BROOKHAVEN
NATIONAL LABORATORY

Quantum entanglement

EPR paradox in high energy colliders

Kong Tu

Initial Stages 2019

BNL

Co-authors: Dmitri Kharzeev, Thomas Ullrich

Manuscript submitted and arXiv:1904.11974

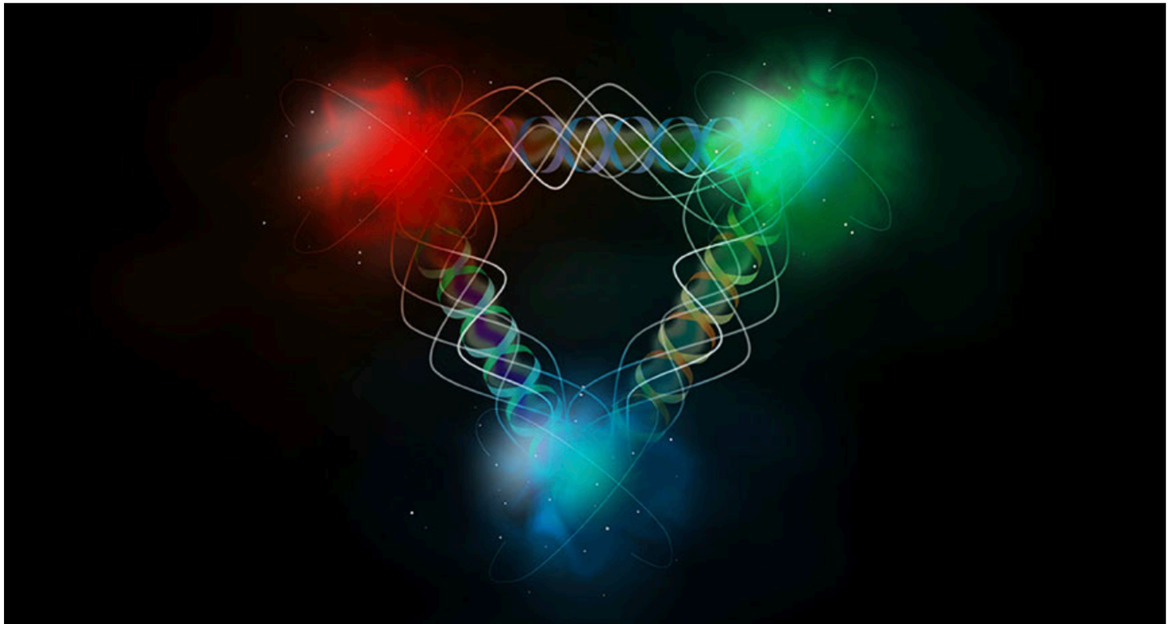
Science News

NEWS QUANTUM PHYSICS, PARTICLE PHYSICS

An experiment hints at quantum entanglement inside protons

LHC data suggests the subatomic particle's constituent quarks and gluons share weird links

BY EMILY CONOVER 11:18AM, MAY 17, 2019



TIED UP Protons contain smaller particles called quarks and gluons (illustrated). Experimental data suggest that quantum entanglement links those particles with one another.

SCIFY/SHUTTERSTOCK

https://www.sciencenews.org/article/experiment-hints-quantum-entanglement-inside-protons

What people are saying…

VoxFox · 12 days ago

More bogus science-news reporting. A theory paper about possible effects on the scale of one-trillionth of a proton is pure fantasy: NOT real news.

1 ^ | ⌄ · Reply · Share ›

What people are saying...

John Turner • 14 days ago

Hmm. Maybe this entanglement is why high-energy particle collisions have failed to produce those all-devouring quark-gluon plasma blobs we were warned about years ago, the ones that were going to grow unstoppably inside particle accelerators and devour the planet.

If so, it would likely be extremely dangerous to go looking for an Off switch to the intraprotonic entanglement.

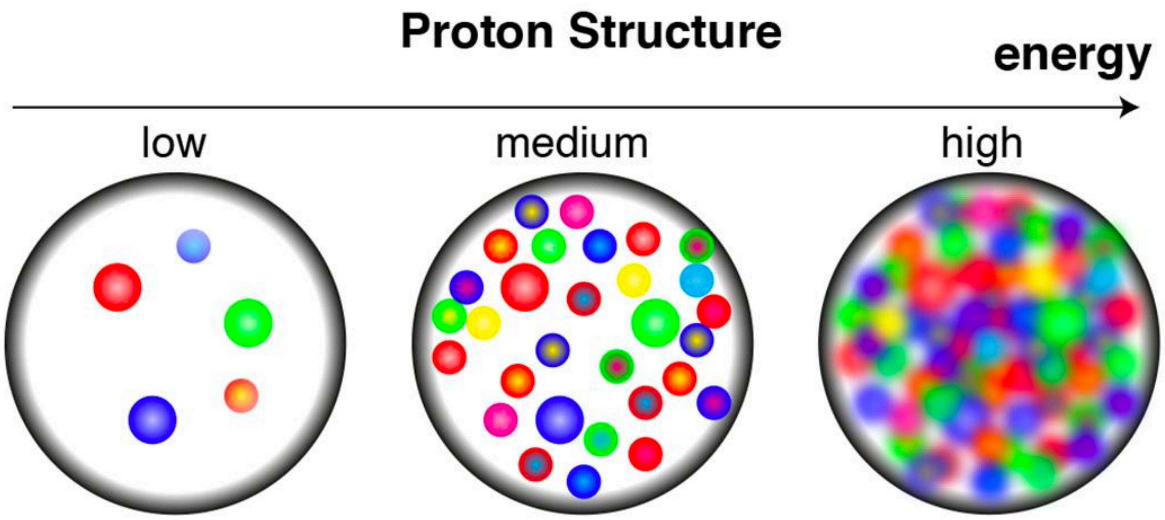
To quote the great Egon Spengler:

"Try to imagine all life as you know it stopping instantaneously, and every molecule in your body exploding at the speed of light."

