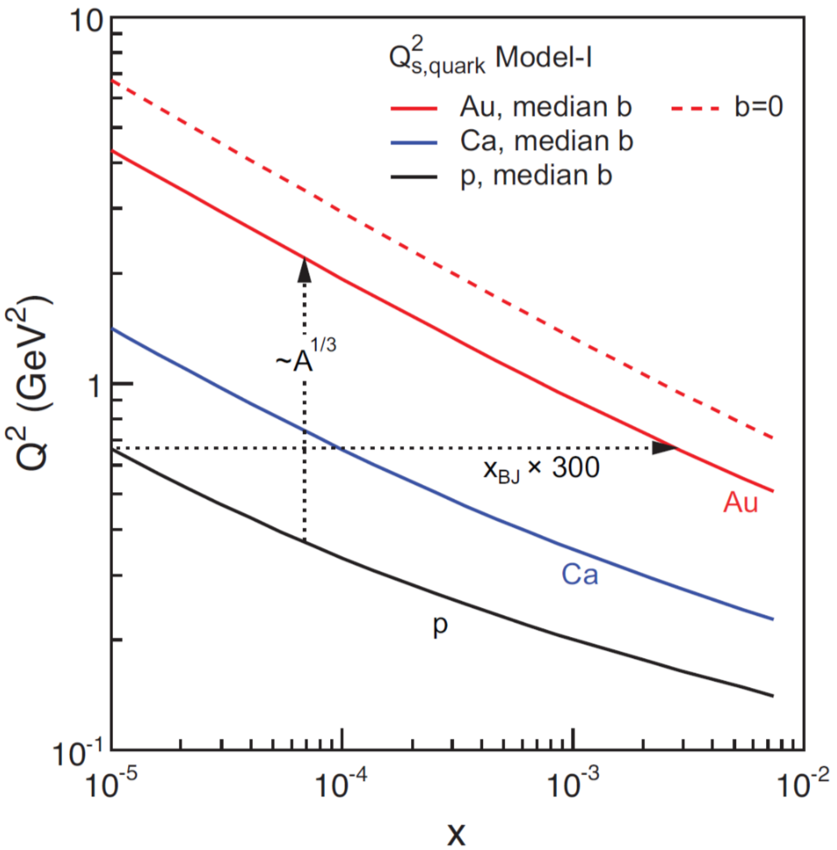
- The total cross section arises from an **interference** of **initial-** and **final-state** radiation

- They interfere with **perfect phase coherence** in eikonal kinematics
  - **Cancellation** of radiation for Abelian theories (QED)
  - **Commutator** for non-Abelian theories (QCD)

$$d\sigma^{\mathrm{rad}} \sim \alpha_s \frac{dx}{x}$$

- The high-energy, small-x limit is **uniquely sensitive** to **non-Abelian** dynamics

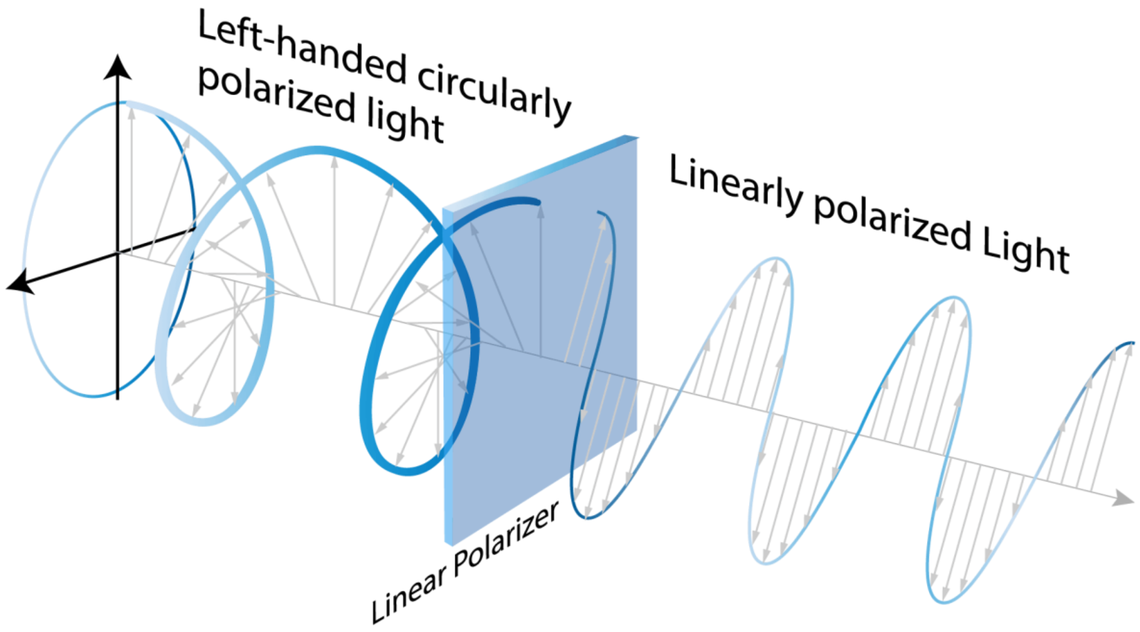


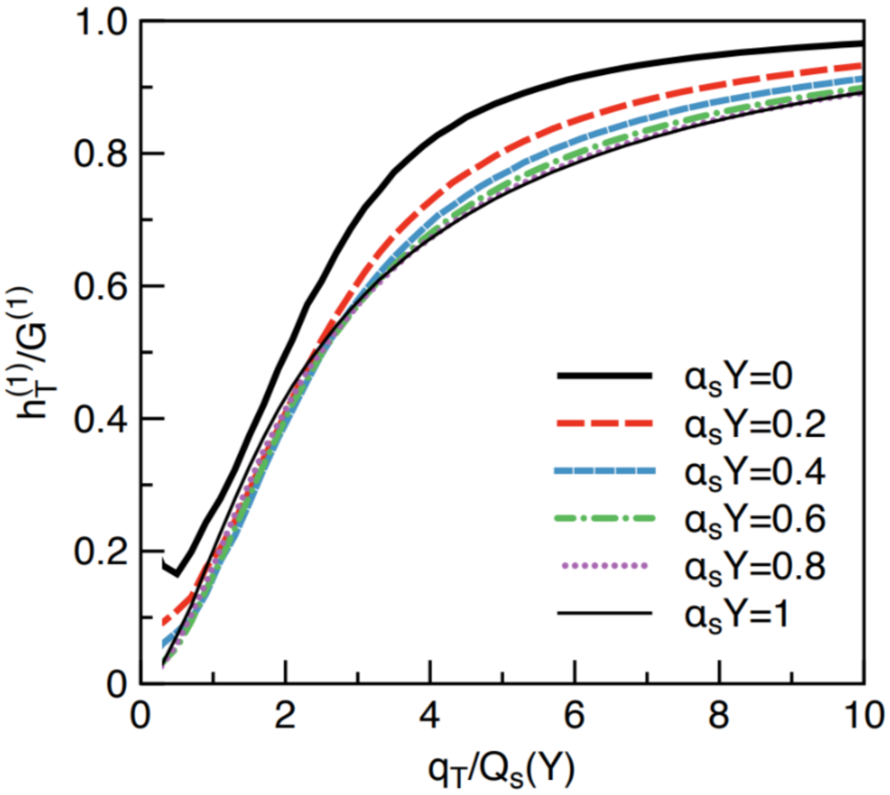


- Dynamical **growth of charge density** drives QCD into the **nonlinear regime**
- Leads to a **semi-hard scale Q_s** which **screens** the **nonperturbative IR behavior**

Looking for Spin at Small x

The Small-x Cascade is Highly Polarized….





