Dynamical Systems and Nonlinear Transient Rheology of the Far-From-Equilibrium Bjorken Flow

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Abstract

For the Bjorken flow, the hydrodynamization of different moments of the distribution function is investigated by analyzing its kinetic equations. The solutions of these equations are written as multi-parameter transseries. At a given order of the perturbative expansion of each mode, the transport coefficients get effectively renormalized by summing over all the non-perturbative sectors appearing in the transseries. A new non-perturbative dynamical renormalization scheme is born out of this formalism that goes beyond the realms of linear response theory. Furthermore, we show that the leading dissipative correction to the distribution function is determined by not only the effective shear viscosity but also a new non-hydrodynamic moment. The survival of this mode is related with the nonlinear mode-to-mode coupling with the shear viscous term.

Kinetic equations and transseries

The Boltzmann equation for the Bjorien flow in the relaxation time approximation is

$$\partial_\tau f\left(\tau, p_T, p_\zeta\right) = -\frac{1}{\tau_r(\tau)}\left[f\left(\tau, p_T, p_\zeta\right) - f_{\rm eq}(-u\cdot p/T)\right]$$

with

$$f\left(\tau, p_T, p_\zeta\right) = f_{eq}\left(\frac{p^\tau}{T}\right)\left[\sum_{n=0}^{N_n}\sum_{l=0}^{N_l} c_{nl}(\tau)\mathcal{P}_{2l}\left(\frac{p_\zeta}{\tau p^\tau}\right)\mathcal{L}_n^{(3)}\left(\frac{p^\tau}{T}\right)\right]$$

The hydrodynamical variables are written as functions of the moments, i.e., the effective shear viscosity $\overline{\pi} \sim c_{01}$. The evolution of the moments c_{nl} is determined by solving their kinetic equations.

($w = \tau T^{1-\Delta}$ is the inverse Knudsen number)

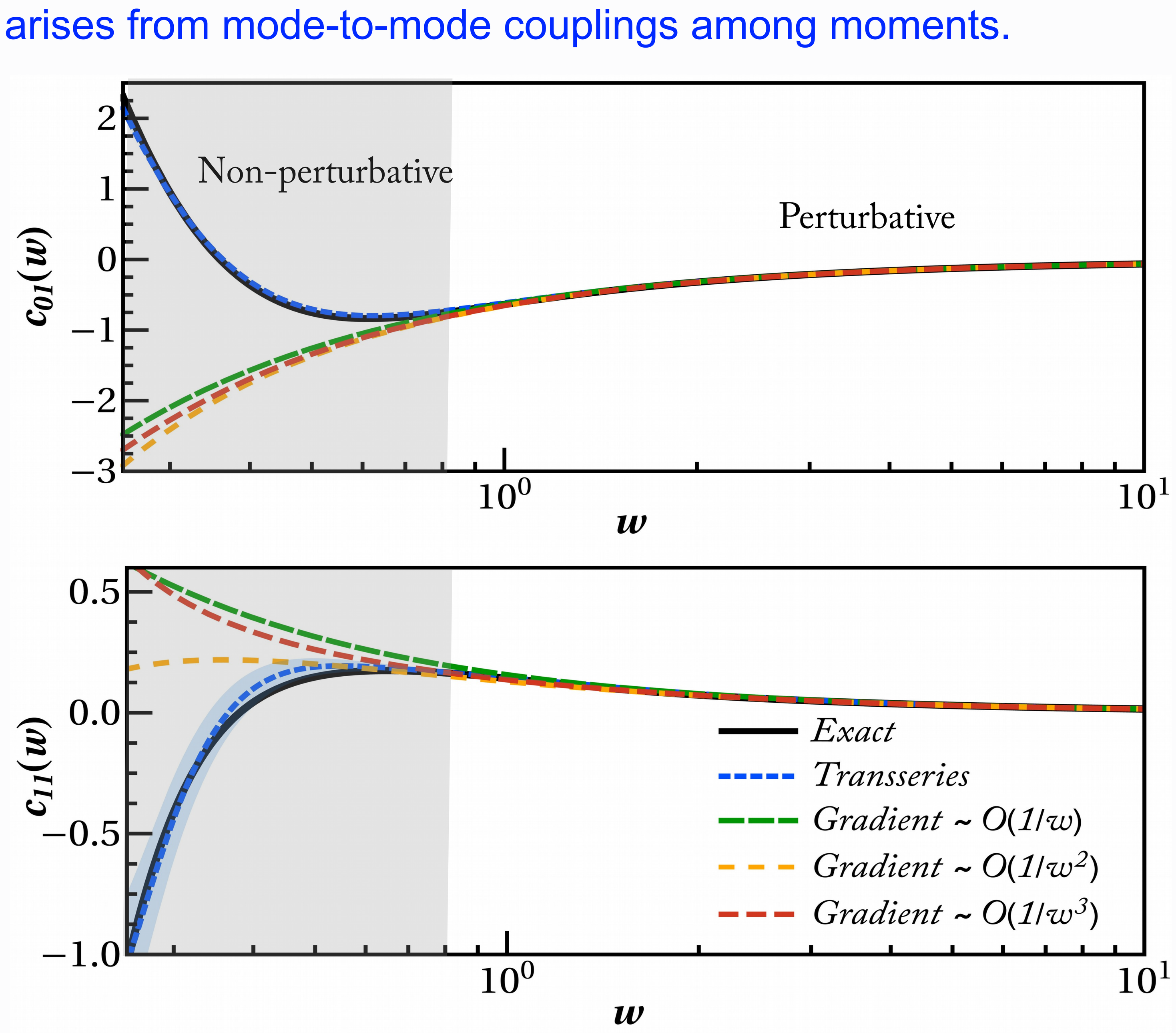
$$\left(1 - \frac{c_1}{20}\right)\frac{d\mathbf{c}}{dw} + \left[\hat{\Lambda}\mathbf{c} + \frac{1}{w}\left(\mathfrak{B}\mathbf{c} + \gamma\right)\right] = 0$$