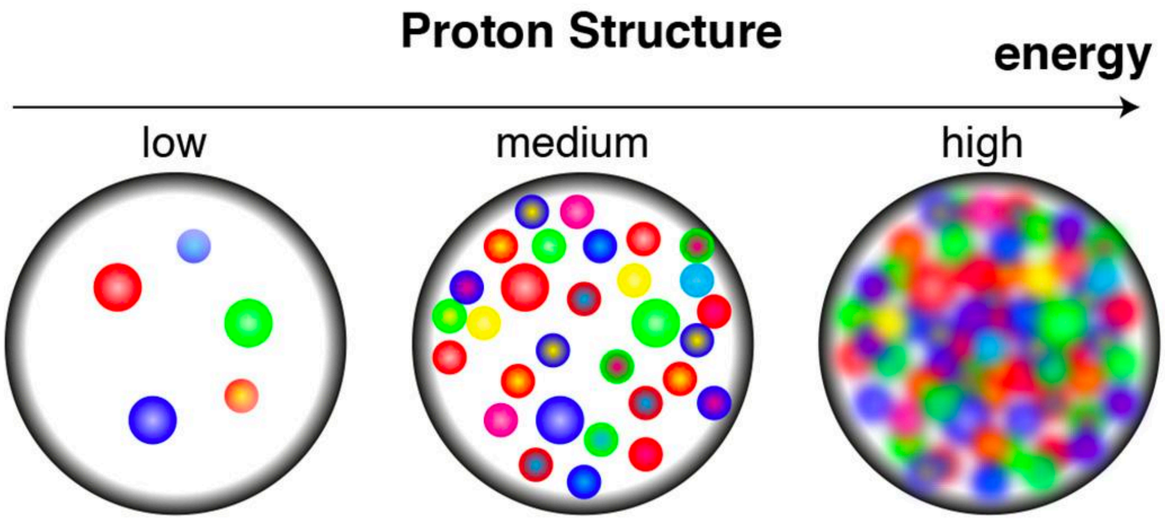
We'll just have to do it quite carefully, of course.

1 ^ | ˅ • Reply • Share ›

Building block - Nucleon



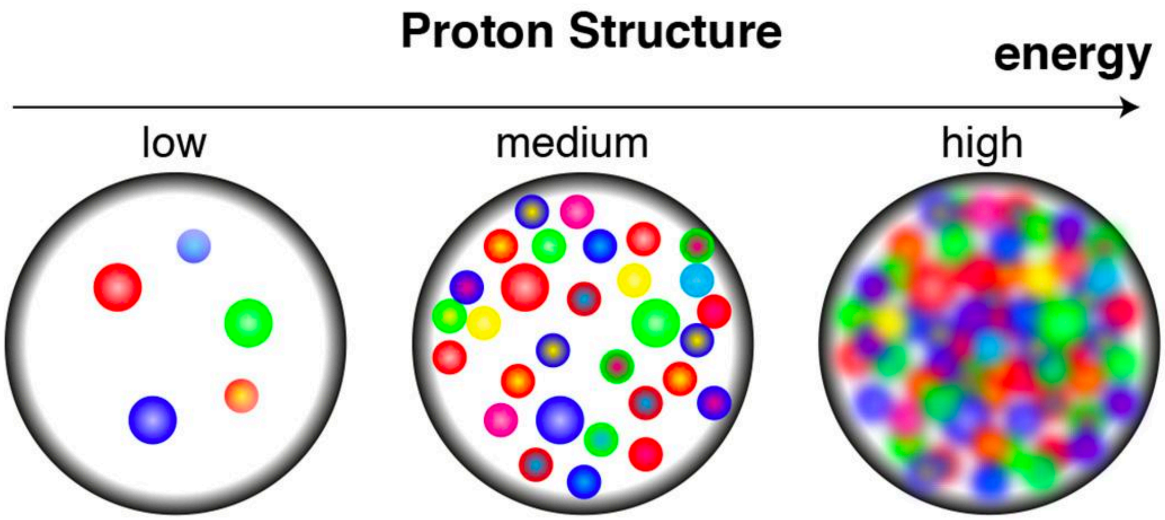
Building block - Nucleon



Parton model

- Based on “quasi-free” partons that are frozen in the Infinite momentum frame.

Building block - Nucleon



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Color confinement

- Partons are not just correlated, they cannot exist as free particles in nature

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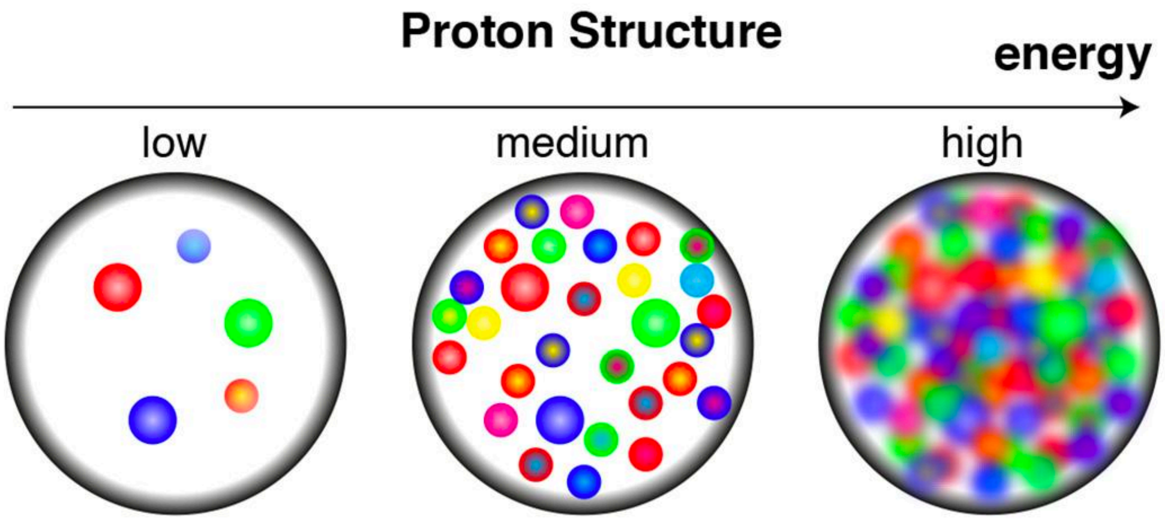

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One conceptual question arises:

- One set of incoherent partons corresponds to a non-zero von Neumann entropy $S \neq 0$

How to understand?

