Investigation of the linear and mode-coupled flow harmonics in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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What are the respective roles of \( \varepsilon_n \) and its fluctuations, flow correlations and \( \eta/s(T) \) on the \( v_n \)?
Introduction

Anisotropic flow
Asymmetry in initial geometry → Final-state momentum anisotropy

What are the respective roles of $\varepsilon_n$ and its fluctuations, flow correlations and $\eta/s(T)$ on the $v_n$?

Higher-order flow harmonics ($v_{n>3}$) have multiple contributions:
- Linear response $\propto \varepsilon_{n>3}$
- Mode-coupled response $\propto \varepsilon_2$ and/or $\varepsilon_3$ and the Event Plane (EP) correlations
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Higher-order flow harmonics ($v_{n>3}$) have multiple contributions:

- Linear response $\propto \varepsilon_{n>3}$
- Mode-coupled response $\propto \varepsilon_2$ and/or $\varepsilon_3$ and the Event Plane (EP) correlations

The focus of this work:
- Separate and study the linear and mode-coupled contributions
- Study the nature of the eccentricity coupling and the EP correlations
Introduction

The Solenoidal Tracker At RHIC

Time Projection Chamber
Tracking of charged particles with:

✔ Full azimuthal coverage
✔ $|\eta| < 1$ coverage
Analysis Method

The two- and three-particle correlations:

\[ C_{k,2n} = \left\langle \cos((2 + n)\varphi_1^A - 2\varphi_2^B - n\varphi_3^B) \right\rangle \]

\[ \nu_n^{Inclusive} = \left\langle \cos(n\varphi_1^A - n\varphi_2^B) \right\rangle^{1/2} \]

\[ n = 2,3 \quad k = n + 2 \]

The four-particle correlations:

\[ \langle v_2^4 \rangle = \left\langle \cos(2\varphi_1 + 2\varphi_2 - 2\varphi_3 - 2\varphi_4) \right\rangle - 2 \left\langle \cos(2\varphi_1 - 2\varphi_2) \right\rangle^2 + 2 \left\langle \cos(2\varphi_1^A - 2\varphi_2^B) \right\rangle^2 \]

\[ \langle v_2^2 v_3^2 \rangle = \left\langle \cos(3\varphi_1 + 2\varphi_2 - 3\varphi_3 - 2\varphi_4) \right\rangle - \left\langle \cos(2\varphi_1 - 2\varphi_2) \right\rangle \left\langle \cos(4\varphi_1 - 3\varphi_2) \right\rangle \]

\[ + \left\langle \cos(2\varphi_1^A - 2\varphi_2^B) \right\rangle \left\langle \cos(3\varphi_1^A - 3\varphi_2^B) \right\rangle \]
Analysis Method

The two- and three-particle correlations:

\[ C_{k,2n} = \left\langle (\cos((2 + n)\varphi_1^A - 2\varphi_2^B - n\varphi_3^B)) \right\rangle \]

The four-particle correlations:

\[ v_{n,2n}^{\text{Inclusive}} = \frac{1}{2} \left( \cos(n\varphi_1^A - n\varphi_2^B) \right) \]

\[ v_n^A = \left\langle \cos(2\varphi_1 + 2\varphi_2 - 2\varphi_3 - 2\varphi_4) \right\rangle - 2 \left\langle \cos(2\varphi_1 - 2\varphi_2) \right\rangle^2 + 2 \left\langle \cos(2\varphi_1^A - 2\varphi_2^B) \right\rangle^2 \]

\[ v_n^B = \left\langle \cos(3\varphi_1 + 2\varphi_2 - 3\varphi_3 - 2\varphi_4) \right\rangle - \left\langle \cos(2\varphi_1 - 2\varphi_2) \right\rangle \left\langle \cos(4\varphi_1 - 3\varphi_2) \right\rangle \]

Assume the orthogonality between linear and non-linear contributions

\[ V_k = V_k^{\text{Linear}} + V_k^{\text{Non-Linear}} \]

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PLB 773 68 (2017)
Analysis Method