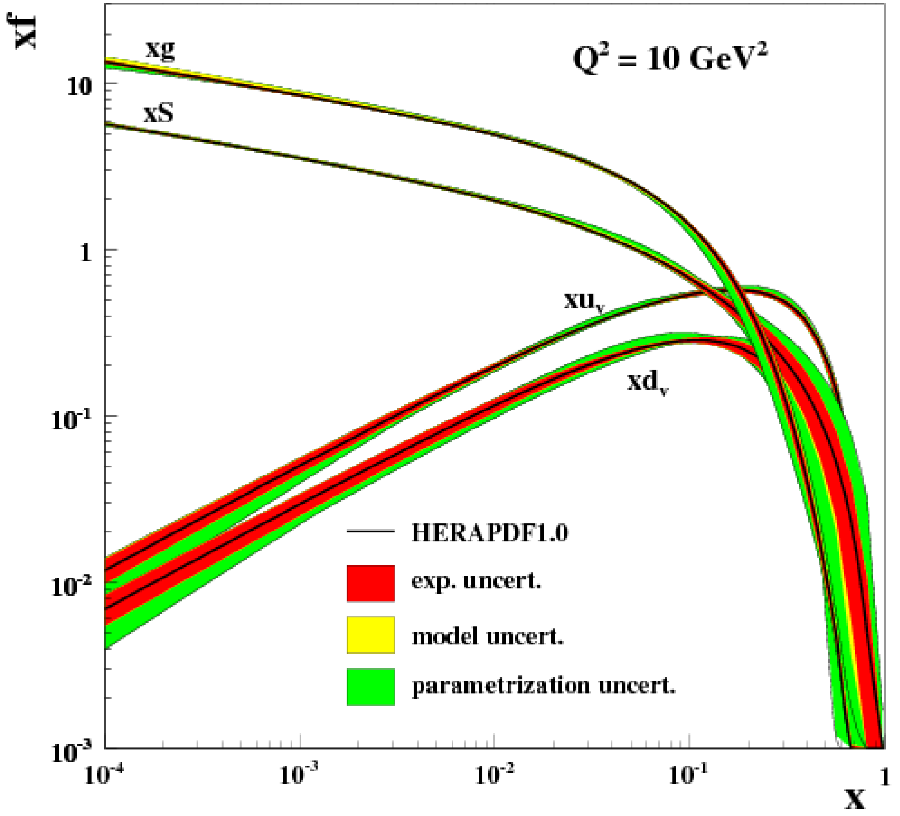
- The gluon cascade possesses a high / maximal degree of **linear polarization**

Metz & Zhou, Phys. Rev. **D84** (2011)

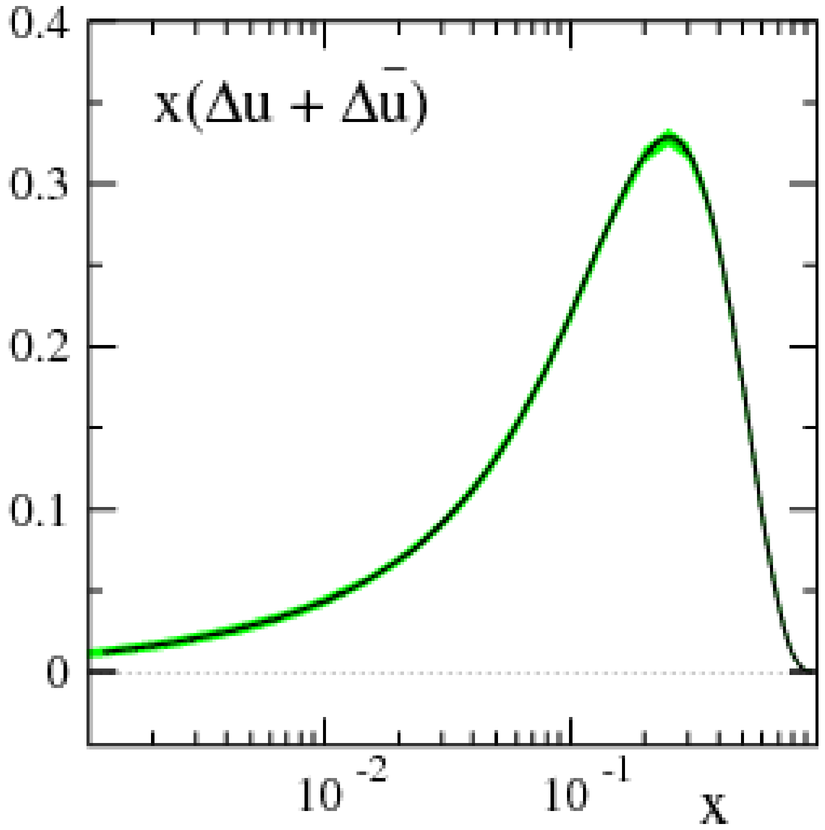
Dumitru, Lappi, & Skokov,
Phys. Rev. Lett. **115** (2015)

R. Boussarie talk

JHEP **1001** (2010) 109



De Florian et al., Phys. Rev. ***D80*** *(2009) 034030*



- While **unpolarized** parton densities **grow** at small x, **helicity** distributions **fall**

