MAGNETIC FIELD IN EXPANDING QUARK-GLUON PLASMA

Kirill Tuchin

1. Time and space dependence of the electromagnetic field in relativistic heavy-ion collisions, arXiv:1305.5806
2. Electromagnetic field and the chiral magnetic effect in the quark-gluon plasma, arXiv:1411.1363
3. Initial value problem for magnetic fields in heavy ion collisions, arXiv:1508.06925
4. Magnetic field in expanding quark-gluon plasma (with Evan Stewart), arXiv:1710.08793

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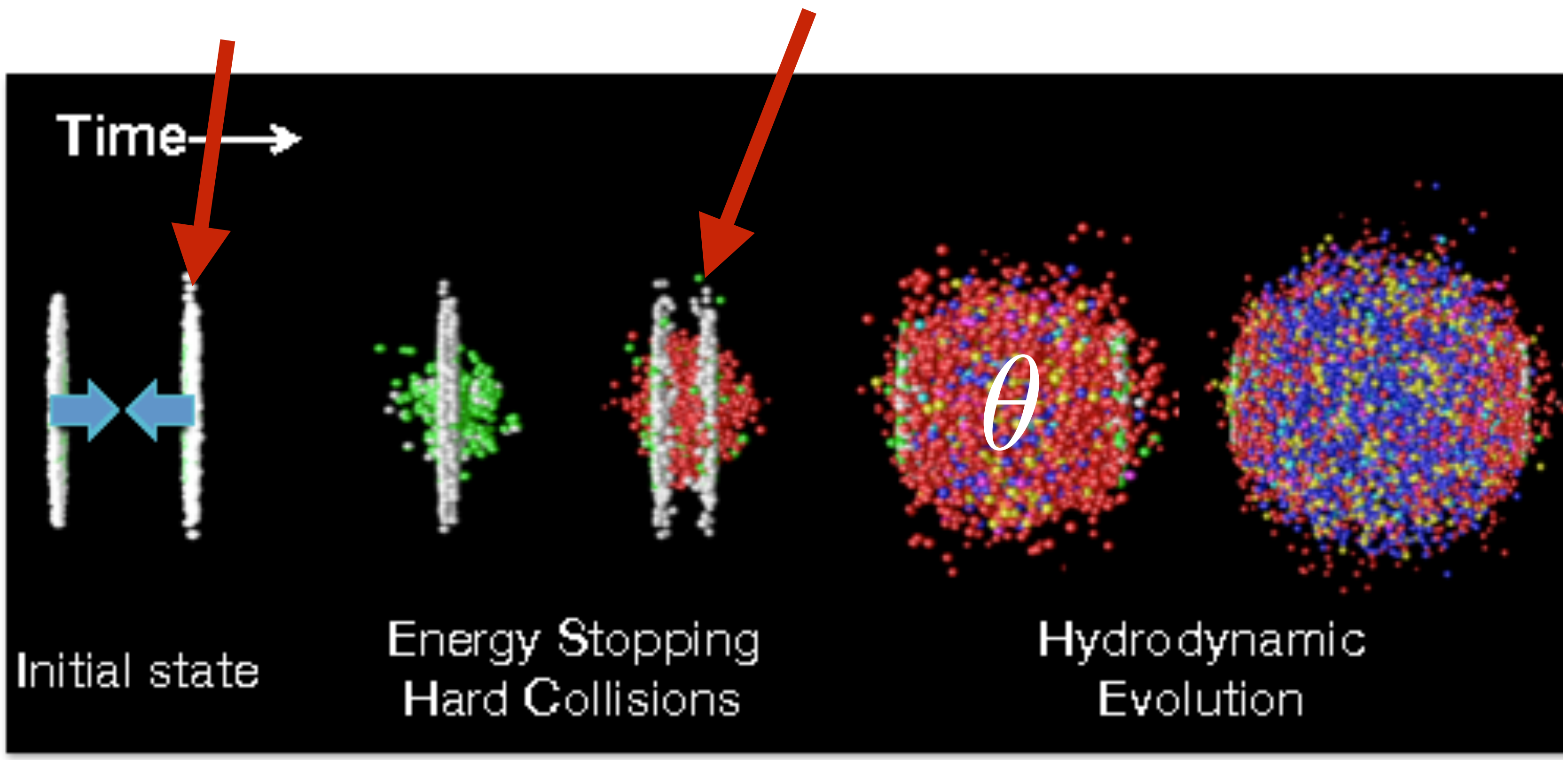
STAGES OF EM FIELD EVOLUTION