The two- and three-particle correlations:

\[ C_{k,2n} = \left\langle \cos((2+n)\phi_1^A - 2\phi_2^B - n\phi_3^B) \right\rangle \]

\[ \nu_n^{Inclusive} = \left( \left\langle \cos(n\phi_1^A - n\phi_2^B) \right\rangle \right)^{1/2} \]

|Δη| > 0.7

The four-particle correlations:

\[ \langle v_2^4 \rangle = \left\langle \cos(2\phi_1 + 2\phi_2 - 2\phi_3 - 2\phi_4) \right\rangle - 2 \left\langle \cos(2\phi_1 - 2\phi_2) \right\rangle^2 + 2 \left\langle \cos(2\phi_1^A - 2\phi_2^B) \right\rangle^2 \]

\[ \langle v_2^2v_3^2 \rangle = \left\langle \cos(3\phi_1 + 2\phi_2 - 3\phi_3 - 2\phi_4) \right\rangle - \left\langle \cos(2\phi_1 - 2\phi_2) \right\rangle \left\langle \cos(4\phi_1 - 3\phi_2) \right\rangle + \left\langle \cos(2\phi_1^A - 2\phi_2^B) \right\rangle \left\langle \cos(3\phi_1^A - 3\phi_2^B) \right\rangle \]

\[ \nu_k^{Non-Linear} \]

carry information about:

✓ Viscous effects, EP angular correlations and Eccentricity coupling

\[ \nu_k^{Non-Linear} = \frac{C_{k,2n}}{\sqrt{\langle v_2^2v_3^2 \rangle}} , \]

\[ = \frac{\langle v_k v_2 v_n \cos(k\Psi_k - n\Psi_n - 2\Psi_2) \rangle}{\langle v_2^2v_3^2 \rangle} , \]

\[ \sim \langle v_k \cos(k\Psi_k - n\Psi_n - 2\Psi_2) \rangle , \]

Assume the orthogonality between linear and non-linear contributions

\[ V_k = V_k^{Linear} + V_k^{Non-Linear} \]

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Analysis Method

The two- and three-particle correlations:

\[ C_{k,2n} = \left\langle \cos((2+n)\phi_1^A - 2\phi_2^B - n\phi_3^B) \right\rangle \]

The four-particle correlations:

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\[ \langle v_2^2 v_3^2 \rangle = \langle \cos(3\phi_1 + 2\phi_2 - 3\phi_3 - 2\phi_4) \rangle - \langle \cos(2\phi_1 - 2\phi_2) \rangle \langle \cos(4\phi_1 - 3\phi_2) \rangle 
+ \left\langle \cos(2\phi_1^A - 2\phi_2^B) \right\rangle \left\langle \cos(3\phi_1^A - 3\phi_2^B) \right\rangle \]

\[ \nu_n^{Inclusive} = \left\langle \cos(n\phi_1^A - n\phi_2^B) \right\rangle^{1/2} \]

\( \Delta \eta > 0.7 \)

\( |\Delta \eta| > 0.7 \)

\[ A \]

\[ B \]

\[ \eta \]

\[ \text{Assume the orthogonality between linear and non-linear contributions} \]

\[ V_k = V_k^{Linear} + V_k^{Non-Linear} \]

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\[ V_k^{Linear} = \sqrt{\left( V_k^{Inclusive} \right)^2 - \left( V_k^{Non-Linear} \right)^2} \]

\[ V_k^{Non-Linear} = \frac{C_{k,2n}}{\sqrt{\langle v_2^2 v_n^2 \rangle}} \]

\[ \sim \langle v_k v_2 v_n \cos(k\Psi_k - n\Psi_n - 2\Psi_2) \rangle \]

\( \nu_k^{Non-Linear} \) carry information about:

- Viscous effects, EP angular correlations and Eccentricity coupling

\[ \rho_{k,2n} = \frac{\nu_{k}^{Non-Linear}}{\nu_{k}^{Inclusive}} = \langle \cos(k\Psi_k - 2\Psi_2 - n\Psi_n) \rangle \]

\( \text{EP angular correlations} \)

\[ \chi_{k,2n} = \frac{\nu_{k}^{Non-Linear}}{\sqrt{\langle v_2^2 v_n^2 \rangle}} \]

\( \text{Eccentricity coupling} \)

\( \text{Weak viscous effect expected} \)

P.Liu, R.Lacey
PRC 98, 021902 (2018)
The short-range non-flow contributions in the three-particle correlations

Three-particle correlations, $C_{4,22}$ and $C_{5,23}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV using the AMPT model

Two-subevents reduce the short-range non-flow effects in the three-particle correlation measurements

$C_{4,22} = \langle \cos(4\phi_1^A - 2\phi_2^B - 2\phi_3^B) \rangle$

$C_{5,23} = \langle \cos(5\phi_1^A - 2\phi_2^B - 3\phi_3^B) \rangle$
Three-particle correlations, $C_{4,22}$ and $C_{5,23}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV compared with different hydrodynamic simulations.