“Spin Stopping” is a Sub-Eikonal Effect

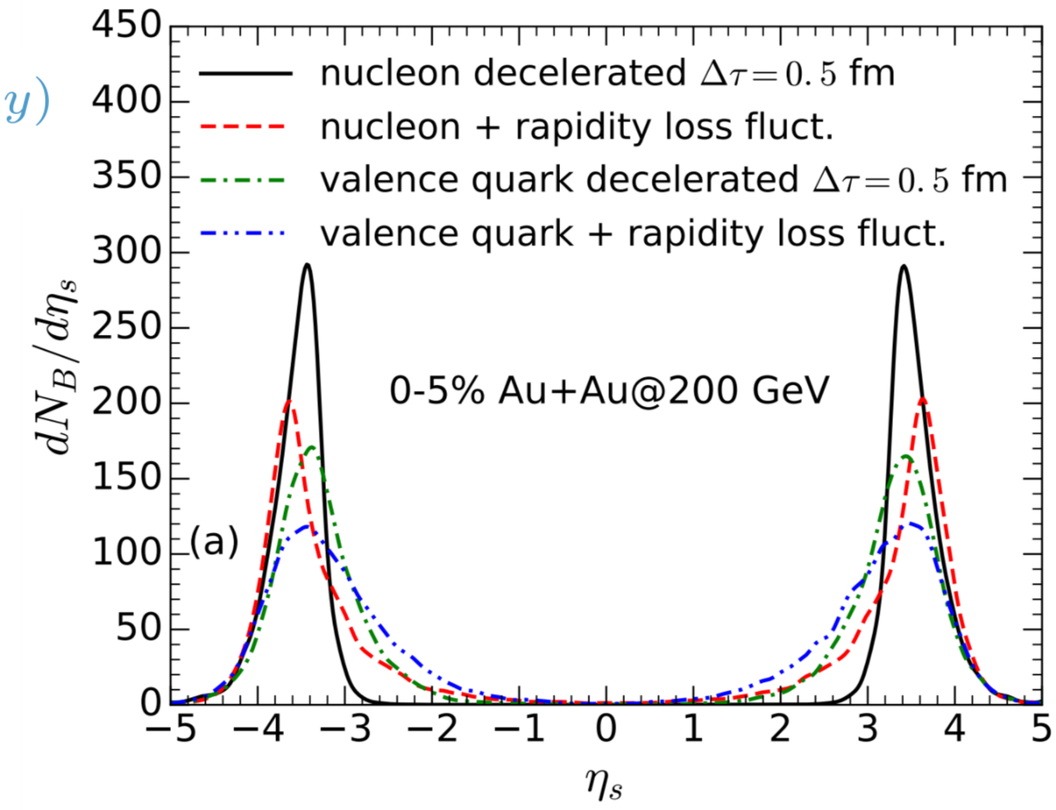
$$\frac{dN_B}{dy} \sim x \sim e^{-(Y_{beam}-y)} \qquad \frac{dN_S}{dy} \sim x \sim e^{-(Y_{beam}-y)}$$

- Just like **baryon stopping**, spin transfer is **power suppressed** at small x.

- Other sub-eikonal corrections:

  1. other components of the target background field $A^\mu_a(x)$
  2. dynamics of the target : x^- dependence of $A^\mu_a(x)$
  3. Finite width L^+ of the target along x^+

T. Altinoluk talk



Shen & Schenke, Phys. Rev. **C97** (2018)

C. Shen talk

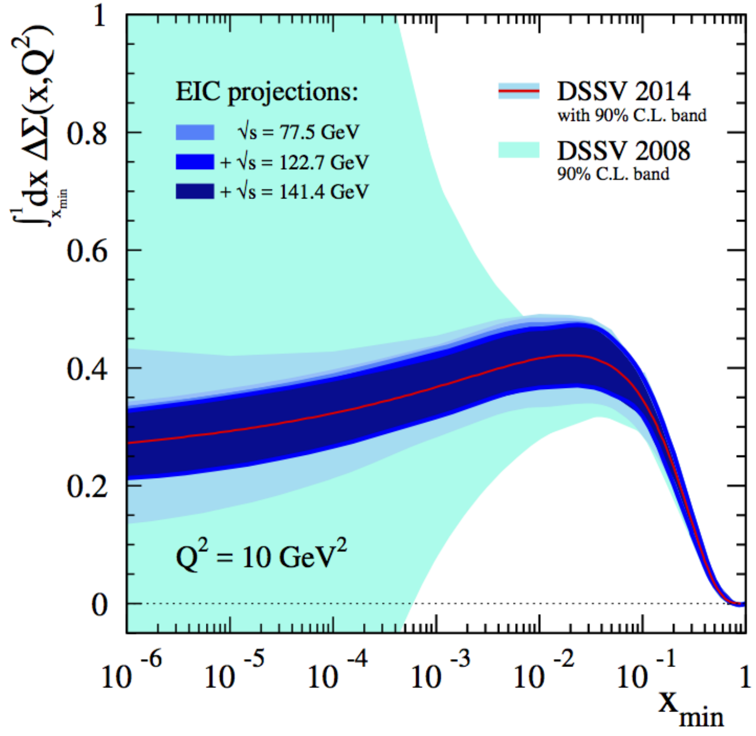
The Need for Small x in the Proton Spin Budget

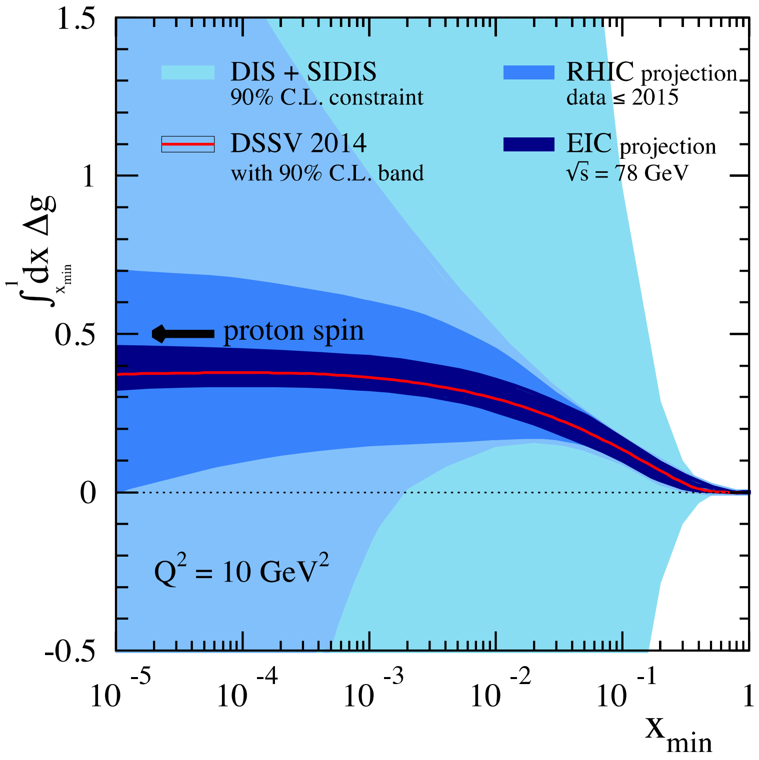
E.-C. Aschenauer et al., Phys. Rev. ***D92*** *(2015) 094030*

