Double Inclusive Gluon Production in a Biased Ensemble

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Introduction

The double gluon inclusive production cross section is computed in a re-weighted ("biased") ensemble of small-x color fields. In particular, we obtain the cross section and the azimuthal harmonics for target configurations corresponding to an increased gluon density from $k_T=\Lambda$ to $k_T=Q$.

Biased Ensemble

Calculating small-x observables with an average over a biased ensemble:

$$\langle \mathcal{O}\rangle_b = \int \mathcal{D}\rho\, W[\rho]\;\; \mathrm{b}[\rho]\, \mathcal{O}[\rho]$$

The bias can be written in terms of the covariant gauge gluon distribution:

$$b[X] \equiv \exp\left(\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\, t(\vec{k})\, X(\vec{k})\right)$$

$$X(\vec{k}) \equiv g^2 \mathrm{tr}\, A^+(\vec{k})A^+(-\vec{k})$$

Constraint Effective Potential

We integrate out fluctuations which do not affect the covariant gauge gluon distribution

$$\int \mathcal{D}\rho\; \delta\left(X(\vec{k}) - \frac{g^4}{k^4}\mathrm{tr}\,\rho(\vec{k})\rho(-\vec{k})\right)\, W[\rho] = e^{-V_{\rm eff}[X]}$$

Assuming Gaussian fluctuations of the color charge density,

$$W[\rho] = \exp\left[-\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\,\frac{\rho(\vec{k})\rho(-\vec{k})}{\mu^2(\vec{k})}\right]$$

one obtains the effective potential

$$V_{\rm eff}[X(\vec{k})] = \int \frac{\mathrm{d}^2 k}{(2\pi)^2}\left[\frac{k^4}{g^4\mu^2(k)} X(\vec{k}) - \frac{1}{2}A_\perp N_c^2 \log X(\vec{k})\right]$$

The bias

$$t(\vec{k}) = \left(1 - \frac{1}{\eta(\vec{k})}\right)\frac{k^4}{g^4\mu^2(k)}$$

leads to the biased gluon distribution, at the stationary point,

$$X(\vec{k}) = \eta(\vec{k})\, X_s(k^2)$$

$$X_s(k^2) = \frac{1}{2}N_c^2 A_\perp \frac{g^4\mu^2(k)}{k^4}$$

Two Gluon Inclusive Cross Section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q} = 16N_c^2(N_c^2-1)\,g^{12}\,\frac{A_\perp\Lambda^2}{p^4q^4\Lambda^4}\,\frac{\mu_T^4\mu_P^4}{(2\pi)^2}\,(\mathcal{A}+\mathcal{B}+\mathcal{C})$$

$\vec{p}, \vec{q}$ are the small x gluon transverse momenta

Λ is an infrared cutoff for Glasma graphs

$\mathcal{A}$ is proportional to the squared uncorrelated cross section

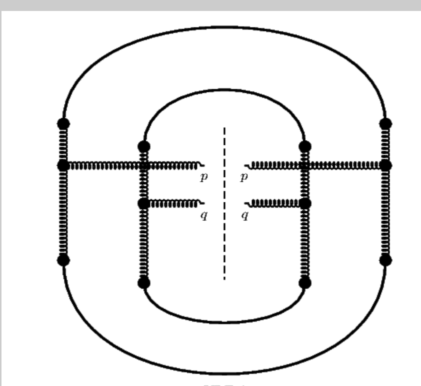
$\mathcal{C}$ are the HBT like contributions $\sim \delta^2(\vec{p}\pm\vec{q})$ (not calculated here)

$\mathcal{B}$ is the rest of the cross section

We'll calculate the angularly averaged cross section (where the angle is between the produced gluons):

$$\langle e^{in\phi}\rangle = \frac{\int \frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q}\, e^{in\phi}\,\mathrm{d}\phi}{\int \frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q}\,\mathrm{d}\phi}$$