- Matrix Λ characterizes the relaxation time scale of the fluid
- Matrix encodes the physics of non-linear mode-to-mode couplings among moments

The nonlinear ODEs for the moments admit solutions which are written as transseries. For instance, when one considers only the solution of slowest non-hydro mode c_{01}

$$c_{01} = \underbrace{\sum_{k=0} a_k w^{-k}}_{\text{Gradient expansion (perturbative)}} + \underbrace{\sum_{\substack{n=1\\k=0}}\left[u_k^n\left(\sigma e^{-Sw} w^{-b_1}\right)^n\right] w^{-k}}_{\text{Non-perturbative sectors}}$$

- The late-time perturbative gradient expansion is divergent and independent of the initial conditions
- Real part of the parameter σ is the initial condition
- Lyapunov exponents Λ characterizes the decay rate of the non-perturbative sector of the solutions for the moments c_{nl}.
- The non-integer power law behavior of the fluctuation w^{-b_1} arises from mode-to-mode couplings among moments.

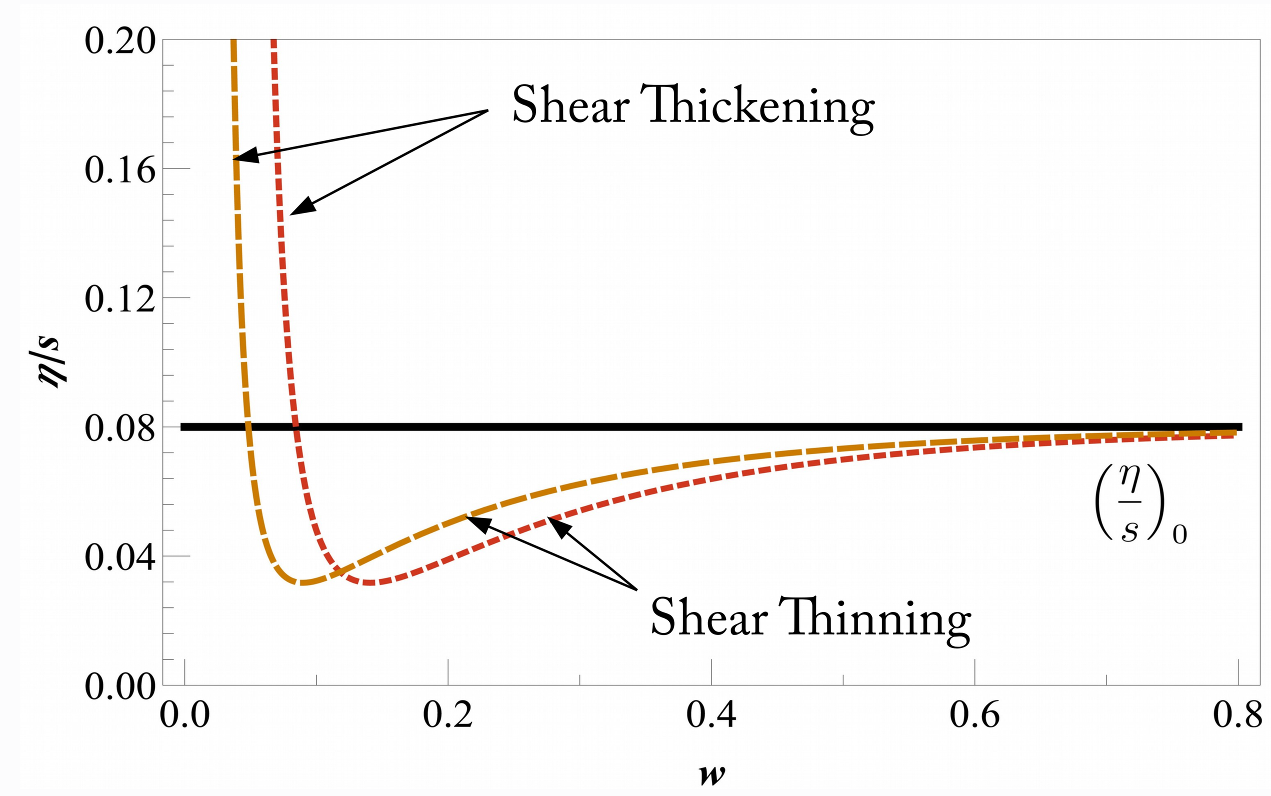


Dynamical renormalization of transport coefficients

The group properties of a RG flow are satisfied by our dynamical system for the moments. Accordingly, we can systematically compute the renormalized transport coefficients using the fact that they appear as expansion coefficients for the resummed moments.

