Understanding p_T-dependent flow fluctuations from initial geometry

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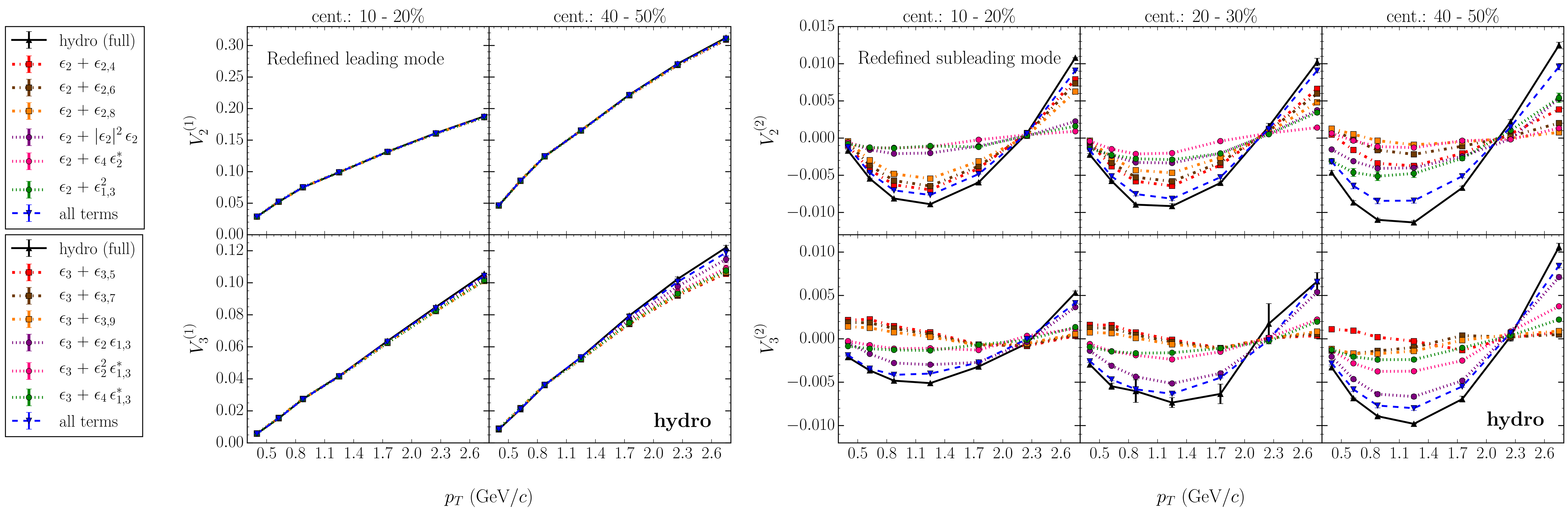

FIGURE 1: How different features of the **initial geometry** contribute to the **redefined PCA modes**. **Full predictions** of the leading (left) and subleading (right) modes are compared to **expectations from estimators**, for $Pb+Pb$ collisions at 5.02 TeV.

1 Introduction

Flow vectors reflect the **hydrodynamic response** of the quark-gluon plasma to the **initial geometry**. Defining

$$E\frac{dN}{d^3p}=\frac{1}{2\pi}N(p_T,y)\sum_{n=-\infty}^{\infty}\boldsymbol{V_n(p_T,y)}\,e^{-in\varphi}\,,$$

we can find approximate relations of the form

$$V_n(\boldsymbol{p})\simeq\kappa(\boldsymbol{p})\,\epsilon_n\,, \tag{1}$$

where the **eccentricities** ϵ_n characterize the initial geometry. To understand **p_T-dependent flow fluctuations**, we **generalize** (1) to multiple **linear and nonlinear terms** [1].

2 Mapping hydrodynamic response

- **Eccentricities** from cumulant-generating function of the transverse (entropy) density $\rho(\boldsymbol{x})$ [1]:

$$W(\vec{k})\equiv\ln\left(\int d^2x\,\rho(\vec{x})e^{i\vec{k}\cdot\vec{x}}\right)\equiv\sum_{\substack{m=0\\ n=-\infty}}^{\infty}W_{n,m}\,k^m e^{-in\phi_k}$$

$$\epsilon_{n,m}\equiv W_{n,m}/(W_{0,2})^{m/2}\,.$$

- **Differential flow**, $V_n(p_T)$, can be predicted by $\epsilon_{n,m}$:

$$V_2(p_T^a)\simeq\kappa_0^a\,\epsilon_{2,2}+\kappa_1^a\,\epsilon_{2,4}+\kappa_2^a\,\epsilon_{2,6}+\kappa_3^a\,\epsilon_{2,8}+\kappa_4^a\,|\epsilon_{2,2}|^2\epsilon_{2,2}+\kappa_5^a\,\epsilon_4\epsilon_2^*+\kappa_6^a\,\epsilon_{1,3}^2+\ldots \tag{2}$$