Independent Production



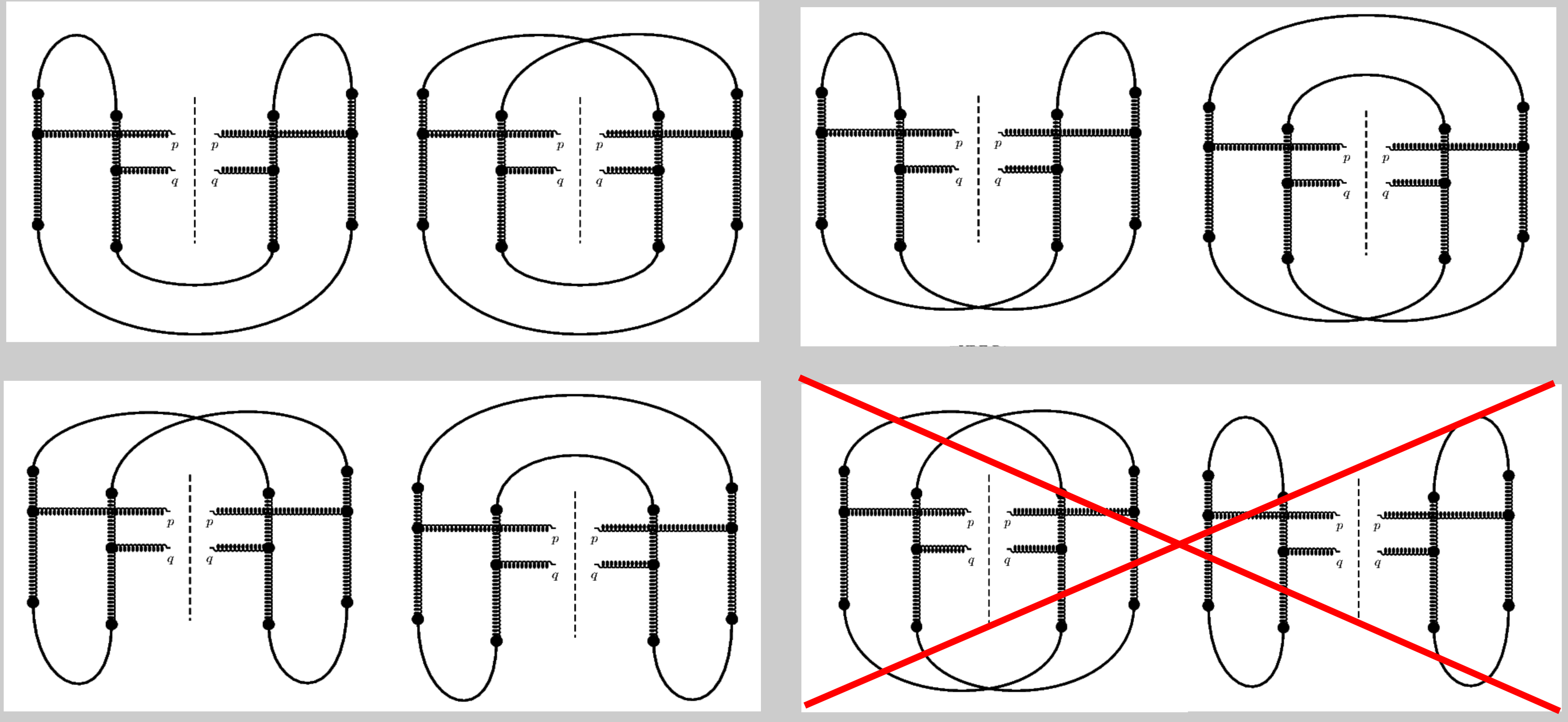
$$\mathcal{A} \propto \frac{(N_c^2-1)A_\perp}{(2\pi)^2}\int_{\Lambda^2}^{\infty}\mathrm{d}^2\vec{k}_1\,\Phi_P(\vec{k}_1)\Phi_T(\vec{k}_1-\vec{p})\cdot$$

$$\cdot\int_{\Lambda^2}^{\infty}\mathrm{d}^2\vec{k}_2\,\Phi_P(\vec{k}_2)\Phi_T(\vec{k}_2-\vec{q})$$

Where

$$\Phi(\vec{k}) = \frac{g^2\mu^2}{k^2}\,\frac{X(\vec{k})}{X_s(k^2)}$$

Correlated Production; Glasma Graphs



$$\mathcal{B} \propto \int_{\Lambda^2}^{\infty}\mathrm{d}^2\vec{k}\,\Phi_T(\vec{k}-\vec{p})\Phi_P(\vec{k})\Big[\Phi_P(\vec{k})\Phi_T(\vec{k}-\vec{q}) + \Phi_T(\vec{k}-\vec{p})\Phi_P(\vec{k}-\vec{p}-\vec{q}) +$$

$$\frac{1}{8}\Phi_T(\vec{k}-\vec{p})\Phi_P(\vec{k}-\vec{p}-\vec{q})\frac{f(\vec{k},\vec{p},\vec{q})}{(\vec{k}-\vec{p}-\vec{q})^2k^2(\vec{k}-\vec{q})^2} + (\vec{q}\to-\vec{q})$$

$$f(\vec{k},\vec{p},\vec{q}) = \left[k^2(\vec{k}-\vec{q})^2 + (\vec{p}+\vec{q}-\vec{k})^2(\vec{p}-\vec{k})^2 - p^2(2\vec{k}-\vec{q}-\vec{p})^2\right]\cdot$$

$$\left[(\vec{p}+\vec{q}-\vec{k})^2(\vec{k}-\vec{q})^2 + k^2(\vec{p}-\vec{k})^2 - q^2(2\vec{k}-\vec{q}-\vec{p})^2\right]$$