For instance, the RG flow equation for the effective η/s is

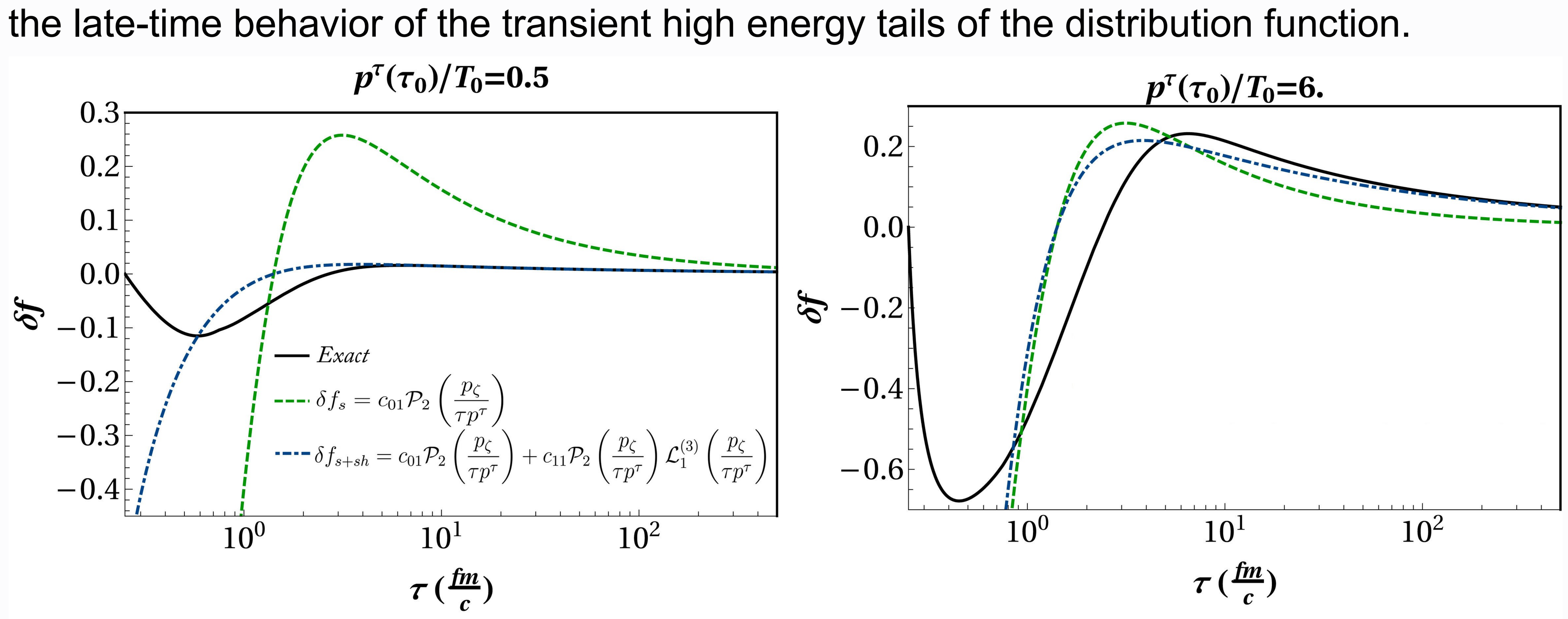
$$\frac{d\,(\eta/s)}{d\log w} = \beta_1(w)$$



The solutions of the RG flow equation for η/s feature a transition to their equilibrium fixed which is a neat characteristic of transient non-Newtonian behavior. In general, in our approach the transport coefficients like η/s become nonlinear functions of the gradient size so they trace back the macroscopic deformation history of the fluid and thus, its rheology.

Transient energy tails and late-time scaling of the distribution function

The kinetic equations of the moments unveils the existence of a new non-hydrodynamic mode $c_{11} \sim (\tau T)^{-1}$ at late times which is the same as the usual Navier-Stokes shear viscous component. The late-time survival of this mode arises from the nonlinear mode-to-mode coupling $c_{01}c_{11}$ which dominates close to the late-time asymptotic fixed point. This non-hydrodynamic mode is the slowest high energetic mode and it determines quantitatively the late-time behavior of the transient high energy tails of the distribution function.



The evolution of the deviation to thermal equilibrium $\delta f = (f - f_{eq})/f_{eq}$ shows that the late-time asymptotic regime of the distribution function is controlled uniquely by two modes: the effective shear viscosity c_{01} and slowest highly energetic mode c_{11}.

Conclusions

We investigate the nonlinear dynamics of a far-from-equilibrium weakly coupled plasma undergoing Bjorken expansion by solving the moments of one-particle distribution function. Using multi-parameter transseries of moments, we develop a new non-perturbative dynamical renormalization scheme that goes beyond the linear response theory. As a result, this provides a new description for the transport coefficients in the far-from-equilibrium regimes and thus hydrodynamics is successfully formulated to describe the transient rheological behavior of the fluid in high energy collisions where Kn and Re^{-1} are large. Furthermore, we discovered a new slowest energy mode c_{11} which characterizes properly the relaxation of transient energy tails at late times.

References

A. Behtash, C. N. Cruz-Camacho, S. Kamata, M. Martinez; arXiv:1805.07881(2018)

A. Behtash, S. Kamata, M. Martinez, H. Shi; PRD 99, 116012 (2019)