pA vs. eA Universality

from the perspective of small-x physics (CGC)

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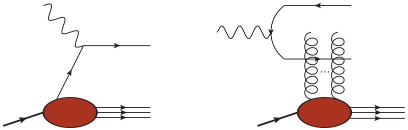
Dual Descriptions of Deep Inelastic Scattering at small-x



Bjorken frame Dipole frame

Bjorken frame

$$F_2(x, Q^2) = \sum_q e_q^2 x \left[f_q(x, Q^2) + f_{\bar{q}}(x, Q^2) \right].$$

Dipole frame [Mueller, 01]

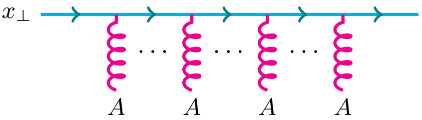
$$F_2(x, Q^2) = \sum_f \frac{e_f^2 Q^2 S_\perp}{4\pi^2 \alpha_{\text{em}}} \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r_\perp \left| \psi\left(z, r_\perp, Q\right) \right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right) \right]$$

- Bjorken: the partonic picture of a hadron is manifest. Saturation shows up as a limit on the occupation number of quarks and gluons.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for Dipole amplitude. Geometrical scaling: [Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]

Wilson Lines in Color Glass Condensate Formalism

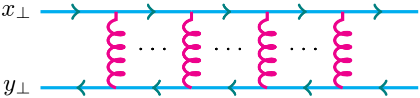
We use Wilson line to represent the multiple scattering between the fast moving quark and target background gluon fields.

$$U(x_\perp) = \mathcal{P} \exp\left(-ig \int dz^+ A^-(x_\perp, z^+)\right)$$



The Wilson loop (color dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$



- Dipole amplitude $S^{(2)}$ then produces the quark k_T spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2 k_\perp} = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle .$$

- How about Quadrupole $\frac{1}{N_c}\left\langle \mathrm{Tr} U U^\dagger U U^\dagger\right\rangle \neq \frac{1}{N_c}\left\langle \mathrm{Tr} U U^\dagger\right\rangle \frac{1}{N_c}\left\langle \mathrm{Tr} U U^\dagger\right\rangle$?

A Tale of Two Gluon Distributions

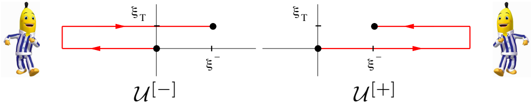
Two gauge invariant gluon definitions: [R. Boussarie, Tuesday] [Bomhof, Mulders and Pijlman, 06]; [Dominguez, Marquet, Xiao and Yuan, 11]

I. Weizsäcker Williams gluon distribution: conventional gluon distributions

$$xG_{\rm WW}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions: not probability density

$$xG_{\rm DP}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



- Modified Universality for Gluon Distributions: $\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

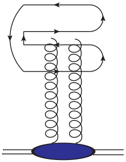
	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
$xG_{\rm WW}$	$\times$	$\times$	$\surd$	$\times$	$\surd$
$xG_{\rm DP}$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

- Gauge links can generates different p_T behaviors. Also see [Gelis and Tanji, 19].
- In large N_c limit and dilute-dense scatterings, generalized universality for small-x gluon.
- Complementary physics missions in measurements in pA and eA collisions.

A Tale of Two Gluon Distributions

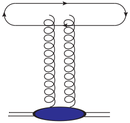
I. Weizsäcker Williams gluon distribution

$$xG_{\rm WW}(x,k_\perp) = \frac{2N_c}{\alpha_S}\int\frac{d^2R_\perp}{(2\pi)^2}\frac{d^2R'_\perp}{(2\pi)^2}e^{iq_\perp\cdot(R_\perp-R'_\perp)}$$
$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[i\partial_i U(R_\perp)\right]U^\dagger(R'_\perp)\left[i\partial_i U(R'_\perp)\right]U^\dagger(R_\perp)\right\rangle$$



