Non-equilibrium Green's functions for energy-momentum perturbations around Bjorken flow from the Boltzmann equation in relaxation time approximation

Syo Kamata, Mauricio Martinez
Physics Department, North Carolina State University, Raleigh, USA

Soeren Schlichting
Bielefeld University, Germany

Abstract

Non-equilibrium Green's functions provide an efficient tool to describe the evolution of the energy-momentum tensor during the early time pre-equilibrium stage, and provide a meaningful to address the question when and to what extent a hydrodynamic description of the system becomes applicable. We present a calculation of the Green's functions describing the evolution of energy density perturbations in the transverse plane, based on the Boltzmann equation in relaxation time approximation. We discuss the approach towards viscous hydrodynamics along with the emergence of various scaling phenomena for conformal systems. By comparing our results obtained in the relaxation time approximation to previous calculations in QCD kinetic theory, we further address the question which macroscopic features of the energy momentum tensor are sensitive to the underlying microscopic dynamics.

Kinetic theory framework

We consider a system whose dynamics can be described as a combination of an homogeneous boost invariant background and a linearized perturbation

$$f(x^\mu, p_i) = f_{BG}(\tau, p_T, |p_\eta|) + \delta f(\tau, \mathbf{x}_T, \mathbf{p}_T, |p_\eta|)$$

The evolution of the background f_{BG} and the perturbation δf is determined from the Boltzmann equation and its linear version, i.e.,

$$\left(\partial_\tau - \frac{p_\eta}{\tau}\partial_{p_\eta}\right) f_{BG} = \mathcal{C}[f_{BG}]$$

$$\left(\partial_\tau - \frac{p_\eta}{\tau}\partial_{p_\eta} + \mathbf{p}_T \cdot \partial_{\mathbf{x}_T}\right)\delta f = \mathcal{C}[f_{BG}, \delta f]$$

where the collisional kernel is $\mathcal{C}[f] = -(f - f_{eq})/\tau_r$

We map the mathematical problem of solving the Boltzmann equation and its linearized version into finding solutions for the following set of moments

$$c_l^m(\tau) = \frac{\pi^2}{3T^4}\int_{\mathbf{p}} E_{\mathbf{p}}^2\, Y_l^{m*}(\theta_{\mathbf{p}}, \phi_{\mathbf{p}})\, f_{BG}(\tau, p_T, |p_\eta|)$$

$$\delta c_l^m(\tau, \mathbf{k}_T) = \frac{\pi^2}{3T^4}\int_{\mathbf{p}} E_{\mathbf{p}}^2\, Y_l^{m*}(\theta_{\mathbf{p}}, \phi_{\mathbf{p}})\, \delta f(\tau, \mathbf{k}_T, \mathbf{p}_T, |p_\eta|)$$

The evolution of the background moments is given by the following set of nonlinear ODEs