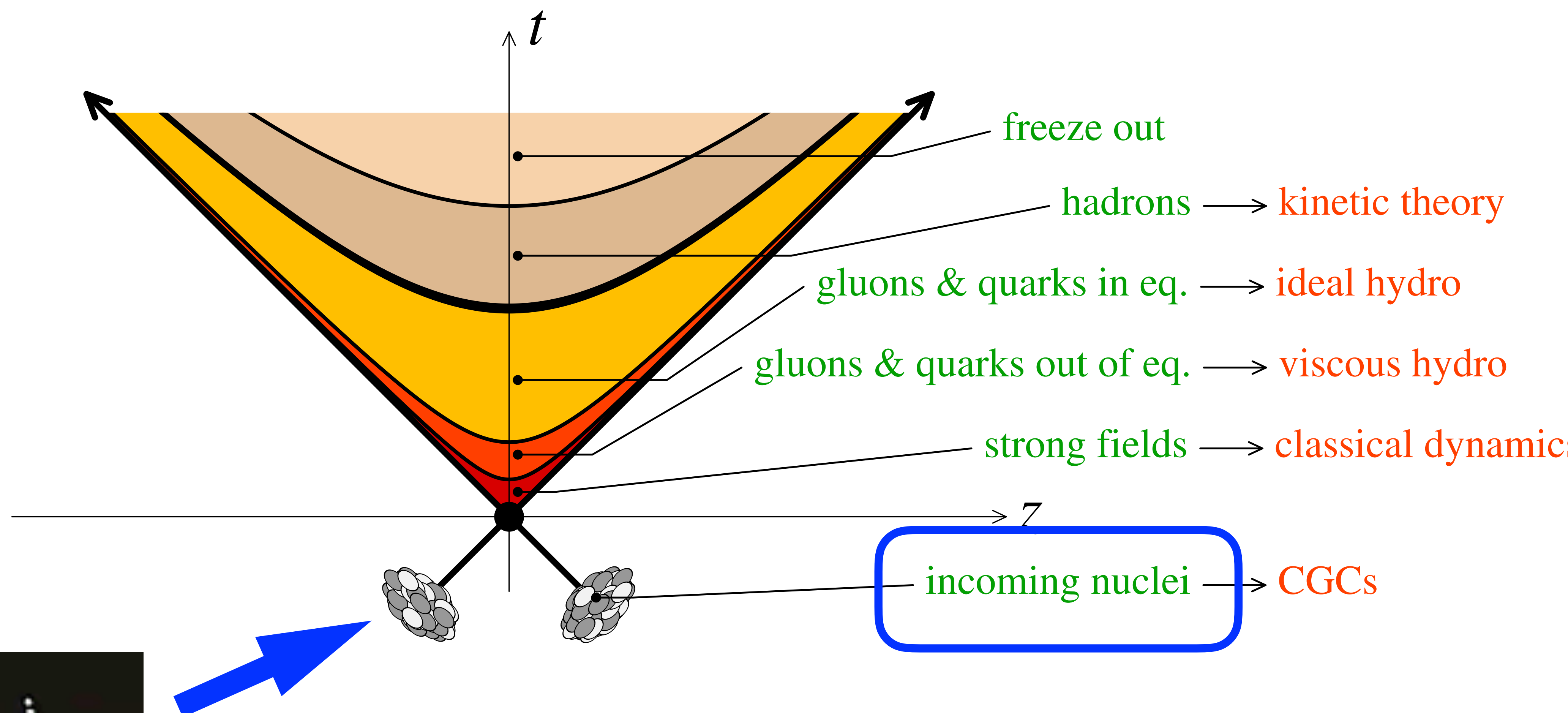
valence electric charges



EM field is generated by Z protons of each heavy-ion moving on the light-cones:

1. Before the collision: ions move towards each other.
2. After the collision, but before the QGP formation: valence quarks move away from each other; there is a small correction due to the baryon stopping.
3. After the QGP formation: valence quarks move away from each other; electric currents in QGP contribute to the field.

STAGE 1 (BEFORE THE COLLISION)



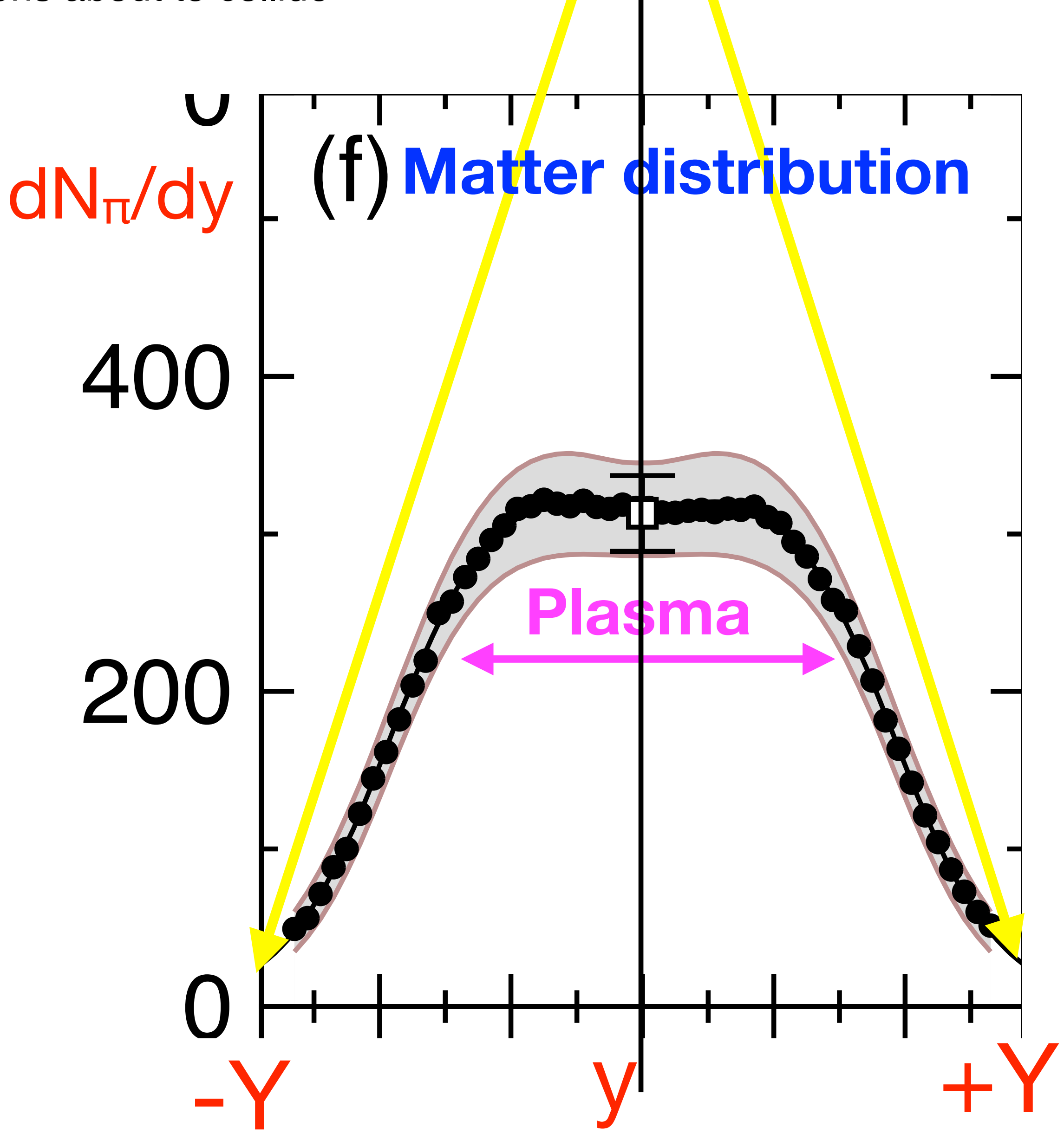
Ions about to collide

EM field of each ion is a boosted Coulomb field

$$\boldsymbol{B}_1 = \frac{\gamma e v \hat{\boldsymbol{\phi}}}{4\pi} \frac{b}{(b^2 + \gamma^2 (vt - z)^2)^{3/2}}$$

STAGE 2 (AFTER THE COLLISION, BEFORE QGP)

Ions about to collide



It is too "expensive" to transfer net baryon and electric charge to the central plateau region.

$$\frac{dN_{\rm val}}{dy} \sim e^{-\Delta_R (Y-y)} + e^{-\Delta_R (Y+y)}$$

$$\Delta_R \approx 0.47$$

Number of valence quarks (μ_B) at y=0 decreases with energy: "baryon stopping".

⇒ The contribution of the "stopped" baryons is exponentially (in y) small

EM field = sum of two boosted Coulomb fields of each ion

STAGE 2 (AFTER THE COLLISION, BEFORE QGP)

Maxwell equations in the Lorentz gauge: $\nabla^2 \boldsymbol{A}_1(\boldsymbol{r},t) = \partial_t^2 \boldsymbol{A}_1(\boldsymbol{r},t) - \boldsymbol{j}(\boldsymbol{r},t)\,,$

$$\boldsymbol{j} = \int_{\text{ion A}} ev\hat{\boldsymbol{z}}\delta(z-vt)\delta(\boldsymbol{b}) - \int_{\text{ion B}} ev\hat{\boldsymbol{z}}\delta(z+vt)\delta(\boldsymbol{b})$$

Solution:

$$\boldsymbol{A}_1(\boldsymbol{r},t) = \int_{\text{ion A}} \frac{\gamma ev\hat{\boldsymbol{z}}}{4\pi}\frac{1}{\sqrt{b^2+\gamma^2(vt-z)^2}} - \int_{\text{ion B}} \frac{\gamma ev\hat{\boldsymbol{z}}}{4\pi}\frac{1}{\sqrt{b^2+\gamma^2(vt+z)^2}}$$

STAGE 3 (AFTER QGP FORMATION)

Assumptions:

1. QGP emerges instantaneously at $t=t_0$

2. QGP is nonmagnetic ($\mu=1$), neutral (no net electric charge), electric currents are Ohmic with constant electric conductivity (σ):

$$\varepsilon = 1 + i\sigma/\omega$$

⇒ Emergent plasma at $t=t_0$ produces no new EM field (which would've been an artifact of the initial conditions) because it is neutral and $\varepsilon_0=1$.

