pA vs. eA Universality
from the perspective of small-$x$ physics (CGC)

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Dual Descriptions of Deep Inelastic Scattering at small-$x$

Bjorken frame

$$F_2(x, Q^2) = \sum_q e_q^2 x \left[ f_q(x, Q^2) + f_{\bar{q}}(x, Q^2) \right].$$

Dipole frame [Mueller, 01]

$$F_2(x, Q^2) = \sum_f \frac{e_f^2 Q^2 S_\perp}{4\pi^2 \alpha_{\text{em}}} \int_0^1 dz \int d^2 r_\perp |\psi(z, r_\perp, Q)|^2 \left[ 1 - S^{(2)}(Q_s r_\perp) \right].$$

- **Bjorken**: the partonic picture of a hadron is manifest. Saturation shows up as a limit on the occupation number of quarks and gluons.
- **Dipole**: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for Dipole amplitude. Geometrical scaling: [Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]
Wilson Lines in Color Glass Condensate Formalism

We use Wilson line to represent the multiple scattering between the fast moving quark and target background gluon fields.

\[ U(x_\perp) = \mathcal{P} \exp \left( -ig \int dz^+ A^- (x_\perp, z^+) \right) \]

The Wilson loop (color dipole) in McLerran-Venugopalan (MV) model

\[ \frac{1}{N_c} \langle \text{Tr} U(x_\perp) U^\dagger(y_\perp) \rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}} \]

- Dipole amplitude \( S^{(2)} \) then produces the quark \( k_T \) spectrum via Fourier transform

\[ \mathcal{F}(k_\perp) \equiv \frac{dN}{d^2 k_\perp} = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} \frac{1}{N_c} \langle \text{Tr} U(x_\perp) U^\dagger(y_\perp) \rangle. \]

- How about Quadrupole \( \frac{1}{N_c} \langle \text{Tr} UU^\dagger UU^\dagger \rangle \neq \frac{1}{N_c} \langle \text{Tr} UU^\dagger \rangle \frac{1}{N_c} \langle \text{Tr} UU^\dagger \rangle ? \]
A Tale of Two Gluon Distributions

Two gauge invariant gluon definitions: [R. Boussarie, Tuesday] [Bomhof, Mulders and Pijdman, 06]; [Dominguez, Marquet, Xiao and Yuan, 11]

I. Weizsäcker Williams gluon distribution: conventional gluon distributions

\[ xG_{WW}(x, k_{\perp}) = 2 \int \frac{d\xi^- d\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_{\perp} \cdot \xi_{\perp}} \text{Tr}(P|F^{+i}(\xi^-, \xi_{\perp})U^{[+]\dagger}F^{+i}(0)U^{[+]}|P). \]

II. Color Dipole gluon distributions: not probability density

\[ xG_{DP}(x, k_{\perp}) = 2 \int \frac{d\xi^- d\xi_{\perp}}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_{\perp} \cdot \xi_{\perp}} \text{Tr}(P|F^{+i}(\xi^-, \xi_{\perp})U^{[-]\dagger}F^{+i}(0)U^{[+]}|P). \]

- **Modified Universality** for Gluon Distributions: \( \times \Rightarrow \) Do Not Appear. \( \sqrt{\ } \Rightarrow \) Appear.

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<th>( xG_{WW} )</th>
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- Gauge links can generate different \( p_T \) behaviors. Also see [Gelis and Tanji, 19].
- In large \( N_c \) limit and dilute-dense scatterings, generalized universality for small-x gluon.
- Complementary physics missions in measurements in pA and eA collisions.
A Tale of Two Gluon Distributions

I. Weizsäcker Williams gluon distribution

\[ xG_{WW}(x, k_{\perp}) = \frac{2N_c}{\alpha_s} \int \frac{d^2 R_{\perp} \, d^2 R'_{\perp}}{(2\pi)^2} \frac{e^{iq_{\perp} \cdot (R_{\perp} - R'_{\perp})}}{(2\pi)^2} \]

\[ \frac{1}{N_c} \left\langle \text{Tr} \left[ i\partial_{i} U(R_{\perp}) \right] U^{\dagger}(R'_{\perp}) \left[ i\partial_{i} U(R'_{\perp}) \right] U^{\dagger}(R_{\perp}) \right\rangle \]