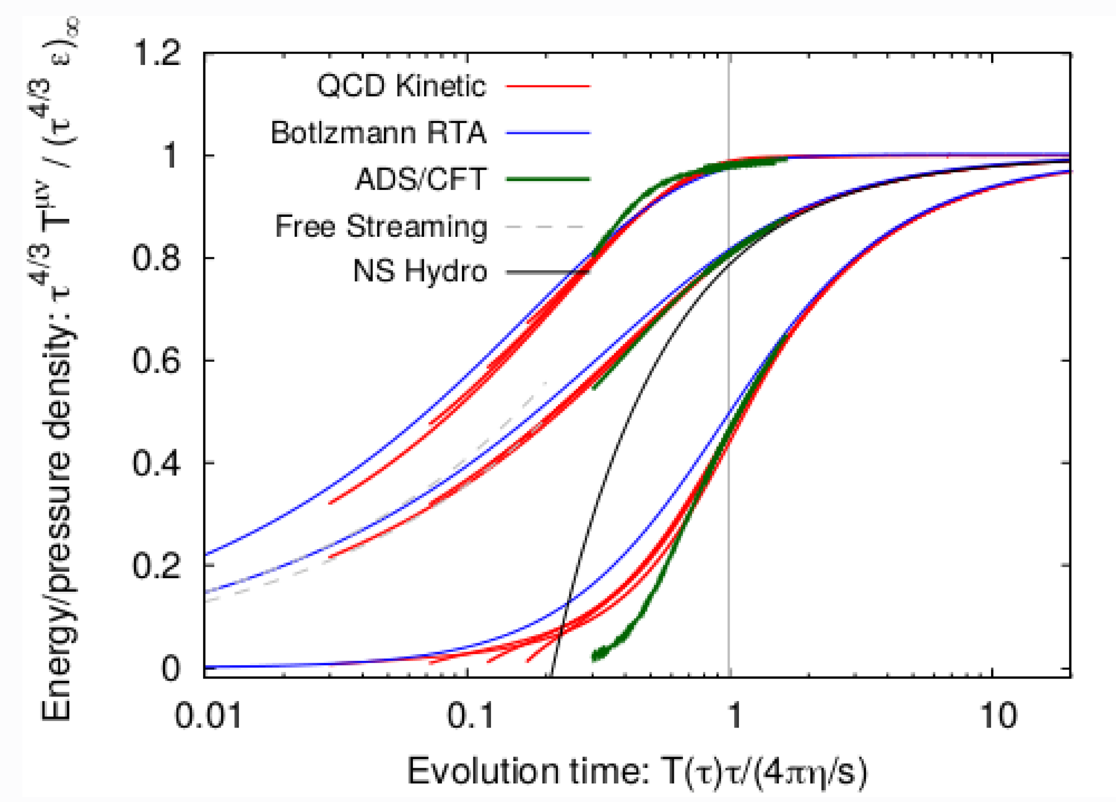
$$\frac{dT}{d\tau} + \frac{T}{3\tau}\left(1 + \frac{c_{20}}{2\sqrt{5}}\right) = 0 \qquad \frac{d\mathfrak{c}}{d\tau} + \frac{2}{3\tau}\left(\mathcal{B}\mathfrak{c} + c_{20}\mathcal{D}\mathfrak{c} + \mathcal{A}\right) = -\frac{\mathfrak{c}}{\tau_r}$$

while the equations of motion of the linearized moments are

$$\frac{\partial \delta c_l^m}{\partial \tau} + 4\frac{\partial T}{\partial \tau}\delta c_l^m + \frac{1}{\tau}\mathcal{F}\delta c_l^m - i\frac{k_T}{2}\left(\mathcal{G}_{l+1}^{m+1}\delta c_{l+1}^{m+1} + \mathcal{H}_{l-1}^{m-1}\delta c_{l-1}^{m-1}\right) = -\frac{1}{\tau_r}\delta c_l^m - \frac{1}{4\tau_r}\mathcal{K}(\tau)\delta c_0^0 - \frac{1}{\tau_r}\left(\mathcal{I}_1^{-1}(\tau)\delta c_1^{-1} + \mathcal{J}_1^1(\tau)\delta c_1^1\right)$$