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_G$$

$$\Delta\Sigma(Q^2) = \sum_{f=q,\bar{q}} \int_0^1 dx\, \Delta q_f(x, Q^2)$$

$$\Delta G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$





- The **proton spin budget** requires integrating spin contributions down to **x = 0**
  - ➢ Will always require **extrapolation** beyond **finite-x** data

Quark and Gluon Helicity Distributions

- **Quarks:**

$$\Delta q(x,Q^2) = \int \frac{dr^-}{2\pi} e^{ixp^+r^-} \left\langle pS_L \middle| \bar{\psi}(0)\, \mathcal{U}[0,r]\, \frac{\gamma^+\gamma^5}{2}\, \psi(r) \middle| pS_L \right\rangle$$

  - ➢ **Massless** quarks always **conserve helicity**

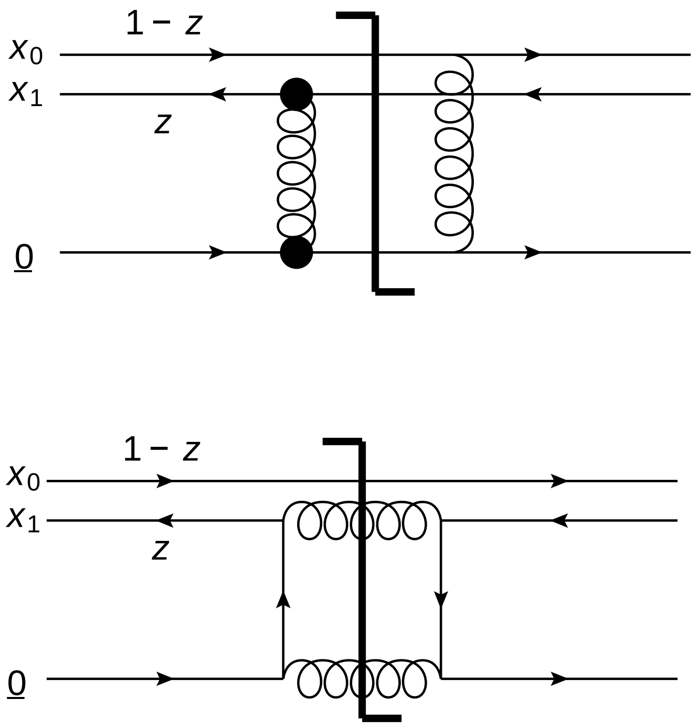
- **Gluons:**

$$\Delta G(x,Q^2) = \frac{-2i}{xp^+} \int \frac{dr^-}{2\pi} e^{ixp^+r^-} \left\langle pS_L \middle| \epsilon_T^{ij}\, \mathrm{tr}\left[F^{+i}(0)\, \mathcal{U}[0,r]\, F^{+j}(r)\, \mathcal{U}'[r,0]\right] \middle| pS_L \right\rangle$$

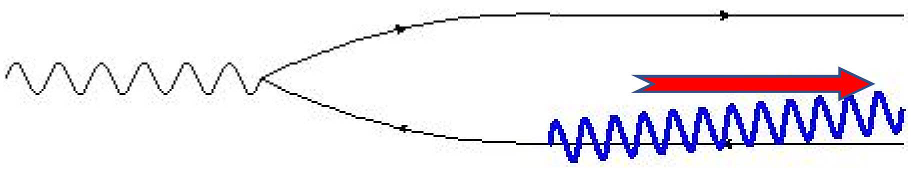
  - ➢ **Circular** polarization requires **azimuthal correlations**

- As a **power-suppressed effect**, spin transfer challenges the **whole small-x framework**
  - A **measurable subset** of all sub-eikonal corrections
  - **Quark exchange** now competes with sub-eikonal gluon exchange
  - Effects which only enter the **unpolarized sector at NLO** are **leading effects** for spin

Tree-Level Spin Transfer



The Crucial Difference: UV Dominance



- **Spin-dependent** scattering **discriminates** between **short-wavelength** fluctuations
  - ➢ Color transparency doesn't guarantee cancellation with spin degrees of freedom

➢ The transverse phase space is **logarithmic** in the UV for spin at small x

$$dP \sim \alpha_s \frac{dx}{x} \frac{d^2 k}{k_T^2}$$

➢ Helicity evolution is **double-logarithmic** at small x

$$\alpha_s \ln^2 \frac{1}{x} \sim \mathcal{O}(1)$$

Implications: The Complexity of Polarized Evolution

- **Double logarithmic** evolution is very sensitive to the UV structure
  - Dominated by **linear** evolution ("Polarized BFKL")
  - Strict **transverse** ordering
  - Elements which only appear at **NLO** in the **unpolarized** sector

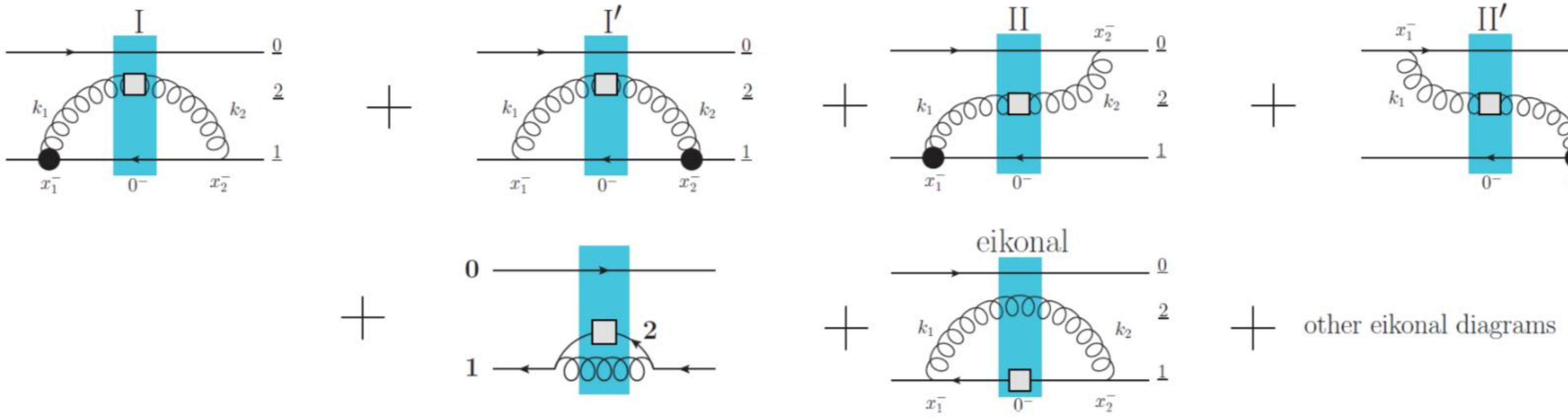
$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int d^2 \boldsymbol{z} \, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \left(S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\right)$$

$$\left\{1 + \bar{\alpha}_s \left[\bar{b} \ln(\boldsymbol{x}-\boldsymbol{y})^2 \mu^2 - \bar{b} \, \frac{(\boldsymbol{x}-\boldsymbol{z})^2 - (\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2} \ln \frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{y}-\boldsymbol{z})^2} + \frac{67}{36} - \frac{\pi^2}{12} \boxed{- \frac{1}{2} \ln \frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2} \ln \frac{(\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}}\right]\right\}$$