II. Color Dipole gluon distribution:

\[ xG_{DP}(x, k_{\perp}) = \frac{2N_c}{\alpha_s} \int \frac{d^2 R_{\perp} \, d^2 R'_{\perp}}{(2\pi)^4} e^{iq_{\perp} \cdot (R_{\perp} - R'_{\perp})} \]

\[ \left( \nabla_{R_{\perp}} \cdot \nabla_{R'_{\perp}} \right) \frac{1}{N_c} \left\langle \text{Tr} \left[ U(R_{\perp}) U^{\dagger}(R'_{\perp}) \right] \right\rangle_x, \]

- Quadrupole ⇒ Weizsäcker Williams gluon; Dipole ⇒ Color Dipole gluon.
- Analytical results in MV model exhibit different \( p_T \) behavior.
- In eA and pA collisions, small-\( x \) processes are probing either DP or QP or both at leading \( N_c \) approximation. More detail see [R. Boussarie, Tuesday].
- Generalized universality in large \( N_c \) in eA and pA collisions for Wilson lines [F. Dominguez, C. Marquet, A. Stasto and BX, 12]
Forward hadron production in \( pA \) collisions

Forward hadron in \( pA \) [Dumitru, Jalilian-Marian, 02] [T. Altinoluk, Monday]

\[
\text{projectile: } x_1 \sim \frac{p_{\perp}e^y}{\sqrt{s}} \sim 1 \quad \text{valence}
\]

\[
\text{target: } x_2 \sim \frac{p_{\perp}e^{-y}}{\sqrt{s}} \ll 1 \quad \text{gluon}
\]

Dilute-dense factorization at forward rapidity

\[
\frac{d\sigma_{LO}^{pA\to hX}}{d^2p_\perp dy} = \int_1^1 \frac{dz}{z^2} \left[ \sum_f x_p q_f(x_p) \mathcal{F}(k_\perp) D_{h/q}(z) + x_p g(x_p) \tilde{\mathcal{F}}(k_\perp) D_{h/g}(z) \right].
\]

\[
\Rightarrow \quad U(x_\perp) = \mathcal{P} \exp \left\{ i g s \int_{-\infty}^{+\infty} dx^+ T^c A_c^- (x^+, x_\perp) \right\},
\]

\[
\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} S^{(2)}_Y (x_\perp, y_\perp).
\]

- Dense gluons at low-\( x \) in the nucleus target is described by \( \mathcal{F}(k_\perp) \) and \( \tilde{\mathcal{F}}(k_\perp) \).
- The universality at higher order. Only need dipole amplitudes at leading \( N_c \).
NLO hadron productions in $pA$ collisions: An Odyssey

Dilute-Dense factorizations: large $x$ proton or $\gamma^* \rightarrow$ as dilute probe:

$$x_p = \frac{k_\perp e^+ y}{\sqrt{s}} \sim 1 \text{ dilute}$$

$$x_A = \frac{k_\perp e^- y}{\sqrt{s}} \ll 1 \text{ dense}$$

- LO [Dumitru, Jalilian-Marian, 02]: probing $xG_{DP}(x, k_\perp)$ at small-$x$.
- NLO Cutoff [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11]
- NLO Complete NLO in DR: [Chirilli, BX and Yuan, 12].
  1. soft, collinear to the target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $F(k_\perp)$. Subtraction scheme is not Unique but highly constrained.
  2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs, $\overline{MS}$ scheme.
  3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs, $\overline{MS}$ scheme.
  4. The importance of subtraction: systematic resummation of large logarithms. $(\alpha_s \ln 1/x_g)$, which allows us to have $H \sim \mathcal{O}(\alpha_s)$. Interesting recent development: RG approach and threshold resummation.