⇒ ***Initial conditions:*** E, B must be continuous at $t=t_0$

STAGE 3 (AFTER QGP FORMATION) CONT.

Assumptions (cont.):

3. Electromagnetic field does not affect QGP flow. This seem to hold to a few % accuracy.

Roy, Pu, Rezzolla, Rischke,   Pang, Endrodi, Petersen   Greif, Greiner, Xu
Voronyuk, Toneev, Cassing, Bratkovkaya, Konchalovski, Voloshin
Gursoy, Kharzeev, Marcus, Rajagopal
(Mohapatra, Saumina, Srivastava disagree)

$\Rightarrow$ Do not need to solve the Relativistic MHD to compute E,B 👍

Using the gauge condition $\partial_t\varphi + \boldsymbol{\nabla}\cdot\boldsymbol{A} + \sigma\varphi = 0$ Maxwell equations read

$$-\nabla^2\varphi_2 + \partial_t^2\varphi_2 + \sigma\partial_t\varphi_2 = \rho\,,$$
$$-\nabla^2\boldsymbol{A}_2 + \partial_t^2\boldsymbol{A}_2 + \sigma\partial_t\boldsymbol{A}_2 - \sigma\boldsymbol{u}\times(\boldsymbol{\nabla}\times\boldsymbol{A}_2) = \boldsymbol{j}\,,$$

plasma velocity

INITIAL VALUE PROBLEM FOR EM FIELD IN QGP

$$-\nabla^2 \boldsymbol{A}_2 + \partial_t^2 \boldsymbol{A}_2 + \sigma \partial_t \boldsymbol{A}_2 - \sigma \boldsymbol{u} \times (\boldsymbol{\nabla} \times \boldsymbol{A}_2) = \boldsymbol{j} \, ;$$

$$A_2^\mu(\boldsymbol{r}, t_0) = A_1^\mu(\boldsymbol{r}, t_0) \equiv \mathcal{A}^\mu(\boldsymbol{r}) \, ,$$

$$\partial_t A_2^\mu(\boldsymbol{r}, t)\big|_{t=t_0} = \partial_t A_1^\mu(\boldsymbol{r}, t)\big|_{t=t_0} \equiv \mathcal{V}^\mu(\boldsymbol{r})$$

Solution:

$$A_2^\mu(\boldsymbol{r}, t) = \int_\tau^{t_0+} dt' \int d^3r' j^\mu(\boldsymbol{r}', t') G_2(\boldsymbol{r}, t|\boldsymbol{r}', t')$$
$$+ \int d^3r' \left[\sigma \mathcal{A}^\mu(\boldsymbol{r}') + \mathcal{V}^\mu(\boldsymbol{r}')\right] G_2(\boldsymbol{r}, t|\boldsymbol{r}', t')\big|_{t'=t_0}$$
$$- \int d^3r' \mathcal{A}^\mu(\boldsymbol{r}') \partial_{t'} G_2(\boldsymbol{r}, t|\boldsymbol{r}', t')\big|_{t'=t_0}$$

see Morse, Feshbach

Need: the retarded Green's function G_2

COMPUTING THE GREEN'S FUNCTION G_2

1. Assume that the u-term is small. For example, in the blast wave model:

$$\boldsymbol{u}(\boldsymbol{r},t) = \frac{u_o}{R_o}\boldsymbol{b}\,\theta(R_o - b) + \frac{\boldsymbol{z}}{t}$$

where $R_o = 7.5$ fm, $u_o = 0.55$ — Teaney

typical distances $b \ll R_o$

Treat the u-term perturbatively: $\boldsymbol{A} = \boldsymbol{A}^{(0)} + \boldsymbol{A}^{(1)}$

$$-\nabla^2 \boldsymbol{A}^{(0)} + \partial_t^2 \boldsymbol{A}^{(0)} + \sigma\partial_t \boldsymbol{A}^{(0)} = \boldsymbol{j}\,,$$

$$-\nabla^2 \boldsymbol{A}^{(1)} + \partial_t^2 \boldsymbol{A}^{(1)} + \sigma\partial_t \boldsymbol{A}^{(1)} = \sigma\boldsymbol{u}\times\boldsymbol{B}^{(0)}\,.$$

$$-\nabla^2 G + \partial_t^2 G + \sigma\partial_t G = \delta(t-t')\delta(\boldsymbol{r}-\boldsymbol{r}')\,.$$

Substitute $G(\boldsymbol{r},t|\boldsymbol{r}',t') = e^{-\sigma t/2}\mathcal{G}(\boldsymbol{r},t|\boldsymbol{r}',t')$ $\Rightarrow$ $-\nabla^2\mathcal{G} + \partial_t^2\mathcal{G} + m^2\mathcal{G} = e^{\sigma t'/2}\delta(t-t')\delta(\boldsymbol{r}-\boldsymbol{r}')\,.$

where $m = i\sigma/2$

Klein-Gordon equation

COMPUTING THE GREEN'S FUNCTION G_2 (CONT.)

$$G_2(\boldsymbol{r},t|\boldsymbol{r}',t') = \frac{1}{4\pi}e^{-\frac{1}{2}\sigma(t-t')}\frac{\delta(t-t'-R)}{R}\theta(t-t')$$