II. Color Dipole gluon distribution:

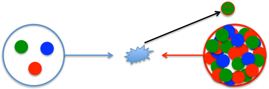
$$xG_{\rm DP}(x,k_\perp) = \frac{2N_c}{\alpha_s}\int\frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4}e^{iq_\perp\cdot\left(R_\perp-R'_\perp\right)}$$
$$\left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right)\frac{1}{N_c}\left\langle\mathrm{Tr}\left[U\left(R_\perp\right)U^\dagger\left(R'_\perp\right)\right]\right\rangle_x ,$$



- Quadrupole ⇒ Weizsäcker Williams gluon; Dipole ⇒ Color Dipole gluon.
- Analytical results in MV model exhibit different p_T behavior.
- In eA and pA collisions, small-x processes are probing either DP or QP or both at leading N_c approximation. More detail see [R. Boussarie, Tuesday].
- Generalized universality in large N_c in eA and pA collisions for Wilson lines [F. Dominguez, C. Marquet, A. Stasto and BX, 12]

Forward hadron production in pA collisions

Forward hadron in pA [Dumitru, Jalilian-Marian, 02] [T. Altinoluk, Monday]

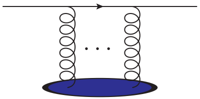


projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

Dilute-dense factorization at forward rapidity

$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p)\mathcal{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z)\right].$$



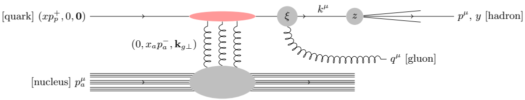
$$\Rightarrow \qquad U(x_\perp) = \mathcal{P}\exp\left\{ig_S\int_{-\infty}^{+\infty} \mathrm{d}x^+\, T^c A_c^-(x^+, x_\perp)\right\},$$

$$\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp).$$

- Dense gluons at low-x in the nucleus target is described by $\mathcal{F}(k_\perp)$ and $\tilde{\mathcal{F}}(k_\perp)$.
- The universality at higher order. Only need dipole amplitudes at leading N_c.

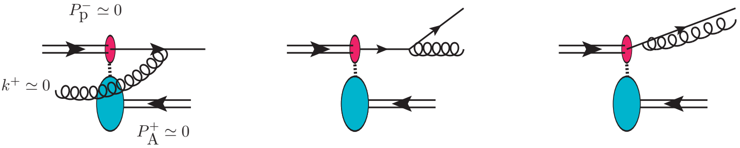
NLO hadron productions in pA collisions: An Odyssey

Dilute-Dense factorizations: large x proton or γ^* $\rightarrow$ as dilute probe:



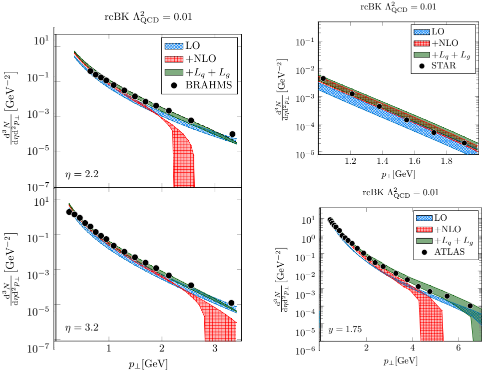
$$x_p = \frac{k_\perp}{\sqrt{s}} e^{+y} \sim 1 \quad \text{dilute}$$
$$x_A = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1 \quad \text{dense}$$

- LO [Dumitru, Jalilian-Marian, 02]: probing $xG_{\rm DP}(x, k_\perp)$ at small-x.
- NLO Cutoff[Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11]
- NLO Complete NLO in DR: [Chirilli, BX and Yuan, 12].
  1. 1. soft, collinear to the target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$. Subtraction scheme is not Unique but highly constrained.
  2. 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs. $\overline{MS}$ scheme.
  3. 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs, $\overline{MS}$ scheme.
  4. The importance of subtraction: systematic resummation of large logarithms. $(\alpha_s \ln 1/x_g)$, which allows us to have $\mathcal{H} \sim \mathcal{O}(\alpha_s)$. Interesting recent development: RG approach and threshold resummation.