- Proton is a pure quantum mechanical state, its entropy is zero $S = 0$

Entanglement

Entanglement is the natural “picture” in addressing this conceptual question

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1. Definition:

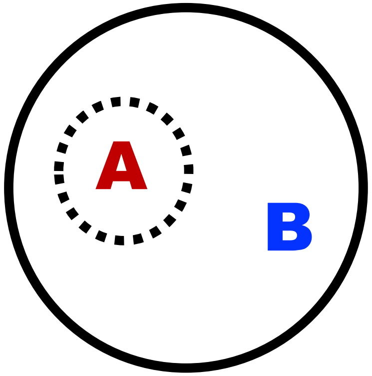
$|\Psi\rangle$ is a pure quantum state, density matrix is therefore $\rho_{tot} = |\Psi\rangle\langle\Psi|$

Entanglement Entropy (EE) is defined:

$$S_A = -\mathrm{Tr}\rho_{\mathrm{A}} \ln \rho_{\mathrm{A}}$$

, where $\rho_A \equiv \mathrm{Tr}_{\mathrm{B}}(\rho_{\mathrm{tot}})$, A and B are two complementary parts of $|\Psi\rangle$

pure quantum state



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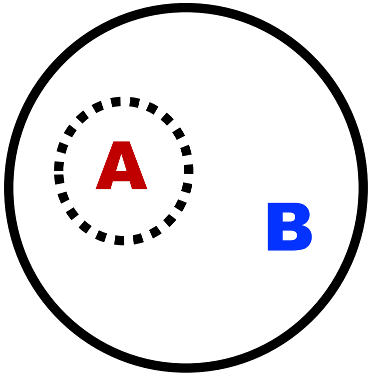
2. Take-home messages:

1) For the whole system ρ_{tot}, von Neumann entropy is zero by definition (i.e., proton)

2) **When measuring A only:**
   i. $S_{EE} > 0$ if A and B are entangled.
   ii. $S_{EE} = 0$ if A and B are independent.

pure quantum state



A two-body example

(i) $|\Psi\rangle = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \otimes \left[|\uparrow\rangle_B + |\downarrow\rangle_B\right]$

$$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right]\cdot\left[\langle\uparrow|_A + \langle\downarrow|_A\right].$$

Not Entangled

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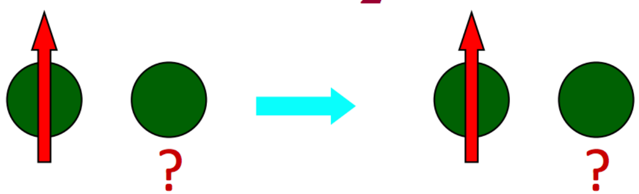

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(ii) $|\Psi\rangle = \left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right] / \sqrt{2}$

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$S_A = \log 2$

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Entangled

$S_A = \log 2$

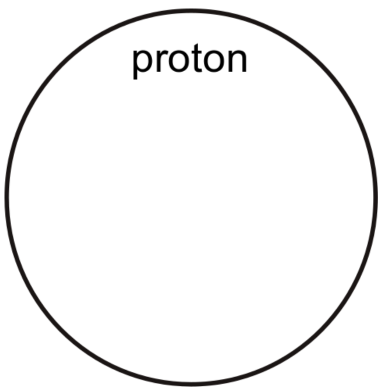
"EE is a measure of how much a given state is quantum mechanically entangled"

Taken from a lecture note of Tadashi Takayanagi, http://www2.yukawa.kyoto-u.ac.jp/~tadashi.takayanagi/CERNEE.pdf

Experiments at Colliders

(a)

before collision



Proton: a pure quantum state (by definition)

$$\mathrm{S}_{\mathrm{EE}} = 0$$

Experiments at Colliders

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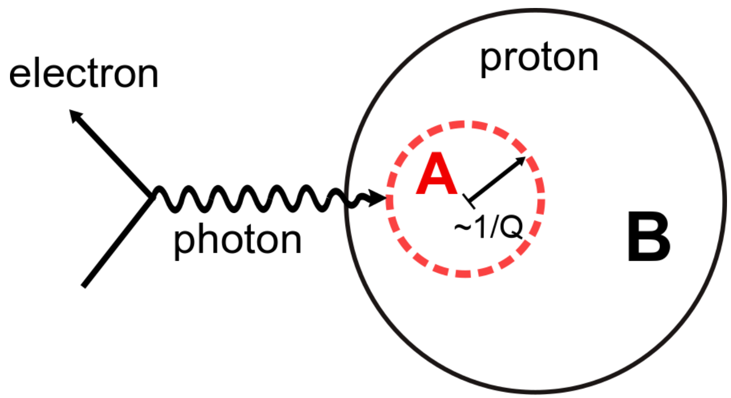
before collision

Proton: a pure quantum state (by definition)

$$S_{EE} = 0$$

(b)

hard collision



Hard interaction, fast enough to test entanglement, e.g.,

$$\frac{1}{Q} \sim 1 \text{ GeV}^{-1} \sim 0.2 \text{ fm}$$

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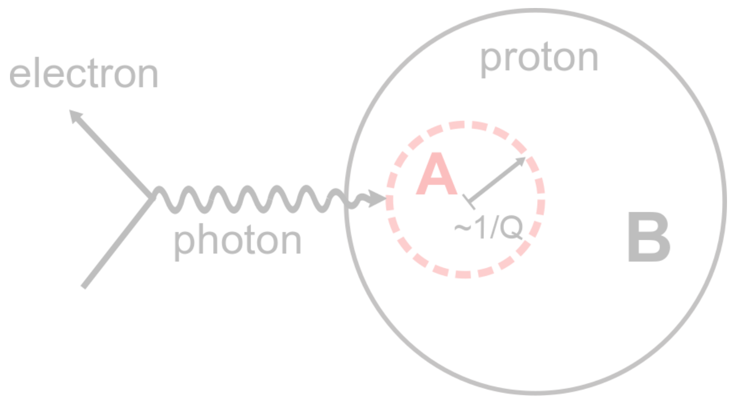