B. Ducloue talk

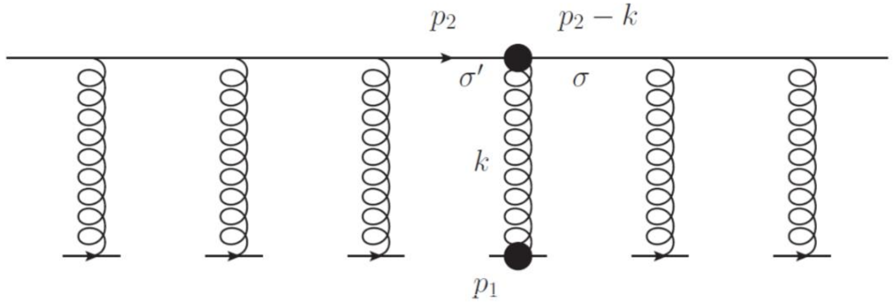
Some Calculations on the Market

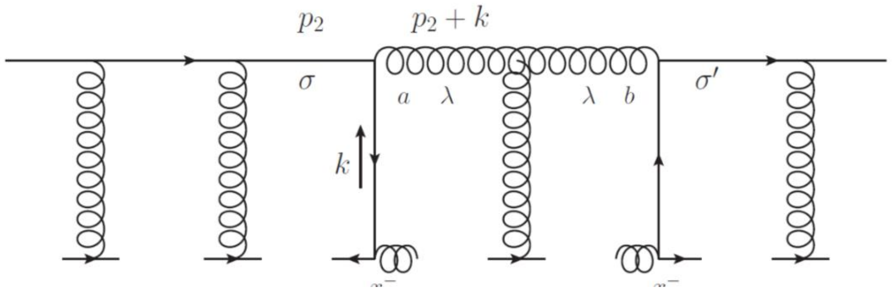
Method: Evolution of Polarized Dipoles



M.S., Kovchegov, & Pitonyak:

JHEP ***1601*** *(2016),*
Phys. Rev. Lett. ***118*** *(2017),*
Phys. Rev. ***D95*** *(2017),*
Phys. Lett. ***B772*** *(2017),*
JHEP ***1710*** *(2017),*
Phys. Rev. ***D99*** *(2019)*

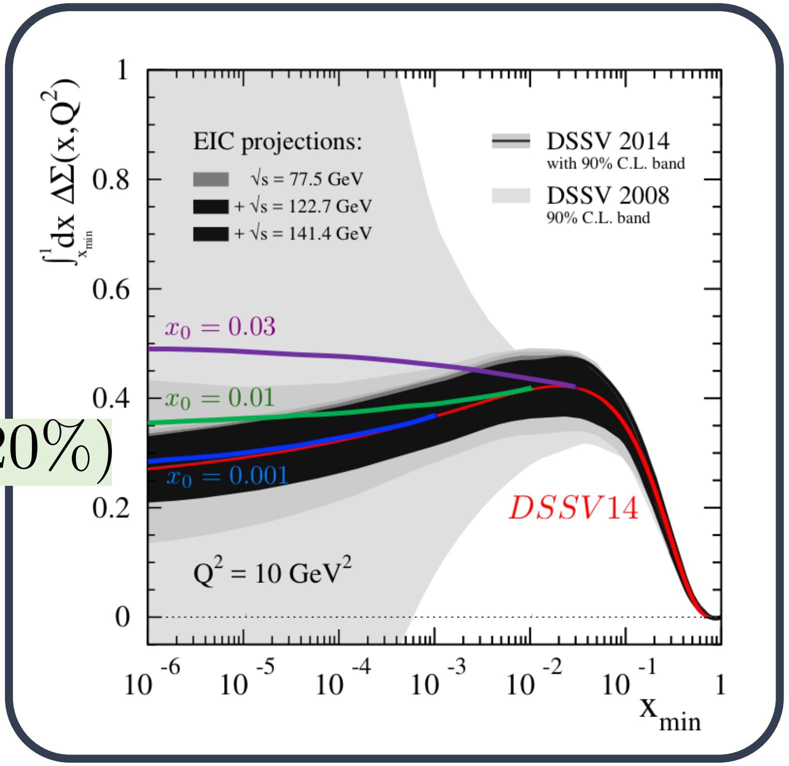




- “Polarized Wilson lines”
- Background field method / rapidity factorization
- Light-cone gauge $A^+ = 0$