NLO hadron productions in pA collisions: An Odyssey

SOLO (Saturation physics at One Loop Order) results [Stasto, Xiao, Zaslavsky, 13; Watanabe, Xiao, Yuan, Zaslavsky, 15]



- Agree with RHIC and LHC data in low $p_\perp \leq Q_s$ region where pQCD does not apply.
- SOLO (1.0 and 2.0) break down in the large $p_\perp \geq Q_s$ region($k_\perp \gg Q_s$).
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14; Kang, Vitev and Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller and Triantafyllopoulos, 16]
- Threshold resummation (Sudakov type)! Approach kinematical threshold at high $k_\perp$. ($\bar{\alpha}_s \ln(1 - x_p) < 0$). [Xiao, Yuan, 18; work in process]

Threshold resummation in the saturation formalism

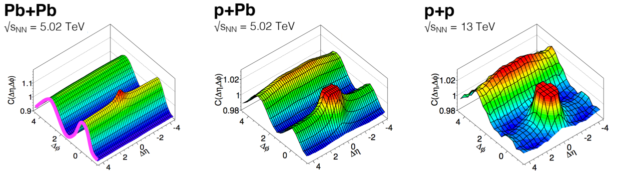
[Xiao, Yuan, 18; numerical work in process]

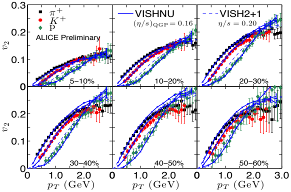
- The objective is to identify large logarithms $\ln(1-x_p)$ and $\ln k_\perp^2/Q_s^2$ in the large $k_\perp$ region ($k_\perp \gg Q_s$) near threshold at fixed rapidity. In fact, these two types of logs seem to always appear together in our calculation and soft-collinear effective theory (SCET) in almost identical pattern.
- Many different threshold resummation formalism. We find remarkable similarities between the threshold resummation in pA collisions in the small-x formalism and threshold resummation in SCET[Becher, Neubert, 06].
- The forward threshold jet function $\Delta(\mu^2, \mu_b^2, z)$ satisfies an almost identical RGE equation. The solution helps to resum threshold logs.

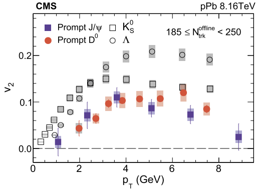
$$
\begin{aligned}
\frac{d\Delta(\mu^2, \mu_b^2, z)}{d\ln\mu} &= -\frac{2\alpha_s N_c}{\pi}\left[\ln z + \beta_0\right]\Delta(\mu^2, \mu_b^2, z) \\
&\quad + \frac{2\alpha_s N_c}{\pi}\int_0^z dz' \frac{\Delta(\mu^2, \mu_b^2, z) - \Delta(\mu^2, \mu_b^2, z')}{z - z'},
\end{aligned}
$$

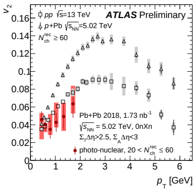
Solution: $$\Delta(\mu^2, \mu_b^2, z = \ln\frac{x}{\tau}) = \frac{e^{(\beta_0 - \gamma_E)\gamma_{\mu,b_\perp}}}{\Gamma[\gamma_{\mu,b_\perp}]} z^{\gamma_{\mu,b_\perp}-1}.$$

Collectivity (correlation) is everywhere!



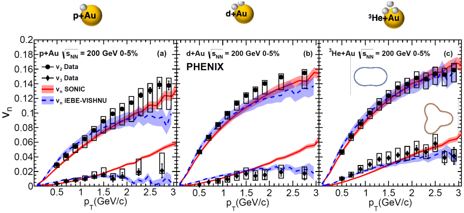




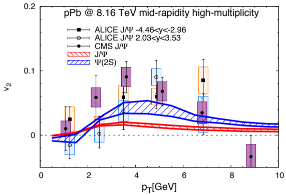


- Collectivity is used to describe the particle correlation. It is observed in both large and small systems and for light and heavy hadrons!
- [Song, Shen, Heinz, 11];[Shen, Heinz, Huovinen, Song, 11]
- New exciting results for $b \to \mu$ and $c \to \mu$ as well as UPC in PbPb collisions. [J. Jia, Monday; K. Gajdosova; B. Seidlitz, Wednesday]

Where does the collectivity come from? Final state vs Initial state.