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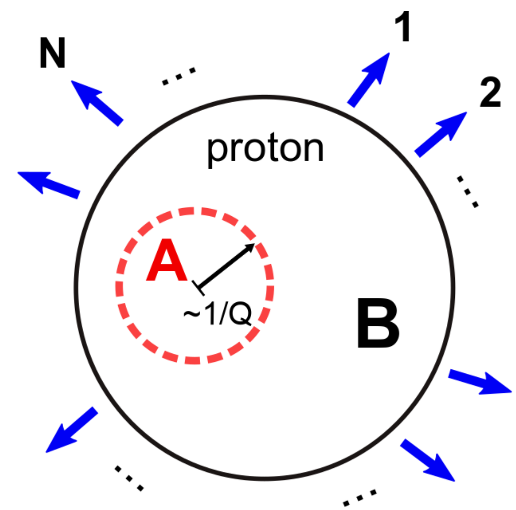
(c)

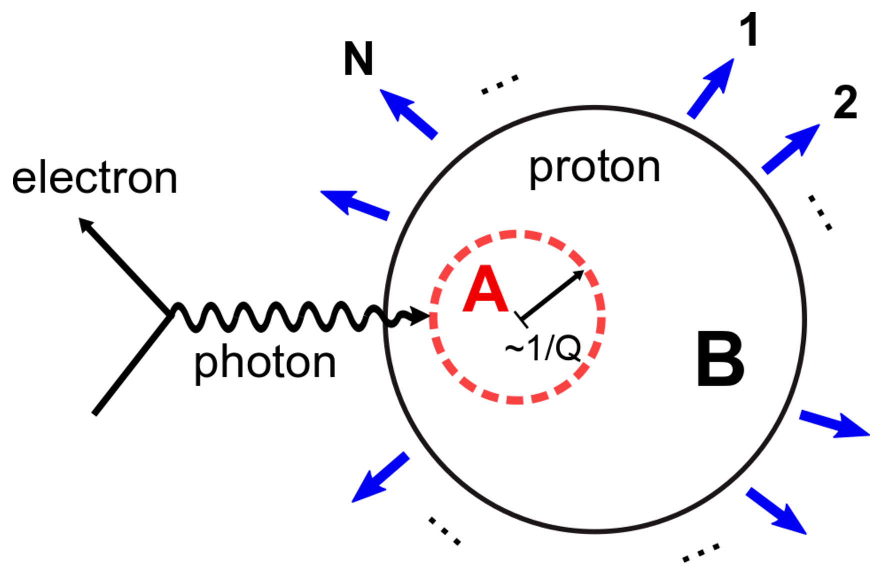
after collision



Hadronization and if A,B are entangled, entropy:

$$S^{A}_{EE} = S^{B}_{EE}$$

Principle and Practice

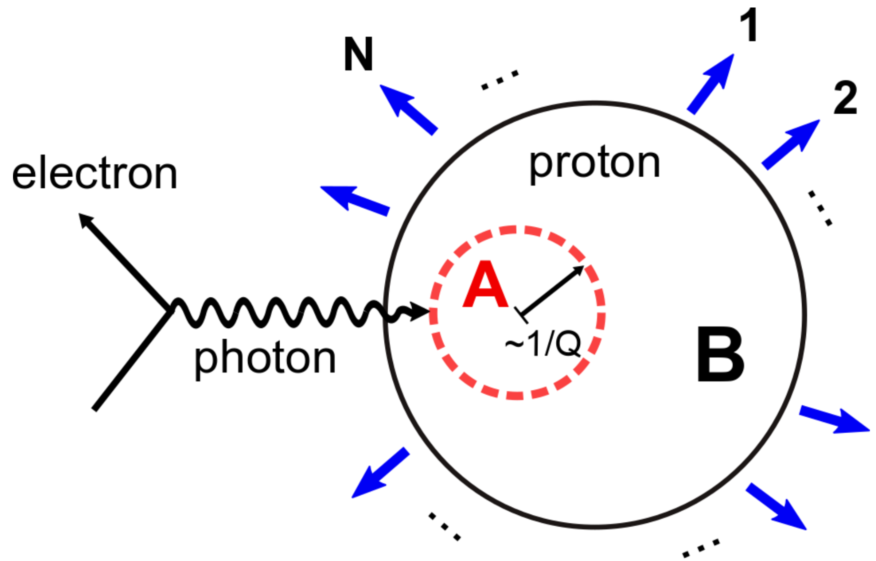


In principle

- Measure $\boldsymbol{S}_{\mathbf{A}}$ **and** $\boldsymbol{S}_{\mathbf{B}}$ independently, and directly test against each other.

- But partons don't live ☹.
- Need all hadrons from A and B

Principle and Practice



In principle

- Measure S_A **and** S_B independently, and directly test against each other.

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In practice

- **Theorists[1] made a prediction**

$$S_{EE} = \ln\left[xG\right]$$

at small x, e.g., $x < 10^{-3}$

- We have well constrained PDFs

1. D. Kharzeev and E.Levin, *Phys. Rev. D 95, 114008 (2017)*

A well-defined test

- At similar kinematics in x and Q^2 (region A), the S_{EE} can be checked from the entropy of finite-state hadron around region A

prediction

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experiment

$$S_{hadron} = -\sum P(N) \ln[P(N)]$$

Assuming entropy doesn't grow much

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- The event kinematics define the region of interest, using relation between x and rapidity,

$$\ln\left(\frac{1}{x}\right) \approx y_{beam} - y_{hadron}$$

(arXiv:hep-ph/9903536)

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For example,

fixed Q^2, and x, e.g., $\mathbf{x} \in (\mathbf{x_1},\mathbf{x_2})$ → Final-state hadrons $\mathbf{y} \in (\mathbf{y_1},\mathbf{y_2})$