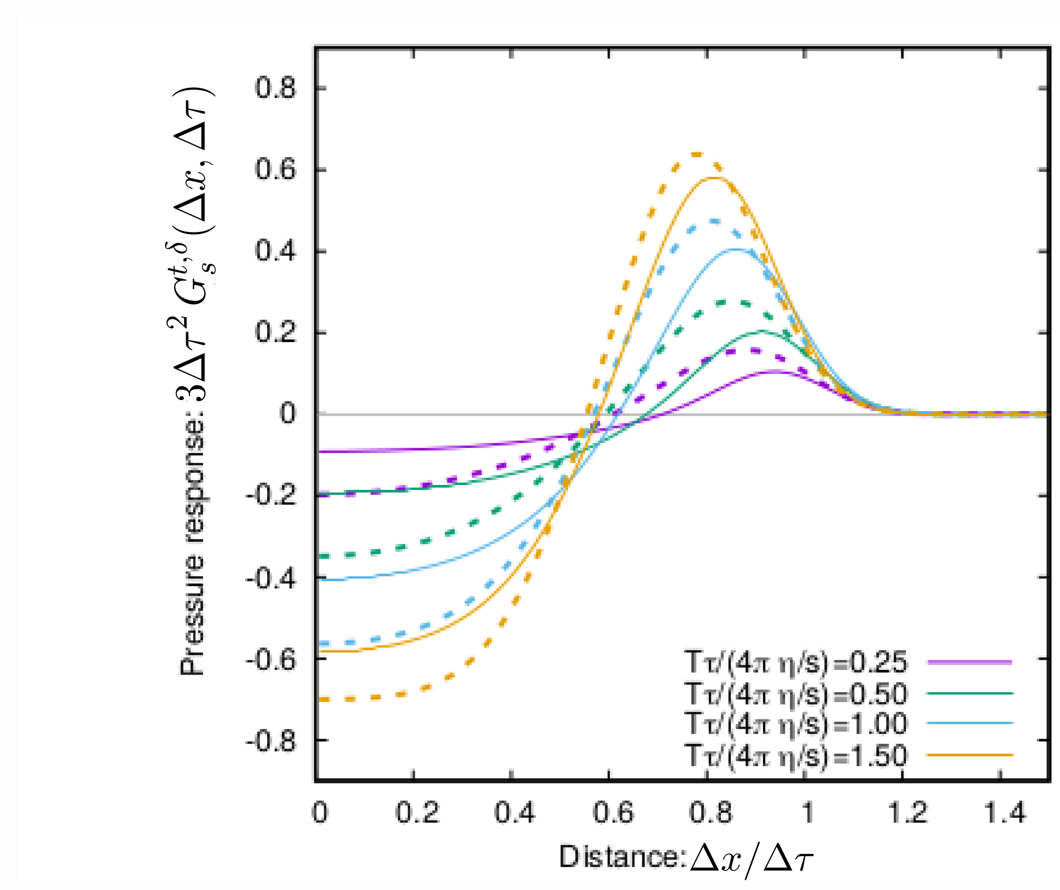
The scaling behavior of Greens functions is manifest at the level of EOMs.

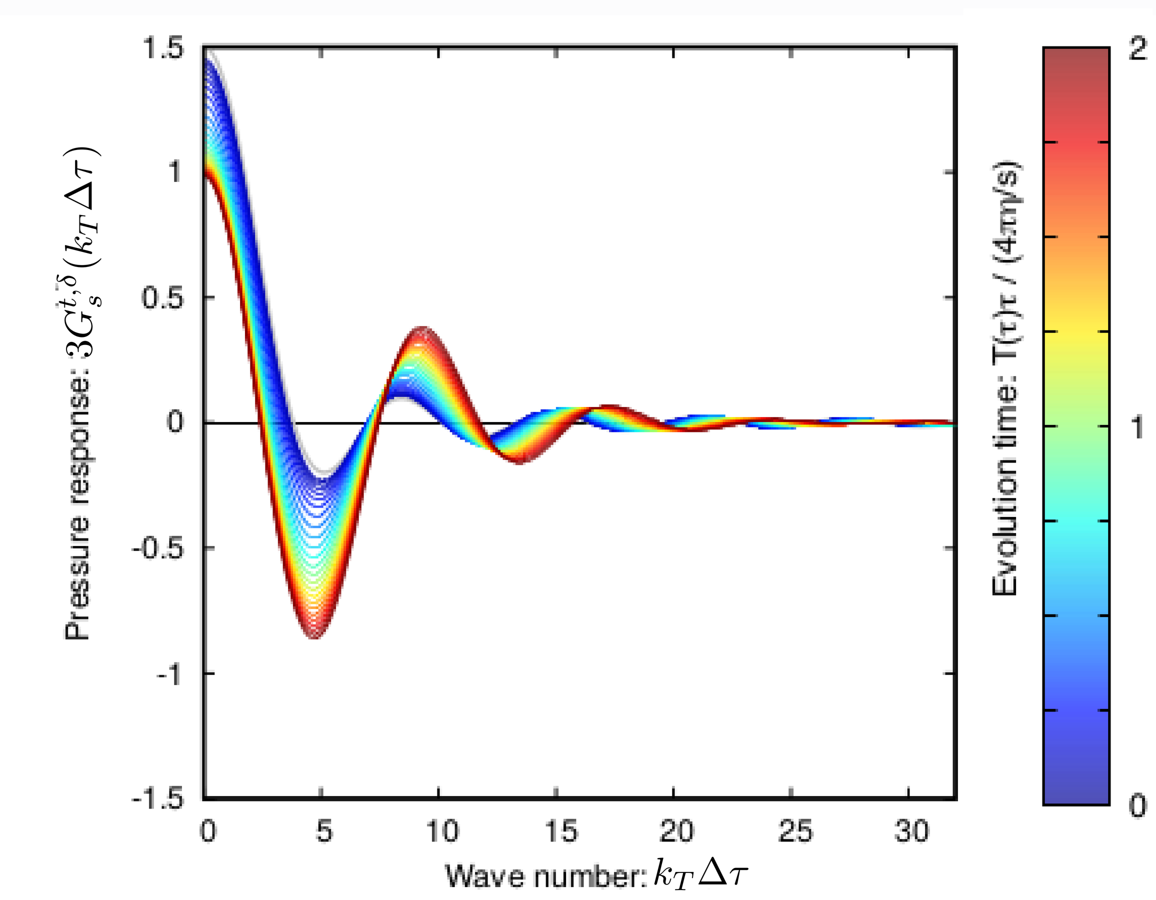
Evolution of the boost invariant homogeneous background