$P^-_p \simeq 0$

$k^+ \simeq 0$

$P^+_A \simeq 0$
NLO hadron productions in $pA$ collisions: An Odyssey

SOLO (Saturation physics at One Loop Order) results [Stasto, Xiao, Zaslavsky, 13; Watanabe, Xiao, Yuan, Zaslavsky, 15]

- Agree with RHIC and LHC data in low $p_{\perp} \leq Q_s$ region where pQCD does not apply.
- SOLO (1.0 and 2.0) break down in the large $p_{\perp} \geq Q_s$ region ($k_{\perp} \gg Q_s$).
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14; Kang, Vitev and Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller and Triantafyllopoulos, 16]
- Threshold resummation (Sudakov type)! Approach kinematical threshold at high $k_{\perp}$.

$(\bar{\alpha}_s \ln(1 - x_P) < 0)$. [Xiao, Yuan, 18; work in process]
Threshold resummation in the saturation formalism

[Xiao, Yuan, 18; numerical work in process]

- The objective is to identify large logarithms $\ln(1 - x_p)$ and $\ln k_{\perp}^2/Q_s^2$ in the large $k_{\perp}$ region ($k_{\perp} \gg Q_s$) near threshold at fixed rapidity. In fact, these two types of logs seem to always appear together in our calculation and soft-collinear effective theory (SCET) in almost identical pattern.

- Many different threshold resummation formalism. We find remarkable similarities between the threshold resummation in pA collisions in the small-$x$ formalism and threshold resummation in SCET[Becher, Neubert, 06].

- The forward threshold jet function $\Delta(\mu^2, \mu_b^2, z)$ satisfies an almost identical RGE equation. The solution helps to resum threshold logs.

\[
\frac{d\Delta(\mu^2, \mu_b^2, z)}{d \ln \mu} = -\frac{2\alpha_s N_c}{\pi} \left[ \ln z + \beta_0 \right] \Delta(\mu^2, \mu_b^2, z) \bigg|_{\mu_b} + \frac{2\alpha_s N_c}{\pi} \int_0^z dz' \frac{\Delta(\mu^2, \mu_b^2, z) - \Delta(\mu^2, \mu_b^2, z')}{z - z'} \bigg|_{\mu_b}.
\]

Solution:

\[
\Delta(\mu^2, \mu_b^2, z = \ln \frac{x}{\tau}) = \frac{e^{(\beta_0 - \gamma_E)\gamma_{\mu,b_{\perp}}}}{\Gamma[\gamma_{\mu,b_{\perp}}]} z^{\gamma_{\mu,b_{\perp}} - 1}.
\]
Collectivity (correlation) is everywhere!

- Collectivity is used to describe the particle correlation. It is observed in both large and small systems and for light and heavy hadrons!
- [Song, Shen, Heinz, 11]; [Shen, Heinz, Huovinen, Song, 11]
- New exciting results for $b \rightarrow \mu$ and $c \rightarrow \mu$ as well as UPC in PbPb collisions. [J. Jia, Monday; K. Gajdosova; B. Seidlitz, Wednesday]
Where does the collectivity come from? Final state vs Initial state.

- Geometry + final state evolution (Hydro + freeze-out) explain data well.
- On the other hand, CGC alone has trouble to describe the PHENIX data.
- Final state effect may not be the full story. For example, $v_2$ of heavy mesons. [Du, Rapp, 18]
- Experimental review on heavy flavor in small systems: [Z. Chen, Thursday]
- Mechanism to generate correlations in CGC. [Lappi, 15; Dumitru, McLerran, Skokov, 15; Lappi, Schenke, Schlichting, Venugopalan, 16; Dusling, Mace, Venugopalan, 17; Fukushima, Hidaka, 17; Mace, Skokov, Tribedy, Venugopalan, 18; Davy, Marquet, Shi, Xiao, Zhang, 18]
Correlations in CGC