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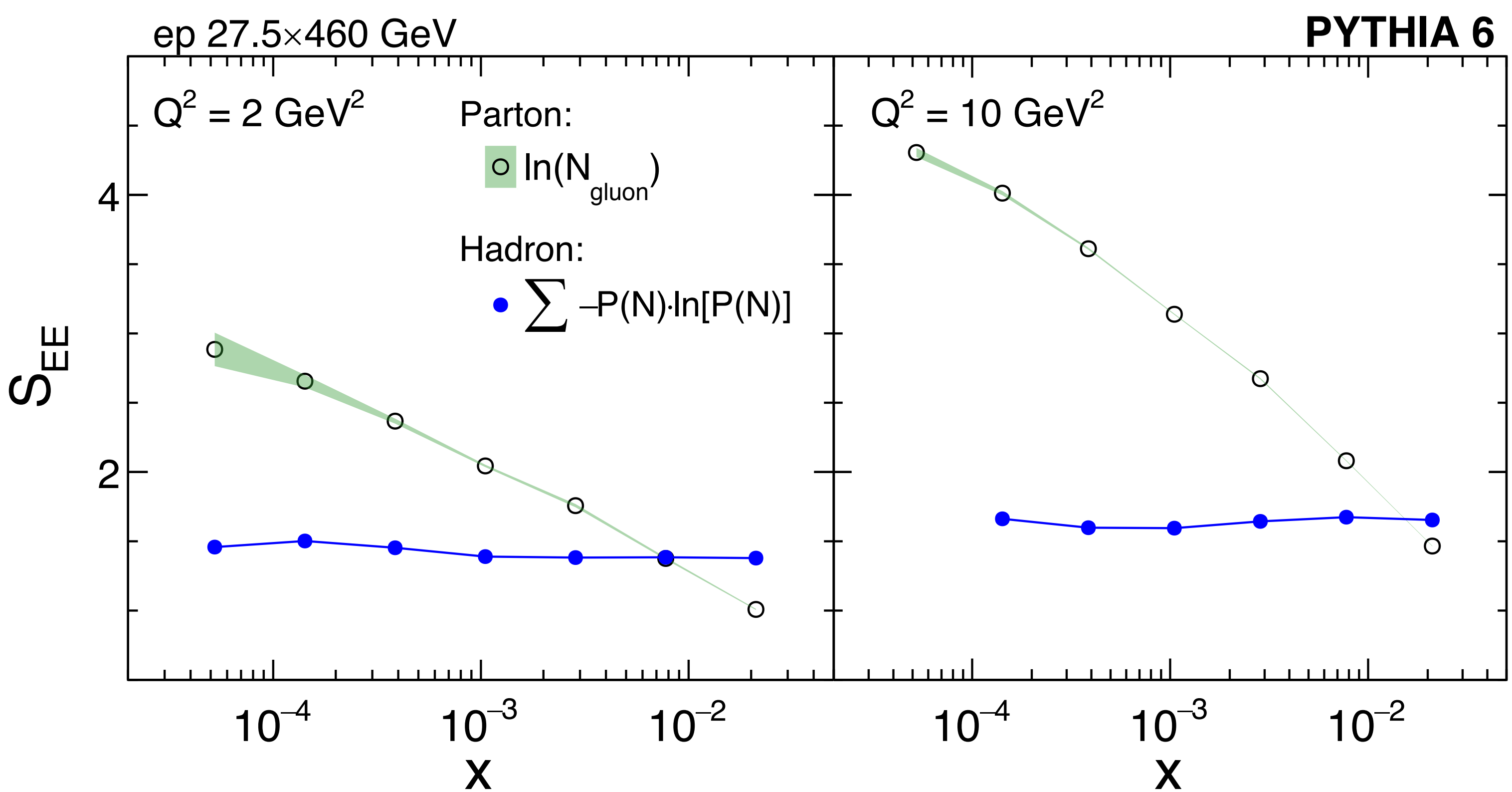
prediction

experiment

$$S_{EE}^{(x_1<x<x_2)} = \ln[xG] \stackrel{?}{=} S_{hadron}^{(y_1<y<y_2)} = -\sum P(N) \ln[P(N)]$$

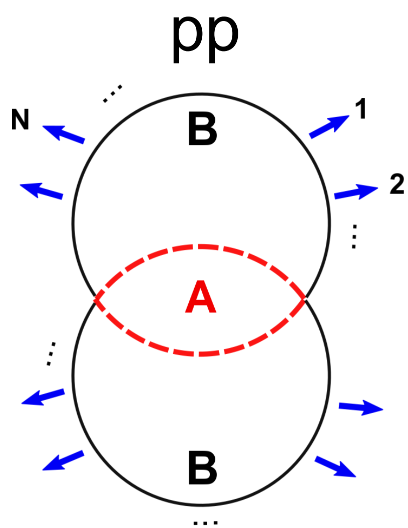
ep



No indication of entanglement in simulation

- x*G(x)* is from LO *MSTW,* no substantial difference from using other PDFs
- Other models, DJANGO, PYTHIA6, and PYTHIA8, same conclusion

High energy pp collisions

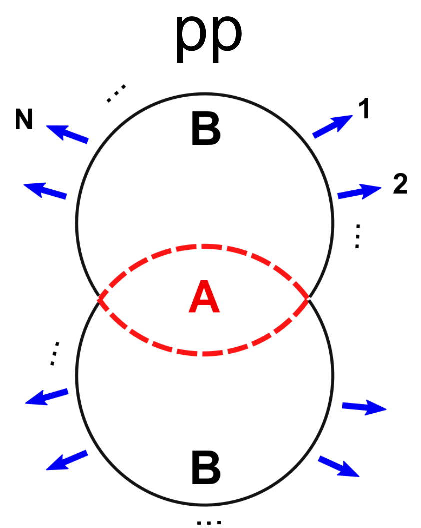


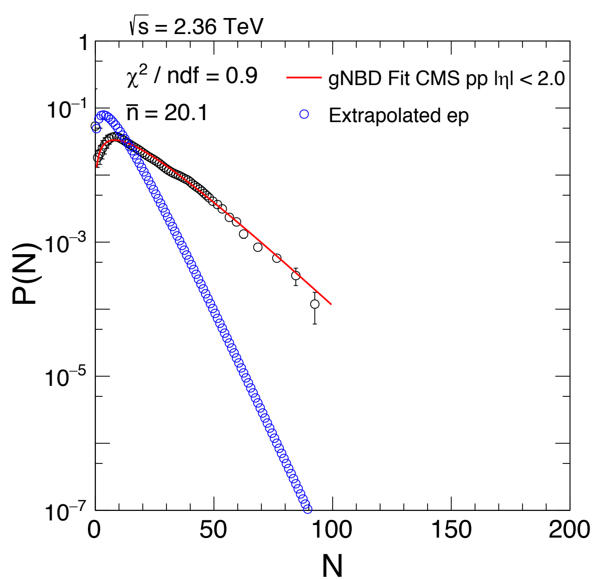
- At high energy, dominated by gluon-gluon interactions, pp collisions could be tested using similar idea.
  - Get the x value from y_{beam} and y_{hadron},