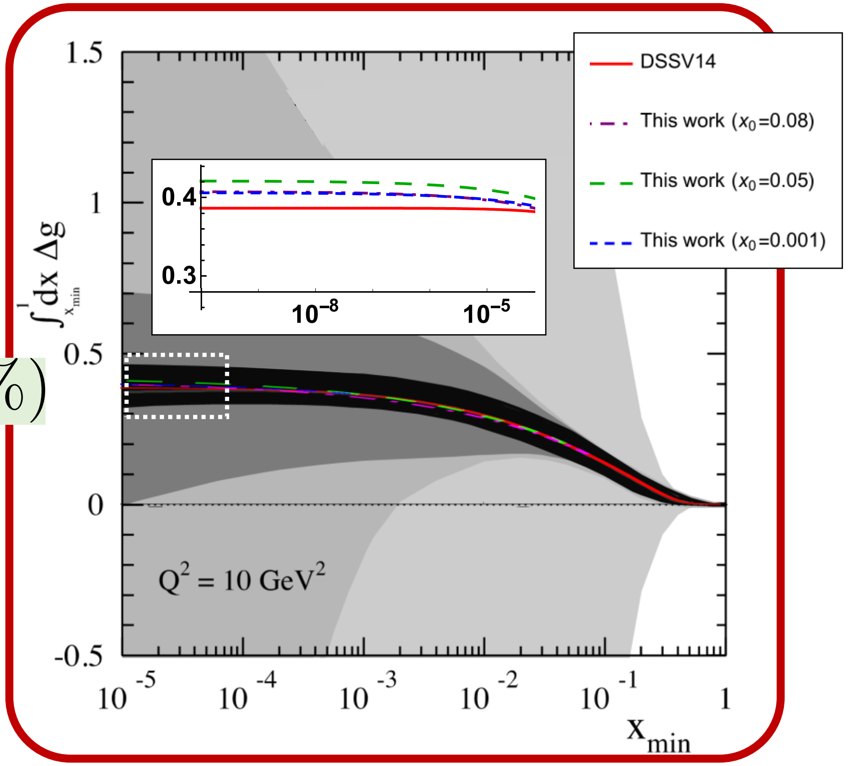
Results: Evolution of Polarized Dipoles

Quarks



$\mathcal{O}(20\%)$

Gluons



$\mathcal{O}(5\%)$

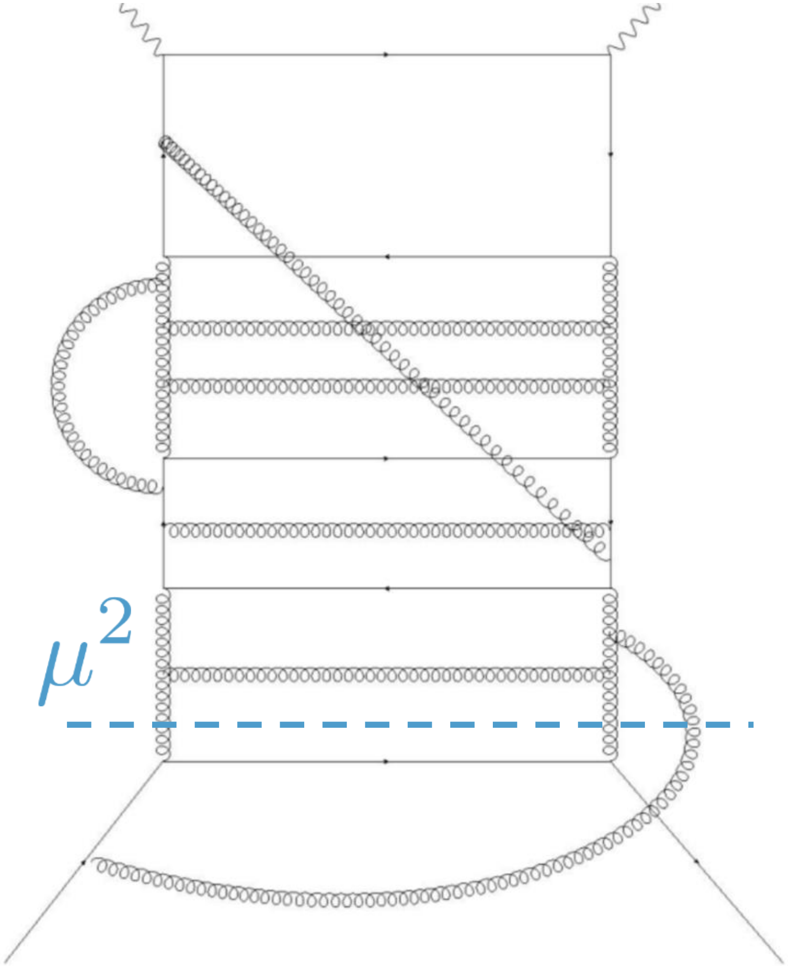
*Y. Kovchegov, JHEP **1903** (2019)*

$$L_q = -\Delta\Sigma$$

$$L_G = +\Delta G$$

- **Significant** but **finite** enhancement for **quark** helicity
- **Much smaller** effect for **gluons**, because they can become azimuthally **decorrelated**

Bartels, Ermolaev, & Ryskin,
Z. Phys. ***C70*** *(1996), Z. Phys.* ***72*** *(1996)*



- Based on identifying where the softest momentum scale lives in a complex diagram

- Goal is to resum **all** double logarithms

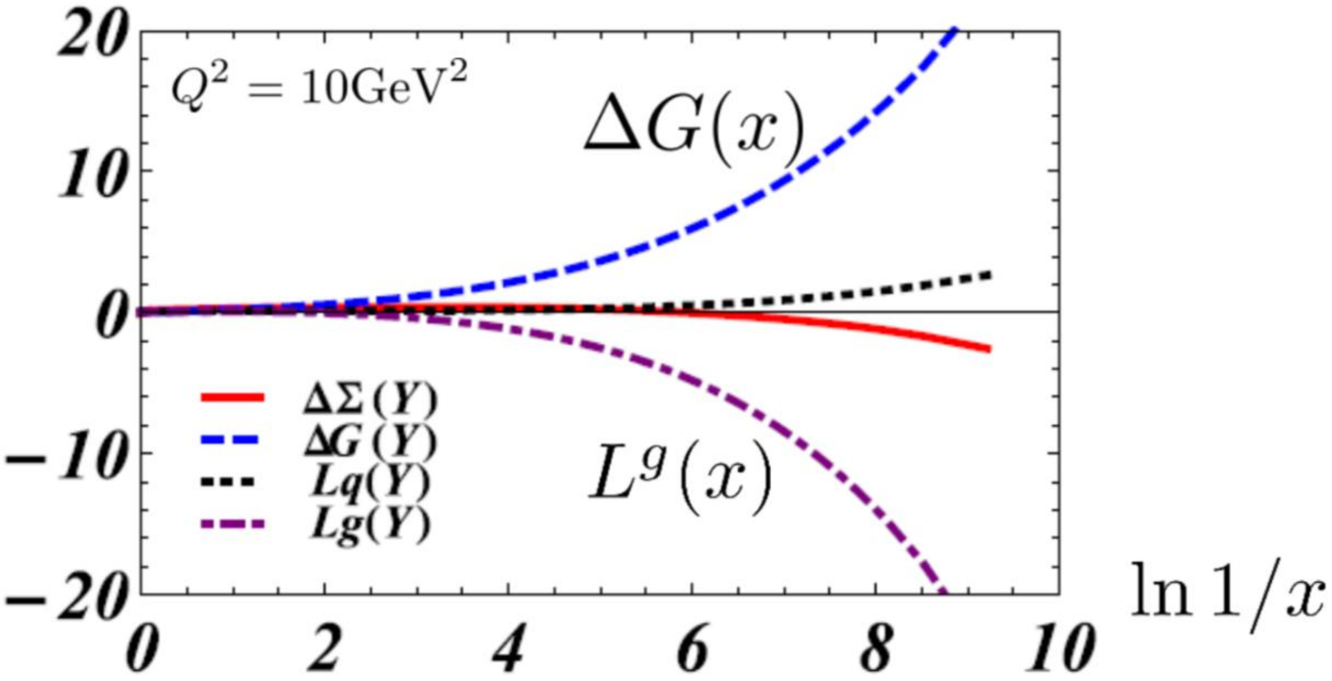
$$\alpha_s \ln^2 \frac{1}{x} \sim \mathcal{O}(1) \qquad \alpha_s \ln \frac{1}{x} \ln \frac{Q^2}{\mu^2} \sim \mathcal{O}(1)$$

- Close relation to DGLAP

- Feynman gauge

Hatta & Yao, arXiv:1906.07744
Boussarie, Hatta, & Yuan, arXiv:1904.02693
Hatta & Yang, Phys. Lett. ***B781*** *(2018)*
Hatta et al., Phys. Rev. ***D95*** *(2017)*

Results: Infrared Evolution Equations



- Individual contributions are separately **divergent** at small x
  - Possible role for nonlinear **saturation** effects to regulate individual divergences?
  - Delicate **cancellation** among competing terms
  - "The solution of the spin puzzle is not at small x" in this picture

A Growing Field

- Very **difficult to directly compare** the "polarized Wilson line" and "infrared evolution equation" methods…
  - But **very important to resolve** the full picture.