Correlations between uncorrelated incoming quarks (gluons) are generated due to quadrupole as $N_c$ corrections. [Lappi, 15; Lappi, Schenke, Schlichting, Venugopalan, 16; Dusling, Mace, Venugopalan, 17; Davy, Marquet, Shi, Xiao, Zhang, 18]

\[
\frac{d^2 N}{d^2 k_{1\perp} d^2 k_{2\perp}} = \int d^2 r_{1\perp} d^2 r_{2\perp} e^{-i k_{1\perp} \cdot r_{1\perp}} e^{-i k_{2\perp} \cdot r_{2\perp}} \times \frac{1}{N_c^2} \langle \text{tr} [V(x_1)V(x_2)^\dagger] \text{tr} [V(x_3)V(x_4)^\dagger] \rangle
\]

where

\[
\frac{1}{N_c^2} \langle \text{tr} [V(x_1)V(x_2)^\dagger] \text{tr} [V(x_3)V(x_4)^\dagger] \rangle \neq \frac{1}{N_c^2} \langle \text{tr} [V(x_1)V(x_2)^\dagger] \rangle \langle \text{tr} [V(x_3)V(x_4)^\dagger] \rangle
\]

\[
= e^{-\frac{q^2}{4}(r_1^2+r_2^2)} \left[ 1 + \frac{(\frac{q^2}{2} \mathbf{r}_1 \cdot \mathbf{r}_2)^2}{N_c^2} \int_0^1 \int_0^\xi d\eta e^{-\frac{N_c^2 q^2}{8}[(\mathbf{r}_1-\mathbf{r}_2)^2 - 4(\mathbf{b}_1-\mathbf{b}_2)^2]} \right]
\]

- At leading $N_c$, \( \frac{d^2 N}{d^2 k_{1\perp} d^2 k_{2\perp}} = \left( \frac{dN}{d^2 k_{1\perp}} \right) \left( \frac{dN}{d^2 k_{2\perp}} \right) \), there are no correlations.
- The correlations only come in as higher order $N_c$ corrections as shown above.

Fierz identity

\[
\begin{array}{c}
\begin{array}{c}
\text{Diagram 1} \quad = \frac{1}{2} \\
\text{Diagram 2} \quad - \frac{1}{2N_c}
\end{array}
\end{array}
\]
Elliptic flow of $J/\psi$ in CGC

$J/\psi$ productions together with a light quark reference

$$v_2[J/\psi] \equiv V_{2\Delta}(J/\psi, \text{ref})/v_2[\text{ref}]$$

$\mathcal{O}(1) + \mathcal{O} \left( \frac{1}{N_c^2} \right)$

- [Zhang, Marquet, Qin, Wei, Xiao, 19] The same CGC model calculations with the additional $g \rightarrow Q\bar{Q}$ splitting to produce heavy quarkonia at small $k_T$.
- Predictions for Upsilon $v_2$. Preliminary results show that $D$ and $B$ mesons have similar $v_2$ at low $k_T$. Please stay tuned.
**DIS dijet**

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]

\[
\frac{d\sigma}{dP.S.} \propto N_c \alpha_{em} e_q^2 \int \frac{d^2 x}{(2\pi)^2} \frac{d^2 x'}{(2\pi)^2} \frac{d^2 b}{(2\pi)^2} \frac{d^2 b'}{(2\pi)^2} e^{-ik_{1\perp} \cdot (x-x')} e^{-ik_{2\perp} \cdot (b-b')} \sum \psi_T^*(x-b) \psi_T(x'-b') \left[ 1 + S_{xg}^{(4)}(x, b; b', x') - S_{xg}^{(2)}(x, b) - S_{xg}^{(2)}(b', x') \right],
\]

- eA collision is golden channel for the **Weiszäcker Williams** gluon at small-x.
- Due to linearly polarized gluon distribution at small-\(x\) [Metz, Zhou, 11], there can be the analog of elliptic flow \(v_2\) as well. [Dumitru, Lappi, Skokov, 15]
Can we measure Wigner distributions?

- Wigner distributions ingeniously encode all quantum information of how partons are distributed inside hadrons. [Ji, 03; Belitsky, Ji, Yuan, 03]

- Small-$x$ gluon $\leftrightarrow$ gluon Wigner distributions? [Ji, 03] [A. Dumitru, Tuesday]

- TMDs and GPDs can be studied and measured in various processes. In condense matter, Wigner distribution of photons can be measured.