$$\ln\left(\frac{1}{x}\right) \approx y_{\text{beam}} - y_{\text{hadron}}$$

  - Saturation scale Q_s is used from NLO BK model [*see backup for other models*]

High energy pp collisions





CMS published data **JHEP 1101 (2011) 079**

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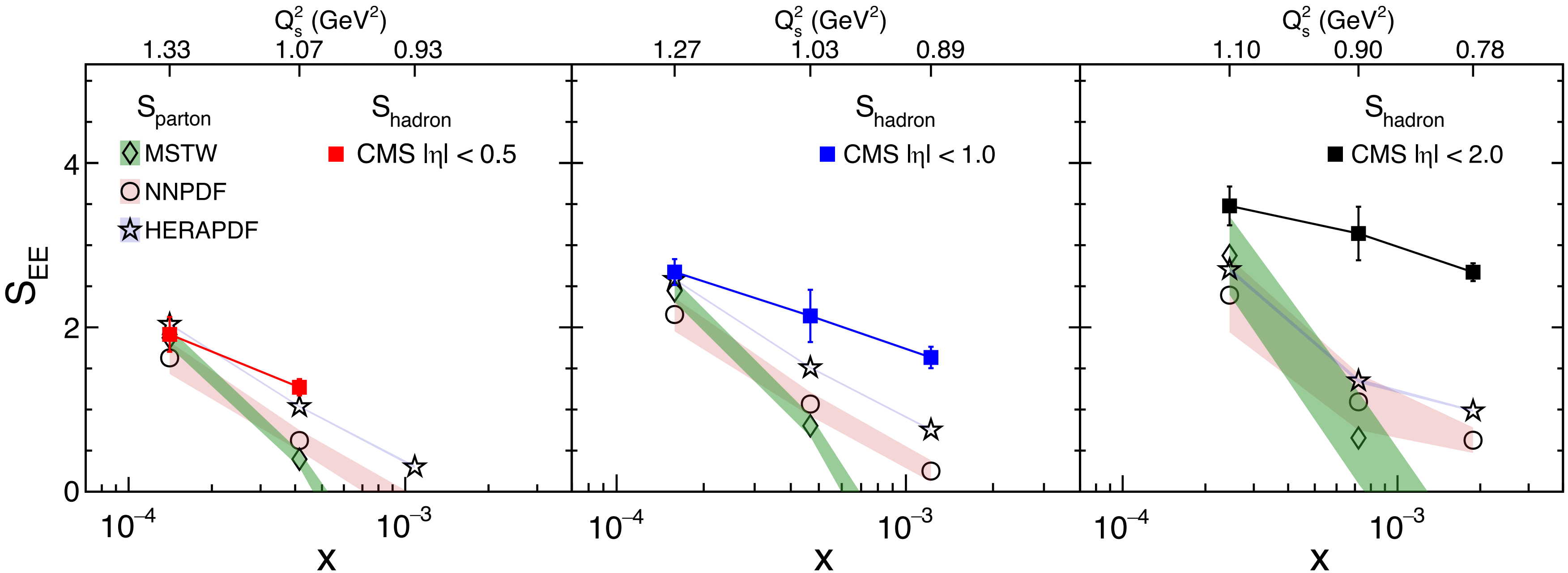
  - Saturation scale Q_s is used from NLO BK model [*see backup for other models*]
  - A negative binomial distribution (NBD) is used to extrapolate P(N) distribution per nucleon, assuming $\langle N \rangle$ is half.

(different fit ranges, double NBDs are used and included as systematics)

pp



A strong indication of quantum entanglement

- EE and its dependence on x are well predicted, e.g., expected only for $x < 10^{-3}$
- Similar at all rapidity ranges. Compatible with different PDFs.