the original pulse

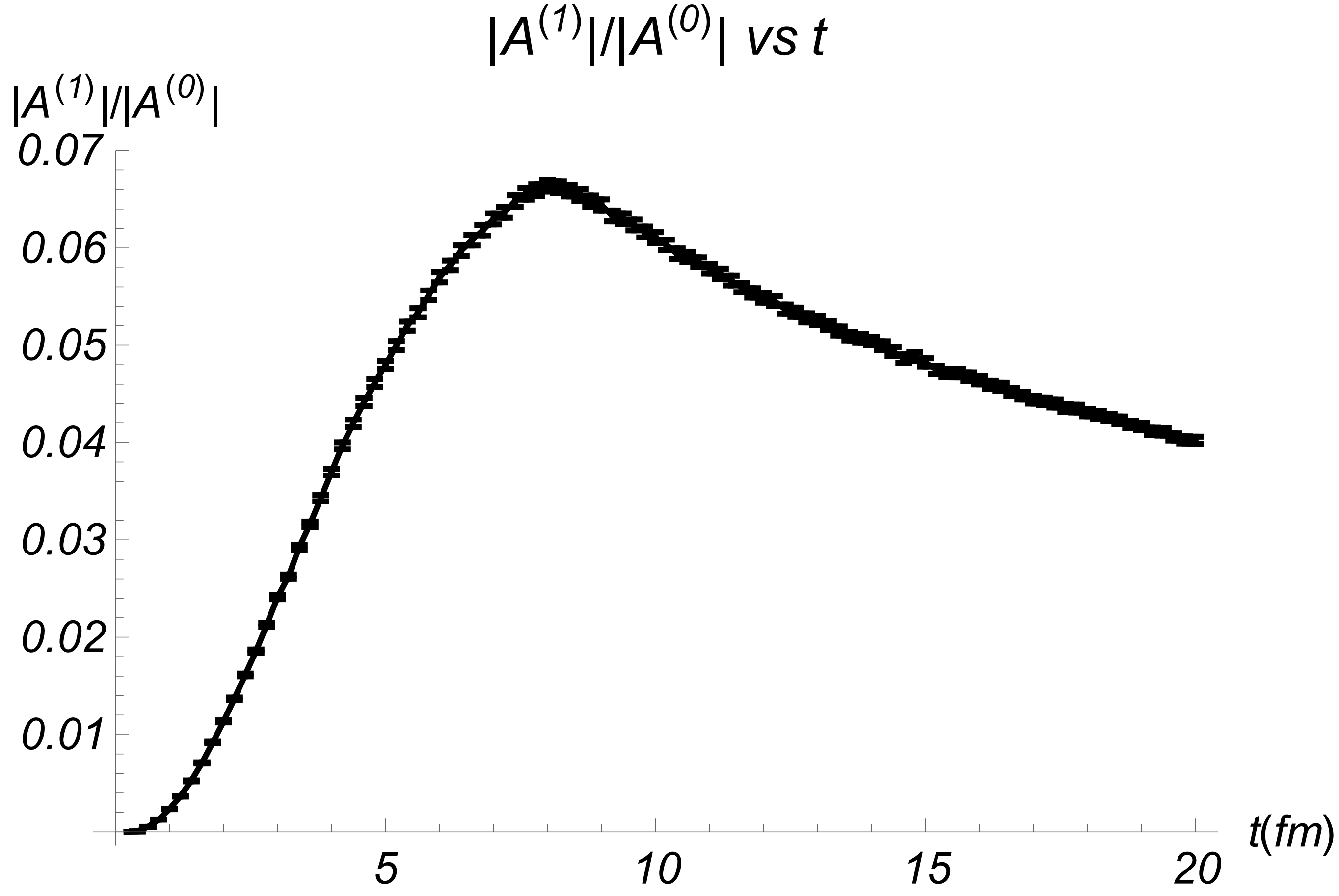
$$+\frac{1}{4\pi}e^{-\frac{1}{2}\sigma(t-t')}\frac{\sigma/2}{\sqrt{(t-t')^2-R^2}}I_1\left(\frac{\sigma}{2}\sqrt{(t-t')^2-R^2}\right)\theta(t-t'-R)\theta(t-t')$$

wake produced by
the induced currents

$\Rightarrow$ solve for $\boldsymbol{A}^0$

$$\underbrace{-\nabla^2\boldsymbol{A}^{(1)}+\partial_t^2\boldsymbol{A}^{(1)}+\sigma\partial_t\boldsymbol{A}^{(1)}}_{\text{same Green's function } G_2}=\sigma\boldsymbol{u}\times\boldsymbol{B}^{(0)}\,.$$

COMPUTING THE GREEN'S FUNCTION G_2 (CONT.)



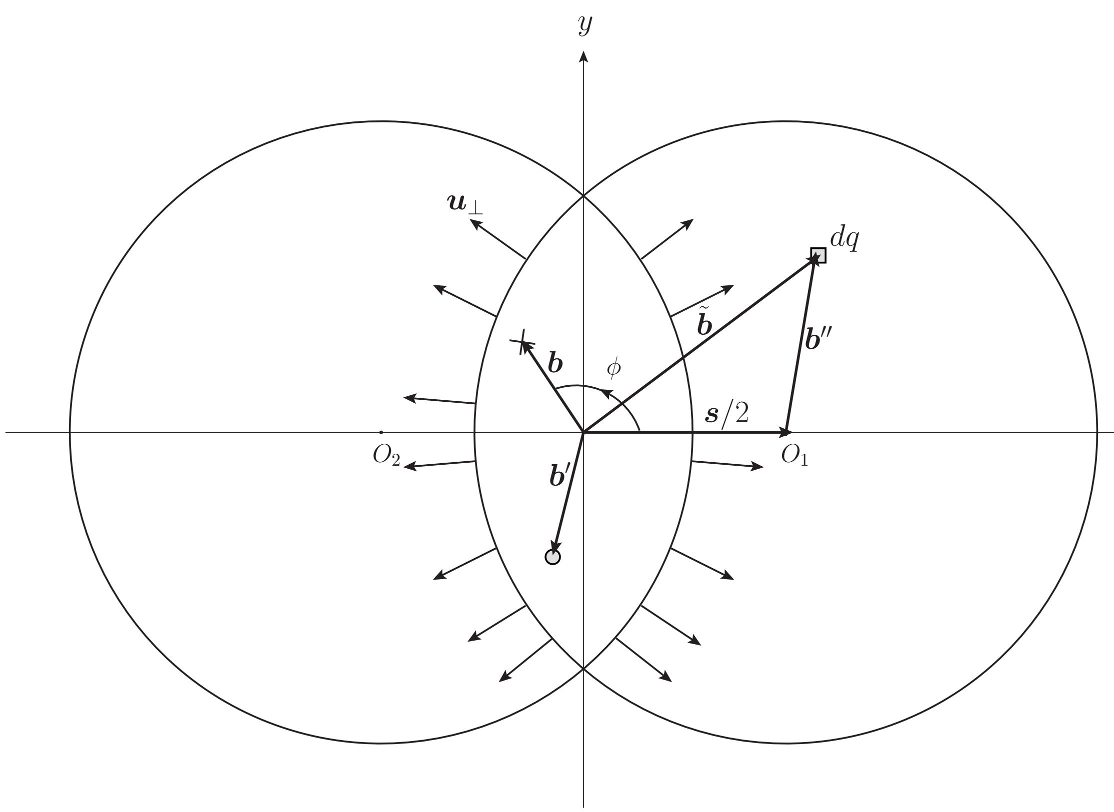


FIG. 3. The vector potential $\boldsymbol{A} = \boldsymbol{A}^{(0)} + \boldsymbol{A}^{(1)}$ created at a representative point $z = 0$, $b = 1$ fm, $\phi = \pi/6$ (see Fig. 2) in QGP by a remnant of the gold ion moving with the boost-factor $\gamma = 100$ ($\sqrt{s} = 0.2$ TeV) and impact parameter $|\boldsymbol{s}| = 3$ fm.