$$V_3(p_T^a)\simeq\kappa_0^a\,\epsilon_{3,3}+\kappa_1^a\,\epsilon_{3,5}+\kappa_2^a\,\epsilon_{3,7}+\kappa_3^a\,\epsilon_{3,9}+\kappa_4^a\,\epsilon_{2,2}\epsilon_{1,3}+\kappa_5^a\epsilon_{2,2}^2\epsilon_{1,3}^*+\kappa_6^a\,\epsilon_4\epsilon_{1,3}^*+\ldots \tag{3}$$

- Correlation between $V_n(p_T)$ and estimator $\Rightarrow$ performance!

2.1 Results

We **test** the estimators above within a hybrid **hydrodynamic model**, using T$_\text{R}$ENTO + MUSIC + ISS + UrQMD, with parameters for $Pb+Pb$ collisions at $\sqrt{s_{NN}}=5.02$ TeV and oversampled events [2].

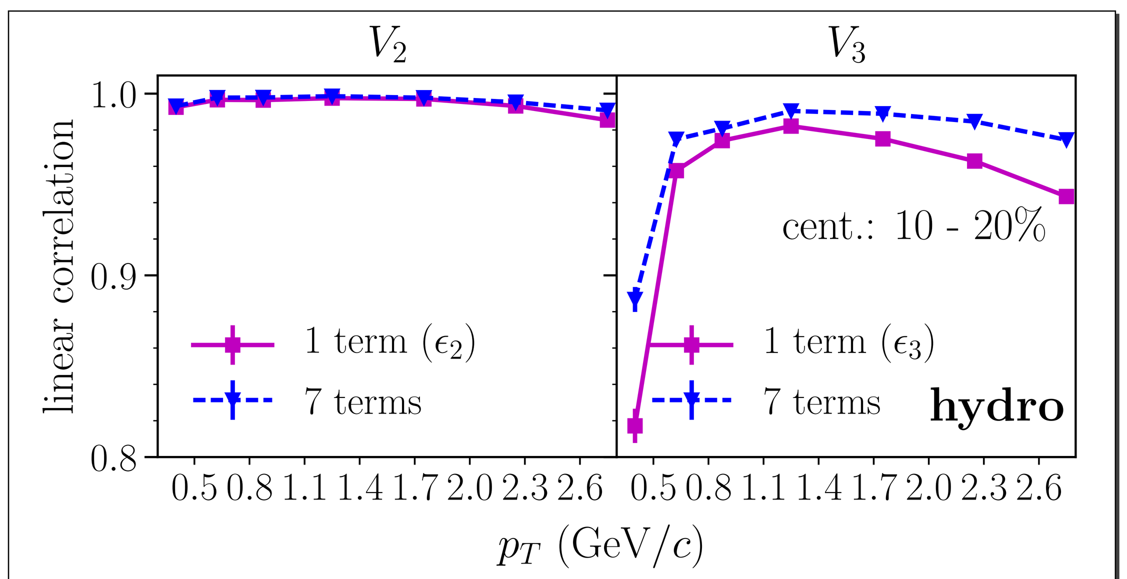

FIGURE 2: Eqs. (2) and (3) present **good predictors** for $V_2(p_T)$ and $V_3(p_T)$, even with a **single term**.

- Up to **few % gain** with $\epsilon_{1,3}^2$ and $\epsilon_{2,2}\epsilon_{1,3}$.
- **Excellent** for $p_T\gtrsim1$ GeV/c and 10–60% centrality.

3 Principal Component Analysis

Principal component analysis (PCA) provides a natural, concise visualization of **p_T-dependent two-particle correlations** [3]. It **isolates** linearly uncorrelated modes of **flow fluctuations**.

The second such mode can be dominated by **multiplicity fluctuations**. We **redefine** this PCA observable to reveal **fluctuations of the initial geometry**.

3.1 Standard definition [3]

- **Diagonalization** of the covariance matrix [3]

$$V_{n\Delta}^N(\mathbf{p}_1,\mathbf{p}_2)\overset{\text{hydro}}{=}\langle N(\mathbf{p}_1)\,N(\mathbf{p}_2)\,V_n(\mathbf{p}_1)\,V_n^*(\mathbf{p}_2)\rangle\,.$$

- **PCA modes** defined from $V_n^{(\alpha)}\equiv\sqrt{\lambda_n^{(\alpha)}}\psi_n^{(\alpha)}/\langle N\rangle$:

$$V_{n\Delta}^N\simeq\sum_{\alpha=1}^{k}\langle N(\mathbf{p}_1)\rangle\langle N(\mathbf{p}_2)\rangle\,V_n^{N(\alpha)}(\mathbf{p}_1)V_n^{N(\alpha)}(\mathbf{p}_2)\,. \tag{4}$$

- Eigenvalues are **strongly ordered** $\Rightarrow$ truncation at $k=2$.
- PCA of **multiplicity fluctuations** from $n=0$.