- **Entanglement provides a new perspective on understanding the proton**

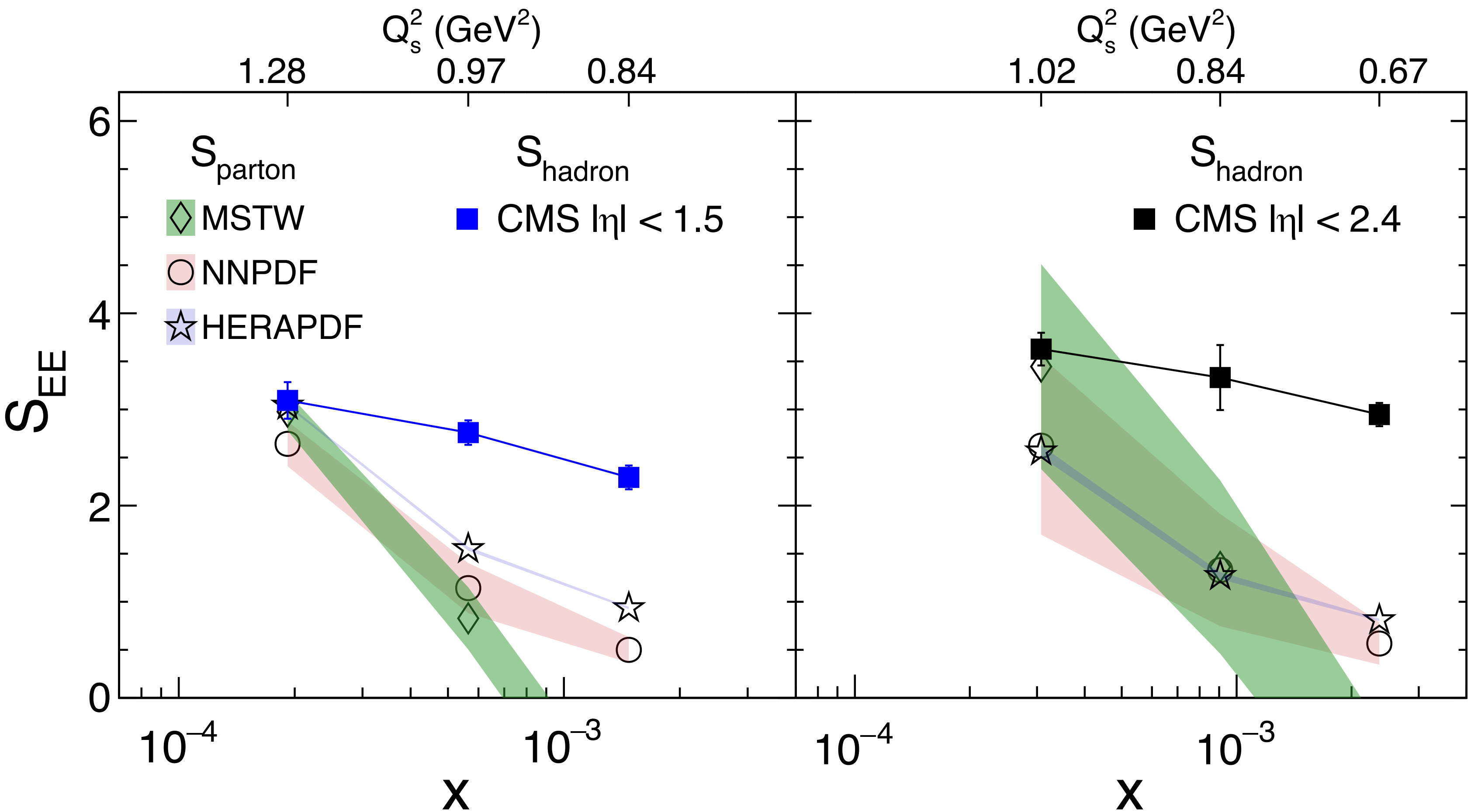
Summary and outlook

- **First indication of quantum entanglement at sub-nucleonic scales,** *encountered EPR paradox using high energy particle colliders*
  - Resolved an “apparent paradox” between the Parton model and quantum mechanics.
  - Opened a new perspective on studying the proton.
  - *Entanglement as a probe of confinement* *(Nucl.Phys.B796:274-293,2008)*
  - Thermalization through entanglement in pp collisions (Phys. Rev. D 98, 054007 (2018))

- **What else can be done?**
  - DIS experiment using ep data, e.g., HERA (published data does not go down to low x)
  - LHC pp data with a different scale?
  - Q2 evolution of entanglement entropy
  - Electron-Ion Collider in the future