Conclusion: QGP expansion is a small correction to the total EM field

THE PULSE FIELD

$$G_2(\boldsymbol{r},t|\boldsymbol{r}',t') = \frac{1}{4\pi} e^{-\frac{1}{2}\sigma(t-t')} \frac{\delta(t-t'-R)}{R}\theta(t-t')$$

Dissipation due to work done by
the field on the medium currents

$$\boldsymbol{A}^{(0)}(\boldsymbol{r},t) = \int G(\boldsymbol{r},t|\boldsymbol{r}',t')\boldsymbol{j}(\boldsymbol{r}',t')d^3r'dt' \;\; = \frac{ev\hat{\boldsymbol{z}}}{4\pi}\frac{1}{\sqrt{\xi^2+b^2/\gamma^2}}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}$$

$$\xi = vt - z$$

A good approximation: $\sqrt{\xi^2+b^2/\gamma^2} \gg 1/\sigma\gamma^2 \sim 10^{-5}\ \mathrm{fm}$ and $b/\gamma \ll \xi$ $\Rightarrow$

$$\boldsymbol{B}_a^{(0)} \approx \frac{ev}{8\pi}\hat{\boldsymbol{\phi}}\frac{\sigma b}{\xi^2}e^{-\frac{\sigma\xi}{2(1+v)}}e^{-\frac{b^2\sigma}{4\xi}}, \quad \xi > 0\,.$$ (the "diffusion approximation")

Note: no dependence on the collision energy!

Full solution:

$$A_2^\mu(\boldsymbol{r},t) = \frac{1}{4\pi}\left\{\int d^3r'\frac{1}{R}e^{-\frac{1}{2}\sigma R}j^\mu(\boldsymbol{r}',t-R)\theta(t-t_0-R)\right.$$
$$+ (t-t_0)e^{-\frac{1}{2}\sigma(t-t_0)}\oint_{S_{\boldsymbol{r}}^{t-t_0}}[\sigma\mathcal{A}^\mu(\boldsymbol{r})+\mathcal{V}^\mu(\boldsymbol{r})]d\Omega$$
$$\left. + \partial_t\left[(t-t_0)e^{-\frac{1}{2}\sigma(t-t_0)}\oint_{S_{\boldsymbol{r}}^{t-t_0}}\mathcal{A}^\mu(\boldsymbol{r})d\Omega\right]\right\}$$

$S_{\boldsymbol{r}}^{t-t_0}$ is a sphere of radius $t-t_0$ with the center at $\boldsymbol{r}$

THE WAKE FIELD

$$\boldsymbol{A}_a^{(0)}(\boldsymbol{r},t) = \frac{ev\hat{\boldsymbol{z}}}{4\pi}\frac{1}{\sqrt{\xi^2+b^2/\gamma^2}}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}$$ pulse

$$\boldsymbol{A}_b^{(0)}(\boldsymbol{r},t) \approx \frac{e\hat{\boldsymbol{z}}}{4\pi}\frac{\sigma v}{4}\exp\left\{-\frac{\sigma\gamma^2}{2}\left(-v\xi+\sqrt{\xi^2+b^2/\gamma^2}\right)\right\}$$ wake

The pulse dominates when $\sqrt{\xi^2+b^2/\gamma^2} \ll 4/\sigma \sim 10^2\ \mathrm{fm}.$

⇒ in QGP the wake plays no role!

HIGH-ENERGY (DIFFUSION) APPROXIMATION

For an ultrarelativistic charge $\partial_z^2 - \partial_t^2 \sim k_z^2/\gamma^2 \ll k_\perp^2, \sigma k_z$

$\Rightarrow$ The retarded Green's function obeys the 2D diffusion equation

$$-\nabla_\perp^2 G_{\mathcal{D}} + \sigma\partial_t G_{\mathcal{D}} = \delta(t-t')\delta(\boldsymbol{r}-\boldsymbol{r}')$$

Solution:

$$G_{\mathcal{D}}(\boldsymbol{r},t|\boldsymbol{r}',t') = \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')+i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{p_\perp^2 - i\omega\sigma} = \frac{1}{4\pi t}\delta(z-z')\theta(t-t')e^{-\frac{\sigma(\boldsymbol{r}_\perp-\boldsymbol{r}'_\perp)^2}{4(t-t')}}$$

$$\Rightarrow \qquad \boldsymbol{A}^{(0)}(\boldsymbol{r},t) = \frac{e\hat{\boldsymbol{z}}}{4\pi(t-z/v)}e^{-\frac{\sigma b^2}{4(t-z/v)}}\theta(t-z/v)$$

We can solve this problem even for time-dependent conductivity!

Introduce a new "time-variable" $\lambda(t) = \int_{t_0}^{t}\frac{dt'}{\sigma(t')}$

MAGNETIC FIELD IN CONFIGURATION SPACE

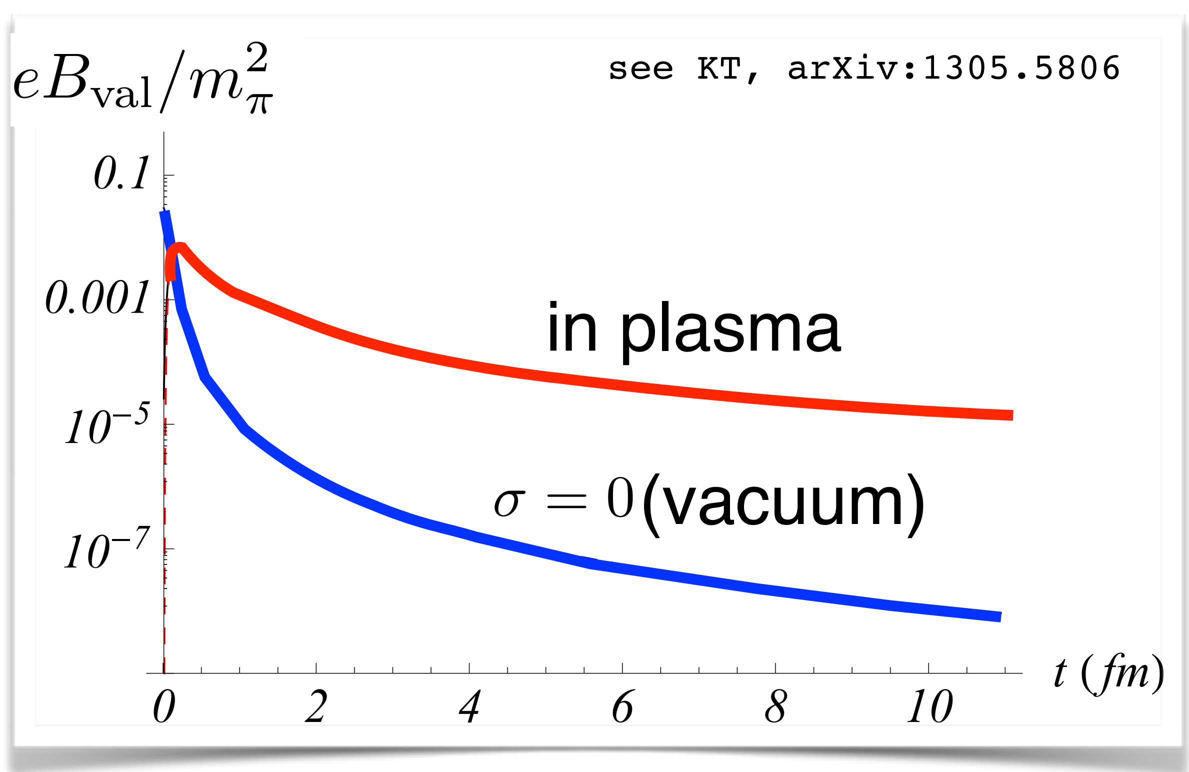
$$\boldsymbol{B} = \boldsymbol{B}_{\text{val}} + \boldsymbol{B}_{\text{init}}$$