[PHENIX, 19]

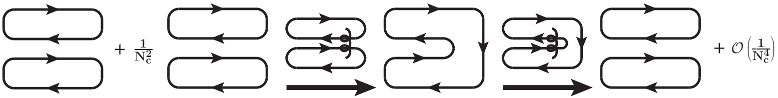


[Du and Rapp, 18]

- Geometry + final state evolution (Hydro + freeze-out) explain data well.
- On the other hand, CGC alone has trouble to describe the PHENIX data.
- Final state effect may not be the full story. For example, v_2 of heavy mesons. [Du, Rapp, 18]
- Experimental review on heavy flavor in small systems: [Z. Chen, Thursday]
- Mechanism to generate correlations in CGC. [Lappi, 15; Dumitru, McLerran, Skokov, 15; Lappi, Schenke, Schlichting, Venugopalan, 16; Dusling, Mace, Venugopalan, 17; Fukushima, Hidaka, 17; Mace, Skokov, Tribedy, Venugopalan, 18; Davy, Marquet, Shi, Xiao, Zhang, 18]

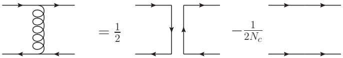
Correlations in CGC

Correlations between uncorrelated incoming quarks (gluons) are generated due to quadrupole as N_c corrections. [Lappi, 15; Lappi, Schenke, Schlichting, Venugopalan, 16; Dusling, Mace, Venugopalan, 17; Davy, Marquet, Shi, Xiao, Zhang, 18]



$$
\begin{aligned}
\frac{d^2N}{d^2k_{1\perp}d^2k_{2\perp}} &= \int d^2r_{1\perp}d^2r_{2\perp}e^{-ik_{1\perp}\cdot r_{1\perp}}e^{-ik_{2\perp}\cdot r_{2\perp}} \\
&\quad\times\frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V(x_1)V(x_2)^\dagger\right]\mathrm{tr}\left[V(x_3)V(x_4)^\dagger\right]\right\rangle \\
\text{where}\quad &\frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V(x_1)V(x_2)^\dagger\right]\mathrm{tr}\left[V(x_3)V(x_4)^\dagger\right]\right\rangle \neq \frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V(x_1)V(x_2)^\dagger\right]\right\rangle\left\langle \mathrm{tr}\left[V(x_3)V(x_4)^\dagger\right]\right\rangle \\
&= e^{-\frac{Q_s^2}{4}(r_1^2+r_2^2)}\left[1+\frac{(\frac{Q_s^2}{2}\boldsymbol{r}_1\cdot\boldsymbol{r}_2)^2}{N_c^2}\int_0^1 d\xi\int_0^\xi d\eta e^{\frac{\eta Q_s^2}{8}[(\boldsymbol{r}_1-\boldsymbol{r}_2)^2-4(\boldsymbol{b}_1-\boldsymbol{b}_2)^2]}\right]
\end{aligned}
$$

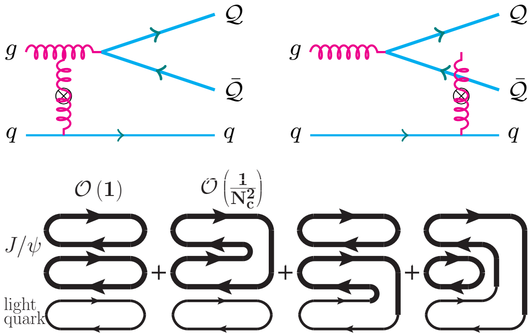
- At leading N_c, $\frac{d^2N}{d^2k_{1\perp}d^2k_{2\perp}}=\left(\frac{dN}{d^2k_{1\perp}}\right)\left(\frac{dN}{d^2k_{2\perp}}\right)$, there are no correlations.
- The correlations only come in as higher order N_c corrections as shown above. Fierz identity

