Self-similarity and spectral functions of non-Abelian plasmas in 2+1D

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In preparation

Physics picture

- In high energy nuclear collisions high density of gluons at the saturation scale Q_s.
- If Q_s large $\rightarrow \alpha_s(Q_s) \ll 1, \rightarrow$ gluons at Q_s overoccupied classical color fields.
- Many systems (non-Abelian gauge theories and scalars in 3D and scalars in 2D etc.) with $f \gg 1 \rightarrow$ universal self-similar attractors.
- At high energy the initial color fields are boost invariant $\rightarrow$ effectively 2D.
- Gauge systems in 3D exhibit self-similarity. Less is known about 2D.

Aims of this talk:

Questions:

- 3D gauge and scalar systems exhibit self-similarity. How about 2D gauge theory?
- Teaser: there are quasiparticles in the self-similar regime in 3D (PRD 98 (2018) no.1, 014006). How about 2D?

Methods - two different theories:

- 2+1D gauge theory.
- Dimensionally reduced 3+1D theory = 2+1D gauge + adjoint scalar. Mimics the boost invariant system.

Answers:

- 2+1D and eff. 2+1D systems exhibit self-similarity.
- Quasiparticle exitations for large p. Small $p \rightarrow$ inverse lifetime $\approx \omega$, which makes interpretation more difficult.

Introduction: self-similarity in 3D

Gauge and scalar systems exhibit self-similar behavior at late times. Dynamics governed by universal scaling exponents.

$$f(t,p) = (Qt)^{\alpha} f_S((Qt)^{\beta} p), \tag{1}$$

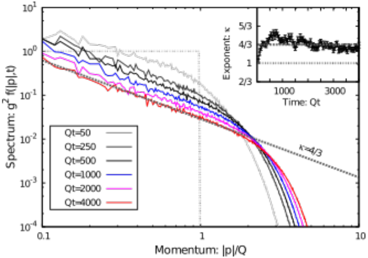


Figure: Occupation number distribution. Phys.Rev. D89 (2014) no.11, 114007

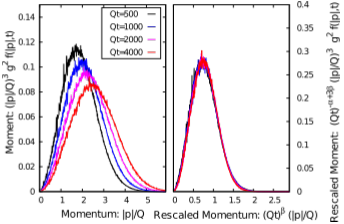


Figure: Third moment of the occupation number distribution, also with rescaling. Phys.Rev. D89 (2014) no.11, 114007

Self-similarity in 2+1D

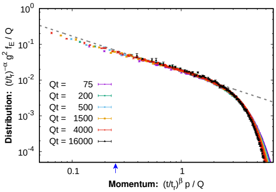


Figure: With rescaling

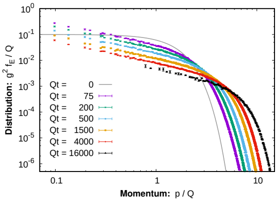


Figure: No rescaling

- Self-similar evolution also in 2+1D!

In 2D results:

$$\alpha = -3/5 \tag{2}$$
$$\beta = -1/5. \tag{3}$$

In 3D we had:

$$\alpha = -4/7 \tag{4}$$
$$\beta = -1/7. \tag{5}$$

- $\alpha = (d+1)\beta \rightarrow$ energy conservation.
- $\beta = -1/5$: kinetic theory + small angle approximation.
- $\rightarrow$ energy cascade to UV.

Self-similarity in eff. 2+1D

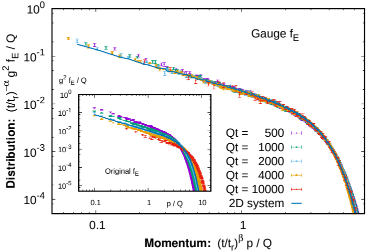


Figure: Gauge distribution

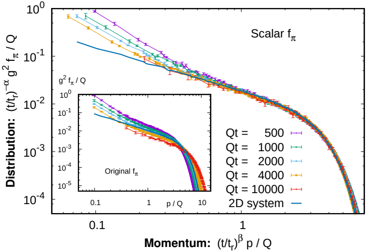


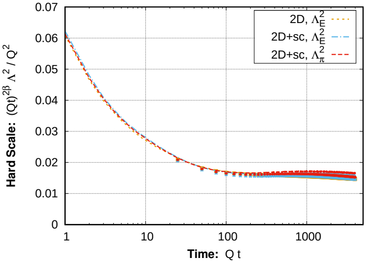
Figure: Scalar distribution

- The same exponents also in eff. 2+1D simulations.
- For scalar distribution f_π, the self-similar scaling is violated for $p < m_D$. One observes enhancement in the IR.