$$e\boldsymbol{B}_{\text{val}}(\boldsymbol{r},t) = \hat{\boldsymbol{\phi}}\frac{\alpha\pi b}{2\sigma(z/v)[\lambda(t)-\lambda(z/v)]^2}\exp\left\{-\frac{b^2}{4[\lambda(t)-\lambda(z/v)]}\right\}\theta(tv-z)\theta(z-vt_0)\,,$$

$$e\boldsymbol{B}_{\text{init}}(\boldsymbol{r},t) = \hat{\boldsymbol{\phi}}\gamma\alpha v\int_0^\infty dk_\perp k_\perp J_1(k_\perp b)\exp\left\{-k_\perp^2\lambda(t)-k_\perp\gamma|z-vt_0|\right\}$$

Remarks:

1. Electric conductivity of QGP plays a crucial role in EM dynamics.



MAGNETIC FIELD IN CONFIGURATION SPACE

$$\boldsymbol{B} = \boldsymbol{B}_{\text{val}} + \boldsymbol{B}_{\text{init}}$$

$$e\boldsymbol{B}_{\text{val}}(\boldsymbol{r},t) = \hat{\boldsymbol{\phi}} \frac{\alpha\pi b}{2\sigma(z/v)[\lambda(t)-\lambda(z/v)]^2} \exp\left\{-\frac{b^2}{4[\lambda(t)-\lambda(z/v)]}\right\} \theta(tv-z)\theta(z-vt_0)\,,$$

$$e\boldsymbol{B}_{\text{init}}(\boldsymbol{r},t) = \hat{\boldsymbol{\phi}}\gamma\alpha v \int_0^\infty dk_\perp k_\perp J_1(k_\perp b) \exp\left\{-k_\perp^2\lambda(t) - k_\perp\gamma|z-vt_0|\right\}$$

Remarks:

1. Electric conductivity of QGP plays a crucial role in EM dynamics.

2. Energy dependence: $B_{init}(r,t) \sim \gamma$, while $B_{val}(r,t) \sim \gamma^0$

3. Late time-dependence: $B_{init}(r,t) \sim 1/t^{3/2}$, while $B_{val}(r,t) \sim 1/t^2$

TWO COUNTER-PROPAGATING CHARGES

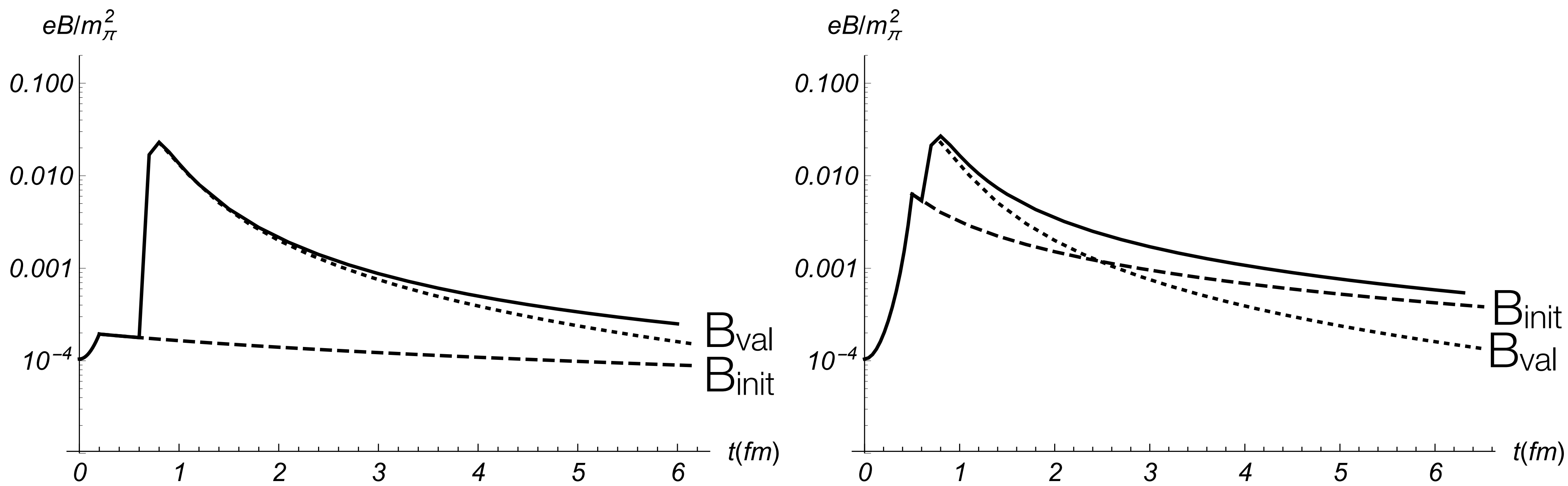


FIG. 3: Magnetic field in units of m_π^2/e. $\sigma = 5.8$ MeV, $z = 0.6$ fm ($\eta = 0.086$). Left panel: $\boxed{t_0 = 0.2 \text{ fm},}$ right panel: $\boxed{t_0 = 0.5 \text{ fm}.}$ Solid, dashed and dotted lines stand for B, B_{init} and B_{val}.