- **Other recent progress:**
  - Background field propagators — *G. Chirilli, JHEP **1901** (2019)*
  - Small-x gluon Sivers function — *Yao, Hagiwara, & Hatta, Phys. Lett. **B790** (2019)*
  - Spin-dependent odderon — *J. Zhou, Phys. Rev. **D89** (2014)*
  - Rapidity evolution framework — *Balitsky & Tarasov, JHEP **1510** (2015)*

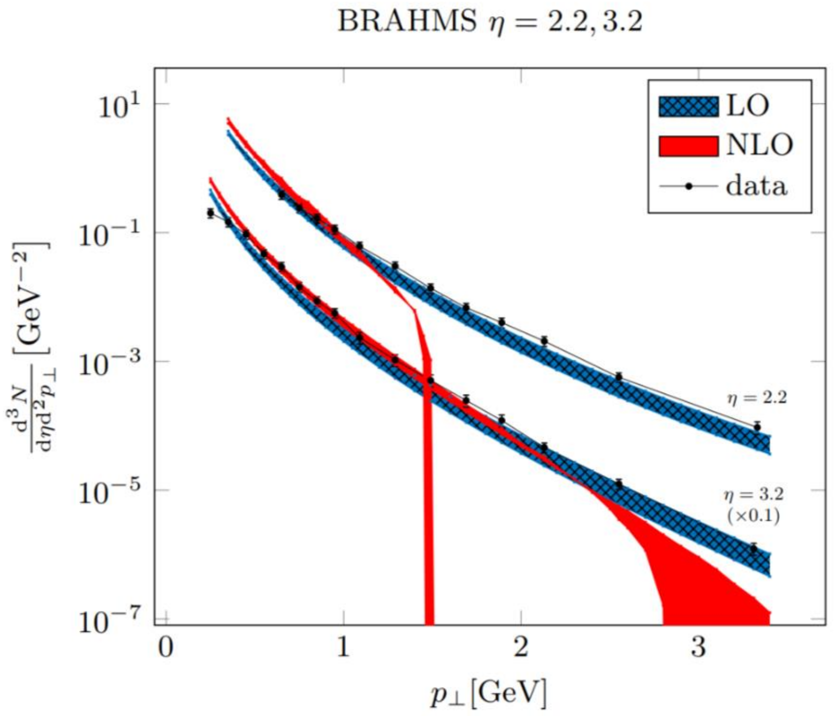
  … and more …

A Take-Home Message:
"Why Should I Care?"

For the **Small-x** Community: We Should Care About **Spin**

- There are many approaches to small x which all agree at **leading order** and **leading power**.
  - ➢ **Crucial differences** appear beyond this simple starting point
  - ➢ These differences challenge us to **broaden** and question the **small-x paradigm**
- **Spin at small x** touches on all of these issues
  - ➢ Concrete, observable **subset** of the corrections