Unbiased Ensemble: $\eta(\vec{k}) = 1$

The LLA cross section (keeping the leading angular and non-angular terms):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q} \approx 16N_c^2(N_c^2-1)g^{12}\frac{S_\perp}{p^4q^4\Lambda^2}\frac{\mu_T^4\mu_P^4}{(2\pi)^2}\Big[(N_c^2-1)(S_\perp\Lambda^2)\log\left(\frac{p^2}{\Lambda^2}\right)\log\left(\frac{q^2}{\Lambda^2}\right) +$$

$$+2\pi\log\left(\frac{\ell^2}{\Lambda^2}\right)\left(\frac{\Lambda^2}{(\vec{q}-\vec{p})^2}+\frac{\Lambda^2}{(\vec{q}+\vec{p})^2}\right)\left(\frac{q^2}{p^2}+\frac{p^2}{q^2}\right)\Big]$$

where $\ell^2 = \min[p^2, q^2, (\vec{p}\pm\vec{q})^2]$. The azimuthal harmonics are

$$\langle e^{2n\phi i}\rangle \approx \frac{4\pi}{(N_c^2-1)S_\perp}\frac{1}{\mid p^2-q^2\mid}\frac{q_<^{2n}}{q_>^{2n}}\left(\frac{q^2}{p^2}+\frac{p^2}{q^2}\right)\frac{\log\left(\frac{\ell^2}{\Lambda^2}\right)}{\log\left(\frac{p^2}{\Lambda^2}\right)\log\left(\frac{q^2}{\Lambda^2}\right)}$$

Boosting the gluon density between Λ and Q

$$\eta(\vec{k}) = 1 + \eta_0\Theta(Q^2-k^2)\Theta(k^2-\Lambda^2)$$

η_0 is a dimensionless parameter, Q is a momentum cutoff

The cross section for $Q^2 \gg p^2, q^2, (\vec{p}\pm\vec{q})^2$ is

$$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q}\right]_b = (1+\eta_0)^2\left[\frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q}\right]_{\rm unb.}$$

For $\Lambda^2 \ll Q^2 \ll p^2, q^2, (\vec{p}\pm\vec{q})^2$ the cross section is

$$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}y_p\mathrm{d}^2p\,\mathrm{d}y_q\mathrm{d}^2q}\right]_b \approx 16N_c^2(N_c^2-1)g^{12}\frac{S_\perp}{p^4q^4\Lambda^2}\frac{\mu_T^4\mu_P^4}{(2\pi)^2}\Big[(N_c^2-1)(S_\perp\Lambda^2)\left(\log\frac{p^2}{\Lambda^2}+\frac{\eta_0}{2}\log\frac{Q^2}{\Lambda^2}\right)\left(\log\frac{q^2}{\Lambda^2}+\frac{\eta_0}{2}\log\frac{Q^2}{\Lambda^2}\right) +$$

$$+2\pi\left(\frac{\Lambda^2}{(\vec{q}-\vec{p})^2}+\frac{\Lambda^2}{(\vec{q}+\vec{p})^2}\right)\left(\frac{q^2}{p^2}+\frac{p^2}{q^2}\right)\left(\log\frac{\ell^2}{\Lambda^2}+\frac{\eta_0}{2}\log\frac{Q^2}{\Lambda^2}\right)+\frac{\pi}{8}\eta_0^2\left(\frac{\Lambda^2Q^2}{(\vec{p}-\vec{q})^4}+\frac{\Lambda^2Q^2}{(\vec{p}+\vec{q})^4}\right)\Big]$$

The elliptic anisotropy is

$$\langle e^{2\phi i}\rangle_b \approx \frac{4\pi}{(N_c^2-1)S_\perp}\frac{1}{\mid p^2-q^2\mid}\frac{q_<^2}{q_>^2}\frac{\left(\frac{q^2}{p^2}+\frac{p^2}{q^2}\right)\left(\log\frac{\ell^2}{\Lambda^2}+\frac{\eta_0}{2}\log\frac{Q^2}{\Lambda^2}\right)+\frac{\eta_0^2}{16}\frac{Q^2}{(p^2-q^2)^2}(3q_>^2-q_<^2)}{\left(\log\frac{p^2}{\Lambda^2}+\frac{\eta_0}{2}\log\frac{Q^2}{\Lambda^2}\right)\left(\log\frac{q^2}{\Lambda^2}+\frac{\eta_0}{2}\log\frac{Q^2}{\Lambda^2}\right)}$$

The choice of η_0 also determines the excess gluon multiplicity,

$$\triangle N_g \approx \frac{1}{8\pi}N_c^2(S_\perp\Lambda^2)\,\eta_0\log\left(\frac{Q^2}{\Lambda^2}\right)$$

as well as the probability of such a gluon distribution in the unbiased ensemble,

$$-\log P[\eta(\vec{k})] = V_{eff}[\eta(\vec{k})] \approx \frac{1}{8\pi}N_c^2(S_\perp\Lambda^2)\,\eta_0\frac{Q^2}{\Lambda^2}$$

Outlook

We will extend the above to other biases (k dependent and anisotropic ones) and relate the parameters η_0 and Q^2 of the biased gluon distribution to observable quantities such as dN/dy and $<p_T>$.

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References: for the single inclusive gluon cross section in a biased ensemble see Dumitru, Kapilevich, Skokov, Nucl. Phys. A 974 (2018) 106

for an overview of azimuthal correlations in double gluon production see Schlichting, Tribedy, Adv. High Energy Phys. (2016) and refs therein