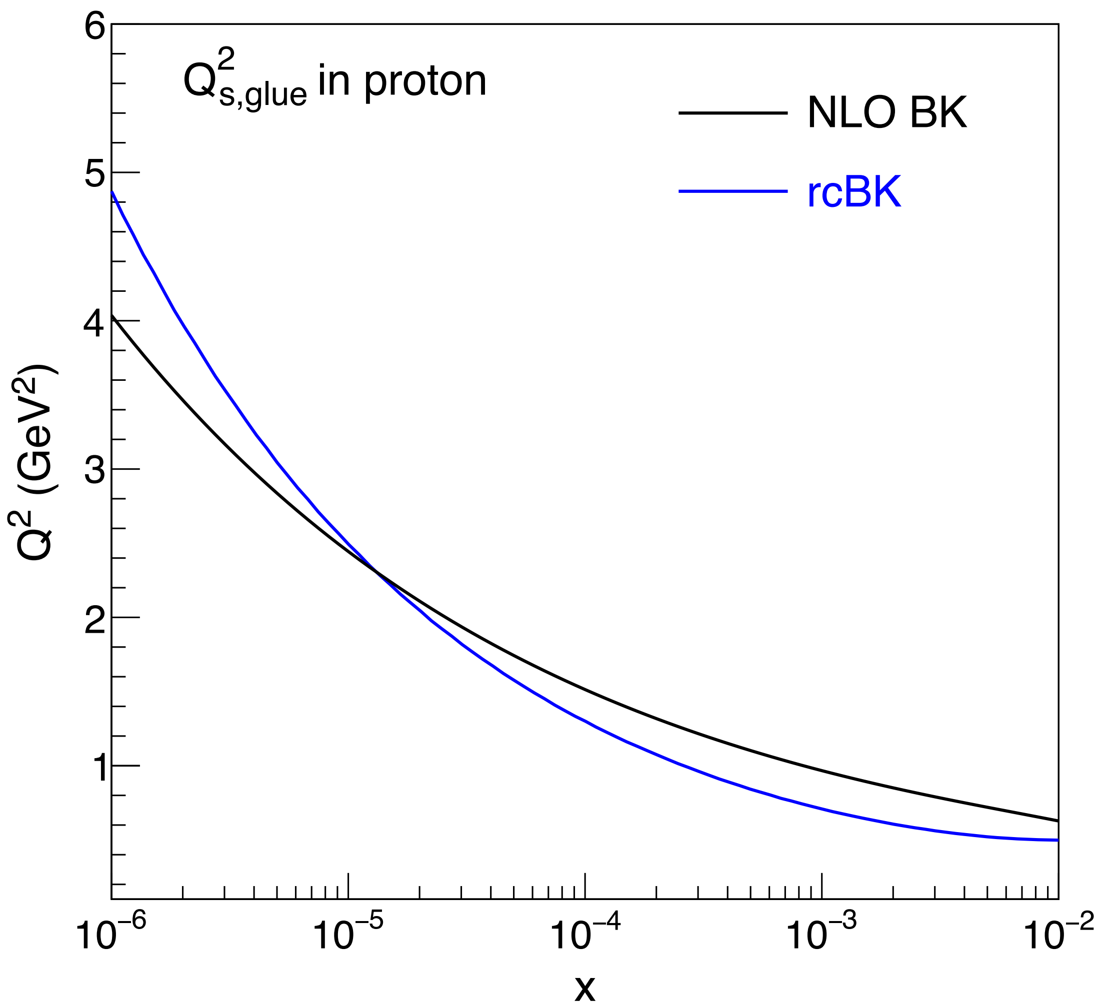
Backup

data

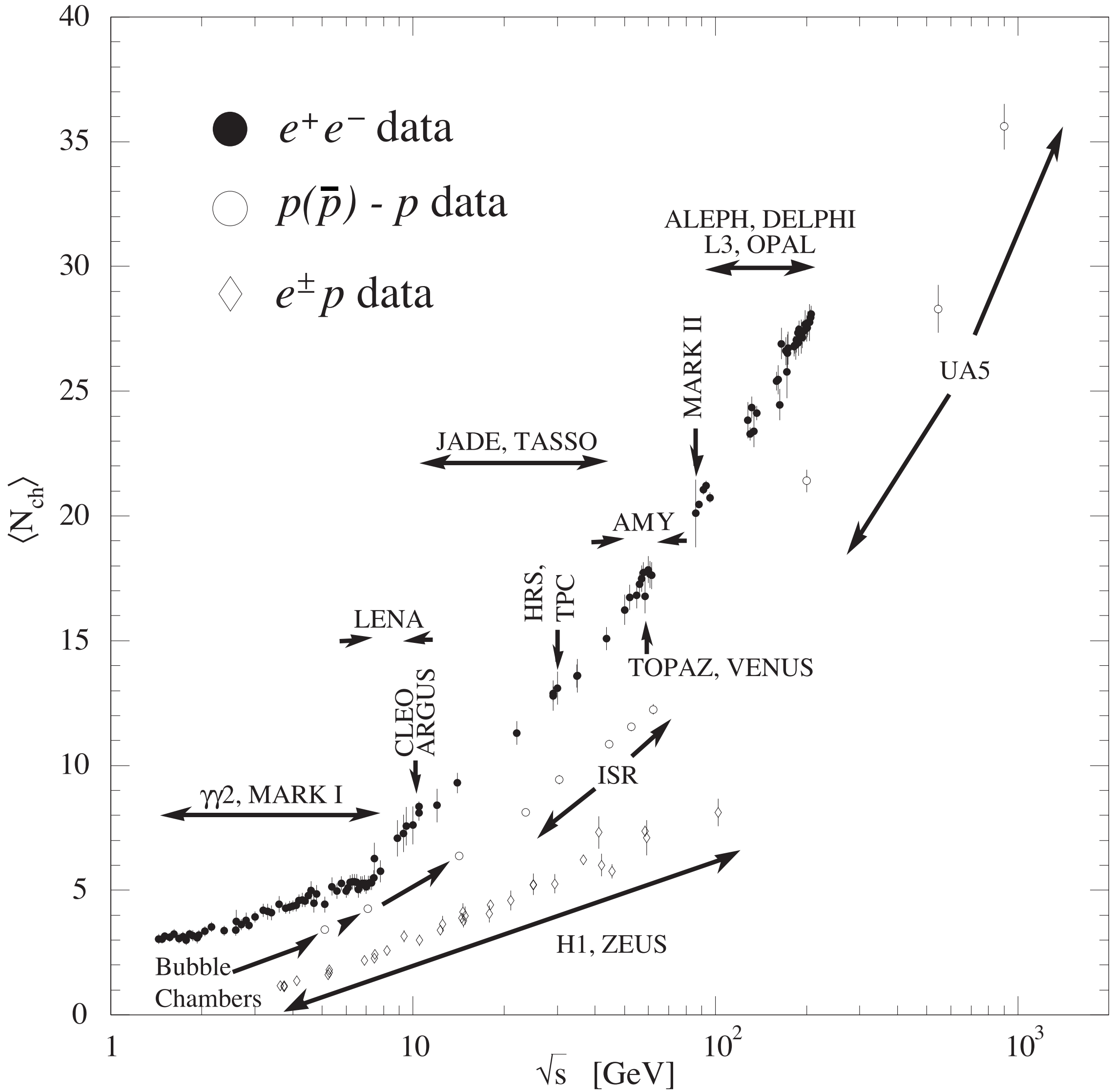


Q_s^2 (GeV^2)
1.28
0.97
0.84
S_{parton}
MSTW
NNPDF
HERAPDF
S_{hadron}
CMS |η| < 1.5
S_{EE}
6
4
2
0
10^{-4}
10^{-3}
x
Q_s^2 (GeV^2)
1.02
0.84
0.67
S_{hadron}
CMS |η| < 2.4
10^{-4}
10^{-3}
x

Saturation scales



ee, ep, and pp multiplicities



EPR paradox

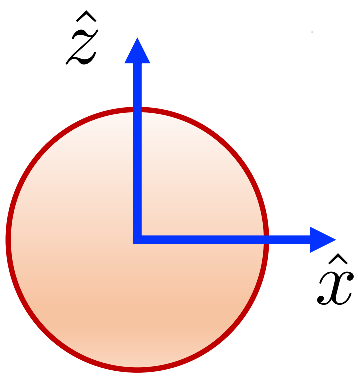
MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

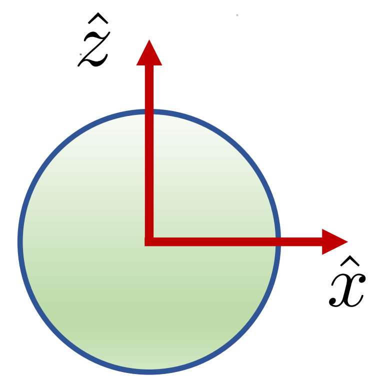
(Received March 25, 1935)

Alice



Entanglement ?

"spooky action at a distance"



Bob

- Many modern experiments have seen evidence of EPR paradox (e.g., in cold atom experiments)