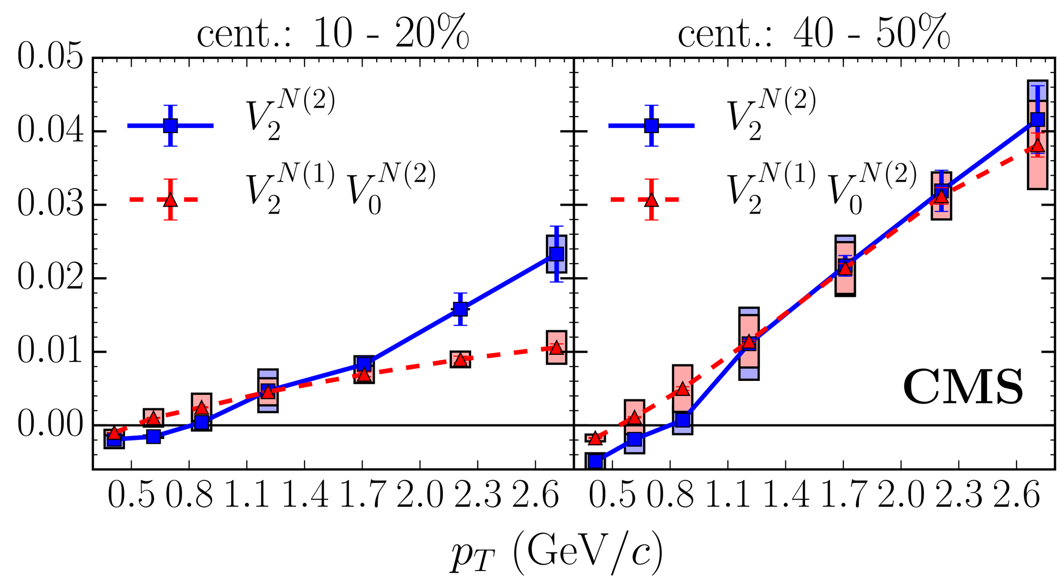

FIGURE 3: The **standard subleading PCA mode** for V_2 seen in **CMS** data can be **dominated by multiplicity fluctuations** [5]. Data for $Pb+Pb$ collisions at 2.76 TeV.

3.2 Multiplicity fluctuations [5]

- $\langle\Delta N(\boldsymbol{p}_1)\Delta N(\boldsymbol{p}_2)\rangle$ not compensated by Eq. (4).
- **Redundancy** between $V_2^{N(1)}$, $V_0^{N(2)}$ and $V_2^{N(2)}$ [5].
- See results from **CMS data** in **Figure 3**.
- Multiplicity fluctuations not predicted by the "mapping".

3.3 New prescription [5]

- Diagonalization of **redefined** covariance matrix [5]

$$V_{n\Delta}(\mathbf{p}_1,\mathbf{p}_2)\overset{\text{hydro}}{=}\langle V_n(\mathbf{p}_1)\,V_n^*(\mathbf{p}_2)\rangle\,.$$

- Most of the information is in the first **PCA modes**:

$$V_{n\Delta}\simeq\sum_{\alpha=1}^{k}V_n^{(\alpha)}(\mathbf{p}_1)V_n^{(\alpha)}(\mathbf{p}_2)\,.$$

- Alternatively, one might use $V_{n\Delta}^N(\mathbf{p}_1,\mathbf{p}_2)/\langle N(\mathbf{p}_1)N(\mathbf{p}_2)\rangle$ [5].
- **Predictions** for these observables are shown in **Figure 1**, along with estimates from **predictors** (2) and (3).

4 Final Remarks

- **Generalization** of $V_n\propto\epsilon_n$, with **p_T dependence** and **multiple terms**. [1].
- Eccentricities of **higher order m $\Rightarrow$ smaller scales**.
- The **standard subleading PCA mode** can be **dominated** by **multiplicity fluctuations** [5].
- We can understand how **p_T-dependent flow fluctuations** arise from **initial geometry** fluctuations.
- **For V_3**, the redefined **subleading PCA mode** is dominated by **nonlinear response**.
- **For V_2**, the redefined **subleading PCA mode** is sensitive to **higher-order eccentricities**.
- Promising probe of small-scale structure.

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