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## Results

Three-particle correlations, $C_{4,22}$ and $C_{5,23}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV compared with different hydrodynamic simulations.

![Graph showing Au+Au 200 GeV, Two-subevent method, and Hydro-1, Hydro-2a, Hydro-2b comparisons for $C_{4,22}$ and $C_{5,23}$ with different centrality and $p_T$ bins.]

<table>
<thead>
<tr>
<th>Centrality %</th>
<th>Hydro-1</th>
<th>Hydro-2a/b</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 2, m = 2</td>
<td>$C_{4,22}$</td>
<td>$C_{5,23}$</td>
</tr>
<tr>
<td>0 - 20</td>
<td>$\times 10^{-4}$</td>
<td>$\times 10^{-4}$</td>
</tr>
<tr>
<td>20 - 40</td>
<td>STAR Preliminary</td>
<td>STAR Preliminary</td>
</tr>
<tr>
<td>40 - 60</td>
<td>(b) 0.2 &lt; $p_T$ &lt; 4 GeV/c</td>
<td>(b) 0.2 &lt; $p_T$ &lt; 4 GeV/c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hydro-1</th>
<th>Hydro-2a/b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta/s$</td>
<td>0.05</td>
</tr>
<tr>
<td>Initial conditions</td>
<td>TRENTO Initial conditions</td>
</tr>
<tr>
<td>Contributions</td>
<td>Hydro + Direct decays</td>
</tr>
<tr>
<td>(a) Hydro + Hadronic cascade</td>
<td>(b) Hydro only</td>
</tr>
</tbody>
</table>

- (2) B.Schenke, C.Shen, and P.Tribedy
  PRC 99, 044908 (2019)

- However both models fit the $\nu_n$ they need additional constrains in order to describe the 3-particle correlations

$$C_{4,22} = \langle \cos(4\phi_1 - 2\phi_2 - 2\phi_3^B) \rangle$$

$$C_{5,23} = \langle \cos(5\phi_1 - 2\phi_2 - 3\phi_3^B) \rangle$$
The \( p_T \)-differential dependence of the inclusive, linear and non-linear \( \nu_4 \) for Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV are shown.

- The inclusive, linear and non-linear \( \nu_4 \) shows a characteristics \( p_T \) dependence.

\[
\nu_4^{\text{Non-Linear}} = \frac{C_{4,22}}{\sqrt{\langle \nu_4^2 \rangle}}
\]

\[
\nu_4^{\text{Linear}} = \sqrt{(\nu_4^{\text{Inclusive}})^2 - (\nu_4^{\text{Non-Linear}})^2}
\]
The $p_T$-differential dependence of the inclusive, linear and non-linear $v_4$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are shown.

- The inclusive, linear and non linear $v_4$ shows a characteristics $p_T$ dependence.
- The linear $v_4$ term dominates in central collisions.

\[ v_{4}^{\text{Non-Linear}} = \frac{C_{4,22}}{\sqrt{\langle v_2^4 \rangle}} \]
\[ v_{4}^{\text{Linear}} = \sqrt{\langle v_4^{\text{Inclusive}} \rangle^2 - \langle v_4^{\text{Non-Linear}} \rangle^2} \]
Results

Centrality dependence of the inclusive, linear and non-linear $v_n$ (n=4,5) for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

The linear $v_n$ (n=4,5) terms dominate in central collisions

$$v_{m+2}^{\text{Non-Linear}} = \frac{c_{m+2,2m}}{\sqrt{\langle v_2^2 v_m^2 \rangle}}$$

$$v_{m+2}^{\text{Linear}} = \sqrt{(v_{m+2}^{\text{Inclusive}})^2 - (v_{m+2}^{\text{Non-Linear}})^2}$$
Results

Centrality dependence of the non-linear mode-coupling coefficients, $\chi_{4,22}$ and $\chi_{5,23}$ and the EP angular correlations $\rho_{4,22}$ and $\rho_{5,23}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