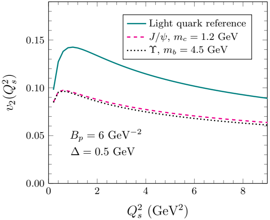


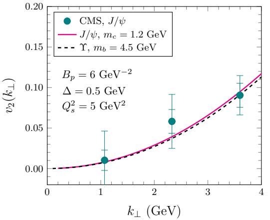
Elliptic flow of J/ψ in CGC

J/ψ productions together with a light quark reference

$$v_2[J/\psi] \equiv V_{2\Delta}(J/\psi, \mathrm{ref})/v_2[\mathrm{ref}]$$



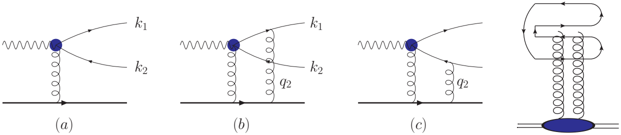




- [Zhang, Marquet, Qin, Wei, Xiao, 19] The same CGC model calculations with the additional $g \to \mathcal{Q}\bar{\mathcal{Q}}$ splitting to produce heavy quarkonia at small k_T.
- Predictions for Upsilon v_2. Preliminary results show that D and B mesons have similar v_2 at low k_T. Please stay tuned.

DIS dijet

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]



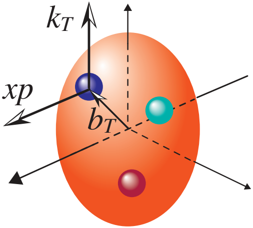
$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} \propto N_c \alpha_{em} e_q^2 \int \frac{\mathrm{d}^2x}{(2\pi)^2}\frac{\mathrm{d}^2x'}{(2\pi)^2}\frac{\mathrm{d}^2b}{(2\pi)^2}\frac{\mathrm{d}^2b'}{(2\pi)^2} e^{-ik_{1\perp}\cdot(x-x')}$$

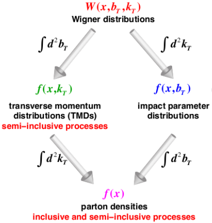
$$\times e^{-ik_{2\perp}\cdot(b-b')}\sum \psi_T^*(x-b)\psi_T(x'-b')$$

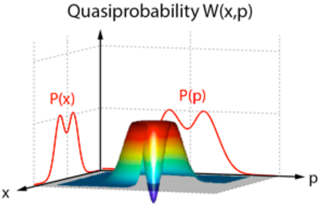
$$\underbrace{\left[1+S_{x_g}^{(4)}(x,b;b',x')-S_{x_g}^{(2)}(x,b)-S_{x_g}^{(2)}(b',x')\right]}_{-u_i u'_j \frac{1}{N_c}\langle \mathrm{Tr}\left[\partial^i U(v)\right]U^\dagger(v')\left[\partial^j U(v')\right]U^\dagger(v)\rangle_{x_g} \Rightarrow \text{Operator Def}},$$

- eA collision is golden channel for the Weizsäcker Williams gluon at small-x.
- Due to linearly polarized gluon distribution at small-x[Metz, Zhou, 11], there can be the analog of elliptic flow v_2 as well. [Dumitru, Lappi, Skokov, 15]

Can we measure Wigner distributions?







- Wigner distributions ingeniously encode all quantum information of how partons are distributed inside hadrons. [Ji, 03; Belitsky, Ji, Yuan, 03]
- Small-x gluon $\Leftrightarrow$ gluon Wigner distributions? [Ji, 03] [A. Dumitru, Tuesday]
- TMDs and GPDs can be studied and measured in various processes. In condense matter, Wigner distribution of photons can be measured.
- Can we measure the gluon Wigner distribution at small-x? Yes, we can!
- GTMD (Fourier transform of Wigner) [Meissner, Metz and Schlegel, 09]