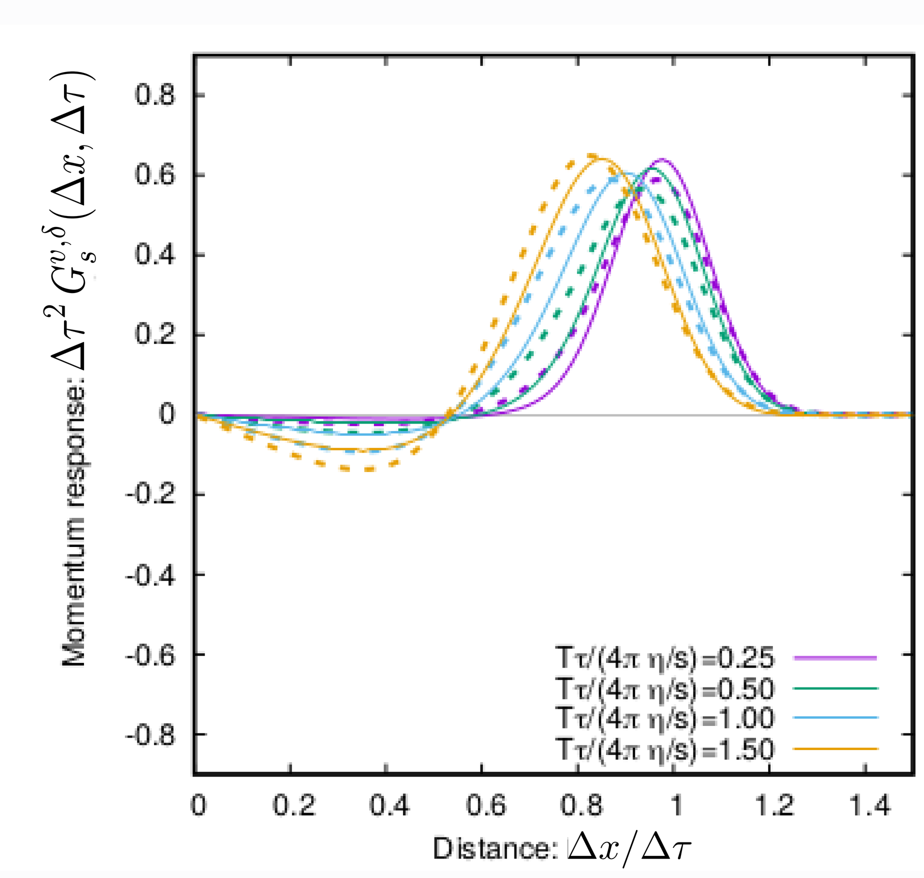


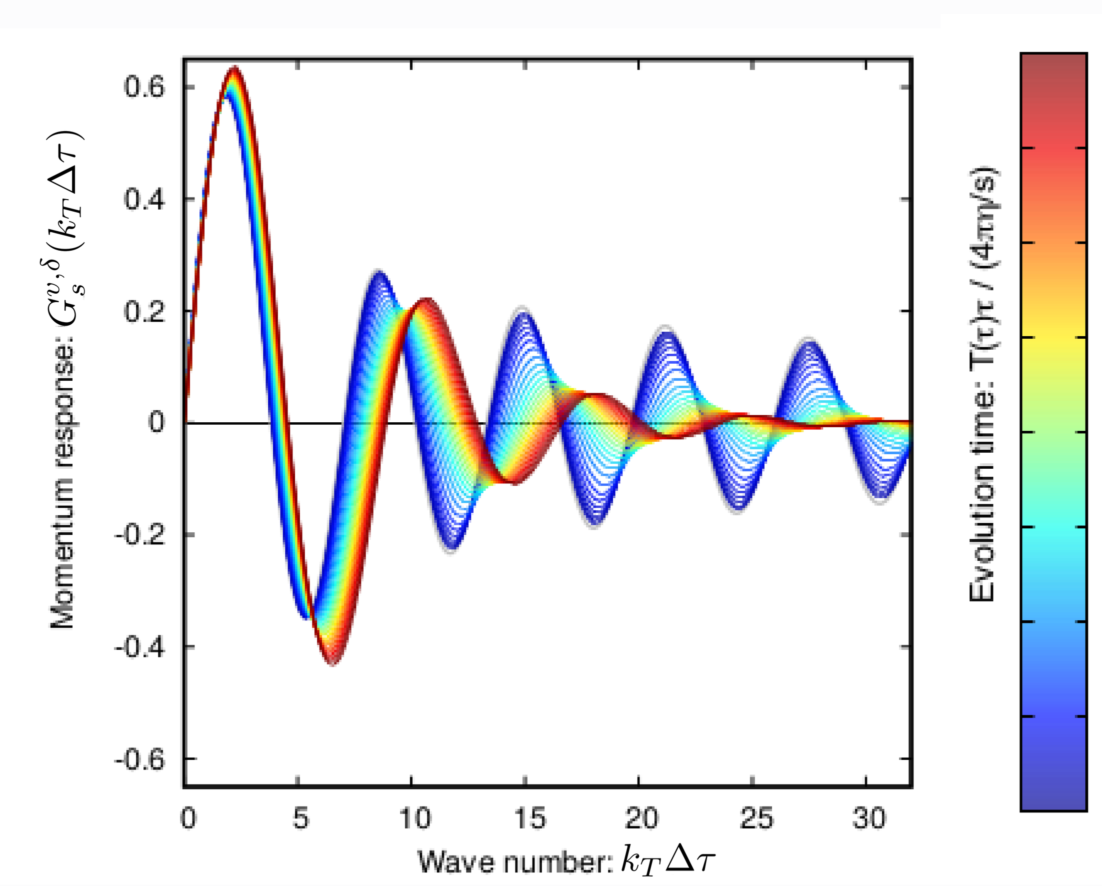
- Irrespective of the microscopic details of the particular theory we observe a smooth transition from the approximate early-time free-streaming behavior towards a universal hydrodynamic attractor at late-times.
- In the intermediate stage prior to the full hydrodynamization there is no universal behavior and a mismatch between different theoretical frameworks is observed.

Energy-momentum perturbations around Bjorken flow









Non-equilibrium Green's function describe linear response to transverse gradients

$$T^{\mu\nu}(\tau, \mathbf{x}) \simeq \underbrace{T^{\mu\nu}_{\mathrm{BG},\mathbf{x}}(\tau)}_{\text{non-eq. evolution of (local) avg. background}} + \underbrace{\int_{\odot} d^2\mathbf{x}_0\, G^{\mu\nu}_{\alpha\beta}(\tau, \tau_0, \mathbf{x}, \mathbf{x}_0)\, \delta T^{\alpha\beta}(\tau_0, \mathbf{x}_0)}_{\text{non-eq. evolution of local fluctuations of energy-momentum tensor}}$$

We observe a similar behavior of background evolution and Greens functions in pQCD and RTA.

Conclusions &Outlook

In this work we develop a new method to calculate non-equilibrium Green functions for far-from-equilibrium weakly coupled systems. The mathematical problem of solving the RTA Boltzmann equation and its linearized version is mapped into solving a coupled set of linear ODEs of their moments. Studying the dynamics of the moments allows us to assess the response to transverse gradients, explore additional features (longitudinal dynamics, …), obtain analytic insights and study the sensitivity to microscopic details of the non-equilibrium dynamics of the QGP with KoMPoST.

References

[1] M. Martinez et. al., PRD99 (2019) 116012, 1805.07881

[2] A. Kurkela et. al., PRC99 (2019) 034910, PRL122 (2019) 122302

[3] S. Kamata, M. Martinez and S. Schlichting, Forthcoming