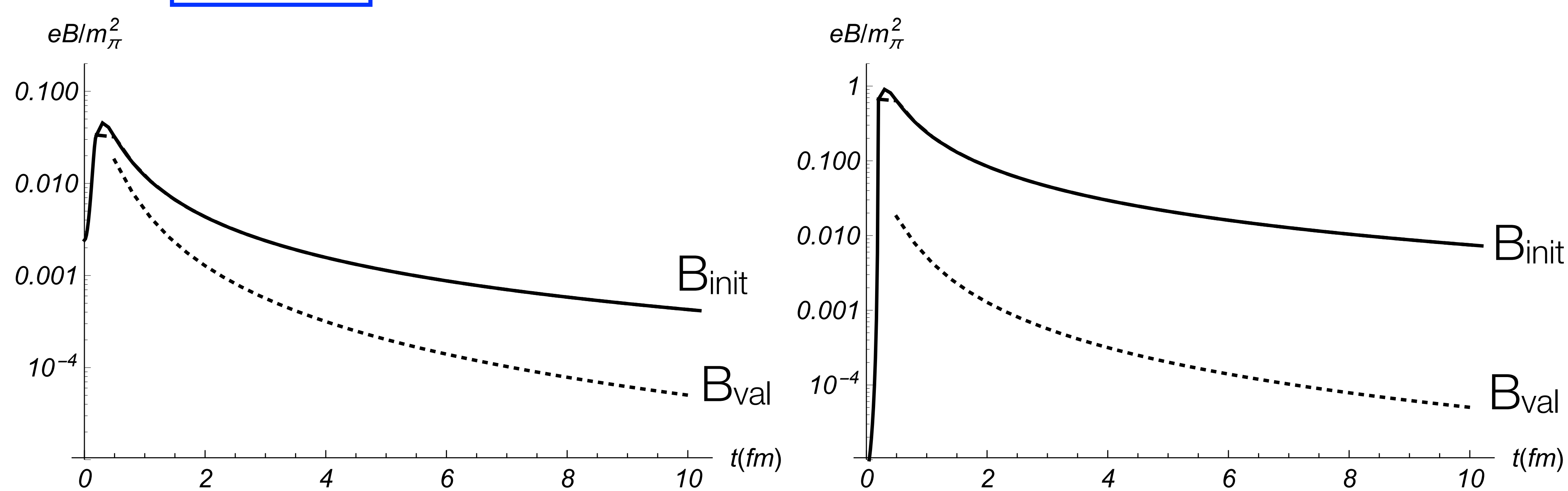


FIG. 4: Magnetic field in units of m_π^2/e. $\sigma = 5.8$ MeV, $z = 0.2$ fm $t_0 = 0.2$ fm. Solid, dashed and dotted lines stand for B, B_{init} and B_{val}. Left panel: $\gamma = 100$ (RHIC), right panel: $\gamma = 2000$ (LHC).

ANOMALOUS CURRENTS IN QGP

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + c_A \left(\boxed{\partial_t \theta\, \boldsymbol{B}} + \boxed{\boldsymbol{\nabla} \theta \times \boldsymbol{E}} \right)$$

The anomalous currents

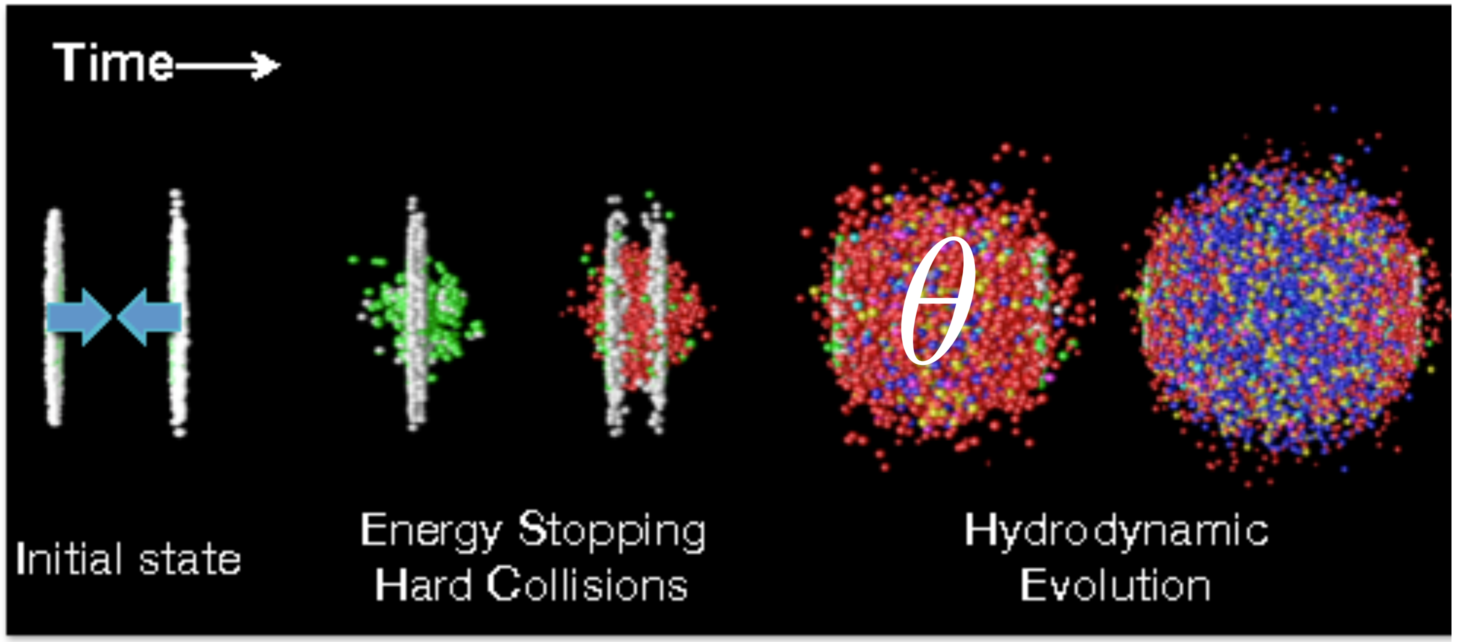
$$\boldsymbol{j}_{\mathrm{CME}} = \sigma_\chi \boldsymbol{B}$$

$$\boldsymbol{j}_{\mathrm{AHE}} = \boldsymbol{b} \times \boldsymbol{E}$$

Chiral magnetic effect

Anomalous Hall Effect

Kharzeev, McLerran, Warringa (2008)



INSTABILITY EM FIELD

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0\,,$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{D} = e\delta(z-vt)\delta(\boldsymbol{b})\,,$$

$$\boldsymbol{\nabla}\times\boldsymbol{E} = -\partial_t\boldsymbol{B}\,,$$

$$\boldsymbol{\nabla}\times\boldsymbol{H} = \partial_t\boldsymbol{D} + \sigma_\chi\boldsymbol{B} + ev\hat{\boldsymbol{z}}\delta(z-vt)\delta(\boldsymbol{b})$$

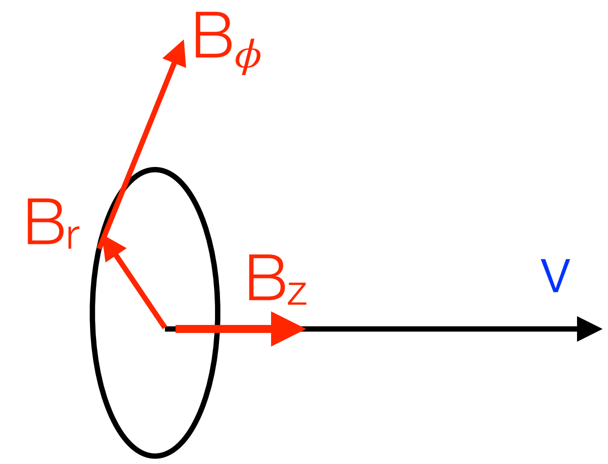
The dispersion relation $\omega_{1,2} = \dfrac{-i\sigma k_\perp^2 \pm k_\perp\sigma_\chi\sqrt{k_\perp^2-\sigma^2-\sigma_\chi^2}}{\sigma^2+\sigma_\chi^2}$

$$\boldsymbol{B} = \int \frac{d^2k_\perp}{(2\pi)^2} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{b}}\left\{\frac{i}{\omega_2-\omega_1}\left[e^{-i\omega_1 x_-}\boldsymbol{f}(\omega_1)\theta(k_\perp-\sigma_\chi) - e^{-i\omega_2 x_-}\boldsymbol{f}(\omega_2)\right]\theta(x_-) - \frac{i}{\omega_2-\omega_1}e^{-i\omega_1 x_-}\boldsymbol{f}(\omega_1)\theta(\sigma_\chi-k_\perp)\theta(-x_-)\right\}$$

acausal term is a manifestation of instability.