The exact connection between dipole amplitude and Wigner distribution

[Hatta, Xiao, Yuan, 16] Def. of gluon Wigner distribution:

$$\begin{aligned} xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) &= \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \\ &\times \left\langle P + \frac{\Delta_\perp}{2} \middle| F^{+i}\left(\vec{b}_\perp + \frac{\xi}{2}\right) F^{+i}\left(\vec{b}_\perp - \frac{\xi}{2}\right) \middle| P - \frac{\Delta_\perp}{2} \right\rangle, \end{aligned}$$

Def. of GTMD [Meissner, Metz and Schlegel, 09]

$$xG(x,q_\perp,\Delta_\perp) \equiv \int d^2b_\perp e^{-i\Delta\cdot b_\perp} xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp).$$

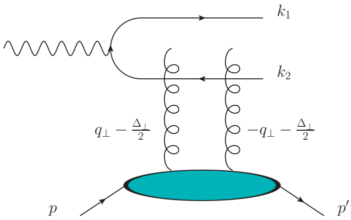
- With dipole like gauge link $[b_\perp = \frac{1}{2}(R_\perp + R'_\perp), r_\perp = R_\perp - R'_\perp] \Rightarrow$

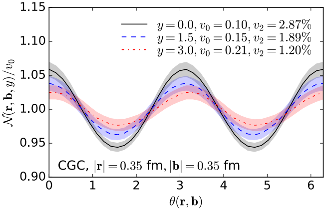
$$\begin{aligned} xG_{\mathrm{DP}}(x,q_\perp,\Delta_\perp) &= \frac{2N_c}{\alpha_s} \int \frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4} e^{iq_\perp\cdot(R_\perp - R'_\perp) + i\frac{\Delta_\perp}{2}\cdot(R_\perp + R'_\perp)} \\ &\times \left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right) \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U(R_\perp)\, U^\dagger(R'_\perp)\right]\right\rangle_x . \end{aligned}$$

- This provides the $3D$ quasiprobabilistic distribution $(x, b_\perp, k_\perp)$ of small-x gluon.

Probing 3D Tomography of Proton at small-x

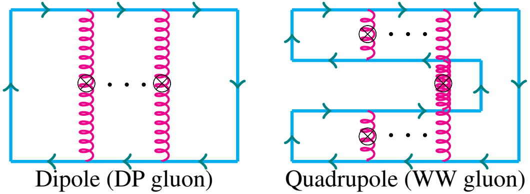
Diffractive back-to-back dijet productions in DIS [Altinoluk, Armesto, Beuf and Rezaeian, 15]; [Hatta, Xiao, Yuan, 16]; [Mantysaari, Mueller, and Schenke, 19]





- Measure final state proton recoil $\Delta_\perp$ as well as dijet momentum $k_{1\perp}$ and $k_{2\perp}$.
- We can approximately access $|xG_{\text{DP}}(x, q_\perp, \Delta_\perp)|^2$ in the back-to-back limit in which $q_\perp \simeq P_\perp \equiv \frac{1}{2}(k_{2\perp} - k_{1\perp}) \gg \Delta_\perp$.
- Cross-Sections are positive-definite, although Wigner distributions may not be.
- Dipole amplitude can have elliptic part $\mathcal{N}(r, b) = v_0\,[1 + 2v_2 \cos 2\theta(r, b)]$.
- WW Wigner (WWW) distribution can be also defined and measured.
- Gluon OAM [Ji, Yuan, Zhao, 16; Hatta, Nakagawa, Yuan, Zhao, 16, Bhatttacharya, Metz, Zhou, 17]; UPC at the LHC [Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 17]

Summary



- Complementary study of eA and pA collisions can provide us more important information on the universal building blocks of small-x physics: dipole and quadrupole.
- This universality has many applications in pA and eA collisional systems. (Dilute-dense) For example, probing of WW and Wigner distribution, etc.
- The same universality implies that initial state interactions (CGC) can generate sufficient amount of collectivity for heavy mesons.
- Maybe the elliptic flow of $\Upsilon(b\bar{b})$ or other heavy mesons (B) at the LHC can be a clear signal for CGC initial state effects.