Gauge-invariant hard scale: self-similar evolution

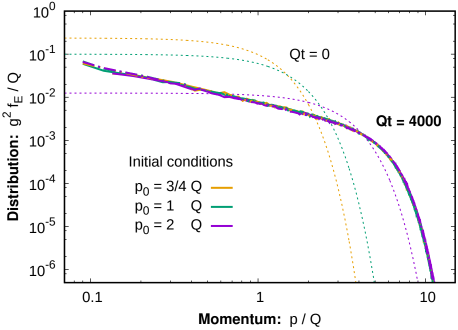
$$\Lambda_E^2(t) = \frac{g^2}{d_A Q^4} \sum_{k,l,i=1,2} \left\langle (D_k^{ab} F_{ki}^b(t,\mathbf{x}))(D_l^{ad} F_{li}^d(t,\mathbf{x})) \right\rangle \tag{6}$$

$$\Lambda_\pi^2(t) = \frac{g^2}{d_A Q^4} \sum_{k,l=1,2} \left\langle (D_k^{ab} D_k^{bc} \phi^c(t,\mathbf{x}))(D_l^{ad} D_l^{de} \phi^e(t,\mathbf{x})) \right\rangle . \tag{7}$$



- Gauge invariant hard scale follows self-similar evolution in both theories for both gauge and scalar excitations.

Attractor in 2+1D



- $Q \approx \sqrt[4]{g^2\epsilon}, \quad f(t=0,p_\perp) = \frac{Q}{g^2} n_0 \, e^{-\frac{p_\perp^2}{2p_0^2}}, \quad g^2\epsilon \sim n_0 p_0^4$
- Dashed lines: initial condition, full lines: 2D theory, dash-dotted lines: 2D + scalar theory.
- 3 different initial conditions. At later times they fall on top of each other. → Dynamics not sensitive to such initial conditions.

Extraction of spectral function: Linear response theory

Use linear response theory, extract retarded propagator as in (PRD 98 (2018) no.1, 014006) for 3D gauge system. Split the gauge field into background field and a fluctuation

$$A_\mu(t,x) \to A_\mu(t,x) + a_\mu(t,x). \tag{8}$$

a evolves according to linearized EOMS. Use

$$\langle \hat{a}_i^b(t,\mathbf{p})\rangle = \int \mathrm{d}t'\, G_{R,ik}^{\;\;bc}(t,t',\mathbf{p})\, j_c^k(t',\mathbf{p}). \tag{9}$$

Source j can be chosen such that G_R can be obtained from $\langle ja\rangle$. Finally obtain the spectral function as

$$\rho = 2\mathfrak{Im} G_R. \tag{10}$$

Spectral function in 2+1D: Preliminary

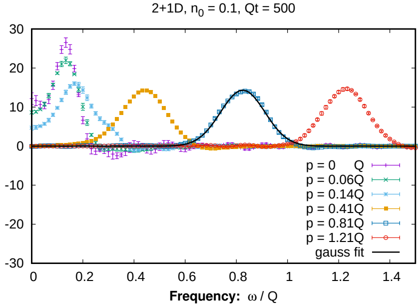


Figure: Numerically extracted spectral function

- Curves correspond to $\omega\rho$
- Peak position $\longleftrightarrow \omega(p)$ dispersion relation.
- Peak width $\longleftrightarrow \gamma(p)$ damping rate (inverse lifetime)
- At small p: $\omega \approx \gamma$ i.e. quasiparticle interpretation problematic. At large p: we see quasiparticle peaks.

Conclusions

We have

- Observed self-similar behavior in 2+1D gauge theories. In both theories both gauge and scalar fields approach a universal attractor that is the same for both.
- The scaling exponents are $\alpha = {}^{-3}/_{5}$ and $\beta = {}^{-1}/_{5}$. Different from 3D.
- Scalar distribution IR enhanced.
- Extracted spectral function from 2+1D simulations. We find that quasiparticles exist for large p. However for small p quasiparticle description becomes problematic.