OSCILLATIONS OF MAGNETIC FIELD

$$B_\phi = \frac{eb}{8\pi x_-^2} e^{-\frac{b^2\sigma}{4x_-}} \left[\sigma \cos\left(\frac{b^2\sigma_\chi}{4x_-}\right) + \sigma_\chi \sin\left(\frac{b^2\sigma_\chi}{4x_-}\right)\right]$$



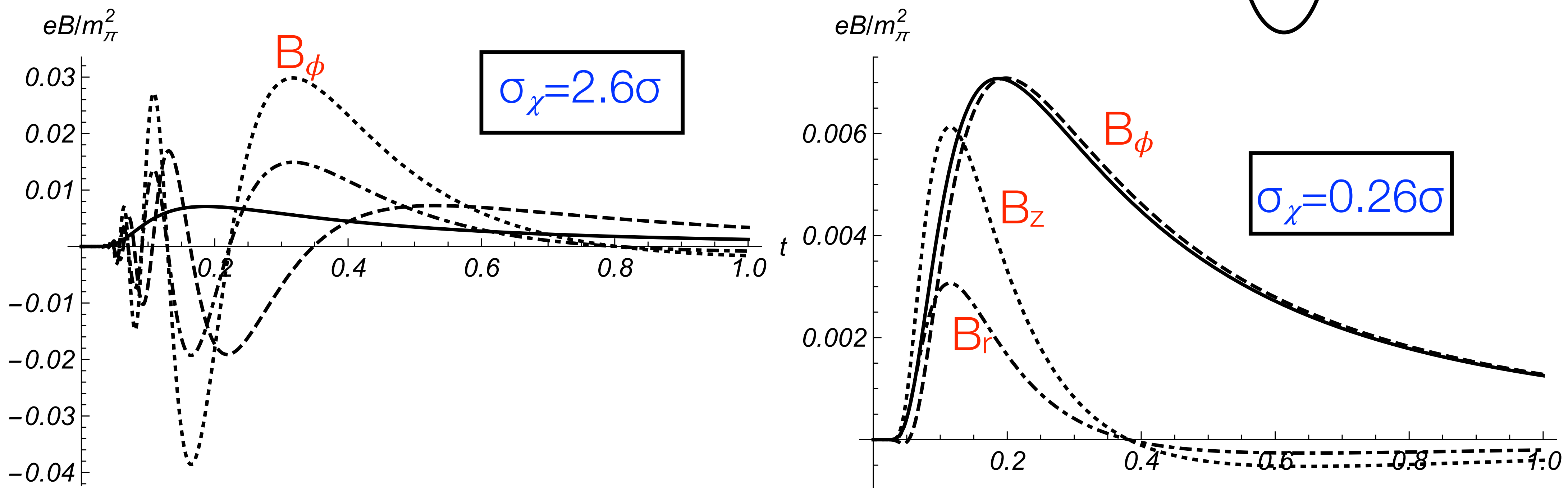


FIG. 2: Magnetic field of a point charge as a function of time t at $z = 0$. (Free space contribution is not shown). Electrical conductivity $\sigma = 5.8$ MeV. Solid line on both panels corresponds to $B = B_\phi$ at $\sigma_\chi = 0$. Broken lines correspond to B_ϕ (dashed), B_r (dashed-dotted) and B_z (dotted) with $\sigma_\chi = 15$ MeV on the left panel and $\sigma_\chi = 1.5$ MeV on the right panel. Note that the vertical scale on the two panels is different.

SUMMARY

1. To compute EM field in HIC one has to solve the initial value problem

2. Electric conductivity of QGP plays a crucial role in EM dynamics:

3. Contribution of the initial conditions increases with energy, while the valence one doesn't.

4. The topological effects may be important

TIME-DEPENDENT CONDUCTIVITY

Two models:

$$\sigma(t) = \frac{\sigma}{2^{-1/3}(1+t/t_0)^{1/3}}\,, \quad \text{Model A.}$$

$$\sigma(t) = \sigma\left(1 - e^{-t/\tau}\right)\,, \quad \text{Model B} \qquad \tau\text{=1fm}$$

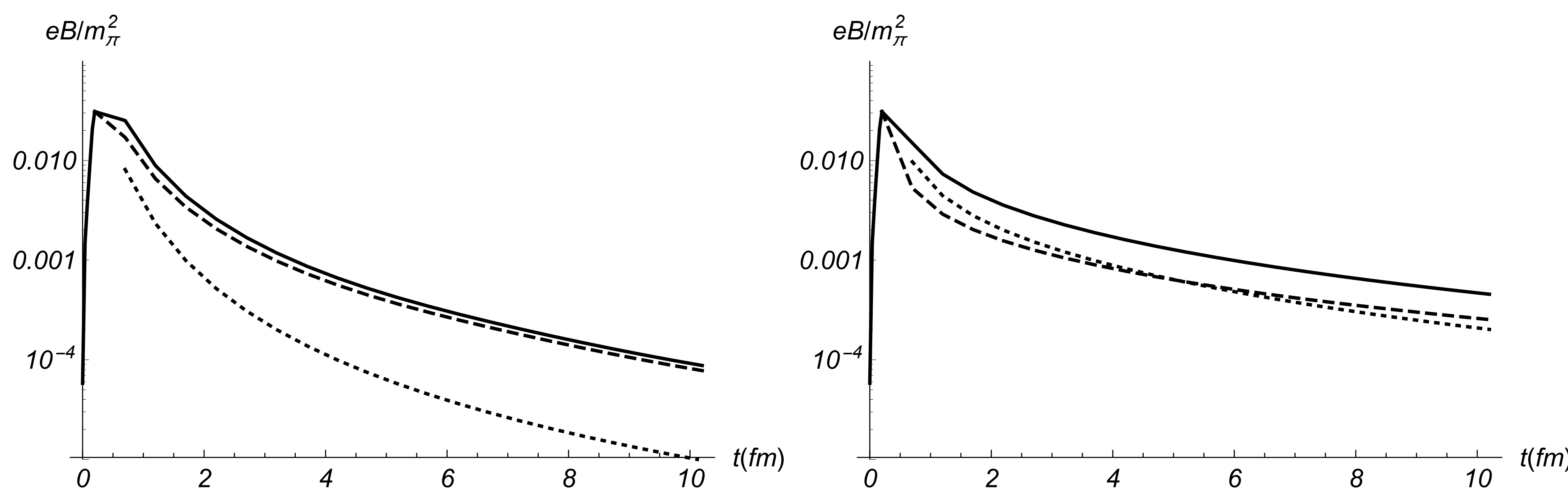


FIG. 5: Magnetic field in units of m_π^2/e. $z = 0.2$ fm $t_0 = 0.2$ fm. Left panel: model A. Right panel: model B. Solid, dashed and dotted lines stand for B, $B_{\rm init}$ and $B_{\rm val}$.

Time-dependence of conductivity is important at later times.