- $\chi_{k,nm}$ shows a weak centrality dependence (weak viscous effect)
- $\rho_{k,nm}$ shows a strong centrality dependence

$$\chi_{m+2,m} = v_{m+2}^{Non-Linear} / \sqrt{(v_2^2 v_m^2)} \quad m = 2,3$$

$$\rho_{m+2,m} = v_{m+2}^{Non-Linear} / v_{m+2}^{Inclusive}$$
Results

Centrality dependence of the non-linear mode-coupling coefficients, $\chi_{4,22}$ and $\chi_{5,23}$ and the EP angular correlations $\rho_{4,22}$ and $\rho_{5,23}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

- $\chi_{k,nm}$ shows a weak centrality dependence (weak viscous effect)

- $\rho_{k,nm}$ shows a strong centrality dependence

- $\rho_{k,nm}$ and $\chi_{k,nm}$ show a weak beam energy dependence

$\chi_{m+2,m} = \frac{v_{m+2}}{\sqrt{v_{2}^{2} + v_{m}^{2}}} \quad m = 2,3$

$\rho_{m+2,m} = \frac{v_{m+2}}{v_{m+2}}$
Results

Centrality dependence of the non-linear mode-coupling coefficients, $\chi_{4,22}$ and $\chi_{5,23}$ and the EP angular correlations $\rho_{4,22}$ and $\rho_{5,23}$ for Au+Au collisions at $\sqrt{S_{NN}} = 200$ GeV

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- $\rho_{k,\text{nm}}$ shows a strong centrality dependence
- $\rho_{k,\text{nm}}$ and $\chi_{k,\text{nm}}$ show a weak beam energy dependence

\[\begin{align*}
\chi_{m+2m} &= \nu_{m+2}^{\text{Non-Linear}} / \sqrt{\nu_{2}^{2} \nu_{m}^{2}} \\
\rho_{m+2m} &= \nu_{m+2}^{\text{Non-Linear}} / \nu_{m+2}^{\text{Inclusive}}
\end{align*}\]

$m = 2, 3$

[1] ALICE Collaboration
PLB 773 68 (2017)
The $p_T$-differential dependence of the non-linear mode-coupling coefficients, $\chi_{4,22}$ and the EP angular correlations $\rho_{4,22}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are shown.

- $\chi_{4,22}$ shows a weak $p_T$ dependence
- $\rho_{4,22}$ shows a weak $p_T$ dependence
- Dynamical final-state effects are significantly less than the initial-state effect.

\[
\chi_{m+2m} = \frac{v_{m+2}^{\text{Non-Linear}}}{\sqrt{\langle v_2^2 v_m^2 \rangle}} \quad m = 2
\]
\[
\rho_{m+2m} = \frac{v_{m+2}^{\text{Non-Linear}}}{v_{m+2}^{\text{Inclusive}}} \]

Results
The influence of event shape selection

- Events are further subdivided into groups with different $q_2$ magnitude:

$$Q_{n,x} = \sum_{i=1}^{M} \cos(n \phi_i) \quad Q_{n,y} = \sum_{i=1}^{M} \sin(n \phi_i)$$

$$|Q_n| = \sqrt{Q_{n,x}^2 + Q_{n,y}^2} \quad q_n = \frac{|Q_n|}{\sqrt{M}}$$
The influence of event shape selection

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- $v_2\{2\}$ increases linearly with $q_2$

$q_2$ is good event-shape selector
The influence of event shape selection

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$$Q_{n,x} = \sum_{i=1}^{M} \cos(n \varphi_i)$$
$$Q_{n,y} = \sum_{i=1}^{M} \sin(n \varphi_i)$$

$$|Q_n| = \sqrt{Q_{n,x}^2 + Q_{n,y}^2}$$
$$q_n = \frac{|Q_n|}{\sqrt{M}}$$

- $v_n(2)$ shows no sensitivity to $q_4$

- $v_2(2)$ increases linearly with $q_2$

$q_2$ is good event-shape selector

ALICE Collaboration
PRC 93, 034916 (2016)
Results

Centrality dependence of the linear and non-linear $v_4$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with different event shape selections ($q_2\%$)

- The non-linear $v_4$ increases with $q_2$ selections

\[ v_4^{Non-Linear} = \frac{C_{4,22}}{\sqrt{\langle v_4^2 \rangle}} \]

\[ v_4^{Linear} = \sqrt{\langle v_4^{inclusive} \rangle^2 - (v_4^{Non-Linear})^2} \]
Results

Centrality dependence of the linear and non-linear $v_4$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with different event shape selections ($q_2\%$)

- The non-linear $v_4$ increases with $q_2$ selections.
- The linear $v_4$ shows a weak sensitivity to $q_2$ selections.