Outlook

- Work in progress also in terms of transport coefficients (κ, poster by JP). We also want to look at plasma instabilities etc.

Correlation functions

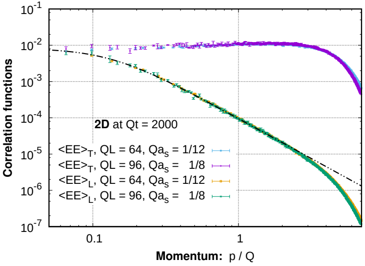


Figure: 2D

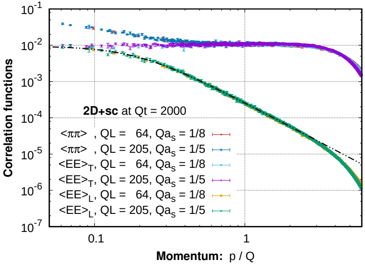


Figure: 2D+Scalar

Scalar correlator is enhanced in the infrared.

Debye mass

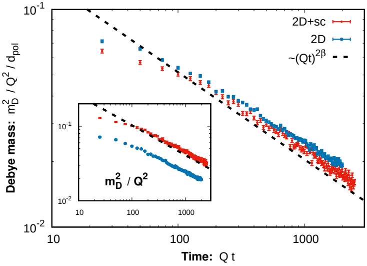


Figure: 2D

The Debye mass extracted from the longitudinal $< EE >$ correlator follows self-similar evolution.

$$\langle E_L E_L^* \rangle \approx \frac{A}{1 + (p^2/m_D^2)^{1+\delta}}, \tag{11}$$

for momenta $p \lesssim \Lambda$. At early times $\delta \approx 0.2 - 0.3$. At $Qt = 2000$ $\delta \approx 0.08 - 0.12$ for both theories.

Scaling exponents

Extract the scaling exponents α and β. Define a rescaled distribution

$$f_{\text{test}}(t,p) = (t/t_r)^{-\alpha} f\left(t, (t/t_r)^{-\beta} p\right). \tag{12}$$

- $f_{\text{test}}(t_r,p) \equiv f(t_r,p)$ for the reference time $Qt_r = 500$.
- Self-similarity $\rightarrow$ $f_{\text{test}}(t,p)$ time-independent.

Quantify the deviation by computing

$$\chi^2_m(\tilde{\alpha},\beta) = \frac{1}{N_t} \sum_i \frac{\int \mathrm{d}\log p \, (p^m \Delta f(t_i,p))^2}{\int \mathrm{d}\log p \, (p^m f(t_r,p))^2}, \tag{13}$$

- $\Delta f(t_i,p) = f_{\text{test}}(t_i,p) - f(t_r,p)$
- $\tilde{\alpha} \equiv \alpha - 3\beta$.
- Momentum integrals are performed in the interval $0.2 \leq p/Q \leq 5$.

Scaling exponents

The deviations χ_m^2 are averaged over the test times $Qt_i = 75, 200, 1500, 4000, 16000$ for different moments with $m = 2, \ldots, 5$.
Define a likelihood function

$$W(\tilde{\alpha}, \beta) = \frac{1}{\mathcal{N}} \exp\left(1 - \frac{\chi^2(\tilde{\alpha}, \beta)}{\chi^2_{\min}}\right). \tag{14}$$

- $\chi^2(\tilde{\alpha}_0, \beta_0) \equiv \chi^2_{\min}$ takes its minimal value.
- The normalization $\mathcal{N}$ satisfies $\int \mathrm{d}\tilde{\alpha}\, \mathrm{d}\beta\, W(\tilde{\alpha}, \beta) = 1$, $W(\beta) = \int \mathrm{d}\tilde{\alpha}\, W(\tilde{\alpha}, \beta)$.
- The uncertainty σ_β for every m, fit $\propto \exp[-(\beta - \beta_0)^2/(2\sigma_\beta^2)]$.
- The statistical error $\sigma_\beta^{\chi^2}$ of the χ^2 fit is the maximumf σ_β among the different m.

Scaling exponents

Best fit values

$$\alpha_{\rm fit} - 3\beta_{\rm fit} = 0.01 \pm 0.02 \qquad (15)$$

$$\beta_{\rm fit} = -0.19 \pm 0.015 \,. \qquad (16)$$