- Can we measure the gluon Wigner distribution at small-$x$? Yes, we can!

- GTMD (Fourier transform of Wigner) [Meissner, Metz and Schlegel, 09]
The exact connection between dipole amplitude and Wigner distribution

[Hatta, Xiao, Yuan, 16] Def. of gluon Wigner distribution:

\[ xW_g^T(x, \vec{q}_\perp; \vec{b}_\perp) = \int \frac{d\xi^-}{(2\pi)^3} \frac{d^2\xi_\perp}{P^+} \int \frac{d^2\Delta_\perp}{2(2\pi)^2} e^{-ixP^+\xi^- - iq_\perp \cdot \xi_\perp} \times \left\langle P + \frac{\Delta_\perp}{2} \left| F^+ \left( \vec{b}_\perp + \frac{\xi_\perp}{2} \right) \right. \left. F^+ \left( \vec{b}_\perp - \frac{\xi_\perp}{2} \right) \right| P - \frac{\Delta_\perp}{2} \right\rangle, \]

Def. of GTMD [Meissner, Metz and Schlegel, 09]

\[ xG(x, q_\perp, \Delta_\perp) \equiv \int d^2b_\perp e^{-i\Delta_\perp \cdot b_\perp} xW_g^T(x, \vec{q}_\perp; \vec{b}_\perp). \]

- With dipole like gauge link \([b_\perp = \frac{1}{2}(R_\perp + R'_\perp), r_\perp = R_\perp - R'_\perp] \Rightarrow \]

\[ xG_{DP}(x, q_\perp, \Delta_\perp) = \frac{2N_c}{\alpha_s} \int \frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4} e^{iq_\perp \cdot (R_\perp - R'_\perp) + i\frac{\Delta_\perp}{2} \cdot (R_\perp + R'_\perp)} \times \left( \nabla_{R_\perp} \cdot \nabla_{R'_\perp} \right) \frac{1}{N_c} \left\langle \text{Tr} \left[ U(R_\perp) U^\dagger(R'_\perp) \right] \right\rangle_x. \]

- This provides the 3D quasiprobabilistic distribution \((x, b_\perp, k_\perp)\) of small-\(x\) gluon.
Probing 3D Tomography of Proton at small-$x$

Diffractive back-to-back dijet productions in DIS [Altinoluk, Armesto, Beuf and Rezaeian, 15]; [Hatta, Xiao, Yuan, 16]; [Mantysaari, Mueller, and Schenke, 19]

- Measure final state proton recoil $\Delta_\perp$ as well as dijet momentum $k_{1\perp}$ and $k_{2\perp}$.
- We can approximately access $|xG_{DP}(x, q_\perp, \Delta_\perp)|^2$ in the back-to-back limit in which $q_\perp \simeq P_\perp \equiv \frac{1}{2}(k_{2\perp} - k_{1\perp}) \gg \Delta_\perp$.
- Cross-Sections are positive-definite, although Wigner distributions may not be.
- Dipole amplitude can have elliptic part $\mathcal{N}(r, b) = v_0 [1 + 2v_2 \cos 2\theta(r, b)]$.
- WW Wigner (WWW) distribution can be also defined and measured.
- Gluon OAM [Ji, Yuan, Zhao, 16; Hatta, Nakagawa, Yuan, Zhao, 16, Bhatttacharya, Metz, Zhou, 17]; UPC at the LHC [Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, 17]
Complementary study of eA and pA collisions can provide us more important information on the universal building blocks of small-\(x\) physics: dipole and quadrupole.

This universality has many applications in \(pA\) and \(eA\) collisional systems. (Dilute-dense) For example, probing of WW and Wigner distribution, etc.

The same universality implies that initial state interactions (CGC) can generate sufficient amount of collectivity for heavy mesons.

Maybe the elliptic flow of \(\Upsilon(b\bar{b})\) or other heavy mesons \(B\) at the LHC can be a clear signal for CGC initial state effects.