The challenges of NLO and sub-eikonal corrections

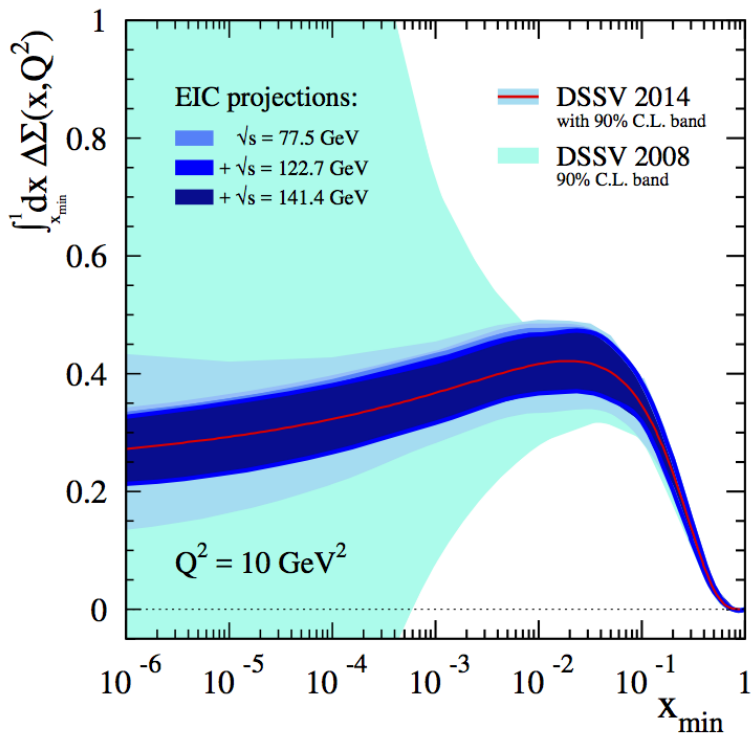


T. Altinoluk & B. Ducloue talks

For the **EIC** Community: We Should Care About **Small x**

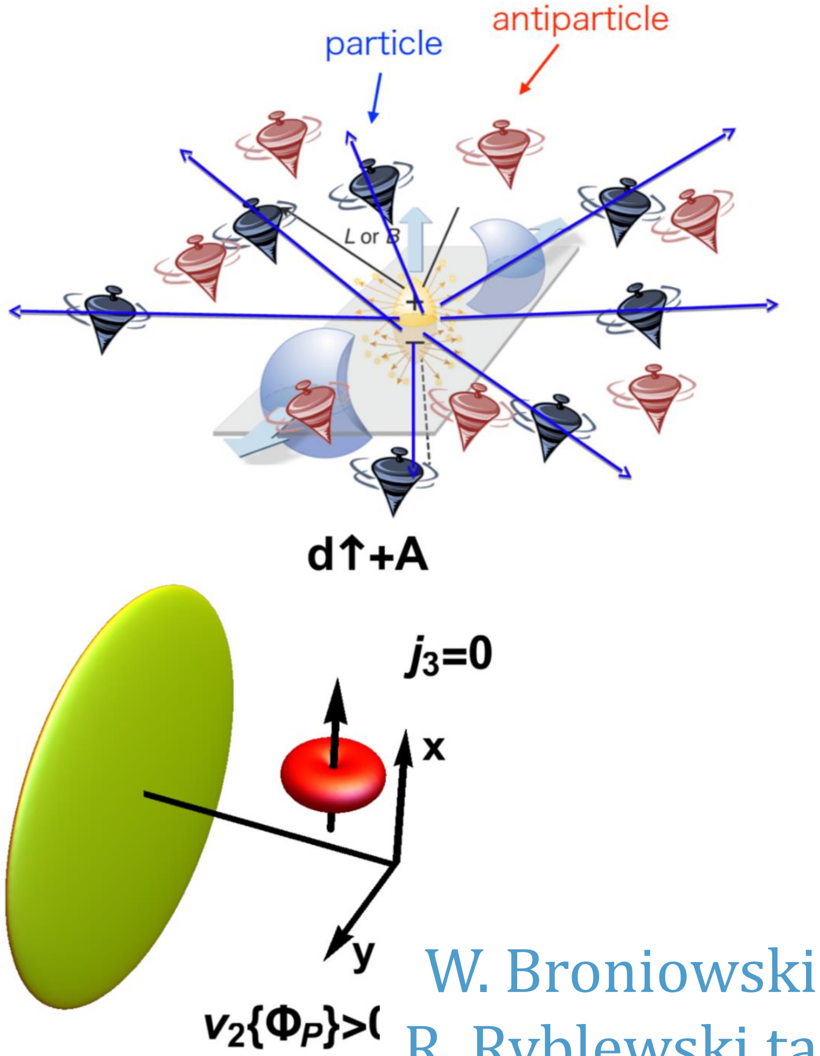
- **Global properties of the proton** are constrained by QCD sum rules
  - Always require information from the **unmeasured small-x regime**
  - Not just for the **proton spin puzzle**, but to connect any **QCD sum rule** to measured **structure functions**
- **Must** be taken into account when formulating the **design needs** for the **EIC**.

Proton structure requires constraints from small x

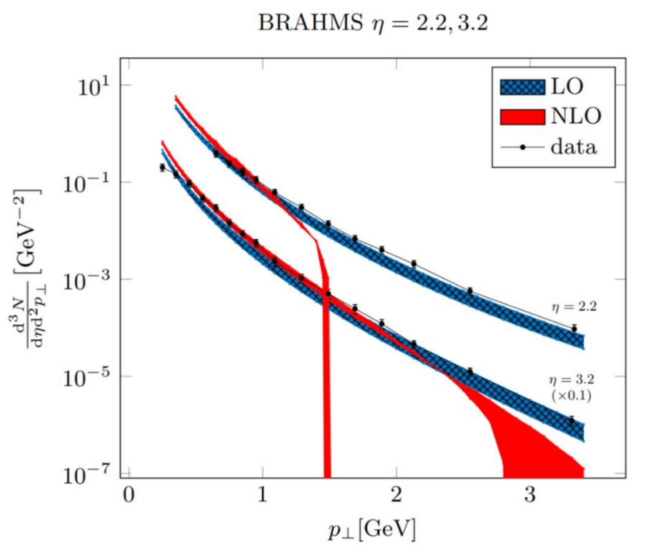


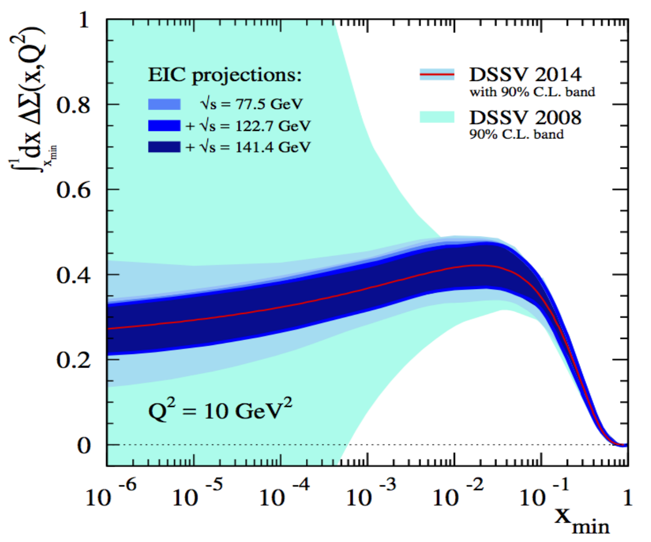
For the **Heavy Ion** Community: We Should Care About **Both**

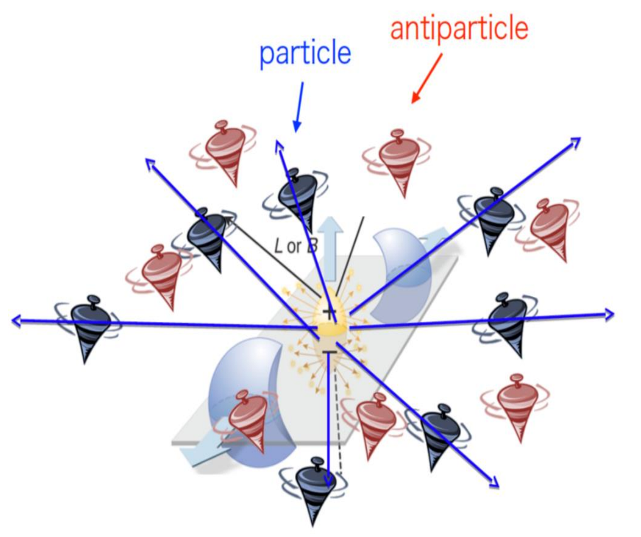
- Interest in **new probes**, **new levers** to study the quark-gluon plasma
  - Orbital angular momentum conversion to spin of **polarized hyperons**
  - **Polarized light ions** as geometry control
- **Small-x** kinematics and hadron structure determine the **initial conditions**
  - At **lower** $\sqrt{s}$, sub-eikonal corrections (**spin, baryon stopping**) become more important



W. Broniowski & R. Ryblewski talks


BRAHMS $\eta = 2.2, 3.2$
LO
NLO
data
$\eta = 2.2$
$\eta = 3.2$ $(\times 0.1)$
$\frac{d^3N}{d\eta d^2 p_\perp}$ [GeV^{-2}]
$p_\perp$ [GeV]


EIC projections:
$\sqrt{s}$ = 77.5 GeV
+ $\sqrt{s}$ = 122.7 GeV
+ $\sqrt{s}$ = 141.4 GeV
DSSV 2014
with 90% C.L. band
DSSV 2008
90% C.L. band
Q^2 = 10 GeV^2
$\int_{x_{min}}^1 dx\, \Delta\Sigma(x,Q^2)$
x_{min}


particle
antiparticle
L or B

Spin at Low x