\[
\begin{align*}
    v_4^{\text{Non-Linear}} &= \frac{C_{4,22}}{\sqrt{\langle v_2^4 \rangle}} \\
    v_4^{\text{Linear}} &= \sqrt{\langle v_4^{\text{inclusive}} \rangle^2 - \langle v_4^{\text{Non-Linear}} \rangle^2}
\end{align*}
\]
Results

Centrality dependence of the $\rho_{4,22}$ and $\chi_{4,22}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with different event shape selections ($q_2$)

- The $\chi_{4,22}$ shows a weak sensitivity to $q_2$ selections

$$\chi_{m+2m} = \frac{v_{m+2}^{\text{Non-Linear}}}{\sqrt{\langle v_2^2 \rangle} v_m^{\text{Linear}}}, \quad m = 2$$

$$\rho_{m+2m} = \frac{v_{m+2}^{\text{Non-Linear}}}{v_{m+2}^{\text{Inclusive}}}, \quad m = 2$$
Results

Centrality dependence of the $\rho_{4,22}$ and $\chi_{4,22}$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with different event shape selections ($q_2$)

- The $\chi_{4,22}$ shows a weak sensitivity to $q_2$ selections
- The $\rho_{4,22}$ increases with $q_2$ selections

\[\chi_{m+2,2m} = \frac{v_{Non-Linear}^{m+2}}{\sqrt{\langle v_2^2 v_m^2 \rangle}} \]
\[\rho_{m+2,2m} = \frac{v_{Non-Linear}^{m+2}}{v_{m+2}^{Inclusive}} \]
Centrality dependence of the linear and non-linear \( v_4 \) and the associated \( \rho_{4,22} \) and \( \chi_{4,22} \) for Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV with different event shape selection (\( q_4 \)).

- The (non)linear \( v_4 \) and the associated \( \rho_{4,22} \) and \( \chi_{4,22} \) show weak sensitivity to \( q_4 \) selections.
Conclusion

The linear and mode-coupled contributions to the higher-order anisotropic flow coefficients $v_4$ and $v_5$, have been studied using two- and multi-particle correlations in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.
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The linear and mode-coupled contributions to the higher-order anisotropic flow coefficients $v_4$ and $v_5$, have been studied using two- and multi-particle correlations in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

- Two-subevents reduce the short-range non-flow effect on in the three-particle correlations.
- The linear $v_n$ (n=4,5) terms dominate in central collisions.
- The $\chi_{k,nm}$ show a weak centrality dependence (weak viscous effects).
- The $\chi_{4,22}$ and $\rho_{4,22}$ show a weak $p_T$ dependence.
  ✓ Dynamical final-state effect are significantly less than the initial-state effect?
Conclusion

The linear and mode-coupled contributions to the higher-order anisotropic flow coefficients $\nu_4$ and $\nu_5$, have been studied using two- and multi-particle correlations in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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  ✓ Dynamical final-state effect are significantly less than the initial-state effect?

- The influence of event shape selection
  - The non-linear $\nu_4$ and $\rho_{4,22}$ increase with $q_2$ selections
  - The linear $\nu_4$, $\chi_{4,22}$ show no sensitivity to $q_2$ selections
  - The (Non)Linear $\nu_4$, $\chi_{4,22}$, $\rho_{4,22}$ and show no sensitivity to $q_4$ selections
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The linear and mode-coupled contributions to the higher-order anisotropic flow coefficients $v_4$ and $v_5$, have been studied using two- and multi-particle correlations in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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- The linear $v_n$ (n=4,5) terms dominate in central collisions
- The $\chi_{k,nm}$ show a weak centrality dependence (weak viscous effects)
- The $\chi_{4,22}$ and $\rho_{4,22}$ show a weak $p_T$ dependence
  - ✓ Dynamical final-state effect are significantly less than the initial-state effect?

- The influence of event shape selection
  - The non-linear $v_4$ and $\rho_{4,22}$ increase with $q_2$ selections
  - The linear $v_4$, $\chi_{4,22}$ show no sensitivity to $q_2$ selections
  - The (Non)Linear $v_4$, $\chi_{4,22}$ $\rho_{4,22}$ show no sensitivity to $q_4$ selections

The integrated and differential measurements, which are compared to viscous hydrodynamic model calculations, will add important constraints for the initial- and final-state models
Thank You
Good agreement with the STAR published measurements